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Social Security and Demographic Uncertainty
The Risk-Sharing Properties of Alternative Policies

Henning Bohn

All over the world, declining population growth rates and rising life expectancy are creating problems for public retirement systems. With a constant population structure, a pay-as-you-go (PAYGO) social security system could operate at constant tax and replacement rates. But, when the ratio of retirees to workers rises, either tax rates must be raised, or the replacement rate must be reduced. These demographic changes are the driving force behind the current social security reform debate.

This paper considers the design of social security from an ex ante perspective. Once a demographic shock is realized, a debate on how to adjust taxes and benefits is necessarily a distributional debate. A lighter burden on one generation implies a heavier burden on other generations. From an ex ante perspective, in contrast, demographics is a stochastic process, and the design questions are about risk sharing. Different realizations of birthrates and survival rates have an effect on the financial status of government programs and, more broadly, on the set of feasible allocations of national resources. Policy questions are then questions of efficiency: How can the financial risks created by demographic uncertainty be shared by different generations? What are the risk-sharing implications of alternative policy rules? Moreover, we can evaluate specific policy actions ("reforms") taken in response to demographic changes in terms of whether they represent efficient responses to the underlying shocks.

I examine demographic changes in a Diamond (1965)-style neoclassical growth model with overlapping generations, building on Bohn (1998). Government policy is potentially welfare improving because future generations are naturally excluded from financial markets. They cannot insure

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themselves against macroeconomic or demographic risks.[1] In this setting, I characterize the general properties of alternative social security systems, with a focus on four specific alternatives: a PAYGO social security system with defined benefits (DB); a PAYGO system with defined contributions (DC); a private/privatized system; and a "conditionally prefunded" system.

The two PAYGO systems are relevant because existing social security systems in many developed countries, including the United States until 1983, are pure PAYGO systems. If the worker-retiree ratio is constant, DB and DC are observationally equivalent. But, when the retiree-worker ratio rises, the key issue for PAYGO social security is whether taxes are held constant and benefits reduced or whether benefits are held constant and taxes increased. This choice is at the heart of the current U.S. policy debate.

The analysis of a privatized system is motivated by the current discussion about systems in which individuals fund their own retirement, at least in part. A fully privatized system represents this policy option in pure form.[2]

Finally, the "conditionally prefunded" social security system is intended to capture key features of the post-1983 U.S. system. The U.S. social security debate is heavily influenced by the Social Security Administration's seventy-five-year extrapolations of current policy. Whenever the seventy-five-year forecast shows a significant revenue gap, public pressure seems to arise to reform the system.[3] If one takes this linkage seriously and assumes that projected funding gaps systematically trigger tax and benefit changes, one obtains a well-defined pattern of intergenerational transfers, namely, a system in which trust funds are accumulated or drawn down in response to demographic shocks. For the stylized representation of such a system, I assume that net benefits are fixed one generational period in advance, at a level that depends negatively on anticipated changes in the retiree-worker ratio.[4]

The paper derives four main sets of results, namely, about the implications of variable birthrates, about variations in longevity, about the different positive effects of alternative policies, and about their efficiency properties.

1. To simplify, I abstract from private risk sharing and from Ricardian bequests.

2. Some of the privatization literature distinguishes between private savings without government intervention and "privatized" social security, meaning a funded system that is mandatory and government regulated. For the intergenerational issues discussed in this paper, this distinction is irrelevant.

3. For example, the 1983 reform was supposed to cover the then-existing revenue gap through tax increases that would accumulate a trust fund sufficient to carry social security through the years of baby-boom retirement. Much of the current debate is also about closing the projected funding gap.

4. There is an apparent consensus that benefit changes ought to be phased in slowly and that the benefits of current retirees cannot be touched. The reform debate is about varying *future* benefit levels in response to anticipated demographic pressures, not about moving to a true PAYGO-DC system with variable benefits to current retirees. McHale (chap. 7 this volume) suggests that social security reforms in other countries follow a similar pattern.

First, members of a small cohort generally benefit from being in a small cohort even if the government operates a DB social security system. This finding deserves emphasis because the main concern in the current reform debate has been about the plight of the baby-bust generation, about the fact that DB imposes relatively high taxes on small cohorts that support preceding larger cohorts. Large cohorts are, however, worse off than small ones if there is no DB social security: their high labor supply drives down the wage rate when the cohort is young; their desire to save reduces the return on capital as they age. Conversely, small cohorts enjoy favorable factor-price movements. They are better off than large cohorts even with a DB social security system unless taxes are so high that the fiscal burden dominates the factor-price effects.

