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*Part I*  
*Changes in Final Demand,*  
*Industry Product, and Prices*



*Factors Affecting the Postwar Industrial  
Composition of Real Product*

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The period since the end of World War II has been characterized by an ever rising level of real gross national product. The sizable postwar increase in real product has not, however, been shared equally by all industry groups in the economy.<sup>1</sup> While real GNP increased by 84 per cent between 1947 and 1964, for some industry groups the increase was over 250 per cent, and for others less than 10 per cent. (Real product in one industry group actually declined during this period of over-all growth.) More than one-fourth of the industry groups analyzed had increases in real product 50 per cent above the average, and one-third had increases of 50 per cent less than the average. What factors explain the sizable differences among industry groups in the extent to which they shared in the over-all postwar expansion of GNP? This paper utilizes the input-output technique to explain industry differences in the postwar growth of real product in terms of two factors—changes in the level and composition of final demand, and changes in the coefficients, which reflect, among other things, the basic technological processes of producing a given basket of final goods.

<sup>1</sup> See "GNP by Major Industries" by Martin L. Marimont in the *Survey of Current Business*, October 1962, and "Comparison of Federal Reserve and OBE Measures of Real Manufacturing Output, 1947-64" by Jack J. Gottsegen and Richard C. Ziemer in this volume.

NOTE: The term real product is used in this paper as a shortcut for the more accurate label, gross national product originating in an industry expressed in constant (1958) prices. The term net product or value added is used to refer to gross national product originating in an industry in current dollars.

*Industry Indexes of Real Product and Direct Sales  
to Final Demand*

The wide dispersion by industry in the postwar movement of real product is evident from Table 1, which presents indexes of real product (1958 = 100) for three points in time (1947, 1958, and 1964) for forty-two industrial groups in the economy.<sup>2</sup> (In general, this industrial grouping corresponds to the two-digit level of the standard industrial classification. In some cases, however, it was necessary to combine two-digit SIC groups, because the classification level of the input-output table was more aggregative than the two-digit level.)

It is interesting to examine the extent to which industry differences in the postwar growth of real product are related to differences among industries in the degree to which they participated in the over-all growth of final demand between 1947 and 1958, and 1958 and 1964. For example, if consumer, government, investment, and foreign demand for the products of a given industry increased at a much slower rate than total final demand, one would expect the increase in production, and consequently value added and real product in this industry, to be slower than average, and vice versa. Indeed this is the case in the food products industry and other industries which sell a large proportion of their total output directly to final users. However, many industry groups sell only a small proportion of their total output directly to final users; for such industries it is unlikely that changes in real product would be highly correlated with changes in direct sales to final users.

For the purpose of systematically examining the relationship between changes in real product and changes in direct final demand, indexes of final demand (in constant 1958 prices) were computed for each of the forty-two industry groups in the economy for the two periods 1947-58 and 1958-64. The individual industry indexes for real product

<sup>2</sup> For the period 1947-58 the industrial distribution of real product upon which this table is based is that derived from input-output data, and as such differs somewhat from that which emerges from OBE's work in real product. In general, these differences are due to various definitional differences in coverage of an industry. For example, in the input-output table, the construction industry is defined to cover all construction activity wherever performed, including both private and public force account activity; in the national accounts the construction industry covers only contract construction. There are numerous other differences of this sort, but it is not essential for the purpose of this paper to catalogue them.

TABLE 1

Indexes of Real Product by Industry Group 1947, 1958, 1964  
(1958=100)

SIC No. <sup>a</sup>	Title	1947 <sup>b</sup>	1964 <sup>c</sup>
All Industries		70.4	129.7
01, 02	Farms	80.1	105.8
07-9	Ag. services, forestry, and fisheries	92.7	106.4
10	Metal mining	91.0	123.2
11, 12	Coal mining	175.9	114.7
13	Crude petroleum and natural gas	83.2	115.0
14	Nonmetallic minerals mining	58.6	123.1
15-17	Construction <sup>d</sup>	59.9	109.8
20	Food and kindred products	82.9	117.1
21	Tobacco manufacturers	71.8	119.0
22	Textile mill products	96.9	130.1
23	Apparel and related products	76.0	128.1
24	Lumber and wood products	121.8	128.9
25	Furniture and fixtures	72.5	128.7
26	Paper and allied products	69.7	140.0
27	Printing and publishing	77.8	125.7
28	Chemicals and allied products	42.6	152.0
29	Petroleum refining and related industries	45.1	144.3
30	Rubber and misc. plastics products	83.2	163.4
31	Leather and leather products	102.3	108.9
32	Stone, clay, and glass products	65.8	131.2
33	Primary metal industries	113.7	130.2
34	Fabricated metal products	74.1	135.2
35	Machinery, except electrical	87.9	153.3
36	Electrical machinery	55.5	167.3
37, 19	Transportation equip. and ordnance	56.0	163.6
38	Instruments and related products	55.2	139.6
39	Miscellaneous manufacturing	81.0	119.3
40-47	Transportation	108.7	127.7
48	Communication	43.4	149.4
49	Electric, gas, and sanitary services	41.0	143.9
50-59	Wholesale and retail trade	69.1	131.6
60-64, 66, 67	Finance and insurance	76.4	119.0
65	Real estate	53.5	136.2
70, 72, 76	Hotels, personal, and repair services	85.4	119.7

(continued)

TABLE 1 (concluded)

SIC No. <sup>a</sup>	Title	1947 <sup>b</sup>	1964 <sup>c</sup>
73, 80-89 (except 88)	Business, medical, etc., services	56.5	133.2
75	Auto repair, etc.	92.4	160.6
78, 79	Amusements	104.2	121.4
--	Federal government enterprises	54.8	145.1
--	State and local government enterprises	77.9	142.9
--	Government industry	68.9	116.8
--	Rest of world	52.7	193.8
88	Household	90.1	118.6

<sup>a</sup>For more complete industry titles and the 1958 Input-Output (I.O.) industry composition of each, see Appendix Table A.

<sup>b</sup>The real-product indexes for 1947 are based on value data from the reworked 1947 input-output table and the 1958 input-output table. The 1947 value-added data were converted to 1958 prices by the use of value-added deflators developed in connection with OBE's work on real product.

<sup>c</sup>The indexes of real product for 1964 are not strictly comparable to those for 1947 since they were derived directly from OBE's real-product data without any adjustments for differences between input-output and real product in industry definitions. These 1964 indexes are preliminary.

<sup>d</sup>The 1947 index includes public and private force account construction as well as contract construction activities. The index for 1964 reflects contract construction activity only.

(from Table 1) and for final demand were then ranked from the lowest to the highest, and a coefficient of rank correlation computed.<sup>3</sup> The coefficient of rank correlation was .705 for the 1947 indexes, and .601 for the 1964 indexes. Thus, even such a crude measure as *rank* correlation does not indicate a marked degree of association between changes in an industry's direct sales to final users and changes in its real product.

It is thus evident that the explanation of industry differences in the movement of real product cannot rest solely upon an examination of changes in an industry's *direct sales* to final users. One must also

<sup>3</sup>Three industries, rest of the world, household, and government—all industries in which value added and final demand are equal by definition—were omitted from the calculation of rank correlation.

examine an industry's *indirect* sales to final users: sales via the utilization of its products in the production of other goods and services which then go directly or indirectly into the satisfaction of final demands. Such an analysis must utilize the input-output technique, for this technique enables one to measure the impact on a given industry's gross output and net output (or value added) not only of changes in the final demand for the products of that industry but of changes in final demand for the output of all other industries as well. Thus, for example, by using an input-output matrix, one can measure the impact on output and value added in the steel industry not only of changes in final demand for steel itself (primarily net export demand) but also of changes in the final demand for motor vehicles, machinery, and all other products which directly or indirectly utilize steel in their production process.<sup>4</sup>

### *Methodology for Factoring Causes of Change in Industry Real Product*

To better understand the methodology used in this paper to factor the causes of change in industry real product, it is helpful to view the industrial distribution of gross national product in any given year as the direct result of a combination of two basic sets of relationships. One set is the level and pattern of industry final demands that prevailed during

<sup>4</sup> Of necessity, it is assumed that the reader of this paper is fairly familiar with input-output analysis and no attempt is made to describe in detail the nature of this technique. Should the reader wish to familiarize himself further, it may interest him to consult the following sources:

1. Wassily W. Leontief, *The Structure of the American Economy, 1919-39*, Oxford, 1951, 2nd ed.
2. Duane Evans and Marvin Hoffenberg, "The Interindustry Relations Study for 1947," *Review of Economics and Statistics*, May 1952.
3. *Input-Output Analysis: An Appraisal*, Studies in Income and Wealth, 18, Princeton University Press for The National Bureau of Economic Research, 1955.
4. Chenery and Clark, *Interindustry Economics*, New York, 1959.

The reader may also wish to consult the three comprehensive input-output bibliographies listed below:

1. V. Riley and R. L. Allen, *Bibliography of Interindustry Economic Studies*, Operations Research Office, Johns Hopkins University, March 1955.
2. Charlotte Taskier, *Input-Output Bibliography, 1955-60*, Harvard Economic Research Project, United Nations, New York 1961.
3. United Nations, *Input-Output Bibliography, 1960-63*, Selected Papers Series M, No. 39, New York, 1964.



the year, and the other is the prevailing technical relationships between individual industry final demands and the gross output and net output (or value added) of each industry. Indeed, the 1958 input-output table prepared by the Office of Business Economics describes the economy in just these terms.<sup>5</sup>

If, in any given year, either the set of industry final demands or the set of technical relationships between final demand and gross and net output had differed from what they actually were, a different set of industry net outputs would have resulted. Thus, for example, if one multiplied a set of 1964 industry final demands (expressed in 1958 prices) by OBE's 1958 input-output inverse matrix, he would derive industry estimates of 1964 gross and net output (in 1958 prices) if 1958, rather than 1964, technical relationships had prevailed. If these *derived* 1964 real-product estimates are then compared with actual real product in 1958, one derives a measure of the change between 1958 and 1964 in real product by industry which is attributable solely to changes in final demand. Further, if these derived estimates of real product in 1964 are compared to actual 1964 real-product estimates, one derives a measure of the amount of change in real product between 1964 and 1958 which is due solely to changes in technical coefficients. (These two sources of change, of course, exhaust the total 1958-64 difference in an industry's real product.)<sup>6</sup>

Likewise, if one multiplied a set of 1947 final demands by industry (in 1958 prices) by the 1958 input-output inverse, he would derive estimates of 1947 real product if 1958 rather than 1947 technical relationships had prevailed. By comparing these derived estimates of 1947 real product to actual real product in 1958, one could measure the amount of change in industry real product which is attributable solely to changing final demand. Similarly, by comparing these derived estimates of real product by industry with actual 1947 real product

<sup>5</sup> See "The Interindustry Structure of the United States," by Morris R. Goldman, Martin L. Marimont, and Beatrice N. Vaccara in the *Survey of Current Business*, November 1964; also "The Transactions Table of the 1958 Input-Output Study," *Survey of Current Business*, September 1965.

