3 Seigniorage and Fixed Exchange Rates: An Optimal Inflation Tax Analysis

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In choosing fixed over flexible exchange rates, a country gives up the right to determine its own rate of inflation and thus the amount of revenue collected by the inflation tax. This constraint imposes an excess burden that should be included in the cost-benefit analysis of the choice of exchange rate regime. If the country goes further by giving up its seigniorage and using a foreign money in place of the domestic money, it loses more tax revenue and has to adjust government spending and other taxes accordingly. The choice of exchange rate regime is thus related to questions discussed in optimal inflation tax analysis.

This paper presents an analysis of the optimal inflation tax in section 3.1.[1] The consequences of a constraint on the rate of money creation are studied in section 3.2, while section 3.3 analyzes the effects of the loss of revenue from the inflation tax. Section 3.4 presents an interpretation of the preceding analysis as applied to alternative exchange rate regimes. The interest in the paper derives from the explicit calculation of optimal inflation taxes for a specific utility function and production function, embodied in an intertemporal framework, as well as from the application to exchange rate regimes.

3.1 The Optimal Inflation Tax

The representative infinitely lived family in the economy is growing at rate n and derives utility from private consumption, from the services

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1. Phelps (1973) is the original reference in this tradition. See also Aghevli (1977), Drazen (1979), and Brock and Turnovsky (1980) for further developments.

provided by holding real balances,[2,3] from leisure, as well as from consumption of a public good. There are no nondistorting taxes, and the government finances its expenditures through the issue of money and taxes on labor income. It is convenient to assume there is no capital.

The utility function of the representative household is

$$(1) \qquad V = \int_0^\infty U(c, m, x, g) \, e^{-\delta s} \, ds,$$

where c is per capita consumption, m is per capita real balances, $x = \overline{\ell} - \ell$ is leisure (ℓ is labor supply), and g is government spending; $\delta > 0$ is the discount rate or rate of time preference.

The household budget constraint is

$$(2) \qquad c + \dot{m} + (\pi + n) \, m = w(1 - t) \, \ell,$$

where π is the rate of inflation, n the growth rate of family population, w is the wage rate, and t the tax rate on labor income. It is assumed throughout that w is constant.[4]

The household maximizes (1) subject to the budget constraint (2), taking g, government spending, as given. The government budget constraint is

$$(3) \qquad g = tw\ell + (\dot{M}/PN) = tw\ell + \dot{m} + (\pi + n) \, m,$$

where $\dot{M}/PN$ is the flow of real resources, per capita, the government obtains by printing money. (N is population.)

The analysis proceeds in stages. First, the household optimization problem, taking π and t as given, is solved. I then note that there is no inherent dynamics in this model, since there is no capital accumulation, and that for a given rate of nominal money growth, θ, the rational expectations solution for the price level will have the economy jump initially to its steady state, in which $\dot{m} = 0$ and $\pi = \theta - n$. The remainder of the analysis is therefore conducted under the assumption that the economy is in steady state.

2. Fischer (1974) discusses the issue of money in the production (and utility) function, which is emphasized by Thomas Sargent in his comments on this paper and the paper by Guillermo Ortiz appearing later in this volume. The essential point is that of revealed preference: putting money in the utility (or production) function is *equivalent* to postulating a demand function for money. Deeper analyses of the demand for money require a more detailed specification of the transactions environment. It is well known that the choice of a medium of exchange in any model of transactions is extremely delicate in that there is no good reason for one asset rather than another to serve as medium of exchange. The Kareken-Wallace (1981) indeterminacy of exchange rates in a multicountry world represents the same logical difficulty as that of accounting for the use of noninterest-bearing currency in a single country where there are alternative assets. This problem was stressed by Keynes (1936) and Samuelson (1947) as the essential difficulty of monetary theory. In the light of these difficulties at the theoretical level, it is remarkable that there is so little difficulty in getting private economic agents to use the domestic currency: it takes extraordinary rates of inflation before there is flight from a given national currency. The theoretical challenge is to explain this phenomenon.

3. For use of a similar framework in a multiasset context, see Fischer (1972).

4. If capital were included in the model, w would become variable.

At the second stage, a Cobb-Douglas utility function is used to study the optimal tax problem. For any given level of g, there is an optimal combination of taxes to finance the spending. The optimal tax combination and its variation as g changes are examined. Finally, I ask what the optimal level of g is, under the assumption that the government maximizes (1), subject to the private sector behavioral functions and its budget constraint (3).

