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On Specification in Simultaneous Equation Models

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1. Introduction

The development of estimation procedures in econometric models over the past 35 years has outpaced the growth of a complementary theory of testing and inference. Problems in examination of alternative versions of large scale models and current debates on forecasting are evidence of this fact. Perhaps this demise is due in part to the relatively informal approach to econometrics adopted by latter day pioneers and the nonexistence of a well-accepted formal axiomatic basis for a theory of econometric modeling.

An econometric model is a complete specification of economic and statistical behavior. Over time, "specification analysis" has come to refer to the rather limited issue of excluded or erroneously included variables, or with some generalization, to questions of stochastic structure. Formalization of the econometric modeling process requires the development of an axiomatic base for a very complex decision procedure. Among the broader range of

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issues to be considered in such a development are linearity versus nonlinearity, exogeneity versus endogeneity, normalization, static response versus dynamic response, measurement error, aggregation, nonexclusion restrictions, (linear and nonlinear) nonrandom behavior and identity relations, stochastic interdependency, and identification structure.

Given the current state of the art, these issues cannot be addressed simultaneously; progress, as documented in the survey by Dhrymes *et al.* (1972), appears slow. In this paper we consider in isolation tests of three parts of a complete specification: exogeneity versus endogeneity, identification, and normalization. In the absence of a comprehensive formalization of specification analysis, these are "early" tests which should be performed before consideration of more familiar issues *within* the context of an identified simultaneous equation model. The concerns of this paper are therefore methodologically similar to the consideration of specification problems by Ramsey (1969) and Wu (1973).

In Section 2 we develop notation for the complete dynamic simultaneous equation model (CDSEM), and in Section 3 discuss the implications, estimation, and testing procedures for the exogeneity specification. Section 4 deals similarly with overidentifying and normalizing restrictions, and Section 5 applies the theory to four classic econometric models. A conclusion reemphasizes our points in the light of the empirical findings.

2. The Complete Dynamic Simultaneous Equation Model

The specifications and tests discussed in this paper are considered within the context of the complete dynamic simultaneous equation model (CDSEM),

$$\underset{g\times g}{B(L)}\underset{g\times 1}{y_t} + \underset{g\times k}{\Gamma(L)}\underset{k\times 1}{x_t} = \underset{g\times 1}{u_t}. \tag{2.1}$$

The vector y_t is the realization of a time series of g endogenous variables, x_t the realization of k exogenous variables, and u_t the realization of g disturbances. The operators $B(L)$ and $\Gamma(L)$ are matrices of polynomials of infinite order in nonnegative powers of the lag operator L, whose defining property is that $L^s w_t = w_{t-s}$ for any time series $\{w_t\}$. The explicit expansions $B(L) = \sum_{s=0}^{\infty} B_s L^s$ and $\Gamma(L) = \sum_{s=0}^{\infty} \Gamma_s L^s$ are sometimes useful, where B_s has typical element β_{ijs}, and Γ_s has typical element γ_{ijs}. We shall ascribe to these lag operators certain properties discussed most conveniently in terms of their respective generating functions $B(z) = \sum_{j=0}^{\infty} B_j z^j$ and $\Gamma(z) = \sum_{j=0}^{\infty} \Gamma_j z^j$, whose domains are the set of all complex numbers. It is assumed throughout that all lag operators possess generating functions which are analytic in the region $\{z : |z| < 1\}$. This permits a more rigorous interpretation of equations involving lag operators; e.g., (2.1) becomes

$$\lim_{m\to\infty} E\left[\sum_{s=0}^{m} (B_s L^s y_t + \Gamma_s L^s x_t - u_t)\right]\left[\sum_{s=0}^{m} (B_s L^s y_t + \Gamma_s L^s x_t - u_t)\right]' = 0.$$

The specification of the CDSEM is completed by four assumptions.[1]

I. $Eu_t = 0$ for all t.
II. $\text{cov}(u_t, u_{t-s}) = 0$ for all t and all $s \neq 0$.
III. $\text{cov}(u_t, x_{t-s}) = 0$ for all t and all $s \geq 0$.
IV. $B(z)$ has no roots in the region $\{z : |z| \leq 1\}$.

Assumption I is made without loss of generality. Assumptions II and III are part of the distinction between the endogenous variables y_t and the exogenous variables x_t. Assumption IV asserts that the lag operator $B(L)$ is stable. It guarantees the existence (Whittle, 1963, Chapter 2) of a one-sided operator $B^{-1}(L)$, whose defining property is $B^{-1}(z) = [B(z)]^{-1}$ and whose operational significance arises from the fact that $y_t = -B^{-1}(L)\Gamma(L)x_t + B^{-1}(L)u_t$. Assumption IV is most familiarly invoked in discussions of statistical inference in the CDSEM. For example, it is a necessary condition in central limit theorems for simultaneous equation estimators. Quite apart from any questions of inference, however, assumption IV is critical in the logical interpretation of the CDSEM. If $B(L)$ is stable, then (2.1) is a system which accepts current and past x_t and u_t as inputs and from them determines current y_t as output, and is therefore complete; if not, then no such interpretation is possible. The importance of this assumption is illustrated by the fact that it is indispensable in the classic theory of economic policy as exposited, for example, by Tinbergen (1955).

In application further assumptions are usually added to these. Such assumptions often take the form of exclusions of elements of $B(L)$ and $\Gamma(L)$ (e.g., $\gamma_{ijs} = 0$ for all s) or limitations on lag length (e.g., $\beta_{ijs} = 0$ for all $s > 0$) made specifically to reflect constraints arising from theory, to identify the system, or reduce the parameter space to a manageable size. Estimation and hypothesis testing then proceed within the context of a set of maintained assumptions which is typically rather large and rarely subjected to empirical verification. Ideally, this procedure should be replaced with one in which the refutable implications of all assumptions—both those usually maintained and those explicitly set forth in the null hypothesis—are explicitly formulated as null hypotheses and tested jointly. We do not attempt to meet this demanding criterion here but proceed in that direction by testing some of the traditionally maintained hypotheses individually.

[1] Our assumptions would be equivalent to those of Theil (1971, pp. 484–486) if the lag operators in (2.1) were of finite order.

3. Testing the Exogeneity Specification

3.1. Refutable Implications

We shall refer to assumptions II–IV as the exogeneity specification. It has long been understood that this specification has refutable implications, but only recently has that understanding become widespread or explicit. Koopmans (1949) and Phillips (1956) were obviously aware of the point, and the engineering literature (e.g., Caines & Chan 1975; Zemanian, 1972) suggests that the implications were being exploited outside economic contexts even earlier. Sims (1972) established tests of "unidirectional causality" as defined by Granger (1969), a concept which is closely related to the exogeneity specification. Geweke (1978) derived the following two sets of restrictions on the covariances of the time series $\{y_t\}$ and $\{x_t\}$ implied by the exogeneity specification.[2]

3.1.1. First Implication of Exogeneity

Suppose that $\{y_t\}$ and $\{x_t\}$ are jointly covariance-stationary with autoregressive representation and that the linear regression of y_t on all current, lagged, and future values of x_t is

$$y_t = \sum_{s=-\infty}^{\infty} K_s x_{t-s} + v_t, \tag{3.1}$$

where $\operatorname{cov}(v_t, x_{t-s}) = 0$ for all s and t. There exists a CDSEM with exogenous x_t and endogenous y_t and no other variables if, and only if, $K_s = 0$ for all $s < 0$.

