I. Introduction

The Federal Reserve and other U.S. governmental bodies undertook a series of extraordinary policy interventions in 2008 to counter the severe financial market disruptions around the Bear Stearns crisis in March of 2008, and following the bankruptcy of Lehman Brothers in September of 2008. Among these interventions, the Federal Reserve introduced liquidity facilities aimed at restoring funding liquidity in various markets, including the commercial paper market via the Commercial Paper Funding Facility (CPFF), the repo market via Primary Dealer Credit Facility (PDFC) and the Term Securities Lending Facility (TSFL), and the asset-backed securities market via the Term Asset-Backed Securities Lending Facility (TALF).1

Ashcraft, Gârleanu, and Pedersen (henceforth Ashcraft et al.) offer a theory of haircut setting that presents a potential rationalization for the functioning of the TALF program. TALF was introduced by the Federal Reserve in response to a standstill in the issuance of asset-backed securities (ABS) in October 2008. This disruption in the ABS market had the potential to adversely affect the supply of credit to the real economy. It was also accompanied by an extraordinary surge in risk premia of ABS securities. The Federal Reserve provided funding for eligible AAA tranches of ABS securities via the TALF program. The TALF supplied term financing to ABS issuers by effectively providing investors with leverage that they were unable to obtain in the marketplace.

Financial institutions’ ability to take on leverage depends on various constraints that include risk management, margin requirements, and haircuts. Such constraints can be imposed in two ways: either on the balance sheet of the entire financial institution, or on a security-by-security basis. The key assumption of Ashcraft et al. is that investors’ leverage is constrained due to security-specific margins that prevail in
In this discussion, we contrast the theory of Ashcraft et al. with an alternative theory of leverage based on Adrian, Etula, and Shin (2009; henceforth Adrian et al.). The comparison of these two approaches allows us to work out in detail which implications of Ashcraft et al.’s theory are due to the security-specific constraints and which parts arise in a world where constraints are imposed on the balance sheets of financial institutions.

The policy experiment that is at the core of Ashcraft et al. is the comparison of haircut policy to interest rate policy. The key result of the paper is that in the economic setting of Ashcraft et al., cutting the risk-free rate can tighten the leverage constraints of some investors and thus induce greater risk premia on some assets. Margin policy—that is, central bank lending at margins below the margins prevailing in the marketplace—can directly influence the leverage available to investors in different assets and can more directly influence the supply of credit.

II. Intermediary Risk Appetite

A. Leveraged Financial Intermediaries

In order to understand the model of Ashcraft et al. in more detail, we contrast it with the closely related model by Adrian et al. Consider a leveraged financial intermediary \(A\) that manages its leverage actively. Denote the return to risky assets in excess of the risk-free rate at time \(t + 1\) by \(r_{t+1}\) and the vector of excess returns by \(r_t\). We suppose that intermediaries are risk neutral and maximize expected portfolio returns subject to a balance sheet constraint related to their value-at-risk (Var). Denoting by \(y_i^A\) the share of the active intermediary’s wealth \(w_t^A\) in position \(i\), the investment problem is

\[
\max_{y_i^A} E_t(y_i^A, r_{t+1}) \quad \text{s.t.} \quad \text{Var}_t \leq w_t^A. \tag{1}
\]

If \(\text{Var}_t\) is a multiple \(\kappa\) of equity volatility, the risk constraint becomes \(w_t^A \sqrt{\text{Var}_t(y_i^A, r_{t+1})} \leq w_t^A\). By risk neutrality, this constraint binds with equality. It follows that the Lagrangian is

\[
L_t = E_t(y_i^A, r_{t+1}) - \phi_t \left[\kappa \sqrt{\text{Var}_t(y_i^A, r_{t+1})} - 1\right]. \tag{2}
\]
The first-order condition for portfolio choice is

\[ y^A_t = \frac{1}{\kappa \phi_t} [\text{Var}_t(r_{t+1})]^{-1} E_t(r_{t+1}). \tag{3} \]