The first order conditions for maximization of (1) subject to (2) are

$$(4) \qquad 0 = U_c - \lambda,$$

$$(5) \qquad 0 = U_x - \lambda w(1 - t),$$

$$(6) \qquad \dot{\lambda} = (\pi + n + \delta)\lambda - U_m,$$

where λ is the multiplier associated with the budget constraint (2) and, from (4), is also the marginal utility of consumption.[5]

Now, consideration of equilibrium paths in which π, the rate of inflation in (6), is equated to the rate of inflation implied by solution of the full system (4)–(6) for given constant θ will show that the only path that converges to a steady state is one that goes immediately to that steady state.[6] Thus we can set $\dot{\lambda} = 0$ and work henceforth with the steady state system, (4), (5), and

$$(6') \qquad 0 = U_m - \lambda (\pi + n + \delta).$$

The general optimal tax analysis approach could now be applied, but I prefer to use a specific, Cobb-Douglas, example to illustrate the relevant considerations.[7] Assume

$$(7) \qquad U(c, m, x, g) = c^\alpha m^\beta x^\gamma g^\epsilon, \text{ with } \quad \alpha, \beta, \gamma, \epsilon > 0,$$
$$\text{and } \alpha + \beta + \gamma + \epsilon < 1.$$

Then, using equations (4)–(6') and the budget constraint (2):

$$(8) \qquad c = \frac{\alpha w(1 - t)\bar{\ell}}{\alpha + (\beta\theta/\theta + \delta) + \gamma}, \qquad \frac{\partial c}{\partial t} < 0, \frac{\partial c}{\partial \theta} < 0,$$

$$(9) \qquad m = \frac{\beta w(1 - t)\bar{\ell}}{(\alpha + \beta + \gamma)\theta + \delta(\alpha + \gamma)}, \qquad \frac{\partial m}{\partial t} < 0, \frac{\partial m}{\partial \theta} < 0,$$

$$(10) \qquad \ell = \bar{\ell}\left[1 - \frac{\gamma}{\alpha + (\beta\theta/\theta + \delta) + \gamma}\right], \qquad \frac{\partial \ell}{\partial t} = 0, \frac{\partial \ell}{\partial \theta} > 0.$$

The properties of the functions (8)–(10) are unsurprising, except for the absence of a wage or labor tax effect on labor supply. This last result is

5. For a similar optimization problem, see Fischer (1979).
6. See Fischer (1979) for the type of argument needed.
7. The Cobb-Douglas form does not permit the level of government spending to affect the rates of substitution between other pairs of variables. Thus a utility function like (7) cannot, for instance, reflect the notion that government and private consumption are close substitutes.

a consequence of the canceling of income and substitution effects and ensures, in this model, that total taxes from labor rise as the income tax rate increases.

Note from (9) that for an interior maximum with $m > 0$, it is required that

$$(11) \qquad \theta > - \frac{\delta(\alpha + \gamma)}{\alpha + \beta + \gamma},$$

which also implies that $\theta + \delta > 0$.

The government budget constraint (3) implies that for any tax rates, t and θ,

$$(12) \qquad g = w\bar{\ell} \left[\frac{\alpha t(\theta + \delta) + \theta\beta}{(\alpha + \beta + \gamma)\theta + \delta(\alpha + \gamma)} \right],$$

where I have substituted from (9) and (10) into (3). It is convenient to define

$$(13) \qquad \mu \equiv \frac{g}{w\bar{\ell}},$$

which is a measure of the share of government spending in potential (full-time work) output.