<sup>6</sup> The two sources of change exhaust the total change in real product only under the particular procedure just described. This procedure in effect employs 1964 weights for the measurement of the technological change factor and 1958 weights for the measurement of the final-demand factor. A procedure which employed the same set of weights for each factor would leave a residual or "interaction" factor. See Appendix Table E. For further discussion of this point see references cited in footnote 7.

by industry, one could derive a measure of the 1947–58 change in real product which is due to changing technical relationships. Thus by the procedures described it is possible to divide the postwar changes in real product, for two periods 1947–58 and 1958–64, into two components, that which is due to changes in technical coefficients, and that due to changes in the level and structure of final demand.

It is also theoretically possible, however, to factor the causes of postwar change in an industry's real product by alternative methods. For example, the change between 1947 and 1958 in real product by industry can be separated into its two components by a procedure which involves multiplying the 1958 final demands by fixed 1947 technical relationships. This process would yield industry estimates of real product in 1958 if 1947 technical relationships prevailed. When these derived estimates of 1958 real product are compared to actual 1958 measures of real product by industry, one would get another estimate of the amount of change in real product between 1947 and 1958 which is attributable to changing technical relationships. Likewise, if these derived estimates of 1958 real product by industry are compared to actual 1947 industry data on real product, one derives an alternative measure of the amount of change in real product between 1947 and 1958 that is attributable to changing final demand. These measures would, in all likelihood, not be the same as those which were derived by the use of fixed 1958 technical coefficients.<sup>7</sup> (Similarly, one could explain the 1958–64 changes in real product by industry by a procedure which involved assuming fixed 1964 rather than 1958 technical relationships for both the 1964 and 1958 final demand.)

It thus becomes obvious that for each subperiod there is no single correct method of measuring the relative importance of the two factors—final demand and technical coefficients—which explain the postwar changes in real product. The best measure of the relative im-

<sup>7</sup> This phenomenon is attributable to the "interaction" factor and occurs whenever one attempts to factor out "causes of change" when alternative weighting schemes are available. For further discussion of this point see comments by Edward F. Denison on the paper by Frank A. Hanna, "Analysis of Interstate Income Differentials: Theory and Practice" in *Regional Income*, Studies in Income and Wealth 21; Princeton for NBER, 1957; *Concepts and Measurement of Production and Productivity* by Irving H. Siegel, pp. 86–92 (a working paper of the BLS); and the Technical Appendix to "Corporate Profits Since World War II" by Harlow D. Osborne and Joseph B. Epstein in the *Survey of Current Business*, January 1956.

portance of the two factors would be an average of the answers that result from the two alternative procedures available for each subperiod, i.e., assuming the fixed technical coefficients of the terminal year and the fixed technical coefficients of the initial year.

In actual practice, however, we were not faced with the immediate possibility of applying the two alternative procedures for the two subperiods 1947–58 and 1958–64. There is, as yet, no input–output inverse matrix for 1964 (the Office of Business Economics is currently working on an input–output table for the year 1963), and the 1947 input–output matrix constructed by the BLS was not conceptually or statistically consistent with the 1958 input–output table or the national accounts estimates of the total and component categories of final demand.

Some progress had been made, however, towards adjusting the 1947 input–output table to the 1958 basis. The Harvard Economic Research Project, under contract to the Interagency Growth Project, had already performed the extremely difficult task of reconciling the 1945 Standard Industrial Classification with the completely revised 1957 Standard Industrial Classification and had collapsed the 1947 table to the more aggregative basis of the 1958 table. It had also converted the 1947 table to a 1958 valuation basis. In addition, considerable work had been done at OBE to make the 1947 estimates of personal consumption expenditures by industry consistent with the 1958 table.<sup>8</sup> It was decided to build upon this work and to develop a 1947 input–output table as far as possible conceptually consistent with the 1958 table and the national income and product accounts.<sup>9</sup> In this way, we would have the necessary working tool for applying two alternative methods of factoring out causes of change in industry real product between 1947 and 1958.

The present unavailability of an input–output table for the mid-1960's meant that for the period 1958–64 only one of the two alternative methods could be applied. Since results for the period 1947–58 indicated that there was a marked difference in the measures of the

<sup>8</sup> See "Personal Consumption Expenditures in the 1958 Input–Output Study" by Nancy W. Simon in the October 1965 issue of the *Survey of Current Business*.

<sup>9</sup> This paper does not examine any of the many complicated statistical and conceptual problems which arose in the reworking of 1947 input–output table to make it conceptually consistent with the 1958 table. It is hoped that at some later date the authors can write a paper explaining these procedures.

relative importance of the two factors, depending upon whether one assumed the fixed technical coefficients of the initial or the terminal year, it was believed that the use of a single method for the period 1958–64 would not yield a true measure of the relative importance of the two factors. It was therefore decided to confine this paper to an analysis of the factors affecting real product in the postwar period 1947–58.

The work plan thus calls for an averaging of the two methods of factoring the total change in industry real product between 1947 and 1958—one method involving the application of a 1958 set of final demands to a 1947 input–output inverse matrix, and the other the application of 1947 final demands to a 1958 input–output inverse matrix.<sup>10</sup> The output totals which result from these matrix multiplications will then be converted to estimates of value added by the application of industry value-added/output ratios which are consistent with the given year's input–output table. Where necessary these value-added estimates will be converted to real-product estimates by the use of value-added price indexes (1958 = 100).<sup>11</sup>

Let us now turn to an analysis of the results obtained by applying the methodology just described.

### *Influence of Changes in Final Demand on Real Product by Industry*

During the period 1947–58, total final demand in constant prices (and its counterpart, total real product) increased by 42 per cent. As indicated earlier, this rising level of final demand did not affect all industry groups in the economy to the same extent. For some industry groups, the direct and indirect effect of the over-all increase in final demand was a slight lowering in the level of real product, while for other

<sup>10</sup> See Appendix Tables C and D for the separate results of each of these two methods. Appendix Table E combines the results obtained from both these procedures and presents alternative measures of each of the two factors which do not exhaust the total change in real product and thus permit the measurement of the "interaction" effect as a separate factor.

<sup>11</sup> The reworked 1947 table has not yet been repriced in 1958 dollars. The methodology of this paper required only the deflation of final demand (1947 and 1958, each in the other year's prices) and 1947 value added. The actual step-by-step procedures for deriving the alternative value-added estimates, together with the necessary data on value-added/output ratios, final-demand deflators, and implicit value-added deflators are presented in Appendix Tables A and B.

industry groups the direct and indirect effect was an increase of more than double the over-all percentage. These facts become clear from an examination of Table 2, which shows the total change in real product (1947-58) for each of forty-two industry groups in the economy and distributes this total change between its two components, final-demand change and change in technical coefficients. Column 6 of this table shows the actual 1958 industry indexes of real product, while column 8 indicates what the 1958 indexes of real product would have been had there been no change in the technical coefficients but merely a change in final demand.

An examination of column 8 shows a wide range in the indexes for individual industry groups—from a low of 90.8 in the amusements group to a high of 206.7 in the electricity and gas group. This range of almost 116 index points is, however, considerably smaller than the range of 187 points in the total indexes of real product shown in column 6. Thus, if only final demand had changed between 1947 and 1958 (while the technical relationships had remained constant), there would have been a considerable narrowing of the extent to which individual industry indexes of real product differed from the average index. For example, the index of real product for the coal mining industry would have been 98.0 rather than the actual index of 56.8 while, at the other extreme, the index for electricity and gas would have been 206.7 instead of 244.2. This narrowing of the difference between individual industry indexes of real product and the average index, if one considers indexes based only on final demand change rather than indexes based on total change, is almost universal and not just the case for the extremes of the array. In thirty-two cases, the index in column 8 is closer to the average index of 142 than is its counterpart index in column 6, and in only seven cases is it further from the average than its counterpart. (For three industries, those where gross product and final demand are equal by definition, the indexes in column 6 and column 8 are identical.) The differences between the various individual industry indexes of real product and the over-all index are shown in Table 3. For the total indexes of real product the differences between individual industry indexes and the over-all index average 34.3 points, for the indexes of real product which assume that only final demand changed, these differences average 20.8 points.

The fact that the range and average variation of the individual in-

TABLE 2

*Change in Real Product by Industry Group, 1947 to 1958, Distributed by Cause of Change*  
(millions of 1958 dollars)

SIC No. <sup>a</sup>	Title	Change in Real Product, 1947-1958					Index of Change in Real Product (1947=100)		
		Real Product		Total	Due to Coeffi- cient Changes	Final- Demand Changes	Total	Coeffi- cient	Final Demand
		1947 <sup>b</sup>	1958 <sup>c</sup>						
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)		
01, 02 07-9	Farms Ag. services, forestry, and fisheries	16,698	20,846	4,148	-1,461	5,609	124.8	91.2	133.6
10	Metal mining	1,172	1,264	92	-240	332	107.8	79.5	128.3
11, 12	Coal mining	832	914	82	-100	182	109.9	88.0	121.9
13	Crude petroleum and natural gas	2,822	1,604	-1,218	-1,161	-57	56.8	58.8	98.0
14	Nonmetallic minerals mining	5,548	6,671	1,123	-1,412	2,535	120.2	74.5	145.7
15-17	Construction <sup>d</sup>	719	1,226	507	80	427	170.5	111.1	159.4
20	Food and kindred products	17,322	28,937	11,615	-226	11,841	167.1	98.7	168.4
21	Tobacco manufacturers	13,734	16,630	2,896	-880	3,776	121.1	93.6	127.5
22	Textile mill products	2,050	2,854	804	333	471	139.2	116.2	123.0
		3,285	3,389	104	2	102	103.2	100.1	103.1

(continued)

TABLE 2 (continued)

SIC No. <sup>a</sup>	Title	Change in Real Product, 1947-1958			Index of Change in Real Product (1947=100)				
		Real Product		Total Changes	Due to Coeffi- cient Changes	Due to Final- Demand Changes	Coeffi- cient		
		1947 <sup>b</sup>	1958 <sup>c</sup>				Col. 2 ÷ 1	Col. 3 ÷ 1	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)		
23	Apparel & related products	4,593	6,042	1,449	-113	1,562	131.5	97.5	134.0
24	Lumber and wood products	3,509	2,880	-629	-1,398	769	82.1	60.2	121.9
25	Furniture and fixtures	1,481	2,042	561	15	546	137.9	101.0	136.9
26	Paper and allied products	3,501	5,020	1,519	38	1,481	143.4	101.0	142.3
27	Printing and publishing	4,646	5,974	1,328	-167	1,495	128.6	96.4	132.2
28	Chemicals and allied products	4,178	9,811	5,633	2,736	2,897	234.8	165.5	169.3
29	Petroleum refining and related industries	1,628	3,608	1,980	767	1,213	221.6	147.1	174.5
30	Rubber and misc. plastics products	2,605	3,131	526	-31	557	120.2	98.8	121.4
31	Leather and leather products	1,693	1,655	-38	-42	4	97.8	97.5	100.2
32	Stone, clay, and glass products	3,225	4,900	1,675	424	1,251	151.9	113.1	138.8
33	Primary metal industries	11,951	10,510	-1,441	-3,892	2,451	87.9	67.4	120.5

