Volume Title: Determinants of Investment Behavior

Volume Author/Editor: Robert Ferber, editor

Volume Publisher: NBER

Volume ISBN: 0-87014-309-3

Volume URL: http://www.nber.org/books/ferb67-1

Publication Date: 1967

Chapter Title: Estimates of the Cost of Capital Relevant for Investment Decisions Under Uncertainty

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Chapter URL: http://www.nber.org/chapters/c1236

Chapter pages in book: (p. 179 - 213)

PART II

Financial Aspects

Estimates of the Cost of Capital Relevant for Investment Decisions Under Uncertainty

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For discussion at the Conference, we distributed a lengthy and detailed paper in which we attempted to develop methods for estimating the cost of capital relevant for investment decisions under uncertainty and to apply these methods to a cross-sectional sample of large electric utilities for 1954, 1956, and 1957. Much of this material had a direct and important bearing on the subject of the conference, particularly the latter sections of the paper which contrasted our estimates with several of the alternative measures of the cost of capital currently used in empirical studies of investment behavior.

Other parts of the paper, however—especially the review of the underlying theory of valuation under uncertainty, the discussion of the various theoretical and practical problems involved in the estimation, and the fairly extensive testing of the basic specification—were clearly of less direct concern to the Conference's central theme, except as supporting material for a critical evaluation by the discussants and those with a direct interest in the area of finance.

In revising our paper for inclusion in this volume, therefore, we decided to confine ourselves primarily to those portions of the original document most closely related to the central purpose of the Conference. Thus we have focused on our specific estimates of the cost of capital, on comparisons with alternative measures, and on the problems inherent in trying to develop continuous historical series. The remaining portions have been summarized and cut down to the bare minimum necessary for explaining and interpreting the results. Readers interested in a fuller development are referred to the *American Economic Review* for June 1966, which contains an unabridged, though slightly revised, version of the original paper (referred to hereafter as the "unabridged version").

I. Introduction

In its simplest form, the central normative proposition of the micro-theory of capital is that the firm should adjust its capital stock until the marginal rate of return on further investment (or disinvestment) is equal to the cost of capital. Under conditions of perfect certainty—which is the assumption on which most of classical theory has been developed—the concept of the cost of capital presents no particular difficulty; it is simply the market rate of interest. Since all securities must have the same yield in equilibrium under certainty, there is only one such rate per period and it is, in principle, a directly observable magnitude. Under real world conditions, however, we are confronted not with one, but with a bewildering variety of securities, with very different kinds and priorities of claims to portions of the (uncertain) future earnings of the firm. Since these securities will also, in general, have different anticipated yields, it is by no means clear which yield or combination of yields is the relevant cost of capital for rational investment planning. Nor, because it is based on anticipations, is the cost of capital any longer a directly observable magnitude. It must, rather, somehow be inferred from what *is* observable, namely, the market prices of the various kinds of claims represented by the different securities.

Although most (but not all) recent studies of investment behavior have shown some awareness of these difficulties, a common approach in empirical work has been simply to ignore the problem and to use, without comment or explicit justification, some standard index of current, nominal yields on high-grade corporate bonds (or even government bonds) as a measure of the cost of capital. Other writers use both a series on current bond yields to represent the cost of debt capital and a current profit series to measure the "availability" and hence, presumably, also the "cost" of equity capital. Still others have tried indexes of share prices, current dividend yields, or current earnings yields alone or in various averages with bond yields along the lines suggested in the standard texts on corporation finance. How much error is involved in the use of such measures is still unknown, though even a cursory survey of the underlying theory suggests many grounds for apprehension on this score; but we cannot be sure. Too little work has yet been done to permit even a rough calibration of these series as proxies for the cost of capital, let alone to provide acceptable alternative series.

The results that follow should be thought of as first steps toward closing this gap in our understanding and measurement of the cost of funds relevant to investment decisions. They are only first steps partly because of the very limited coverage of the estimates (three years for one industry)[1] and partly because the underlying model from which the estimates are derived is a special and still incompletely tested one. In particular, the interpretation of our estimates as the "cost of capital" rests on the assumptions of perfect capital markets and rational behavior by investors and by the corporate managers responsible for the actual investment decisions. Neither of these assumptions, needless to say, is likely to be enthusiastically accepted by those working in this field. In their defense, however, the following points are perhaps worth noting. (1) Our concern in this paper (and in the series of earlier papers on which it builds) is almost entirely with large, well-established firms. Though relatively few in number, these firms account for a disproportionately large share of total investment and in some major industries (such as our utilities or the steel industry) for virtually all of it. For such firms, we feel that the assumption of perfect capital markets—which implies, among other things, that, over the relevant range of funds requirements and except for very short intervals of time, there is no constraint on the total of funds from all sources that a firm can obtain at the going "cost of capital" to finance its investment outlays—cannot be ruled out as implausible, at least to a first approximation. For smaller firms, on the other hand, which are known to face severe limitations in their ability to expand their equity capital, the assumption may be largely inappropriate, and we would regard other kinds of models (stressing "availability" considerations) as more promising. (2) Insofar as rational behavior is concerned, the great virtue in that assumption is that it leads to a direct and simple connection between

[1] The sample consists of sixty-three separate firms representing all of the (consolidated) systems classified as of 1950 as Class A by the Federal Power Commission; plus those of the smaller Class B systems, whose assets devoted to electricity generation were at least $15 million in 1950. The sample years are 1954, 1956, and 1957.

the cost of capital and market valuation. To some, of course, such a defense will smack of looking for a missing wallet under the lamppost because the light is better there. But we do not yet know that the wallet is not there! There will be time enough to try working with more complicated behavioral assumptions when the evidence shows that the assumption of rationality fails (and precisely how it fails). As it turns out, some of the implications of the assumption of rational investor behavior stand up quite well when confronted with the data (see the unabridged version). The issue of the rationality of investment policy by corporate management is a more delicate one, and the final answer will have to wait until estimates of the cost of capital of the kind developed here have been tried out in studies of investment behavior.

With tests of this sort kept in mind as the ultimate goal, we turn now to the immediate task at hand. We shall begin in section II by providing an operational definition of the cost of capital and developing therefrom a link between the cost of capital and market valuation. The perfect market and rational behavior model of valuation will then be sketched out and the basic estimates of market-required rates of return and their relation to the cost of capital will be presented along with a very brief account of the estimating methods employed. Section III compares the estimates of the cost of capital with various popular alternative measures, with particular emphasis on average yield measures of the kind emphasized in the conventional literature on corporation finance. We conclude in section IV with some tentative suggestions on the problem of developing time series estimates of the cost of capital.

II. Valuation and the Cost of Capital

As used throughout, the term cost of capital (C) will be taken to mean the minimum prospective rate of yield that a proposed investment in real assets must offer to be worthwhile undertaking from the standpoint of the current owners of the firm. Under conditions of perfect capital markets, there is a one-for-one correspondence between "worthwhileness" in the above sense and the current market value of the owners' interest. If the management of the firm takes as its working criterion for investment (and other) decisions, "maximize the market value of the shares held by the current owners of the firm," then it can be shown (see, e.g., Hirshleifer [6])[2] that this policy is also equivalent to maximizing the economic welfare or utility of the owners. Thus, under the assumptions, valuation and the cost of capital are intimately related.

[2] Numbers in brackets refer to Bibliography at end of paper.

1. THE SIMPLE CERTAINTY MODEL

The precise relation between them is most easily seen in the context of a simple certainty model, in which all real assets are assumed to yield uniform, sure income streams in perpetuity and in which the market rate of interest (r) is given and constant over time. If, in addition, we assume perfect capital markets, rational investor behavior, and no tax differentials on different sources of income, then it can readily be shown that the equilibrium current market value (V) of any firm (i.e., the sum of the market values of all securities or other claims to its future earnings) is given by

$$V = \frac{1}{r}X \tag{1}$$

where X is the (uniform) income per period generated in perpetuity by the assets presently held. The term $1/r$ in (1)—the reciprocal of the interest rate—is commonly referred to as the market "capitalization rate" for sure streams since it represents the factor the market applies to a unit income flow to convert it to a capital stock.

