7 Staggered Contracts and Exchange Rate Policy

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7.1 Introduction

The main objective of this paper is to provide a simple framework to analyze exchange market policies in the presence of staggered nominal contracts (see Phelps 1978; Taylor 1979, 1980). Although this is essentially a theoretical essay and is, therefore, hopefully applicable to a variety of circumstances, I will put special emphasis on the policies and problems associated with the recent experience of the semi-industrialized countries of the "Southern Cone" (Argentina, Chile, and Uruguay), where a centerpiece of the inflation stabilization packages was having a preannounced (not necessarily constant) exchange rate against the dollar.

The assumption of short-run sticky prices and wages has been amply explored in the balance of payments literature (see Dornbusch 1980). However, because the latter has typically been studied under the further constraint that expectations follow some kind of "adaptive" scheme, little is known about the effects of announcements that involve the future change of a policy variable. An important exception is the work of Wilson (1979); however, this effort is, in principle, not fully satisfactory because the price adjustment equation involves only contemporaneous variables (like "present" excess demand). Thus, an advantage of the Phelps-Taylor staggered contracts approach is that present "new" contracts (an obviously important ingredient for a change of the average price level) are based on (rational) forecasts about the future course of all relevant variables.

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But, of course, the recognition that a theory has an obvious limitation is not enough justification for generalizing it, unless one feels that the theory leads to implications that are bluntly invalidated by the facts, or are potentially so significant for policy purposes that a search for robustness is warranted.

Our motivation for extending the Dornbusch-Wilson assumption to account for staggered (or overlapping) contracts is partly related to the latter consideration. Recently, Krugman and Taylor (1978) have revived the Hirschman (1949) and Diaz Alejandro (1963) view that a devaluation could be contractionary in the short run. Thus, for instance, Díaz Alejandro argues that a devaluation will result in a change in income distribution in favor of profitmakers (the “high” savers) and against workers (the “low” savers), leading to a contraction of aggregate demand, and hence (with sticky prices) output. Since these models are of the income-expenditure variety with no explicit consideration of wealth effects, the above implication can more generally be stated by saying that a short-run appreciation (depreciation) of the real exchange rate will be expansionary (contractionary).

A bit more formally, let $P$ denote the (log of) the nominal price of domestically produced goods, and $E$ the (log of) the exchange rate (i.e., the price of foreign in terms of domestic currency). Hence, the above models imply that (given the price of importables, which we henceforth assume constant and equal to one) aggregate demand is an increasing function of:

\[(1) \quad p = P - E.\]

In other words, if $f(p)$ denotes the excess aggregate demand function, the implication is that:

\[(2) \quad f'(p) > 0.\]

The literature on this matter stops here, but imagine that we wish to (plausibly) continue the story along the Dornbusch-Wilson lines and postulate

\[(3) \quad \dot{p} = f(p),\]

that is, we assume that the growth rate of the price of importables with respect to domestic goods is an increasing function of aggregate demand. Clearly, then, by (2) and (3), the system is unstable.

To couch the above finding in more familiar terms, let us (as usual) say that the (log of the) real exchange rate ($-p$) appreciates if $p$ increases.

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1. $f' > 0$ is not an inevitable implication of those models (see Krugman and Taylor 1978), but, for the time being, we will concentrate on it because it is the most interesting one.
2. A dot over a variable indicates that it is a time derivative.
Consequently, the model implies that if the currency is initially "over-valued" (i.e., $p > \bar{p}$, where $f(\bar{p}) = 0$), the real exchange rate will keep appreciating over time. A sudden devaluation may restore equilibrium if it sets $p = \bar{p}$, but if $p$ overshoots in the opposite direction ($p < \bar{p}$) a cycle of increasing unemployment and real depreciation will follow. (Observers of a country like Argentina in the 1979–81 period could not but be impressed by the predictive power of this [apparently] simple model!) It is obvious, however, that these findings deserve a closer look.

In section 7.2 we discuss a continuous time, perfect capital mobility version of the staggered contracts approach with rational expectations, where, for the sake of generality, aggregate demand is also made a function of the "real" interest rate. In section 7.3 we study the existence and uniqueness of stable equilibria for a predetermined path of the exchange rate. An important result in this respect will be to show that when $f'(p) > 0$ (at steady state) there exists a multiplicity (in fact, a "continuum") of equilibria—thus contradicting the implications of the Dornbusch-Wilson-type model. On the other hand, when $f'(p) < 0$ a unique, stable equilibrium exists, and in section 7.3 we proceed to characterize it under the following conditions: (i) a constant rate of devaluation, and (ii) a future expected discontinuous jump in (a) the level and (b) the rate of growth of the exchange rate. The common outcome is that (starting at steady state) there is a transitory appreciation of the real exchange rate.

Section 7.4 studies the possibility of restoring uniqueness when $f'(p) > 0$ by adopting a crawling peg rule by which the rate of devaluation is made a function of past rates of inflation ("minidevaluations"), and the answer is found to be in the negative. However, the same question is explored for the case where a government is able to impose controls on capital mobility; it is shown that uniqueness is now a definite possibility if the ratio of the derivative of aggregate demand with respect to the real rate of interest to the interest semielasticity of the demand for money is sufficiently large (in absolute value).

Section 7.5 analyzes the model with a perfectly flexible exchange rate and perfect capital mobility. Conditions for uniqueness are found to be the same as before, thus showing that the difficulties associated with $f'(p) > 0$ cannot be cured by allowing the exchange rate to float. Furthermore, when a unique stable equilibrium exists, results similar to those in Dornbusch (1976) are shown to hold. The paper concludes in section 7.6 with a discussion of the central results.