Different combinations of θ and t can be used to finance any feasible level of government spending. Locus BB in figure 3.1 shows those combinations in (t, θ) space for a given value of μ. The locus does not necessarily cross the t and θ axes, since there is a maximum μ that can be financed through exclusive use of either the income tax or seigniorage. In particular, if there is no use of seigniorage ($\theta = 0$), then it is required that

$$t = \frac{\alpha + \gamma}{\alpha} \mu.$$

When $t = 1$, the government is using the income tax to appropriate all income, and government spending is given by:[8]

$$(14) \qquad \mu_1 = \frac{\alpha}{\alpha + \gamma}.$$

In the case of nonuse of the income tax, maximum g is achieved as θ goes to infinity, and

$$(15) \qquad \mu_2 = \frac{\beta}{\alpha + \beta + \gamma}.$$

Since β is likely to be small relative to α and γ, the maximum steady state

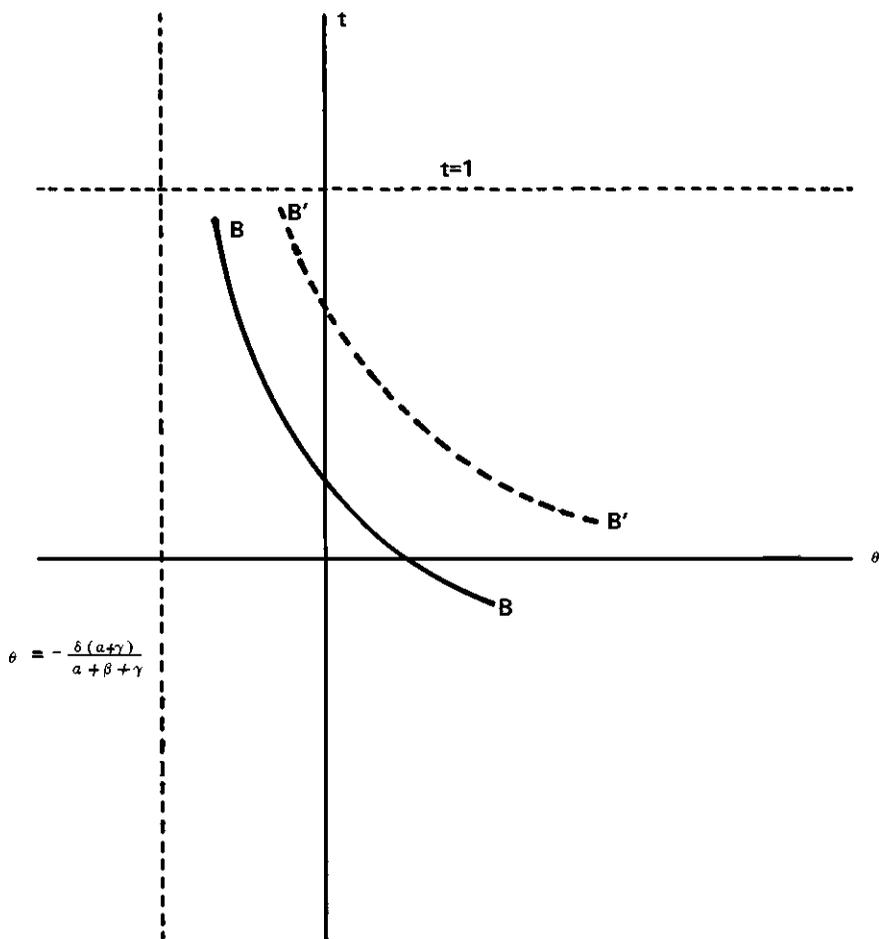8. I am grateful to Olivier Blanchard for correcting an error at this point in a previous draft.

Fig. 3.1   Alternative tax combinations to finance a given level of government spending.

$(g/w\bar{\ell})$ that can be financed by seigniorage alone is also likely to be small. Use of the inflation tax does increase the level of output through its effect on labor supply; thus when the inflation tax alone is used to finance government spending, the level of output is higher than when the income tax is used to finance the same level of government spending.

The maximum attainable level of government spending when both taxes are used, μ_3, is obtained by setting $t = 1$ and letting θ go to infinity in (12):

(16)
$$\mu_3 = \frac{\alpha + \beta}{\alpha + \beta + \gamma}.$$

Whether the *BB* locus crosses the t and θ axes as shown in figure 3.1 depends on the value of μ. For the *BB* locus in figure 3.1, μ is less than both μ_1 and μ_2. As μ increases, the locus shifts up to the right. The $B'B'$ locus applies for a level of government spending larger than μ_2 but smaller than μ_1 and μ_3.