In the model, the magnitude of the factor-price effects relative to the fiscal burden depends on the elasticity of factor substitution and on the level of social security taxes. With Cobb-Douglas technology (as the benchmark), the factor-price effects dominate if the ratio of tax rate (θ) to one minus the tax rate, $\theta/(1 - \theta)$, is below the capital share in output. For the United States, this condition is satisfied by a wide margin, suggesting that the factor-price effects of birthrate changes should dominate the fiscal effects. The current debate about social security reform, in contrast, focuses on fiscal pressures and virtually ignores factor-price effects.[5]

One may wonder, of course, to what extent the results from the two-period model are empirically realistic. The empirical evidence is unfortunately very limited, largely because it takes decades of data to obtain a single generation-length observation. Empirical evidence in related areas—cross-country growth and studies of relative wages—suggests, however, that demographic changes have wage effects broadly consistent with the overlapping-generations model (see sec. 6.5 below).

The second set of results is about unexpected changes in old-age mortality. The implications for the allocation of risk depend significantly on the individual predictability of death, on the availability of fair annuities, and on who might receive any accidental bequests. Under a variety of assumptions, lower old-age mortality increases the need for retirement consumption. The efficient response to a longer retirement period is then to increase social security benefits. This argument applies if deaths are individually foreseeable, or if savings are annuitized so that accidental bequests are small, or if accidental bequests are distributed within a cohort.

5. The Social Security Administration's long-run projections of the social security system's financial status are, e.g., based on extrapolating historical trends. Neither the linkage between cohort size and factor prices nor the insurance role of DB social security is a new idea. Easterlin (1987) provides much broader arguments about the advantages of being in a small cohort. Smith (1982) provides a numerical example illustrating the insurance role of DB social security. The point here is that the factor-price effects are large relative to the fiscal effects under empirically plausible assumptions and therefore important for social security reform.

Reduced benefits might be efficient, however, if lower old-age mortality reduces the accidental bequests received by workers.[6]

Third, a comparison of alternative policies shows that a fully privatized system has essentially the same risk-sharing properties as a DC PAYGO system. This is because neither a DC PAYGO nor a privatized system imposes higher taxes on the young when the retiree-worker ratio rises, whereas a DB system does. For risk-sharing purposes, a partially privatized system (say, combining a smaller DC plan with individual accounts) is therefore equivalent to a mixture of DB and DC systems. A conditionally funded DB system mimics a partially privatized system with regard to anticipated demographic changes, but it behaves like a pure DB system when unexpected changes occur.

Fourth, none of the above systems is fully efficient. Efficient policy responses (if any) should take place as soon as a demographic shock is revealed. Moreover, efficiency requires that all risks are shared by all generations, making no exception for current retirees. This requirement is violated by DB and DC systems because both fail to vary current retiree benefits in anticipation of future changes in the retiree-worker ratio, for example, when the current birthrate changes. I have argued elsewhere (Bohn 1999b) that the *political* viability of social security requires at least a one-period-ahead commitment to retiree benefits (see also McHale, chap. 7 in this volume). This may explain why the political debate takes for granted that current retirees are exempt from reforms. From a risk-sharing perspective, such an exemption is nonetheless a glaring inefficiency.

Although this paper focuses on demographic risks, I should briefly comment on other sources of uncertainty, notably on productivity risk and stock market risk.[7] Productivity shocks are arguably the most important source of long-run uncertainty about wages and capital income (Bohn 1999a). In an overlapping-generations setting, productivity risk is not necessarily allocated efficiently across cohorts. Policy tools such as government debt and social security implicitly shift risk across cohorts (Bohn 1998). Social security, especially a wage-indexed system, has an important role in this context because it provides a means of intergenerational redistribution that is more "neutral" with regard to risk shifting than government debt.

6. In the current reform debate, increased longevity is often cited to justify an increased "normal" retirement age, i.e., reduced benefits for a given retirement age. Some proposals even call for an indexing of the retirement age to life expectancy. The efficiency considerations of this paper provide support for such proposals only if the accidental bequest channel is empirically important. This is an open question.

7. There is also a huge literature on how social security helps share individual-level risks such as disability, mortality, and cross-sectional income uncertainty (see, e.g., Storesletten, Telmer, and Yaron 1999). Such risks may well be responsible for the existence and popularity of social security, but they are beyond the scope of this paper.

Stock market risk has recently received considerable attention in the social security literature. Here, one should distinguish work on "privatized" retirement (investment options in "individual accounts") from work on intergenerational risk sharing through the social security trust fund. Individual accounts are essentially irrelevant from a generational perspective because the returns accrue to the contributors (Bohn 1997). Trust fund investments, on the other hand, reallocate risk across generations because future taxpayers are the residual claimants in any DB system. Bohn (1997, 1999a), Smetters (1997, chap. 3 in this volume), Shiller (1999), and Abel (1998, chap. 5 in this volume) discuss some of the positive and normative implications of alternative trust fund investments. This paper abstracts from most financial market issues to focus on demographics. But I include a simple productivity shock to demonstrate that shocks to the labor force have very different welfare implications than productivity shocks even though both have the same effect on the effective capital-labor ratio. The productivity shock also illustrates how easily other shocks could be added.