3. See Calvo (1981) for a sketch of the 1976–80 period, which was characterized by a remarkable increase in $p$ (over 100 percent) and full employment. In the period of January to June 1981 a devaluation of more than 200 percent occurred, but it was associated with an increase in the rate of unemployment of over three times (from about 2 percent to more than 6 percent according to educated guesses).
7.2 A Model of Staggered Contracts

In this section we will assume the more general case where aggregate demand is also a function of the real rate of interest. More specifically,

\[ \text{Aggregate Excess Demand for Domestic Goods} = f(p, r), \]

where \( r \) is the real rate and,

\[ f_p \geq 0, f_r < 0. \]

Phelps (1978) and Taylor (1980) assume a contract technology by which contracts are determined in nominal terms and are supposed to last for an exogenous amount of time. In the present version, we basically adopt the same assumptions except, to simplify the mathematics, we suppose that contract length is stochastic and identically and independently distributed across contracts; furthermore, if a contract has not yet expired or is being renewed at time \( t \), the probability (density) that it will last for \( s \) more periods is:

\[ \delta e^{-\delta s}, \delta > 0, \]

and is, therefore, independent of \( t \) and of the amount of time the contract has lasted at \( t \).

We envision a situation where there is a very large number of identical firms (in fact, a "continuum"), each one setting the price of its own supply of domestic goods. Thus, letting \( V_t \) denote the (log of) price set on contracts renewed (or signed) at time \( t \), we get

\[ P_t = \delta \int_{-\infty}^{t} V_s e^{-\delta(t-s)} ds. \]

Following Phelps (1978) and Taylor (1980), we assume that price setters are rational (equivalent to "perfect foresight" in the present context) and set their prices taking into account the average price level and excess demand during the length of the contract. Given our previous assumptions, it appears natural to make

\[ V_t = \delta \int_{t}^{\infty} [P_s + \beta f(p_s, r_s)] e^{-\delta(s-t)} ds, \beta > 0. \]

Therefore, at every point in time \( t \) where \( p_t \) and \( r_t \) are continuous, we can differentiate (7) and (8) with respect to time to get

4. The following assumption is essentially the one in Phelps (1978) and Taylor (1980) for "new" contracts.

5. With a nonzero real interest rate, (8) could also naturally incorporate a factor to reflect it. This addition would, however, make steady states (in the ensuing analyses) sensitive to permanent changes in the rate of devaluation or expansion of money supply. Without denying the existence, and maybe importance, of such effects, we will stick to form (8) for the sake of simplicity. See Phelps (1978) and Taylor (1979, 1980) for a similar procedure.
(9) \[ \dot{P}_t = \delta(V_t - P_t), \]
(10) \[ \dot{V}_t = \delta[V_t - P_t - \beta f(P_t, r_t)]. \]

This constitutes our basic system. The following sections will close the model by making different alternative assumptions on the exchange rate regime and the degree of capital mobility.

7.3 Prefixed Rates and Perfect Capital Mobility

In this section we will assume that the (log of) the exchange rate, \( E \), is announced at time zero for the whole future; thus at any point in time we have a regime of fixed exchange rates, but the latter may not be constant over time (this is, in stylized form, the type of policy recently followed in Southern Cone countries). Furthermore, we assume that the country is small in the world financial markets and that there exists perfect capital mobility. Thus, letting \( i \) denote the domestic nominal interest rate, and assuming (to simplify notation) that the international interest rate is equal to zero, we have:

(11) \[ i_t = \dot{E}_t \]
at every \( t \) at which the (right-hand) derivative is defined.

We define:

(12) \[ r_t = i_t - \dot{P}_t^+, \]
where \( \dot{P}_t^+ \) is the right-hand derivative of \( P \) at \( t \) (i.e., the expected rate of inflation of the domestic price level at \( t \) with perfect foresight).

Consider now the case where the exchange rate is constant over time and equal to \( \bar{E} \). By (9), (10), (11), and (12) the system becomes:

(13) \[ \dot{P}_t = \delta(V_t - P_t), \]
(14) \[ \dot{V}_t = \delta[V_t - P_t - \beta f(P_t - \bar{E}, \delta(P_t - V_t))]. \]

At time \( t = 0 \), the "present," \( P_0 \), is given by the outstanding contracts and, thus, becomes a "predetermined" variable; \( V_0 \), however, is free to take any value. Consequently, the system is as yet unable to provide us with a unique solution. To try to close the model, we will further impose the condition that the solution converges to a steady state. Figure 7.1 illustrates the determination of equilibrium for the case:

6. After \( t = 0 \), and as long as the policy announced at \( t = 0 \) remains unchanged, (8) implies that \( V_t \) will be continuous. Thus along an equilibrium path starting at \( t = 0 \), \( V_t \) can jump with respect to its past values only at \( t = 0 \). Incidentally, notice that by the last observation and (14), in the present case \( \dot{P} \) exists (and hence equals \( \dot{P}^+ \)) for all \( t > 0 \) (the "present").

7. This is an already standard procedure in rational expectations models. For a (partial) justification, see Brock (1974) and Calvo (1979), and for earlier applications to the present field, see Kouri (1976), Dornbusch (1976, 1980), and Calvo and Rodriguez (1977).
1 + \beta f_P(\bar{P} - \bar{E}, 0) + \delta f_r(\bar{P} - \bar{E}, 0) < 0, \text{ and } 1 + \beta \delta f_r(\bar{P} - \bar{E}, 0) > 0

(hereafter a bar over a variable indicates its steady-state value). It is interesting to note that under the assumptions underlying figure 7.1, equilibrium $V$ is a function of $P$; thus if we denote it $V^*(P)$, we get, by (13), that in equilibrium

\begin{equation}
\dot{P}_t = \delta [V^*(P_t) - P_t] = \phi(P_t),
\end{equation}

and, thus, the model boils down to a fixed-rate version of Dornbusch (1976).