(continued)

TABLE 2 (continued)

SIC No. <sup>a</sup>	Title	Change in Real Product, 1947-1958				Index of Change in Real Product (1947=100)			
		Real Product		Total	Due to Coefficient Changes	Due to Final-Demand Changes	Total	Coefficient	Final Demand
		1947 <sup>b</sup>	1958 <sup>c</sup>						
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)		
34	Fabricated metal products	6,099	8,231	2,132	154	1,987	135.0	102.5	132.4
35	Machinery, except electrical	9,767	11,112	1,345	78	1,267	113.8	100.8	113.0
36	Electrical machinery	5,327	9,602	4,275	1,760	2,515	180.3	133.0	147.2
37, 19	Transportation equip. and ordnance	8,883	15,868	6,985	110	6,875	178.6	101.2	177.4
38	Instruments and related products	1,372	2,484	1,112	353	759	181.0	125.7	155.3
39	Miscellaneous manufacturing	1,805	2,229	424	-32	456	123.5	98.2	125.3
40-47	Transportation	22,386	20,600	-1,786	-4,141	2,355	92.0	81.5	110.5
48	Communication	3,821	8,800	4,979	2,016	2,963	230.3	152.8	177.5
49	Electric, gas, and sanitary services	4,060	9,914	5,854	1,522	4,332	244.2	137.5	206.7
50-59	Wholesale and retail trade	47,658	69,006	21,348	2,531	18,817	144.8	105.3	139.5
60-64, 66, 67	Finance and insurance	11,338	14,831	3,493	-1,728	5,221	130.8	84.8	146.0
65	Real estate	23,998	44,821	20,823	4,287	16,536	186.8	117.9	168.9
70, 72, 76	Hotels, personal, and repair services	6,321	7,402	1,081	-568	1,649	117.1	91.0	126.1

(continued)



TABLE 2 (concluded)

SIC No. <sup>a</sup>	Title	Change in Real Product, 1947-1958					Index of Change in Real Product (1947=100)		
		Real Product		Total Changes (3)	Due to Coeffi- cient Changes (4)	Due to Final- Demand Changes (5)	Total Col. 2 ÷ 1 (6)	Coeffi- cient ÷ 1 (7)	Final Demand ÷ 1 (8)
		1947 <sup>b</sup>	1958 <sup>c</sup>						
73, 80-89 (except 88)	Business, medical, etc. services	15,288	27,082	11,794	1,814	9,980	177.1	111.9	165.3
75	Auto repair, etc.	3,519	3,808	289	-1,274	1,563	108.2	63.8	144.4
78, 79	Amusements	3,116	2,989	-127	158	-285	95.9	105.1	90.8
--	Federal government enterprises	980	1,788	808	238	570	182.4	124.3	158.2
--	State and local govern- ment enterprises	2,028	2,604	576	-550	1,126	128.4	72.9	155.5
--	Government industry	26,889	39,029	12,140	0	12,140	145.1	100.0	145.1
--	Rest of world	1,069	2,030	961	0	961	189.9	100.0	189.9
88	Household IVA <sup>e</sup>	3,156	3,503	347	0	347	111.0	100.0	111.0
		-944	-311	633	0	633	n.a.	n.a.	n.a.
	All industries	315,128 <sup>f</sup>	447,330	132,202	0	132,202	142.0	100.0	142.0

## Notes to Table 2

n.a. not applicable.

<sup>a</sup>For more complete industry titles and the 1958 Input-Output (I.O.) industry composition of each, see Appendix Table A.

<sup>b</sup>The real-product estimates shown in this column are based on value-added data from the reworked 1947 input-output table. These value-added figures were converted to 1958 prices by the use, with some slight modifications, of the two-digit value-added deflators developed in connection with OBE's work on real product.

<sup>c</sup>These data are derived from the published 1958 input-output flow table. See *Survey of Current Business*, September 1965.

<sup>d</sup>In addition to contract construction activity this industry group includes public and private force account construction.

<sup>e</sup>This is not an actual industry but rather a balancing entry required by the fact that in the 1958 input-output table the inventory valuation adjustment was made in total only—not industry by industry as in the national income accounts. In computing inventory change for the original 1947 input-output table a mixture of procedures was employed. For some industries, notably trade, inventory change was computed on a revalued basis, for others, inventory change was computed before revaluation. The information which was required to redo the 1947 inventory change data on a basis consistent with the 1958 input-output table was not available. Thus, the 1947 entry for the inventory valuation adjustment is not the total of this adjustment in the national accounts but only that portion of the total which was not taken account of in the original 1947 procedure for calculating inventory change by industry.

<sup>f</sup>The grand total for real product shown in this table is slightly different from the published 1947 GNP in constant prices of \$314.4 billion. In reworking the 1947 BLS input-output table it was not possible to agree precisely with the published figures on GNP because minor statistical revisions in the basic data were not taken account of in the reworking of the 1947 table.

dustry indexes of real product is considerably narrowed when one constructs indexes which reflect solely changes in final demand indicates that during the period 1947 and 1958 the influence of changing technical coefficients was to increase the variability of the actual industry indexes of real product. This would thus imply that, in general, those industries which had above average increases in final demand (direct and indirect) were also faced with increasing demand for their product because of changing technical requirements, while those industries

**TABLE 3**  
*Divergence of Industry Indexes of Real Product From Average  
 Index for All Industries*  
 (1947=100)

SIC No.	Index of Real Product <sup>a</sup>		
	Total	Final-Demand Change Only	Coefficient Change Only
01, 02	-17.2	-8.4	-8.8
07-9	-34.2	-13.7	-20.5
10	-32.1	-20.1	-12.0
11, 12	-85.2	-44.0	-41.2
13	-21.8	3.7	-25.5
14	28.5	17.4	11.1
15-17	25.1	26.4	-1.3
20	-20.9	-14.5	-6.4
21	-2.8	-19.0	16.2
22	-38.8	-38.9	.1
23	-10.5	-8.0	-2.5
24	-59.9	-20.1	-39.8
25	-4.1	-5.1	1.0
26	1.4	.3	1.1
27	-13.4	-9.8	-3.6
28	92.8	27.3	65.5
29	79.6	32.5	47.1
30	-21.8	-20.6	-1.2
31	-44.2	-41.8	-2.5
32	9.9	-3.2	13.1
33	-54.1	-21.5	-32.6
34	-7.0	-9.6	2.5
35	-28.2	-29.0	.8
36	38.3	5.2	33.0
37, 19	36.6	35.4	1.2
38	39.0	13.3	25.7
39	-18.5	-16.7	-1.8
40-47	-50.0	-31.5	-18.5
48	88.3	35.5	52.8
49	102.2	64.7	37.5
50-59	2.8	-2.5	5.3
60-64, 66, 67	-11.2	4.0	-15.2
65	44.8	26.9	17.9
70, 72, 76	-24.9	-15.9	9.0
73, 80-89 (except 88)	35.1	23.3	11.9
75	-33.8	2.4	-36.2
78, 79	-46.1	-51.2	5.1
Fed. gov't enterprises	40.4	16.2	24.3
State and local gov't enterprises	-13.6	13.5	-27.1
Gov't industry	3.1	3.1	0
Rest of world	47.9	47.9	0
Household	-31.0	-31.0	0
All industries average <sup>b</sup>	34.3	20.8	16.2

<sup>a</sup>Based on indexes shown in columns 6-8 of Table 2.

<sup>b</sup>Signs ignored in computation of this average.

which experienced below average increases, or actual decreases, in final demand were also industries whose output was in lessened demand because of changes in the technical requirements of production. (These facts will become clear in a later section in which we examine the inter-relationship between changing technical requirements and changing final demand.)

### *Influence of Changing Technical Requirements on Real Product by Industry*

The amount of change in each industry's real product (1947-58) which is attributable solely to changes in the technical relationships reflected in the input-output matrices for the two years 1947 and 1958 is shown in column 4 of Table 2. Within the fixed column total of zero,<sup>12</sup> there is a wide range in the individual entries from large positives to large negatives, with the number of cases of negative values about equal to the number of positive cases. Perhaps a more interesting way to analyze the impact of changing technical relationships is to examine the indexes in column 7 of Table 2. The entries in this column show what the 1958 industry indexes of real product (1947 = 100) would have been had there been no change in final demand during the period but only a change in the technical relationships reflected in the input-output tables. An index of 100 indicates that there was a neutral effect of technical change, an index of under 100 indicates that there was a decline between 1947 and 1958 in the output and real product of a given industry that would be required in order to produce a given basket of final goods, and an index of over 100 indicates an increase between 1947 and 1958 in the output requirements from a given industry to produce a fixed basket of final goods.

Before we proceed with a detailed examination of the changing technical relationships, it should be pointed out that the changes in technical requirements reflected in columns 4 and 7 cannot be equated with *technological change*. Changing technology is only one of the many possible causes of change between two periods in the technical coefficients reflected in an input-output table. One of the important

<sup>12</sup> Since the total final demand is always equal to the total value added, no matter what the technical relationships, changing technical relationships can only affect the industry distribution of real product, not its total.

factors that could cause a difference between any two periods in the technical coefficients for a given industry is changing product mix. In such a highly aggregative picture of the U.S. economy as is reflected in the eighty-five-order classification system of the 1958 input-output table, an individual industry cannot represent a single or even a homogeneous set of commodities. Shifts between 1947 and 1958 in the product composition of an individual industry could thus affect the 1947-58 comparison of the technical relationship for that industry. Over a time span as long as eleven years, changing product mix could be an important cause of change in the technical relationships.

Another factor which could contribute to changes in technical relationships between two time periods is the divergence of actual technical relationships from a linear-homogeneous function. For some inputs, particularly those which reflect overhead-type costs, there undoubtedly is not a strictly proportional relationship between changes in outputs and changes in inputs. Thus the 1947 coefficients for a given industry might differ from those of 1958 merely because the scale of operation or degree of capacity utilization was much greater in 1958 than 1947 (or vice versa).

The problem of nonproportionality of some, if not most, inputs could be particularly important in the present analysis since we are comparing technical relationships in two years which reflect different phases of the business cycle. The year 1947 was generally one of over-all expansion of the business economy while the year 1958 was a year of a business cycle trough. Hence, for some industries, the two time periods would reflect markedly different degrees of capacity utilization—a phenomenon which could seriously influence the comparison of technical coefficients in the two years.

Finally, it should be pointed out that some of the difference in technical coefficients may reflect random factors such as differences between the two input-output tables in the statistical methods of estimating the technical relationships, as well as some errors that may have cropped up in our work in revising the 1947 BLS table to make it conceptually consistent with the 1958 table. Thus we note several factors which could contribute to changes between 1947 and 1958 in the technical coefficients which are not related to changes in the technological requirements for producing a fixed basket of final goods.