From equation (3), we see that the asset demands of the leveraged intermediaries are identical to the standard capital asset pricing model (CAPM) choices, but where the risk-aversion parameter is the scaled Lagrange multiplier \( \kappa \phi_t \) associated with the balance sheet constraint. Even though the intermediary is risk-neutral, it effectively behaves as if it were risk-averse. In other words, the intermediary’s “risk appetite” (the inverse of its effective risk aversion) fluctuates with shifts in \( \kappa \phi_t \). As the balance sheet constraint binds harder, leverage must be reduced. Danielsson, Shin, and Zigrand (2009) solve for the rational expectations equilibrium of a continuous time dynamic model along these lines.

There are two key differences between the setup of Ashcraft et al. and Adrian et al. First, the leverage constraint imposed on intermediaries by Ashcraft et al. is asset specific, while the constraint imposed by Adrian et al. is institution specific. Second, the constraint in Adrian et al. is proportional to the riskiness of the intermediaries’ balance sheet, while it is a simple leverage constraint in the case of Ashcraft et al. A minor difference between the models is that Ashcraft et al. assume that both agents are risk-averse, while Adrian et al. assume that financial intermediaries are risk neutral. As a result, the constraint is always binding in Adrian et al., but only sometimes binding in Ashcraft et al.

Because the constraint in Adrian et al. is proportional to the risk of the intermediaries’ balance sheet, the Lagrange multiplier on the constraint plays the role of effective risk aversion. In contrast, in Ashcraft et al., the Lagrange multiplier on the constraint enters the asset demand additively, so it is more similar to a hedging demand (see eq. [10] of Ashcraft et al.). While changes in the tightness of the constraint shift the demand for risky assets in Adrian et al., they result in twists of the demand curve in Ashcraft et al.

B. Equilibrium Returns

Adrian et al. assume that passive (\( P \)) investors have constant relative risk aversion \( \gamma \), so that their portfolio choice is

\[ y^P_t = \frac{1}{\gamma} [\text{Var}_t(r_{t+1})]^{-1} E_t(r_{t+1}). \tag{4} \]
Market clearing then implies that the vector $s_t$ of net supply of assets is

$$s_t = y_t^A w_t^A / (w_t^A + w_t^P) + y_t^P w_t^P / (w_t^A + w_t^P).$$

Plugging the two asset demands (3) and (4) in the market-clearing condition and rearranging, Adrian et al. obtain an expression for the equilibrium expected returns:

$$E_t(r_{t+1}) = \text{Cov}_t(r_{t+1}, r_{t+1}^W) \Gamma_t$$

$$= \beta_t \lambda_t,$$

where $r_{t+1}^W = s^t r_{t+1}$ is the return on the aggregate wealth portfolio and

$$\Gamma_t = (w_t^A + w_t^P) / [w_t^A / (\kappa \phi_t) + w_t^P / \gamma]$$

is the economy’s effective risk aversion. The market risk premium per unit of exposure is $\lambda_t = \text{Var}_t(r_{t+1}^W) \Gamma_t$.

Adrian et al.’s expression for equilibrium expected returns (eq. [5]) should be contrasted with equation (15) of proposition 1 in Ashcraft et al. There are two main differences. First, Ashcraft et al. do not explicitly solve for the price of market risk. Second, Ashcraft et al. have an additional term that determines expected returns. This additional term is proportional to the Lagrange multiplier of the constrained investors. While the theory of Adrian et al. is cast in terms of a CAPM with time-varying expected returns, the theory of Ashcraft et al. is cast as a deviation from CAPM.