From figure 7.1 and considerations given above, it should be clear at this point that existence and uniqueness of equilibrium require that (13) and (14) be characterized by saddle-path stability. Assuming away the borderline case where the determinant of the linear approximation of (13)–(14) vanishes at steady state, a necessary and sufficient condition for existence of a saddle path (the locus of equilibrium $[P, V]$ pairs) is that,

\begin{equation}
\det A_1 < 0,
\end{equation}

8. For technical reasons, the borderline case will be assumed away in what follows.

9. For the linear system, a necessary and sufficient condition for saddle-path stability is that the characteristic roots of $A_1$ have opposite signs. But since $\det A_1 = \lambda_1 \lambda_2$, where $\lambda_1 \lambda_2$ are the roots of $A_1$, it is easily seen that (16) is equivalent to the above condition.
where

\[ A_1 = \left( \frac{\partial \dot{P}}{\partial P}, \frac{\partial \dot{P}}{\partial V} \right) = \left( -\delta \frac{\partial}{\partial P}, -\delta \frac{\partial}{\partial V} \right) = \left( -\delta [1 + \beta f_p(\bar{P} - \bar{E}, 0)] \right)
\]

Hence,

\[ \det A_1 = \beta \delta^2 f_p(\bar{P} - \bar{E}, 0), \]

and condition (16) is satisfied if and only if

\[ f_p(\bar{P} - \bar{E}, 0) < 0. \]

Consequently, existence of a unique converging equilibrium path is linked to the condition that, at steady state, a devaluation (i.e., increase in \( \bar{E} \)) be expansionary, not contractionary as in the Hirschman-Díaz Alejandro-Krugman-Taylor case. If, contrariwise, we have:

\[ f_p(\bar{P} - \bar{E}, 0) > 0 \]

(i.e., if a devaluation is contractionary), then the system is locally stable because, by (5) and (17),

\[ \text{tr} A_1 = \beta \delta^2 f_p(P - E, 0) < 0, \]

meaning that for \( P_0 \) sufficiently close to \( \bar{P} \) there exists an infinite number (in fact, a "continuum") of \( V_0 \) values from which the system converges to the steady state.

This "embarrassment of riches" simply entails that the model is not powerful enough to convey information about the future course of the economy (see Calvo 1978 for a discussion of this type of difficulty with rational expectations models, and section 7.6 of this paper). Thus we are led to the conclusion that, although contrary to the Dornbusch-type model (1976), the present model exhibits stability when (20) holds, and the economy will, instead, be immersed in complete confusion if individuals try to base their expectations on predictions derived from the model.

Another point that is worth noting is that the condition for uniqueness, (19), is independent from the interest elasticity of aggregate demand\(^{10,11}\) and, most remarkably, from the average contract length.

Consider now the case where there is a unique solution. First, we notice that at steady state the real exchange rate \((-\bar{p})\) is independent of

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10. Notice that when this elasticity is sufficiently high (in absolute terms) a model similar to the one discussed in section 7.1 would become stable.

11. To gain further insight into the uniqueness question, the reader is invited to study the case where \( P \) is perfectly flexible and \( f(p, r) = 0 \) at all times. Interestingly enough, when there is perfect capital mobility, the uniqueness condition is also exactly given by (19).
Imagine now an unanticipated once-and-for-all devaluation at \( t = 0 \), that is, \( \Delta \bar{E} > 0 \) starting from steady state;\(^{12}\) in terms of figure 7.1 this implies that we start from a point like \( a \). After the devaluation, the \( V = 0 \) curve shifts up, thus the new equilibrium (with same initial \( P \)) will move to a point like \( b \); \( P \) will start increasing until the initial \( \bar{P} \) is restored. In other words, an unanticipated devaluation will originally depreciate the real exchange rate, but the latter will steadily recover its initial level.\(^ {13}\)

Suppose now the perhaps more intriguing situation where at \( t = 0 \) people find out that a once-and-for-all devaluation will occur at \( t_0 > 0 \). Thus, say,

\[
E_t = \bar{E}_0 \quad \text{for} \quad 0 \leq t < t_0, \\
E_t = \bar{E}_1 \quad \text{for} \quad t \geq t_0,
\]

where \( \bar{E}_1 > \bar{E}_0 \). Figure 7.2 exhibits the dynamics of this case; the phase diagram corresponds to \( E = \bar{E}_0 \), except for the dashed curve which is the saddle path associated with \( E = \bar{E}_1 \). For equilibrium, \( V \) has to be continuous after \( t = 0 \) (the moment of the announcement), and the system must be on some point of the dashed saddle path at \( t = t_0 \). Thus it is clearly seen that if the system was originally at \( a \), it must jump to a point like \( b \) and proceed along the dotted line until it exactly connects with \( c \) at \( t = t_0 \). Therefore, \( P \) starts rising before the devaluation occurs, meaning that:

\[
p_t > \bar{P}, \quad 0 < t < t_0,
\]

that is, the real exchange rate appreciates during the transition before devaluation actually occurs. Furthermore, since \( P \) keeps rising at \( c \) and, as argued before, \( p \) at \( a \) equals \( p \) at \( d \), then,

\[
p_t < \bar{P}, \quad t_0 < t,
\]

that is, despite its being anticipated, a devaluation results in a depreciation of the real exchange rate below \( \bar{P} \).

Intuitively this is easy to understand; because contracts made today have to take into account the devaluation at \( t = t_0 \), it is natural to expect that before the devaluation actually occurs \( V_t \) will be higher than it would be otherwise, thus giving \( P \) an upward push. Furthermore, at the time of the devaluation (\( t = t_0 \)), there will be some outstanding contracts that took into account the exchange rate prevailing before \( t = t_0 \), and thus the corresponding \( V_t \)'s would be smaller than if the old exchange rate had been equal to the new one; this explains the depreciation of the real exchange rate at \( t = t_0 \).

\(^{12}\) For the sake of concreteness all the comparative dynamics exercises of this paper will assume that the system was originally at steady state.

\(^{13}\) Although we are basing our conclusions on figure 7.2, the reader can easily show that this is a general feature. Incidentally, in order not to encumber the exposition with technical details, propositions derived from specific examples will be meant to have general validity, unless the contrary is asserted.
Recalling (12), notice that the real interest rate, \( r \), is always well defined because \( \dot{P}^+ \) exists for all \( t \geq 0 \), and notice that under the present assumptions,

\[
(25) \quad r_t = -\dot{p}^+_t, \quad t \geq 0.
\]

Hence, both here and in the previous example, the real rate is negative for all \( t \) and converges to zero (its steady-state value).

