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Erratum

DO EQUILIBRIUM REAL BUSINESS CYCLE THEORIES EXPLAIN POSTWAR U.S. BUSINESS CYCLES?

In the article "Do Equilibrium Real Business Cycle Theories Explain Postwar U.S. Business Cycles?" in *NBER Macroeconomics Annual 1986* we deduced the equilibrium laws of motion for output in the context of a monetary model with a cash-in-advance constraint. This law of motion, and the law of motion for the money supply, were then used to investigate the money-output linkages implied by our monetary business cycle model. The derived law of motion for the intermediate good underlying this analysis is correct only in the absence of taste shocks and monetary shocks. In this erratum we present the general solution.

Briefly, the economic environment is as follows. A representative firm produces a nondurable consumption good y_t, using input x_t, according to the production function

$$y_t = x_t\alpha\lambda_t, \; 0 < \alpha < 1, \quad (1)$$

where λ_t is the shock to the consumption good technology at date t. The firm buys the storable intermediate good x_t from consumers at the unit real price w_t. The representative consumer has an initial endowment of the intermediate good of k_0. The law of motion for the consumer's holding of the intermediate good at the beginning of period t is

$$k_t = \theta_t[k_{t-1} - x_{t-1}]. \quad (2)$$

In (2), θ_t represents a stochastic shock to the storage technology.

Money enters the model, because consumers are subject to a cash-in-advance constraint:

We are grateful to Lars Hansen for pointing out the error in Eichenbaum and Singleton (1986). Helpful comments were also provided by Larry Glosten and Ravi Jaganathan.

$$P_t c_t \leqslant M_t, \quad (3)$$

where M_t is the predetermined cash balance at the beginning of period t, c_t is the level of consumption during period t, and P_t is the time t price level. This constraint is assumed to be binding each period. In addition to money, the consumer holds z_t shares of claims to the dividends of the representative firm at time t.

Realizations of all time t random variables are learned at the beginning of period t. Then the level of x_t to sell to the firm at the dollar price W_t and the level of c_t to purchase at the dollar price P_t are chosen. Payment for the sale of x_t is received in cash after the consumption decision, so the proceeds (if any) cannot be used to relax the cash-in-advance constraint in period t. After the goods market is closed, the consumer receives his share of cash dividends and a lump sum monetary transfer J_t. Thus, in real terms, the period t budget constraint is

$$M_{t+1}/P_t + q_t^s z_{t+1} \leqslant [q_t^s + d_t]\, z_t + w_t x_t + J_t/P_t, \quad (4)$$

where q_t^s is the real price per share of equity.

The consumer is assumed to choose contingency plans for c_t, x_t, M_{t+1}, and z_{t+1} to maximize

$$E_o \sum_{t=o}^{\infty} \beta^t \nu_t \ln c_t, \quad (5)$$

subject to constraints (2) and (4). In (5), E_t denotes the expectation conditioned on agent's information at date t, ν_t is a taste shock, and $\beta \in (0,1)$ is the subjective discount factor.

To deduce the equilibrium law of motion for x_t implied by this model we first solve the analogous problem for a finite lifetime of T periods, subject to the terminal condition $k_{T+1} = 0$, and then let $T \rightarrow \infty$. Specifically, suppose that at the end of period T the capital shock is zero $[k_{T+1} = 0]$, so that $k_T = x_T$ (see (2)). Also, $q_T^s = 0$, since consumers will not purchase shares in the final period of this economy. For there to be a monetary equilibrium in the finite horizon economy, either consumers or firms must want to hold nominal cash balances in the final period. Consumers purchase consumption goods in the last period in the amount $P_T c_T = P_T y_T$. After the close of the goods market, the firm distributes profits in the form of cash dividends, $[P_T y_T - W_T x_T]$, and pays consumers for the intermediate good. That is, consumers' cash receipts are

$$W_t x_T + [P_T y_T - W_t x_t] = P_T y_T. \quad (6)$$

Following Townsend (1982), we assume that it is optimal for agents to hold these balances in the final period by imposing a tax of $P_T y_T$ at the end of period T; $J_T = -P_T y_T$ or, equivalently, $M_{T+1} = J_T$. As $T \to \infty$, the present discounted value of this terminal tax approaches zero and has no impact on the equilibrium for the infinite horizon version of our economy.