For an expansion of real assets to be worth undertaking from the standpoint of the current owners of such a firm, the investment must lead to an increase in the market value of their holdings. If we let dA equal the purchase cost of the assets acquired, dS^o the change in value of the holdings of the original owners, and dS^n the market value of the additional securities issued to finance the investment, then differentiating (1) with respect to A yields:

$$\frac{dV}{dA} = \frac{dS^o}{dA} + \frac{dS^n}{dA} = \frac{dS^o}{dA} + 1 = \frac{dX}{dA}\frac{1}{r}. \tag{2}$$

It follows that the cost of capital (C) must be the reciprocal of the market capitalization rate for earnings since from (2), $dS^o/dA \geq 0$ if and only if $dX/dA \geq r$, i.e., if and only if the rate of return on the new investment is equal to or greater than the market rate of interest.[3]

2. EXTENSION TO THE CASE OF UNCERTAINTY

When we turn from a world of certainty to one of uncertainty, the problem of relating the cost of capital to market valuation becomes a

[3] We have here stated (and shall continue to state) the conditions for optimality of investment decisions in terms of the rate of return or internal yield on investment. Although it is well known that there may be cases in which such a rate of return cannot be adequately or unambiguously defined (see, e.g., Hirshleifer [6]), such cases are largely ruled out by our additional simplifying assumption.

much more formidable one for which no completely general solution is yet available. We have at least been able to show, however (see [10]), that if we retain the assumptions of perpetual streams, rational investor behavior, perfect markets, no taxes (and no "growth" in a sense to be more precisely defined later), then an analog of the certainty valuation formula (1) does carry through to the case of uncertainty. In particular, if we restrict attention to what we have called a "risk equivalent class" of firms, then the equilibrium market valuation of any firm in such a class can be expressed as

$$V = \frac{1}{\rho_k}\bar{X} \tag{3}$$

for all firms in class k, where V is the sum of the market values of all the firm's securities, $\bar{X}$ is the *expected* level of average annual earnings generated by the assets it currently holds, and $1/\rho_k$ can be interpreted as the market's capitalization rate for the expected value of uncertain, pure equity earnings streams of the type characteristic of class k. Hence, by a straightforward extension of the reasoning in the previous section, the cost of capital for a proposed expansion in scale by any firm in the class is simply ρ_k. The precise value of ρ_k will, of course, be different from class to class, presumably increasing with the market's uncertainty about the level of future long-run earnings in the class (and reflecting also the nature and extent of the covariation with the returns in other classes). Though the various ρ's themselves are not directly observable, they can, in principle, be inferred from the market valuations, e.g., by regressing the observed V on estimates of $\bar{X}$ over a cross section of firms within a class.

An important implication of (3) is that the market value of a firm depends only on its real earning power and on the market capitalization rate for pure equity streams of its class and not at all upon the particular mix of security types that characterize its financial structure. This independence of value and financial structure is basically a reflection of the assumption of perfect capital markets—an assumption implying, among other things, that for comparable collateral, the supply curve of borrowed funds for individuals is the same as that for corporations. Hence, if corporations making heavy use of borrowed funds should sell, say, at a premium relative to unlevered corporations in the same class, rational investors could always obtain a more efficient portfolio by selling the "overvalued" levered shares, purchasing the "undervalued" unlevered shares, and restoring the previous degree of leverage by borrowing against the shares on personal account. And the converse is true if levered shares should sell at a discount, in which case the "arbitrage" operation involves selling the unlevered shares, buying the levered shares, and unlevering them by also buying a pro rata share of the firm's debts.[4]

With reference to the cost of capital, the independence of market value and financial policy implies, of course, that the cost of capital relevant for investment decisions is also independent of how the investment is to be financed, even though the particular securities considered may, and in general will, have very different expected yields. This seeming paradox disappears as soon as it is recognized that the independence property also requires that the common shares in levered corporations have higher expected yields than those of less levered corporations in the same class—a differential which can be thought of as compensation for the greater "riskiness" attaching to levered shares. Thus, the apparent gain in terms of the cost of capital coming from the ability of a firm to finance an investment with "cheap" debt capital is offset (and with rational behavior in a perfect market exactly offset) by the correspondingly higher cost of equity capital.

3. THE EFFECT OF CORPORATE INCOME TAXES

When we extend the analysis to allow for the existence of corporate income taxes and the deductibility of interest payments, the picture changes in a number of respects, the most important being that market value and financial structure are no longer completely independent. To see what is involved, let us again denote by $1/\rho_k$ the capitalization rate in a given class for pure equity streams available to investors (i.e., streams of expected net profits after taxes in unlevered firms), by $\bar{X}$ a firm's expected total earnings (now to be taken as earnings before taxes as well as interest), and by τ the (constant) marginal and average rate of corporate income taxation. Then the market value of an unlevered firm can be expressed as:

$$V_u = S_u = \frac{\bar{X}(1 - \tau)}{\rho_k}, \tag{4}$$

where $\bar{X}(1 - \tau)$ is the unlevered firm's earnings after taxes. The value of a levered firm with D of debt or other securities whose payments are tax

[4] A fuller account of the arbitrage mechanism and proof of the independence proposition is given in our [10]. It is perhaps worth noting that the independence proposition can be proved under assumptions much weaker than those necessary to develop equation (3). In particular, neither the perpetuity assumption nor the concept of a risk equivalent class is essential (see, e.g., the discussion in our [9], pp. 429–430, and also Hirshleifer [5]).

deductible and P of preferred stock or other nondeductible senior securities can be shown to be (see our [12]):

$$V \equiv S + D + P = \frac{\bar{X}(1-\tau)}{\rho_k} + \tau D. \tag{5}$$

Note that in (5), the expression $\bar{X}(1-\tau)$ no longer represents the firm's earnings after taxes or any other standard accounting concept when D is not zero and hence when $\bar{X}$ includes some tax deductible interest. To avoid confusion, therefore, we shall hereafter refer to $\bar{X}(1-\tau)$ as "tax adjusted" earnings using the symbol $\bar{X}^{\tau}$ for earnings after taxes in the ordinary accounting sense (i.e., for the sum of expected net profits after taxes, preferred dividends, and interest payments as they actually come on to the market for sale to the various security purchasers).[5]

As to the meaning of (5), it says, in effect, that the government pays a subsidy to firms using certain sources of capital which under current law would include bonds, notes, and other firm contractual obligations of indebtedness but not preferred stocks (with some minor exceptions) or common stocks. The addition to the present worth of the firm occasioned by these tax savings is the corporate tax rate times the market value of the debt—the latter being, of course, the present worth, as judged by the market, of the future stream of tax deductible payments.

Since the deductibility of interest payments thus makes the value of the firm a function of its financial policy, it must also make the required yield or cost of capital a function of financial policy. To see the precise nature of this dependence, let dS^o, as before, stand for the change in the market value of the shares held by the current owners of the firm, dS^n for the value of any new common shares issued, dP for the value of any new preferred stock issued, and dD for the value of any new tax-deductible debt issued, with $dS^n + dP + dD = dA$. Then from (5) we have:

$$\frac{dV}{dA} = \frac{dS^o}{dA} + \frac{dS^n}{dA} + \frac{dP}{dA} + \frac{dD}{dA} = \frac{dS^o}{dA} + 1 = \frac{d\bar{X}(1-\tau)}{dA}\frac{1}{\rho_k} + \tau\frac{dD}{dA}, \tag{6}$$

from which it follows that the cost of capital or required yield on a tax-adjusted basis is

$$C = \rho_k\left(1 - \tau\frac{dD}{dA}\right) \tag{7}$$

[5] The relation between the various concepts can easily be established by observing that expected taxes paid will be $\tau(\bar{X} - \bar{R})$ so that $\bar{X} = \bar{X}^{\tau} + \tau(\bar{X} - \bar{R})$ and hence $\bar{X}(1-\tau) = \bar{X}^{\tau} - \tau\bar{R}$. Further discussion of these concepts along with the basic proofs for the tax case will be found in our [12].

since $(dS^o/dA) \geq 0$ if and only if $(d\bar{X}(1-\tau)/dA)$ is equal to or greater than the right-hand side of (7).

In connection with (7), the two extreme cases of financing methods are of particular interest. For an investment financed entirely by equity capital—and in this context equity capital includes nondeductible preferred stock—dD/dA will equal zero. Hence the required tax-adjusted yield or "marginal cost of equity capital" is ρ_k. For an investment financed entirely by debt or other sources of capital whose payments are tax deductible, dD/dA is unity, implying that the "marginal cost of debt capital" is $\rho_k(1-\tau)$.

The term marginal cost has been put in quotation marks to emphasize that, while these extreme cases serve to illuminate the meaning of (7), neither is directly relevant for actual decision-making at the level of the firm. For companies with reasonable access to the capital markets, as would certainly be true of those in our sample, investment and financing decisions (including decisions to retire outstanding securities) are made continually and largely independently. Since particular investment projects thus are not, and in general cannot be, linked to particular sources of financing, the relevant cost of capital to the firm must be thought of as essentially an average of the above costs of debt and equity capital, with weights determined by the long-run average proportions of each in the firm's program of future financing. If we denote this "target" proportion of debt as L, then the weighted average cost of capital can be expressed as $C = C(L) = \rho_k(1-\tau)L + (1-L)\rho_k$, or, more compactly, as $C = C(L) = \rho_k(1-\tau L)$, where the notation $C(L)$ will be used when we want to emphasize that the cost of capital is a *function* of the target debt ratio L.