It is instructive to compare our model with the one discussed in section 7.1. Although, recalling figure 7.2, \( V \) is still functionally linked to \( P \) in equilibrium, the function itself depends on the nature and timing of the anticipations. In fact, even the functional relationship would cease to hold in slightly more complicated examples. Thus, for instance, if we assume:

\[
E_t = \bar{E}_0 \text{ for } 0 \leq t < t_0,
\]
\[
E_t = \bar{E}_1 \text{ for } t_0 \leq t < t_1,
\]
\[
E_t = \bar{E}_0 \text{ for } t \geq t_1,
\]

and \( t_1 > t_0 \), the equilibrium path will look like the arrowed curve in figure 7.3; clearly there we find values of \( P \) for which there are two \( V \)'s. Point \( b \) is reached at \( t = 0 \), \( c \) at \( t_0 \), \( d \) at \( t_1 \), and \( a \) as \( t \to \infty \).\(^{14}\)

\(^{14}\) The pattern shown in figure 7.3 corresponds to the case depicted in figure 7.1 and is not generalizable.
We now turn to the case where $E_t$ is continuous and $E_t$ is constant, or is expected to have a one-step future jump (akin to the Southern Cone experiences). Let us denote,

$$v_t = V_t - E_t.$$  \hfill (26)

Thus, subtracting $E_t$ from both sides of (9) and (10), and recalling (11) and (12), we get:

$$\dot{p}_t = \delta (v_t - p_t) - \dot{E}_t,$$  \hfill (27)

$$\dot{v}_t = \delta [v_t - p_t - \beta f(p_t, - \dot{p}_t)] - \dot{E}_t.$$  \hfill (28)

We will, first, consider a situation where,

$$\dot{E}_t = \epsilon, \text{ for } 0 \leq t.$$  \hfill (29)

The following results are easily shown: (a) steady-state $p, \bar{p}$, is independent of $\epsilon$ (the rate of devaluation), and (b) the uniqueness condition is (19), the same as before. Assuming uniqueness, consider now the case where $\epsilon$ is expected to take a once-and-for-all jump at $t = t_0 > 0$. This is illustrated in figure 7.4 under the same assumptions as in figure 7.1. The phase diagram corresponds to the situation before the jump in $\epsilon$, and the dashed curve is the new saddle path after the jump. Thus, the system first jumps from $a$ to $b$ and then smoothly travels from $b$ to $c$; it reaches $c$ at $t = t_0$, and then abruptly starts moving toward $d$ (in a smooth fashion). In
other words, suppose the system was at a rest point under the expectation that \( \epsilon \) would be constant forever, and then it is announced that \( \epsilon \) will have a once-and-for-all increase at \( t = t_0 > 0 \). Then our analysis has shown that the real exchange rate will start appreciating (\( p \) rises) until the change in \( \epsilon \) actually occurs; then \( p \) will start to go down until eventually the real exchange rate returns to its initial level.

Notice that unlike the one-step depreciation experiment considered before, here \( p \) always stays above \( \bar{p} \). Furthermore, an anticipated future change in \( \epsilon \) sharply contrasts with the case where \( \epsilon \) suffers a once-and-for-all change at \( t = 0 \) (the "present"), because the latter results in no change in \( p \) (assuming, as usual, that \( p_0 = \bar{p} \)).

It is interesting to note in this connection that the Dornbusch-type model discussed in section 7.1 will exhibit no impact on the real exchange rate whether or not the change in the rate of devaluation is anticipated.

### 7.4 Crawling Peg and Controls on Capital Mobility

In this section we briefly examine the significance of a familiar crawling peg system and of capital mobility controls for eliminating the non-uniqueness problem that was discovered in section 7.3.
Let us first consider the case where $\dot{E}_t$ is chosen according to a rule that sets it as a function of some average of past rates of inflation (this appears to be the rule followed for a considerable amount of time by several countries, like Colombia and Brazil, and has apparently, and very recently, been adopted by Argentina). Specifically, we assume,\(^{15}\)

\begin{equation}
\dot{E}_t = \epsilon_t = \gamma \int_{-\infty}^{t} \dot{P}_s e^{-\gamma(t-s)} ds,
\end{equation}

implying

\begin{equation}
\epsilon_t = \gamma (\dot{P}_t - \epsilon_t) = \gamma \dot{P}_t,
\end{equation}

and, hence,

\begin{equation}
\epsilon_t = \epsilon_0 + \gamma \dot{P}_t,
\end{equation}

where $\epsilon_0$ is given by history at $t = 0$.

Now substituting the expression for $\epsilon_t$ given by (32) in equations (27)-(28) we obtain a differential equation system in $v$ and $\dot{P}_t$, and we are thus able to check the uniqueness condition as we did in section 7.3. However, as is readily verifiable, uniqueness is also insured in this case if $f_\gamma < 0$ at steady state. Thus the crawling peg system does not eliminate the nonuniqueness problem.

Let us now study the implications of imposing controls on capital mobility. For the sake of simplicity, let us consider the case of total capital immobility, where the money market equilibrium is given by

\begin{equation}
M_t - P_t = -k i_t,
\end{equation}

where the right-hand side is the money demand function, $M$ denotes (the log of) money supply, and $k$ is a positive parameter. Assuming that both the money supply and the exchange rate are constant (and equal to $\bar{M}$ and $\bar{E}$, respectively), we get

\begin{equation}
r_t = i_t - \dot{P}_t = \frac{P_t - \bar{M}}{k} - \dot{P}_t.
\end{equation}

Thus by (9), (10), and (34),

\begin{equationa}
\dot{P}_t = \delta (V_t - P_t),
\end{equationa}

(35b)

\begin{equationb}
\dot{V}_t = \delta \left[ V_t - P_t - \beta f \left( P_t - \bar{E}, \frac{P_t - \bar{M}}{k} - \delta (V_t - P_t) \right) \right].
\end{equationb}

Applying the procedure of section 7.3, the condition for uniqueness now becomes

\text{15. More precisely (30) should have $\dot{P}^-$ (the left-hand derivative of $P$) in the integrand, but this would not affect our implications.}
which shows that (36) could be satisfied even when \( f_p > 0 \), thus implying that controlling capital mobility may be an effective way to prevent nonuniqueness. It should be noted, however, that (36) implies that a devaluation is not contractionary.