C. An Estimate of Effective Risk Aversion

Adrian et al. demonstrate that the economy’s effective risk aversion $\Gamma_t$ has a reduced form representation that can be expressed as a function of the leverage of the active financial intermediaries and passive investors. Denoting the leverage of the active investors by $\text{lev}_t^A \equiv 1 + \text{debt}_t^A / w_t^A = \sum y_{i,t}^A$, and the leverage of the market by $\text{lev}_{t}^{A&P} \equiv 1 + (\text{debt}_t^A + \text{debt}_t^P) / (w_t^A + w_t^P) = \sum s_{i,t}$, Adrian et al. obtain

$$\Gamma_t = \gamma \left[ 1 + \frac{w_t^A}{w_t^P} \left( 1 - \frac{\text{lev}_t^A}{\text{lev}_{t}^{A&P}} \right) \right].$$

Equation (7) states that the time variation in the effective risk aversion is represented by fluctuations in the leverage of active intermediaries relative to the leverage of the market, scaled by the wealth of active intermediaries relative to the wealth of passive investors. Specifically, an increase in active intermediaries’ leverage is associated with a decrease in effective risk aversion. The greater the wealth share of the active intermediaries, the greater the impact of their leverage on $\Gamma_t$. 
It is important to emphasize that the time variation in the effective risk aversion in Adrian et al. is not due to the specification of preferences but rather to the time variation in the tightness of the intermediaries’ risk constraints. As a result, the theory of Adrian et al. yields an expression for the effective risk aversion as a function of observable balance sheet components. Figure 1 displays an estimate of the effective risk aversion for the universe of U.S. investors between 1986Q1 and 2009Q4. Adrian et al. proxy the actively managed, leveraged financial intermediaries by the universe of U.S. primary dealers and let the rest of the Center for Research in Security Prices (CRSP) universe proxy for passive investors. The data are obtained from the CRSP/Compustat merged database.

The measure of effective risk aversion of Adrian et al. has a number of intuitive features. For instance, the series reaches its highest level during the latest financial crisis in the fall of 2008. It also exhibits a pronounced increase following the long-term capital management crisis, and following the savings and loans crisis prior to the recession of 1991.

D. Impact on Investment

Adrian et al.’s theory can be complemented with a real sector and real investment, following the setup presented by Ashcraft et al. In particular, assume that productivity shocks $A_{t+1}$ are independently and identically distributed (i.i.d.), and that firms fully depreciate after one period, so that the one period return is pinned down by the liquidating

![Effective Risk Aversion](image.png)

Fig. 1. Effective risk aversion of Adrian, Etula, and Shin (2009)
dividend. Under these assumptions, real investment directly inherits the persistence of the time-varying risk aversion; that is, investment is no longer i.i.d. Specifically, if we assume that the production technology is given by the Cobb-Douglas function

\[ A_i^{1/2} K_i^{1/2} L_i^{1/2}, \]

the investment in Adrian et al. is

\[
(I_t^{1/2} = \frac{(1/2)E_t(A_t^{1/2})}{1 + r_f + (1/4) \operatorname{Var}_t(A_t^{1/2})} \Gamma_t).
\]

In contrast, the equivalent equation in Ashcraft et al. is

\[
(I_t^{1/2} = \frac{(1/2)E_t(A_t^{1/2})}{1 + r_f + (1/4) \operatorname{Var}_t(A_t^{1/2})} \gamma + \psi_i x_m^i).
\]

The key difference is that in Adrian et al., the equilibrium investment is affected by systematic shifts in effective risk aversion \( \Gamma_t \), whereas in Ashcraft et al. the equilibrium investment in project \( i \) fluctuates with the asset-specific term \( \psi_i x_m^i \). The shifts in effective risk aversion in Adrian et al. are amplified by the volatility of the productivity shock \( \operatorname{Var}_t(A_t^{1/2}) \).

In both theories, investment is persistent, and a function of the tightness of financial constraints of the leveraged investors. Ashcraft et al. call this property the “margin-constraint accelerator” in their proposition 2.6

E. Leverage-Return Trade-off

In equilibrium, the theory of Adrian et al. implies a trade-off between returns and leverage. This can be most easily seen by plugging equation (7) into equation (5) for the case of the market return \( r_{t+1}^W = r_{t+1}^W \):

\[
E_t(r_{t+1}^W) = \operatorname{Var}_t(r_{t+1}^W) \gamma \left[ 1 + \frac{w_i^A}{w_f^P} \left( 1 - \frac{\text{lev}_i^A}{\text{lev}_i^{A&P}} \right) \right].
\]

In other words, the theory of Adrian et al. states that the expected market return is proportional to market variance (this is the usual risk-return trade-off), but this trade-off is magnified by the amount of intermediary leverage. Graphically, figure 2 shows this trade-off between returns and leverage for two different levels of market variance: higher leverage is associated with lower equilibrium returns; the impact of leverage on equilibrium returns is greater when the volatility of the market is higher.