Turning to the optimum problem faced by the consumer at date $T-1$, let $\mu_T \equiv M_{T+1}/M_T$, where the dating of μ_T reflects the fact that M_{T+1} is chosen at date T. Then, after some manipulation, the first-order conditions to this optimum problem lead to

$$E_{T-1}\left[\frac{\nu_T}{x_{T-1}\mu_{T-1}}\right] = E_{T-1}\left[\frac{\beta\,\theta_T\,\nu_{T+1}}{x_T\,\mu_T}\right]. \quad (7)$$

Expression (7) is identical to the first-order condition (3.18) in Eichenbaum and Singleton (1986). Using the fact that $x_T = k_T$, substituting (2) for k_T and solving for x_{T-1} gives

$$x_{T-1} = \frac{k_{T-1}\,E_{T-1}\,[\nu_T/\mu_{T-1}]}{E_{T-1}\left[\dfrac{\nu_T}{\mu_{T-1}} + \beta\,\dfrac{\nu_{T+1}}{\mu_T}\right]}. \quad (8)$$

Solving recursively backwards and using the law of iterated expectations gives the following solution for x_{T-j},

$$x_{T-j} = k_{T-j}\frac{E_{T-j}[\nu_{T-j+1}/\mu_{T-j}]}{E_{T-j}\left[\sum_{k=1}^{j+1}\beta^{k-1}\nu_{T-j+k}/\mu_{T-j+k-1}\right]},\ j=1,2,\ldots. \quad (9)$$

To obtain the corresponding solution for the case of an infinitely lived, representative consumer we let $T \to \infty$, which yields

$$x_t = k_t\frac{E_t[\nu_{t+1}/\mu_t]}{E_t\left[\sum_{j=0}^{\infty}\beta^j\dfrac{\nu_{t+j+1}}{\mu_{t+j}}\right]} = \delta_t\,k_t. \quad (10)$$

Note that under our assumption that $E_t\,\nu_{t+1} > 0$, $0 < \delta_t < 1$ for all t which implies that $k_t > 0$, for all t. In contrast, the solution for x_t given in Eichenbaum and Singleton (1986) does not guarantee a positive stock of capital in the presence of taste or monetary shocks. The latter solution presumes that δ_t in (10) is a constant.

The class of monetary growth rate rules which is consistent with the assumption that the cash-in-advance constraint is binding for all t in equilibrium is the same as in Eichenbaum and Singleton (1986). Namely, μ_t must satisfy

$$\mu_t > \beta\ E_t[\nu_{t+1}]/\nu_t. \quad (11)$$

An alternative expression for x_t is obtained by solving (10) for k_t and substituting this expression into (2):

$$x_t = \theta_t\ \delta_t[1/\delta_{t-1} - 1]x_{t-1}, \quad (12)$$

or in logarithmic form

$$\ell n x_t = \ell n x_{t-1} + \ell n\theta_t + \ell n\delta_t^*,\ \delta_t^* = \delta_t[1/\delta_{t-1} - 1]. \quad (13)$$

Notice that $(\ell n x_t)$ continues to embody a unit root in this general solution under the assumption that $(\ell n\theta_t)$ and $(\ell n\delta_t^*)$ are stationary stochastic processes.

The logarithm of the money growth rate does not in general enter (13) or the corresponding equation for output in a simple additive form as in Eichenbaum and Singleton (1986). Consequently, the circumstances under which monetary policy affects output, but in which money growth does not Granger cause output, also differ from our previous analysis. There is a simple and nontrivial money supply rule that yields this result in context of the general solution (13). Suppose the monetary growth rate is set so as to offset the effects of taste shocks on intermediate goods decisions:

$$\mu_t = E_t[\nu_{t+1}], \quad (14)$$

and $\nu_t > \beta$ for all t so that (11) is satisfied. Then $\delta_t = (1 - \beta)$ and $x_t = \beta\theta_t x_{t-1}$. This result has a very intuitive interpretation. A positive taste shock at date $t + 1$ increases the marginal utility of consumption at date $t + 1$. Since consumption goods must be purchased at $t + 1$ with cash balances carried over from period t, consumers will want to hold more cash balances at the end of period t (desired M_{t+1} is larger). If the monetary authorities follow the accommodative rule (14), then these desires are met in a manner such that taste shocks do not affect output. Effectively, the monetary authorities are being perfectly accommodating to "demand" shocks. If ν_t is uncorrelated with the other shocks in the system, then (14) implies that μ_t is not Granger caused by the growth rate of output. Eichenbaum and Singleton (1986) present substantial evidence against this null hypothesis. It follows that the implied law of motion for the μ_t under this policy is counterfactual unless ν_t is correlated with the other shocks in the system.

Another interesting observation is that the monetary model under (14) is observationally equivalent to the corresponding RBC model without taste shocks. That is, the general solution to the RBC model in Eichenbaum and Singleton (1986) is:

$$x_t = k_t \frac{\nu_t}{E_t\left[\sum_{j=0}^{\infty} \beta^i \nu_{t+j}\right]}. \quad (15)$$

If $\nu_t = 1$ for all t (taste shocks are absent), then $x_t = k_t(1 - \beta)$. This, in turn, implies that $x_t = \beta\theta_t x_{t-1}$. Thus, an RBC model with technology shocks alone is indistinguishable from a monetary model with taste and technology shocks and the nontrivial money supply rule (14). Clearly, money is not neutral in the latter environment.

Of course, if the monetary authorities do not follow the rule (14)—for example, they respond to expected future technology shocks as well as taste shocks—then in general monetary growth will have predictive power for output growth. In this case, the failure to find Granger causality from money to output must be interpreted in the context of (13) as resulting from measurement problems, low power of the tests in small samples, or misspecification of the time-series representations of money and output.

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