In closing, it is worth noting that any policy that makes \( f_p < 0 \) will tend to restore uniqueness. In particular, the removal of import quotas will work toward that end because it will increase the effective substitutability of domestic for imported goods.

7.5 Flexible Exchange Rate and Capital Mobility

The main purpose of this section is to sketch out the dynamics of the flexible exchange rates case and show that under perfect capital mobility the uniqueness condition is similar to that under (pre-) fixed rates.

Again we assume that the money market equilibrium is given by (33). Thus, recalling (11), we have

\[
\dot{E}_t = \frac{1}{k} \tilde{P}_t - \mu ,
\]

where a tilde indicates that \( M_t \) is subtracted from the corresponding variable (e.g., \( \tilde{P}_t = P_t - M_t \)), and we assume \( M_t = \mu \), a constant (i.e., money supply grows at a constant rate \( \mu \)). Contrary to the previous sections, now the exchange rate is market determined under the condition that the central bank does not accumulate or reduce reserves.

On the other hand, by (9), (10), (11), and (12), we have

\[
\begin{align*}
\dot{P}_t &= \delta (V_t - \tilde{P}_t) - \mu , \\
\dot{V}_t &= \delta \left( V_t - \tilde{P}_t - \beta f \left[ \tilde{P}_t - E_t, \left( \frac{1}{k} + \delta \right) \tilde{P}_t - \delta \tilde{V}_t \right] \right) - \mu .
\end{align*}
\]

Therefore, at steady state,

\[
A_2 = \begin{bmatrix}
\frac{\partial \dot{P}}{\partial \dot{P}}, & \frac{\partial \dot{P}}{\partial \dot{V}}, & \frac{\partial \dot{P}}{\partial \dot{E}} \\
\frac{\partial \dot{V}}{\partial \dot{P}}, & \frac{\partial \dot{V}}{\partial \dot{V}}, & \frac{\partial \dot{V}}{\partial \dot{E}} \\
\frac{\partial \dot{E}}{\partial \dot{P}}, & \frac{\partial \dot{E}}{\partial \dot{V}}, & \frac{\partial \dot{E}}{\partial \dot{E}}
\end{bmatrix} = \begin{bmatrix}
-\delta & \delta & 0 \\
-\delta \left[ 1 + \beta f_p + \beta f_r \left( \frac{1}{k} + \delta \right) \right] & \delta (1 + \beta \delta f_r) & \delta \beta f_p \\
\frac{1}{k} & 0 & 0
\end{bmatrix}.
\]
Clearly, then, letting the characteristic roots be denoted \( \lambda_i, i = 1, 2, \text{ and } 3 \), we get

\[
\text{det} A_2 = \frac{\delta^2}{k} \beta f_p = \lambda_1 \lambda_2 \lambda_3, \tag{40}
\]

\[
\text{tr} A_2 = \delta^2 \beta = \lambda_1 + \lambda_2 + \lambda_3 < 0, \tag{41}
\]

and,

\[
\delta^2 \beta \left( \frac{f_p}{k} + \frac{f_z}{k} \right) = \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3. \tag{42}
\]

Consider first the case \( f_p > 0 \), which implies nonuniqueness in the fixed rates case. By (40) and (41) we have either (a) two roots are negative and one is positive, or (b) the three roots are positive. However, since in system (37)-(38) only \( \tilde{P}_t \) is a predetermined variable, uniqueness requires only one root be negative; in case (a) there is a continuum of converging equilibria, while in case (b) there is none.

Let us now study the situation where \( f_p < 0 \). Again, by (40) and (41), there are two possibilities: either (a') one root is negative and the other two have nonpositive real parts, or (b') one root is negative and the other two are positive. We will show now that (a') can never happen, thus proving that, similarly to the (pre-) fixed rates case, the uniqueness condition is \( f_p < 0 \) at steady state. We prove it in the following way: If \( f_p < 0 \), then, by (42),

\[
\lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3 < 0, \tag{43}
\]

which contradicts case (a') if all the roots are real numbers. Alternatively, suppose now that \( \lambda_1 \) is a real number and

\[
\lambda_1 < 0, \quad \lambda_2 = a + bi, \quad a \leq 0, \quad \lambda_3 = a - bi \tag{44}
\]

(where \( i \) is the imaginary unit). Then

\[
\lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3 = \lambda_1 (a + bi) + \lambda_1 (a - bi) + (a + bi)(a - bi) = 2 \lambda_1 a + a^2 + b^2 > 0,
\]

contradicting (43). This proves that (a') cannot hold.

Let us now assume that there exists a unique converging solution (i.e.,

16. These relationships between roots and functions of \( A_2 \) are well known (see Gantmacher 1959).
17. We say that a root is negative (positive) according to whether its real part is negative (positive).
18. As before, we will abstract from the case \( f_p = 0 \).
As is well known (see Coddington and Levinson 1955, chap. 4), the qualitative features of the solution of system (35)–(37) in a "sufficiently" small neighborhood of the steady state can be ascertained on the basis of its linear approximation. Let $\lambda_1$ be the negative root (a real number, of course), and let $x$ be its associated characteristic vector, that is,

$$A_2 x = \lambda_1 x.$$  

Therefore, converging solutions of the linear system satisfy

$$\begin{pmatrix} \bar{P}_t - \bar{P}, \bar{V}_t - \bar{V}, \bar{E}_t - \bar{E} \end{pmatrix} = \eta x e^{\lambda_1 t},$$

where, as usual, a bar indicates steady-state levels, and $\eta$ is a constant that makes the left-hand side equal to the right-hand side of (46) at $t = 0$.

