This PDF is a selection from an out-of-print volume from the National Bureau of Economic Research

Volume Title: A General Equilibrium Model for Tax Policy Evaluation

Volume Author/Editor: Charles L. Ballard, Don Fullerton, John B. Shoven, and John Whalley

Volume Publisher: University of Chicago Press

Volume ISBN: 0-226-03632-4

Volume URL: http://www.nber.org/books/ball85-1

Publication Date: 1985

Chapter Title: The Single Period Submodel

Chapter Author: Charles L. Ballard, Don Fullerton, John B. Shoven, John Whalley

Chapter URL: http://www.nber.org/chapters/c11215

Chapter pages in book: (p. 25 - 54)

3 The Single Period Submodel

3.1 Overview of Structure of the Model

In this chapter we describe the basic structure of the model that we use to calculate each equilibrium. We begin with an overview of the entire model, and then we present each feature in greater detail. The appendix to this chapter provides a glossary of notation, including each symbol in order of appearance in the text, the definition of that symbol, the equation where it is first used, and the data or source of its derivation. The model's structure is summarized diagrammatically in figure 3.1. We begin our discussion with an overview of the production side of the model, which is represented on the left-hand side of the diagram.

The model includes nineteen profit-maximizing producer good industries, which are listed in table 3.1. Each industry uses labor and capital in a constant elasticity of substitution (CES) value-added function. We choose substitution elasticities for each industry from the econometric literature. Values typically range from 0.6 to 1.0, as described in chapter 6. The *Survey of Current Business* (*SCB*) and unpublished data from the National Income Division of the Bureau of Economic Analysis are used to obtain each industry's payments for labor and capital in the 1973 benchmark equilibrium. Like Harberger (1966), we determine the quantities of primary inputs (labor and capital) by using the convention that a unit of each primary factor is that amount which earns one dollar net of taxes in the benchmark year. For intermediate inputs of other commodities, we derive a fixed-coefficient input-output matrix from Bureau of Economic Analysis tables.

As indicated in figure 3.1, final demands for each producer good include direct demands by government, demands for export, and demand for investment. Producer goods for consumption, however, are only

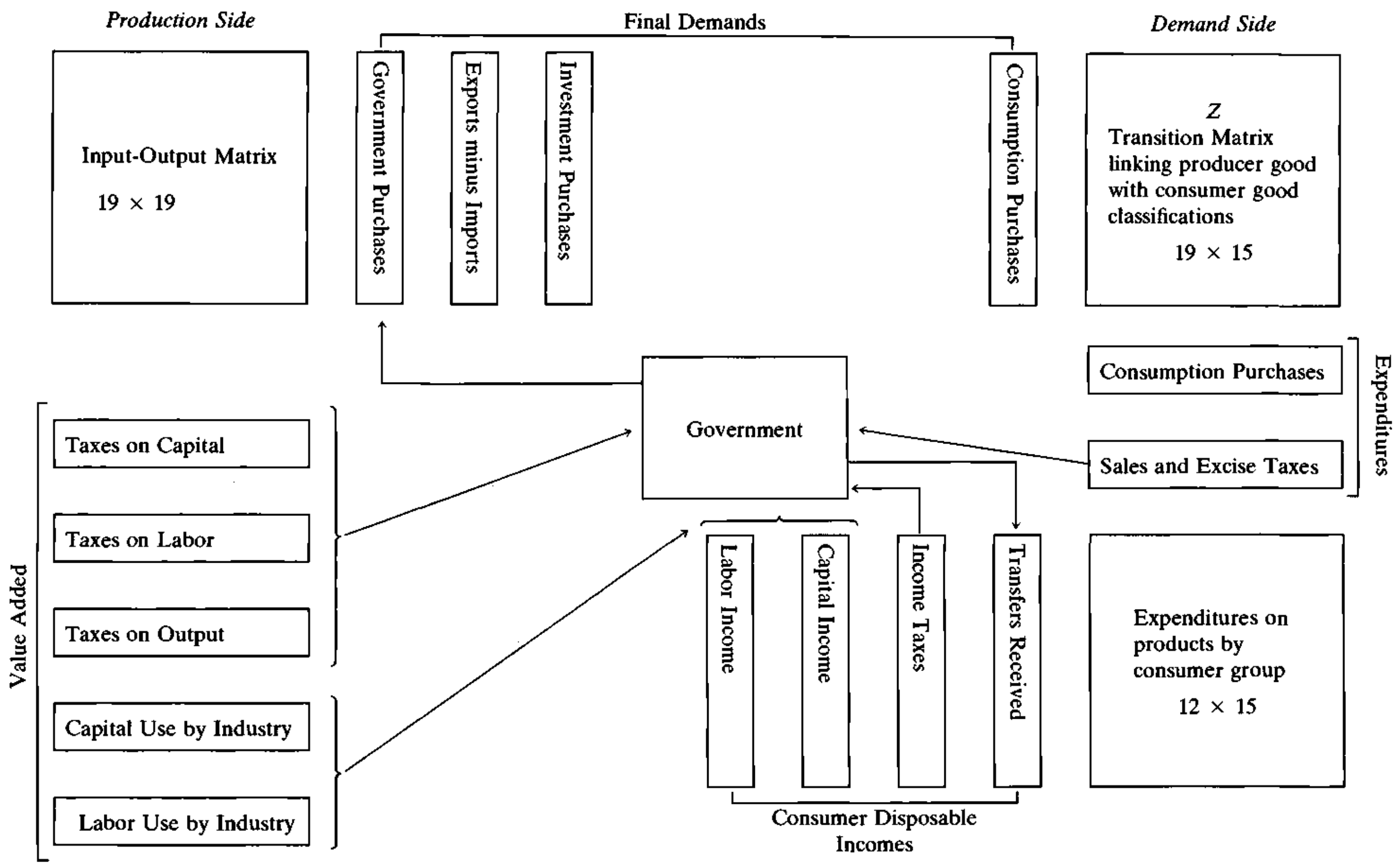

Figure 3.1 Basic structure of the single-period submodel.

demanded indirectly. We use a fixed-coefficient Z matrix of transition between the nineteen producer goods and the fifteen consumer goods. The Z matrix is shown in figure 3.1, and the fifteen consumer goods are listed in table 3.1. This transition is necessary because the Commerce Department data include industries such as mining, machinery, and trade, while the Labor Department's Survey of Consumer Expenditures provides data on purchases of goods like furniture, appliances, and recreation.

Table 3.1 **Classification of Industries, Consumer Expenditures, and Consumer Groups**

Producer Goods (Industries)	Consumer Goods (Expenditure Categories)
1. Agriculture, forestry, and fisheries	1. Food
2. Mining	2. Alcoholic beverages
3. Crude petroleum and gas	3. Tobacco
4. Contract construction	4. Utilities
5. Food and tobacco	5. Housing
6. Textiles, apparel, and leather	6. Furnishings
7. Paper and printing	7. Appliances
8. Petroleum refining	8. Clothing and jewelry
9. Chemicals, rubber, and plastics	9. Transportation
10. Lumber, furniture, stone, clay, and glass	10. Motor vehicles, tires, and auto repair
11. Metals, machinery, instruments, and miscellaneous manufacturing	11. Services
12. Transportation equipment and ordnance	12. Financial services
13. Motor vehicles	13. Reading, recreation, and miscellaneous
14. Transportation, communications, and utilities	14. Nondurable, nonfood household items
15. Trade	15. Gasoline and other fuels
16. Finance and insurance	
17. Real estate	
18. Services	
19. Government enterprises	

Household Consumer Groups
(classified by gross money income in 1973 dollars)

1. 0–2,999	5. 6,000–6,999	9. 12,000–14,999
2. 3,000–3,999	6. 7,000–7,999	10. 15,000–19,999
3. 4,000–4,999	7. 8,000–9,999	11. 20,000–24,999
4. 5,000–5,999	8. 10,000–11,999	12. 25,000+

Table 3.1 also shows the 1973 income levels that define the twelve household classes. Industry and government payments to buy labor and capital services are matched by total household receipts from the supply of each factor. The Treasury Department's Merged Tax File provides information on labor and capital income for each of the twelve consumer classes. The Merged Tax File also provides data on tax payments and an estimate of the average marginal income tax rate for each group. These marginal tax rates range from 1 percent for the first income classes to 40 percent for the highest income class. As discussed later in this chapter, we model the graduated income tax system as a series of linear schedules, one for each group. Then, as discussed in chapter 5, we estimate that 30 percent of savings flow through tax-free vehicles such as pensions, Individual Retirement Accounts, and Keogh plans. This fact is reflected in the model by exempting a fixed 30 percent of savings from personal income taxation.