Notice, finally, that while the definition of the cost of capital has become a good deal more complex as a result of the introduction of income taxes, the problem of estimation remains essentially the same. It still involves only the estimation of a single capitalization rate—in this case, $1/\rho_k$, the capitalization rate for unlevered, pure equity streams in the class. The difference between the cost of equity and debt capital introduces no new difficulties because the cost of debt capital does not depend on the market rate of interest on bonds, but only on the above capitalization rate and the tax rate. Hence $1/\rho_k$ remains a sufficient parameter both for economists seeking to explain rational investment behavior and for firms planning their investment programs on the basis of given financial policies.[6]

4. GROWTH AND VALUATION[7]

Up to this point, we have focused attention entirely on the role of current earning power and financial policy as determinants of the value of the firm. There are, of course, very many other factors that influence real world valuations and some that may well be large enough and systematic enough to warrant incorporating them directly into the model rather than impounding them in the general disturbance term. Of these, one of the most important is "growth potential," in the sense of opportunities the firm may have to invest in real assets in the future at rates of return greater than "normal" (i.e., greater than the cost of capital).

Clearly, translating such a concept into operational terms is a task of formidable proportions and one for which no universally applicable solution can be expected. For industries such as the electric utilities, however—where the growth in earnings has been (and will presumably continue to be) reasonably steady—rough, but tolerable, approximations to growth potential can probably be obtained by exploiting the so-called constant-growth model. In particular, suppose that a firm has the opportunity to invest annually an amount equal to 100 k per cent of its tax-adjusted earnings ($k \lesseqgtr 1$), on which investments it will earn a tax-adjusted rate of return of ρ^*, greater than $C = C(L)$, its average cost of capital. (These assumptions imply, among other things, that earnings will grow at the constant rate of $k\rho^*$ per year.) And suppose further that these especially profitable opportunities are expected to persist over the next T years, after which only normally profitable opportunities will be available. Then, by analogy to the solution we have derived for the

[6] The independence of the cost of equity capital (and hence also of debt capital) from the interest rate is, of course, an independence only within a partial equilibrium framework. In a general equilibrium setting, there is necessarily a very direct connection between the interest rate (which may be regarded, to a first approximation, as the yield on assets generating sure streams) and the various ρ_k (which are the expected yields on assets generating streams of various degrees of uncertainty). But while the connection is direct (since they are mutually determined in the process of market clearing and jointly reflect such underlying factors as the level of wealth, the composition of the stock of real assets, and attitudes toward risk), there is no reason to believe that they will always move closely together over time.

[7] Since all the main earnings and cost of capital concepts have now been introduced, we shall hereafter, in the interests of simplicity, drop all subscripts and superscripts on the variables where there is no danger of ambiguity.

certainty case (see [9], footnote 15) the current market value of the firm can be expressed as

$$V = \frac{1}{\rho}\bar{X}(1 - \tau) + \tau D + k\bar{X}(1 - \tau)\left[\frac{\rho^* - C}{C(1 + C)}\right]T, \tag{8}$$

where the first two terms, as before, represent the capitalized value of the current tax-adjusted earning power plus the tax benefits on debt, and the last term is the contribution to value of the future growth potential.

Despite the heroic simplifications invoked in its derivation, the above expression for growth potential is still by no means a simple one. It is the *product* of three separate elements: the profitability of the future opportunities as measured by the difference between ρ^* and $C(L)$; the size of these opportunities $(k\bar{X}(1-\tau))$; and how long they are expected to last (T). None of these component terms is directly observable, though some such as $k\bar{X}(1-\tau)$ (and possibly ρ^*) might be approximated by extrapolating recent past experience. In this paper, we take the simplest way out by focusing on the most tractable component $k\bar{X}(1 - \tau)$, the level of investment opportunities, and impounding the others in its regression coefficient.

As an empirical estimate of investment opportunities, we have used in the subsequent estimating equations the quantity $(1/5(A_t - A_{t-5})/A_{t-5}) \cdot A_t$. That is, we have used a linear five-year average growth rate of total assets times current assets denoted for simplicity hereafter as $\Delta\bar{A}$. This particular form of average, for reasons still not entirely clear to us, yields consistently higher gross and net correlations with total value than other simple averages we have tried. But the differences are not large and the estimates of the other coefficients are not sensitive to the specific measure used.

5. DIVIDEND POLICY, VALUATION AND THE COST OF CAPITAL

Under ideal conditions of perfect capital markets, rational investor behavior, and no tax discrimination between sources of income, dividend policy would present no particular problem. In such a setting, we have shown [9] that, given a firm's investment policy, its dividend policy will have no effect whatever on the current market value of its shares or on its cost of capital; and that despite the impressions of some writers to the contrary (see, e.g., Lintner [7]), this conclusion is equally valid whether one is considering a world of certainty or uncertainty. Dividend policy serves to determine only the division of the stockholders' return between current cash receipts and capital appreciation, and the division of the firm's equity financing between retained earnings and external flotations.

The picture becomes considerably more complicated, however, as soon as we weaken the assumptions to allow for the present tax subsidy to capital gains and for the existence of brokerage fees and flotation costs. Under these conditions, a firm's dividend policy can, in general, be expected to have some effect on its market value though the precise amount of the effect is impossible to determine a priori.

Given this uncertainty as to the size and, to some extent, even the direction of the dividend effect, the indicated course might seem to be simply to add a dividend term with an unspecified coefficient to the structural equation (8) and let the sample determine its value. From such a valuation equation we could, of course, also go on to derive an extension of the cost of capital formula (7) running in terms of dividend policy as well as debt policy.[8]

The trouble with such an approach, however, is that if it is applied in a straightforward fashion, as in Gordon [3] or Durand [2], the resulting estimate of the dividend coefficient will inevitably be strongly biased upward (and the key earnings coefficient correspondingly biased downward). Since the precise mechanism generating this bias has been described at length in our [11] and will be further referred to below, we need not dwell on the matter further at this point beyond observing that the difficulty arises from the widespread practice of dividend stabilization. With current dividends based in large part on management's expectations of long-run future earnings, the dividend coefficient in the regression equation will reflect this substantial informational content about $\bar{X}(1-\tau)$ along with the true effect, if any, of dividends per se on valuation.

Because of this confounding of the earnings and dividend coefficients, our approach here will be to omit the dividend variable entirely and to focus on the problem of estimating the earnings coefficient (which is, of course, to be interpreted as the capitalization rate for earnings for companies following the sample average dividend policy). As it turns out, tests of the dividend effect (presented in detail in the unabridged

[8] Although the procedure for deriving the marginal and average costs of capital in the dividend case is analogous to that for the leverage case, the derivation is considerably more complicated. Further difficulties arise from the fact that, in such a setting, maximizing market value is no longer always equivalent to maximizing the economic welfare of the owners. Since these and related problems are largely peripheral to the main concerns of this paper, further discussion of them will be deferred to a separate paper.

version) indicate that it is quite small in this industry for the years under study and can safely be neglected for our main purposes here.

6. SIZE AND VALUATION

All the valuation equations considered so far have been written as linear homogeneous functions of the independent variables, implying, among other things, that a given proportionate change in the values of all the independent variables leads to an equal proportionate change in the market value of the firm. The results of previous valuation studies (see, e.g., Gordon [4]) suggest, however, that the true market capitalization rate for the expected earnings of large firms may tend to be larger than that of small firms in the same industry.

As was true of the growth effect, there are a number of possible ways of incorporating this size or scale effect into the model. By far the simplest is merely to add a constant term to the valuation equation. The resulting nonhomogeneous equation must then be interpreted as the linear approximation over the sample range to the underlying nonlinear relation, and the coefficient of the earnings variable as the (constant) marginal capitalization rate in the industry. The magnitude and direction of the scale effect would be indicated by the size and sign of the constant term. A negative constant term would confirm that the average capitalization rate is less than the marginal rate and hence that the average capitalization rate tends to rise with increasing size of firm. A positive value for the constant term, on the other hand, would imply decreasing returns to scale in valuation.

7. THE ECONOMETRIC MODEL AND THE METHOD OF ESTIMATION

Our analysis of the theory of valuation thus leads to the following structural equation:

$$(V - \tau D) = a_0 + a_1\overline{X}(1 - \tau) + a_2\overline{\Delta A} + U, \tag{9}$$

where a_1 is the marginal capitalization rate for pure equity streams in the class and hence the key parameter for deriving the cost of capital, a_0 is an intercept term whose size and sign will measure any effects of scale on valuation, a_2 is a measure of the effects of growth potential on value, and U is a random disturbance term. Note that since the theory implies that the coefficient of the leverage variable D is equal to the marginal corporate tax rate τ, we shall, to increase the efficiency of estimation, so constrain it by incorporating it with the dependent variable.

Least-squares estimates of the coefficients of (9) will be efficient and unbiased only if, among other things, the variance of the disturbance term U is a constant, independent of the size of the firm, and the disturbances are not correlated with the independent variables. Unfortunately, neither of these conditions can reasonably be expected to hold in our sample.

As for the variance of the disturbances, one would certainly suppose that the errors in a valuation equation, including errors in measuring $(V - \tau D)$, are of the multiplicative rather than the additive variety. And indeed, check of the simple scatter of value on measured earnings suggests that the error term is approximately proportional to the size of firm. Any attempt to fit (9) directly, therefore, would be highly inefficient and in our sample (where the largest firm is on the order of 100 times the smallest) the results would be completely dominated by a handful of giant companies.