An implication of (46) is that all variables converge in a monotonic fashion.

Consider now the effect of a once-and-for-all increase in money supply at $t = 0$. Clearly, starting at steady state, it makes

$$P_0 < \bar{P}$$

(notice that $\bar{P}$ and all other steady-state values remain unchanged by this experiment).

Consequently, by (37),

$$\dot{E}_0 < 0.$$

But since $\bar{E}_t$ is monotonic, we get

$$\dot{E} < 0 \quad \text{for all } t.$$  

In equilibrium, variable $\bar{E}$ (as all the others) has to return to its steady-state level. Hence, by (49), $\dot{E}_0$ will jump above $\bar{E}$; after that, $\dot{E}$ will monotonically return to $\bar{E}$. Interestingly enough, the overshooting and eventual monotonic return to $\bar{E}$ are implications that coincide with those in Dornbusch (1976) based on the model discussed in section 7.1.

Finally, let us consider the impact of a once-and-for-all increase in the rate of expansion of money supply (i.e., an increase in $\mu$). First, we see from (37) that,

$$\frac{d\dot{E}}{d\mu} = < \frac{d\bar{P}}{d\mu} = k > 0.$$  

Since, as indicated above, equilibrium paths are monotonic, and (having started at steady state)

$$\dot{E}_0 < 0$$

(recall [37] and that $\frac{d\bar{P}_0}{d\mu} = 0$), we are able to conclude, as in the previous experiment, that $\dot{E}_t$ will be monotonically decreasing over time; thus, by (50), for the path to converge to steady state we have
implying that after a higher \( \mu \) is announced at \( t = 0 \), \( \tilde{E}_0 \) will overshoot its new, and higher, steady-state level, after which \( \tilde{E} \) will monotonically return to its new \( \tilde{E} \). Furthermore, monotonicity of equilibrium paths plus (50) immediately leads us to the conclusion that \( \tilde{P} \) will steadily rise to its new, and higher, steady-state level.

7.6 Summary and Conclusions

This paper has analyzed a very simple disequilibrium model with staggered contracts. Its main advantage is that it provides us with a framework where questions about uniqueness and stability can be examined for alternative exchange rate systems. Furthermore, despite its elementary nature, our model represents a level of greater complexity and richness than those of standard models.

We will not repeat here all the results associated with specific regimes. Instead, we will concentrate on the issues of uniqueness and stability with particular reference to the contractionary devaluation literature.

As pointed out in section 7.1, our interest in the issues studied in this paper stemmed from the observation that if devaluations are contractionary, then a simple-minded, but not implausible, adjustment mechanism (like [3]) would lead one to conclude that the system is unstable. Although there is no sacred law by which a system could not be unstable (Samuelson’s “correspondence principle” notwithstanding), instability is not an implication that should easily be accepted because it may simply be reflecting a flaw in the model’s structure. This is so, in particular, because as the economy travels along an unstable path, people will eventually become conscious that reality departs more and more from long-run equilibrium, and hence the very nature of the adjustment mechanisms may change, possibly reverting the instability implications.

The model studied in this paper goes a long way to cover itself against the above-mentioned difficulty. In the first place, we assume perfect foresight, thus not simply under unusual circumstances, but always is the public aware of the structure of the economy. Second, slow price adjustment is not merely postulated but derived from a price-setting technology. True, the latter is not as yet fully based on conventional microeconomic principles, but it should be recalled that our existence and uniqueness results are independent of the average contract length (i.e., independent of \( \bar{\delta} \)). Thus for our analysis to be relevant, we only need to essentially assert that there are nominal contracts lasting (on average) a nonnegligible amount of time.
In connection with the contractionary devaluation case, our model revealed that there will be a "continuum" of converging equilibria if capital mobility is perfect. This was shown to be so independently of whether the exchange rate is fixed or flexible. One possible outcome of such an "embarrassment of riches" is "confusion" on the part of rational individuals, given that their predictions will not only have to depend on the structure of the model but on what they think the others think, the others think, etc. Moreover, even when the private sector finds a rational way to resolve which one of the multiple solutions to choose, we as economists would be at a loss trying to make our predictions on the basis of the model, unless, of course, there was a systematic way to identify the chosen solution. Since there is as yet no generally accepted theory to recover uniqueness in situations where the model does not provide it, we think that nonuniqueness situations should in principle be linked to potential confusion for the private sector, and assured disconcert for economists.

A major policy implication of this inquiry is that uniqueness will tend to be restored the stronger the expenditure-switching effects of a devaluation, and the tighter the controls on capital mobility. In particular, the former could be enhanced by the elimination of quantitative trade barriers (quotas).

References


19. Taylor (1977) has suggested that in these cases one should choose the solution characterized by having the minimum (asymptotic) variance. His criterion is, however, not useful for us here because all solutions have zero asymptotic variance. Furthermore, unlike the Sargent-Wallace (1973) convergence conditions, Taylor's condition has not yet been justified on microeconomic grounds.


**Comment**

John B. Taylor

An important methodological innovation of Guillermo Calvo's paper is his continuous time formulation of the staggered contracts model. I found the analysis fascinating. The approach should have many applications, not only in the open economy setting emphasized by Calvo, but also in closed economy models. In this respect the general, staggered contracts approach—focusing not so much on contracts per se but on unsynchronized wage or price setting—has a number of advantages as a technique...
for modeling slow adjustment of prices, and these come out in Calvo's continuous time treatment. Among these I would emphasize that (1) there is no long-run trade-off between inflation and unemployment; (2) there is a short-run trade-off even if policy is fully anticipated, so that an announced monetary deceleration can have real effects; and (3) anticipations of the future matter for wage behavior since firms and workers look ahead when setting prices and wages. Calvo's continuous time treatment permits a convenient analytic development of these properties of wage and price dynamics. In discussing the paper, I will concentrate on the specifications of the price and contract equations as well as the non-uniqueness property—a result emphasized in Calvo's discussion.