Table 3.2 provides a brief description of the treatment of taxes in the model. The corporation income tax, state corporate franchise tax, and local property taxes are modeled as ad valorem taxes on each industry's use of capital. The Social Security payroll tax and workmen's compensation tax are modeled as ad valorem taxes on industry use of labor. Various federal excise taxes and indirect business taxes are modeled as output taxes for each of the nineteen industries. State and local sales taxes apply to each of the consumer goods.

Another important feature of our model is the *personal factor* tax. This construct is described more thoroughly later, but it is designed to capture the features of the personal income tax that discriminate among industries. For each industry we calculate the fraction of capital income that is fully taxable at the personal level. This fraction is determined using data on dividends, capital gains, interest, and rent. The personal factor tax acts as a withholding tax at the industry level. We model personal taxes on capital as if they were collected at the industry level at the overall average marginal personal income tax rate. At the consumer level, rebates are given to groups with lower rates, while additional taxes are collected from those with higher than average marginal tax rates. The model thus captures the favorable treatment of industries with a high proportion of retained earnings, industries that receive large amounts of noncorporate investment tax credits, and the housing industry.

Consumer demands are based on budget-constrained maximization of a nested CES/Cobb-Douglas utility function. In the first stage of the maximization process, consumers save some income for future consumption. They allocate the rest to a subutility function defined over present consumption and leisure. The elasticity of substitution between future and present consumption is based on estimates of the elasticity of saving with respect to the net-of-tax rate of return. These estimates, and our choice among them, are discussed in section 6.4.3. In our standard case we use Michael Boskin's (1978) estimate of 0.4 for the uncompensated saving elasticity, but we also perform sensitivity analyses using different values. Consumers make their saving decisions under the (myopic) expectation that all present prices, including the rental price of capital, will prevail in all future periods. In the second stage of the consumer's maximization problem, consumers allocate income between current consumption goods and leisure. The elasticity of substitution between leisure and consumer goods is based on estimates of the elasticity of labor supply with respect to the net-of-tax wage. These are discussed in section 6.4.2. In our standard case we use an estimate of 0.15 for the uncompensated labor supply elasticity, but again we perform sensitivity analyses. Expenditures on individual consumption goods are based on a Cobb-Douglas subutility function.

Consumer decisions regarding factor supplies are made jointly with consumption decisions. Demands for leisure and for saving depend on the prices of both factors and goods. The model simultaneously considers the uses of income and the sources of income in determining the utility of any group.

Saving is converted immediately into investment demand for producer goods. The distribution of investments among producer goods is based on national accounting data for fixed private investment and inventories. We model the foreign trade sector with constant elasticity export supply and import demand functions. This treatment closes the model, maintains the trade balance, and makes it easy for us to calculate trade quantities, once prices are known.

We complete the static model by specifying the government sector. The government uses revenues from the various taxes for transfer payments and for purchases of labor, capital, and producer goods. The government budget is always in balance. Lump-sum transfers to each consumer group are based on unpublished Treasury Department data for Social Security receipts, welfare, government retirement, food stamps, and similar programs. We assume that the government demands factors and commodities according to fixed expenditure shares.

3.2 Treatment of Taxes

In table 3.2 we give a brief summary of the ways in which we model the components of the United States tax system. These treatments fall squarely within the Harberger tradition, but they also reflect our best summary judgments regarding the incentive effects of each tax. Controversies exist, however, with respect to the appropriate treatment of

Table 3.2 United States Taxes and Their Treatment in the Model

Tax	Treatment
1. Corporate taxes (including state and local) and corporate franchise taxes	Ad valorem tax on use of capital services by industry
2. Property taxes	Ad valorem tax on use of capital services by industry
3. Social Security taxes, unemployment insurance, and workmen's compensation	Ad valorem tax on use of labor services by industry
4. Motor vehicles tax	Ad valorem tax on use of motor vehicles by producers
5. Retail sales taxes	Ad valorem tax on purchases of producer goods
6. Excise taxes	Ad valorem tax on output of producer goods
7. Other indirect business taxes and nontax payments to government	Ad valorem tax on output of producer goods
8. Personal income taxes (including state and local)	Linear function for each consumer; 30 percent of savings currently tax sheltered

many parts of the tax system. In this book we do not test the sensitivity of our results to most of these treatments, but we mention some of the alternative treatments that economists have proposed.

We treat corporate taxes as ad valorem taxes on the use of capital services, with different rates across industries. For each industry we calculate an average effective tax rate by looking at the ratio of observed taxes to a measure of observed capital income. In the standard model we assume that the rate on marginal investment is equal to this average rate in each industry. Joseph Stiglitz (1973) has emphasized that interest payments are deductible from the corporate tax, so there would be no corporate tax on the normal income earned by a debt-financed investment. Stiglitz then argues that if corporations use debt finance at the margin, the corporate tax will be nondistortionary. Whereas our simulations in chapter 8 imply that the corporate tax leads to serious welfare losses, this would not be the case if all investments were debt financed at the margin. Since we do not model the choice among different financial assets, we do not deal explicitly with the different tax treatments of debt and equity.[1]

1. Tax rates equal the ratio of taxes to capital income in each industry, so they reflect lower corporate taxes in debt-intensive industries. Since these tax rates apply to marginal uses of capital in the model, we implicitly assume that marginal investments are financed by the average proportions of debt and equity.

A second issue concerning the corporate tax arises from our assumption that marginal and average capital tax rates are equal. A cost-of-capital approach can be used to calculate tax distortions at the margin, as done by Fullerton and Gordon (1983). This procedure leads to tax rate estimates that are different from those used in most of this book. The choice between the marginal cost of capital approach and the average effective tax rate approach is a difficult one. The advantage of the former is that it is more consistent with the microeconomic theory, while a disadvantage is that it cannot capture the extreme complexity of the tax code.

Finally, as emphasized by Fullerton and Gordon (1983), an explicit treatment of risky investments can greatly alter the standard results. If the real risk-free return to corporate investment is only a small part of total corporate income, then most of the tax applies to a risk premium. Since government would take an equal share of risk and the premium, these taxes would not distort investment behavior at the margin. Only the tax on the real risk-free return would be distorting. In contrast to Fullerton and Gordon, Bulow and Summers (1984) argue that the corporate tax does not share risk proportionately. The appropriate treatment of risk is therefore not clear. For further discussion, see Slemrod (1982, 1983).

We also treat property taxes as ad valorem taxes on the use of capital services. Charles Tiebout (1956) shows that, under a set of special assumptions about the provision of local public goods, local property taxes may be thought of as benefit-related charges. Therefore, these taxes could also be modeled as nondistortionary. We doubt, however, that the restrictive assumptions necessary for the Tiebout model are matched very closely in the actual economy.[2]

Some would model the property tax as an excise tax rather than as a tax on capital income. Mieszkowski (1972) reconciles these two views by pointing out that a common tax rate in all jurisdictions would operate as a factor tax while deviations from this common rate would have excise tax effects. Our model captures the fact that property tax rates on capital income differ by industry, so our general equilibrium results include excise tax effects on the prices of industry outputs. (Since we model the government as a single jurisdiction, we do not measure the effects of the differences in property taxes among different local governments.)

We treat Social Security payroll taxes as ad valorem taxes on the use of labor services by industry, and we treat Social Security benefits as lump-sum transfers. It could be argued that the Social Security system is a contributor-financed insurance scheme that has no distorting effects by

2. Fullerton and Gordon (1983) also investigate results in a model with nondistorting property taxes.

industry. Our treatment abstracts from these controversies.[3] Also, we do not capture the effect of Social Security on saving through the substitution of Social Security wealth for private sector capital accumulation, or the effect of Social Security on retirement decisions. For a discussion of these issues, see Feldstein (1974c).