3.1.2. Second Implication of Exogeneity

Suppose that $\{y_t\}$ and $\{x_t\}$ are jointly covariance-stationary with autoregressive representation and that the linear regression of x_t on all past values of x_t and y_t is

$$x_t = \sum_{s=1}^{\infty} F_s x_{t-s} + \sum_{s=1}^{\infty} G_s y_{t-s} + \varepsilon_t, \tag{3.2}$$

[2] The conditions given in Geweke (1978) are weaker than those employed here; in particular, covariance-stationarity is not required. However, covariance-stationarity or a very similar assumption is required in the development of the asymptotic distribution theory appropriate for testing either of these implications, so its reintroduction loses us very little. In addition, notation is thereby simplified.

where $\text{cov}(\varepsilon_t, x_{t-s}) = 0$ and $\text{cov}(\varepsilon_t, y_{t-s}) = 0$ for all $s > 0$ and all t. There exists a CDSEM with exogenous x_t and endogenous y_t and no other variables if, and only if, $G_s = 0$ for all $s > 0$.

The first implication arises from the fact that the CDSEM accepts current and past x_t and u_t as inputs and generates current y_t as output. The path of y_s for $s \leq t$ will be the same for the two input sequences

$$\{\ldots, (x_{t-2}, u_{t-2}), (x_{t-1}, u_{t-1}), (x_t, u_t), (x'_{t+1}, u'_{t+1}), (x'_{t+2}, u'_{t+2}), \ldots\},$$
$$\{\ldots, (x_{t-2}, u_{t-2}), (x_{t-1}, u_{t-1}), (x_t, u_t), (x''_{t+1}, u''_{t+1}), (x''_{t+2}, u''_{t+2}), \ldots\}.$$

Since this result is true for any choice of (x'_{t+s}, u'_{t+s}) and (x''_{t+s}, u''_{t+s}), it must be that $K_s = 0$ for all $s < 0$ in (3.1). The second implication embodies the notion that, however $\{x_t\}$ is determined, this determination does not depend on y_t.[3] To test this idea we need not specify the appropriate model for x_t: under the null hypothesis of exogeneity x_t is correlated with past values of y_t only through their mutual dependence on lagged x_t and hence only the latter need be included in the equations for x_t.[4]

It is important to note that the testable restrictions in the first and second implications are equivalent to the existence of *some* CDSEM in which the time series x_t is exogenous. If the implications are false, then there can exist no CDSEM with exogenous x_t. Just because the implications are true, however, does not mean that in every CDSEM relating x_t and y_t the variables x_t must be exogenous: an obvious counterexample is the "static" model $Bx_t + \Gamma y_t = \varepsilon_t$ in which the vector $(x'_t, y'_t)'$ is serially uncorrelated. More

[3] This notion appeared early in the simultaneous equation literature. In discussing a simultaneous equation model with endogenous variables p and q, exogenous variable r, and disturbances u and v, Koopmans came close to stating the second implication:

> It should further be noted that we postulate independence between r and (u, v), not between r and (p, q), although we wish to express that r "is not affected by" p and q. The meaning to be given to the latter phrase is that in other equations explaining the formation of r the variables (p, q) do not enter [Koopmans (1949, p. 130)].

The second implication sets Koopmans' remarks in a dynamic context and shows that exact specification of the "other equations" to which he refers is unnecessary for purposes of testing.

[4] Hence single equation tests like the one developed by Sims (1972) do not require the appropriateness of single equation macroeconomic models. It is in fact entirely possible that "exogenous" variables in a given application are endogenous variables in the smaller block of a larger, dynamically recursive system. For example, the system

$$\begin{bmatrix} B_{11}(L) & B_{12}(L) \\ 0 & B_{22}(L) \end{bmatrix} \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \Gamma_{1t}(L) \\ \Gamma_{2t}(L) \end{bmatrix} x_t + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}, \qquad \text{cov}(\varepsilon_{1t}, \varepsilon_{2t}) = 0$$

decomposes into two CDSEM's: in the first y_{1t} is endogenous and y_{2t} and x_t are exogenous; in the second, y_{2t} is endogenous and x_t is exogenous. Both implications will be true for the system as a whole and for each of the two subsystems.

generally, it is clear from the second implication that the testable restrictions will be consistent with a model with endogenous x_t and exogenous y_t if, and only if, y_t influences only that part of x_t which is unpredictable given only the history of x_t. While this condition is not likely in an interesting dynamic model, it cannot be ruled out. Rejection of the implications implies rejection of any specification with exogenous x_t and endogenous y_t, but their acceptance does not foreclose alternative specifications.

3.2. Test Procedures

Since the exogeneity specification places restrictions on the population parameters K_s of (3.1) and G_s of (3.2), tests of the specification may be conducted by attempting to refute one or the other or both of these sets of restrictions. Actual tests are complicated by the fact that the number of parameters in both (3.1) and (3.2) is infinite. This problem arises in the more familiar system of structural equations (2.1) as well, where it frequently has been resolved by *a priori* restriction of lag lengths. If the latter approach is applied to (3.1) and (3.2), these two equations are replaced by

$$y_t = \sum_{s=-m}^{n} K_s^* x_{t-s} + v_t^*, \tag{3.3}$$
$$\operatorname{cov}(v_t^*, x_{t-s}) = 0 \quad \text{for all } t \text{ and } s = -m, \ldots, n;$$

and

$$x_t = \sum_{s=1}^{h} F_s^* x_{t-s} + \sum_{s=1}^{l} G_s^* y_{t-s} + \varepsilon_t^*,$$
$$\operatorname{cov}(\varepsilon_t^*, x_{t-s}) = 0 \quad \text{for all } t \text{ and } s = 1, \ldots, h; \tag{3.4}$$
$$\operatorname{cov}(\varepsilon_t^*, y_{t-s}) = 0 \quad \text{for all } t \text{ and } s = 1, \ldots, l;$$

respectively. So long as x_t and y_t exhibit serial correlation, only if the chosen lag lengths are correct will it be the case that for all s, $K_s^* = K_s$, $F_s^* = F_s$, and $G_s^* = G_s$. If we are to treat a test of the null hypothesis $K_s^* = 0$, $s = -m, \ldots, -1$, as a test of the restriction $K_s = 0$, $s < 0$, then it is important to choose n generously to avoid a severe omitted variables bias. Similarly, treating a test of the null hypothesis $G_s^* = 0$, $s = 1, \ldots, l$, as a test of the restriction $G_s = 0$ for all s demands a generous choice of h. In all applications reported in the next section our choices of n and l were found to be acceptable as constraints on longer lag distributions. In an effort to avoid loss of power, however, only a few (in some cases, one) K_s^*, $s < 0$, and G_s^* were permitted under the respective alternative hypotheses.

In estimating equation (3.3) and testing the null hypothesis $K_s^* = 0$, $s = -m, \ldots, -1$, we must cope with the fact that v_t^* is a covariance-stationary but serially correlated process. To do this, the extension of Hannan's efficient estimator (Hannan, 1963) to multiple equations systems proposed in Geweke (1978) was employed. When $g = 1$, (3.3) consists of a single equation and the estimator is the same as Hannan's. In the special case in which one has *a priori* information that v_t^* is serially uncorrelated, the estimator reduces to the Zellner–Aitken estimator (Zellner, 1962). In the general case, the estimator exploits the fact that the complication of serial correlation in the time domain becomes the numerically more tractable problem of heteroscedasticity in the frequency domain. The estimator is asymptotically efficient and normally distributed and yields a consistent estimator of its own variance, so that a test of the restriction $K_s^* = 0$ for $s < 0$ may be conducted in the usual way. In addition, it imposes the constraints on K_s^* implied by structural equations in (2.1) which are identities, and the reduced rank of $\mathrm{var}(v_t^*)$ resulting from these identities is reflected in the distributions of the reported test statistics.

Estimation of (3.4) is simplified by the fact that ε_t is serially uncorrelated by virtue of being uncorrelated with all lagged values of x_t, and if h is chosen large enough in (3.4), then ε_t^* is very nearly serially uncorrelated. Equation (3.4) may therefore be estimated by the standard Zellner–Aitken procedure, and test statistics may be constructed in the usual way.