Some of the analytic convenience that Calvo obtains comes from the exponential distribution assumption used in the price equation: the number of periods a set price will last before termination is random and exponentially distributed. While allowing for random contract length is in principle an improvement over the existing literature, I have several reservations about the stochastic assumptions in this model.

Casual observation suggests that many contracts in the real world are not of uniform length; for example, recent wage concessions in the United States have lead to termination of union contracts before the customary three-year contract period is completed. Clearly it would be an improvement over many existing contract models to allow for variable contract length so that such behavior can be studied. But to model phenomena like union concessions, it is necessary to capture the economic rationale for the early termination by making the contract period endogenous, or depending on endogenous variables. By making contract length random, it might appear that Calvo's approach can capture such phenomena. However, because the randomness is exogenous in the Calvo model, contract length is no more responsive to endogenous events than in the fixed-length contract models. Early termination of contracts arising for endogenous reasons is ruled out by the assumption that the randomness is exogenous.

When we consider the microeconomic problem of a firm or a worker setting wages or prices, this exogenous randomness raises further questions. One of the rationales that has been given for implicit contracts is the ongoing relationship between a worker and a firm, or a firm and its customers. For example, it is a convenience for a customer to know with near certainty the price charged by a firm for its product. Similarly, risk-averse workers prefer a stable wage. Part of the implicit contract is a (limited) guarantee that the price or wage will be relatively stable. In this sense, adding pure randomness to the length of the contract seems counter to the rationale for the implicit contract. If there were to be early termination of a contract, one would expect this to be the result of the occurrence of an endogenous contingency rather than to some purely random factor.
Fortunately, I feel that there is another way to interpret the equations arising from this randomness in Calvo’s equations. Rather than each individual contract having a random length, there could be a distribution of contracts by length across firms due to heterogeneity of markets or products. Some contracts could be short while others could be long, even though no contract would necessarily be random. If over the economy as a whole there is a stable distribution of firms by contract length, then the Calvo methods would give the same results but with a different interpretation.

However, if this interpretation is made, then Calvo’s exponential assumption might not be realistic. In some empirical work I have done to estimate distributions of contracts by length, I found that the distribution is humped-shaped rather than exponential. For example, there are more one-year wage contracts than one-quarter contracts. Hence, from an empirical perspective it would be interesting to consider an extension of the Calvo model to allow for more general distribution. Unfortunately, this would likely be at the cost of some of the analytic simplicity which makes Calvo’s approach so attractive.

From an open economy perspective, another potential difficulty with the Calvo price-setting formulation is that the aggregate price index in the economy is based solely on domestic firm’s pricing decisions. This is seen most clearly in Calvo’s equation (7). In an open economy, the aggregate price index would be influenced by foreign price decisions and exchange rate movements either through the channel of imported intermediate inputs to production, or through escalation clauses. The omission of such influences from the aggregate price equation could be misleading if one were interested in how alternative exchange rate policies influence domestic inflation and unemployment trade-offs.1 Another interesting extension of Calvo’s model would be to consider such influences on price by making the aggregate price in equation (7) a weighted average of domestic firms’ prices and a foreign price index converted into domestic currency units using the exchange rate.

As currently formulated, Calvo’s model potentially has a continuum of solutions arising from the self-fulfilling properties of rational expectations. Calvo shows that this nonuniqueness can arise if the parameters of the model are such that a devaluation causes a decline in aggregate demand. It should be pointed out, however, that this nonuniqueness does not arise because of the existence of contracts in the model. The replacement of Calvo’s contract equations with perfectly flexible prices would

still result in nonuniqueness for similar parameter values. In fact, the reason for the nonuniqueness is best understood in a flexible price version of the model. Substituting the real interest rate \( r = i - \dot{P} \) and the “interest rate parity” condition \( i = \dot{E} \) into the aggregate excess demand for goods, \( f(P - \dot{E}, r) \), results in the aggregate demand equation, \( f(p) = \dot{p} \), where \( p = P - \dot{E} \). If a devaluation causes a decline in aggregate demand, then \( f_{p} > 0 \). Solving the implicit function \( f \) for \( \dot{p} \) results in \( \dot{p} = g(p) \), where \( g' < 0 \) iff \( f_{p} > 0 \). As is now well known in rational expectations modeling, nonuniqueness occurs in a simple first-order differential equation system if any initial “price” \( p \) leads to convergence. Clearly this is the situation if \( g' < 0 \), and it arises whether or not contracts are in the system. Whether \( f_{p} > 0 \) is an econometric issue related to the relative importance of income and substitution effects. As in other examples of nonuniqueness in rational expectations models, the conditions for nonuniqueness arise when the demand or supply curves slope the “wrong way.” In Calvo’s open economy model, the aggregate demand curve must be upward sloping in the relevant price because substitution effects are dominated by wealth or income effects.

Comment Michael Mussa

Guillermo Calvo has presented us with a paper that is both ingenious in its formal analysis and interesting in its substantive economic content. I am especially impressed by Calvo’s modeling of the mechanism of price adjustment in a situation where the prices of individual commodities are fixed by long-term contracts. The differential equation system (equations [9] and [10]) that describes the dynamic behavior of the general price level and the individual commodity price for a newly negotiated contract is simple and intuitively appealing. As Calvo demonstrates, this model of price adjustment is easily applied to interesting issues in open-economy macroeconomics. I believe it will find many other interesting applications.

My concerns with Calvo’s paper arise primarily in connection with his discussion of, and his emphasis upon, situations in which a devaluation of a country’s currency is contractionary with respect to aggregate demand. In Calvo’s model of macroeconomic behavior (as distinct from his model

2. In J. B. Taylor, “Conditions for Unique Solutions in Stochastic Macroeconomic Models with Rational Expectations,” *Econometrica* 45 (September 1977):1377–85; for example, it is shown that the IS—LM curves must cross in an unusual way to get nonuniqueness.

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of price adjustment) this situation arises only when a country is at an "unstable" equilibrium in the Walrasian sense that an increase in the relative price of that country's output in terms of the output of the rest of the world (which is the impact effect of devaluation in Calvo's model) creates an excess supply of that country's output.