It may be noted from Table 2 that the greatest positive impact of

coefficient change occurred in the chemicals and allied products industry (SIC 28) where the increase in real product between 1947 and 1958 (based solely on coefficient change) was 65.5 per cent. A strong positive influence of changing technical relationships was also evident in the communications industry (SIC 48) and in the petroleum refining industry (SIC 29) with indexes of real product of 152.8 and 147.1 respectively. The largest negative impact on real product occurs in the coal mining industry (SIC 11 and 12) where there was a 41 per cent decline in real product between 1947 and 1958 due solely to changes in the technical coefficients. Large negative impacts of changes in technical coefficients also occur in the lumber and wood products (SIC 24) and primary metal (SIC 33) industries where there are reductions in real product of 40 and 33 per cent, respectively.

It may be noted (from Table 3) that the average industry impact on real product of changes between 1947 and 1958 in the technical coefficients was 16.2 per cent (ignoring sign), a somewhat smaller average impact than that of changes in final demand. Moreover, in only five industries did the absolute magnitude of the impact on real product of changes in technical coefficients exceed that of changes in final demand. In thirty-four industry groups the change in real product attributable to changes in final demand exceeded that which was attributable to changes in technical coefficients. (In three industries, by definition, the entire change in real product is attributable solely to changes in final demand.) Thus for the period 1947-58 the changing level and pattern of final demand was a somewhat more important factor than changes in technical coefficients in explaining industry changes in real product.

#### *Interrelationship Between Changes in Final Demand and Technical Coefficients*

It may be noted from the tables presented that in most cases the individual industry indexes of real product which reflect changes in *both* final demand and technical coefficients vary from the average index to a considerably greater extent than do either of the corresponding indexes which consider solely changes in final demand or technical coefficients. There is thus an indication that, in general, the two elements of change in an industry's real product reinforced rather than

offset one another. This is especially true of the extremes in the array of indexes of total change in real product. The industries with the largest increase in final demand are generally those with the largest increase in technical coefficients and correspondingly the industries with the smallest increase in final demand are generally those which also show a negative influence of coefficient change.

In an attempt to systematically study the interrelationships between the various industry indexes of real product—total real product, final demand, and technical coefficients—the individual industry indexes of total real product have been cross-classified by their corresponding indexes for the two separate factors. These relationships are displayed in Table 4 in the form of a frequency distribution where the various individual industry indexes of total real product have been combined into five groups. Each of these groups is then distributed according to whether the indexes of real product based solely on final demand or technical coefficient were average, above, or below average. A final-demand index of real product was considered below average if it was below 128.0 and above average if it exceeded 155.9. An index of real product for technical coefficients was classed as below average if it was under 90.0, and above average if it exceeded 110.0.

It may be noted from Table 4 that all six industries which had the lowest indexes of total real product were in the group with below average final-demand indexes, and that four of these six had below average technical coefficient indexes as well. Of the twelve industries which had total-real-product indexes in the below average range, none were included in the group with above average final-demand indexes, and eight were in the group with below average final-demand indexes. Furthermore, all of these twelve industries had average or below average technical coefficient indexes. Within the group of eleven industries with average indexes of total real product, seven had average indexes for *both* final-demand and technical coefficients. There was only one industry in the average group (SIC 21—tobacco manufactures) which had offsetting indexes, below average final demand and above average technical coefficients. All but two of the seven industries in the subgroup with above average indexes of total real product were industries with above average indexes of final demand and none of these industries had below average indexes of technical coefficients. Five of the six industries with the highest indexes of total real product fell into

TABLE 4

*Frequency Distribution of 1958 Industry Indexes of Total Real Product  
Cross-Classified by Separate Factor Indexes<sup>a</sup>  
(1947=100)*

Total Index of Real Product	No.	Final-Demand Index – Above Average		Final-Demand Index – Average		Final-Demand Index – Below Average		
		Technical Coefficients		Technical Coefficients		Technical Coefficients		
		Above	Below	Above	Below	Above	Below	
Lowest (Under 100.0)	6	--	--	--	--	--	2	4
Below average (100.0 – 127.9)	12	--	--	--	1	3	--	7
Average (128.0 – 155.9)	11	--	--	1	7	2	1	--
Above average (156.0 – 183.9)	7	3	2	--	2	--	--	--
Highest (184.0 and above)	6	5	1	--	--	--	--	--
Total	42	8	3	0	3	8	5	9

<sup>a</sup>The index of real product based solely on technical coefficient change was considered *above average* if it exceeded 111.0, *average* if it was between 90.0 and 110.0, inclusive, and *below average* if it was less than 90.0. The index of real product based solely on final demand change was considered *above average* if it exceeded 155.9, *average* if it was between 128.0 and 155.9 inclusive, and *below average* if it was less than 128.0.



the category of above average indexes for both final demand and technical coefficients. Thus, a fairly regular pattern of relationships between the component indexes of an industry's real product and its total index seems to emerge.

The fact that the two factors, technical coefficient change and final demand change, tended to reinforce one another makes it seem likely that there are some basic underlying trends which influenced both these movements in the same direction. For example, in the chemicals industry (SIC 28) the above average real-product indexes for both final-demand and technical coefficients may be regarded as a reflection of the growing importance of chemicals in all aspects of our modern day life. Similarly, the below average indexes for technical coefficients and final demand in the coal industry are indications of the declining importance of coal as both an industrial and residential fuel. One would logically expect such strong tendencies to continue beyond 1958.

If these observed trends were to continue, one would expect a fair degree of success in predicting the 1964 pattern of individual industry indexes of real product based on the pattern evidenced during the 1947-58 period. Thus we would expect the six industries that had above average real-product indexes in the 1947-58 period to be the industries with above average indexes of total real product in the 1958-64 period. This should be particularly true for the five industries in this group which had above average indexes for *both* final demand and technical coefficients. Correspondingly, we would expect the four industries with 1958 indexes below average, both for final demand and for technical coefficients, to be in the group with below average 1964 indexes of total real product.

#### *Relationship Between 1958 and 1964 Indexes of Total Real Product*

In this section we examine the relationship between individual industry indexes of real product in the 1947-58 period and the 1958-64 period. This relationship can be observed from Table 5, which distributes the various industries according to both their 1958 and 1964 indexes of total real product. This table is in the form of a two-way frequency distribution in which the total array of real-product indexes are divided into five broad categories for each time period.

TABLE 5

*1958 Industry Indexes of Total Real Product (1947 = 100),  
Distributed by 1964 Indexes of Total Real Product (1958 = 100)<sup>a</sup>*

1958 Index (1947 = 100)	1964 Indexes of Total Real Product (1958 = 100) <sup>b</sup>					
	No.	Lowest (110 and under)	Below Average (110.1-123.1)	Average (123.2-136.2)	Above Average (136.3-149.3)	Highest (149.4 and over)
Highest (184.0 and over)	6		65	(1)	29,49	(2) 28,48,R. of W(3)
Above average (156.0-183.9)	7	15 (1)	(1) 73	(1)	38, fed. enterprises	(2) 36,37 (2)
Average (128.0-155.9)	11		21,60,gov't	(3) 23,25,27,32, 34,50	(6) 26, state & local enterprises	(2)
Below average (100.0-127.9)	12	01,07(2)	13,20,39,70,88(5)	10,22	(2)	30,35,75 (3)
Lowest (under 100)	6	31 (1)	11,78	(2) 24,33,40	(3)	
Total number	42	4	11	13	6	8

<sup>a</sup>The entries (except those in the "Total" row) identify the industry by the first SIC number associated with it; the numbers in parenthesis opposite the SIC numbers are the total number of industries in question. Numbers in the "Total" row are the sum of industries in each column.

<sup>b</sup>Because the over-all range of the 1964 real-product indexes was considerably smaller than that of the 1958 indexes, the same absolute class intervals could not be used for the two distributions. The class intervals for the 1964 indexes were designed to represent approximately the same *percentage deviation* from the average index as the 1958 class intervals.

It can be noted from this table that there is some positive correlation between the relative standing of an industry index in 1958 and in 1964, but the correlation is not as marked as one might expect. Though none of the industries which were in the lowest group in 1958 can be found in the highest group for 1964, and vice versa, there is nevertheless fairly wide dispersion.<sup>13</sup>

Of the six industries which were in the lowest group in the 1947-58 period only one was in the lowest group in the 1958-64 period; two were in the below average group; and three were in the group with average change in the second period. What is more surprising is that the three industries which diverged most from their expected position in the frequency distribution had below average indexes for both final demand and technical coefficients in the 1947-58 period. However, for some of them, particularly the primary metals industry (SIC 33), cyclical rather than trend factors may have caused the 1947-58 final-demand indexes to be below average. (The year 1947 was a period of expanding activity, while 1958 was a year of diminished growth and low demand for durable goods.) It would not be expected that these cyclical influences would continue into the 1960's.

Of the twelve industries which showed below average growth in real product in the earlier period, only five were in the corresponding class in the 1958-64 period. Contrary to expectations, three of the twelve industries (SIC's 30, 35, and 75) were in the highest group in the 1958-64 period. However, since these were industries where there was mixed influence of technical coefficients and final demand in the earlier period, one might expect the 1958-64 index of change to be less predictable. The automobile repair industry (SIC 75) had an average final-demand index but a below average coefficient index in the 1947-58 period, while the two others, rubber and miscellaneous plastics (SIC 30) and machinery, except electrical (SIC 35), had below average final-demand indexes and average coefficient indexes. For at least one of these, the machinery industry, the low level of invest-

<sup>13</sup> One possible explanation for this dispersion may be that the indexes of real product for 1964 are not strictly comparable to those for 1947 since they were derived directly from OBE's real-product data without any adjustments for differences between input-output and real product in industry definitions. In addition, all of the 1964 indexes are preliminary in the sense that they have not benefited from a bench-mark revision.

ment demand in 1958 might have contributed to the low final-demand index in the 1947–58 comparison.

Towards the other end of the scale in Table 5, only two of the seven industries which showed above average increases in the 1947–58 period were in the corresponding class in 1958–64. Two industries differed markedly from their expected position in the table, construction (SIC 15–17), which had one of the lowest indexes, and nonmetallic minerals mining (SIC 14), which showed a below average increase in the latter period. For construction, the failure to include force account activity in the 1958–64 indexes may have influenced the comparison of the two periods. For nonmetallic minerals mining, there is no apparent reason for its different relative standing in the two periods, particularly since this industry falls in the above average groups of Table 4 on both counts, technical coefficients and final demand. Here is a case where one would have expected the above average increase to have continued into the 1958–64 period.