4. Testing the Overidentifying Restrictions

Identification may be comprised of several components, the usual list of which involves (i) exclusion restrictions on structural parameters, (ii) covariance behavior, and (iii) extraneous restrictions on coefficients. Elements of these components represent, in fact, assumptions about economic and statistical behavior. Such assumptions have a variety of implications for the economic and statistical properties of a model, aside from the obvious and direct relationships they describe.

Identification restrictions indirectly affect (i) the stationarity and stability requirements of the model with consequent emphasis on forecasting properties, (ii) the forms and existence of certain interim and final multipliers and associated economic dynamics interpretations, and (iii) estimation techniques and computational procedures. There seems little point in detailing these effects—rather we reemphasize the procedure for testing identification restrictions.

Tests of covariance behavior are generally made through the likelihood ratio technique except in trivial circumstances. Since there is no standard framework which contains all possible forms of covariance patterns we pursue this no further, except to note a desire to see more tests on variance structure carried out in applications.

Tests of extraneous linear restrictions on coefficients in a prescribed equation, or across equations, may be performed using the well-known F test. We note that the assumed existence of such restrictions affects the usual rank condition for identification.

Our prime concern is with the exclusion restrictions on parameters. Exclusion restrictions are testable, equation by equation. To simplify the exposition, we shall study the special case of the CDSEM

$$By_t + \Gamma x_t = \varepsilon_t,$$

whose reduced and final form is

$$y_t = -B^{-1}\Gamma x_t + B^{-1}\varepsilon_t = \Pi' x_t + v_t.$$

The jth equation of this system may be written

$$Y_j^+ \boldsymbol{\beta}_j^+ + X_j \gamma_j + \varepsilon_j = \mathbf{0}, \tag{4.1}$$

where Y_j^+ is a $T \times (L_j + 1)$ matrix of observations on $L_j + 1$ of the g endogenous variables held in Y, X_j is a $T \times K_j$ matrix of observations on K_j of the k predetermined variables held in X, $\varepsilon_j' = (\varepsilon_{j1}, \ldots, \varepsilon_{jT})$ is a vector of T disturbances, $\boldsymbol{\beta}_j^{+\prime}$ is the jth row of B, and γ_j' is the jth row of Γ. Assuming $\boldsymbol{\beta}_j^+$ and γ_j are each nonnull, equation j is identified if and only if,

$$\Pi_j^+ \boldsymbol{\beta}_j^+ = 0 \tag{4.2}$$

yields a unique solution for $\boldsymbol{\beta}_j^+$, where Π_j^+ is the $(k - K_j) \times (L_j + 1)$ submatrix of Π corresponding to the $k - K_j$ excluded predetermined variables and the $L_j + 1$ included endogenous variables. A nontrivial solution of (4.2), unique up to a scale factor, exists if, and only if, $\operatorname{rank}(\Pi_j^+)$ is L_j.

As an aside we note that in the usual version of this restriction the matrix Π_j^+ is partitioned as $[\pi_j^* \vdots \Pi_j^*]$, where Π_j^* is $(k - K_j) \times L_j$, and the equation is identified if

$$[\pi_j^* \vdots \Pi_j^*] \begin{bmatrix} -1 \\ \boldsymbol{\beta}_j \end{bmatrix} = \mathbf{0} \tag{4.3}$$

has a unique solution for $\boldsymbol{\beta}_j$, which requires

$$\operatorname{rank}[\pi_j^* \vdots \Pi_j^*] = \operatorname{rank}(\Pi_j^*) = L_j. \tag{4.4}$$

In this version normalization has been assumed, requiring the first element of $\boldsymbol{\beta}_j^+$ in (4.1) to be nonzero. This is, itself, another assumption which requires verification—specifically, in the dependent linear combination of the $L_j + 1$ columns of Π_j^+ in (4.2), the coefficient in the first column must be nonzero. Aigner & Sawa (1974) have recently commented on this issue.

For each equation of the system to be identified, a submatrix of Π must satisfy restrictions on its member elements. In the multivariate distribution of the endogenous variables Y, the mean $X\Pi$ is unique, as is the remainder of the stochastic structure of Y. While the stochastic format is fixed, a variety of structural economic behavior is permissible unless B, Γ, and Π are restricted sufficiently.

As it stands, the identification condition (4.2) suggests examination of the hypothesis that the rank of Π_j^+ is exactly L_j, against the alternative that its rank is $1, 2, \ldots, L_j - 1$, *or* $L_j + 1$. A formal test of this hypothesis can only be powerful against the alternative of rank $L_j + 1$, because the set of Π_j^* with rank *less* than L_j has measure zero if the set of Π_j^+ with rank L_j or less has positive, finite measure and the measure is smooth. The best we can do is to test for rank L_j against the alternative of rank $L_j + 1$, and then test the restriction that the rank is $L_j - 1$ against the alternative that the rank is L_j. In the second test, we try to refute the existence of an $\boldsymbol{\eta}_j^+ (\boldsymbol{\eta}_j^+ \neq c\boldsymbol{\beta}_j^+$, c a constant) such that

$$\Pi_j^+(\boldsymbol{\beta}_j^+, \boldsymbol{\eta}_j^+) = (\mathbf{0}, \mathbf{0}).$$

The results of these tests must be given their proper interpretation. If the first null is rejected, then under classical testing procedures the hypothesized set of over identifying restrictions should be scrapped. Failure of the second test will lead to rejection of the identifying restrictions only if one thought the equation in question was underidentified in the first place, in which case the first test would never be conducted.

Under the assumption of normality on the disturbances $\boldsymbol{\varepsilon}_j$ in (4.1) the limited information maximum likelihood (LIML) estimator of the coefficients $\boldsymbol{\beta}_j^+$ is found from consideration of the determinantal equation

$$|Y_j^{+\prime} M_j Y_j^+ - \hat{\mu} Y_j^{+\prime} M Y_j^+| = 0,$$

where

$$M_j = I - X_j(X_j'X_j)^{-1}X_j', \qquad M = I - X(X'X)^{-1}X'.$$

Let the $L_j + 1$ roots of this relation be denoted by $1 \leq \hat{\mu}_1 \leq \hat{\mu}_2 \leq \cdots \leq \hat{\mu}_{L_j+1}$. An efficient computational procedure for determination of these roots is given by Dent (1976, 1977).

Consider now several hypotheses on Π_j^+:

H_1: Π_j^+ has full column rank $L_j + 1$,

H_2: $\Pi_j^+ \boldsymbol{\beta}_j^+ = \mathbf{0}$, $\quad$ (rank $\Pi_j^+ = L_j$),

H_3: $\Pi_j^+(\boldsymbol{\beta}_j^+, \boldsymbol{\eta}_j^+) = (\mathbf{0}, \mathbf{0})$, $\quad$ (rank $\Pi_j^* = L_j - 1$), $\quad \boldsymbol{\beta}_j^+ \neq c\boldsymbol{\eta}_j^+$.

Maximization of the likelihood of observed included endogenous variables under the alternative hypotheses leads to the following results for the true parameters:

$$H_1\colon\ \mu_i > 1, \qquad i = 1,\ldots,L_j + 1$$
$$H_2\colon\ \mu_1 = 1,\ \ \mu_j > 1, \qquad j = 2,\ldots,L_j + 1$$
$$H_3\colon\ \mu_1 = 1,\ \ \mu_2 = 1,\ \ \mu_l > 1, \qquad l = 3,\ldots,L_j + 1$$

and to the following decision procedures:

(i) choose H_2 over H_1 if $T \ln \hat{\mu}_1 \le \chi^2_{k-K_j-L_j}(1-\alpha)$ for sample size T and significance level α

(ii) decide against H_3 in favor of H_2 if $T \ln (\hat{\mu}_1 + \hat{\mu}_2) \ge \chi^2_{2(k-K_j-L_j+1)}(1-\alpha)$.

