3 Interdealer Trade and Information Flows in a Decentralized Foreign Exchange Market

William Perraudin and Paolo Vitale

How trading arrangements within the foreign exchange market might affect the behavior of prices and allocative efficiency is largely unexplored territory. In a recent survey article, Flood (1991) stresses how little is known about basic features of the foreign exchange market. For example, why is the magnitude of interdealer trading (80 percent of total volume) so great? Why is half of that trading intermediated by brokers?

Of the few studies of foreign exchange market institutional arrangements, Grossman and Zhou (1991) study the effect of stop-loss trading rules typically used by foreign exchange dealers on optimal portfolio strategies of individual traders. Krugman and Miller (1993) adduce various consequences of such rules for the behavior of the foreign exchange market as a whole, arguing that they may provoke market crashes, which could in turn provide a justification for commonly observed intervention behavior by central banks like target zones. Bossaerts and Hillion (1991) examine the implications of dealer behavior for unbiasedness tests in foreign exchange markets.

Finally, an interesting series of papers by Lyons (1992, 1993, 1995) directly analyzes the microstructure of the foreign exchange market. Lyons concentrates particularly on the role of brokers, viewing them as a means by which order flow information is aggregated and then disseminated among dealers.

William Perraudin is the Woolwich Professor of Financial Economics at Birkbeck College, London, a research associate of the Department of Applied Economics in Cambridge University, and a research fellow of the Centre for Economic Policy Research. Paolo Vitale is a Ph.D. student in economics at King's College, Cambridge, and a research economist at the Bank of England.

The authors thank Silviero Foresi, Alan Kirman, Maureen O'Hara, Kathy Rudd, and seminar participants at the central bank of Sweden and the Perugia conference for their comments. Perraudin's research was supported by grants from the Economic Social Research Council (under grant R000 23 4641) and the Newton Trust, while some of the work was completed while he was visiting the Stockholm School of Economics. Vitale's research was supported by a Luciano Iona scholarship of the San Paolo Bank, Italy.

While this is certainly a fruitful line of research, it is not clear in such models why individual dealers do not have an incentive to deal directly rather than through brokers. Also, conversations with traders suggested to us that brokers are primarily important because of the efficient access that they provide to large numbers of other market participants.

For these reasons, the analysis of foreign exchange trading that we attempt in this paper has a somewhat different focus from that of Lyons. We concentrate on the consequences for efficiency and exchange rate behavior of the market's decentralized nature, that is, the fact that dealers are ignorant of the order flow of other marketmakers. Interbank trading is modeled as a means by which marketmakers "sell" each other information about their transactions with outside customers. We show that, under these assumptions, decentralized market arrangements are privately efficient for the group of marketmakers.

The reason that a decentralized market works well in this case is that it allows marketmakers to capture, through interdealer trade, the informational rents associated with receiving outside orders and hence gives them an incentive to adjust their spreads optimally to maximize those rents. If marketmakers are able to transact with only a fraction of other dealers between customer orders, a centralized market in which order flow information is freely and instantaneously available may be preferable to decentralized arrangements. Although incentives to adjust bid-ask spreads efficiently are diluted in a centralized market, at least dealers can observe customer orders and rationally update their subjective probabilities.

An important aspect of the relative efficiency of different market arrangements is their robustness to extreme informational asymmetries. Glosten and Milgrom (1985) discuss the market crashes that may occur when dealers suspect that large numbers of informed agents are present. During such crashes, volume dries up as spreads widen and the informativeness of prices is lost. One of our more interesting findings is that such crashes happen much less in decentralized than in centralized markets such as those studied by Glosten and Milgrom (1985). The reason is that dealers have an incentive to maintain at least some turnover in order to elicit information that they can us in future trading.

As well as supplying results, our analysis sheds light on the implications of decentralized markets for the time-series behavior of exchange rates. Most notably, we find, first, that the usual martingale properties of prices are absent, in that, in the decentralized market, the bid and ask on average increase and decline, respectively, as trades reveal information. Second, the unconditional variance of changes in bid and ask quotes is greater in the decentralized than in the centralized market. We also prove a series of comparative static results, demonstrating that, as one might expect, in the decentralized market spreads widen as the proportion of informed traders increases, while the bid price is increasing in the probability of higher exchange rate values.

Previous theoretical work on decentralized markets has been quite limited, although recently several authors have begun to address the issue. Biais (1993) studies the effects of deviations from transparency in an inventory model of microstructure. Our approach, concentrating as it does on informational asymmetries, may be viewed as complimentary to Biais's work. An interesting recent paper by Neuberger, Naik, and Viswananthan (1993) examines the effect of trade publication delays on price formation in the London Stock Exchange. Although their modeling approach is quite different from ours,[1] they emphasize as we do the informativeness of customer trades.

The structure of our paper is as follows. In section 3.1, we set out the model, studying first a static model of dealing with informed and uninformed outside customers and then showing how this fits into a more complicated dynamic framework with two periods of customer trading separated by a period of interbank transactions. Section 3.2 describes the results that we obtain with the model. These include results on efficiency, statistics of bid and ask prices, comparative statics for dealer prices and informational rents, and a result on interdealer market volume. An appendix provides a complete account of the proofs of the various lemmas and propositions.

3.1 The Model

3.1.1 Basic Assumptions

Suppose that there are n identical dealers and four periods, denoted 0, 1, 2, 3. In period 0, each dealer selects a bid and ask and then trades with a customer if one presents himself. In period 1, dealers may trade among themselves if they so wish. In period 2, dealers trade again with customers if there are any. In period 3, all uncertainty is resolved.

Assume for simplicity that all agents are risk neutral and that the interest rate is zero.[2] The structure of trading and of the stochastic order flow from customers is as follows (for a summary, see figure 3.1). The value of the exchange rate in period 3 is the realization of a random variable, z. Let the unconditional distribution of z be binomial in that z takes the values 1 and -1 with probabilities q and $(1 - q)$.

In period 0, a single customer arrives and is allocated with probability $1/n$ to a given dealer. With probabilities α and $1 - \alpha$, respectively, the trader is either informed or a liquidity trader. Informed traders transact if and only if their observation of the true value, z, exceeds the ask quoted by the dealer, s_A

1. Their model has a batch trading structure with a single price rather than a bid-ask spread, and risk pooling plays an important role.

2. The latter is simply a normalization as we could value assets relative to the value of a safe bond.

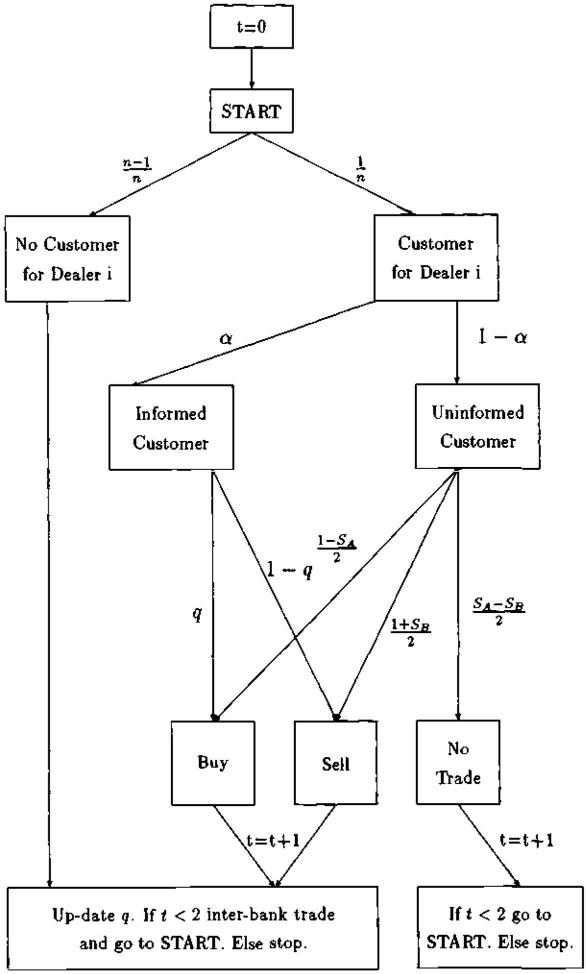

Fig. 3.1 Stochastic order flow structure

(in which case they buy), or falls below the bid price, s_B (in which case they sell). We suppose that liquidity traders buy, sell, or do not trade with probabilities $(1 - s_A)/2$, $(s_B + 1)/2$, and $(s_A - s_B)/2$.

Note that in our model the price sensitivity of liquidity trader orders receives more stress than it does in much of the market microstructure literature. The dealers in our formulation set their spread in order to exploit their monopoly power over part of the uninformed order flow. In stressing this aspect, our study resembles the important early paper by Copeland and Galai (1983).

In period 1, dealers may trade among themselves in such a way as to convey information about the customer order received in period 0. In period 2, each dealer again receives a customer order with probability $1/n$ that once more originates with an informed or an uninformed trader with probabilities α and $1 - \alpha$. Finally, we adopt the following definition:

Definition 1. *A centralized market is one in which information on dealers' order flow is freely and instantly available to other dealers. A decentralized market is one in which this is not the case.*

The above structure of orders implies that our model is applicable to an extremely short period of time. To be specific, we analyze interdealer transactions that can occur in the moments between two substantial and possibly informative customer trades. Perhaps the best way to think of the model is as a description of the situation faced by dealers immediately following some event affecting the foreign exchange market.

3.1.2 The Static Problem

Before considering the more complicated dynamic problem of the dealers described above, let us study the static problem of a dealer who has a single opportunity to trade with informed and uninformed customers. Again assume that informed traders arrive with probability α, and suppose that q is the dealer's current conditional probability for the event $\{z = 1\}$.

The intuition for some of the more important points that emerge from our analysis may be understood even within this simple framework. It is also important to understand the structure of the static problem since this is what dealers face in period 2 even when they have a fully dynamic problem to solve in period 0.