We also treat unemployment compensation taxes as an ad valorem tax on the use of labor services, and unemployment benefits as lump-sum transfer receipts. Once again the relationship between benefits and contributions is inexact. Most states have minimum and maximum tax rates, such that firms with high or low rates of unemployment do not have actuarially fair tax rates.

We model sales and excise taxes as consumer purchase taxes on the fifteen goods. For most of the consumer goods, this treatment is probably not controversial. However, taxes on alcohol and tobacco could be viewed as Pigovian externality-correcting taxes. Gasoline taxes are used to support highway construction, and thus might be modeled as if they were related to benefits.

We model income taxes as linear functions of income for each of the twelve consumer groups. Each group has a negative intercept and a single positive marginal tax rate. Thus, although we capture the fact that the tax system is progressive, we do not capture the fact that each consumer faces a graduated rate schedule. We also do not account for the high implicit marginal tax rates faced by recipients of transfers who might be able to work (see Aaron 1973).

Clearly, our model of the tax system is not the only model that one could adopt. We have tried to adopt a simple treatment for each tax, while at the same time recognizing the diverse controversies that exist in the literature.

3.3 Value Added and Intermediate Production

We assume that there are two primary factors of production—capital and labor—each of which is homogeneous, mobile among sectors,[4] and internationally immobile.[5] Capital, K, is owned by the twelve consumer

3. Blinder, Gordon, and Wise (1980) find that the Social Security system provides a net benefit for older workers. See Burkhauser and Turner 1981 for a reply. While the degree of net tax or benefit may depend greatly on age and other personal characteristics, our model differentiates labor by industry of employment. To our knowledge, no study has measured net incentive effects by industry.

4. In an extension of this model, Fullerton (1983) considers cases in which only new capital investments are mobile across sectors. The interindustry adjustments resulting from tax changes thus occur more slowly, and the welfare changes from tax reforms are typically slightly smaller than in the model with instantaneous capital mobility.

5. This assumption is conventional in general equilibrium tax models, but it is very important for the results they produce. If we were to consider the extreme alternative case of a small, open, price-taking economy facing a perfectly elastic foreign supply of capital,

groups and by government. We denote endowments by $K_j (j = 1, \ldots, 12)$ and K_g. Capital can be used in any of the nineteen producer industries or in the general government sector. These uses of capital are denoted by the i subscript in K_i $(i = 1, \ldots, 20)$. Only consumers have endowments E_j $(j = 1, \ldots, 12)$ of labor, but due to consumption of leisure, their actual supplies are L_j $(j = 1, \ldots, 12)$. This factor can be used in any of the twenty sectors as labor L_i $(i = 1, \ldots, 20)$ or can be retained by consumers for leisure, ℓ_j $(j = 1, \ldots, 12)$. For each consumer, then, we have $E_j = L_j + \ell_j$. In total, we have

$$(3.1) \qquad E = \sum_{j=1}^{12} E_j = \sum_{i=1}^{20} L_i + \sum_{j=1}^{12} \ell_j = L + \ell .$$

We define each of these factors in service units per period. When a unit of capital services is rented out for one period, the owner receives a price, P_K, which is net of factor taxes and net of depreciation. In addition to the rental prices, P_L and P_K, which are paid to factor owners, producers are required to pay ad valorem taxes at rates t_{Li} and t_{Ki}. These taxes differ by sector. The i^{th} factor user thus faces gross-of-tax factor costs of P^*_{Li} and P^*_{Ki}, which equal $P_L(1 + t_{Li})$ and $P_K(1 + t_{Ki})$, respectively.

Capital and labor appear in a constant elasticity of substitution (CES) value-added function of the form

$$(3.2) \qquad VA = \phi \left[\delta L^{\frac{\sigma-1}{\sigma}} + (1-\delta) K^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad \text{for each industry,}$$

where ϕ and δ are production parameters, and σ is the elasticity of substitution.[6] For expositional simplicity, we have suppressed the i subscripts of all variables and parameters in these expressions.

The model uses a 19×19 fixed coefficient input-output matrix, denoted by A, with columns giving the intermediate input requirements per unit of output. The industry outputs are represented as Q_i $(i = 1, \ldots, 19)$. In the standard version of the single period submodel we do not allow for substitution between intermediate inputs and value-added.

A single output is produced by each industry, under constant returns to scale. Producer behavior is characterized by cost minimization for each unit of output. Minimization of factor costs $(P^*_L L + P^*_K K)$ subject to the constraint that $VA = 1$ in equation (3.2) yields the factor demand requirements per unit of value-added. For each industry, these are:

---

results could be changed significantly. In this case, capital would not bear the burden of capital taxes. We return to these issues in chapter 11.

6. Chapter 6 specifies σ for the nineteen industries. In cases where $\sigma = 1$, equation (3.2) reduces to a Cobb-Douglas production function. Also, we define capital costs as net of depreciation. In the Cobb-Douglas case, the constant fraction of value added used for expenditures on capital excludes depreciation expenditures.

$$R_L = \phi^{-1}\left[(1-\delta)\left(\frac{\delta P_K^*}{(1-\delta)P_L^*}\right)^{1-\sigma} + \delta\right]^{\frac{\sigma}{1-\sigma}}, \tag{3.3}$$

$$R_K = \phi^{-1}\left[\delta\left(\frac{(1-\delta)P_L^*}{\delta P_K^*}\right)^{1-\sigma} + (1-\delta)\right]^{\frac{\sigma}{1-\sigma}}. \tag{3.4}$$

Given the parameters δ, ϕ, and σ for each industry, we use the net-of-tax factor prices together with tax rates to calculate each producer's gross-of-tax price for each factor. Thus the tax system distorts factor input decisions.

Consumer goods, X_m ($m = 1, \ldots, 15$), are produced from producer goods, Q_i ($i = 1, \ldots, 19$), through the fixed-coefficient Z matrix shown in the upper right of figure 3.1. Each coefficient, z_{im}, gives the amount of producer goods i needed to produce one unit of consumer good m. For example, a unit of "alcoholic beverages" will include parts of the outputs of three industries: food and tobacco; transportation, communications, and utilities; and trade.

We can impose different ad valorem tax rates on each industry's intermediate purchases from each other industry. State and local motor vehicle registration fees, for example, are modeled as a tax on intermediate use of the motor vehicle industry's output, t_{MVi} ($i = 1, \ldots, 19$). Each industry also pays an output tax at rate t_{Qi} on its own output, regardless of where the output is used.

Because of perfect competition, producers make zero profits after making payments for factors, factor taxes, intermediate inputs, motor vehicle input taxes, and output taxes. The zero-profit prices of the nineteen producer outputs are P_i ($i = 1, \ldots, 19$). Zero-profit conditions also apply to production of consumer goods. Cost-covering consumer good prices are given by:[7]

$$P_m = \sum_{i=1}^{19} z_{im} P_i \qquad m = 1, \ldots, 15. \tag{3.5}$$

The expenditure matrix is shown in the lower right of figure 3.1. When consumers purchase the consumer goods, X_m, they must pay additional ad valorem taxes. We model sales taxes on the purchase of each good at rates t_m ($m = 1, \ldots, 15$). Gross-of-tax prices paid by consumers are $P_m^* = P_m(1 + t_m)$.

7. An unfortunate effect of using this Z matrix is that differential price effects are dampened. Each of the fifteen consumer goods is a weighted average of the nineteen producer goods, with weights given by each column of the Z matrix. The implicit capital/labor ratios in the construction of each consumer good must therefore vary less than the capital/labor ratios of producer goods. When factor prices vary, consumer good prices will

3.4 Household Saving, Labor Supply, and Commodity Demands

The submodel described in this chapter refers only to a single period. In chapter 7 we will consider a sequence of equilibrium periods by incorporating the effects of current savings on the future capital stock and household income. Within a single period, however, individuals make savings decisions based on expectations about the resulting increment to future consumption. We assume that expectations are myopic in the sense that individuals expect all current prices, including the return to capital, to remain constant through all future periods. With this assumption, we can calculate the savings of individuals based only on current prices.[8]

Saving decisions are based on the maximization of a nested utility function, where the outer nest is defined over present consumption and the expected future consumption stream made possible from saving. Bequests are excluded, as is any explicit life-cycle structure. Consider the general case of a consumer who faces the decision of choosing between consumption today (H) and consumption in future periods ($C_1, C_2, \ldots$).