Figure 2 should be compared with figures 2–4 of Ashcraft et al. Note that leverage is inversely proportional to haircuts, so the negative
leverage-return trade-off of Adrian et al. corresponds to the positive return-haircut trade-off of Ashcraft et al. The key difference between the theories can be summarized as follows: in Ashcraft et al., the product of exogenous margin and the Lagrange multiplier of the financial constraint determine the slope of each haircut-return curve, whereas in Adrian et al., the slope of the leverage-return trade-off is determined by the riskiness of each security. By implication, in Ashcraft et al., changes in the risk-free rate directly change the return-haircut trade-off for some securities (see proposition 3 and fig. 4 of Ashcraft et al.). In contrast, in Adrian et al., changes in the risk-free rate do not directly affect the leverage return trade-off. The leverage-return trade-off is affected only if the risk-free rate alters the equilibrium variance-covariance matrix of returns. The extent to which changes in the risk-free rate impact equilibrium second moments is a fruitful direction for future research.

Empirically, the trade-off between equilibrium expected returns and haircuts can be seen in figure 3, which is taken from Adrian and Shin (2009). During the financial crisis, the trade-off between haircuts and option-adjusted yield spreads steepened substantially. This is exactly what equation (9) predicts: as market volatility increased with the deepening of the financial crisis, the slope of the trade-off between required returns and the haircut (e.g., the inverse of leverage) was expected to increase.

III. Policy Implications

Ashcraft et al. assume that asset-specific haircuts are determined exogenously. As a result, Ashcraft et al. interpret TALF as an exogenous
change in the haircut on ABS. The Federal Reserve offered lower haircuts than the ones that prevailed in the marketplace, thereby increasing the overall leverage of TALF investors and changing the leverage-return trade-off for eligible assets. This is paramount to a change in the equilibrium haircut-return trade-off, as figure 5 and proposition 4 of Ashcraft et al. show.

The assumptions in Adrian et al. are not sufficient to pin down the impact of TALF on equilibrium asset prices. Thus, additional assumptions are needed to determine the degree to which TALF changes equilibrium pricing in the model of Adrian et al. We will next discuss such assumptions.

At one extreme are assumptions that make TALF have no effect on equilibrium asset prices. If it is assumed that the proceeds from TALF are transferred from the central bank back to the economy in a manner that is not state contingent, then programs such as TALF (or asset purchases) will have no equilibrium asset pricing implications in the setting of Adrian et al. Cúrdia and Woodford (2010) provide a detailed discussion of the conditions under which quantity policies will have no effect on equilibrium risk premia. However, there are two related assumptions underlying the Cúrdia-Woodford argument. First, the central bank is just a veil, a part of the government that serves as a channel for redistribution. Second, any distributive transfers that result from realized profits or losses made by the central bank are conducted in a non-state-contingent manner.
We emphasize that the Cúrdia-Woodford assumptions are quite restrictive, particularly in the midst of financial crises. The central bank is itself a player in the market with its own balance sheet. Via programs such as TALF, the Fed increases the size of its balance sheet to support prices in economically important parts of the market that suffer from disruptions. The Fed does this in a time frame that matters for investors—a period shorter than the long-run equilibrium in which the Fed’s net profits have been redistributed to the economy. At an extreme, assuming that the central bank is just a channel for redistribution is equivalent to saying that when the Fed makes a profit on its holdings of agency mortgage-backed securities, financial intermediaries such as Citi and Bank of America get recapitalized immediately. Or, alternatively, why did the Treasury need TARP when the Fed’s purchases would have automatically recapitalized all the banks? Relatedly, fiscal redistributions (via taxes and other means of transfer) are likely to depend not only on realized asset payoffs but also on a number of other considerations linked to the state of the economy. Hence, the non-state contingency of fiscal transfers is also a strong assumption that underlies the result that quantity policies have no equilibrium effects.