The dealer's static value function $\Pi(s_A, s_B)$ may be written as

$$\Pi(s_A, s_B) \equiv \Pi_A(s_A) + \Pi_B(s_B), \tag{1}$$

where

$$\begin{aligned}\Pi_A(s_A) &\equiv (1 - \alpha)\frac{(1 - s_A)}{2}(s_A - E(z)) + \alpha E\{(s_A - z)1\{z = 1\}\} \\ &= (1 - \alpha)\frac{(1 - s_A)}{2}(s_A - (2q - 1)) + \alpha q(s_A - 1),\end{aligned} \tag{2}$$

$$\begin{aligned}\Pi_B(s_B) &\equiv (1 - \alpha)\frac{(1 + s_B)}{2}(E(z) - s_B) + \alpha E\{(z - s_B)1\{z = -1\}\} \\ &= (1 - \alpha)\frac{(1 + s_B)}{2}((2q - 1) - s_B) - \alpha(1 - q)(1 + s_B),\end{aligned} \tag{3}$$

where we use the fact that $E(z|q) = 2q - 1$. The static value function is quadratic in the quotes, s_A and s_B. Maximizing this function with respect to s_A and s_B yields the maximizing arguments

$$s_A^* = \min\left\{\frac{q}{1 - \alpha}, 1\right\}, \quad s_B^* = \max\left\{-\frac{1 - q}{1 - \alpha}, -1\right\}. \tag{4}$$

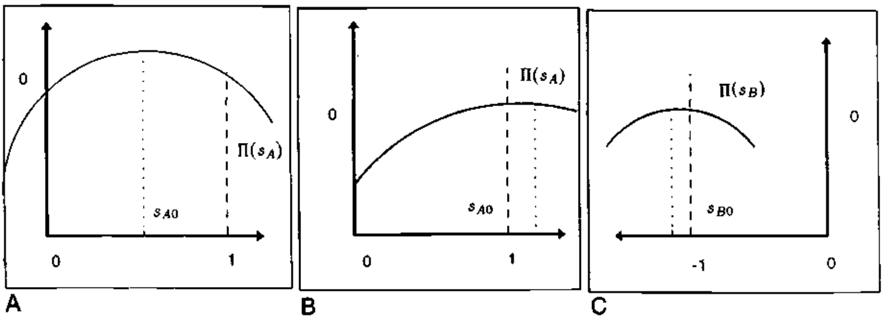

Fig. 3.2 Static value maximization

The min and max operators in equation (4) appear because the maximum of the unconstrained value function may lie outside the interval $[-1, 1]$. In this case, the situation is as depicted in figure 3.2b, c. This feature of the model will generate subcases for the various propositions that we develop below.

In the absence of asymmetric information, that is, when $\alpha = 0$, the optimal static quotes, s_A^* and s_B^*, equal q and $q - 1$ respectively. In this case, the dealer's calculation is motivated solely by the desire to exploit optimally the downward-sloping demand curve for liquidity trader orders that he faces. For $\alpha > 0$, the absolute magnitude of the optimal quotes for a static dealer increases as he is now obliged to protect himself against informed trades by widening his spread.

Substituting for $s_A^* = s_A(q)$ and $s_B^* = s_B(q)$ in (2) and (3), we obtain that, for $\alpha \leq 1/2$,

$$\Pi(s_A(q), s_B(q)) = \tag{5}$$
$$\begin{cases} ((1-\alpha) - q)^2/(2-2\alpha) & \text{if } 0 \leq q < \alpha, \\ (2q(q-1) + \alpha^2 + (1-\alpha)^2)/(2-2\alpha) & \text{if } \alpha \leq q \leq (1-\alpha), \\ (q-\alpha)^2/(2-2\alpha) & \text{if } (1-\alpha) < q \leq 1. \end{cases}$$

One may note that Π is quadratic and continuous in q and that it has an interior minimum at $q = 1/2$. The form of Π, which will be important for our results below, is shown in figure 3.3a.

3.1.3 Filtering

Suppose now that the dealer trades in more than one period. In this case, he may be able to use the information that he has gained from period 0 trades. The information is potentially valuable, first, because, in his own period 2 trading, it may permit him to quote a bid-ask spread that yields higher expected profits. Second, he may be able to "sell" the information to other dealers in that they may be willing to trade with him at advantageous terms in the interbank market in order to learn about his order flow.

Suppose that dealers use Bayes's rule to update their probability assess-

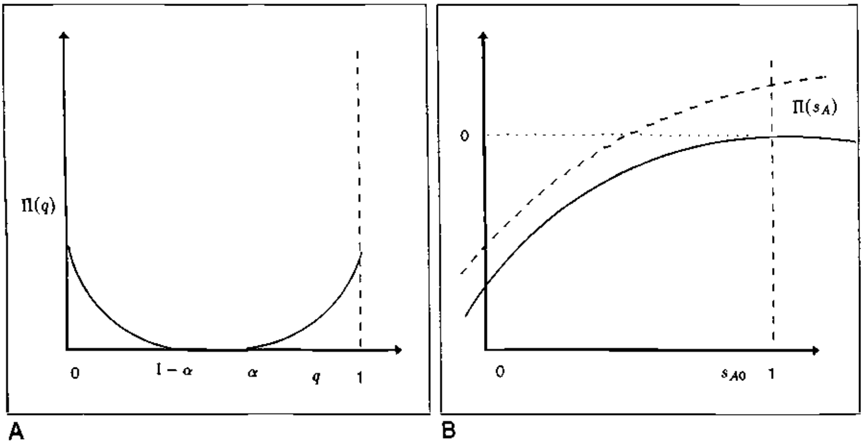

Fig. 3.3 Static profits and binomial probabilities

ments. In this case, the conditional probability of the even $\{z = 1\}$ following a buy order at 0, q_b, will be

$$(6)\qquad q_b \equiv \text{Prob}[z = 1|\text{buy}]$$
$$= \frac{\text{Prob}[z = 1 \text{ and buy}]}{\text{Prob}[\text{buy}]} = \frac{\alpha q + (1 - \alpha)q(1 - s_{A0})/2}{\alpha q + (1 - \alpha)(1 - s_{A0})/2}.$$

The updated probability for $\{z = 1\}$ following a sell, q_s, may be similarly derived as

$$(7)\qquad q_s \equiv \text{Prob}[z = 1|\text{sell}] = \frac{(1 - \alpha)q(1 + s_{B0})/2}{\alpha(1 - q) + (1 - \alpha)(1 + s_{B0})/2}.$$

The important point to note about the updated probabilities is that they depend on the bid and ask quotes in the first period, s_{B0} and s_{A0}. This dependency means that the dealer's choice of period 0 quotes will be influenced by the effect of order flow information on period 2 profits. Since order flow information may be valuable to other dealers, the possibility of "selling" this information through trading in the interbank market will also affect the dealer's optimal choice of period 0 quotes.

In standard, dynamic microstructure models such as those of Glosten and Milgrom (1985) or Easley and O'Hara (1987), this particular link between trading in different periods is broken by the fact that order flow information is assumed to be public knowledge.

3.1.4 Information Rents

Consider a dealer who has received no orders in period 0. If he trades again with outside customers in period 2 without receiving any information about other dealers' trades, his expected profits will be $\Pi(q)$. On the other hand, if he can buy information from another dealer who has received, for example, a buy order, his expected profits will be $\Pi(q_b)$.

The total increase in his expected profits when he learns of a buy order, $\Pi(q_b) - \Pi(q)$, may be decomposed into a news effect and a "feedback rent." Let $\Pi[s_A(q), s_B(q)|q_b]$ be the expected profits that the dealer obtains under updated probabilities, q_b, but under the assumption that he sets s_A and s_B as if the probability of $\{z = 1\}$ were q. Now, one may write

$$\begin{aligned} \Pi(q_b) - \Pi(q) \equiv\ & (\Pi(q_b) - \Pi(s_A(q), s_B(q)|q_b)) \\ & + (\Pi(s_A(q), s_B(q)|q_b) - \Pi(q)). \end{aligned} \tag{8}$$

The first bracketed term on the right-hand side is the expected value to the dealer of being able to adjust his quotes in response to the information. We refer to this extra value as the feedback rent.[3] This represents the increase in expected profits that the dealer can achieve by using the information to select his period 2 bid and ask quotes more efficiently. The second bracketed term in (8) is the pure news value of the information, that is, the change in the dealer's expected profits in a case in which he were, for some reason, unable to adjust his period 2 quotes. A similar decomposition can, of course, be performed for the change in expected profits due to information on a sale.

As mentioned above, our formulation of the interdealer market will entail informed dealers passing information to each other through their period 1 trades. The surplus over which they may be expected to bargain will then be the feedback rent, $\Pi(q_b) - \Pi(s_A(q), s_B(q)|q_b)$. Note that, even if $\Pi(q_b) - \Pi(q)$ is negative, dealers will be willing to pay each other for the information so long as the feedback rent is positive. This is analogous to the willingness of someone to pay for news that he is going to die so as long as the knowledge will allow him to take actions that prolong his life at least a little. Now, in our case, the feedback rent, $\Pi(q_b) - \Pi(s_A(q), s_B(q)|q_b)$, is nonnegative, as one may see from the fact that $\Pi(q_b) = \max_{q\in[0,1]}\{\Pi(s_A(q), s_B(q)|q_b)\}$. Hence, dealers will always be willing to pay for information.

We shall suppose the following:

Assumption 1. *Feedback rent associated with information on a customer trade is captured by the dealer who performs the trade.*

This assumption has the merit of substantially simplifying the analysis. Although it represents a polar case, we think it unlikely that our results would be substantially affected if a more even division of feedback rents were allowed for.

3. As in any dynamic programming problem under uncertainty, the dealer will do better if he can employ "feedback" controls that adjust according to information received.

3.1.5 The Dynamic Model

Suppose that a dealer who receives an order in period 0 is able to transact with a fraction, k, of other marketmakers in period 1. We think it reasonable to assume that k is closer to unity than zero. A substantial marketmaking operation with a sufficiently large dealing personnel can arrange simultaneous trades with fifteen to twenty other dealers. The number of international banks that trade in substantial size does not greatly exceed this figure.