The *BB* locus shows combinations of t and θ that *can* be used to finance a given level of government spending. But of course only one of these combinations will be the optimal tax combination for given μ. Given t, θ, and g, the consumer demand and supply functions (8)–(10) imply the flow of utility

$$(17) \qquad U^* = \xi[(\alpha + \beta + \gamma)\theta + \delta(\alpha + \gamma)]^{-(\alpha + \beta + \gamma)}$$
$$\times (\theta + \delta)^{\alpha + \gamma}(1 - t)^{\alpha + \beta}g^\epsilon,$$

where ξ is a constant of no significance.

The marginal disutilities of the two tax rates, and hence the slope of an indifference curve in (θ, t) space, are obtained from (17), treating g as given. Then, equating the slope of an indifference curve to that of the budget constraint *BB* and solving, pairs of θ and t that are optimal for each level of government spending are obtained. This *optimal tax locus* is given by:

$$(18) \qquad \alpha t[(\alpha + \beta + \gamma)\theta + \delta(\alpha + 2\gamma)] - \alpha\theta(\alpha + \beta + \gamma)$$
$$= \delta[\alpha(\alpha + \gamma) - \beta\gamma].$$

The optimal tax locus, *TT*, is shown in figure 3.2. Its slope is

$$(19) \qquad \frac{dt}{d\theta} = \frac{(1 - t)(\alpha + \beta + \gamma)}{\theta(\alpha + \beta + \gamma) + \delta(\alpha + 2\gamma)},$$

which is positive. Thus both the seigniorage and the labor income tax increase as government spending rises. Corresponding to each point on *TT* is a level of government spending. Whether the *TT* locus crosses the $\theta = 0$ axis at $t > 0$, as shown, depends on the sign of $\alpha(\alpha + \gamma) - \beta\gamma$, the right-hand side of (18). Since β is related to the share of spending on real balance rentals, it is likely to be small; thus the case shown in figure 3.2 is more likely.

The optimum government policy is found by choosing the best point on figure 3.2. This is done by maximizing (17) with respect to θ and t after substituting for g from the government budget constraint (12): the resultant expression for the optimal rate of seigniorage is

$$(20) \quad \theta^2(\alpha + \beta)(\alpha + \beta + \gamma) + \theta\delta[(\alpha + \beta)(\alpha + \gamma) + \alpha(\alpha + \beta + \gamma) - \gamma\epsilon]$$
$$+ \delta^2[\alpha(\alpha + \gamma) - \gamma(\beta + \epsilon)] = 0.$$

Three comments about (20) are in order. First, assuming that the coefficient of θ in the equation is positive, there will be no positive root of (20) unless
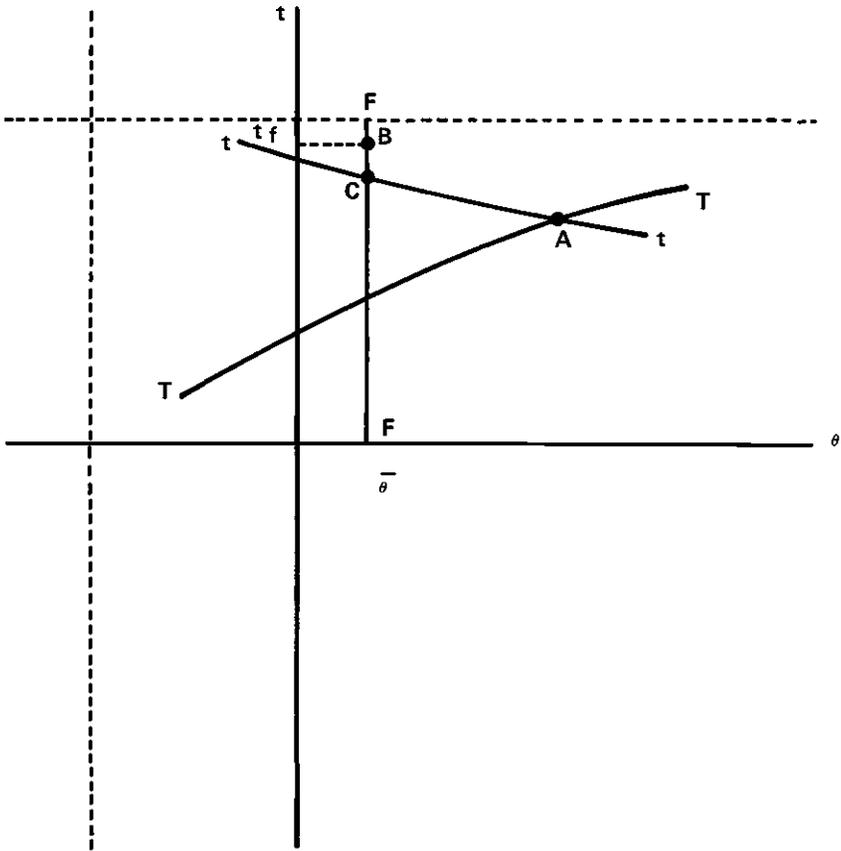
Fig. 3.2   The optimal tax locus TT.