Of the six industries in the group with the highest increases from 1947 to 1958, only one, real estate (SIC 65), differed considerably from what one might expect. It showed only an average increase in the 1958–64 period, despite the fact that it had above average indexes for both final-demand effect and technical coefficient effect in the 1947–58 period. The above average 1958 indexes for final demand, however, may have reflected cyclical rather than longer-range factors since consumer rental payments are not as sensitive to cyclical changes in the level of economic activity as is the demand for other goods and services. Similarly, rent is a production cost which is likely to be non-proportional and, consequently, to show a higher ratio to total costs in times of reduced activity than in times of expanding activity, thus explaining the above average position of the 1958 technical coefficient index for the real estate and rental industry.

A varied picture thus emerges from Table 5. Only seventeen industry groups were in their expected positions in the array of indexes for the 1958–64 period, sixteen industry groups differed slightly from their expected positions, and nine differed markedly. In some cases, these differences may be explained in terms of cyclical influences on the component factor indexes. In other cases, random factors or statistical noncomparabilities may have affected the comparison of indexes of

real product for the two periods. When the 1963 OBE input-output table becomes available and it becomes possible to factor out the causes of change in real product for the period 1958-64, one can then determine to what extent the tendencies which operate in the 1947-58 period have continued into the latter period.

*Appendix: Step by Step Procedure for Factoring Change in Real Product by Industry Group (1947-58)*

- I. *Compute total change in real product 1947-58.*
  - A. Combine industry estimates of value added from 1958 input-output table to two-digit level.
  - B. Combine industry estimates of value added from reworked 1947 input-output table to two-digit level.
  - C. Convert 1947 value added to 1958 prices by use of implicit value added deflators.
  - D. Subtract 1947 real product from 1958 real product.
- II. *Compute 1958 real product if 1947 technical coefficients and 1958 final demand had prevailed.*
  - A. Convert 1958 final demand to 1947 prices by use of special final-demand deflators for each industry developed for this paper.
  - B. Apply 1958 final demand to 1947 inverse to yield 1958 output in 1947 prices.
  - C. Multiply derived 1958 outputs by value added/output ratios from reworked 1947 I.O. Table to yield 1958 value added in 1947 prices—combine to two-digit level.
  - D. Convert estimated 1958 value added in 1947 prices to 1958 prices by use of implicit deflator for value added.
- III. *Factor total change in real product 1947-58 (fixed 1947 coefficient method).*
  - A. For each industry group, subtract derived 1958 real product (from Step II, D above) from the actual 1958 input-output estimate of real product—to yield amount of real product difference (1947-58) which is due to changes in coefficients.
  - B. For each industry group, subtract actual 1947 real product from derived 1958 real product (from Step II, D above) to

yield amount of real product change (1947–58) which is due to changing final demand.

- IV. *Compute 1947 real product if 1958 technical coefficients and 1947 final demand had prevailed.*
  - A. Convert 1947 final demand to 1958 prices by use of special final-demand deflators by industry developed for this paper.
  - B. Apply 1947 final demand to 1958 inverse to yield 1947 output in 1958 prices.
  - C. Apply 1958 value added/output ratios from 1958 I.O. Table to yield 1947 value added in 1958 prices (real product)—combine to two-digit level.
- V. *Factor total change in real product, 1947–58 (fixed 1958 coefficient method).*
  - A. For each industry group, subtract derived 1947 real product (from Step IV, C, above) from actual 1958 input–output estimate of real product to yield amount of real product difference (1947–58) which is due to changing final demands.
  - B. For each industry group, subtract actual 1947 input–output real product from derived 1947 real product (from Step IV, C, above) to yield amount of real product change (1947–58) which is due to change in coefficients.
- VI. *Averaging of factor shares—two methods.*
  - A. For each industry group, average values obtained in Step III, A, and Step V, B, to yield amount of real product change (1947–58) due to coefficient changes.
  - B. For each industry group, average values obtained in Step III, B, and V, A, to yield amount of real product change (1947–58) due to final demand changes.

**TABLE A**  
*Standard Industrial Classification Groups and Their 1958  
 Input-Output Industry Composition*

SIC Number	1958 Input- Output Industry Number	Title
01, 02	1, 2	Farms
07-9	3, 4	Agricultural services, hunting and trapping, forestry and fisheries <sup>a</sup>
10	5, 6	Metal mining
11, 12	7	Coal mining
13	8	Crude petroleum and natural gas <sup>b</sup>
14	9, 10	Mining and quarrying of nonmetallic minerals, except fuels
15-17	11, 12	Construction <sup>bc</sup>
20	14	Food and kindred products
21	15	Tobacco manufactures
22	16, 17	Textile mill products <sup>d</sup>
23	18, 19	Apparel and related products <sup>de</sup>
24	20, 21	Lumber and wood products, except furniture
25	22, 23	Furniture and fixtures
26	24, 25	Paper and allied products
27	26	Printing and publishing
28	27-30	Chemicals and allied products <sup>f</sup>
29	31	Petroleum and related industries
30	32	Rubber and miscellaneous plastics products
31	33, 34	Leather and leather products
32	35, 36	Stone, clay, and glass products
33	37, 38	Primary metal industries <sup>f</sup>
34	39-42	Fabricated metal products
35	43-52	Machinery, except electrical
36	53-58	Electrical machinery
37, 19	13, 59-61	Transportation equipment and ordnance
38	62-63	Instruments and related products
39	64	Miscellaneous manufacturing <sup>e</sup>
40-47	65	Transportation
48	66, 67	Communication

(continued)

TABLE A (concluded)

SIC Number	1958 Input- Output Industry Number	Title
49	68	Electric, gas, and sanitary services
50-59	69	Wholesale and retail trade
60-64, 66, 67	70	Finance and insurance
65	71	Real estate <sup>g</sup>
70, 72, 76	72	Hotels, personal, and repair services, except auto repair <sup>h</sup>
73, 80-89 (except 88)	73, 74, 77	Business, medical, professional, and educational services, and nonprofit organizations <sup>agh</sup>
75	75	Auto repair, auto services, and garages
78, 79	76	Amusements
--	78	Federal government enterprises <sup>i</sup>
--	79	State and local government enterprises <sup>i</sup>
--	84	Government industry <sup>j</sup>
--	85	Rest of the world industry <sup>k</sup>
88	86	Household industry <sup>l</sup>

<sup>a</sup>SIC 0722 – Offices of veterinarians and animal hospitals is included with SIC 80, medical and other health services.

<sup>b</sup>SIC 138, oil and gas field services is included with SIC 15-17, construction.

<sup>c</sup>In addition to contract construction activity, as specified by the SIC, this industry group includes public and private force account construction.

<sup>d</sup>SIC 225, knitting mills is included with SIC 23, apparel and related products.

<sup>e</sup>SIC 3922, furs, dressed and dyed, is included with SIC 23, apparel.

<sup>f</sup>Alumina, part of SIC 2819, is included with SIC 33, primary metal industries.

<sup>g</sup>SIC 6541, title abstract companies is included with SIC 73, business services.

<sup>h</sup>SIC 7694, armature rewinding shops and SIC 7699, repair shops nec are included with SIC 73, business services.

<sup>i</sup>This industry includes those activities of government agencies, with separate accounting records, that cover over half of their current operating costs by the sale of goods and services to the general public.

<sup>j</sup>Value added in this industry is measured by employee compensation of federal, state, and local government employees.

<sup>k</sup>Value added in this industry reflects income originating in the rest of the world.

<sup>l</sup>Value added in this industry reflects employee compensation of domestic servants.



TABLE B  
*Basic Data on Final Demand, Prices, and Value Added*  
 by Input Output Industry, 1947 and 1958

I.O. Industry	Final Demand (current dollars thousands) <sup>a</sup>		1947 Final - Demand Price Deflator (1958=100)	Value-Added/Output Ratio (current dollars)		1947 Value-- Added Deflator (1958=100)
	1947	1958		1947	1958	
1	1,668,613	2,526,740	151.4 <sup>b</sup>	.29011	.34531	120.8
2	3,668,957	5,419,010	89.4 <sup>b</sup>	.60268	.51155	
3	8,765	-108,116	46.3	.39281	.49260	63.1
4	-36,272	233	65.9	.48184	.44629	
5	-93,932	-431,180	60.9	.72607	.55276	77.9
6	-150,698	-124,816	79.8	.75320	.45975	
7	1,092,808	628,220	71.0	.78629	.58348	85.8
8	-93,121	-1,195,846	62.2	.73358	.68994	55.7
9	-92,879	-74,969	60.6	.67503	.61671	80.5
10	32,317	-1,640	67.3	.78606	.61284	
11	20,329,109	52,416,000	68.7	.39339	.35493	
12	1,712,959	4,419,828	61.3	.44987	.61233	69.5
13	131,914	2,520,411	59.7	.56641	.34837 (inc. with 59-61)	
14	32,629,360	46,328,371	89.1	.25401	.26039	83.9
15	2,873,862	4,633,996	75.1	.42491	.48226	82.3
16	2,160,725	624,212	113.5	.33004	.26117	
17	607,086	489,152	83.9	.40051	.28086	127.7

(continued)

TABLE B (continued)

I.O. Industry	Final Demand (current dollars thousands) <sup>a</sup>		1947 Final- Demand Price Deflator (1958=100)	Value-Added/Output Ratio (current dollars)		1947 Value- Added Deflator (1958=100)
	1947	1958		1947	1958	
18	9,184,971	11,280,502	108.6	.38407	.38666	98.1
19	841,488	1,213,233	111.1	.23141	.23094	
20	651,088	-188,800	79.2	.47881	.34258	81.8
21	77,862	-9,728	82.5	.36340	.36854	
22	1,574,737	2,634,185	79.2	.41039	.41606	72.0
23	529,150	1,098,951	61.6	.37521	.44749	
24	-207,291	252,148	69.2	.39388	.38383	69.8
25	196,770	56,859	77.7	.34243	.37452	
26	1,676,748	2,892,509	63.6	.50920	.47364	70.9
27	532,115	1,511,692	84.5	.36341	.39711	
28	2,776	271,675	77.6	.40023	.39830	91.8
29	2,158,877	4,357,513	105.8	.32295	.41990	
30	178,588	42,239	81.0	.29489	.36446	92.2
31	3,146,938	8,181,243	76.0	.18788	.20804	
32	1,142,087	1,700,449	68.0	.43172	.45761	58.2
33	33,919	-11,442	103.4	.29533	.32577	
34	2,247,308	2,689,431	82.3	.32626	.44072	70.5
35	255,628	133,591	57.8	.49973	.57063	
36	328,781	248,890	67.1	.47267	.48961	61.6
37	972,616	224,824	50.2	.37961	.40100	
38	64,895	-269,861	50.6	.24986	.30994	48.8

(continued)

TABLE B (continued)