Let $\mathrm{P}(A) \equiv [\alpha q + (1 - \alpha)(1 - s_{A0})/2]$ and $\mathrm{P}(B) \equiv [\alpha(1 - q) + (1 - \alpha)(1 + s_{B0})/2]$ denote the probabilities, respectively, of receiving an order at the ask or the bid in period 0. The period 0 value function for the dynamically optimizing dealer is

$$(9) \qquad V_0(s_{A0}, s_{B0}) = \frac{1}{n}\Bigg\{\Pi_0(s_{A0}) + \Pi_0(s_{B0}) + \mathrm{P}(A)\left[\frac{1}{n}\Pi_2(s_{A0}) + \frac{k(n-1)}{n}\mathrm{rent}(s_{A0})\right]$$

$$+ \mathrm{P}(B)\left[\frac{1}{n}\Pi_2(s_{B0}) + \frac{k(n-1)}{n}\mathrm{rent}(s_{B0})\right] + (1 - \mathrm{P}(A) - \mathrm{P}(B))\frac{1}{n}\Pi_2(s_{A2}, s_{B2})\Bigg\}$$

$$+ \frac{1}{n}\frac{n-1}{n}\{\mathrm{Prob}(A)\Pi(s_A(q), s_B(q)|q_b) + \mathrm{Prob}(B)\Pi(s_A(q), s_B(q)|q_s)$$

$$+ (1 - \mathrm{P}(A) - \mathrm{P}(B))\Pi(s_{A2}, s_{B2})\},$$

where s_{A2} and s_{B2} are the optimal static, uninformed ask and bid quotes given in equation (4).

When dealers can trade with all other dealers in period 1, the dynamic value function simplifies to

$$(10) \quad V_0(s_{A0}, s_{B0}) = \frac{1}{n}\{\Pi_0(s_{A0}) + \Pi_0(s_{B0}) + \mathrm{P}(A)\Pi_2(s_{A0}) + \mathrm{P}(B)\Pi_2(s_{B0}) + (1 - \mathrm{P}(A) - \mathrm{P}(B))\Pi_2(s_{A2}, s_{B2})\}.$$

Since all dealers are the same, by the symmetry of the problem, each dealer's value function equals $1/n$ times the expected value of the total market order flow. When $k = 1$, this is the above, simple, bracketed expression.

It is possible to obtain analytic solutions for the ask and bid quotes that maximize this value function since the first-order conditions turn out to be cubic functions of the prices.[4] In fact, the complexity of the resulting expressions means that they are not of much practical use. However, as we show in the results section below, one can learn a considerable amount by analyzing the first-order conditions and examining properties of the period 2 problem than by solving directly for bid and ask prices.

4. Although complicated, closed-form solutions to cubic equations are available (see Abramovitz and Stegun 1964).

3.2 Results

3.2.1 Information and Expected Profits

In this section, we establish two propositions on the value of information in our model. We start with the following:

Proposition 1. *The following three statements are equivalent:*
1. *News of a buy order increases total expected profits in period 2.*
2. *News of a buy order decreases dealers' estimates of the conditional variance of the exchange rate.*
3.

$$(11) \qquad q \geq 1/2, \quad \text{or} \quad \alpha > \frac{(1/2 - q)(1 - q)}{1/2 - q + q^2}.$$

A similar result holds for news of a sell order.

To understand what drives this result, examine figure 3.3 above. Panel b of the figure shows the total increase in expected profits associated with a buy order, $\Pi(q_b) - \Pi(q)$. One may easily demonstrate that $q_b \geq q$ and $q_s \leq q$. In the case depicted, the initial unconditional probability q equals q_0, while $q_b > q_0 > 1/2$. The fact that $\Pi(q)$ is quadratic and has a minimum at 1/2 immediately implies that $\Pi(q_b) - \Pi(q) > 0$; that is, information of a buy order implies higher expected profits. On the other hand, since in the diagram $\Pi(q_s) < \Pi(q)$, it follows that news of a sale lowers expected profits.

Why does more information in the latter case lead to lower expected profits? The reason is as follows. One may show that the variance of the binomially distributed random variable, z, equals $4q(1 - q)$ and that this has a maximum at $q = 1/2$. Thus, any information that implies a filtered, updated probability, q_u, that lies closer to 1/2 than the original unconditional q also implies a increase in variance.[5]

But higher variance lowers the dealer's expected profits for the reason that the profit function (see eqq. [2] and [3]) is made up of kinked functions of the underlying payoff, z. In this respect, the dealer's profits resemble a short position in call and put options. Such claims are concave in the random payoff, so, by Jensen's inequality, adding uncertainty lowers expected value.

The pure news value of information in our model takes the simple form

$$(12) \qquad \text{pure news value} \equiv (\Pi(q_b) - \Pi(q)) - (\Pi(q_b) - \Pi(s_A(q), s_B(q)|q_b)) = \frac{2q - 1}{1 - \alpha}(q_b - q).$$

This immediately implies the following result:

5. Readers more familiar with normal filtering problems may find this slightly surprising as updating, in that case, reduces uncertainty. In the present context, for certain values of q, updating actually increases the conditional variance.

Proposition 2. *When buy and sell orders are equally probable, the pure news value of information is zero, and the feedback rent associated with information equals the change in expected profits.*

In other words, when $q = 1/2$, the pure news value is zero. To see the intuition behind this finding, suppose that the dealer cannot adjust his quotes in response to the information. Recall that the only three possible events are a buy order, a sell order, and no trade. If there is no trade, the dealer still has $\Pi(q)$. When $q = 1/2$, the problem is completely symmetric, and, hence, information of either a bid or an ask order must change expected profits by the same amount. But, if all news has the same effect on expected value, it must be that the effect is zero. Hence, the only possible increase in value from the news must come from the dealer's ability to adjust his quotes conditional on information. That is, the increase in total value equals the feedback rent.

3.2.2 Bid-Ask Spreads

In this section, we show that the ask of a dynamically optimizing dealer is greater than that of a static dealer. Since, when n is large, dealers in a centralized market behave as though they are static, profit maximizers, this statement may be regarded as a statement about the behavior of dealer prices in centralized versus decentralized markets. We state the result formally as follows:

Proposition 3. *The optimal period 0 ask of a dynamically optimizing dealer in the decentralized market exceeds that of a static solution. A corresponding result holds for decentralized market bids that exceed static bids in absolute magnitude.*

By increasing the bid-ask spread from its static level, marketmakers sacrifice short-term expected profits. On the other hand, however, they improve the quality of the information that they derive from period 0 trades since uninformed trades are discouraged from transacting. Using the improved information, dealers can earn higher expected profits in subsequent trading.

Corollary 1. *As* n $\rightarrow \infty$, *the bid-ask spread is wider in a decentralized than in a centralized market.*

3.2.3 Efficiency

In this section, we compare the efficiency of centralized and decentralized markets. It should be apparent that the two market organizations each have advantages and disadvantages. In a centralized market, dealers observe all period 0 customer trades so that they can always update their subjective probability assessments and correspondingly adjust their period 2 bid and ask.

On the other hand, in a centralized market, the incentives of dealers to adjust their period 0 bid and ask so as to elicit an efficient amount of information are diluted. In the limit, as the number of dealers $n \rightarrow \infty$, individual dealers in a centralized market will set s_{A0} and s_{B0} in a way that totally ignores the informational rents associated with period 0 trading. In other words, they will act as though they are static profit maximizers.

In a decentralized market, the opportunity to sell information improves incentives to elicit information by optimal adjustment of the bid-ask spread. In the limit, when $k = 1$ and dealers can transact with all other marketmakers, the entire market feedback rent associated with a period 0 customer order is captured by the dealer who receives the order. In this case, marketmakers will optimally adjust their spreads to elicit information, and hence the decentralized market will be private efficient from the dealers' collective point of view.

However, if dealers can transact with only a fraction of other marketmakers, the advantage of better incentives will be reduced as it will then not be possible to capture the feedback rent associated with information. In addition, the decentralized market will suffer from the fact that dealers who do not receive interdealer trades will be unable to update their probabilities in response to period 0 order flow.

To analyze this trade-off formally, we start with the following lemma:

Lemma 1. *The unconditional expectation of period 2 profits is greater when dealers are able to update probabilities on the basis of observation of period 0 order flow.*

Lemma 1 demonstrates that the ability to adjust quotes in response to information on customer trades in period 0 increases the unconditional expectation of period 2 profits. Using this lemma, one may prove the following proposition:

Proposition 4. *If dealers are able to transact with all other marketmakers in the interval between customer trades, that is, if* $\mathrm{k} = 1$, *a decentralized market is fully efficient. If* k *is small, however, total expected dealer profits are higher in a centralized than a decentralized market.*

In the remainder of the paper, we shall assume for simplicity that $k = 1$.

3.2.4 Market Crashes

A point stressed by Glosten and Milgrom in their classic 1985 paper on dealer behavior was that markets with too many informed traders may collapse as dealers will be unable to make positive profits given the adverse selection problems that they face. Such market crashes will involve collapses in volume as bid-ask spreads increase until the market closes. Note that it is possible for the market to close on one or both sides of the bid-ask spread. Crashes are costly because they undermine the informativeness of prices.

In our discussion of the static model in section 3.1, we have already implicitly considered such market collapses by discussing cases in which optimal ask and bid prices equal, respectively, plus and minus unity. Let us suppose, as seems reasonable, the following:

Assumption 2. *If* $\mathrm{s}_A = 1$, *informed traders never buy, while, if* $\mathrm{s}_B = 1$, *informed traders never sell.*

Of course, in these cases, informed customers will be indifferent between trading and not trading, but any slight friction would be enough to make them strongly prefer not to trade.

The expressions for ask and bid prices in the static model, $s_A = \min\{q/(1-\alpha), 1\}$ and $s_B = \max\{-(1-q)/(1-\alpha), -1\}$, immediately suggest under what conditions markets with static dealers will collapse. For any given q, if α is large enough, $s_A = 1$, and $s_B = -1$. On the other hand, for any given α, if $q \to 1$, eventually, $s_A = 1$, while, if $q \to 0$, eventually, $s_B = 1$.