The paper is organized as follows. Section 6.1 describes the model. Section 6.2 examines the risk-sharing implications of alternative social security policies. Section 6.3 studies the implications of missing annuities markets and of accidental bequests. Section 6.4 derives necessary conditions for efficient risk sharing and their implications for social security policy. Section 6.5 comments on extensions of the model and on empirical issues. Section 6.6 concludes.

6.1 A Model with Stochastic Population Growth

This section examines risk sharing in a modified Diamond (1965)-style overlapping-generations model with stochastic population growth and stochastic total factor productivity.

6.1.1 Population Dynamics and Preferences

In the Diamond model, generation t enters as working-age adults in period t and retires in period $t + 1$. For modeling demographic uncertainty, it is important, however, that individuals are born long before they enter the labor force. In terms of generational time units, society has about one period advance notice about changes in the retiree-worker ratio. Hence, I will assume that generation t is born in period $t - 1$, works in period t, and retires in period $t + 1$. At time t, N_t^C is the number of generation $t + 1$ children, N_t^W the number of generation t workers, and N_t^R the number of generation $t - 1$ retirees.

To limit the scope of the paper, I assume throughout that childbearing is exogenous. Each of the N_t^W workers of generation t has b_t children so that $N_t^C = N_t^W \cdot b_t$. To make the future workforce somewhat unpredictable, I assume that only a fraction μ_{1t+1} of children survives into adulthood.[8] Then the growth rate of the workforce, $N^W_{t+1}/N^W_t = \mu_{1t+1} \cdot N^C_t/N^W_t = \mu_{1t+1} \cdot b_t = 1 + n^W_{t+1}$, is partially predictable, but not perfectly. The variables μ_{1t} (survival rate) and b_t (birthrate) are assumed independently and identically distributed (i.i.d.). Throughout, individuals in a cohort are identical, individual survival probabilities equal the aggregate survival rate, and all variables are treated as continuous, including b_t.

Parents care about their children's consumption when the children live in their household. Their preferences do not include an altruistic bequest motive, however. This assumption is important because fiscal policy would be irrelevant if all generations were linked through Ricardian bequests. Since Altonji, Hayashi, and Kotlikoff (1996) find that private intergenerational risk sharing is highly imperfect empirically, it is a reasonable assumption in this context. Bequests may nonetheless occur "accidentally" if mortality is stochastic and annuity markets are imperfect, as I will explain below.

Parents make decisions about their own consumption c^W_t and about their childrens' consumption c^0_t (per child). Throughout, I assume homothetic (constant relative risk aversion [CRRA]) preferences to obtain balanced growth. Let

$$u^1_t = \frac{1}{1-\eta} \cdot [\rho^W \cdot (c^W_t)^{1-\eta} + b_t \cdot \rho_0(b_t) \cdot (c^0_t)^{1-\eta}]$$

be the parent's period t utility, where $\eta > 0$ is the inverse elasticity of intertemporal substitution. The per child weight $\rho_0(b_t)$ may depend on the number of children: it seems reasonable to assume that $0 < \rho_0(b_t) \leq \rho^W$ and that $b_t \cdot \rho_0(b_t)$ is nondecreasing in the number of children. For any level of household consumption $c^1_t = c^W_t + b_t \cdot c^0_t$, the parent's optimality condition

$$b_t \cdot \rho^W \cdot (c^W_t)^{-\eta} = \rho_0(b_t) \cdot (c^0_t)^{-\eta}$$

then implies that u^1_t can be written as an indirect utility over household consumption,

$$u^1_t(c^1_t) = \rho_1(b_t) \cdot (c^1_t)^{1-\eta}/(1-\eta),$$

where

$$\rho_1(b_t) = \rho^W \cdot \{1 + b_t \cdot [\rho_0(b_t)/b_t/\rho^W]^{1/\eta}\}^\eta$$

8. Otherwise, $N^W_{t+1} = N^C_t$ would be known at time t. One may also interpret μ_{1t+1} as reflecting uncertainty about immigration. But, since immigration would raise subtle welfare questions (how to include immigrants in the welfare function), I will not address immigration explicitly, and I interpret all uncertainty about N^W_{t+1} as survival uncertainty.

depends on the number of children. Under the assumptions outlined above, the elasticity of the weight ρ_1 with respect to the birthrate, $\gamma_\rho = \rho_1/b_t \cdot (b_t/\rho_1)$, is in the interval $0 \leq \gamma_\rho \leq \eta$.

Overall, children matter for the analysis for two reasons. Their birth provides advance notice about the size of future adult cohorts, and they affect their parents' spending needs. Thus, the model accounts not only for old-age dependency but also for variations in youth dependency. Otherwise, the model with children works just like Diamond's two-period overlapping-generations model.