In the present context, there are at least two approaches worth considering as possible solutions for this problem of heteroscedasticity: (i) dividing (9) through by $(V - \tau D)$ and re-expressing the structural relation in so-called "yield" form; or (ii) weighting each observation in inverse proportion to the size of the firm and hence to the size of the standard deviation of the error. The former leads to the estimating equation

$$\frac{\overline{X}(1 - \tau)}{V - \tau D} = a'_1 + a'_0 \frac{1}{V - \tau D} + a'_2 \frac{\overline{\Delta A}}{V - \tau D} + u', \tag{10}$$

where $a'_1 = \rho =$ the reciprocal of the capitalization rate for pure equity streams (or, equivalently, the "marginal cost of equity capital"), $a'_0 = a_0\rho$, $a'_2 = -a_2\rho$, and $u' = -\rho(U/V - \tau D)$, with Var (u') approximately a constant for all firms.

While an approach of this kind has the virtue of simplicity, it suffers from the fact that the variable $(V - \tau D)$ enters into the denominator of the ratios on both sides of the equation. This is not only somewhat unesthetic—since we are, in effect, using V to explain V—but will lead to biased estimates to the extent that $(V - \tau D)$ contains stochastic elements independent of those in the numerator of the ratios. In the present case, this will mean that the coefficients of the growth and size variables will be too high (i.e., less negative) and that the estimate of the cost of capital (from the intercept term a_1) will be correspondingly too low. Since $(V - \tau D)$ certainly does have a stochastic component—impounded in the term U in (9)—and since we have, at this stage, no basis for judging how large the resulting bias really is, we obviously cannot afford to rely on estimating equations of this form. We shall therefore rely primarily on the weighted regression approach.

Assuming that the standard deviation of the error term in (9) is roughly proportional to size of firm, the required weighting can be effected by the relatively simple expedient of deflating each of the variables by the book value of total assets, denoted by A. Our reason for using total assets as a deflator rather than, say, total sales (as, e.g., in Neilsen [13]) is mainly that in the utility industry at least such deflated terms as V/A, D/A, or $\bar{X}(1-\tau)/A$ have natural and useful economic interpretations in their own right. The equation to be fitted, then, will be of the form

$$\frac{V-\tau D}{A} = a_0\frac{1}{A} + a_1\frac{\bar{X}(1-\tau)}{A} + a_2\frac{\overline{\Delta A}}{A} + u, \tag{11}$$

with $u = U/A$ and Var $(u) =$ a constant.

One question that immediately arises in connection with (11) is the status of the constant term. Recall that we are interpreting the basic valuation equation (9) in the original, undeflated variables as a linear approximation over the sample range, with its constant term a_0 serving as a measure of the effect of scale on valuation. To preserve this interpretation, we must, therefore, regard the derived deflated regression (11) as homogeneous, that is, as being fitted with no constant term and with the coefficient of the variable $1/A$ now measuring the size effect.

A potentially much more serious problem than heteroscedasticity is that posed by the lack of independence between the disturbance term in (11) and the independent variables, particularly the key earnings variable $\bar{X}(1-\tau)/A$. That variable is defined, it will be recalled, as the market's expectation of the long-run, future earning power of the assets currently held by the firm. Since it is an expectation, it is not directly observable or measurable and the best that can normally be done is somehow to approximate it from the firm's published accounting statements. This best, unfortunately, is likely to be none too good even in an industry, such as the electric utility industry, where there is substantial uniformity of accounting conventions among firms, where there are (at least in our sample period) no firms suffering net losses, and where large, year-to-year random fluctuations in reported earnings seem to be relatively rare.

The implications of these inevitable errors in the measurement of earnings for the problems at hand are perhaps most easily seen by expressing the underlying structure as the following *system* of equations (where, to simplify the notation, we let $V^* = (V-\tau D)/A$, $X^* =$

$\bar{X}(1-\tau)/A$ = the "true" unobservable expected earnings, X = deflated earnings as measured from the accounting statements, and Z_i, $i = 1 \ldots m$ stand for all other relevant variables (including constants, where appropriate):

$$V^* = \alpha X^* + \sum_{i=1}^{m} \beta_i Z_i + u \tag{12a}$$

$$X = X^* + v \tag{12b}$$

$$X^* = \sum_{i=1}^{m} \gamma_i Z_i + w \tag{12c}$$

where some γ_i and β_i may be zero, and with the error terms assumed to be independent of each other and to have mean zero and (constant) variances σ_u^2, σ_v^2, and σ_w^2 respectively. In other words, the value of the firm depends on expected earnings and certain additional explanatory variables; measured earnings are merely in approximation to true expected earnings, the error of measurement being v; and lastly, at least some of the explanatory variables are also correlated with (and hence convey information about) the true but unobservable X^*.

The equations above thus constitute a simultaneous system in which V^* and X are, in effect, the endogenous variables, and the Z_i are the exogenous variables. It follows, then, that if we attempt to fit by direct least squares the single equation.

$$V^* = aX + \sum_{i=1}^{m} b_i Z_i + u' \tag{13}$$

in which V^* is regressed on the Z_i and the endogenous, measured earnings X, the error term u' will not be independent of X and the coefficients of (13) will be biased. More concretely, it can readily be shown (see, e.g., Chow [1], esp. pp. 94–98) that, in the limit for large samples, the coefficients of X will be given by

$$a = \alpha \frac{\sigma_w^2}{\sigma_w^2 + \sigma_v^2}$$

which is less than the true value α and the more so the larger the variance of the error of measurement σ_v^2 and the better proxies the included exogenous variables are for earnings (i.e., the smaller the value of $\sigma_w{}^2$).[9] As for the other variables, the coefficients will be given by

$$b_i = \beta_i + \gamma_i\alpha \frac{\sigma_v{}^2}{\sigma_w{}^2 + \sigma_v{}^2} = \beta_i + \gamma_i[\alpha - a]$$

and thus may be larger or smaller than their true values (β_i) depending on the direction of correlation with X^* (i.e., on the sign of γ_i).[10]

Recasting the original structure in the form of (12) not only serves to clarify the nature of the biases introduced by errors of measurement, but also suggests a remedy, namely, an instrumental variable approach. For reasons of computational simplicity as well as ease of interpretation in the present context, we shall implement this approach by means of a two-stage procedure formally equivalent to the two-stage least-squares method of Theil [14]. (See also Madansky [8].) Operationally, this means first regressing the endogenous variable X on all the instrumental variables Z_i, thereby obtaining estimates g_i of the coefficients γ_i in (12c). From these estimates, a new variable $\bar{X}$ is formed, defined as $\sum_{i=1}^{m} g_i Z_i$, and thus constituting an estimate of X^* from which, if our assumptions are correct, the error of measurement v will have been purged. If $\bar{X}$ is then used in the second stage as the earnings variable in (13) in place of X (and if the conditions for identification are met), the resulting estimates of a and the b_i can be shown to be consistent estimates of α and the β_i in the basic structural equation (12a).

As for the specific exogenous or instrumental variables to be used, we have already considered two, growth and size. In addition—for reasons discussed in detail in the unabridged version—we shall use total assets, two capital structure variables (D/A and P/A), and total dividends paid.

[9] The above expression for the bias in the earnings coefficient was derived on the assumption that (13) was fitted *with* a constant term. If the equation were fitted without a constant term (and if, as we have assumed, there is no constant term in the true specification), then the apparent bias will be considerably smaller. The reason is, of course, that the bias, by flattening the slope of the regression, tends to produce a positive intercept even where none really belongs. Hence forcing the regression through the origin and eliminating the artificial intercept offsets some of the distortion. The offset is only partial, however, and forcing the regression through the origin cannot be regarded as a satisfactory substitute for the more elaborate methods for eliminating the bias to be introduced below.

[10] Note that even if some $\beta_i = 0$ (implying that the corresponding Z_i really has no effect on market value) its estimate b_i might still be positive if $\gamma_i > 0$. And the b_i might be quite large if γ_i is large and if the measurement error in X is substantial (so that a is considerably smaller than α). This is, of course, precisely the "information effect" or proxy variable bias we were concerned about in connection with the dividend variable (see section II 5).