I.O. Industry	Final Demand (current dollars thousands) <sup>a</sup>		1947 Final- Demand Price Deflator (1958=100)	Value-Added/Output Ratio (current dollars)		1947 Value- Added Deflator (1958=100)
	1947	1958		1947	1958	
39	70,799	66,359	57.2	.27699	.33556	70.8
40	716,161	916,163	73.3	.42792	.38482	
41	393,131	279,975	60.3	.43862	.44276	70.8
42	621,868	803,101	59.7	.48398	.43540	
43	364,746	1,089,086	57.2	.35579	.42864	70.8
44	1,177,004	1,739,853	63.2	.36597	.37679	
45	1,216,990	2,055,234	56.4	.39482	.44131	70.8
46	378,937	575,135	56.1	.43621	.37188	
47	931,573	1,522,695	52.9	.56017	.51144	62.1
48	1,591,900	1,788,388	61.0	.50318	.44560	
49	859,653	1,438,877	58.1	.47684	.43590	62.1
50	45,274	64,197	60.9	.53176	.53623	
51	579,389	1,317,570	75.0	.63453	.57375	62.1
52	760,137	1,382,725	76.3	.39842	.34193	
53	1,310,068	1,909,886	62.3	.48863	.49755	80.7
54	2,087,847	2,628,806	86.2	.37985	.37257	
55	393,300	387,939	61.5	.46218	.46874	80.7
56	1,480,101	3,919,141	90.5	.44969	.44607	
57	344,366	450,667	80.1	.44517	.49717	80.7
58	329,241	492,752	64.4	.40988	.42699	

(continued)

TABLE B (continued)

I.O. Industry	Final Demand (current dollars thousands) <sup>a</sup>		1947 Final- Demand Price Deflator (1958=100)	Value-Added/Output Ratio (current dollars)		1947 Value- Added Deflator (1958=100)
	1947	1958		1947	1958	
59	7,538,200	13,276,432	65.1	.32658	.29839	(inc. 13) 65.8
60	920,361	7,123,289	59.7	.46245	.47223	
61	1,949,741	2,761,445	62.0	.41127	.38625	67.2
62	782,879	1,577,675	67.1	.41394	.46941	
63	378,256	803,314	75.2	.53355	.54604	89.1
64	2,073,105	2,962,099	84.5	.49232	.42009	
65	10,083,936	13,480,940	61.8	.67341	.61880	65.4
66	1,886,468	4,693,956	78.0	.79313	.85159	
67	8,839	9,000	75.6	.58104	.57263	77.3
68	3,085,278	8,892,883	92.0	.55638	.48951	
69	41,011,501	68,258,759	82.7	.75029	.72447	84.3
70	4,513,923	12,073,809	57.8	.54948	.56042	
71	16,487,886	41,771,742	70.6	.65591	.72401	70.3
72	5,470,319	9,787,904	69.2	.65572	.60817	
73	1,331,935	3,183,596	53.6	.47395	.45866	(inc. 77) 62.6
74	365,261	5,177,000	59.9	--	.07679	
75	1,858,589	4,598,596	67.3	.60244	.48123	68.0
76	2,915,899	3,516,158	71.8	.58780	.53185	
77	7,877,617	20,880,011	66.3	.66192	.68106	(inc. with 73 & 74) 127.3
78	340,901	816,554	69.6	.68308	.43557	
79	72,662	434,213	100.0	.70063	.54431	48.6

(continued)

TABLE B (concluded)

I.O. Industry	Final Demand (current dollars thousands) <sup>a</sup>		1947 Final- Demand Price Deflator (1958=100)	Value-Added/Output Ratio (current dollars)		1947 Value- Added Deflator (1958=100)
	1947	1958		1947	1958	
80	-2,164,565	-3,369,362	76.8	--	1.00000	--
81	--	-160,000	--	--	0	--
82	--	0	--	--	0	--
83	-1,251,870	-669,714	110.5	--	1.00000	--
84	15,730,000	39,029,000	58.5	1.00000	1.00000	58.5
85	824,000	2,030,000	77.1	1.00000	1.00000	77.1
86	2,348,000	3,503,000	74.4	1.00000	1.00000	74.4
87	-763,965	-311,000	80.9	1.00000	1.00000	80.9

<sup>a</sup>The final demands for both 1947 and 1958 are designed to be used with an input-output table with gross domestic output for each industry as the base for the input coefficients. They therefore incorporate imports, at domestic port value, as a negative final-demand column. The 1958 final demand differs in this respect from the flow table published September 1965, which showed competitive imports destined for intermediate use transferred into their domestic counterpart industry. The 1958 final-demand column also differs from the published input-output table in that the demand for office supplies is shown as demands on the various industries which produce these products instead of a single demand on the office supply dummy industry.

<sup>b</sup>The deflators shown here were used to put 1947 final demands in 1958 prices. In converting 1958 final demands to 1947 prices, different deflators were used for these industries because of extreme differences in product mix. For this latter conversion, the price index for industry 1 is 116.3, and for industry 2, 108.1.

TABLE C  
Factoring of Total Change in Real Product 1947-1958, Fixed 1958 Coefficient Method  
(millions of 1958 dollars)

SIC No.	Actual 1958 Real Product (1)	Derived 1947 Real Product (2)	Change Due To Final Demand (col. 1 - col. 2) (3)	Actual 1947 Real Product (4)	Change Due To Coefficients <sup>a</sup> (col. 2 - col. 4) (5)
All Industries	447,330	315,857	131,473	315,128	729
01, 02	20,846	15,561	5,285	16,698	-1,137
07-9	1,264	1,003	261	1,172	-169
10	914	811	103	832	-21
11, 12	1,604	1,905	-301	2,822	-917
13	6,671	4,482	2,189	5,548	-1,066
14	1,226	772	454	719	53
15-17	28,937	17,295	11,642	17,322	-27
20	16,630	13,018	3,612	13,734	-716
21	2,854	2,349	505	2,050	299
22	3,389	3,283	106	3,285	-2
23	6,042	4,497	1,545	4,593	-96
24	2,880	2,377	503	3,509	-1,132
25	2,042	1,492	550	1,481	11
26	5,020	3,616	1,404	3,501	115
27	5,974	4,546	1,428	4,646	-100
28	9,811	6,487	3,324	4,178	2,309
29	3,608	2,233	1,375	1,628	605
30	3,131	2,531	600	2,605	-74
31	1,655	1,666	-11	1,693	-27
32	4,900	3,585	1,315	3,225	360
33	10,510	8,840	1,670	11,951	-3,111
34	8,231	6,245	1,986	6,099	146

(continued)

TABLE C (concluded)

SIC No.	Actual 1958 Real Product (1)	Derived 1947 Real Product (2)	Change Due To Final Demand (col. 1 - col. 2) (3)	Actual 1947 Real Product (4)	Change Due To Coefficients <sup>a</sup> (col. 2 - col. 4) (5)
35	11,112	9,680	1,432	9,767	-87
36	9,602	6,631	2,971	5,327	1,304
37, 19	15,868	8,927	6,941	8,883	44
38	2,484	1,618	866	1,372	246
39	2,229	1,772	457	1,805	-33
40-47	20,600	19,212	1,388	22,386	-3,174
48	8,800	5,509	3,291	3,821	1,688
49	9,914	5,370	4,544	4,060	1,310
50-59	69,006	49,846	19,160	47,658	2,188
60-64, 66, 67	14,831	10,011	4,820	11,338	-1,327
65	44,821	27,371	17,450	23,998	3,373
70, 72, 76	7,402	5,862	1,540	6,321	-459
73, 80-89 (except 88)	27,082	16,780	10,302	15,288	1,492
75	3,808	2,543	1,265	3,519	-976
78, 79	2,989	3,202	-213	3,116	86
Fed. gov't enterprises	1,788	1,203	585	980	223
State and local gov't enterprises	2,604	1,556	1,048	2,028	-472
Gov't industry	39,029	26,889	12,140	26,889	--
Rest of world	2,030	1,069	961	1,069	--
Household	3,503	3,156	347	3,156	--
IVA	-311	-944	633	-944	--

<sup>a</sup>Theoretically the sum of this column should be zero but it is not precisely zero because of rounding errors and a slight discrepancy which arises in connection with the scrap and by-product adjustment.

TABLE D  
Factoring of Total Change in Real Product 1947-1958, Fixed 1947 Coefficient Method  
(millions of 1958 dollars)

SIC No.	Actual 1958 Real Product (1)	Derived 1958 Real Product (2)	Change Due To Coefficients <sup>a</sup> (col. 1 - col. 2) (3)	Actual 1947 Real Product (4)	Change Due To Final Demand (col. 2 - col. 4) (5)
All Industries	447,330	448,059	-729	315,128	132,931
01, 02	20,846	22,631	-1,785	16,698	5,933
07-9	1,264	1,576	-312	1,172	404
10	914	1,092	-178	832	260
11, 12	1,604	3,009	-1,405	2,822	187
13	6,671	8,429	-1,758	5,548	2,881
14	1,226	1,119	107	719	400
15-17	28,937	29,362	-425	17,322	12,040
20	16,630	17,674	-1,044	13,734	3,940
21	2,854	2,487	367	2,050	437
22	3,389	3,383	6	3,285	93
23	6,042	6,172	-130	4,593	1,579
24	2,880	4,544	-1,664	3,509	1,035
25	2,042	2,023	19	1,481	542
26	5,020	5,058	-38	3,501	1,557
27	5,974	6,208	-234	4,646	1,562
28	9,811	6,647	3,164	4,178	2,469
29	3,608	2,679	929	1,628	1,051
30	3,131	3,119	12	2,605	514
31	1,655	1,711	-56	1,693	18
32	4,900	4,412	488	3,225	1,187
33	10,510	15,183	-4,673	11,951	3,232
34	8,231	8,069	162	6,099	1,970

(continued)



TABLE D (concluded)

SIC No.	Actual 1958 Real Product (1)	Derived 1958 Real Product (2)	Change Due To Coefficients <sup>a</sup> (col. 1 - col. 2) (3)	Actual 1947 Real Product (4)	Change Due To Final Demand (col. 2 - col. 4) (6)
35	11,112	10,869	243	9,767	1,102
36	9,602	7,385	2,217	5,327	2,058
37, 19	15,868	15,692	176	8,883	6,809
38	2,484	2,025	459	1,372	653
39	2,229	2,261	-32	1,805	456
40-47	20,600	25,709	-5,109	22,386	3,323
48	8,800	6,456	2,344	3,821	2,635
49	9,914	8,179	1,735	4,060	4,119
50-59	69,006	66,132	2,874	47,658	18,474
60-64, 66, 67	14,831	16,961	-2,130	11,338	5,623
65	44,821	39,621	5,200	28,998	15,623
70, 72, 76	7,402	8,079	-677	6,321	1,755
73, 80-89 (except 88)	27,082	24,946	2,136	15,288	9,658
75	3,808	5,381	-1,573	3,519	1,862
78, 79	2,989	2,759	230	3,116	-357
Fed. gov't enterprises State and local gov't enterprises	1,788	1,535	253	980	555
Gov't industry	2,604	3,231	-627	2,028	1,203
Rest of world	39,029	39,029	0	26,889	12,140
Household	2,030	2,030	0	1,069	961
IVA	3,503	3,503	0	3,156	347
	-311	-311	0	-944	633

<sup>a</sup>Theoretically the sum of this column should be zero but it is not precisely zero because of rounding errors and a slight discrepancy which arises in connection with the scrap and by-product adjustment.