Now consider retirement. As old-age survival improves, more workers survive into the retirement period, and those who survive live longer. For social security, these changes matter only through their combined effect on the ratio of retirees to workers.[9] For individual behavior, however, an anticipated longer life span may have different implications than a reduced probability of a sudden death. For a known life span, retiree consumption needs are presumably proportional to the length of the retirement period. Retiree consumption needs will also increase if the rate of unanticipated deaths declines in a setting with fair annuities. This is because individuals without a bequest motive will place all their assets into annuities. The return on fair annuities is inversely related to the average survival rate. A rising survival rate will therefore require more savings to support a given consumption level, as in the case of a longer life span. If annuities are unavailable, however, or too expensive to be commonly used, a rising survival rate increases the probability that retirees consume their assets and do not leave accidental bequests. The cases with and without annuities have different policy implications and therefore deserve to be modeled carefully.

To capture a variable life expectancy in the overlapping-generations setting, I model the retirement period as a fractional period. At the start of period t, a fraction $1 - \mu_{2t}$ of all generation $t - 1$ workers dies. The remainder, μ_{2t}, learn that they will live for a period of length $\phi_t \in (0, 1]$. Both the survival probability and the conditional length of life have predictable and unanticipated components: $\mu_{2t} = \mu^e_{2t-1} \cdot \mu^u_{2t}$ and $\phi_t = \phi^e_{t-1} \cdot \phi^u_t$, where μ^u_{2t} and ϕ^u_t are i.i.d. shocks revealed at the start of period t, and μ^e_{2t-1} and ϕ^e_{t-1} are i.i.d. shocks revealed in period $t - 1$.[10] The product

9. The two changes may have different effects if the social security replacement rate varies with age or if one accounts for Medicare. In the United States, social security is fixed in real terms at retirement so that the replacement rate tends to fall with age, but the value of Medicare is rising with age. In the model, the replacement rate is assumed constant within each generational period.

10. For simplicity, I treat ϕ_t and μ_{2t} as level-stationary even though technical progress in medical technology suggests an upward drift. Drift terms would require an analysis of "unbalanced" growth paths. This could be done (for a deterministic analysis, see Bohn 1999b), but it would be cumbersome and would not provide new insights about risk sharing. Autocorrelation could also be accommodated, but it would not affect the main results and is therefore omitted.

$\mu^e_{2t-1} \cdot \phi^e_{t-1}$ may be interpreted as the life expectancy at retirement. Conditional on survival, the period t utility of the old is assumed proportional to the length of life, $u^2_{t+1} = \phi_t \cdot (c^2_{t+1})^{1-\eta}/(1-\eta)$.[11]

Finally, generation t's overall preferences combine the utility over working-age consumption $u^1_t(c^1_t)$ and retirement consumption $u^2_{t+1}(c^2_{t+1})$:

$$(1) \quad U_t = I_{1t} \cdot \left[u^1_t(c^1_t) + I_{2t+1} \cdot \rho_2 \cdot u^2_{t+1}(c^2_{t+1})\right]$$

$$= \frac{1}{1-\eta} \cdot I_{1t} \cdot \left[\rho_1(b_t) \cdot (c^1_t)^{1-\eta} + \rho_2 \cdot \phi_{t+1} \cdot I_{2t+1} \cdot (c^2_{t+1})^{1-\eta}\right],$$

where the random variables I_{1t} and I_{2t+1} are 0-1 indicators for individual survival into adulthood and retirement, and ρ_2 captures time preference. In expectation, $E[I_{1t}] = E[\mu_{1t}] = \mu_1$ and $E_t[\phi_{t+1} \cdot I_{2t+1}] = \phi^e_t \cdot \mu^e_{2t}$ are equal to the respective aggregate values.

Overall, the population dynamics are such that the future labor force and the future worker-retiree ratio are quite predictable one period ahead, but not perfectly. This limited predictability is important for modeling social security because it motivates why policy reforms are debated with some lead time before demographic changes actually take place.

6.1.2 The Macroeconomic Setting

The macroeconomic setting is intentionally kept simple to focus on the demographics. Each working-age person inelastically supplies one unit of labor. Output is produced with capital K_t and labor N^W_t:

$$(2) \qquad Y_t = K^\alpha_t \cdot (A_t \cdot N^W_t)^{1-\alpha},$$

where α is the capital share, and A_t is the economy's total factor productivity. Productivity follows a stochastic trend $A_t = (1 + a_t) \cdot A_{t-1}$ with i.i.d. growth rate a_t. Capital depreciates at the rate δ, implying a national resource constraint

$$(3) \qquad Y_t + (1-\delta) \cdot K_t = c^1_t \cdot N^W_t + c^2_t \cdot \phi_t \cdot \mu_{2t} \cdot N^W_{t-1} + K_{t+1}.$$

Some extensions are examined in section 6.5.[12]

The wage rate $w_t = (1-\alpha) \cdot A_t \cdot [K_t/(A_t \cdot N^W_t)]^\alpha$ and the return on capital $R^k_t = \alpha \cdot [K_t/(A_t \cdot N^W_t)]^{\alpha-1} + (1-\delta)$ both depend on the capital-labor ratio. Since K_t is known in period $t-1$, it is convenient to define

11. One may interpret u^2_t as an indirect utility obtained by maximizing $\int_0^{\phi t} [c(s)]^{1-\eta}/(1-\eta)ds$ over a continuous consumption stream $c(s)$, subject to a resource constraint limiting $\int_0^{\phi t} c(s)ds$. Implicitly, this abstracts from within-period interest and discounting.