III. The Results

1. THE VALUATION EQUATION

The two-stage least-squares estimates of (11) for the three sample years are presented in Table 1. Since our concern here is primarily with

TABLE 1

Two-Stage Least-Squares Estimates of the Basic Valuation Model (Dependent Variable: $(V - \tau D)/A$)

Year	Coefficients of: Earnings $(\bar{X}^{\tau} - \tau\bar{R})/A$	Size $1/A \cdot 10^7$	Growth $\overline{\Delta A}/A$	Mult. R	Adjusted Standard Error	Ratio of Adjusted Standard Error to Mean, V/A
1957	16.1 (.46)	−.280 (.08)	1.36 (.23)	.88	.057	.052
1956	16.7 (.40)	−.114 (.07)	.896 (.21)	.87	.057	.051
1954	19.7 (.45)	−.244 (.07)	.299 (.18)	.73	.063	.053

the cost of capital rather than valuation per se, we shall not comment on these estimates in any detail. Suffice it to say that the results compare quite favorably in terms of explanatory power—as measured, for example, by the ratio of the standard error of the regression to the mean value of V/A—with those that have been obtained in other valuation studies using very different (and to us, at least, very unsatisfactory) specifications. Of particular interest, of course, is the behavior of the growth variable. Future growth potential (though small relative to current earning power and the tax subsidy to debt in terms of contribution to total market value) apparently increased steadily in absolute and relative importance over this period and by 1957 accounted for something over 10 per cent of total market value for the average firm in the sample.

For further reference and comparison, we present in Table 2 two alternative sets of estimates of the valuation equation. The first is the direct least-squares regression of $(V - \tau D)/A$ on measured earnings, with the constant term suppressed. As can be seen, the differences from the two-stage estimates are generally quite small, a result not really

TABLE 2

Direct Least-Squares Estimates with Measured Earnings

Part A: Value Form (Dependent Variable: $(V - \tau D)/A$)

Year	Coefficients of: Earnings $(\bar{X}^{\tau} - \tau\bar{R})/A$	Size $1/A \cdot 10^7$	Growth $\overline{\Delta A}/A$	Mult. R	Adjusted Standard Error	Ratio of Adjusted Standard Error to Mean, V/A
1957	16.0 (.44)	−.277 (.08)	1.39 (.23)	.88	.057	.052
1956	16.6 (.39)	−.111 (.07)	.926 (.21)	.87	.057	.051
1954	19.2 (.43)	−.205 (.07)	.466 (.17)	.75	.063	.053

Part B: Yield Form (Dependent Variable: $(\bar{X}^{\tau} - \tau\bar{R})/V - \tau D$)

Year	Coefficients of: Constant	Size $(1/V - \tau D) \cdot 10^6$	Growth $\overline{\Delta A}/V - \tau D$	Mult. R	Reciprocal of Constant Term and Its Implied Standard Error	Ratio of Standard Error of Regression to Mean of Dependent Variable
1957	.0592 (.002)	.166 (.04)	−.0516 (.02)	.58	16.8 (.44)	.07
1956	.0582 (.001)	.066 (.04)	−.0325 (.01)	.39	17.4 (.38)	.07
1954	.0506 (.001)	.121 (.04)	−.0124 (.01)	.45	19.8 (.45)	.07

very surprising in the light of the suppression of the constant term.[11] The differences turn out to be considerably larger (particularly in 1954) when the constant term is not suppressed and, indeed, it is only in the case of the two-stage estimates that the constant term really does approach zero. The differences become larger still when other tests of the basic specification (notably those concerned with the leverage and dividend variables) are considered.

The second panel shows the estimates obtained by direct least-squares regressions in the "yield form" of equation (10) above. The main drawback of this approach, it will be recalled, comes from the presence of $V - \tau D$ in the denominators of variables on both sides of the equation, which imparts an upward bias to the coefficients of the independent variables and a consequent downward bias to the crucial constant term. Since the direction of the bias is known, however, we can use equations of this form to provide at least a rough check on the reasonableness of the estimates obtained by the more roundabout, two-stage approach.

To facilitate comparison with the estimates in Table 1, a column has been added showing the reciprocal of the constant term, which is the estimate of the capitalization factor for earnings implied by the observed constant terms in the yield equations. As predicted, the capitalization factors obtained via the yield equations are indeed all higher than those obtained via the two-stage approach. The gap between the two sets of estimates tends to widen somewhat over time, but the differences are never very large. This close agreement should remove any lingering fears that major distortions in the estimates may somehow have been introduced in the two-stage approach. At the same time, it suggests that the simpler yield equations may still have a useful role to play in valuation studies, particularly where the interest is mainly in determining the direction of changes in the cost of capital over time rather than developing precise estimates or testing the basic specification as developed here.

2. THE COST OF EQUITY CAPITAL

Turning now from valuation to the other side of the coin, the cost of capital, we show in Table 3 the estimates of the cost of equity capital implied by the earnings coefficients of Table 1. For comparison, the table also shows two other measures of the cost or "ease of acquisition" of equity capital frequently used by economists in investment studies, namely, the average earnings-to-price ratio and the reciprocal of the

[11] See footnote 9.

TABLE 3

Estimated Cost of Equity Capital and Some Alternative Measures of Equity Costs

	Estimated Cost of Equity Capital (ρ)		Average Earnings Yield on Shares ($\bar{\pi}^{\tau}/S$)		Reciprocal of Ratio of Price to Book Value (B/S)		Average Tax and Leverage Adjusted Total Earnings Yield, $(\bar{X}^{\tau} - \tau\bar{R})/(V - \tau D)$	
Year	Amt. (1)	As Per Cent of 1954 (2)	Amt. (3)	As Per Cent of 1954 (4)	Amt. (5)	As Per Cent of 1954 (6)	Amt. (7)	As Per Cent of 1954 (8)
1957	.062 (.002)	122	.070	106	.64	105	.056	110
1956	.060 (.001)	118	.070	106	.63	103	.056	110
1954	.051 (.001)	100	.066	100	.61	100	.050	100

ratio of the average price to the book value of the shares. Notice that all three measures indicate a rise in the cost of equity capital between 1954 and 1957, but our measure indicates a steeper and more substantial increase over the interval. The causes and implications of this apparently lesser responsiveness of the standard measures will become clear in subsequent discussion.

Insofar as levels are concerned, notice that the average earnings yield happens to be consistently higher than our estimate of the cost of equity capital. We say happens to be to emphasize that, under our model of valuation, there is no "normal" or even simple relation to be expected between the two concepts. The earnings yield for any company is not a given fixed number for each member of the class, but rather a function whose arguments include the cost of equity capital for the class, the firm's growth potential, its leverage policy, and its size. The sample mean earnings yield shows only the combined effect of these different and, to some extent, offsetting influences.

3. VALUATION, GROWTH, AND THE COST OF EQUITY CAPITAL

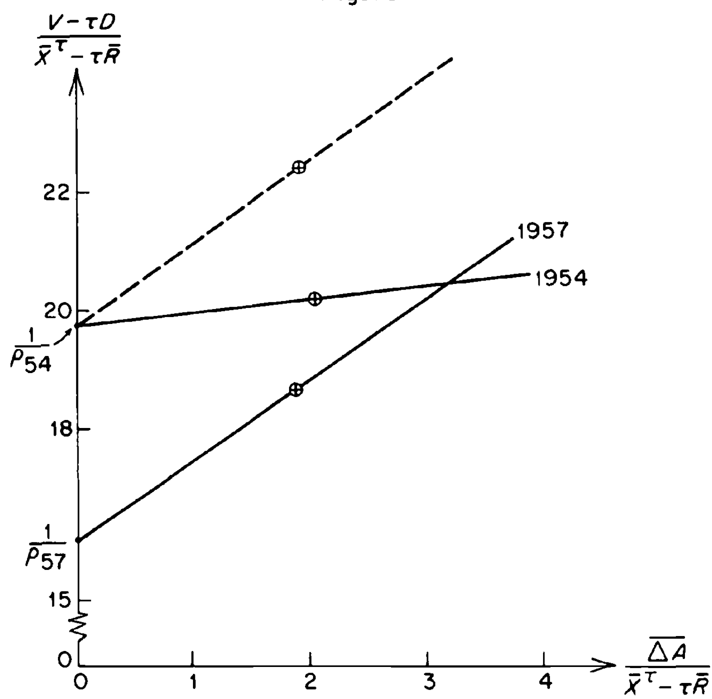
Because of the distortions resulting both from leverage effects and from the fact that the market value of shares incorporates the capitalized value of growth opportunities, we must conclude that the earnings-to-price ratio—which together with long-term interest rates is the most widely used measure of the cost of capital in investment studies—is unlikely to provide an adequate approximation.

The first of these distortions—that arising from leverage—could be handled by falling back on a measure of market yield somewhat different from the earnings-to-price ratio, though related to it. Let us suppose first that there were no corporate income taxes. Then we know that, in the absence of valuable growth opportunities, the fundamental valuation equation, expressed in yield form, would be simply $\bar{X}/V = \rho$. The ratio of expected total earnings to total market value—which may be thought of as a "leverage-corrected" yield—would thus provide a direct estimate of ρ. In principle, any firm could so approximate its cost of equity capital from its own company data, although, of course, as a practical matter, a better estimate would be obtained by averaging over a large group of similar firms so as to wash out any random noise in $\bar{X}$ or V. When we allow for taxes and the consequent tax subsidy on debt the picture becomes slightly more complicated, but a direct approximation of ρ still exists. The appropriate yield—see (5)—now becomes $(\bar{X}^{\tau} - \tau\bar{R})/(V - \tau D)$, the ratio of total tax-adjusted earnings to total market value minus the value of the tax subsidy.