If H_2 cannot be chosen over H_1 at the first step, then the equation is misspecified with respect to parameter exclusions. The implication in applied work is that too many coefficients have been excluded in the population regression. Even if H_2 is preferred as a result of this test, it is still necessary to test uniqueness of the unit root. Thus we desire also the preference of H_2 over H_3. We note that if H_2 cannot be chosen over H_3, then $\boldsymbol{\beta}_j^+$ is not unique, and $\mu_1 = 1$ does not necessarily follow. In this case $\mu_1 = \mu_2 = 1$ which is equal to k_1, the smallest k value of the inadmissible k-class (see Dent, 1976, Eq. 14, p. 93; Farebrother, 1974, Eq. 12, p. 535).

The usual interpretation of the identification tests is concerned with whether or not the correct *number* of variables has been properly excluded in the equation in question. The broader issue, however, is more concerned with the overall structure of the system. For example, the tests are never applied in practice to a just-identified equation. In this instance the matrix Π_j^+ is $L_j \times (L_j + 1)$ and has rank L_j with probability one, so that H_2 must, by default, always be chosen over H_1. This does not preclude, however, the possibility that, in the population relationships, $\boldsymbol{\beta}_j^+$ is not unique and the model is underidentified. The issue is whether the exclusions in an equation are consistent or inconsistent with the structure of specification information in the remainder of the model. Fisher (1976, p. 195), in a postscript, presents the alternative view that restrictions tests in specific equations are more useful than identification tests which relate to the complete system, yet claims that both should be performed because of modern computational power.

Note, in general, if H_1 were chosen over H_2, a contradiction would result. When rank Π_j^+ is $L_j + 1$, then $\Pi_j^+ \boldsymbol{\beta}_j^+ = \mathbf{0}$ has unique solution $\boldsymbol{\beta}_j^+ = \mathbf{0}$, whence the coefficients on included exogenous variables are also zero and the equation makes no sense. Since in this case necessarily $k - K_j \ge L_j + 1$, it follows that with probability one too many variables have been excluded from the relation.

The first test is then a test of the adequacy of the specification of variables in the equation relative to the information that is fully available in the system and relates to the order condition of identification; an equation which is too overidentified (large $k - K_j - L_j$) may not use enough information. The second test is a direct test of the rank condition of identification and is the more important of the two tests, but is rarely effected. Given the usual difficulties of checking the rank condition algebraically for a specified model, and the presumption that it will be satisfied if the order condition holds, this test should be in heavy demand as an indicator of potential problems.

We note that the two test statistics, $T \ln \hat{\mu}_1$, $T \ln \hat{\mu}_1 \hat{\mu}_2$ respectively are well approximated by $T(\hat{\mu}_1 - 1)$, $T(\hat{\mu}_1 + \hat{\mu}_2 - 2)$ when T is large. When $K_j + L_j = k$ and the first test is inappropriate, the second test degenerates to consideration of $T \ln \hat{\mu}_2$ or $T(\hat{\mu}_2 - 1)$, which is approximately distributed in large samples as χ_1^2 when H_2 is true.

If one chooses to believe that $\operatorname{rank}(\Pi_j^+)$ is L_j on the basis of test results, then it is possible to examine any hypothesized normalization of the included endogenous variables. In particular, if it is presumed that $\boldsymbol{\beta}_j^+ = \begin{bmatrix} -1 \\ \boldsymbol{\beta}_j \end{bmatrix}$ as in (4.3), then one may test whether $\operatorname{rank}(\Pi_j^*) = L_{j-1}$ against the alternative that $\operatorname{rank}(\Pi_j^*) = L_j$. Since Π_j^* has L_j columns, a rank of $L_j - 1$ would imply the existence of a nontrivial linear combination of the columns of Π_j^* which would yield the null vector. Thus in $\Pi_j^+ \boldsymbol{\beta}_j^+ = \mathbf{0}$ the remaining coefficient must have value zero, and normalization on the variable corresponding to π_j^* would be impossible. Thus, for any partitioning $\Pi_j^+ = (\pi_j^* \,\vdots\, \Pi_j^*)$, the test that the endogenous variable corresponding to π_j^* is included in the relation (and normalization on it is therefore possible) involves contrasting hypotheses

$$H_4: \quad \operatorname{rank} \Pi_j^* = L_j,$$

and

$$H_5: \quad \Pi_j^* \boldsymbol{\beta}_j = 0, \qquad (\operatorname{rank} \Pi_j^* = L_j - 1)$$

conditional on $\operatorname{rank}(\Pi_j^+) = L_j$.

By symmetry, a direct analog is seen with the LIML estimation process (when L_j, not $L_j + 1$, endogenous variables enter the equation). The determinantal equation

$$|Y_j' M_j Y_j - \hat{k} Y_j' M Y_j| = 0,$$

where $Y_j^+ = [\mathbf{y}_j \,\vdots\, Y_j]$, defines L_j solutions, $\hat{k}_1 \leq \hat{k}_2 \leq \cdots \leq \hat{k}_{L_j}$, which are values of the inadmissible members of the k-class (Dent, 1976, Eqs. 11 and 13). H_4 implies $k_i > 1$, $i = 1, \ldots, L_j$, and H_5, $k_1 = 1$, $k_j > 1$, $j = 2, \ldots, L_j$. The analog of the test between H_1 and H_2 suggests choice of H_4 over H_5 when $T \ln \hat{k}_1 \geq \chi^2_{k - K_j - L_j + 1}(1 - \alpha)$. This test is proposed by Farebrother & Savin (1974, p. 383) as a test of the rank condition of identification in the original equation. Rather, it is a test of identification in a subequation resulting from nonnormalization and is partial in nature since it is conditional on identification in a generalized model.

All the above tests are asymptotic in nature and are valid in the presence of lagged endogenous variables (Anderson & Rubin, 1950, Theorem 5, p. 581). Exact tests of identification for any sample size are available, conditional on $\boldsymbol{\beta}_j^+$ being known (Fisk, 1967, p. 56). Thus the computed value of $(T-k)(\hat{\mu}_1 - 1)/(k - K_j)$ may be compared with the significance points of an $F(k - K_j, T - k)$ variate. If μ_1 is too large, then, whatever the true value of $\boldsymbol{\beta}_j^+$ is, the equation is incorrectly specified. Since the test is biased toward correct specification, the significance level should be higher than usual. Similarly, to test the rank condition of identification, if $(T-k)(\hat{\mu}_2 - 1)/(k-K_j)$ is greater than the same critical F value, whatever the true value of $\boldsymbol{\beta}_j^+$, the evidence is against underidentification. This test is also obviously biased toward identification. Basmann's (1960) work on finite sample sizes suggested that the dependency on $\boldsymbol{\beta}_j^+$ being known in the above tests could be mitigated by using an $F(k - K_j - L_j, T - k)$ distribution. The degrees of freedom correction has not been further supported however.

A summary of a decision procedure concerning identification and normalization is as follows:

(1) Compute $\hat{\mu}_1$, $\hat{\mu}_2$, and $\hat{\kappa}_1$ (if the latter two exist).

(2) If $T(\hat{\mu}_1 - 1)$, $T \ln \hat{\mu}_1 > \chi^2_{k-K_j-L_j}(1-\alpha)$, or $(T-k)(\hat{\mu}_1 - 1)/(k - K_j) > F_{k-K_j,T-k}(1-\alpha)$, more variables need to be added to the equation.

(3) If $T(\hat{\mu}_1 + \hat{\mu}_2 - 2)$, $T \ln \hat{\mu}_1 \hat{\mu}_2 \leq \chi^2_{k-K_j-L_j+1}(1-\alpha)$, or

$$(T-k)(\hat{\mu}_2 - 1)/(k - K_j) \leq F_{k-K_j,T-k}(1-\alpha),$$

the population equation is underidentified.

(4) If $K_j + L_j = k$, step 2 is not performed, and 3 is replaced by consideration of $T(\hat{\mu}_2 - 1)$, $T \ln \hat{\mu}_2$ against χ^2_1.

(5) Given the passage of the two identification tests conditional normalization on the selected endogenous variable is permissible if $T \ln \hat{\kappa}_1$, $T(\hat{\kappa}_1 - 1) \geq \chi^2_{k-K_j-L_j+1}(1-\alpha)$.