Notice from the discussion in the last paragraph that there are two reasons why the static market may collapse, either (i) too many informed traders (α large for q around 1/2) or (ii) too little uncertainty about the value of the exchange rate (q close to zero or unity). These two reasons lead to qualitatively different outcomes in that in the first both sides of the market close while in the second only one side of the market crashes.

Figure 3.4a illustrates the way in which in the static model, for a given α, different assumptions on q may generate crashes. In the case illustrated (in which $\alpha > 1/2$), for $\alpha < q < (1-\alpha)$, the market crashes on both sides, and expected profits are zero. For $q > \alpha$, the market crashes only on the ask side, while, for $q < (1-\alpha)$, it crashes only on the bid.

One of the most interesting implications of our model is the following:

Proposition 5. *The decentralized market never collapses in period 0.*

The interest of this result is that it suggests that decentralized markets are significantly more robust to the asymmetric information problems that provoke collapses in the static model. Recall that, as $n \to \infty$, dealers in centralized markets behave like static profit maximizers, so once again this sheds light on the differences between centralized and decentralized market arrangements.

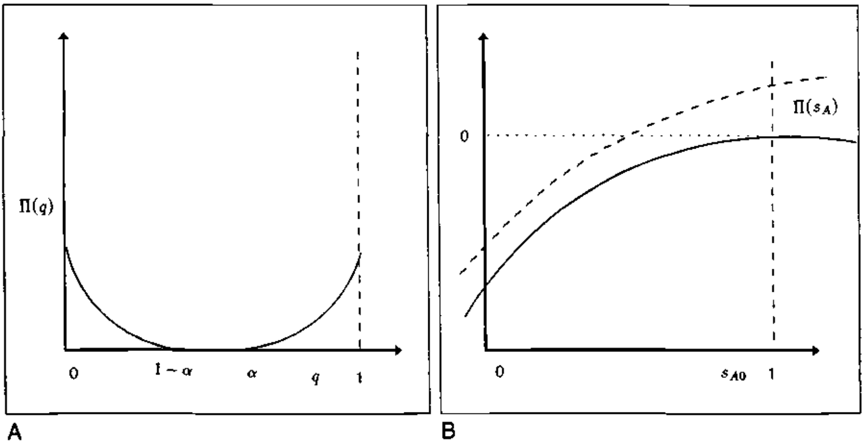

Fig. 3.4 Market crashes and information

The intuition behind proposition 5 is that in a decentralized market dealers have an incentive to provide a small but sufficient incentive for informed traders to transact and hence reveal their information. Given our assumptions about the price elasticity of orders by uninformed traders, if $s_{A0} = 1 - \varepsilon$ or $s_{B0} = -1 + \varepsilon$ for small, positive ε, dealers can obtain very good information in the event of a buy or sell order, respectively. As long as there is some rent to be extracted from this information in the form of higher period 2 profits, dealers will always have an incentive to open the market by adjusting their quotes enough to elicit trades from informed customers.

The above proposition is illustrated by figure 3.4b, which shows expected ask-side profits in the static and dynamic models as a function of the period 0 quote. The static expected profits equal zero at $s_{A0} = 1$, and unity is clearly the maximizing argument. The dynamic expected profits, which appear as a dotted line in the figure, are positive and increasing for ask quotes in an open interval below 1 and then, in fact, drop to zero at 1. The fact that they are positive for $s_{A0} = 1 - \varepsilon$ for small, positive ε is what gives the above result.

3.2.5 Martingale Properties and Volatility

A feature of standard market microstructure models with competitive marketmakers (see, e.g., Glosten and Milgrom 1985; and Easley and O'Hara 1987) is that bid and ask prices are martingales with respect to the information available to dealers. In this section, we shall see that, in our decentralized market model, this is no longer the case and that, in fact, bid and ask prices exhibit mean reversion as information is revealed.

Assumption 3. *Suppose that interior solutions exist for the static model in period 0, that is, that* $\alpha < q < 1 - \alpha$.

First, consider the unconditional expectation of the difference between ask prices in periods 0 and 2.

Proposition 6. *In the static model,*

$$E[s_{A2} - s_{A0}|S] = 0, \quad E[s_{B2} - s_{B0}|S] = 0, \tag{13}$$

while, in the dynamic model,

$$E[s_{A2} - s_{A0}|D] < 0, \quad E[s_{B2} - s_{B0}|D] > 0. \tag{14}$$

This proposition shows that weak-form market efficiency does not hold in our dynamic decentralized market while, in a centralized market with a large number of dealers, bids and asks will be martingales with respect to marketmakers' information. The basic feature of the model that permits deviations from martingale behavior is the market power that we assume for dealers. O'Hara (1994) comments on the fact that monopolistic elements can give such deviations.

Decentralized and centralized markets also differ in the amount of volatility that they imply. One may show that the unconditional variance of changes in bid and ask is greater in the decentralized case.

Proposition 7. *The unconditional variance of quote changes is greater in the dynamic than in the static case, that is,*

(15) $$\mathrm{Var}(s_{A2} - s_{A0}|S) < \mathrm{Var}(s_{A2} - s_{A0}|D),$$

(16) $$\mathrm{Var}(s_{B2} - s_{B0}|S) < \mathrm{Var}(s_{B2} - s_{B0}|D).$$

3.2.6 Comparative Statics

Proposition 3 simply states that the bid-ask spread is larger in a decentralized than in a centralized market, without solving for the values taken by the bid and ask quotes. Although exact solutions for these quotes can be found, they are so complex that little more can be deduced. One may still, however, analyze the first-order condition of the maximization problem to learn more about these solutions.

Proposition 8. *Let* s^*_{A0} *be an internal optimum for the ask price. Then the following results hold:*

(17) $$\frac{\partial s^*_{A0}}{\partial q} > 0, \quad \frac{\partial s^*_{A0}}{\partial \alpha} > 0, \quad \text{for } \alpha < q\,.$$

As a first result, proposition 8 shows that the ask price always increases when the probability of the event $\{z = 1\}$ rises. Furthermore, an increase in the proportion of informed traders produces a rise in the ask. The reasons for these results are two: first, the need for protection against the informed traders is stronger; second, there is an incentive to increase the size of the bid-ask spread because with more informed traders it is possible to get more information.

Proposition 9. *Suppose that* $\alpha < q < 1 - \alpha < q_b$ *and that the quotes chosen by the dealer are internal optima. Then the following results hold:*

(18) $$\frac{\partial\,\text{rent}}{\partial q} < 0,$$

(19) $$\frac{\partial\,\text{rent}}{\partial \alpha} > 0.$$

The result on $\partial\text{rent}/\partial q$ is interesting as it indicates that, for given α, when there is less uncertainty (q further from 1/2), there is less ex ante information in the order flow, that is, less opportunity to sell information.

3.2.7 Interdealer Market Volume

When a marketmaker has received some information, we assume that he can sell it in the second period to all the remaining marketmakers. He accomplishes this by transacting on favorable terms through the interdealer market.

We suppose that marketmakers will quote other dealers a bid-ask spread that (i) is "regret free" according to the definition of Glosten and Milgron, in that after a transaction marketmakers do not regret having completed it, (ii) transfers feedback rent to the informed marketmaker, (iii) creates no incentives for uninformed marketmakers to pretend to possess information, and (iv) minimizes the quantity transacted.

The last property requires some comment. A given rent can be transferred between dealers by various combinations of price and quantity. In this sense, the prices quoted between dealers are indeterminate. However, point iv implies unique interdealer quotes since, if the transaction size is reduced too far and the spread made too generous, eventually uninformed dealers will be able to make profits masquerading as informed. Assuming that the size of trades is reduced to a minimum implies that the incentive constraint implicit in point iii must hold as an equality and hence determines interdealer quotes.

Now, suppose that marketmaker 1 has received a buy order at time 0 (the same reasoning applies for a sell order) and wants to sell the information to marketmaker 2. As he is willing to buy the currency and his expected value of the currency is $2q_b - 1$, marketmaker 2 will quote an ask price in the interval $(2q - 1, 2q_b - 1)$, and a transaction will be completed for a quantity Δx that conveys the rent of a buy order to marketmaker 1 (subject to the assumed properties of interdealer trade (points i–iv above). Proposition 10 reports an interesting result concerning this value:

Proposition 10. *Let us define Δx^* as the minimum quantity transacted among two marketmakers. Suppose that the regularity conditions of proposition 9 are satisfied. The following result holds:*

$$\frac{\partial \Delta x^*}{\partial \alpha} > 0 . \tag{20}$$

The proposition simply indicates that, given these conditions, the volume of transactions between marketmakers is indicative of the informativeness of the order flow as the minimum quantity transacted among two marketmakers is an increasing function of α.

3.3 Conclusion

This paper has provided a theoretical analysis of a decentralized dealer market. Although our results are relevant to a broad category of markets in which order flow information is not publicly available the primary motivation for our study was the desire to understand price formation and efficiency in the foreign exchange market. Our main findings are the following:

1. Bid ask spreads are wider in the decentralized market. The intuition here is that, by posting wider spreads, dealers can discourage price-sensitive liquid-

ity traders and hence improve the informativeness of their order flow. The information embodied in orders can in turn be used to earn higher future profits and can be "sold" to other marketmakers through interbank transactions at advantageous prices.

2. Decentralized markets are privately efficient from the collective point of view of marketmakers when it is possible for dealers to transact with all other dealers in between potentially informative customer trades. This point underlines the potential importance of brokers as a way of facilitating large numbers of simultaneous transactions with other marketmakers.

3. Decentralized markets are much less subject to market crashes than centralized markets. Information on order flow may be used to update subjective estimates of the underlying value of exchange rates. Even in circumstances in which static or centralized markets would crash owing to excessive numbers of informed traders, dealers will have an incentive to preserve some turnover in the decentralized market as they can employ the information in the order flow in subsequent trading. Our model allows only two periods of trading with customers, but we would conjecture that our results on crashes would hold in a multiperiod model, in that dealers would always have an incentive to preserve at least some order flow to gain information.