This method of direct approximation breaks down, however, in the presence of growth. The leverage adjusted yield will be systematically too low as an estimate of ρ for any company with growth potential as will be the group average yield for any sample that contains significant numbers of growth companies. Nor will the movements of the yield over time conform well with changes in ρ to the extent that the market's evaluation of future growth potential changes over time (and, of course, much of the short-term variation visible in share prices stems precisely from this source). Some idea of how sizable the distortions of level and movement of the yield relative to ρ can be—even in such a low-growth industry as our electric utilities and even over such a short span of time—is provided by a comparison of our estimates of the cost of equity capital in the first column of Table 3 with those of the tax and leverage adjusted yield, $[(\bar{X}^\tau - \tau\bar{R})/(V - \tau D)]$, in the last column of that table.

One somewhat surprising aspect of this comparison, already noted above, is the relative stability of the leverage-adjusted yield series over this period. Because of the many uncertainties surrounding estimates of

Figure 1



future growth potential and because of the sensitivity of current market values to even small changes in projected growth rates, one would expect the growth component in the denominator of the yield ratio to be quite volatile, and hence that the market yield would tend to swing quite substantially in response to these continuing re-evaluations. Some idea of why this "normal" pattern did not obtain during our sample period can be gained from Figure 1. The solid-line functions plotted there are the basic value regressions of Table 3 for the beginning and ending years, 1954 and 1957, expressed in ratio form as

$$\frac{V - \tau D}{\overline{X}^{\tau} - \tau \overline{R}} = a_1 + a_2 \frac{\overline{\Delta A}}{\overline{X}^{\tau} - \overline{\tau R}}$$

(and hence ignoring the minor size effect). The dependent variable is thus the reciprocal of the tax- and leverage-adjusted yield; the intercept a_1 is our estimate of $1/\rho$; and the slope a_2 is the coefficient of growth in the V equation.[12]

Notice that in 1954, at the beginning of the period, the market's estimate of the growth potential of the industry was quite low. Because the slope was so flat, the approximate sample mean value of $(V - \tau D)/(\overline{X}^{\tau} - \tau\overline{R})$—indicated by the circled cross—differed only very slightly from the estimate of $1/\rho$ implied by the intercept. By 1957, however, a striking increase had taken place in the market's valuation of the prospects for continuing profitable growth in the industry. As can be seen from the broken line—which has been plotted with 1954 intercept and 1957 slope—this large revaluation would have pushed the average value of $(V - \tau D)/(\overline{X}^{\tau} - \tau\overline{R})$ up by nearly 15 per cent to 22.4 (equivalent to a yield of about .044) if no other changes had occurred. But instead of this "pivoting" around a stable intercept of $1/\rho_{54}$, our estimates indicate that a simultaneous and quite substantial drop in the intercept took place (i.e., rise in the cost of equity capital). So substantial was this drop, in fact (when combined with the slight fall in the mean value of the growth variable itself), that the upward push of the revaluation of growth was more than offset; and the mean value of $(V - \tau D)/(\overline{X}^{\tau} - \tau\overline{R})$ actually fell by about 10 per cent.

Although these compensating movements in ρ and the market's evaluation of growth "explain" the relative stability of the tax- and leverage-adjusted yield during the sample period, the explanation may

[12] The use of the reciprocal of the yield rather than the yield itself is simply a matter of convenience since the presence of growth impounded in V would lead to a nonlinear relation between $(\overline{X}^{\tau} - \tau\overline{R})/(V - \tau D)$ and growth as measured by $\overline{\Delta A}/(V - \tau D)$.

strike the reader as somewhat paradoxical. Growth potential, after all, is the opportunity to invest in the future in projects whose rates of return exceed the cost of capital. One would expect, therefore, that a rise in the cost of capital would normally be associated with a *fall* in growth potential. There are a number of possible explanations for the opposite behavior in the present instance, but discussion of them is perhaps best postponed until we have first provided estimates of the average cost of capital relevant for investment decisions.

4. THE REQUIRED YIELD OR AVERAGE COST OF CAPITAL

As emphasized earlier, the relevant cost of capital for investment decisions at the level of the firm is the average cost of capital, $\rho(1 - \tau L)$, where L measures the "target" proportion of debt in future financing. The average cost is thus not a fixed number, but a schedule or function whose arguments are ρ (which is an "external" property of the class or industry determined by the market) and L (which is a matter of "internal" company policy). Although the average cost of capital, unlike the cost of equity capital, is thus in principle different for each firm in the industry, we can get some idea of its value and behavior for the typical electric utility by using a typical or average value for L. The obvious candidate, of course, is the actual sample average of D/A for each year, since D/A measures the average proportion of debt in past financing and this proportion is likely to be quite stable (particularly when averaged over the industry). Estimates with these values for L are shown in Table 4, columns 1 and 2. (For notational convenience, we shall hereafter refer to these estimates as $C(\bar{D}/\bar{A})$, using $C(L)$ to mean the function itself and $C(D/A)$, i.e., without the bar on D/A, to refer to the function evaluated not at the industry mean, but at a particular company's value of D/A.)

These estimates $C(\bar{D}/\bar{A})$ of the average cost of capital are, of course, always below the corresponding estimates of ρ (Table 3, column 1); but the movements over time of the two series are closely similar since, as expected, the sample mean value of $\bar{D}/\bar{A}$ is quite stable. Notice also that the estimates of ρ and hence of $C(\bar{D}/\bar{A})$ conform quite closely in their movements with the average yield on AAA bonds in the industry (Table 4, columns 3 and 4)—probably the most popular surrogate for the cost of capital in investment studies. This conformity is particularly interesting since the rate of interest on bonds enters only very indirectly into our calculations of ρ and $C(\bar{D}/\bar{A})$ and, as can be seen from columns 5 and 6, the implied average rate of interest in our sample does not even seem to conform well with the AAA series. From the

TABLE 4

Average Cost of Capital and Some Comparison Series

	Average Cost of Capital, $C(\overline{D}/\overline{A})$		Yields on AAA Public Utility Bonds[a]		Sample Mean Yield on Debt (R/D)		Sample Mean Yield on Preferred Stock (Pdv/P)		Weighted Average Yields: With Book Value Weights $\overline{C}_B(\overline{D/A})$		Weighted Average Yields: With Market Value Weights $\overline{C}_M(\overline{D/V})$		Growth-Adjusted Average Yield[b] $(\overline{X}^{\tau}-\tau\overline{R})/V-G$	
Year	Amt. (1)	As Per Cent of 1954 (2)	Amt. (3)	As Per Cent of 1954 (4)	Amt. (5)	As Per Cent of 1954 (6)	Amt. (7)	As Per Cent of 1954 (8)	Amt. (9)	As Per Cent of 1954 (10)	Amt. (11)	As Per Cent of 1954 (12)	Amt. (13)	As Per Cent of 1954 (14)
1957	.046	128	.043	137	.029	112	.047	115	.034	104	.042	108	.047	122
1956	.045	125	.039	126	.029	112	.046	112	.035	106	.043	110	.045	115
1954	.036	100	.031	100	.026	100	.041	100	.033	100	.039	100	.039	100

[a] Monthly average for December (from *Federal Reserve Bulletin*).

[b] With G equal to growth coefficient in Table 1 times sample mean value of $\overline{\Delta A/A}$.

economic point of view, this parallelism between movements in ρ and the AAA yields would seem to suggest that, over this short interval at least, the movements of both series were dominated by factors affecting the supply of and demand for capital generally. Changes, if any, in investors' tastes for risk-bearing or in their evaluation of the riskiness of this industry in relation to others were apparently not large enough (except possibly in 1957) to cause any significant divergence of movement in the period under study.

It is also instructive to contrast our estimates of the average cost of capital with those that would be obtained by following the prescriptions laid down in much of the traditional literature of corporation finance. Essentially, these call for computing the weighted sum of the market yields of each type of security, the weights being the "target" proportions of each security in the capital structure. That is, if we let i equal the earnings-to-price ratio (our $\bar{\pi}^{\tau}/S$), p the preferred yield (our $\overline{Pdv}/P$), r the average rate of interest on bonds (our $\bar{R}/D$), l the target debt ratio, and l' the target preferred ratio, then the weighted average cost of capital function under the traditional view can be expressed as $i(1-l-l')+p(l')+r(1-\tau)(l)$. Where the target weights l and l' are computed at book value as is usually recommended (i.e., with $l=D/A$ and $l'=P/A$ and $(1-l-l')=B/A$ in our notation), we shall refer to the resulting average as $\bar{C}_B(D/A)$; where they are taken at market value (i.e., with $l=D/V$ and $l'=P/V$ and $1-l-l'=S/V$), we shall refer to the average as $\bar{C}_M(D/V)$, with unbarred values of the argument standing as before for a single company value and barred values for industry means. Estimates of both $\bar{C}_B(\overline{D/A})$ and $\bar{C}_M(\overline{D/V})$ for the typical firm in the sample, using actual sample mean values of i, p, and r, as well as of the book and market value measures of l and l' in each case, are shown in Table 4, columns 9–12.