12. Bohn (1998) has shown how this setting can be generalized, e.g., to include a variable labor supply, temporary productivity, government spending, and a production function with an elasticity of substitution different from one, but such complicating features would be distracting here.

the state variable $k_{t-1} = K_t/(A_{t-1} \cdot N^W_{t-1})$ that scales the capital stock by lagged productivity and the lagged labor force. Wages and interest rates then depend on k_{t-1}, on current productivity growth, and on the current workforce growth.

To model policy, I abstract from all government activity but social security.[13] The government collects payroll taxes on wages w_t at a rate θ_t from all workers and pays benefits to retirees at a replacement rate β_t. The cost of social security is the product of the number of surviving retirees $N^R_t = \mu_{2t} \cdot N^W_{t-1}$, their length of life ϕ_t, and the level of benefit $\beta_t \cdot w_t$. The system's revenues are $\theta_t \cdot w_t \cdot N^W_t$. For given replacement rate β_t, the PAYGO budget constraint therefore implies a payroll-tax rate of

$$\text{(4)} \qquad \theta_t = \beta_t \cdot \phi_t \cdot \mu_{2t} \cdot \frac{N^W_{t-1}}{N^W_t} = \beta_t \cdot \frac{\phi_t \cdot \mu_{2t}}{b_{t-1} \cdot \mu_{1t}}.$$

The ratio $(\phi_t \cdot \mu_{2t})/(b_{t-1} \cdot \mu_{1t})$ can be interpreted as the "average" retiree-worker ratio (after smoothing over ϕ_t).

Interesting special cases of the PAYGO system are the defined-benefit (DB) system with $\beta_t = \beta^*$ and the defined-contribution (DC) system with $\theta_t = \theta^*$ and $\beta_t = (1 + n^W_t)/(\phi_t \cdot \mu_{2t}) \cdot \theta^*$. Since individuals are not liquidity constrained, government-mandated savings (sometimes called *privatized* or *individual accounts* systems) would simply reduce private savings (Bohn 1997). A privatized social security system is therefore equivalent to $\theta^* = 0$. In a mixed system consisting of individual accounts plus a PAYGO component, one should interpret θ_t and β_t as the taxes and benefits of the PAYGO component.

A system with government-run trust funds is somewhat more complicated if the system promises benefits that do not depend on the performance of the trust fund (as in the United States). Generational accounting implies that each cohort's net benefits are equal to the system's PAYGO component, that is, to the statutory benefits minus the proceeds from the trust fund built up by the same cohort's payroll taxes (see Bohn 1997). In the United States, the buildup of the current trust fund started in 1983 in response to a funding gap in the Social Security Administration's long-run projections. Projected funding gaps are similarly influencing the current debate. Such gaps arise from two principal sources, rising life expectancy and reduced birthrates. Hence, one may interpret the current U.S. system as a defined-benefits system that accumulates trust funds in response to either a rise in life expectancy, $\mu^e_{2t} \cdot \phi^e_t$, or a fall in the birthrate, b_t. Since a trust fund buildup is equivalent to a reduction in net benefits, such a

13. This approach is nonetheless quite general because government transfers matter only through different cohorts' generational accounts. Hence, social security can be interpreted broadly as a stand-in for other intergenerational transfers.

"conditionally prefunded" system can be represented parsimoniously by a benefit function $\beta_t = \beta(\mu^e_{2t}, \phi^e_t, b_t)$ with $\partial\beta/\partial\mu^e_2 < 0$, $\partial\beta/\partial\phi^e < 0$, and $\partial\beta/\partial b > 0$.

McHale's (chap. 7 in this volume) analysis of recent pension reforms around the world suggests that a variable-benefit function of this type is empirically realistic for other countries, too. In the countries studied by McHale, reforms were generally triggered by anticipated funding gaps. Benefits to current retirees remained virtually unchanged, but benefits to future generations were reduced. This implies a benefit function with the same features as in the conditionally prefunded system.

More generally, a variety of social security systems with and without prefunding can be reinterpreted as PAYGO systems with an appropriately state-contingent benefit function. Hence, I will use the PAYGO notation throughout the paper.

6.1.3 Individual Behavior

Individuals maximize their expected utility (1) subject to their budget constraints. The main complications are potential imperfections in the market for private annuities.