4. The time-series behavior of exchange rates in our model differs according to whether trading is organized on a centralized or a decentralized basis. When dealers maximize profits in a static fashion (which they will do in a centralized market containing large numbers of marketmakers), bid and ask quotes are martingales with respect to the information available to dealers. In the decentralized market, bid-ask spreads on average shrink as order flow reveals information.

It is very interesting to note that this implication of the model is consistent with the findings of Goodhart and Figliuoli (1991). Their study suggests that, prior to jumps in exchange rates, there is an increase in the negative autocorrelation. If we regard jump times as moments at which significant information becomes public knowledge (i.e., corresponding to our period 3), then our model would suggest that, in the immediately preceding period, a small number of agents will know the information and dealers will be adjusting quotes so that the bid-ask spread is contracting on average.

5. Another implication of the model for the statistical properties of exchange rates is that changes in rates will be more variable in the decentralized than in the centralized market. It is perhaps not clear quite what is the quantitative significance of this difference in variance, but, given the widely acknowledged volatility of exchange rates, it is at least reassuring that our model predicts greater variance in decentralized markets.

Appendix

Proofs are stated for the ask side of the market throughout. Similar arguments apply to the bid side.

Proof of Proposition 1

Proposition 1. *The following three statements are equivalent:*

1. *News of a buy order increases total expected profits in period 2.*
2. *News of a buy order decreases dealers' estimates of the conditional variance of the exchange rate.*
3.

$$q \geq 1/2, \quad \text{or} \quad \alpha > \frac{(1/2 - q)(1 - q)}{1/2 - q + q^2}. \tag{21}$$

A similar result holds for news of a sell order.

Proof. The equivalence of the first two statements is obvious; in fact, as $\Pi(s_A(q), s_B(q))$ is symmetric around 1/2, we have a gain in the expected profits from a buy order if $q_b - 1/2 > 1/2 - q$; that corresponds to a reduction in the conditional variance of the exchange rate. Moreover, $q_b - 1/2 > 1/2 - q$ holds if

$$\alpha q^2 + (1 - \alpha)(1 - s_{A0})(q - 1/2) > 0. \tag{22}$$

Then, for $s_{A0} = q(1 - \alpha)$, this condition becomes

$$\alpha q^2 > (1 - \alpha - q)(1/2 - q).$$

It is immediately obvious that this condition holds for the values of α and q respecting the condition (21). This completes the proof.

Proof of Proposition 3

Proposition 3. *The optimal period 0 ask of a dynamically optimizing dealer in the decentralized market exceeds that of the static solution. A corresponding result holds for decentralized market bids that exceed static bids in absolute magnitude.*

Proof. Let V_0' be that part of the dynamic value function that depends on the ask price, s_{A0}, multiplied by the constant n. V_0' equals

$$V_0'(s_{A0}) \equiv \Pi_0(s_{A0}) + \frac{(n-1)+1}{n}\text{Prob}(A)\Pi_2(s_{A0}) - \frac{1}{n}\left[(n-1)\text{Prob}(A) + \frac{1 - s_{A0}}{2}(1-\alpha)\right]\Pi_2(s_{A2}, s_{B2}). \tag{23}$$

Assuming that $\alpha < 1/2$, and given the different form of the static value function for different configurations of q and α, we consider six cases: $q < q_b < \alpha$;

$q < \alpha \le q_b < 1 - \alpha$; $q < \alpha < 1 - \alpha \le q_b$; $\alpha < q < q_b < 1 - \alpha$; $\alpha < q < 1 - \alpha \le q_b$; and $q > 1 - \alpha$; so that $q_b > 1 - \alpha$. In all six cases, we have

$$(24)\quad \frac{\partial V'}{\partial s_{A0}} = \frac{\partial \Pi_0(s_{A0})}{\partial s_{A0}} + \left(\frac{\partial}{\partial s_{A0}}[\text{Prob}(A)\Pi_2(s_{A0})]\right) + \frac{1-\alpha}{2}\Pi_2(s_{A2}, s_{B2})$$
$$= q - (1-\alpha)s_{A0} + \Gamma = 0.$$

We show that in all cases Γ is positive and therefore

$$(25)\quad s^*_{A0} \ge \text{the static solution.}$$

In particular, we show that in the first five cases

$$(26)\quad s^*_{A0} > \frac{q}{1-\alpha} = \text{the static solution,}$$

while in the last case the static and the dynamic solutions are both equal to one.

Case 1. $q < q_b < \alpha$. The components of Γ are as follows:

$$(27)\quad \frac{1-\alpha}{2}\Pi_2(s_{A2}, s_{B2}) = \frac{[(1-\alpha)-q]^2}{4},$$

$$(28)\quad \text{Prob}(A)\frac{\partial \Pi_2(s_{A0})}{\partial s_{A0}} = \frac{(q_b - q)\,[q_b - (1-a)]}{2},$$

$$(29)\quad \frac{\partial\, \text{Prob}(A)}{\partial s_{A0}}\Pi_2(s_{A0}) = -\frac{[(1-\alpha)-q_b]^2}{4}.$$

Since

$$(30)\quad \frac{\partial q_b}{\partial s_{A0}} = \frac{(q_b - q)(1-\alpha)}{2\,\text{Prob}(A)},$$

$$(31)\quad \frac{\partial\, \text{Prob}(A)}{\partial s_{A0}} = -\frac{1-\alpha}{2}.$$

It then follows that

$$(32)\quad \Gamma \equiv \frac{(q_b - q)^2}{4}.$$

Case 2. $q < \alpha \le q_b < 1 - \alpha$.

$$(33)\quad \frac{1-\alpha}{2}\Pi_2(s_{A2}, s_{B2}) = \frac{[(1-\alpha)-q]^2}{4},$$

$$(34)\quad \text{Prob}(A)\frac{\partial \Pi_2(s_{A0})}{\partial s_{A0}} = \frac{(2q_b - 1)(q_b - q)}{2},$$

$$(35) \qquad \frac{\partial \text{ Prob}(A)}{\partial s_{A0}} \Pi_2(s_{A0}) = \frac{(1-q_b)q_b}{2} - \frac{\alpha^2 + (1-\alpha)^2}{4}.$$

We end up with

$$(36) \qquad \Gamma \equiv \frac{2(q_b - q)^2 - (q - \alpha)^2}{4}.$$

This is positive as $q_b > q$ and $\alpha > q$.
Case 3. $q < \alpha < 1 - \alpha < q_b$. Using the fact that

$$(37) \qquad \frac{1-\alpha}{2}\Pi_2(s_{A2}, s_{B2}) = \frac{[(1-\alpha) - q]^2}{4},$$

$$(38) \qquad \text{Prob}(A) \frac{\partial \Pi_2(s_{A0})}{\partial s_{A0}} = \frac{(q_b - \alpha)(q_b - q)}{2},$$

$$(39) \qquad \frac{\partial \text{ Prob}(A)}{\partial s_{A0}} \Pi_2(s_{A0}) = -\frac{(q_b - \alpha)^2}{4}.$$

It follows that

$$(40) \qquad \Gamma \equiv \frac{\{(q_b - \alpha)^2 + 2(q_b - \alpha)(\alpha - q) + [(1-\alpha) - q]^2\}}{4}.$$

That is positive as $q_b > \alpha$ and $\alpha > q$.
Case 4. $\alpha < q < q_b < 1 - \alpha$. We now have

$$(41) \qquad \frac{(1-\alpha)}{2}\Pi_2(s_{A2}, s_{B2}) = \frac{\alpha^2 + (1-\alpha)^2}{4} - \frac{(1-q)q}{2},$$

$$(42) \qquad \text{Prob}(A) \frac{\partial \Pi_2(s_{A0})}{\partial s_{A0}} = \frac{(2q_b - 1)(q_b - q)}{2},$$

$$(43) \qquad \frac{\partial \text{ Prob}(A)}{\partial s_{A0}} \Pi_2(s_{A0}) = \frac{(1-q_b)q_b}{2} - \frac{\alpha^2 + (1-\alpha)^2}{4}.$$

Hence

$$(44) \qquad \Gamma \equiv \frac{(q_b - q)^2}{2}.$$

Case 5. $\alpha < q < 1 - \alpha \leq q_b$. We have that

$$(45) \qquad \frac{1-\alpha}{2}\Pi_2(s_{A2}, s_{B2}) = \frac{\alpha^2 + (1-\alpha)^2}{4} - \frac{(1-q)q}{2},$$

$$(46) \qquad \text{Prob}(A) \frac{\partial \Pi_2(s_{A0})}{\partial s_{A0}} = \frac{(q_b - \alpha)(q_b - q)}{2},$$

$$\frac{\partial\,\mathrm{Prob}(A)}{\partial s_{A0}}\Pi_2(s_{A0}) = -\frac{(q_b-\alpha)^2}{4}. \tag{47}$$

This implies that

$$\Gamma \equiv \frac{\{2q_b(q_b-q) + [(1-\alpha)-q]^2\}}{4}. \tag{48}$$

But, since $q_b > 1-\alpha > q$, it follows that $q_b^2 > -(1-\alpha)[2q-(1-\alpha)]$. So $\Gamma > 0$ as required. Finally, consider the last case:
Case 6. $q > 1-\alpha$ so that $q_b > 1-\alpha$. Here we have

$$\frac{1-\alpha}{2}\Pi_2(s_{A2}, s_{B2}) = \frac{(q-\alpha)^2}{4}, \tag{49}$$

$$\mathrm{Prob}(A)\frac{\partial\Pi_2(s_{A0})}{\partial s_{A0}} = \frac{(q_b-\alpha)(q_b-q)}{2}, \tag{50}$$

$$\frac{\partial\,\mathrm{Prob}(A)}{\partial s_{A0}}\Pi_2(s_{A0}) = -\frac{(q_b-\alpha)^2}{4}. \tag{51}$$

Therefore

$$\begin{aligned}\Gamma &\equiv \frac{[2(q_b-\alpha)(q_b-q) + (q-\alpha)^2 - (q_b-\alpha)^2]}{4}\\ &= \frac{[2q_b(q_b-q) + q^2 - q_b^2]}{4} = \frac{[q_b-q]^2}{4} > 0.\end{aligned} \tag{52}$$

This completes the proof.