As can be seen from Table 4, both the levels and the time paths of $\bar{C}_B(\overline{D/A})$ and $\bar{C}_M(\overline{D/V})$ differ significantly from those of $C(\overline{D/A})$. The largest discrepancies arise in the case of the widely advocated $\bar{C}_B(\overline{D/A})$ measure, which is substantially below $C(\overline{D/A})$ in all three years and which shows only a very slight rise over the period. The market value estimates, $\bar{C}_M(\overline{D/V})$, are considerably closer to those of $C(\overline{D/A})$, but they too fail to indicate the sizable increase in the cost of capital which seems to have occurred during this period.

5. RECONCILIATION WITH CONVENTIONAL AVERAGES

To understand precisely why these three methods of estimating the average cost of capital gave such different answers for the years under study (and why they are likely to continue to diverge for other years and other industries), it is helpful to begin by showing how these estimates would relate to each other in a much simpler world in which no growth potential ever existed. In such a world, we have seen that the ratio $(\bar{X}^\tau - \tau\bar{R})/(V - \tau D)$—the tax- and leverage-adjusted yield of the previous section—would be a measure of our ρ, the cost of equity capital. Hence, from the standpoint of any individual firm, the $C(L)$ function can be expressed as

$$C(L) = \frac{\bar{X}^\tau - \tau\bar{R}}{V - \tau D}(1 - \tau L) = \frac{\bar{X}^\tau - \tau\bar{R}}{V} \cdot \frac{1 - \tau L}{1 - \tau D/V}. \tag{14}$$

The weighted average cost of capital function with market value weights is

$$\bar{C}_M(D/V) = \frac{\bar{\pi}^\tau}{S} \cdot \frac{S}{V} + \frac{\overline{Pdv}}{P} \cdot \frac{P}{V} + \frac{\bar{R}(1 - \tau)}{D} \cdot \frac{D}{V} = \frac{\bar{X}^\tau - \tau\bar{R}}{V} \tag{15}$$

and with the book value weights

$$\bar{C}_B(D/A) = \frac{\bar{\pi}^\tau}{S} \cdot \frac{B}{A} + \frac{\overline{Pdv}}{P} \cdot \frac{P}{A} + \frac{\bar{R}(1 - \tau)}{D} \cdot \frac{D}{A}. \tag{16}$$

Notice first that for the special case of $L = D/V$—i.e., when the target leverage coincides with current leverage at market value, D/V, the function $C(L)$ takes the value $C(D/V) = (\bar{X}^\tau - \tau\bar{R})/V = \bar{C}_M(D/V)$. In other words, if a firm's current and future target leverage is D/V, it will get precisely the same estimate for its average cost of capital regardless of whether it chooses to multiply its current tax- and leverage-adjusted yield by $(1 - \tau D/V)$, to compute the weighted average of the current yields of its outstanding securities with market value weights for each, or to simply use the ratio of expected tax-adjusted earnings to total market value. A similar equivalence of estimates (at least to a very close degree of approximation) would also hold, of course, for economists concerned with "typical" values for the industry and using industry mean values of D/V and of the various yields, i.e., $C(\overline{D/V}) \cong \bar{C}_M(\overline{D/V})$.[13]

[13] Although the equivalence holds for individual company data and for industry averages, there is one important case in which the equivalence very definitely does *not* hold. This is the common case of the firm following the weighted average approach of (15) or (16) with current (or prospective future) company weights, but using industry-wide averages of the component yields so as to obtain more reliable estimates. The trouble here is that the market yield on shares (and to some extent the yields on preferred and bonds as well) are increasing functions of leverage. Hence, for a firm whose target leverage is greater (smaller) than the average for the industry mean yield will be an underestimate (overestimate) of its own yield and the resulting average cost of capital will be too low (high). This problem does not arise under our (14), of course, since $(\bar{X}^\tau - \tau R)/(V - \tau D)$ is not a function of firm policy (as is $\bar{\pi}$ $/S$) but an estimate of the external, market-given parameter, ρ.

Note also that if V equals A—which would tend to be the case if there were no growth past or future—then $\overline{C}_B(\overline{D}/A)$ becomes the same function as $\overline{C}_M(D/V)$ and, by extension, as $C(L)$. In this special case of no growth, therefore, all three company and industry-wide estimates will coincide.

This simple picture changes quite drastically, however, as soon as growth potential is introduced. The function $C(L)$ must now be expressed as

$$C(L) = \frac{\overline{X}^\tau - \tau\overline{R}}{V - \tau D - G}(1 - \tau L) = \frac{\overline{X}^\tau - \tau\overline{R}}{V - G} \cdot \frac{1 - \tau L}{1 - \tau\left(\dfrac{D}{V - G}\right)}, \quad (17)$$

where G is the market's current valuation of future growth potential. Hence, as can be seen by referring back to (15), there no longer exists any concept of L for which the function $C(L)$ will be the same as $\overline{C}_M(D/V)$. Note also that in the special case in which future growth potential constitutes the only major source of divergence between V and A, $(\overline{D/A}) \cong (\overline{D/V - G})$ so that our estimates of the average cost of capital $C(D/A)$ would be closely approximated by the ratio $(\overline{\overline{X}^\tau - \tau\overline{R}/V - G})$. The actual sample mean values of that ratio (with G taken as the product of the growth coefficient in Table 1 and the mean value of our growth variable $\overline{\Delta A/A}$) are shown in columns 13 and 14 of Table 4. As can be seen, the approximation to $C(\overline{D/A})$ is indeed quite close in 1956 and 1957; but it is less satisfactory in 1954 where the growth contribution is small, both in absolute terms and relative to the other sources of divergence between V and A.

Where the ratio $(\overline{\overline{X}^\tau - \overline{R}/V - G})$ is a good approximation to $C(\overline{D/A})$, it will, of course, also follow that both measures will exceed $\overline{C}_M(\overline{D/V})$ which, as we saw above, is given approximately by $(\overline{\overline{X}^\tau - \tau\overline{R}/V})$. As for the relation between the popular $\overline{C}_B(\overline{D/A})$ and $C(\overline{D/A})$, note that we can express the ratio $(\overline{\overline{X}^\tau - \tau\overline{R}/V - G})$ approximately as

$$\left(\overline{\frac{\overline{X}^\tau - \tau\overline{R}}{V - G}}\right) = \left(\overline{\frac{\pi^\tau}{S - G} \cdot \frac{S - G}{V - G}}\right) + \left(\overline{\frac{Pdv}{P} \cdot \frac{P}{V - G}}\right) + \left(\overline{\frac{\overline{R}(1 - \tau)}{D} \cdot \frac{D}{V - G}}\right) \quad (18)$$

$$\cong \left(\overline{\frac{\pi^\tau}{S - G}}\right)\left(\overline{\frac{B}{A}}\right) + \left(\overline{\frac{Pdv}{P}}\right)\left(\overline{\frac{P}{A}}\right) + \left(\overline{\frac{\overline{R}(1 - \tau)}{D}}\right)\left(\overline{\frac{D}{A}}\right),$$

since the assumption $V - G \cong A$ implies $D/V - G \cong D/A$, $P/V - G \cong$

P/A and $S - G/V - G \cong B/A$. Comparison with (16) shows that the weights in the two expressions are essentially the same; but since $(\overline{\pi\tau}/S) < (\overline{\pi\tau}/S - G)$, $\overline{C}_B(\overline{D/A})$ too will necessarily fall short of $C(\overline{D/A})$ when growth is present, and the gap will be larger, the larger is the contribution of growth to the value of the shares.

Once again, then, we see that attempts to infer the cost of capital directly from market yields rather than by the more detailed, cross-sectional estimating procedures developed in this paper break down in the face of growth. Where growth is present, all the popular, short-cut approximations will underestimate the cost of capital; and where the market changes its evaluation of growth potential over time (as is inevitable in view of the nature of growth), the time path of the yield measures may give a quite misleading picture of the true changes in the cost of capital. In particular, in our sample it happens that the market's evaluation of growth increased substantially over the period, thereby causing the yield measures to understate seriously the rise in capital costs that appears to have been taking place at the same time. As noted earlier, it is somewhat paradoxical that these two changes should have occurred simultaneously, since an increase in the cost of capital should tend to reduce what the market is willing to pay for given investment opportunities. We can perhaps throw some light on this paradox by taking a closer look at our growth coefficients and their implicit components.

6. A FURTHER ANALYSIS OF THE VALUATION OF GROWTH

The growth term in our basic valuation equation (see section II 4) is of the form

$$k\overline{X}(1 - \tau)\left[\frac{\rho^* - C}{C(1 + C)}\right]T,$$

where k is the ratio of investment to tax-adjusted earnings, C is the average cost of capital, ρ^* is the tax-adjusted rate of return on new investment, and T is a measure of the length of time for which the opportunities to invest at the rate ρ^* are expected last. In the actual estimating equations, we have taken as our growth variable an estimate of $k\overline{X}(1 - \tau)$, the level of future investment opportunities. Hence, if one accepts the underlying model, the observed coefficients of the growth variable can be interpreted as an approximation to $[\rho^* - C/C(1 + C)]T$.