**TABLE E**  
**Computation of "Interaction" Effect by Use of 1947 and 1958 Weights**  
(millions of 1958 dollars)

	1947 Weights			1958 Weights		
	Total Difference in Real Product (col. 1 minus col. 4, Table C) (1)	Change due to:		Final Demand (col. 3, Table C) (5)	Change due to:	
		Final Demand Coefficients (col. 5, Table D) (2)	Interaction minus cols. 2+3 (4)		Coefficients (col. 3, Table D) (6)	Interaction minus cols. 5+6 (7)
All industries	132,202	132,931	729	131,473	-729	1458
01,02	4,148	5,933	-1,137	5,285	-1,785	648
07-9	92	404	-169	261	-312	143
10	82	260	-21	103	-178	157
11, 12	-1,218	187	-917	-301	-1,405	488
13	1,123	2,881	-1,066	2,189	-1,758	692
14	507	400	53	454	107	-54
15-17	11,615	12,040	-27	11,642	-425	398
20	2,896	3,940	-716	3,612	-1,044	328
21	804	437	299	505	367	-68
22	104	98	-2	106	6	-8
23	1,449	1,579	-96	1,545	-130	34
24	-629	1,035	-1,132	503	-1,664	532
25	561	542	11	550	19	-8
26	1,519	1,557	115	1,404	-38	153
27	1,328	1,562	-100	1,428	-234	134
28	5,633	2,469	2,309	3,324	3,164	-855
29	1,980	1,051	605	1,375	929	-324
30	526	514	-74	600	12	-86
31	-38	18	-27	-11	-56	29

(continued)

TABLE E (concluded)

	1947 Weights				1958 Weights			
	Total Difference in Real Product (col. 1 minus col. 4, Table C) (1)	Change due to:		Interaction (col. 1 minus cols. 2+3) (4)	Final Demand Coefficients (col. 5, Table D) (2)	Change due to:		Interaction (col. 1 minus cols. 5 + 6) (7)
		Final Demand Coefficients (col. 5, Table C) (3)	Coefficients (col. 1 minus cols. 2+3) (4)			Final Demand Coefficients (col. 3, Table C) (5)	Coefficients (col. 3, Table D) (6)	
32	1,675	1,187	360	128	1,315	488	-128	
33	-1,441	3,232	-3,111	-1562	1,670	-4,673	1562	
34	2,132	1,970	146	16	1,986	162	-16	
35	1,345	1,102	-87	330	1,432	243	-330	
36	4,275	2,058	1,304	913	2,971	2,217	-913	
37, 19	6,985	6,809	44	132	6,941	176	-132	
38	1,112	653	246	213	866	459	-213	
39	424	456	-33	1	457	-32	-1	
40-47	-1,786	3,323	-3,174	-1935	1,388	-5,109	1935	
48	4,979	2,635	1,688	656	3,291	2,344	-656	
49	5,854	4,119	1,310	425	4,544	1,735	-425	
50-59	21,348	18,474	2,188	686	19,160	2,874	-686	
60-64, 66, 67	3,493	5,623	-1,327	-803	4,820	-2,130	803	
65	20,823	15,623	3,373	1827	17,450	5,200	-1827	
70, 72, 76	1,081	1,758	-459	-218	1,540	-677	218	
73, 80-89 (except 38)	11,794	9,658	1,492	644	10,302	2,136	-644	
75	289	1,862	-976	-597	1,265	-1,573	597	
78, 79	-127	-357	86	144	-213	230	-144	
Fed. gov't enterprises	808	555	223	30	585	253	-30	
State & local gov't enterprises	576	1,203	-472	-155	1,048	-627	155	
Gov't industry	12,140	12,140	---	0	12,140	---	0	
Rest of world	961	961	---	0	961	---	0	
Household	347	347	---	0	347	---	0	
IVA	633	633	---	0	633	---	0	

COMMENT

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It is always delightful to read a paper by Bea Vaccara. Her papers are neat, direct, methodical, and meticulous. The procedures employed are stated explicitly, and one can be sure that the calculations have been carried out with great care. This paper by Vaccara and Simon is no exception.

The idea involved is a simple one. The authors present it in verbal form; for additional clarity, I shall restate it mathematically and diagrammatically. They begin with the basic input-output relation:

$$(1) \quad F = (I - A)S$$

- where  $F$  = an  $n$  component column vector of industry final demands;
- $I$  = an  $n \times n$  identity matrix;
- $A$  = an  $n \times n$  matrix of coefficients with elements  $a_{ij}$  which are the dollars of input from industry  $i$  required to produce a dollar of gross output of industry  $j$ ;
- $S$  = an  $n$  component column vector of gross industry output.

Then, a substitution is made for  $S$ :

$$(2) \quad S = CX$$

- where  $C$  = an  $n \times n$  diagonal matrix whose elements are the dollars of gross output of industry  $i$  per dollar of real product (value added) of that industry;
- $X$  = an  $n$  component column vector of industry real product.

Therefore,

$$(3) \quad F = (I - A)CX$$

and

$$X = C^{-1}(I - A)^{-1}F$$

A time subscript can be attached to the  $X$  and  $F$  vectors. They must, however, be measured in the same prices as  $C$  and  $A$ , although then they can be transformed into quantities with prices of other periods. Thus,

$$(4) \quad P_X^{58-1}_{1947} X^{47}_{1958} = P_X^{58-1}_{1947} C^{-1} (I - A)^{-1}_{1947} P_F^{58}_{1947} F^{58}_{1958}$$

where *superscripts* are the base year of price indexes or the constant dollars in which monetary flows are measured; *subscripts* are the period of observation. The price variables are defined as follows:

$P_X^{58}$  = an  $n \times n$  diagonal matrix of indexes of current-dollar value added per constant dollar of real product, 1958 = 1.00

$P_F^{58}$  = an  $n \times n$  diagonal matrix of indexes of prices of industry gross output, 1958 = 1.00.

Now, call the left hand variable  $\hat{X}_{1958,1947}^{58}$ , or "estimated output originating measured in superscript (1958) dollars in year  $i$  (1958) using input-output matrix for year  $j$  (1947)." That is:

$$\hat{X}_{1958,1947}^{58} = P_X^{58-1} X_{1947,1958}^{47}$$

Vaccara and Simon use this quantity to divide the total change in gross product originating in each industry into two components: (a) that due to changes in final demand of *all* industries; and (b) that due to shifts in input-output coefficients of *all* industries. That is:

$$\text{Total change} = X_{1958}^{58} - X_{1947}^{58}$$

$$(5) \quad \text{Due to change in coefficients} = X_{1958}^{58} - \hat{X}_{1958,1947}^{58}$$

$$(6) \quad \text{Due to change in final demand} = \hat{X}_{1958,1947}^{58} - X_{1947}^{58}$$

Similarly, it is possible to derive the estimated output originating in 1947, measured in 1958 dollars, using the input-output matrix for 1958, i.e.,  $\hat{X}_{1947,1958}^{58}$ . Then an alternative set of change measures is

$$\text{Total change} = X_{1958}^{58} - X_{1947}^{58}$$

$$(7) \quad \text{Due to change in final demand} = X_{1958}^{58} - \hat{X}_{1947,1958}^{58}$$

$$(8) \quad \text{Due to change in coefficients} = \hat{X}_{1947,1958}^{58} - X_{1947}^{58}$$

While the total change is the same for either measure, the distribution of the output difference between that due to a change in coefficients or a change in final demand is not likely to be identical. Therefore, Vaccara and Simon take the simple arithmetic average as their indicator, or:

Average change in final demand =

$$\frac{1}{2}[(X_{1958}^{58} - X_{1947}^{58}) + (\hat{X}_{1958,1947}^{58} - \hat{X}_{1947,1958}^{58})]$$

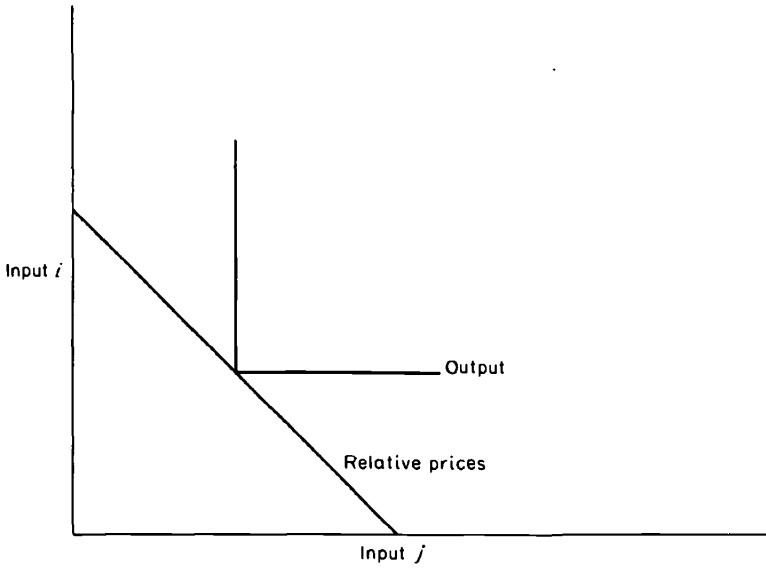


FIGURE 1-a  
1947 Production

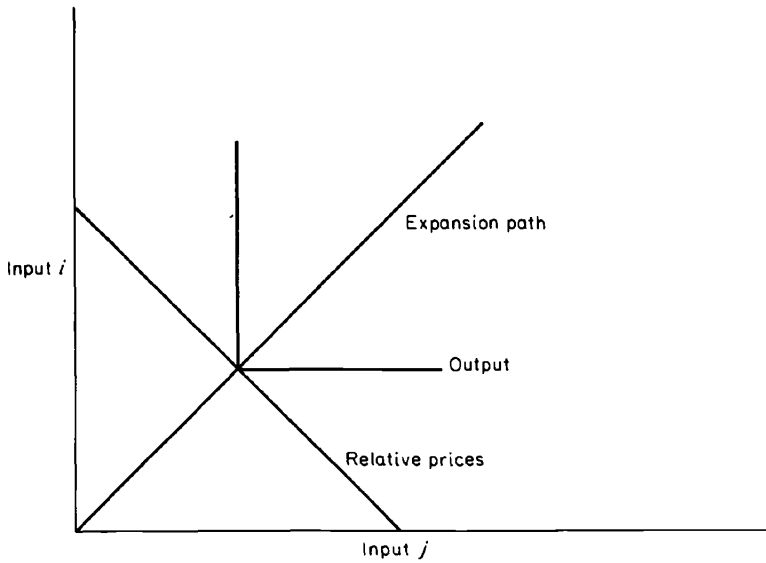


FIGURE 1-b  
1947 Technology Expansion Path

Average change in coefficients =

$$\frac{1}{2}[(X^{58}_{1958} - X^{58}_{1947}) + (X^{58}_{1947,1958} - X^{58}_{1958,1947})]$$

Summing the two changes gives the actual difference between output originating in 1958 and 1947 (in 1958 prices),  $X^{58}_{1958} - X^{58}_{1947}$ . On first blush this all seems straightforward and irreproachable. But, there are a number of difficulties. Vaccara and Simon cite three: (1) There may be aggregation problems—both at the final-demand level and within individual input cells. (2) The underlying production functions may not be linear and homogeneous. Ergo, scale increases and differential capacity utilization can lead to biased estimates of the impact of demand vs. coefficient changes. (3) There may be errors of observation, estimation, and computation.