When working, individuals earn an after-tax wage income $w_t \cdot (1 - \theta_t)$ and possibly receive accidental bequests Q^1_t (defined below). Denoting savings by s_t, the first-period budget equation is

$$c^1_t = w_t \cdot (1 - \theta_t) + Q^1_t - s_t. \tag{5}$$

If fair annuities exist, they offer a return R^k_{t+1}/μ_{2t+1} that is above the return on nonannuitized savings.[14] Hence, all savings should be annuitized. Empirically, however, private annuities are so costly that the bulk of private savings is not annuitized (Congressional Budget Office 1998).

To gauge the significance of this apparent market imperfection, first consider the case with fair annuities.[15] If all assets are annuitized, surviving retirees will spend their private resources $R^k_{t+1}/\mu_{2t+1} \cdot s_t$ at the rate $1/\phi_{t+1}$, and there are no bequests. Retirement consumption (including receipts from social security) is then

14. Either one may assume that individual annuity payoffs are indexed to the ex post survival rate μ_{2t+1}; or, if annuity contracts promise a payoff R^k_{t+1}/μ^e_{2t} linked to the expected survival rate, one may note that annuity firms, like all other firms, are owned by the old so that the annuity firms' aggregate profit $R^k_{t+1} - \mu_{2t} \cdot R^k_{t+1}/\mu^e_{2t}$ accrues to the old. In either case, the old bear the risk of unexpected mortality changes.

15. Ideally, one might want to include a model of why private annuities are so costly (e.g., a model of adverse selection), but this would excessively complicate the analysis. Hence, I focus on two simple polar cases, fair annuities and prohibitively costly private annuities. In the latter case, I implicitly assume that social security has a cost advantage. This is perhaps plausible because a mandatory system avoids adverse selection.

(6a) $$c^2_{t+1} = \frac{R^s_{t+1}}{\mu_{2t+1} \cdot \phi_{t+1}} \cdot s_t + \beta_t \cdot w_{t+1},$$

and savings are determined by the individual optimality condition

(7a) $$\rho_1(b_t) \cdot (c^1_t)^{-\eta} = \rho_2 \cdot E_t[\phi_{t+1} \cdot I_{2t+1}] \cdot E_t\left[\frac{R^s_{t+1}}{\mu_{2t+1} \cdot \phi_{t+1}} \cdot (c^2_{t+1})^{-\eta}\right]$$
$$= \rho_2 \cdot E_t[R^k_{t+1} \cdot (c^2_{t+1})^{-\eta}].$$

Note that mortality cancels out in (7a). Also, all individual and policy constraints depend on the length of life and on the survival rate only through their product $\phi_t \cdot \mu_{2t}$. Hence, under the assumption of perfect annuities, survival uncertainty μ_{2t} can be subsumed into ϕ_t and does not have to be examined separately.

Second, suppose that annuities do not exist (or are prohibitively costly). Then those who die at the start of their retirement period must leave accidental bequests. On aggregate, bequests of

(8) $$R^k_{t+1} \cdot s_t \cdot (1 - \mu_{2t+1}) \cdot N^W_t = Q^1_{t+1} \cdot N^W_{t+1} + Q^2_{t+1} \cdot N^R_{t+1}$$

accrue either to workers (the next generation, Q^1_{t+1}) or to other retirees (the same generation, Q^2_{t+1}).

The surviving retirees will spend their private resources $R^k_{t+1} \cdot s_t$ at the rate $1/\phi_{t+1}$. Including bequests and social security, retirement consumption is

(6b) $$c^2_{t+1} = \frac{R^k_{t+1}}{\phi_{t+1}} \cdot s_t + \frac{Q^2_{t+1}}{\phi_{t+1}} + \beta_t \cdot w_{t+1}.$$

Savings are determined by the first-order condition

(7b) $$\rho_1(b_t) \cdot (c^1_t)^{-\eta} = \rho_2 \cdot E_t[\phi_{t+1} \cdot I_{2t+1}] \cdot E_t\left[\frac{R^s_{t+1}}{\phi_{t+1}} \cdot (c^2_{t+1})^{-\eta}\right]$$
$$= \rho_2 \cdot \mu^e_{2t} \cdot E_t[R^k_{t+1} \cdot (c^2_{t+1})^{-\eta}].$$

Savings decisions now involve the probability of survival, μ^e_{2t}, and they are distorted because individuals do not value bequests. Moreover, accidental bequests affect the distribution of resources across cohorts to the extent that they go to the young (if $Q^1_t > 0$).[16]

Despite this multitude of effects, annuities turn out to be relatively un-

16. If all bequests go to the old, missing annuities have only an incentive effect but no redistributional effect because (6b) would then imply that the retirement income $R^k_{t+1}/\phi_{t+1} \cdot s_t + Q^2_{t+1}/\phi_{t+1} = [R^s_{t+1}/(\mu_{2t+1} \cdot \phi_{t+1})] \cdot s_t$ is the same as with annuities.

important except for studying time-varying survival probabilities per se (see sec. 6.3 below). Intuitively, savings distortions ($\mu^e_{2t} < 1$) affect the level of economic activity, but they leave the propagation of other shocks and their effect on the different cohorts largely unchanged. And bequests ($Q_1 > 0$) give the young some exposure to shocks affecting capital income, but the effect is proportional to the size of such bequests relative to wage income, which is likely small.