Now that we have estimates of C, the average cost of capital for a typical firm, we can attempt some further decomposition of these growth coefficients and, in the process, we hope, gain some additional insights into the market's appraisal of the growth potential in this industry. In particular, it should be possible, from what we know about past earnings and about the regulatory process governing earnings in the industry, to make at least a rough approximation of ρ^*.

An obvious first candidate as an approximation to ρ^* is, of course, the current, tax-adjusted rate of return on assets, $(\bar{X}^\tau - \tau\bar{R})/A$. Such a measure, however, is almost certainly an underestimate of ρ^* since we know that there are components of total assets—actually, of total liabilities—that regulatory commissions systematically exclude from the rate base.[14] Some idea of the extent of this underestimate is provided by our knowledge that during these years most of the state commissions were still setting the "reasonable return on the rate base" in the neighborhood of the classical 6 per cent. By contrast, the sample average values of $\bar{X}^\tau/A$—$\bar{X}^\tau$ rather than tax-adjusted earnings $\bar{X}(1-\tau)$ being the relevant earnings concept in rate setting—were only .054, .056, and .055 per cent in 1954, 1956, and 1957, respectively. One simple adjustment, therefore, would be to blow up each sample mean value of tax-adjusted earnings by the ratio of 6 per cent to the sample mean value of $\bar{X}^\tau/A$. The rates of return thus adjusted, as well as the original unadjusted rates of return, are presented in Table 5, along with the estimates of T they imply.

These results would seem to suggest the following as the resolution of the paradox described in the previous section. The observed rise in the market's valuation of the industry's growth potential, in the face of the sharp rise in the cost of capital during the period, cannot reasonably be attributed to any compensating increase in the expected rate of return on future investment. No sharp upward trend in earnings rates, adjusted or unadjusted, is visible in the data; nor would such a trend be expected in view of the regulatory controls over the level of earnings. What seems to have been happening rather is that early in the period investors came to recognize that the regulatory authorities were setting rates at levels

[14] A further word of caution is necessary because the so-called accounting rate of return (earnings after depreciation divided by net assets) is not the same as the ordinary internal rate of return when a firm is growing. This discrepancy does not seem likely to create any very serious problems insofar as the valuation equations or estimates of the cost of capital are concerned since in those equations assets appear only as a deflator and since an explicit growth variable is included. It may, however, raise difficulties for comparisons of the kind being attempted here. We say may, because the ρ^* in our formula is not the usual internal rate of return, but the so-called "perpetual rate of return" (see [9], p. 416), and the relations between that rate and the accounting rate of return have, to our knowledge, nowhere yet been explored. We are indebted to Sidney Davidson and Robert Williamson for some helpful discussions on this general point.

that would probably permit firms in the industry to earn somewhat more than the cost of capital on any new capital invested. The subsequent rise in the cost of capital narrowed the margin of gain somewhat; but its effects on valuation were more than offset by an increase in the length of time that favorable terms for new investment were expected to persist. The actual numerical estimates of the expected duration of growth opportunities, as presented in Table 5, are not, of course, to be taken seriously in view of the many approximations, theoretical and empirical, involved in their computation. But upward revaluation of growth prospects (or at least in increasing awareness of growth potential on the part of investors) is very definitely indicated.

IV. Some Concluding Observations

As emphasized at the outset, this paper should be regarded by economists as a first step toward developing historical estimates of the cost of capital relevant for investment decision-making by business firms. It will have adequately fulfilled its objectives if it has succeeded in convincing economists working with investment functions that there *is* a cost of capital problem, that some of the major econometric problems that have prevented progress in the area to date can be overcome, and that the averages and yield measures of the kind recommended in much of the traditional literature on corporation finance as measures of the cost of capital are likely to be treacherous and unreliable.

As for the direction to be taken by future research, clearly one urgent need is further testing of the basic, rational, behavior-perfect, capital market specification. Some confirmatory evidence for the model is provided in our unabridged paper and further tests on the same sample will be provided in sequel papers. But it would obviously be desirable to have independent tests by others and on samples which are a little fresher, both in age and extent of handling.

Even after a basic specification is agreed upon, there remain numerous perplexing problems of estimation, notably those connected with the crucial growth effect. If the market's expectation of future growth opportunities changed seldom, slowly, or only in response to movements of other more readily measurable variables such as sales, profits, or even dividends, then the task would be the difficult but still essentially straightforward one of extrapolation. In practice, however, the market's valuation of growth potential often changes abruptly, substantially, and with little readily apparent relation to changes in observable economic series.

TABLE 5

Analysis of the Growth Effect

Year	Growth Coefficient[a] (G)	Average Cost of Capital[b] $C(\overline{D/A})$	Average Tax-Adjusted Return on Assets (ρ_1^*)	Average Return Assuming 6 Per Cent Return After Taxes (ρ_2^*)	Implied Value of T[c] (in years) For ρ_1^*	For ρ_2^*
1957	1.36	.046	.047	.052	51	11
1956	.90	.045	.048	.052	12	6
1954	.30	.036	.046	.052	1	1

[a] From Table 1.

[b] From Table 4, column 1.

[c] Computed as $G\left[\frac{(C)(1+C)}{(\rho^*-C)}\right]$.

While we, and we hope others, will continue to experiment with the extrapolative approach, we suspect that a somewhat more indirect attack may in the long-run prove more fruitful. In particular, instead of attempting to correct for the elusive and changing growth component in the value of growth companies, effort might be directed to finding and identifying companies that the market seems to regard as essentially no-growth companies—and growth here does not mean expansion, but opportunities to invest large sums at rates of return above the cost of capital. The search for such no-growth firms might proceed either by further decomposing the growth term along the lines of the last part of the previous section, with a view to finding the firms for which $\rho^* \cong C$, or by constructing scatters similar to those in Figure 1 for a series of widely spaced years and observing which firms seem continually to cluster in the near neighborhood of the "pivots." Once a sample of no-growth firms has thus been obtained, estimates of ρ can be made relatively simply and quickly via the leverage-corrected yield route discussed in section III. This approach, or variants relying on a judicious interplay of time series and cross-sectional estimation, should enable us, within a reasonable span of time and with a reasonable amount of effort, to obtain a usable time series of the estimated cost of capital.

Needless to say, even these hoped-for time series estimates would have to be handled quite gingerly in investment studies. It is not to be expected, for example, that the desired capital stock will always adjust quickly to the current level of the cost of capital. Because of the substantial decision-making costs involved, the cut-off rate or required minimum yield on new investments is likely to be changed only infrequently by large firms (typically, but by no means exclusively, on the occasions when external financing is contemplated). By suitably smoothing or lagging the series, however, it should be possible to incorporate at least the major changes in the level of the cost of capital that have occurred as the economy has swung over the last forty years from boom to severe depression, to a postwar prosperity widely regarded as temporary, and finally to a long period of sustained prosperity interrupted by only minor recessions and with fears of major future depressions largely absent.

Finally, we should like to stress once more that the measure of the cost of capital described in this paper should prove primarily relevant for the investment behavior of large corporate enterprises. For smaller firms, other measures—including more conventional measures of interest rates and of those rather elusive factors that may be lumped under the heading of availability of funds—might be a good deal more to the point.

BIBLIOGRAPHY

1. Chow, G., *The Demand for Automobiles in the United States,* Amsterdam, 1957.
2. Durand, D., *Bank Stock Prices and the Bank Capital Problem,* Occasional Paper 54, New York, NBER, 1954.
3. Gordon, M., "Dividends, Earnings and Stock Prices," *Review of Economics and Statistics,* May 1959.
4. ———, *The Investment, Financing and Valuation of the Corporation,* Homewood, Ill., 1962.
5. Hirshleifer, J., "Investment Decision Under Uncertainty," *Quarterly Journal of Economics* (forthcoming).
6. ———, "On the Theory of Optimal Investment Decision," *Journal of Political Economy,* August 1958.
7. Lintner, J., "Dividends, Earnings, Leverage, Stock Prices and the Supply of Capital to Corporations," *Review of Economics and Statistics,* August 1962.
8. Madansky, A., "On the Efficiency of Three-Stage Least-Squares Estimation," *Econometrica,* January-April 1964.
9. Miller, M., and Modigliani, F., "Dividend Policy, Growth and the Valuation of Shares," *Journal of Business,* October 1961.
10. Modigliani, F., and Miller, M., "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review,* June 1958.
11. ———, "Reply," *ibid.,* September 1959.
12. ———, "Corporate Income Taxes and the Cost of Capital: A Correction," *ibid.,* June 1963.
13. Neilsen, S., *Market Value and Financial Structure in the Railroad Industry,* Hartford, 1961.
14. Theil, H., *Economic Forecasts and Policy,* 2d rev. ed., Amsterdam, 1961.