At the other extreme is the assumption that the central bank does not distribute the proceeds from TALF at all. In that case, the only effect of TALF will be to change the net supply of risky assets. The change in the net supply of risky assets, in turn, changes the market-clearing conditions and the effective risk aversion prevailing in the economy.

Under the assumption that the Federal Reserve does not redistribute the proceeds from TALF, or does so after a period that is beyond the investment horizon of investors, or does so in states that are irrelevant to investors’ portfolio choice, variations in TALF lending can be interpreted as changes in the net supply of risky assets. In the setting of Adrian et al., such changes in \( s_t \) lead to changes in the relative leverage of active financial intermediaries (the term \( \frac{\text{lev}^A_t}{\text{lev}^{A&P}_t} \) in [7]). Since an increase in financial intermediaries’ leverage results in a decrease in the economy’s effective risk aversion, an increase in TALF lending is expected to result in a decline in risk premia (via [5]) and an increase in investment (via [8]).

A simple back-of-an-envelope calculation illustrates magnitudes for the securitized mortgage market. Suppose that active investors hold $2 tr assets (say, with equity $0.1 tr) and passive investors hold $10 tr assets (say, with equity $2 tr). Suppose that the government funds $0.2 tr in a way that it only transacts with active investors. The leverage of the active investors thus increases from 2/0.1 = 20 to 2.2/0.1 = 22. At the same time, the leverage of the overall financial sector increases from 12/2.1 = 5.7
to $(12 + 0.2)/2.1 = 5.8$. The immediate change in effective risk aversion is then

$$\Gamma_1 - \Gamma_0 \over \Gamma_0 = -{(0.1/2)(22/5.8 - 20/5.7)} \over [1 + (0.1/2)(1 - 20/5.7)] = -1.63\%.$$ 

The government-induced releveraging of active investors results in a 1.63% reduction in the private sector’s effective risk aversion. With a market beta for AAA credit returns of 0.8, the decline in required returns due to the government intervention is 1.3%. This is smaller than the 3% that Ashcraft et al. estimate, which might reflect that the TALF program targeted very specific assets, while our back-of-the-envelope calculation is aimed at the overall leveraged financial sector.

An alternative policy tool to the liquidity injections is changes in the capital requirements of financial institutions. This can be viewed as variation in $\kappa$. The parameter $\kappa$ directly affects the risk appetite of the levered financial institution and hence equilibrium risk premia. Systematic changes in $\kappa$ as a function of macroeconomic state variables can be interpreted as macroprudential policy, which is a primary topic of ongoing research.

**Endnotes**

The views expressed here are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of New York or the Federal Reserve System.

1. Details on the Federal Reserve’s liquidity facilities are provided by Adrian, Marchioni, and Kimbrough (2010) for the CPFF; Adrian, Burke, and McAndrews (2009) for the PDCF; Fleming, Hrung, and Keane (2009) for the TSLF; and Ashcraft, Malz, and Pozsar (forthcoming) for TALF.

2. Adrian et al. was first circulated publicly as a Federal Reserve Bank of New York Staff Report in January 2009; Ashcraft et al. rely on Gârleanu and Pedersen (2009), which was first circulated in September 2009. Adrian and Etula (2010) solve an intertemporal extension of Adrian et al. in continuous time.


4. The “passive” investors of Adrian et al. correspond to the risk-averse $a$ investors in Ashcraft et al.

5. The time variation of effective risk aversion generated by institutional constraints can be contrasted to the time variation of effective risk aversion due to habit formation in preferences (see, e.g., Campbell and Cochrane 2000).

6. Empirically, the prediction that investment varies with risk premia fits nicely with the finding of Philippon (2009) that the investment-to-capital ratio of nonfinancial corporations is closely correlated to risk premia in corporate bond markets.


**References**