One other special case emerges, viz; when $L_j = 0$ and $\hat{\kappa}_1$ and $\hat{\mu}_2$ are nonexistent. In that case the jth equation is in reduced form, Π_j^+ has only one column and rank will therefore be unity or zero ($\Pi_j^+ = \mathbf{0}$), and H_3 and the second test of identification are irrelevant. The equation cannot be underidentified, and if Π_j^+ is possibly the null vector in the population, then its coefficient in $\Pi_j^+ \beta_0 = \mathbf{0}$ may be taken as nonzero and normalization is trivially available.

We reemphasize that the only truly valid test on an axiomatic justification is that outlined in step 2 above. In each of the remaining tests the null hypothesis is one of misspecification, and the test results must be considered in this light. They should be interpreted with caution and used as indicators of potential expository problems. As a further caveat, the tests are valid only asymptotically.

5. Empirical Illustrations

5.1. Tintner's Meat Market Model

Tintner's two-equation model of the U.S. meat market (Tintner, 1965, p. 176) has been used by many authors for illustrative purposes. Demand and supply are represented by

$$\begin{aligned} y_1 + b_{12}y_2 + c_{11}z_1 \qquad\qquad\qquad\qquad &+ c_{14} = u_1, \\ y_1 + b_{22}y_2 \qquad\quad + c_{22}z_2 + c_{23}z_3 &+ c_{24} = u_2, \end{aligned}$$

where y_1 is the quantity of meat consumed, y_2 is the price of meat, z_1 is per capita disposable income, z_2 is the cost of processing meat, and z_3 the cost of producing agricultural products. Annual data (23 observations) over the period 1919–1941 are used for estimation purposes. A summary of the data is found in Tintner (1965, pp. 177–178) with the exact observations given in French (1950).

As the model stands, the demand equation is overidentified and the supply equation just identified. Interest in the model has been aroused because of the vastly differing estimates of the coefficient b_{12} available from distinct methods of estimation (LIML yields an estimate of 4.84293, 3 Stage Least Squares (and 2SLS) 1.57903 implying price elasticities related by a multiplicative factor of 3).

Tintner originally proposed two versions of the model; in the original version the variable z_3 was excluded from the supply equation, and both equations were just-identified. Drèze (1972, 1976) has made use of the model and has shown, when z_3 is included that the likelihood surface as a function of the coefficient, b_{12} is very flat over a wide range of b_{12} values. Applying a Bayesian analysis, he finds that the posterior density for b_{12} is extremely sensitive to the inclusion of the variable z_3 and concludes (Drèze, 1976, p. 1072) that "the presence of z_3 tends to reduce the coefficient of z_1 in the demand equation, and this calls for a 10-fold compensation in coefficient of y_2." Further, if "z_3 is to be included (for reasons pertaining to the supply side), it seems important to offset the resulting multicollinearity by prior information" on c_{11}. Multicollinearity apparently refers to sample correlations between z_3 and z_1 of 0.82, between z_3 and y_2 of 0.61, and between z_1 and y_2 of 0.73 respectively.

TABLE 1

TESTS OF EXOGENEITY SPECIFICATION,

TINTNER'S MEAT MARKET MODEL[a]

First implication[b]

Explanatory variable	Lag	Quantity equation: Coefficient	Quantity equation: t ratio	Price equation: Coefficient	Price equation: t ratio
Disposable income	−1	−.0435	− .4655	.0787	.8271
	0	.0104	.1218	.1332	1.5270
	1	.0091	.1372	−.0396	−.5839
Processing costs	−1	.2573	1.1076	−.3382	1.4294
	0	−.3358	−1.6624	.2308	1.1220
	1	−.4461	−1.9835	.2524	1.1022
Agricultural costs	−1	.6360	1.325	−.7816	1.5984
	0	.0215	.0543	.1919	.3193
	1	.0077	.0130	.0342	.0753

Second implication[c]

Explanatory variable	Lag	Disposable income equation: Coefficient	Disposable income equation: t ratio	Processing cost equation: Coefficient	Processing cost equation: t ratio	Agricultural cost equation: Coefficient	Agricultural cost equation: t ratio
Disposable income	1	1.5329	3.9926	.0749	1.2438	.1416	2.4416
Processing cost	1	.8405	.7665	.8556	4.9755	.2137	1.2898
Agricultural cost	1	−1.3797	−.5928	−.5880	−1.6110	.3114	.8856
Quantity	1	−.8234	−.2734	.2583	.5469	.1497	.3291
Price	1	−3.6398	−1.1662	−.1054	−.2154	−.4273	−.9062

[a] Data source: French (1950).

[b] The period of observation for the dependent variable was 1912–1940. All data were prefiltered by $(1 - .4L)$, but no other correction for serial correlation was made. Test statistic: $F(6,20) = 1.12$.

[c] The period of observation for the dependent variable was 1920–1941. Test statistic: $F(6,48) = 1.25$.

We would dispute the idea that correlations of these magnitudes even remotely suggest problems of multicollinearity, and consider the specification of the model to be questionable on other grounds. As an initial test of specification we examined the issue of exogeneity for the variables z_1, z_2, and z_3.

Test results for the two implications of exogeneity are reported in Table 1. A lag length of 1 was chosen for each test, and the hypothesis that lag length is 1 was not rejected when tested against the alternative of a lag length of 2 or 3. Modest positive serial correlation of the disturbances was indicated in ordinary least squares residuals of the equation estimated for the first implication. This was eliminated by application of the prefilter $(1 - .4L)$ to all data to obtain the results presented. Once this transformation is made, the purely contemporaneous nature of the relations (5.1) and (5.2) cannot be rejected. Neither implication of exogeneity can be rejected. The results of the first test, however, would do little to sway one who doubted the exogeneity specification since coefficients on future values of the exogenous variables are at least as important as those on current and past values. Nevertheless, the exogeneity specification is not rejected, and tests of the identifying and overidentifying restrictions may logically proceed.

Interpretation of the test results in Table 2 reveals no problems of specification with the supply equation, but does indicate potential problems in the demand equation. Both rank tests are passed with little difficulty, but the conditional test for normalization fails. We note that the null hypothesis for this test is based on the premise that the considered model is invalid, and that the power of the test is unknown. Further, the test is best justified asymptotically. Even so, $\hat{k}_1$ is sufficiently close to unity in the sample to warrant questioning of the demand equation normalization. The suggested alternative demand equation is

$$y_2 + c_{11}^* z_1 + c_{14}^* = u_1^*,$$

where the price paid for meat is unrelated to the quantity consumed. Without extra information this may seem unreasonable, but we note that the actual quantity variable y_1 "is a sum of the per capita consumption of meat, poultry, and fish [Tintner (1967, p. 177)]," and so the variable includes consumption of several substitutable items. Similarly, y_2 is an index of "retail prices of meat" and was determined from the "Bureau of Labor Statistics Index Price Series, including poultry and fish, . . . deflated by the Index of Consumer Prices for Moderate Income Families in Large Cities [Tintner (1965, p. 178)]." Under these definitions it is not so unreasonable to suggest that the price people were willing to pay for this highly aggregated "meat" commodity related primarily to changes in income during the period 1919–1941.