Proof of Efficiency Results

> **Lemma 1.** *The unconditional expectation of period 2 profits is greater when dealers are able to update probabilities on the basis of observation of period 0 order flow.*

Proof. Define $E\Pi_u$ as the unconditional expectation of period 2 profits when probabilities are updated at 1 after the observation of a trade should one occur:

$$\begin{aligned}E\Pi_u = {} & \mathrm{Prob(buy)}\left[\max_{s_{A2}, s_{B2}} \Pi(s_{A2}, s_{B2} \mid q_b)\right]\\ & + \mathrm{Prob(sell)}\left[\max_{s_{A2}, s_{B2}} \Pi(s_{A2}, s_{B2} \mid q_s)\right]\\ & + [1 - \mathrm{Prob(buy)} - \mathrm{Prob(sell)}]\left[\max_{s_{A2}, s_{B2}} \Pi(s_{A2}, s_{B2} \mid q)\right].\end{aligned} \tag{53}$$

The unconditional expectation of profits without updating is denoted $E\Pi$. By the iterative property of conditional expectations, this can be expressed in the following way:

$$\text{E}\Pi = \max_{s_{A2}, s_{B2}} \Big[\text{Prob(buy)}\ \Pi(s_{A2}, s_{B2} \mid q_b)$$

$$(54) \qquad + \text{Prob(sell)}\ \Pi(s_{A2}, s_{B2} \mid q_s)$$

$$+ [1 - \text{Prob(buy)} - \text{Prob(sell)}]\ \Pi(s_{A2}, s_{B2} \mid q) \Big] .$$

It is then clear that $\text{E}\Pi_u > \text{E}\Pi$ for Prob(buy) + Prob(sell) > 0.□

Proposition 4. *If dealers are able to transact with all other marketmakers in the interval between customer trades, that is, if* k $= 1$, *a decentralized market is fully efficient. If* k *is small, however, total expected dealer profits are higher in a centralized than a decentralized market.*

Proof. Follows from lemma 1.

Proof of Proposition 5

Proposition 5. *The decentralized market never collapses in period 0.*

Proof. We can prove this statement in two steps. In the first, we prove that the market cannot be always completely closed in period 2 and at least on one side in period 0. In the second, we prove that, if the market is open at least on one side in the second period, it cannot be closed on either side in period 0.

Step 1. Suppose that the marketmaker's strategy implies that the market is always closed in period 2 and is closed on the sell side in period 0; we show there exists another strategy that dominates it. Suppose that we fix in period 0 $s_{A0} - 1$ and $s_{B0} = -1 + \varepsilon$, for $\varepsilon > 0$ and small. Suppose that a dealer will set $s_{A2} = 1/2$ and $s_{B2} = -1$ if he receives a sell order and $s_{A2} = 1$ and $s_{B2} = -1$ otherwise. As the sell order in period 0 for a given dealer will occur with probability $\alpha(1 - q)/n$, and as the bid-ask spread will have a negligible effect on period 0 profits, the total expected profits will be $(1 - \alpha)(1 - s_{A2})(1 + s_{A2})/2 = 3(1 - \alpha)/8$; this implies that the second strategy dominates the first one. A similar argument works for the ask side.

Step 2. Suppose that the ask side is closed in period 0. Lowering s_{A0} slightly hardly affects period 0 expected profits but means that, in the event of a buy order, the dealer receiving it knows almost surely that $z = 1$. The profit function with the ask side closed in period 0 is

$$(55) \qquad V_0(s_{A0}, s_{B0}) = V_0(1, s_{B0}) = \frac{1}{n} \Big\{ \Pi_0(s_{B0}) + P(B)\Pi_2(s_{B0})$$

$$+ (1 - P(B))\Pi_2(s_{A2}, s_{B2}) \Big\} .$$

With $s_{A0} = 1 - \varepsilon$, we have

$$(56) \quad V_0(s_{A0}, s_{B0}) = V_0(1 - \varepsilon, s_{B0}) = \frac{1}{n} \Big[\Pi_0(s_{B0}) + P(A)\Pi_2(s_{A0} \,|_{\, s_{A0} = 1 - \varepsilon})$$

$$+ P(B)\Pi_2(s_{B0}) + (1 - P(A) - P(B))\Pi_2(s_{A2}, s_{B2})\Big] .$$

Hence, the result follows if $\Pi_2(s_{A0}|_{s_{A0}=1-\varepsilon}) > \Pi_2(s_{A2}, s_{B2})$. But, as long as q is different from 1,

$$\Pi_2(s_{A0}|_{s_{A0}=1-\varepsilon}) = \Pi_2(\hat{q}|_{\hat{q}=1}) > \Pi_2(q) = \Pi_2(s_{A2}, s_{B2}) . \tag{57}$$

Therefore, the ask side of the market will not be always closed in period 2; a similar argument holds for the bid side. This completes the proof. □

Statistics of Quote Changes

Proposition 6. *In the static model,*

$$E[s_{A2} - s_{A0}|S] = 0, \quad E[s_{B2} - s_{B0}|S] = 0, \tag{58}$$

while, in the dynamic model,

$$E[s_{A2} - s_{A0}|D] = 0, \quad E[s_{B2} - s_{B0}|D] > 0 . \tag{59}$$

Proof. In the dynamic model, we can write the difference in expectations as:

$$\begin{aligned} E[s_{A2} - s_{A0}|D] &= \text{Prob(sell)}\left(\frac{q_s}{1-\alpha} - s_{A0}\right) + \text{Prob(buy)}\left(\frac{q_b}{1-\alpha} - s_{A0}\right) \\ &\quad + \text{Prob(no trade)}\left(\frac{q}{1-\alpha} - s_{A0}\right) \\ &= \frac{q}{1-\alpha} - s_{A0} < 0, \end{aligned} \tag{60}$$

where we use the fact that $\text{Prob(sell)}(q_s - q) = -\text{Prob(buy)}(q_b - q)$ and $s_{A0} > q/(1 - \alpha)$.□

Proposition 7. *The unconditional variance of quote changes is greater in the dynamic than in the static case, that is,*

$$\text{Var}(s_{A2} - s_{A0}|S) < \text{Var}(s_{A2} - s_{A0}|D),$$
$$\text{Var}(s_{B2} - s_{B0}|S) < \text{Var}(s_{B2} - s_{B0}|D).$$

Proof. Consider volatility with the two sets of quote-setting behavior. In both static and dynamic cases,

$$\begin{aligned} \text{Var}(s_{A2} - s_{A0}) &= \text{Prob(sell)}\left[\frac{q_s - q}{1-\alpha}\right]^2 + \text{Prob(buy)}\left[\frac{q_b - q}{1-\alpha}\right]^2 \\ &= \left[\frac{\alpha q(1-q)}{1-\alpha}\right]^2\left(\frac{1}{\text{Prob(buy)}} + \frac{1}{\text{Prob(sell)}}\right) . \end{aligned} \tag{61}$$

The result then follows from the fact that Prob(buy) and Prob(sell) are larger in the static than in the dynamic case.□

Proof of Comparative Statics

Proposition 8. *Let s^*_{A0} be an internal optimum for the ask price. Then the following results hold:*

$$\frac{\partial s^*_{A0}}{\partial q} > 0, \quad \frac{\partial s^*_{A0}}{\partial \alpha} > 0, \quad \text{for } \alpha < q \,. \tag{62}$$

Proof. We can use the first-order condition to study the effect of a change in any parameter of the model, β, on s^*_{A0}, as the second-order condition guarantees that we still have an internal solution; therefore we consider

$$\frac{\partial s^*_{A0}}{\partial \beta} = -\frac{\partial^2 V'/\partial\beta\partial s^*_{A0}}{\partial^2 V'/\partial s^{*2}_{A0}} \,. \tag{63}$$

Now, as from the second-order condition it follows that $\partial^2 V'/\partial s^{*2}_{A0} < 0$, the sign of the derivative of s^*_{A0} with respect to β corresponds to that of the numerator. Hence, consider $\partial^2 V'/\partial q\partial s^*_{A0}$, which is equal to $1 + \partial\Gamma/\partial q$. To prove that this is always positive, we have to consider five of the six cases discussed in proposition 2 because for $q > 1 - \alpha$ we do not have an internal optimum. It is easy to show that

$$\frac{\partial(q_b - q)}{\partial q} = \frac{\alpha(1 - q - q_b)}{\Delta}, \tag{64}$$

where $\Delta = \alpha q + (1 - \alpha)(1 - s_{A0})/2$; for $q + q_b > 1$, as $1 - q_b > 0$ and $\Delta > \alpha q$ this derivative is negative but larger than -1; while, for $q + q_b < 1$, this derivative is positive. This permits us to show that in all cases $\partial\Gamma/\partial q > -1$.

In case 1,

$$\frac{\partial\Gamma}{\partial q} = \frac{\partial(q_b - q)}{\partial q}\frac{q_b - q}{2} \,. \tag{65}$$

Therefore $\partial\Gamma/\partial q > 0$ as $q + q_b < 1$.

In case 2,

$$\frac{\partial\Gamma}{\partial q} = \frac{\partial(q_b - q)}{\partial q}(q_b - q) + \frac{\alpha - q}{2} \,. \tag{66}$$

Therefore $\partial\Gamma/\partial q > 0$ as $q + q_b < 1$ and $\alpha > q$.