The consumer choice problem can be represented as the maximization of

$$U = U(H, C_F), \tag{3.6}$$

subject to a budget constraint. Here, H is a composite of present consumption goods and leisure, and C_F is a composite of the future consumption stream ($C_1, C_2, \ldots$). We describe specific functional forms below. Implicit in these forms is a rate-of-time preference between H and C_F. In the calculations below, we assume C_F to be the annual consumption of a perpetual annuity made possible through the current period's saving.

In a more complete model of life-cycle behavior, households would calculate, in each period, the discounted present value of resources over their remaining lifetimes. In this model, by contrast, households in each period only concern themselves with the allocation of current income between consumption and saving. While a full lifetime model is beyond the scope of this book, this important contrast should be emphasized.[9]

The structure of our nested CES/Cobb-Douglas utility functions is outlined in figure 3.2. Each consumer starts with a budget, I, which

---

vary less than producer prices; consumer purchases will vary less than they would if consumers bought producer goods directly. However, the weights differ enough to capture substantial effects.

8. Ballard and Goulder (1982) have investigated the effect of giving consumers foresight into the movements of relative prices over time. When a capital-deepening tax change is introduced, consumers will save less if they have foresight, because they see that the return to capital will decrease over time. The results of our simulations change somewhat, depending upon the expectational structure, but the magnitude of the change is not great.

9. See Summers 1981 and Auerbach, Kotlikoff, and Skinner 1983 for numerical life-cycle models incorporating tax effects.

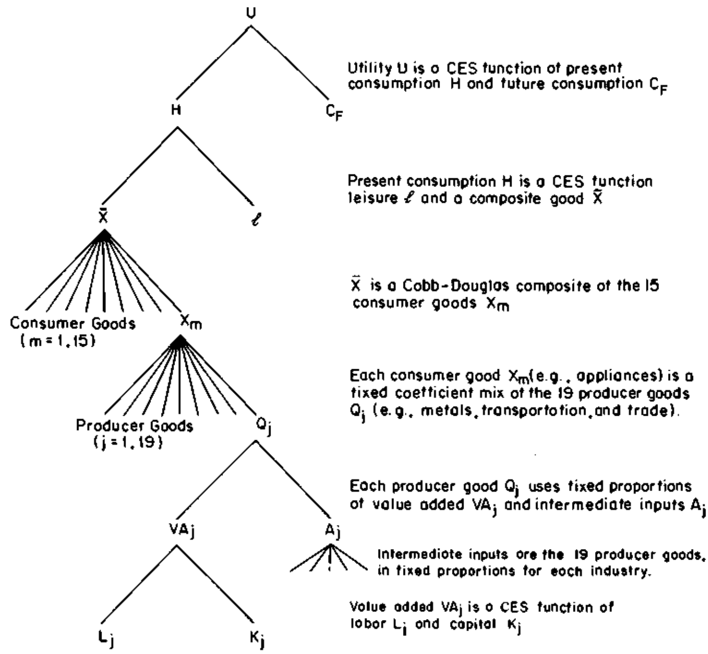

Fig. 3.2 The structure of consumption and production in the model.

equals the rental value of his capital and labor endowments (whether sold or retained as leisure), plus transfers, minus taxes. We refer to I as expanded income to differentiate it from observed money income. Because of the nested CES form, we can divide consumer decisions into stages. In the first stage, the consumer divides this income between present consumption, H (costing P_H as described later), and future consumption, C_F. Next, the consumer decides how to divide present consumption, H, into consumption of leisure time, ℓ (costing P_ℓ), and a composite consumption commodity, $\bar{X}$ (costing $\bar{P}$). When we subtract the values of savings and leisure from I, we have the earned income available for present goods consumption. Since the composite good $\bar{X}$ has a composite price $\bar{P}$, expenditure on consumption goods is $\bar{P}\bar{X}$. In the final utility nest shown in figure 3.2, individuals divide these expenditures among the fifteen consumer goods, X_m ($m = 1, \ldots, 15$), according to a Cobb-Douglas function. Consumers face gross-of-tax prices, P_m^* ($m = 1, \ldots, 15$), on these consumer goods. (Figure 3.2 goes on to show how each consumer good is a combination of nineteen producer goods, and how each producer good is a combination of primary factors and intermediate inputs.)

We assume that consumers use their saving to purchase a saving good, S. The implicit assumption is that consumer groups own real capital and rent it directly to the ultimate users in industry and the government. When individuals save, they must add to their stocks of real capital. The model therefore assumes that the household sector buys investment goods with their saving. This saving-investment commodity, S, is actually a composite of the nineteen industry outputs. The nineteen outputs go into the composite in fixed proportions. The proportions are given by the observed 1973 total investment purchases from each industry.[10]

Capital services, K, are measured in units of asset rental per time period (like machine-hours, except that our capital asset is homogeneous). Since we define a unit of K as that which earns P_K in the benchmark period, *net* of taxes and depreciation, the net saving of consumers corresponds to the net investment purchases of the nineteen outputs.

This treatment excludes variations in the composition of investment that might occur in response to variations in tax rates. For example, corporate tax reductions might imply a reallocation of capital from real estate to incorporated industries. The fixed-coefficient composition of S would not, however, reflect the decreased investment purchases from the construction industry, or the increased purchases of machinery. Of course, the model does capture the reallocation of K itself.

The savings commodity can be interpreted as a composite of newly produced capital goods, since saving is invested immediately. The price of saving, P_S, can also be interpreted as the composite price of investment goods. The capital goods purchased with savings will yield a flow of capital services in the future. This flow can, in turn, be sold for future consumption. Each unit of S is assumed to yield γ units of capital services in each future period, and each of these capital service units is expected to earn P_K per period. (Because $P_K = 1$ in the benchmark year, γ is the initial real after-tax rate of return.) The capital income in each future year finances planned future consumption, which is expected to cost $\bar{P}$. Therefore, $P_K \gamma S = \bar{P} C_F$. If we multiply both sides of this equality by P_S and rearrange, we have

$$(3.7) \qquad P_S S = \frac{P_S \bar{P}}{P_K \gamma} C_F, \text{ for each consumer.}$$

This equation states that the value of saving matches the discounted present value of expected future consumption. The parameter γ denotes the physical service flow per unit of capital goods purchased. We specify γ exogenously. A given value of savings, $P_S S$, earns a return of $P_K \gamma S$ in every future period. Therefore, the endogenous after-tax rate of return is

10. Firms decide how much capital to use in the current period, but they do not make explicit intertemporal investment decisions in our model. The intertemporal decisions of consumers determine future capital stocks.

$P_K\gamma/P_S$, which we denote by r. Since the price of consumption goods, $\bar{P}$, is not expected to change, r is also the expected real rate of return.