TABLE 2

TINTNER'S MEAT MARKET MODEL

TEST STATISTICS FOR OVERIDENTIFICATION AND NORMALIZATION

	T $\hat{\mu}_1$	$\hat{k}_1$ $\hat{\mu}_2$	$T(\hat{\mu}_1 - 1)$ $T \ln \hat{\mu}_1$	$T(\hat{\mu}_1 + \hat{\mu}_2 - 2)$ $T \ln \hat{\mu}_1 \hat{\mu}_2$	$(T - k)(\hat{\mu}_1 - 1)/(k - K_j)$ $(T - k)(\hat{\mu}_2 - 1)/(k - K_j)$	$T(\hat{k}_1 - 1)$ $T \ln \hat{k}_1$
Demand equation	23 1.12939	1.15748 1.76947	2.976 2.799	38.373 24.229	1.487 7.306	3.622 3.364
Test variable significance point			$\chi_1^2(.95)$ 3.841	$\chi_4^2(.95)$ 9.488	$F_{2,19}(.95)$ 3.52	$\chi_2^2(.95)$ 5.991
Supply equation	23 1.00000	1.38362 1.74362	— —	17.103[a] 12.787[b]	— —	8.823 7.468
Test variable significant point				$\chi_1^2(.95)$ 3.841		$\chi_1^2(.95)$ 3.841

[a] $T(\hat{\mu}_2 - 1)$
[b] $T \ln \hat{\mu}_2$

5.2. Haavelmo Multiplier Model (Prewar)

Haavelmo (1947) proposed and estimated two simple multiplier models of U.S. macroeconomic activity in the interwar period. The more elaborate of these two distinguishes between autonomous gross investment (gross private domestic investment plus government net deficit, denoted x_t) and induced business saving r_t. The model

$$c_t = \alpha y_t + \beta + u_t, \tag{5.1}$$

$$r_t = \mu(c_t + x_t) + \nu + w_t, \tag{5.2}$$

$$y_t = c_t + x_t - r_t \tag{5.3}$$

was constructed, where c_t is consumers' expenditures, y_t is disposable personal income, and x_t is assumed to be exogenous. The disturbances u_t and w_t were assumed to be serially uncorrelated and independent of x_t, but not necessarily of each other. This assumption implies that u_t and w_t are uncorrelated with lagged values of c_t, r_t, and y_t as well, and (5.1)–(5.3) is therefore a special case (albeit a simple one) of the CDSEM. Both the contemporaneous nature of the model and the exogeneity specifications are testable, however, and here we have several options.

If a CDSEM (2.1) is in fact purely contemporaneous, then the lag distribution in (3.1) degenerates to a contemporaneous relation as well, and the parameterization chosen in (3.3) becomes exact. If ε_t in (2.1) is serially uncorrelated, then one has the additional simplification that v_t^* in (3.3) is also serially uncorrelated. The first implication of the exogeneity specification may then be tested by choosing $m = 1$ and $n = 0$ in (3.3), applying the Zellner–Aitken estimator accounting for (5.3), and constructing a test statistic in the usual way. The result of this procedure is

$$\begin{aligned}
y_t &= \underset{(16.409)}{2.2501x_t} - \underset{(-1.950)}{.2953x_{t+1}} + v_{1t},\\
c_t &= \underset{(10.868)}{1.6436x_t} - \underset{(-.714)}{.1193x_{t+1}} + v_{2t},\\
r_t &= \underset{(8.042)}{.3934x_t} + \underset{(3.257)}{.1760x_{t+1}} + v_{3t},
\end{aligned}$$

where intercepts and trend terms were included but not reported and t statistics are shown parenthetically. The period of estimation was 1929–1940. The test statistic for exogeneity is significant at the 0.5% level.[5]

[5] In this and all other results reported in Section 5.2, all Durbin–Watson statistics were insignificant or inconclusive at the 5% level when evaluated as if they had been obtained from an ordinary least squares regression.

The weakness in this procedure is obvious. If x_t is indeed exogenous, but the model is not contemporaneous, then a result similar to the one reported might be expected because of the omitted lagged variables. At the same time the very small sample size places a premium on keeping lag lengths short. In experimentation with 0, 1, 2, and 3 lags, with or without a single future value of x_t, no shorter lag was rejected, at the 10% level of significance, as a constraint on a longer lag. The largest test statistic obtained was $F(2, 10) = 1.341$, for 0 lags as a constraint on 1 lag, and on the basis of this test statistic a lag length of 1 was chosen. This type of strategy is not optimal but appears unavoidable in such a small sample; it seems reasonable to conjecture that test procedures using the usual test statistics conditional on the outcome of this lag length choosing process are biased toward non-rejection of the null hypothesis that the first implication is true. This bias notwithstanding, the test results reported in Table 3 show that the first implication is rejected.

TABLE 3

TESTS OF EXOGENEITY SPECIFICATION, HAAVELMO'S MULTIPLIER MODEL (PREWAR)[a]

First implication[b]

Explanatory variable	Lag	Income equation: Coefficient	Income equation: t ratio	Consumption equation: Coefficient	Consumption equation: t ratio	Business saving equation: Coefficient	Business saving equation: t ratio
	-1	$-.4898$	-2.536	$-.3881$	-2.086	.1017	1.914
Investment	0	2.2723	8.607	1.6114	6.300	.3391	1.598
	1	.0110	.062	$-.0724$	$-.422$	$-.0835$	-1.705

Second implication[c]

Explanatory variable	Lag	Investment equation: Coefficient	Investment equation: t ratio
Investment	1	2.3979	2.549
Income	1	-3.0718	-3.660
Consumption	1	2.8781	3.260

[a] Data source: Haavelmo (1947)

[b] The period of observation for the dependent variable was 1932–1940. No correction for serial correlation was made. Test statistic: $F(2, 8) = 4.035$.

[c] The period of observation for the dependent variable was 1930–1940. Test statistic: $F(2, 5) = 5.613$.

In testing the second implication one is confronted with the problem of choosing lag length in a small sample whether the model is contemporaneous or not since the parameters in (3.2) are not functions of those in (2.1) alone. Under the null hypothesis of exogeneity, x_t is a function of lagged values of itself, and y_t, c_t, and r_t do not contribute further to variation in x_t. In autoregressions for x_t, it was found that lag lengths 1 and 2 were rejected marginally at about the 15% significance level as special cases of lag length 3. The long lag was retained, and (3.4) was estimated with one lagged value for each of c_t and y_t, r_t being eliminated by virtue of (5.3). The estimates and test statistic for the second implication, presented in Table 3, show that the second implication cannot be rejected.

On the basis of these test results, the exogeneity specification in Haavelmo's model appears dubious. Certainly, the contemporaneity *and* exogeneity assumptions are not jointly tenable on the basis of the same 1929–1941 annual data which were used in the original estimation of the model. When contemporaneity is not assumed, this rejection of exogeneity is not conclusive by the usual standards, a result which (as always, for believers in the alternatives) can be ascribed to the smallness of the sample. In the hope of obtaining more conclusive results, we turn to the larger sample afforded by the postwar period.[6]

5.3. Haavelmo Multiplier Model (Postwar)[7]

Zellner & Palm (1974) respecified Haavelmo's model more generally for postwar quarterly U.S. data:

$$c_t = \alpha(L)y_t + \beta + u_t, \tag{5.4}$$

$$r_t = \mu(L)(c_t + x_t) + \nu + w_t, \tag{5.5}$$

$$y_t = c_t + x_t - r_t. \tag{5.6}$$

The variables were defined as in the prewar model, with x_t remaining exogeneous. The lag operators $\alpha(L)$ and $\mu(L)$ were assumed to be of finite order, and u_t and w_t were assumed to have a finite order, joint moving average representation

$$\begin{bmatrix} u_t \\ w_t \end{bmatrix} = \begin{bmatrix} f_{11}(L) & f_{12}(L) \\ f_{21}(L) & f_{22}(L) \end{bmatrix} \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = F(L)e_t.$$

[6] The indecisive nature of the exogeneity assumptions in this model and the findings for the model in its postwar form suggest that there is little value in analyzing identification issues. Results of identification test procedures when exogeneity is rejected are discussed in Section 5.4.

[7] Some of the results reported in this subsection may also be found in Geweke (1978).

The model can easily be cast as a special case of (2.1). We drop the assumption that $\alpha(L)$ and $\mu(L)$ have the finite parameterization asserted and premultiply the first two equations by $F^{-1}(L)$ to produce serially uncorrelated disturbances.[8] The two testable implications of exogeneity discussed in Section 3.1 then follow.