In case 3,

$$\frac{\partial\Gamma}{\partial q} = \{\frac{\partial q_b}{\partial q}(q_b - q) - (q_b - \alpha) - [(1 - \alpha) - q]\}/2 > -1. \tag{67}$$

In case 4,

(68)
$$\frac{\partial \Gamma}{\partial q} = (q_b - q)\,\frac{\partial (q_b - q)}{\partial q} > -1.$$

Finally, in case 5, we have

(69)
$$\frac{\partial \Gamma}{\partial q} = \{(q_b - q)\,\frac{\partial (q_b - q)}{\partial q} + q_b\frac{\partial q_b}{\partial q} - (1 - \alpha)\}/2 > -1.$$

Let us consider $\frac{\partial^2 V'}{\partial \alpha \partial s^*_{A0}} = s^*_{A0} + \partial\Gamma/\partial\alpha$. In this case, we have that

(70)
$$\frac{\partial q_b}{\partial \alpha} = \frac{(1 - s^*_{A0})q(1 - q)}{2\Delta^2} > 0.$$

For $\alpha < q$ and $q_b < 1 - \alpha$, we have to discuss only case 4; we can easily prove that $s^*_{A0} + \partial\Gamma/\partial\alpha$ is positive. In fact, we have

(71)
$$\frac{\partial \Gamma}{\partial \alpha} = (q_b - q)\,\frac{\partial q_b}{\partial \alpha} > 0.$$

Proposition 9. *Suppose that* $\alpha < q < 1 - \alpha < q_b$ *and that the quotes chosen by the dealer are internal optima. Then the following results hold:*

$$\frac{\partial \text{ rent}}{\partial q} < 0,$$

$$\frac{\partial \text{ rent}}{\partial \alpha} > 0.$$

Proof. We assume that α is such that $\alpha < q < q_b < 1 - \alpha$ so that we concentrate on case 4. The rent from a buy order is given by

(72)
$$\text{rent(buy)} = \frac{(q_b - q)^2}{1 - \alpha}.$$

In case 4, $q + q_b > 1$ so that $\partial(q_b - q)/\partial q$ is negative. This is sufficient to prove that $\partial\text{rent(buy)}/\partial q$ is negative. Conversely, as $\partial q_b/\partial\alpha > 0$, it is immediately obvious that $\partial\text{rent(buy)}/\partial\alpha$ is positive. □

Proof of Gearing Effect Results

Proposition 10. *Let us define* Δx^* *as the minimum quantity transacted among two marketmakers. Suppose that the regularity conditions of proposition 9 are satisfied. The following result holds:*

$$\frac{\partial \Delta x^*}{\partial \alpha} > 0.$$

Proof. We assume that α is such that $\alpha < q < q_b < 1 - \alpha$ so that we concentrate on case 4. If a buy order has been received, the minimum value of the transacted quantity is

$$\Delta x^* = \frac{\text{rent(buy)}}{2(q_b - q)}. \tag{73}$$

We know that the rent of a buy order is given in case 4 by

$$\text{rent(buy)} = \frac{(q_b - q)^2}{1 - \alpha}. \tag{74}$$

This implies that

$$\Delta x^* = \frac{(q_b - q)}{2(1 - \alpha)}. \tag{75}$$

Therefore, as $\partial q_b/\partial\alpha > 0$, it follows that $\partial\Delta x^*/\partial\alpha > 0$. This completes the proof.□

References

Abramovitz, M., and I. A. Stegun. 1964. *Handbook of mathematical functions with formulas, graphs, and mathematical tables.* Applied Mathematics Studies no. 55. Washington, D.C.: National Bureau of Standards.

Biais, B. 1993. Price information and equilibrium liquidity in fragmented and centralized markets. *Journal of Finance* 48, no. 1 (March): 157–85.

Bossaerts, P., and P. Hillion. 1991. Market microstructure effects of government intervention in the foreign exchange market. *Review of Financial Studies* 4, no. 3:513–41.

Copeland, T., and D. Galai. 1983. Information effects and the bid-ask spread. *Journal of Finance* 38:1457–69.

Easley, D., and M. O'Hara. 1987. Price, trade size and information in securities markets. *Journal of Financial Economics* 19:69–90.

Flood, M. D. 1991. Microstructure theory and the foreign exchange market. *Federal Reserve Bank of St. Louis Review* 73, no. 6 (November/December): 52–70.

Glosten, L. R., and P. R. Milgrom. 1985. Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics* 14:71–100.

Goodhart, C. A. E., and L. Figliuoli. 1991. Every minute counts in financial markets. *Journal of International Money and Finance* 10, no. 1:23–52.

Grossman, S., and Z. Zhou. 1991. Investment strategies for controlling drawdowns. University of Pennsylvania, Wharton School of Economics. Mimeo.

Krugman, P., and M. Miller. 1993. Why have a target zone. *Carnegie-Rochester Conference Series on Public Policy* 38:279–314.

Lyons, R. K. 1992. Private belief and information intermediation in the foreign exchange market. University of California, Berkeley, Business School, November. Mimeo.

———. 1993. Equilibrium microstructure in the foreign exchange market. University of California, Berkeley, Business School, June. Mimeo.

———. 1995. Tests of microstructure hypotheses in the foreign exchange market. *Journal of Financial Economics* 39:321–51.

Neuberger, A., N. Naik, and S. Viswananthan. 1993. Disclosure and trading with large market makers: An analysis of the London Stock Exchange. London Business School, January. Mimeo.
O'Hara, M. 1994. Market microstructure theory. Cornell University. Mimeo.

Comment Silverio Foresi

Perraudin and Vitale's paper explores the implications for equilibrium prices of a multidealer market in which dealers cannot see each other's order flow. Their main result is that a decentralized market dealers' market is less prone to market crashes than a centralized market. Given its policy implications, this result is very important and deserves closer scrutiny.

Glosten and Milgrom (1985) show that the market closes down if the specialist needs to post too wide a spread to break even when the informational asymmetry is too severe: the dealer prefers not to trade rather than trading at a disadvantage with an informed customer. The failure to trade is an externality on future trades that is not accounted for by the dealer. So Glosten and Milgrom (1985) go on to conjecture that a Pareto improvement would result from a dealer who could retain some monopoly power.

Perraudin and Vitale propose an interesting mechanism that may give dealers incentives to trade even when they face severe informational asymmetries. If dealers can share information with each other, and, more important, if they can agree to act as a monopolist before trades start, they can extract the surplus from liquidity traders, and, by appropriating this rent, they have an incentive not to let the market break down. The possibility that dealers could learn from other dealers' quotes is quite appealing. (It was probably first presented formally by Garbade, Pomrenze, and Silber [1979], who tested it in the market for U.S. Treasury securities.)

The main elements of the model are as follows. There are three classes of traders: informed, uninformed, and marketmakers or dealers. Everybody knows that the value of the underlying is $S = 1$ with probability q and $S = -1$ with probability $(1 - q)$. The informed traders know the realization of S before the first round of trades starts. Everybody else learns about it after the last round of trade. The model features three trading periods: in the intermediate trading period, dealers trade among themselves, sharing the information they (may have) received in the first round of trading in order to trade again in the last round of trades.

Uninformed trades are $(1 - \alpha)/\alpha$ as numerous as informed traders. Interestingly, their net demands are sensitive to prices. This is crucial because dealers choose prices to maximize their profit from trading with uninformed traders

Silverio Foresi is visiting associate professor of finance at Columbia University and assistant professor of finance at the Stern School of Business, New York University.

in a manner similar to a monopolist choosing quantities taking as given the demand function,

$$\max_{s} sQ(s);$$

formally, because of the price sensitivity of the net demand, the static profit becomes quadratic in the quoted bid (ask) prices. The price sensitivity of the demand of uninformed traders delivers the concavity of the profit function in the price s and its convexity in the probability of high outcomes q.

It is not immediately obvious how to justify the assumption that informed dealers can appropriate the rent by trading with uninformed dealers in period 1. The issue is related to the question, Why aren't quotes in period 1 fully revealing? In standard models with noise that is because noise trade provides camouflage to informed traders. But there are only dealers trading in period 1. What provides camouflage to the "informed dealer" in period 1? Since the uninformed dealer cannot see the informed dealer trade, he must learn from other dealers' quotes and possible trades with the informed ones. Assume that dealers do not act strategically. If dealer A, who saw no customer in period 0, calls dealer B in period 1, she learns from the quote whether A saw a sell, a buy, or nothing. But, having seen the quote, she does not need to trade. But why would informed dealers want to post bid-ask prices that reveal any information? There are two effects at play here. On the one hand, if information is better diffused, all dealers may agree to narrow their spread in period 2. This increases liquidity trades and presumably dealers' profits. On the other hand, larger liquidity volumes provide additional camouflage to informed traders. This reduces dealers' profits. It is unclear whether the balance of the two effects is positive.

While the rent-sharing rule assumed in the paper is not essential, it is essential to show that some rule is viable. Consider the following story, which may justify the assumption in the paper that 100 percent of the rent is captured by the informed dealers. Only dealers who did not see a trade need to get information; let us assume that they call around to all other dealers and ask to trade, giving their (uninformed) bid-ask spread. If the called dealer is informed, there is a trade, and they get information. If the called dealer is uninformed, there is no trade. At this point, however, there are two types of dealers: the ex-uninformed, who have learned all the trades, and the ex-informed, who know only their period 0 trade. In a second round of interdealer trade, the ex-informed dealers may all call each other to share information or trade with one of the ex-uninformed dealers. While complicated, this story is appealing for two reasons. First, the information is transferred credibly. Second, period 1 volume may far exceed period 0 volume, which agrees with the observation that a large volume of trade is not customer driven.

This rent-sharing mechanism also justifies the hypothesis that there is trade in period 1, as assumed in the paper. The interdealers' period 1 trades are otherwise not essential for the main result of the paper. If we did not have period 1 trades, there would be four types of traders in period 2: uninformed and informed traders and uninformed and informed dealers. Dealers do not know ex ante whether they will be informed or uninformed. However, since they are risk neutral, they will leave the market open in period 0 more often than competitive marketmakers in a centralized market, provided again that they agree to act collusively.