The consumer's budget constraint is given by

$$I = P_H H + P_S S, \tag{3.8}$$

where I is current expanded income after taxes and transfers, $P_S S$ is the value of saving, and P_H is the price of composite present consumption H. If we use equation (3.7), we can write the consumer's maximization problem as

$$\text{Max } U[H, C_F], \text{ subject to } I = P_H H + \frac{P_S \bar{P}}{P_K \gamma} C_F. \tag{3.9}$$

Each consumer group has its own parameters and values in the CES form of this utility function, but we suppress indexes for expositional simplicity. The consumer utility function is:

$$U = \left[\alpha^{\frac{1}{\sigma_2}} H^{\frac{\sigma_2 - 1}{\sigma_2}} + (1 - \alpha)^{\frac{1}{\sigma_2}} C_F^{\frac{\sigma_2 - 1}{\sigma_2}}\right]^{\frac{\sigma_2}{\sigma_2 - 1}}, \tag{3.10}$$

where α is a weighting parameter and σ_2 is the elasticity of substitution between H and C_F. Constrained maximization of this utility function yields:

$$H = \frac{\alpha I}{P_H^{\sigma_2} \Delta_2} \tag{3.11}$$

and

$$C_F = \frac{(1 - \alpha) I}{\left(\frac{P_S \bar{P}}{P_K \gamma}\right)^{\sigma_2} \Delta_2}, \text{ where} \tag{3.12}$$

$$\Delta_2 = \alpha (P_H)^{1 - \sigma_2} + (1 - \alpha)\left(\frac{P_S \bar{P}}{P_K \gamma}\right)^{\sigma 1 - \sigma_2}. \tag{3.13}$$

We discuss P_H below. Using equation (3.7) from above, we translate the demand for C_F into the demand for saving:

$$S = \frac{(1 - \alpha) I}{P_S^{\sigma_2} \left[\frac{\bar{P}}{P_K \gamma}\right]^{(\sigma_2 - 1)} \Delta_2}. \tag{3.14}$$

After saving $P_S S$, consumers have $I - P_S S$ to spend on consumption of H. In the second stage, they maximize

(3.15) $$H = \left[(1-\beta)^{1/\sigma_1}\bar{X}^{(\sigma_1-1)/\sigma_1} + \beta^{1/\sigma_1}\ell^{(\sigma_1-1)/\sigma_1}\right]^{\sigma_1/(\sigma_1-1)},$$

subject to

(3.16) $$I - P_S S = \bar{P}\bar{X} + P_\ell \ell,$$

where β is a weighting parameter, and σ_1 is the elasticity of substitution between $\bar{X}$ and ℓ. The price of leisure, P_ℓ, is taken to be the after-tax return to labor of each group. Since a unit of labor earns P_L after factor taxes, $P_\ell = P_L(1 - \tau_j)$, where τ_j is the j^{th} consumer's personal marginal tax rate. Constrained maximization of the subutility function, H, provides the demand functions:

(3.17) $$\bar{X} = \frac{(1-\beta)(I - P_S S)}{\bar{P}^{\sigma_1}\Delta_1},$$

and

(3.18) $$\ell = \frac{\beta(I - P_S S)}{P_\ell^{\sigma_1}\Delta_1}, \text{ where}$$

(3.19) $$\Delta_1 = (1-\beta)\bar{P}^{(1-\sigma_1)} + \beta P_\ell^{(1-\sigma_1)}.$$

We will discuss $\bar{P}$ below.

After spending $P_\ell \ell$ on leisure, consumers have $I - P_S S - P_\ell \ell$ to spend on the consumption components of $\bar{X}$. In the third stage, they maximize a Cobb-Douglas form for the subutility function

(3.20) $$\bar{X} = \prod_{m=1}^{15} X_m^{\lambda_m},$$

subject to

(3.21) $$I - P_S S - P_\ell \ell = \sum_{m=1}^{15} X_m \cdot P_m^*.$$

The λ_m weighting parameters are the Cobb-Douglas expenditure shares. Constrained maximization of the subutility function, $\bar{X}$, provides the demand functions

(3.22) $$X_m = \frac{\lambda_m(I - P_S S - P_\ell \ell)}{P_m^*}, \qquad m = 1, \ldots, 15.$$

An important property of the nested Cobb-Douglas and CES utility functions is that we can derive the indirect utility functions and expenditure functions easily. In the Cobb-Douglas case just described, for example, we form the indirect utility function by substituting the demand functions (3.22) into the direct utility function (3.20). If we use I_X to denote $I - P_S S - P_\ell \ell$, then

$$\bar{X} = \prod_{m=1}^{15} \left(\frac{\lambda_m I_X}{P_m^*} \right)^{\lambda_m} . \tag{3.23}$$

The Cobb-Douglas function is defined such that the sum of the fifteen coefficients, λ_m, is unity. Thus we have

$$\bar{X} = I_X \prod_{m=1}^{15} \left(\frac{\lambda_m}{P_m^*} \right)^{\lambda_m} . \tag{3.23'}$$

The indirect utility function in this case expresses subutility, $\bar{X}$, as a function of income, prices, and preference parameters. From here, it is easy to solve for the expenditure function, which is the income solution of the indirect utility function.

$$I_X = \bar{X} \cdot \prod_{m=1}^{15} \left(\frac{P_m^*}{\lambda_m} \right)^{\lambda_m} . \tag{3.24}$$

The expenditure function gives the income necessary to reach a given level of utility under a given configuration of prices.

Note that we can rewrite equation (3.21) as

$$I_X = \bar{X}\bar{P}. \tag{3.25}$$

Combining equations (3.24) and (3.25), we see that

$$\bar{P} = \prod_{i=1}^{15} \left(\frac{P_m^*}{\lambda_m} \right)^{\lambda_m} . \tag{3.26}$$

We have used the expenditure function to create a composite price index, $\bar{P}$, from the individual prices, P_m^*. An especially convenient property of this kind of price index for both the Cobb-Douglas and CES functions is that the composite price can be calculated without knowing the actual quantities, X_m. This property simplifies our calculations considerably.

We use similar procedures to derive the expenditure functions for the CES nests of the utility functions. The function H is a composite of $\bar{X}$ and ℓ, and the composite price is

$$P_H = \left[(1 - \beta)\bar{P}^{(1-\sigma_1)} + \beta P_\ell^{(1-\sigma_1)} \right]^{\frac{1}{1-\sigma_1}} . \tag{3.27}$$

If we use I_H to denote $I - P_S S$, the income available for expenditure on H, we have the expenditure function,

$$I_H = P_H H. \tag{3.28}$$

As with the Cobb-Douglas nest, the quantity of a composite good times the composite price equals the expenditure on the good.

The function U is a composite of H and C_F, and its composite price is

$$(3.29) \qquad P_U = \left[\alpha P_H^{(1-\sigma_2)} + (1-\alpha)\left[\frac{P_S \bar{P}}{P_K \gamma}\right]^{(1-\sigma_2)}\right]^{\frac{1}{1-\sigma_2}}.$$

The overall indirect utility function is $U = I/P_U$, and the overall expenditure function is $I = P_U \cdot U$.

3.5 Household Income and Taxes

The U.S. personal income tax (PIT) has graduated marginal rates that differ among income groups. It also includes special features that discriminate by industry. Some industries, for example, are more heavily incorporated than others, with a higher proportion of capital income in the form of retained earnings. These industries are more lightly taxed at the personal level than are other industries in which capital income is more heavily comprised of dividends and interest. The housing industry is favored by the PIT, because the imputed net rents of owner-occupied homes are not taxed. This model incorporates both the increasing marginal income tax rates by income class and the industrial discrimination of the PIT.

In order to describe the discriminatory aspects of the personal and corporate tax systems in more detail, we first calculate each industry's capital income net of corporate income tax, corporate franchise tax, and property tax. We denote these figures by CAP_i ($i = 1, \ldots, 19$). The government's payments for privately owned capital are represented by CAP_{20}. The sum of this capital income is received by the twelve consumer classes in the model. Therefore,

$$(3.30) \qquad \sum_{i=1}^{20} CAP_i = \sum_{j=1}^{12} CAP_j ,$$

where CAP_j is the capital income received by the j^{th} consumer class.

Each of the twelve consumer classes has a marginal tax rate on all capital and labor income, denoted by τ_j ($j = 1, \ldots, 12$). We can then calculate τ, which is the weighted average of the marginal tax rates on capital income.

$$(3.31) \qquad \tau = \frac{\sum_{j=1}^{12} CAP_j \tau_j}{\sum_{j=1}^{12} CAP_j}.$$

For each of the nineteen industries and government, we define a fraction, f_i, which is the proportion of that sector's capital income subjected to full personal income taxation. This fraction will differ across industries for a number of reasons, including the variance in dividend/retention policies and differences in the degree to which unincorporated capital qualifies for the investment tax credit.

In order to capture these intersectoral differences in the taxation of capital income at the personal level, we employ a construct that we call the personal factor tax, PFT_i ($i = 1, \ldots, 20$). Total capital tax in each industry is the sum of corporate taxes, property taxes, and the personal factor tax. For each sector, the total personal factor taxes paid are given as

$$PFT_i = f_i \, CAP_i \tau, \qquad i = 1, \ldots, 20, \tag{3.32}$$

where the personal factor tax rate on CAP_i is $f_i\tau$.