$$\alpha(\alpha + \gamma) < \gamma(\beta + \epsilon).$$

This condition requires government spending optimally to take a relatively large share of output. For values of the parameters that generate approximately the observed ratios of consumption to income, consumption to real balances, labor to leisure, and government spending to consumption in the U.S. economy, the condition is not satisfied. Thus the current analysis does not give support to the notion that optimal rates of seigniorage can be high. In part, no doubt, this is a result of the functional form being used.[9] It may also reflect the absence of a banking system in this model.[10]

9. For instance, the likelihood of positive use of seigniorage at the optimum would be increased if real balances entered the utility function in Stone-Geary form as $(m - \bar{m})$. See Diamond and Mirrlees (1971) for an example. In the present paper use of the Stone-Geary form turns (20) into a quartic equation and thus is not appealing. Barro (1971) argues that optimal rates of inflation are low.

10. In unpublished work, Guillermo Calvo and Jacob Frenkel have shown that the

Second, nowhere has the analysis had occasion to enter the variables θ and π separately. Thus in this example the optimal use of seigniorage is independent of the rate of population growth.[11] The optimal steady state rate of inflation therefore falls one for one as the population growth rate rises.

Third, the optimal rate of seigniorage use, θ, is directly proportional to δ, the rate of time preference. If optimal θ is positive, it increases proportionately with δ, which may be thought of in this context as the interest rate. If optimal θ is negative, then higher δ would mean a lower optimal rate of inflation, which is consistent with the optimal quantity of money argument.

3.2   Constrained Optimal Taxation

The optimal position for this economy is a point like A in figure 3.2. In this section I consider the effects of constraining the rate of money growth, θ, to a level $\bar{\theta}$. Such a constraint would apply, for example, if the exchange rate were kept fixed. In terms of figure 3.2, the government is constrained to the locus FF. Two questions about the rate of income tax to be used are considered.

First, we could ask what rate t_f would be necessary to maintain any specified level of government spending, for instance, the optimal level associated with point A. That is a purely technical question to be answered using the budget constraint (12). The implied point B is shown illustratively, in figure 3.2.

The second question is: What, given the constraint $\bar{\theta}$, is the optimal level of government spending? The answer is found by maximizing (17) with respect to t, after substituting in for g from the government budget equation (12) and treating θ as a constant. The resultant locus, giving optimal t (by implication from (12), also g) as a function of $\bar{\theta}$, is

$$(21) \qquad t = \frac{\epsilon}{\alpha + \beta + \epsilon} - \frac{\bar{\theta}\beta(\alpha + \beta)}{\alpha(\alpha + \beta + \epsilon)(\bar{\theta} + \delta)} .$$

This *optimal income tax locus*, *tt*, in figure 3.2 is negatively sloped and lies above TT to the left of the optimal point A. When some seigniorage is taken away from the government, it optimally reduces government spending and increases its use of the labor income tax. Given the constraint on θ in figure 3.2, the optimal point is C.

