Appendix D
Technical Aspects of the United States Tax System

This appendix describes a procedure to take $A_z$, the present value of depreciation allowances on a dollar of investment, and average it over different assets. Capital stocks by themselves do not provide correct weights, because short-lived assets require relatively more reinvestment that also qualifies for depreciation deductions. In particular, a dollar of asset type $j$ can be maintained in real terms by reinvesting, at time $u$, a nominal amount equal to $\delta_j e^{\mu u}$, where $\delta_j$ is the $j$th asset's economic depreciation rate and $\mu$ is the inflation rate. Each dollar of that reinvestment receives $A_z$ of depreciation allowances in present value terms at time $u$. Discounting those nominal amounts by $\rho$, the nominal discount rate, provides

$$ PV(A_z) = A_z + A_z \int_0^\infty \delta_j e^{\mu u} e^{-\rho u} du $$

(D.1)

as the present value of depreciation on a maintained real dollar of capital. Note that $[1 + \delta_j/(\rho - \pi)]$ is the present value of expenditures necessary to keep a real dollar of capital stock. These present values can be averaged over the twenty equipment types or fourteen structure types, weighting by capital stock in each type, $K_j$, to get the present value of depreciation for an aggregate asset in each industry:

$$ \bar{PV}(A_z) = \sum_{j=1}^N K_j PV(A_z) / \sum_{j=1}^N K_j $$

(D.2)

where $N$ is twenty for equipment and fourteen for structures (industry subscripts are suppressed for simplicity, but $K$ is actually a thirty-four-by-three matrix). We can now ask the following question: Suppose a main-
tained real dollar of a single aggregate asset (either equipment or structures) had an aggregate economic depreciation rate \( \delta \) and was allowed exponential tax depreciation at rate \( \bar{a} \). What rate \( \bar{a} \) would provide the same present value of allowances as (D.2) above?

The present value of depreciation for a current dollar of investment in this aggregate asset is

\[
(D.3) \quad \overline{A}_z = \int_0^\infty \bar{a} e^{-\bar{a}u} e^{-\rho u} du = \frac{\bar{a}}{\bar{a} + \rho}.
\]

The present value of depreciation for a maintained real dollar of this aggregate asset is

\[
(D.4) \quad PV(\overline{A}_z) = \frac{\bar{a}}{\bar{a} + \rho} \left[ 1 + \frac{\delta}{\rho - \pi} \right].
\]

Setting (D.4) equal to (D.2) implies that

\[
(D.5) \quad \overline{A}_z = \frac{\bar{a}}{\bar{a} + \rho} = \frac{\sum_{j=1}^N K_j A_{z_j} \left[ 1 + \frac{\delta_j}{\rho - \pi} \right]}{\sum_{j=1}^N K_j \left[ 1 + \frac{\delta_j}{\rho - \pi} \right]}.
\]

The right-hand side of (D.5) is used to calculate \( \overline{A}_z \) from the disaggregate \( A_{z_j} \) described in the text. Since \( \delta \), derived in section 6.2.4, is just the capital-weighted average of \( \delta_j \), the denominator of equation (D.5) can be written as \( \sum K_j \left[ 1 + \delta_j / (\rho - \pi) \right] \). Thus the desired aggregate present value \( \overline{A}_z \) is just a weighted average of the \( A_{z_j} \), but the weights are \( K_j \left[ 1 + \delta_j / (\rho - \pi) \right] \) rather than \( K_j \). These weights can be interpreted as capital plus the present value of economic depreciation. Assets that depreciate faster than the average are given more weight because they will require more than the average amount of reinvestment receiving \( A_{z_j} \) in future years. Or, if depreciation rates \( \delta_j \) were the same for all assets, then \( A_z \) would be a simple capital-weighted average of the \( A_{z_j} \).

Now consider the averaging of \( g_j \), the rate of grant for the \( j \)th asset. For the same permanent one-dollar increase in the stock of this asset, nominal replacement investment at time \( u \) is \( \delta_j e^{\pi u} \). These nominal amounts receive credits at rate \( g_j \) and can be discounted by the nominal rate \( \rho \) to obtain \( PV(g_j) \), the present value of credits for the original investment as well as for the subsequent reinvestment:

\[
(D.6) \quad PV(g_j) = g_j + g_j \int_0^\infty \delta_j e^{(\pi - \rho)u} du = g_j \left[ 1 + \frac{\delta_j}{\rho - \pi} \right].
\]
These present values can be weighted by \(K_j\), the stock of capital in the \(j\)th asset, to obtain,

\[
(D.7) \quad \bar{PV}(g) = \frac{\sum_{j=1}^{N} K_j \, PV(g_j)}{\sum_{j=1}^{N} K_j},
\]

where \(N\) is twenty for equipment and fourteen for structures. If an aggregate asset of all equipment or of all structures were to depreciate at rate \(\bar{\delta}\) and receive a credit at rate \(\bar{g}\), then the present value of its credits would be:

\[
(D.8) \quad PV(\bar{g}) = \bar{g} + \bar{g} \int_{0}^{\infty} \bar{\delta} e^{(\pi - \rho)u} \, du
\]

\[
= \bar{g} \left[ 1 + \frac{\bar{\delta}}{\rho - \pi} \right].
\]

Set (D.7) equal to (D.8) and solve for \(\bar{g}\) as:

\[
(D.9) \quad \bar{g} = \frac{\sum_{j} K_j g_j \left[ 1 + \frac{\delta_j}{\rho - \pi} \right]}{\sum_{j} K_j \left[ 1 + \frac{\bar{\delta}}{\rho - \pi} \right]}
\]

Since \(\bar{\delta}\) is the capital-weighted average of \(\delta_j\), equation (D.9) just weights \(g_j\) by \(K_j [1 + \delta_j/(\rho - \pi)]\), the capital stock plus present value of actual depreciation.