My first concern is that operation of an economy in the neighborhood of such an equilibrium is not the only possible or reasonable explanation for observing that devaluations are frequently associated with contractions of national outputs of the devaluing countries. Another possible explanation for such a relationship is that a devaluation has contractionary effects because it reduces the real value of the domestic money supply. In Calvo's model, this effect is not present because it is implicitly assumed that only the prices of domestic goods (which do not respond immediately to a devaluation) are relevant in determining the real value of the domestic money supply. However, in a model that allowed import prices to enter the price index relevant for measuring real money balances, or that allowed some domestic prices to respond essentially immediately to a devaluation, there would be an immediate contractionary effect of devaluation from the reduction in the real value of domestic money balances. In addition, the impact effect of a devaluation in raising the prices of imported inputs and perhaps in raising the costs of some domestic inputs (possibly including wage rates that respond to the actual and anticipated price level effects of a devaluation) would shift the aggregate supply curve upward and have a contractionary effect on national output for any given position of the aggregate demand curve. Further, if a devaluation is combined, as it frequently is, with more restrictive monetary and fiscal policies designed to remove the basic cause for the devaluation and the need for further devaluations, then these more restrictive policies are likely to induce an observed relationship between devaluations and contractions of the outputs of devaluing countries.

My second concern is that if there is an equilibrium that is unstable in the Walrasian sense, it should be bracketed by two equilibria that are stable in the Walrasian sense, and the issue arises of which is the relevant equilibrium for comparative static and dynamic analysis. Existence of an odd number of alternating stable and unstable equilibria is implied by continuity of the excess demand function $f(p, r)$ with respect to $p$, for $r$ equal to the world real interest rate $r^*$, and by the usual boundary conditions that $f(p, r^*)$ must be negative for all sufficiently high values of $p$ and must be positive for all sufficiently low values of $p$.

For the case of three equilibria, under the assumptions of Calvo's model (with a fixed exchange rate), the dynamic system governing the behavior of the general price level, $P_t$, and the price for a newly negotiated contract, $V_t$, must have three steady-state points in the phase space
of the state variables $P_t$ and $V_t$. At the two outer steady-state points, which correspond to the two Walrasian-stable equilibria where $\frac{df}{dp} < 0$, the dynamic system governing $P_t$ and $V_t$ has one positive and one negative characteristic root. There is a unique stable branch of this dynamic system that converges to each of these steady-state points. This is consistent with the normal saddle-point stability property that one expects in a rational expectations model that has both a forward-looking dynamic process and a backward-looking dynamic process. Paths that start outside the region between these two stable branches eventually diverge at an explosive rate from these steady-state points. These paths may be excluded from consideration by assuming that the economically sensible choice for the initial value of the price of a newly negotiated contract should not produce such explosively divergent behavior. The paths that start in the region between the stable branches leading to the two outer steady state points also do not converge to either of these steady state points. This is not peculiar in a model in which we should normally expect saddle-point stability. What is peculiar is that all these paths that start at points between the two stable branches ultimately converge to the middle steady-state point, which is the steady-state point that is associated with the Walrasian-unstable equilibrium.

Calvo refers to this result of a continuum of paths converging to an equilibrium where $\frac{df}{dp} > 0$ as "an embarrassment of riches." I find it simply an embarrassment that a whole continuum of paths should converge to this equilibrium which is unstable in the Walrasian sense, while only a single path converges to each of the equilibria that are stable in the Walrasian sense. In my view the way to avoid this embarrassment is to recognize that none of the paths converging to the Walrasian-unstable equilibrium represents an economically sensible solution for the dynamic system governing $P_t$ and $V_t$.

To develop this point, it is important to note that the economic specification of Calvo's model requires that the solutions for $P_t$ and $V_t$ should have forward-looking components; that is, there should be a part of the solutions for $P_t$ and $V_t$ that depends on a discounted sum of expected future economic conditions relevant for determining the behavior of prices. The need for these forward-looking components arises because $V_t$ is defined to depend on a weighted average of expected future values of $P_s + \beta \cdot f(P_s, r_s)$ and because the domestic interest rate depends on the forward-looking expected inflation rate. In the neighborhood of a Walrasian-stable equilibrium, it is easy to construct the forward-looking components of the solutions for $P_t$ and $V_t$ for the linearized form of the dynamic system governing these two variables. In these solutions, the positive characteristic root of the dynamic system at the steady-state point is used as the discount rate applied to expected future economic conditions in constructing the forward-looking components of these solu-
tions. In the neighborhood of a Walrasian-unstable equilibrium, however, there is no way to construct solutions for the linearized form of the dynamic system governing \( P_t \) and \( V_t \) that have forward-looking components because both of the characteristic roots of the dynamic system at such an equilibrium have negative real parts. In the neighborhood of a Walrasian-unstable equilibrium, it is possible to construct solutions for \( P_t \) and \( V_t \) that are wholly backward-looking, but these wholly backward-looking solutions do not meet the requirements for an economically sensible solution for Calvo's model. Since the continuum of solutions that converge to a Walrasian-unstable equilibrium all ultimately come within a small neighborhood of this equilibrium, it may be concluded that none of these solutions meets the requirements of economic sensibility.

Unfortunately, this conclusion requires that we sacrifice some of the more interesting parts of Calvo's paper that deal explicitly with equilibria where \( \frac{df}{dp} \) is positive and focus instead on equilibria that are stable in the usual Walrasian sense. In my view, this sacrifice is preferable to accepting the proposition that Calvo's ingeniously constructed and intuitively appealing model of price adjustment leads to the obviously unsatisfactory conclusion that a Walrasian-unstable equilibrium is "superstable" in the sense that there is a whole continuum of economically sensible paths of \( P_t \) and \( V_t \) that converge to such an equilibrium and that there is literally no escape from the neighborhood of such an equilibrium. Moreover, even without the discussion of the case where \( \frac{df}{dp} \) is positive, a great deal that is of interest and value remains in Calvo's excellent paper.