Figure 3.3 shows the utility implications of giving up control over θ. The curve describes the level of utility traced out along *tt*. Point C shows

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introduction of a banking system with fractional reserves in an optimal inflation tax analysis increases the optimal inflation rate.
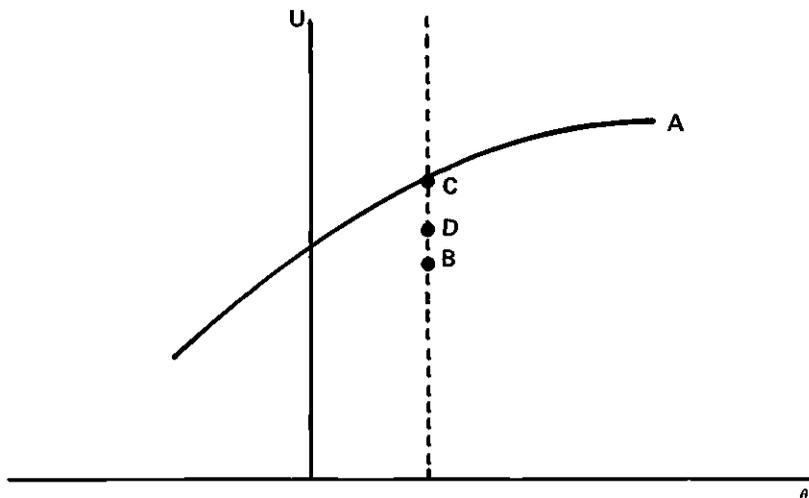
11. Cf. Friedman (1971).

Fig. 3.3   Utility implications of alternative monetary and exchange arrangements.

the utility level corresponding to optimal policy when θ is fixed at $\bar{\theta}$. The utility level corresponding to B, where government spending is held to the level that obtains at A, lies below C. The utility loss from A to C can be compensated for by some amount of resources, which is not in general equal to the amount of seigniorage lost in moving from A to C. In the context of discussion of fixed exchange rates, that amount of compensation is the excess burden of accepting fixed exchange rates.

3.3   Losing the Inflation Revenue

Finally, suppose that the revenue generated by seigniorage is no longer available to the government. This would occur if, for instance, a country used a foreign money. The maximal attainable level of utility is certainly less than that shown by C in figure 3.3. The government loses a source of revenue and will again optimally reduce government spending below its level at C and increase the income tax rate above its level at C. The optimal tax rate is now

$$(22) \qquad\qquad t = \frac{\epsilon}{\alpha + \beta + \epsilon},$$

which is independent of $\bar{\theta}$; however, government spending optimally increases with $\bar{\theta}$. This occurs because an increase in $\bar{\theta}$, which may be thought of as an increase in the rate of inflation, increases labor supply and thus income tax revenue.

Corresponding to the higher rate of income tax when the government loses seigniorage, optimal holdings of real balances will be lower than the

level corresponding to point C in figure 3.3, even though the inflation rate is the same. Point D in figure 3.3 represents the maximum utility attainable when the government loses its seigniorage.

3.4   Exchange Rate Regimes

The above analysis is relevant to one aspect of the differences among exchange rate regimes. The full optimal tax analysis presented in section 3.1 describes the options available when the exchange rate is flexible. The excess burden imposed by the constraint on θ in section 3.2 describes one of the costs of adopting a fixed rate regime which, however, uses domestic money. The rate of money creation $\bar{\theta}$ in section 3.2 is that rate required to maintain fixity of the exchange rate. Section 3.3 calculates the further cost of giving up domestic money, using instead a foreign money. The rate of inflation in this case will also be consistent with the rate of money growth $\bar{\theta}$.

The ranking of utilities of these sets of arrangements is unambiguous. Free choice of the rate of money growth is preferred to the situation where the rate of money growth is fixed at θ, with use of domestic money. Utility at point A in figure 3.3 is undoubtedly above (or no lower than) that at C. Use of a foreign money imposes a further cost, implying that point D is below C in figure 3.3.

It is entirely reasonable to ask whether the seigniorage considerations emphasized in this paper are of any empirical significance. The possibly surprising answer is yes. For the industrial countries over the period of 1960–1978, seigniorage provided 5.7 percent of government revenue, representing 1 percent of GNP on average. In some other countries, such as Greece and Spain, seigniorage revenue exceeded 10 percent of government revenue.[12] Such high rates of seigniorage are perhaps nonoptimal in the light of the preceding analysis, but they certainly indicate that seigniorage is a factor to be taken into account in the choice of an exchange rate regime. But of course it is not the only consideration determining the desirability of alternative exchange arrangements.

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12. More detailed estimates of the amount of revenue collected from seigniorage are presented in Fischer (1982), along with a less formal discussion of the analysis presented in this paper.

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