It is then possible to define net capital income, $NCAP_i$, as capital income net of the corporate taxes, property taxes, and the personal factor tax on capital income in that industry:

$$NCAP_i = CAP_i - PFT_i = CAP_i(1 - f_i\tau). \tag{3.33}$$

The average fraction of CAP_i that is fully taxable by the personal income tax is

$$\bar{f} = \frac{\sum_{i=1}^{20} CAP_i f_i}{\sum_{i=1}^{20} CAP_i}. \tag{3.34}$$

If we define CAP and $NCAP$ as the sums of CAP_i and $NCAP_i$ over twenty sectors, then the last two equations imply:

$$NCAP = CAP(1 - \bar{f}\tau). \tag{3.35}$$

This expression provides an average conversion from capital income net of corporate and property taxes to capital income net of all taxes.

Although consumers in fact receive CAP_j ($j = 1, \ldots, 12$) and pay their own personal income taxes, we model the PIT on capital income as if it were paid at the industry level. Since tax at rate τ has been paid on an average $\bar{f}$ of capital income CAP_j, however, there must be a correction for differences among the marginal rates at the personal level. The personal factor tax at the industry level can be viewed as a withholding tax. For consumer j, with capital income of CAP_j, the amount of tax paid at the industry level is $\tau\bar{f}CAP_j$. Consumer j should actually pay a tax of

$\tau_j \bar{f} CAP_j$, however, so consumers for whom τ_j exceeds τ must pay additional taxes at the personal level (in addition to the personal factor tax at the industry level). Those for whom τ_j falls below τ get rebates. Thus the correction at the personal level is

$$\Gamma_j = (\tau_j - \tau) CAP_j \bar{f}. \tag{3.36}$$

Since τ is the capital-weighted average of the marginal tax rates, the sum of these corrections at the personal level is zero. Since $NCAP_j = CAP_j (1 - \bar{f}\tau)$, personal tax correction can also be described as

$$\Gamma_j = (\tau_j - \tau) NCAP_j \frac{\bar{f}}{1 - \bar{f}\tau}. \tag{3.37}$$

This rearrangement is necessary because our endogenously determined rental price P_K is defined as the amount earned by each unit of capital, net of *all* taxes. Net capital income $P_K K_j$ is used for $NCAP_j$ in equation (3.37) for our model calculations.

Many transfer payments are not subject to the income tax. In our model we assume that all transfers are tax-exempt. Labor income is fully taxable. Therefore we have the following formula for income taxes paid by group j:

$$T_j^I = B_j + \tau_j P_L L_j + (\tau_j - \tau) P_K K_j \frac{\bar{f}}{1 - \bar{f}\tau}. \tag{3.38}$$

The intercept of each linear tax function, B_j, is negative to reflect the fact that marginal tax rates exceed average tax rates. While marginal changes in income are taxed at the appropriate marginal rate for each group, this marginal rate does not change as income changes. Expanded income, I_j, equals transfers plus labor and capital income, plus the value of leisure, minus income taxes. Since $E_j = L_j + \ell_j$, we have

$$I_j = T_j^R - B_j + E_j P_L (1 - \tau_j) + P_K K_j \left[1 - (\tau_j - \tau) \frac{\bar{f}}{1 - \bar{f}\tau}\right], \tag{3.39}$$

where T_j^R are the lump-sum transfers. Transfer payments are held constant in real terms by a price index on each consumer group's consumption purchases. If the value of leisure, $P_\ell \ell_j$, were subtracted from this expression, we could rearrange it using equation (3.38) to obtain a more usual definition of income:

$$I_j - P_\ell \ell_j = T_j^R + P_L L_j + P_K K_j - T_j^I. \tag{3.40}$$

The price of leisure, P_ℓ, is equal to $P_L(1 - \tau_j)$.

3.6 Government Receipts and Expenditures

We divide government activities into two broad categories. Some publicly supplied goods and services are offered free of charge. We refer to these as general government activities. Other goods and services are subject to user charges, even though the charges may not cover costs (e.g., postal services and some utilities). We refer to these as government enterprise or industry 19. This industry is modeled like the eighteen private industries, and its particular data is described in chapter 4. Consequently, we will not describe it in detail here. The remainder of this section covers the modeling of general government activities.

Expenditures by government other than those for public enterprises are an element of final demand. We model the government as if it were a single consumer, with a Cobb-Douglas utility function defined over all nineteen producer goods, capital, and labor.[11] These government expenditures do not enter the utility functions of consumers as public goods. When tax rates are changed for a simulation, the equal yield feature ensures that enough tax revenue is obtained from an alternative source so that government expenditures at the new equilibrium prices leave the government with the utility level it had in the old equilibrium. Consequently, we only need to be concerned with changes in consumer utility when we want to calculate the total welfare change from some policy.

The government obtains income by collecting taxes and by renting out its endowment of capital services. It makes redistributive transfer payments to consumers in a lump-sum fashion; we use data for Social Security, food stamps, Aid to Families with Dependent Children, and similar programs to determine the amounts of these transfers. These transfers are held constant in real terms, using a Laspeyres price index for each consumer group. The government uses the remaining revenues to buy producer goods at the prices P_i ($i = 1, \ldots, 19$), to buy labor at the gross-of-tax price $P_L(1 + t_L^G)$, and to buy capital at the gross-of-tax price $P_K(1 + t_K^G)$.

The tax rate paid for labor is based on Social Security and railroad retirement taxes paid by the government and its employees. When the government pays these taxes on its use of labor, it pays the taxes to itself. Consequently, the income effects cancel out. However, the price effects measure correctly the opportunity cost to government of hiring additional labor.

The tax rate on capital used by government, t_K^G, is more problematic. Governments in the United States do not typically pay corporate income taxes or property taxes. If we were to model t_K^G as only the personal tax on

11. This formulation allows government to purchase quantities that depend at least somewhat on output prices, but in any case it does not greatly affect the results of the model that pertain to structural tax reform.

that capital income, the government's tax rate on K would be substantially less than the private sector's tax rate. The benchmark equilibrium would imply a misallocation of capital in favor of government use. Any reduction in the capital taxes faced by the private sector would imply reallocation flows from the government sector to the private sector. Since the gross-of-tax capital price in the private sector reflects the marginal product of capital, this capital flow from the public to the private sector would imply (possibly large) welfare gains.

We do not want to contaminate our calculations of the welfare effects of distorting taxes in this way. Therefore, in our model the entire government sector faces a price for capital that is equal to $P_K(1 + \Phi)$, where Φ is the weighted average tax rate on capital used in industry. Then, if the industry tax rates were to change, the government's price would change accordingly. The new price of capital faced by the government would be $P_K \cdot 1$ plus the new weighted average industry tax rate. For example, if industry tax rates were reduced through corporate tax integration, the price of capital used in government would not change relative to the price faced by producers. Thus capital would not flow from the government to the private sector.[12]

3.7 External Sector

We treat the foreign trade activity of the United States in a simple manner, so as to close the model.[13] In our standard model we do not differentiate between commodities on the basis of origin, i.e., U.S.-produced cars and imported cars are considered to be identical.

Foreign trade introduces a difference between the demands of consuming groups in the United States (broadly defined to include business investment and government purchases) and the demands for products faced by U.S. domestic industries. We can represent this distinction by introducing a vector of imports and a vector of exports, using the producer good classification of the model. These vectors account for differences between the demands of U.S. groups and the demands facing U.S. industries.

12. If the government acts to maximize social welfare, it would recognize that each unit of capital taken out of the private sector reduces general welfare by the gross-of-tax price paid by the private purchasers of capital. When government uses another unit of capital, it gives up not only P_K but also the tax revenue that a private producer would pay if that unit of capital were to be used in the private sector. If the government realizes this and acts to maximize social welfare, it would charge itself a shadow price equal to $P_K(1 + \Phi)$.