The previous discussion shows that the main result of the paper hinges on dealers' collusion. I now look at how the collusion is achieved a bit more closely. The model is designed to mimic a market like the foreign exchange market in which there is no consolidated information on the order flow. Nobody knows whether other trades were executed at bid or ask prices, if at all. The model assumes, however, that customers are served at random. This cuts off the feedback from prices to demand and ensures that there is no incentive to deviate from the dealers' cartel in period 0. It is realistic, however, that liquidity traders see the quotes posted by all dealers and prefer to trade at the narrower bid-ask spread. In this case, a dealer has an incentive to post a bid-ask spread just a bit narrower than her competitors to monopolize information, and in so doing she will break the cartel.

The assumption that makes the cartel self-sustaining in the model of Perraudin and Vitale is that dealers cannot attract more customers by offering better prices: the probability of serving a customer is fixed and equal to $(1/n)$. But this assumption is too strong for a market like the foreign exchange market, where there is consolidated information on quotes.

Dealers' markets offer services that auction markets cannot offer, such as the certainty of execution of trade, which may be essential to liquidity traders in the foreign exchange market. These are services that make dealers' markets undoubtedly desirable. This paper contains a different argument for the desirability of a decentralized dealers' system: a dealers' market is less prone to market crashes than a centralized one.

I have argued, however, that the result does not depend on information sharing. It is essential that dealers agree to collude and act as a monopolist. It is reasonable that in a dealers' market with a small number of players it is easier to collude and agree on rent-sharing rules. But, if we are ready to trade off the market's robustness for liquidity traders' happiness, why not have a single monopolist dealer? A monopolist dealer may be better than a cartel of dealers if there are any costs in monitoring the coalition and sharing the rent.

References

Garbade, Kenneth D., Jay L. Pomrenze, and William L. Silber. 1979. On the information content of prices. *American Economic Review* 69, no. 1:50–59.

Glosten, Lawrence R., and Paul R. Milgrom. 1985. Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics* 14:71–100.

Comment Alan Kirman

This paper is a particularly interesting contribution since it attempts to model certain specific aspects of the microstructure of the foreign exchange market and to explain a particular phenomenon, the large amount of interdealer trading. Many papers on this subject either simply describe the functioning of the market, which is, of course, interesting in itself, or build macro models in which allusion is made to certain features of the microstructure. The latter are thus used to justify rather than to analyze the macroeconomic characteristics.

The main line of the early theoretical literature that did analyze the microstructure and that can be identified with Garman (1976) concentrated on how marketmakers would adjust inventories and bid-ask spreads in response to a stream of orders. Later, the problem of asymmetric information became the dominant concern (see, e.g., Hsieh and Kleidon, chap. 2 in this volume), and the current paper fits in this category.

In the first part of this comment, I make some specific remarks about the model developed by the authors and in the second suggest other potential modeling strategies to capture the phenomenon in which they are interested.

The authors' model can be thought of as one in which bookmakers are faced with a two-horse race. There are a number of experts around who know which horse will win, and the bookmakers know how many of these there are, but not their identity, and have a prior probability as to which horse will win and give odds as a function of this. In the first stage, a bettor arrives and places a bet on one of the horses. This event provides information for the bookmaker, who can adjust his odds, and, since odds are not posted, he can pass this information on to his fellow bookmakers at a price. Thus, a transaction will occur between bookmakers. Bookmakers can now accept further bets from customers, if any are forthcoming. The race is then run, and the whole procedure starts again for the next race. The obvious objection here, and one to which I come back later, is that in the foreign exchange market the "race" is never run, although one could assimilate it to the arrival of some news about which people had prior ideas.

Interdealer trading in the model permits the flow of information about customer orders. The fixed horizon keeps the analysis tractable. Leaving this on one side for a moment, two features are striking. First, the position that a dealer holds does not enter into the analysis. It is, of course, often said that many traders, as in this model, do not trade off their positions, but this cannot hold all the time if, as in reality, they are constrained to be in a zero net position at the end of the trading day. One should therefore stick to the authors' interpretation that they are dealing with a very short interval of time, but one that cannot be too near the close of the market.

Alan Kirman is professor of economics at the European University Institute, Florence, on leave from the GREQAM (Groupe de Recherche en Economie Quantitative d'Aix Marseille).

Second, the probability with which liquidity traders act does not depend on q, which I take to be common knowledge. (In the model, the realization of the exchange rate is a random variable z that takes on the values 1 and -1 with probabilities q and $[1 - q]$.) Indeed, the first customer, however misinformed, could infer the prior q from the bid and ask that he is offered. One would have expected there to be some sensitivity to the probability q in the reaction of liquidity traders.

In the model, one dealer receives a trade, and he captures the rent from that trade by doing the smallest transaction size and charging the appropriate price to exact the rent from the purchaser. The relation between the potential profit to be made from receiving a customer and the number of traders is interesting. While it seems reasonable that with a small number of dealers only one would receive a customer in some short period of time, it would seem that as the number of firms rose the number of customers arriving would rise also and therefore that both the probability and the profitability of such an encounter would change.

Finally, in the context of this model, agents trade only with each other once an order has been received from an outside customer, but this need not be the case in general. Furthermore, the bids and asks that a new customer faces in period 2, that is, after interdealer trades have taken place, will depend on whether the dealer he meets has just been engaged in such trades. Thus, the sequence of prices will be influenced by this and the properties of the price series further complicated.

Incidentally, while the properties of the stochastic price process are of theoretical interest, they are difficult to test precisely because transactions data are not generally available.

To turn now to alternative approaches, the most obvious of these is to suggest that dealers holding open positions are aware that they will have to close them by the end of the day in general and will therefore adjust their bids and asks accordingly. This would suggest that an approach based on risk sharing and inventory management (see Lyons 1995) might be appropriate. This is what is suggested by Suvanto (1993) when he says, "Transactions the dealer undertakes in the role of a customer with a market maker are called *cover transactions*." He also says, "The motive for this kind of transaction, in general, is position adjustment, not trading income as such." Two things feature here: one is the position adjustment because of risk, and the other is adjustment to close the position as the end of day horizon approaches. The horizon problem is thus different from that in Perraudin and Vitale's model and is linked no longer to the arrival of some realization of a random variable but rather to the closing of the market. As I suggested earlier, the "race" in my analogy to their model is never actually run, and for this reason the other sort of horizon seems more plausible.

Another problem is that of where information comes from. In reality, a foreign exchange dealer is faced with a continual barrage of information. He sees screens full of indicative quotes, and he hears the quotes of brokers through loudspeakers as well as observing the electronic broking system. Now, it can be argued that the indicative quotes do not reflect transaction prices clearly, that an actual trade conveys much more information, and that this will change behavior and resultant prices. The difficulty with this is that, having thus obtained theoretical results concerning the characteristics of prices derived from a model of information-generating transactions, it is very difficult to test them since most of the data available correspond to indicative quotes, not to transaction prices.

Another observation is that interdealer trading may simply be due to different expectations (see, e.g., Frankel and Froot 1990), which may not be irrational (see, e.g., Kurz 1994) or which may involve agents learning (see Lewis 1989a, 1989b).

A last way of looking at exchange rate evolution is as a process in which dealers infer from or are influenced by the actions of others, which leads to "herd behavior" (see, e.g., Banerjee 1992; Kirman 1993; and Sharfstein and Stein 1990) or to "informational cascades" (see Bikhchandani, Hirshleifer, and Welch 1992). Indeed, one can interpret Perraudin and Vitale's contribution as a special case of this type of model, in which one piece of information is passed along sequentially to other dealers.

However, in fact what seems to be important is that numbers of dealers are trading with and taking account of the trades of their usual network of partners. How traders react will depend on a combination of their current position and their interpretation of the information contained in a trade. In such a framework the stochastic reactions of the agents may or may not generate a shift in an exchange rate, but there is not necessarily any fundamental information contained in trades. Thus Perraudin and Vitale view interdealer trading as involving the sale and passage of information contained in orders, while an alternative view developed in Kirman (1995) is that interdealer trading can, of itself, generate exchange rate movements without any exogenous information.

In conclusion, the present paper offers an interesting contribution to the literature showing how variations in a particular structure in a model lead to changes in the prices in that model. Whether the aspect that the authors choose—information transmission—is the most important in explaining interdealer trading is an open question, but their contribution provides a way of making a more precise analysis of the question.

References

Banerjee, A. 1992. A simple model of herd behavior. *Quarterly Journal of Economics* 108:797–817.

Bikhchandani, S., D. Hirshleifer, and I. Welch. 1992. A theory of fads, fashion, custom, and cultural change as informational cascades. *Journal of Political Economy* 100, no. 5:992–1026.

Frankel, J. A., and K. A. Froot. 1990. The rationality of the foreign exchange rate: Chartists, fundamentalists, and trading in the foreign exchange market. *American Economic Review* 80:181–85.
Garman, M. B. 1976. Market microstructure. *Journal of Financial Economics* 3:257–75.
Kirman, A. P. 1993. Ants, rationality and recruitment. *Quarterly Journal of Economics* 108 (February): 137–56.
Kirman, A. P. 1995. The behaviour of the foreign exchange market. *Bank of England Quarterly Bulletin* 15 (August): 286–93.
Kurz, M. 1994. On the structure and diversity of rational beliefs. *Economic Theory* 4:877–900.
Lewis, K. K. 1989a. Can learning affect exchange-rate behavior? The case of the dollar in the early 1980's. *Journal of Monetary Economics* 23:79–100.
Lewis, K. K. 1989b. Changing beliefs and systematic rational forecast errors with evidence from foreign exchange. *American Economic Review* 79:621–36.
Lyons, R. K. 1995. Tests of microstructural hypotheses in the foreign exchange market. *Journal of Financial Economics* 39:321–51.
Sharfstein, D. S., and J. C. Stein. 1990. Herd behaviour and investment. *American Economic Review* 80, no. 3:465–79.
Suvanto, A. 1993. *Foreign exchange dealing: Essays on the microstructure of the foreign exchange market.* Helsinki: Research Institute of the Finnish Economy (ETLA).