Because of these complications and the fact that annuitized survival risk is economically equivalent to length-of-life risk, I abstract from old-age survival risk for much of the analysis and focus instead on length-of-life uncertainty (setting $\mu_{2t} \equiv \mu^e_{2t} \equiv 1$). Since shocks to survival uncertainty *with* fair annuities can be subsumed into ϕ_t, the ϕ_t shocks in this analysis can be interpreted as reflecting both shocks to the length of life and "diversifiable" (through annuitization) survival uncertainty. When I explicitly add survival uncertainty later (sec. 6.3), it will be sufficient to model the case *without* annuities because annuitized survival uncertainty is already covered under ϕ_t.

With either assumption about annuities, the basic dynamics are similar to the Diamond (1965) model. Each period, the young divide their wage income (and bequests, if any) between consumption and savings. Savings determine the next period's capital stock, $K_{t+1} = N^W_t \cdot s_t$, which determines the wage rate for the next young generation. Since I am not interested in issues of dynamic inefficiency, I assume that $\rho_2 \cdot \mu^e_{2t}/\rho_1(b_t)$ is low enough (for all μ^e_{2t}, b_t) that the economy is dynamically efficient.

With all the shocks and flexibly parametrized preferences, the model does not generally have a closed-form solution. As in Bohn (1998), I therefore follow the real business cycle and finance literature and examine log-linearized solutions—analytically derived ones, however, not numerically simulated ones. To ensure balanced growth, I assume a stationary policy rule for the replacement rate β_t. Without government, the model would have a Markov structure with K_{t-1} and the shocks $Z = \{b_t, b_{t-1}, \mu_{1t}, \phi^u_t, \phi^e_t, \phi^e_{t-1}, \mu^u_{2t}, \mu^e_{2t}, \mu^e_{2t-1}, a_t)$ as state variables. Adding more state variables would be uninteresting. I assume, therefore, that the policy rule is a function of at most these variables so that the model with government has the same structure.[17]

Given the Markov structure, the log deviation of any variable (y) from the perfect foresight path is an approximately linear function of the log deviations of the state variables. Unless otherwise noted, let symbols without the time subscript refer to steady states and hats (ˆ) denote log devia-

17. Without government, one could treat n^W_t and ϕ_t as state variables instead of their components. The components will have different effects, however, if policy treats expected and unexpected changes differently, e.g., in the conditionally prefunded system. Hence, I treat the components of n^W_t and ϕ_t as distinct state variables throughout.

tions.[18] The log-linearized law of motions for any variable y can be written as[19]

$$\hat{y}_t = \pi_{yk} \cdot \hat{k}_{t-1} + \sum_{z \in Z} \pi_{yz} \cdot \hat{z}_t, \tag{9}$$

where π_{yz} denotes the coefficient for state variable z. The π_{yz} coefficients can be interpreted as elasticities of y with respect to z.

The main variables of interest are the consumption of workers and retirees and the level of capital investment. Since the young divide their labor income between consumption and savings, c_t^1 and k_t depend on all shocks affecting the wage rate, on the incentives to save (R_{t+1}^k), and on the payroll tax. The consumption of the old depends on all shocks affecting capital income and social security benefits (see eqq. [6a] and [6b]). The resulting elasticity coefficients for various specifications of the model are listed in several tables discussed in the following sections.

To illustrate the practical implications of the model, I also provide the elasticity coefficients implied by a simple numerical example. For example, assume a capital share of $\alpha = 1/3$, full depreciation ($\delta = 1$), payroll taxes of $\theta = 0.15$, zero population growth ($n = 0$), a steady-state productivity growth factor of $1 + a = 1.35$ (1 percent annual growth for a thirty-year generational period), and an elasticity of substitution of $1/\eta = 1/3$. The effective retirement period—length times probability—is $\phi \cdot \mu_2 = 1/2$ (where $\phi = 1/2$ and $\mu_2 = 1$, except in sec. 6.3 below), and the time preference ρ_2 is set such that, in steady state, workers save 25 percent of their disposable income.[20]

6.2 The Risk-Sharing Properties of Alternative Systems

This section examines the positive effects of demographic shocks on the fortunes of different cohorts. The main sources of demographic uncertainty are shocks to the workforce and shocks to the number of retirees. For this section, I abstract from shocks that would trigger accidental be-

18. For example, $\hat{c}_t^1 = \ln(c_t^1) - \ln(c^1)$. When growth rates are involved, the "1+" is suppressed for notational convenience, as in $\hat{n}_t^W = \ln(1 + n_t^W) - \ln(1 + n^W)$.

19. An intercept term could be added to reflect average "displacements" from the deterministic paths caused, e.g., by risk aversion and precautionary savings (see Bohn 1998). But, since the focus here is on fluctuations, not level variables, intercept terms are omitted.