Aside from these difficulties, there are index number problems. These can best be illustrated with a diagram. In order to simplify matters, only two inputs and a single output will be used in the example.

Figure 1-a shows the results of applying 1947 technology and inputs (in 1958 prices) for the production of actual 1947 final demand. The isoquant depicted is that of a fixed proportion, linear, homogeneous production function, i.e., the type implicit in the input-output assumptions above. In Figure 1-b, an output expansion ray showing the effect of using more inputs is added.

Similarly, Figure 2-a depicts 1958 technology, inputs, and production (also in 1958 prices), and the expansion path. Figure 2-b superimposes Figures 1-b and 2-a in a single diagram.

Now, by adding two circular arcs with the origin as the center and the intersection of the input-output isoquants with the expansion paths as radii, it is possible to decompose the 1947-58 output change à la Vaccara-Simon.

In Figure 3,  $OA$  is the 1947 output (in 1958 prices) along the 1947 expansion path. Similarly,  $OH$  is the 1958 output (in 1958 prices) along the 1958 expansion path. The difference between  $OH$  and  $OA$  is  $AD = EH$ , or  $X^{58}_{1958} - X^{58}_{1947}$ .

Now, suppose that in 1947 the 1958 technology had been available. Then, the 1958 expansion path and 1958 relative prices of inputs would have prevailed. Production could then have taken place at the point  $G$  on the 1958 expansion path ( $aa'$  is a line through point  $A$  parallel to 1958 prices at point  $H$ ). Output (in 1958 dollars) would be  $OG$ . Actual 1947 output (in 1958 dollars) was  $OA$  and, scribing off the same distance on the

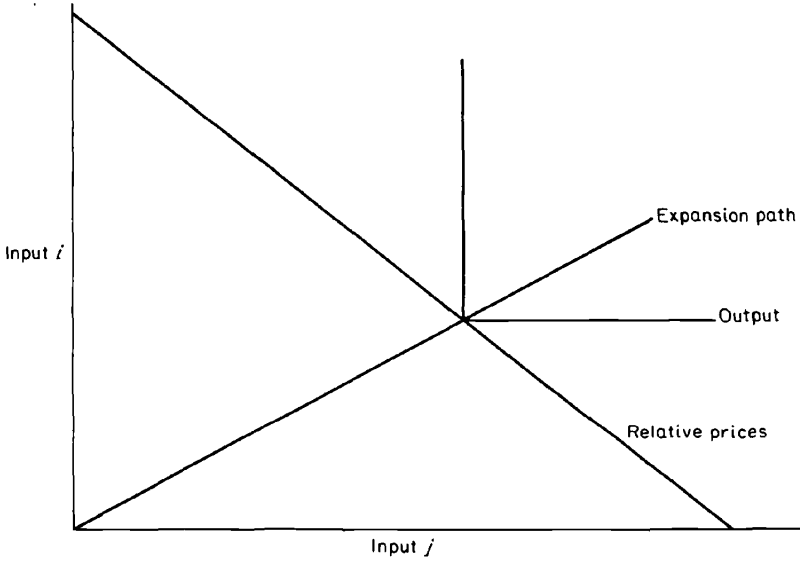


FIGURE 2-a  
1958 Production

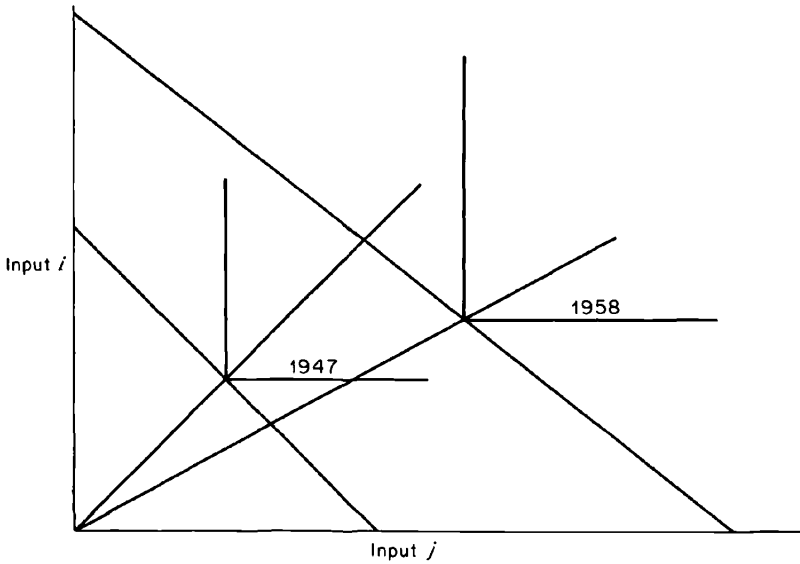


FIGURE 2-b  
1947 and 1958 Technology Expansion Paths



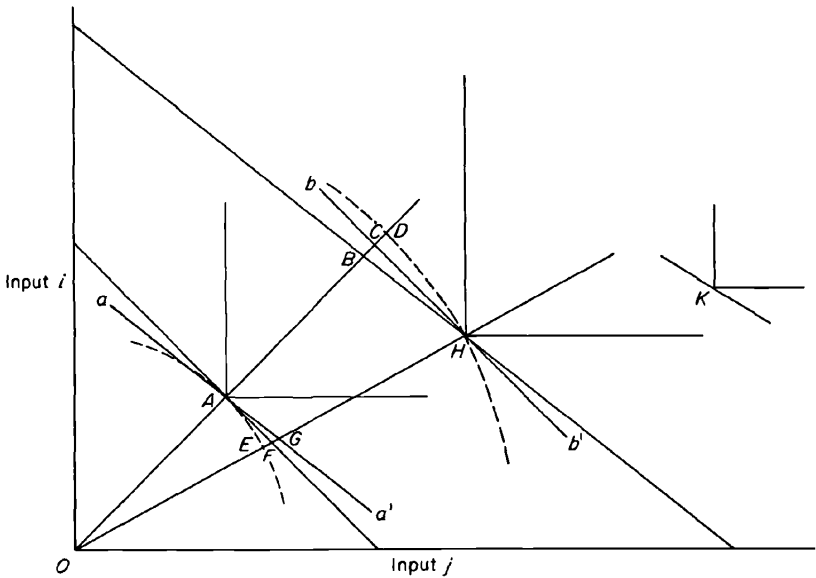


FIGURE 3  
Derivation of Final Demand and Technology Contributions  
to Output Increments

1958 expansion path, this is equal to  $OE$ .<sup>1</sup> Thus, the result of applying 1958 technology and relative factor prices to 1947 production yields an output increment of  $OG - OE = EG$ , or  $X_{1947,1958}^{58} - X_{1947}$ . This leaves  $GH$  as the change in output due to the increase in final demand from 1947 to 1958, or  $X_{1958}^{58} - X_{1947,1958}^{58}$ .

In the same fashion, it is possible to consider the effect of producing the 1958 output with 1947 technology and relative costs of factor inputs.  $CD$  equals the loss in output that would have ensued, or  $X_{1958}^{58} - X_{1958,1947}^{58}$  ( $bb'$  is a line through point  $H$  parallel to 1947 prices at point  $A$ ).  $AC$  corresponds to  $X_{1958,1947}^{58} - X_{1947}^{58}$ , i.e., the outputs due to shifts in final demands between 1947 and 1958.

That there is an index number problem should clearly be evident.  $CD$  does not equal  $EG$ , nor does  $AC$  equal  $GH$ . The Vaccara-Simon solution to this dilemma is to take arithmetic averages of  $CD$  and  $EG$  and  $AC$  and  $GH$ . This does preserve the measure of total change between 1947 and 1958, but it does little else.

<sup>1</sup> Stated alternatively,  $A$  is an interior point in the 1958 production possibility set. To get the same output as  $A$ , at lower cost, production would have taken place at  $E$ .

The distance *EG* is essentially a Paasche indicator of the effects of technological change between the end points of the period while *CD* is a Laspeyres indicator.<sup>2</sup> Averaging the two gives a bastard measure of beginning- and end-point quantities and prices.

Moreover, when a third year, say 1964, is added, matters become even worse. Then, assuming further relative price and technological shifts, using the Vaccara-Simon technique leads to extremely ambiguous results. If, for example, 1964 production took place at point *K* in Figure 3, it would turn out that the sum of their coefficient change measures for 1947-58 and 1958-64 would not equal their measure for 1947-64.

There are other ramifications of the index number problem. The impact of applying 1958 technology to the production of 1947 final demands was calculated assuming 1958 relative input prices. It could just as well have been done with 1947 price relatives, yielding the measure of coefficient change *EF*. (Again, this does not necessarily equal *CD*.) Similarly, the loss of output from applying 1947 technology to 1958 production could have been computed with 1958 instead of 1947 relative input prices. This gives the output amount due to technical progress and coefficient shifts *BD* (instead of *CD*).

Finally, outputs are measured in 1958 prices using 1958 relative importance weights. The answers would be different if the weights employed were for 1947 or some other year.

Thus, for all these reasons, the need for explicitly confronting the index number problem should be clear. It should not be casually left in a gray area of implicit ignorance.

There are further issues. First, there is the minor point that Vaccara and Simon cast their analysis in terms of total rather than partial derivatives. They compute the change in output of an industry due to *all* coefficient changes and *all* final demand shifts. In order to better understand what actually took place, it would seem desirable to compute and show two triangular matrices: (1) a set of partial derivatives of output with respect to changes in final demand of each industry (this might also be put in the form of total increments over the period); and (2) a set of partial derivatives of output with respect to shifts in coefficients of each industry.

<sup>2</sup> The term technological change here and above encompasses both the output increment due to improvement in production processes and that due to choice of inputs in response to shifting relative prices.

More importantly, the Vaccara-Simon computations give no clue as to why output and coefficient shifts have taken place. They do not reveal to what extent the changes are due to technical progress or to reactions to altered relative prices. No information is gained as to whether the technical progress is capital or labor augmenting, whether it is embodied or disembodied, or whether it is merely the consequence of younger average vintages of equipment or labor education. Nothing is learned about capital-labor substitutions or their consequences. These, of course, are all questions revolving about knowledge of production functions. Unfortunately, the input-output assumptions of Vaccara and Simon preclude acquiring any insights in that regard. Nevertheless, their paper is an interesting, although simplistic, initial attempt to provide first approximations of the output changes associated with shifting technical coefficients and final demands. It is hoped that the authors will continue their study and make it more powerful and definitive.