The exogeneity test was conducted using quarterly data, with the range of the dependent variable in each case being 1951–1971. In tests of the first implication, (3.3) was estimated with twelve lagged and four leading values of x_t used on the right side and the vector $(c_t, r_t, y_t)'$ on the left. In tests of the second implication, (3.4) was estimated with twelve lagged values of x_t and two each of y_t and c_t on the right side and current x_t on the left. Estimation

TABLE 4

TESTS OF EXOGENEITY SPECIFICATION, HAAVELMO MULTIPLIER MODEL (POSTWAR)[a]

First implication[b]							
		Income equation		Consumption equation		Business saving equation	
Explanatory variable	Lag	Coefficient	t ratio	Coefficient	t ratio	Coefficient	t ratio
Investment	−4	.0423	.562	.1564	1.943	.1141	1.895
	−3	.2680	3.574	.2792	3.471	.0113	.195
	−2	.0788	1.090	.1160	1.487	.0372	.649
	−1	.3084	3.883	.4199	4.616	.1114	1.797
	0	.6787	8.687	.1185	1.393	.4398	7.326
	1	.3930	5.015	.4653	5.123	.0722	1.133
	2	.3704	4.678	.3789	4.127	.0085	.133
	3	.1503	1.780	.1622	1.739	.0120	.191
	4	.0573	.672	.0041	.043	−.0533	−.803
	5	.1638	1.891	.0901	.927	−.0736	−1.065
	6	.4340	5.632	.3914	4.675	−.0426	−.683
	7	.2715	3.513	.2052	2.516	−.0664	−1.057
	8	.2519	3.304	.2402	3.089	−.0117	−.184
	9	.2330	3.202	.1738	2.380	−.0593	−1.003
	10	.2476	3.353	.2019	2.635	−.0458	.783
	11	.1430	2.040	.0902	1.258	−.0528	−.947
	12	.0498	.761	.0201	.297	.0296	−.569

(continued)

[8] The existence of $F^{-1}(L)$ is a subtle issue which is never explicitly addressed by Zellner & Palm (1974). However, in their discussion of forecasting they assume that e_t can be recovered from complete historical records of y_t, c_t, and x_t if all the parameters of the model are known; this requires that $F(L)$ be invertible.

TABLE 4 (*continued*)

Second implication[c]

Explanatory Variable	Lag	Investment equation: Coefficient	t ratio
Investment	1	.8155	4.245
	2	−.1424	−.681
	3	.1199	.925
	4	−.2866	−2.076
	5	.4025	2.979
	6	−.2144	−1.508
	7	−.0210	−.142
	8	−.1320	−.996
	9	.0077	.060
	10	.1032	.806
	11	.1362	1.083
	12	−.0888	−.883
Income	1	−.1228	−.457
	2	.1691	.671
Consumption	1	.5075	2.222
	2	−.4718	−1.972

[a] Data sources: See appendix of Zellner & Palm (1974) or Geweke (1978). Data is quarterly.

[b] The period of observation for the dependent variable was 1951–1971. The estimator is the extension of the Hannan efficient estimator described in Geweke (1978). Test statistic: $F(8,130) = 4.11$.

[c] The period of observation for the dependent variable was 1951–1971. Test statistic: $F(4,66) = 2.27$.

of (3.3) is in this case complicated by the joint serial correlation and degenerate distribution of v_t^*. The estimator applied to cope simultaneously with these problems is the one discussed in Section 3.2 and derived in Geweke (1978). Estimation of (3.4) is much simpler since u_t^* is approximately serially uncorrelated and (5.6) implies only that we must drop one variable from the right side of the equation.

Test results are presented in Table 4. In the case of the first implication, notice that several coefficients on leading values of x_t are individually significant, and all twelve are jointly significant at the 0.1% level. (The eight numerator degrees of freedom are a consequence of (5.6.) The lag distribution is long, but a fourth year of lags proved to be insignificant. The second implication is not refuted at the 5% level of significance, although it is at 10%;

it appears that investment responds to consumption but not to saving, as one might expect if a simple accelerator model were more appropriate. The same fundamental misspecification of the model therefore seems to appear in both the prewar and the postwar sample, with rejection being more decisive in the latter because of its size.

5.4. Klein's Model I

Klein (1950) proposed a small macroeconometric model of the U.S. which is used widely to illustrate inference procedures in simultaneous equation models (e.g., Kmenta, 1971, pp. 594–596; Maddala, 1977, pp. 237–242; Theil, 1971, pp. 432–437). The model pertains to aggregate, annual data for the period 1919–1941. While it is well known, we shall restate it here for reference.

$$C_t = \alpha_0 + \alpha_1(W_{1t} + W_{2t}) + \alpha_2\Pi_t + u_{1t}, \tag{5.7}$$

$$I_t = \beta_0 + \beta_1\Pi_t + \beta_2\Pi_{t-1} + \beta_3 K_{t-1} + u_{2t}, \tag{5.8}$$

$$W_{1t} = \gamma_0 + \gamma_1 X_t + \gamma_2 X_{t-1} + \gamma_3 t + u_{3t}, \tag{5.9}$$

$$X_t = C_t + I_t + G_t, \tag{5.10}$$

$$\Pi_t = X_t - W_{1t} - T_t, \tag{5.11}$$

$$K_t - K_{t-1} = I_t. \tag{5.12}$$

The endogenous variables are C_t (consumption), W_{1t} (wage bill paid by private industry), I_t (net investment), Π_t (profits), K_t (capital stock), and X_t (total production of private industry); the exogenous variables are W_{2t} (government wage bill), T_t (taxes) and G_t (government nonwage expenditure), in addition to the trend term, t, and intercept. All variables are measured in constant prices.

Before proceeding with our test of the specification that W_{2t}, T_t and G_t are exogenous, the model may be simplified somewhat. Substituting (5.12) in (5.8), we obtain

$$I_t = \left[\frac{1 - L}{1 - (\beta_3 + 1)L}\right](\beta_0 + \beta_1\Pi_t + \beta_2\Pi_{t-1} + u_{2t}). \tag{5.13}$$

The model is thus dynamically recursive, with K_t determined after a five equation CDSEM consisting of (5.7), (5.9), (5.10), (5.11) and (5.13), with endogenous variables C_t, W_{1t}, I_t, Π_t and X_t. Because it is smaller and all identities are contemporaneous, our computations are much simpler in this system. A caveat is in order, however. One may verify that, in the original system, Assumption IV of the CDSEM implies that the spectral density of K_t is everywhere finite. The spectral density of I_t is therefore zero at frequency zero, leading to restrictions on the parameters of the equation for I_t in (3.1) and implying that the rank of the spectral density matrix of v_t is one dimension less at the zero frequency than elsewhere. (Similar restrictions are reflected in (5.13)). We have not imposed these restrictions in our estimates and test statistics for the first implication since to do so would involve a complicated, nonlinear procedure; it is difficult to conjecture how this imposition would affect our results. Fortunately, the test of the second implication does not require estimation of (3.3), and so our evaluation need not be contingent on the unknown effect of ignoring the peculiar behavior of the spectral density matrix of v_t.

The tests of the first implication were conducted by estimating (3.4) with endogenous variables C_t, W_{1t}, I_t, Π_t and X_t, and exogenous variables T_t, W_{2t} and G_t, subject to the restrictions on the coefficients and disturbance variance matrix implied by (5.10) and (5.11). Comparison of lag lengths 0, 1, 2, and 3 showed that no constraint on lag length could be rejected. In the spirit of Klein's parameterization of the structural equations of his model, we chose a lag length of 1. For reasons discussed in Section 3.1, only one future value of each exogenous variable was allowed in each equation. All equations also included constant and trend terms although we have not reported estimates of their coefficients.