13. For modeling and results of several alternative trade and international capital flow specifications in this context, see chapter 11. The treatment of foreign trade in this chapter is based on Whalley and Yeung 1948. Alternatives include the use of the Armington assumption (that imports differ from domestically produced goods) and the possibility of imbalanced commodity through international capital flows. The capital flows might be in capital goods or in capital services.

The demand for U.S. exports by foreigners has a negative price elasticity, while the supply of imports to the United States has a positive price elasticity. The relative prices of traded goods are determined endogenously in the model. Trade balance is assured, since the export demand and import supply functions satisfy budget balance.

For each of the nineteen producer goods, we specify foreign export demand and import supply functions. These functions incorporate parameters that determine constant price elasticities of import supply and export demand:

$$M_i = M_i^0(P_{Mi}^w)^{\mu}, \qquad 0 < \mu < \infty, \quad i = 1, \ldots, 19; \tag{3.41}$$

$$E_i = E_i^0(P_{Ei}^w)^{\nu}, \qquad -\infty < \nu < 0, \quad i = 1, \ldots, 19;$$

where M_i and E_i are import demand and export supply, M_i^0 and E_i^0 are constants, P_{Mi}^w is the world price of imports, and P_{Ei}^w is the world price of U.S. exports. These equations imply that the i^{th} commodity can be both imported and exported. This phenomenon of *crosshauling* is evident from the trade statistics, even with highly disaggregated data, and it underlies much of the recent literature on intraindustry trade (see Grubel and Lloyd 1975). There are many reasons for this phenomenon. One explanation asserts that foreign commodities are qualitatively different from domestic goods. For example, U.S. and foreign cars are close but not perfect substitutes. This assumption, first discussed by Armington (1969), is considered explicitly in chapter 11. Crosshauling can also be explained by reference to geography and transportation costs. For example, it may be perfectly sensible for the United States to export Alaskan oil to Japan and at the same time import the identical product through ports on the East Coast and the Gulf of Mexico, given the cost of delivering Alaskan oil to the eastern United States.

In order to close the system and solve the general equilibrium model, we add the trade balance constraint:

$$\sum_{i=1}^{19} P_{Mi}^w M_i = \sum_{i=1}^{19} P_{Ei}^w E_i. \tag{3.42}$$

If we substitute for M_i and E_i from equation (3.41) into equation (3.42), we have

$$\sum_{i=1}^{19} P_{Mi}^w M_i^0(P_{Mi}^w)^{\mu} = \sum_{i=1}^{19} P_{Ei}^w E_i(P_{Ei}^w)^{\nu}. \tag{3.43}$$

We define the relationship between U.S. and world prices through an exchange rate term, e, as $P_{Ei}^{US} = eP_{Ei}^w$ and $P_{Mi}^{US} = eP_{Mi}^w$. The model is, of course, a real trade model and has no financial exchange rate variables, but the use of this construct enables us to write foreign import supply and export demand functions as functions of U.S. prices rather than of world prices. U.S. prices are determined endogenously in the model. If we substitute these U.S. prices into (3.43), we have:

$$e = \left(\frac{\omega_2}{\omega_1}\right)^{\frac{\mu}{\nu-\mu}}, \text{ where} \tag{3.44}$$

$$\omega_1 = \sum_{i=1}^{19} (P_{Mi}^{US})^{\mu+1} M_i^0, \tag{3.45}$$

$$\omega_2 = \sum_{i=1}^{19} (P_{Ei}^{US})^{\nu+1} E_i^0.$$

Finally, substituting these results into equation (3.41), gives

$$M_i = M_i^0 (P_{Mi}^{US})^{\mu} \left(\frac{\omega_2}{\omega_1}\right)^{\frac{\mu}{\mu-\nu}}, \tag{3.46}$$

$$E_i = E_i^0 (P_{Ei}^{US}) \left(\frac{\omega_2}{\omega_1}\right)^{\frac{\nu}{\mu-\nu}}.$$

Note that ω_1 and ω_2 are themselves functions of U.S. import and export prices. Equations (3.46) can be thought of as foreign import supply and export demand functions, written as functions of U.S. prices, and incorporating zero trade balance. Thus, while equations (3.41) specify import and export behavior, the μ and ν parameters are not supply and demand elasticities that incorporate trade balance conditions. To derive expressions for an import supply elasticity and export demand elasticity that do satisfy trade balance, consider a simplified two commodity case in which each country exports one item and imports the other. Let us say that the foreigner demands our exports of good 1. Then, suppressing the *US* superscript and substituting equations (3.45) into the export equation (3.46) we have:

$$E_1 = E_1^0 (P_{E_1})^{\nu} \left(\frac{(P_{E_1})^{\nu+1} E_1^0}{(P_{M_2})^{\mu+1} M_2^0}\right). \tag{3.47}$$

It is simple to differentiate with respect to P_{E_1} and get the own-price elasticity of export demand:

$$\epsilon_E^{FD} = \frac{\nu(\mu + 1)}{\mu - \nu}. \tag{3.48}$$

Similarly, we can find the own-price elasticity of import supply as:

$$\epsilon_M^{FS} = \frac{-\mu(1+\nu)}{(\mu-\nu)}. \tag{3.49}$$

We would like to restrict μ and ν so that the export demand curve slopes downward and the import supply curve slopes upward. These conditions will be met if $\mu \geq 0$ and $\nu \leq -1$.

In the two-good case, equations (3.48) and (3.49) can be used to set values for μ and ν that are consistent with econometric estimates of ϵ_E^{FD} and ϵ_M^{FS}. We follow the same procedure in our model with nineteen commodities (see chapter 6).

Appendix

Glossary of Notation

These variables are defined in approximately the order they appear in the text of chapter 3. (Some do not appear in equations until considerably after they are defined and used in the text.)

Table 3.A.1

Symbol	Definition	Equation Number of First Appearance	Data Source or Derivation
Section 3.3			
K_j	Capital endowment, in service units, of the j^{th} consumer		Basic data, from Treasury Department Merged Data File
K_G	Capital endowment, in service, of general government		Basic data, imputed from capital stocks in Kendrick 1976
E_j	Labor endowment, in service units of the j^{th} consumer	(3.1)	Basic model uses 1.75 of observed labor supply
ℓ_j	Leisure demand, in service units, of the j^{th} consumer	(3.1)	Basic model starts with .75 of observed labor supply
L_j	Labor supply, in service units, of the j^{th} consumer	(3.1)	Basic data, from Treasury Department Merged Tax File
VA_i	Value added in i^{th} industry	(3.2)	From basic data
Q_i	Output of the i^{th} industry		From basic data on value added and intermediate use

Table 3.A.1 (continued)

Symbol	Definition	Equation Number of First Appearance	Data Source or Derivation
ϕ	CES production normalization parameter, for each industry	(3.2)	From calibration, chapter 6
δ	CES factor weighting parameter for each industry	(3.2)	From calibration, chapter 6
K_i	Capital use, in service units, of the i^{th} industry	(3.2)	From Commerce Department data, and procedures of chapter 5
L_i	Labor use, in service units, of the i^{th} industry	(3.1)	From Commerce Department data, and procedures of chapter 5
σ	CES elasticity of substitution between K and L for each industry	(3.2)	Econometric estimates, surveyed in Caddy 1976
P_K	Price of capital, in net rents per unit each period		Units convention in the benchmark, endogenous in any simulation
P_L	Price of labor, in net rents per unit each period		Units convention in the benchmark, endogenous in any simulation
t_{Ki}	ad valorem tax rate on capital for the i^{th} industry		From Commerce Department data, and procedures of chapter 5
t_{Li}	ad valorem tax rate on labor for the i^{th} industry		From Commerce Department data, and procedures of chapter 5
P^*_{Ki}	Gross-of-tax cost of capital to the i^{th} industry	(3.3)	From P_K and t_{Ki}
P^*_{Li}	Gross-of-tax cost of labor to the i^{th} industry	(3.3)	From P_L and t_{Li}
R_L	Requirement of labor per unit of output, for each industry	(3.3)	From equation (3.3)
R_K	Requirement of capital per unit of output, for each industry	(3.4)	From equation (3.4)
A	19×19 fixed-coefficient input-output matrix		From Commerce Department Bureau of Economic Analysis
a_{ik}	Element of the A input-output matrix		From Commerce Department Bureau of Economic Analysis
t_{MVi}	Tax on intermediate use of motor vehicles of the i^{th} industry		From Commerce Department data, and procedures of chapter 5
t_{Qi}	ad valorem rate of tax on the output of the i^{th} industry		From Commerce Department data, and procedures of chapter 5
P_i	Price of the output of the i^{th} industry	(3.5)	Units convention in the benchmark, endogenous in any simulation