The test results, presented in Table 5, indicate that the first implication should be rejected. The estimated coefficients on the government wage bill in each equation are particularly unsatisfactory: coefficient sums are of the wrong sign, interim multipliers are implausible, and the coefficient on the future values of the government wage bill is in each case the largest in absolute value of all coefficients in the equation. The estimated coefficients on taxes and government nonwage expenditures suffer from some of the same problems although not quite so severely. One is tempted to speculate about whether the estimates are more consistent with a model in which policy is purely passive, but to do so would go beyond the illustrative purposes of this example.

The second implication fares much worse, as the results presented in the second half of Table 5 show. Lag lengths 0, 1, and 2 were each very nearly rejected as restrictions on lag length 3 at the 10% significance level, and so 3 years of lags were retained. Because of the identities (5.10) and (5.11), only three endogenous variables could be included in each equation. The latter are highly correlated and no estimated coefficient for any one of them is significant, yet the hypothesis that they jointly make no contribution to the variance of the specified exogenous variables is easily rejected, the test statistic being significant at the 0.1% level. We may therefore reject the exogeneity specification using a test which is not complicated by the problem with the investment variable discussed above.

TABLE 5

TESTS OF EXOGENEITY SPECIFICATION, KLEIN MODEL I[a]

First implication[b]

Explanatory variable	Lag	Investment equation	Wage equation	Output equation	Consumption equation	Profit equation
Taxes	−1	.7921	.6548	1.5110	.7188	.8562
		(.810)	(.547)	(.669)	(.559)	(.776)
	0	.0216	−.4187	−.1883	−.2099	−.7696
		(−.026)	(−.412)	(−.098)	(−.192)	(−.821)
	1	.0941	.2911	.5977	.4997	.3026
		(.107)	(.272)	(.296)	(.434)	(.306)
Government nonwage expenditures	−1	1.6034	1.8354	3.6007	1.9973	1.7653
		(1.935)	(1.811)	(1.883)	(1.831)	(1.888)
	0	−1.4872	.2034	−.4707	.0165	−.6741
		(−.832)	(.093)	(−.114)	(.007)	(−.335)
	1	−2.1014	−2.9320	−5.6177	−3.5162	−2.6857
		(−1.251)	(−1.427)	(−1.449)	(−1.591)	(−1.417)
Government wage bill	−1	−6.3214	−8.3784	−16.2177	−9.8963	−7.8394
		(−1.455)	(−1.577)	(−1.618)	(−1.731)	(−1.600)
	0	−1.5808	−2.6785	−2.9116	−1.3308	−.2331
		(−.487)	(−.675)	(−.389)	(−.311)	(−.064)
	1	4.3151	3.4319	7.3469	3.0318	3.9150
		(1.078)	(.701)	(.795)	(.575)	(.867)

Second implication[c]

Explanatory variable	Lag	Tax equation		Government wage equation		Government expenditure equation	
		Coefficient	t ratio	Coefficient	t ratio	Coefficient	t ratio
Taxes	1	−.4603	−1.291	−.0749	−.823	−.0979	−.239
	2	−.3463	−1.542	.0377	.658	.1252	.485
	3	−.7287	−3.530	−.0621	−1.179	−.0785	−.331
Government wage	1	−.9575	−.720	−.4435	−1.307	.1025	.067
	2	−.4880	−.391	.4118	1.292	−.6173	−.430
	3	1.8738	1.934	.4715	1.907	2.5280	2.271
Government expenditure	1	−.0834	−.173	−.1003	−.812	.8369	1.506
	2	.7329	1.229	.0869	.571	.3075	.449
	3	.2438	.582	−.1365	−1.277	1.2062	2.508
Output	1	.5278	1.178	.1383	1.209	.2585	.502
Consumption	1	−.7239	−1.150	−.1954	−1.216	−.1252	−.173
Profits	1	−.1894	−.554	−.0979	−1.123	−.0823	−.210

[a] Data source: Klein (1950).

[b] The period of observation for the dependent variable was 1921–1940. No correction for serial correlation was made. t ratios are in parentheses. Test statistic: $F(9, 27) = 2.26$.

[c] The period of observation for the dependent variable was 1923–1941. Test statistic: $F(9, 15) = 7.774$.

The failure of the tests of the exogeneity implications for this model suggests that the issue of identification is irrelevant. In Table 6 however we report a number of values of test statistics related to identification issues which arise when one ignores the exogeneity misspecification dilemma. Superficially, there appear to be no problems with identification of the investment equation (5.8), and one would record only slight hesitancy with the first identification test on the consumption equation (5.7).

In the labor demand equation (5.9), however, emphatic rejection of identification is suggested. The usual reaction would probably involve the addition of exogenous or endogenous variables to the right hand side of this equation. In fact one should reject the complete system.

It is possible that this particular test result is due to the exogeneity misspecification; the exogeneity of output or the endogeneity of the government wage bill may be the basic issue. Such adjustments would necessarily change test statistics in the other equations also.

6. A Suggestion for Standards in Empirical Work

We have seen that the specifications of exogeneity, normalization, and overidentifying restrictions made in virtually all simultaneous equation models may be tested. The tests we propose have the very practical advantage that they may be applied in most econometric models which are sufficiently small that estimation by classical consistent methods is also possible. Their disadvantage is that they have not been integrated into a single, joint test of specification. Our argument in Section 4 that such integration is impossible is simply a reaffirmation of the observation of Koopmans & Hood (1953) that these difficulties can be overcome only in a theory of simultaneous choice of model and estimator. Such a theory appears no more imminent now than it did a quarter century ago. In deciding whether to reject a given specification or proceed with estimation, one must therefore devise a decision strategy whose a formal properties are not known. An example of such a strategy was presented at the end of Section 4.

Despite these formal problems, in practice, tests of specification of the type discussed here yield information about simultaneous equation models which is vitally important in their evaluation. This point is supported by the four examples studied in Section 5. In greater or lesser degree, each example is a classic model which still retains some academic interest and respectability. Yet, of the four models studied, none escaped serious question when subjected to the specification tests we have proposed. This fact suggests

TABLE 6

KLEIN'S MODEL I

TEST STATISTICS FOR OVERIDENTIFICATION AND NORMALIZATION

	T μ_1	k_1 μ_2	$T(\mu_1 - 1)$ $T \ln \mu_1$	$T(\mu_1 + \mu_2 - 2)$ $T \ln \mu_1 \mu_2$	$(T - k)(\mu_1 - 1)/(k - K_j)$ $(T - k)(\mu_2 - 1)/(k - K_j)$	$T(k_1 - 1)$ $T \ln k_1$
Consumption equation	21 1.49875	2.33542 7.61754	10.474 8.497	149.44 51.137	1.081 14.388	28.044 17.812
Test variable significance point			$\chi_4^2(.95)$ 9.488	$\chi_{10}^2(.95)$ 18.307	$F_{6,13}(.95)$ 2.92	$\chi_5^2(.95)$ 11.07
Investment equation	21 1.08595	1.74404 4.56885	1.805 1.732	76.752 33.636	0.223 9.279	15.625 11.68
Test variable significance point			$X_4^2(.95)$ 9.488	$X_{10}^2(.95)$ 18.307	$F_{5,13}(.95)$ 3.02	$\chi_5^2(.95)$ 11.07
Labor demand equation	21 2.46858	3.02718 3.12923	30.840 18.977			
Test variable significance point			$\chi_4^2(.95)$ 9.488			

that tests of the specifications of exogeneity, normalization, and overidentifying restrictions are both conceptually important and very useful criteria for the evaluation and use of econometric models.

It is our hope that future research will be addressed to integration of the tests proposed here, so that more formal evaluation of model specification is possible. Monte Carlo work on the small sample properties of these tests is also needed. In the interim, tests of the type presented here ought to be made and reported whenever possible, as part of the presentation of any estimated econometric model. The reader of the empirical literature may then evaluate the test statistics for himself in much the same way that he now informally incorporates estimates with his prior knowledge. As the standard becomes widespread, fewer but more durable simultaneous equation models ought to appear.

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