Table 3.A.1 (continued)

Symbol	Definition	Equation Number of First Appearance	Data Source or Derivation
X_m	Quantity of the m^{th} consumer good		From Commerce and Consumer Expenditure Survey data
Z	19 × 15 fixed-coefficient matrix converting industry outputs to consumer goods		February 1974 *Survey of Current Business*
z_{im}	Element of the coefficient Z matrix	(3.5)	February 1974 *Survey of Current Business*
P_m	Price of the m^{th} consumer good	(3.5)	From equation (3.5)
t_m	ad valorem sales tax rate on the m^{th} consumer good, for each consumer		From Commerce Clearing House's *State Tax Handbook*
P_m^*	cum-tax prices paid for the m^{th} consumer good, for each consumer	(3.21)	From P_m and t_m
Section 3.4			
H	Composite of present consumption, consumer goods, and leisure, for each consumer	(3.6)	From equation (3.15)
C_F	Composite of future consumption, $C_1, \ldots, C_T$, for each consumer	(3.6)	From definition as the annual consumption made possible by one unit of saving
U	Utility of each consumer	(3.6)	Defined in equation (3.10)
I	Expanded income of each consumer, from endowments and transfers after taxes	(3.8)	Defined in equation (3.39). Data from Treasury Department Merged Tax File, and procedures of chapter 5
P_H	The price of composite consumption H, for each consumer	(3.8)	Defined in equation (3.27)
P_ℓ	Price of leisure, ℓ, for each consumer	(3.16)	Defined as net-of-tax wage
$\bar{X}$	Composite of consumer goods, X_m, for each consumer	(3.15)	Defined in equation (3.20)
$\bar{P}$	Price of the composite $\bar{X}$	(3.7)	Defined in equation (3.26)
S	"Quantity" of savings-investment good, for each consumer	(3.7)	Basic data adjusted from Consumer Expenditure Survey
P_S	Price of the savings good, a composite of the 19 output prices, as they are used for investment	(3.7)	Units convention in benchmark, endogenous in any simulation
γ	The fixed constant that converts savings-investment into capital service units	(3.7)	Chosen as the real rate of return to capital in the benchmark

Table 3.A.1 (continued)

Symbol	Definition	Equation Number of First Appearance	Data Source or Derivation
α	CES utility-weighting parameter for H and C_F, for each consumer	(3.10)	From calibration, chapter 6
σ_2	CES utility elasticity of substitution between H and C_F, for each consumer	(3.10)	From η saving elasticity and calibration, chapter 6
Δ_2	Notational shorthand for part of H and C_F demands for each consumer	(3.11)	Defined in equation (3.13)
β	CES utility-weighting parameter for $\bar{X}$ and ℓ, for each consumer	(3.15)	From calibration, chapter 6
σ_1	CES utility elasticity of substitution between $\bar{X}$ and ℓ, for each consumer	(3.15)	From labor supply elasticity and calibration, chapter 6
τ_j	The marginal personal income tax rate of the j^{th} consumer	(3.31)	From Treasury Department Merged Tax File
Δ_1	Notational shorthand for part of $\bar{X}$ and ℓ demands, for each consumer	(3.17)	Defined in equation (3.19)
λ_m	Cobb-Douglas utility-weighting parameter on the m^{th} consumer good, for each consumer	(3.20)	From Consumer Expenditure Survey and calibration, chapter 6
I_X	Income after leisure and savings	(3.23)	Defined in equation (3.25)
I_H	Income after savings	(3.28)	Defined in equation (3.28)
P_U	Price index for composite utility units	(3.29)	Defined in equation (3.29)
Section 3.5			
CAP_i	Capital payments from the i^{th} industry, net of corporate and property taxes	(3.30)	Commerce Department and procedures of chapter 5
CAP_j	Capital income of the j^{th} consumer, net of corporate and property taxes	(3.30)	Treasury Department Tax File data, procedures of chapter 5
CAP	Sum of 20 CAP_i or of 12 CAP_j	(3.35)	See above, CAP_i or CAP_j
τ	Capital-weighted average of the 12 consumers' marginal tax rates	(3.31)	Defined in equation (3.31)
f_i	The proportion of the i^{th} sector's CAP_i that is subject to the personal income tax	(3.32)	Defined in equation (4.1)
PFT_i	Personal factor tax in the i^{th} sector	(3.32)	Defined in equation (3.32)
$\bar{f}$	The capital-weighted average of the 20 sectors' f_i parameters	(3.34)	Defined in equation (3.34)
$NCAP_i$	Capital payments from the i^{th} industry, net of all taxes	(3.33)	Defined in equation (3.33); equals K_iP_K in simulations

Table 3.A.1 (continued)

Symbol	Definition	Equation Number of First Appearance	Data Source or Derivation
$NCAP_j$	Capital income of the j^{th} consumer, net of all taxes	(3.36)	Treasury Department Merged Tax File; equals K_jP_K in simulations
$NCAP$	Sum of 20 $NCAP_i$ or of 12 $NCAP_j$	(3.35)	See above; defined in equation (3.35)
Γ_j	Personal factor tax of the j^{th} consumer	(3.36)	Defined in equations (3.36) and (3.37)
T_j^I	Personal income tax of the j^{th} consumer	(3.38)	Treasury Department Merged Tax File data, as defined in equation (3.38)
B_j	Negative intercept in the linear personal tax function of the j^{th} consumer	(3.38)	Treasury Department Merged Tax File and calibration, chapter 6
T_j^R	Lump-sum government transfers to the j^{th} consumer group	(3.39)	Treasury Department Merged Tax File data
t_L^G	ad valorem tax rate on the labor purchased by government		From Commerce Department data on Social Security and railroad retirement taxes paid by government
t_K^G	ad valorem tax rate on the capital purchased by government		Set to Φ as discussed in section 3.6
Φ	Weighted average industry tax rate		From t_{Ki}, weighted by K_i, for $i = 1, \ldots, 19$
Section 3.7			
M_i	Quantity of imports of the i^{th} producer good	(3.41)	Benchmark data from OECD trade statistics
M_i^O	Constant in import supply	(3.41)	Based on benchmark imports
P_{Mi}^w	World price of imports of i^{th} producer good	(3.41)	Not used in actual calculations
P_{Mi}^{US}	Domestic price of imports of i^{th} producer good	(3.45)	Equal to P_i which is from the units convention in the benchmark, endogenous in simulations
μ	Price elasticity of import supply	(3.41)	From Stern, Francis, and Schumacher 1977
E_i	Quantity of exports of the i^{th} producer good	(3.41)	Benchmark data from OECD trade statistics
E_i^O	Constant in export demand	(3.41)	Based on benchmark exports
P_{Ei}^w	World price of exports of the i^{th} producer good	(3.41)	Not used in actual calculations
P_{Ei}^{US}	Domestic price of exports of the i^{th} producer good	(3.45)	Equal to P_i which is from the units convention in the benchmark, endogenous in simulations

Table 3.A.1 (continued)

Symbol	Definition	Equation Number of First Appearance	Data Source or Derivation
e	Exchange rate between domestic and foreign prices	(3.44)	Solved out in equation (3.44)
ν	Price elasticity of export demand	(3.41)	From chapter 6
ω_1	Notational shorthand functions of import and export prices	(3.44)	Defined in equation (3.45)
ω_2	Notational shorthand functions of import and export prices	(3.44)	Defined in equation (3.45)
ϵ_E^{FD}	Foreign price elasticity of demand for U.S. exports	(3.50)	Defined in equation (3.50)
ϵ_M^{FS}	Foreign price elasticity of supply for U.S. imports	(3.51)	Defined in equation (3.51)