20. The example is motivated by the calibrated overlapping-generations model in Bohn (1999a), which can be consulted for a discussion of calibration issues. The assumed full depreciation is a convenient simplification, but it implies a caveat: setting $\delta = 1$ reduces the autocorrelation of capital (π_{kk}) and therefore understates the propagation of shocks. This is acceptable here because the analysis focuses on the impact effects. Setting $\delta = 1$ also reduces the level of R^k, which I offset by raising ρ_2 enough that the savings rate roughly matches the empirical investment share in GDP. This is why I calibrate savings, not the time preference.

quests (setting $\mu_{2t} \equiv \mu_{2t}^e \equiv 1$) and assume that all variations in old-age mortality are either changes in the known length of life or annuitized.

6.2.1 Defined Benefits

To start, consider an economy with constant social security benefits (DB). It will provide a benchmark for studying variable benefits below. Table 6.1 summarizes the log-linearized equilibrium responses of workers and retirees to various shocks.

First, consider an unanticipated shock to the number of workers ($\hat{n}_t^W = \hat{\mu}_{1t}$; panel A). A large number of workers has a clear positive effect on the old ($\pi_{c2\mu1} > 0$) because the reduced capital-labor ratio increases the old generation's capital income. The effect on the young is in principle ambiguous. With a defined-benefit system, members of a large cohort pay less social security tax (θ). But a large workforce also reduces the wage rate, as captured by negative α terms. The negative effects dominate whenever $\alpha > \theta/(1 - \theta)$. For plausible capital shares (0.3–0.4), this inequality holds unless the tax rate is well over 20 percent. If $\alpha > \theta/(1 - \theta)$, workers' income, consumption, and savings decline in response to a positive shock to the workforce, whereas retiree consumption rises. This is also true in the numerical example: $\alpha = 1/3 > \theta/(1 - \theta) = 0.176$; $\pi_{c1\mu1} = -0.131$ and $\pi_{k\mu1} = -0.235$ are negative; and $\pi_{c2\mu1} = 0.436$ is positive.

The main conclusion, to be reexamined below, is that, for plausible parameters, *large cohorts tend to be demographically disadvantaged.* Conversely, being in a small cohort is beneficial. Even though small cohorts face relatively high taxes under a defined-benefit system, they also enjoy high wages and high returns on savings.

Second, consider shocks to the current birthrate b_t (table 6.1, panel B). If one ignores children's expenses (setting $\gamma_\rho = 0$ for this argument), shocks to the birthrate are like shocks to the labor force that become known one period in advance. With defined benefits, such shocks have no effect on the old ($\pi_{c2b} = 0$). News about next period's labor force is relevant for the young, however, because they expect to be alive when the shock actually hits the retiree-worker ratio. Looking forward, they know that changes in b_t have the same effect in period $t + 1$ as the μ_{1t+1} shocks discussed above. A high birthrate b_t has a positive effect on retired generation t workers. But, provided $\alpha > \theta/(1 - \theta)$, it has a negative effect on generation $t + 1$ workers.

The response of period t workers is most likely an increase in current consumption and a reduction in savings. Specifically, table 6.1 shows that the elasticities π_{c1b} and π_{kb} depend on the interaction of three effects. First, expected retirement income rises because a high future workforce reduces next period's capital-labor ratio and raises the return on current savings. This income effect is captured by the positive γ_{c2nw} term in π_{c1b} and π_{kb}. Second, the increased return triggers a substitution effect in the opposite

Table 6.1 **Macroeconomic Dynamics with Defined Benefits**

Effect on:	Elasticity Coefficients	Numerical Example
	A. Shocks to the Current Workforce, μ_{1t} *and* b_{t-1}	
Retirees	$\pi_{c2\mu1} = \pi_{c2b1} = \gamma_{c2nw} > 0$	.436
Workers	$\pi_{c1\mu1} = \pi_{c1b1} = -\Delta_c \cdot \left[\alpha - \frac{\theta}{1-\theta}\right]$ is negative, provided $\alpha > \frac{\theta}{1-\theta}$	−.131
Investment	$\pi_{k\mu1} = \pi_{kb1} = -\Delta \cdot \left[\alpha - \frac{\theta}{1-\theta}\right] < 0$	−.235
	B. Shocks to the Current Birthrate, b_t	
Retirees	$\pi_{c2b} = 0$	0
Workers	$\pi_{c1b} = [1 - \Delta_c \cdot (c^1/A)y^1] \cdot (\gamma_{c2nw} - \pi_{Rk}/\eta + \gamma/\eta)$	.080
Investment	$\pi_{kb} = -\Delta_k \cdot (c^1/A)/y^1 \cdot (\gamma_{c2nw} - \pi_{Rk}/\eta + \gamma_\rho/\eta)$	−.240
	C. Shocks to the Current Length of Life, ϕ_t^u and ϕ_{t-1}^e	
Retirees	$\pi_{c2\phi u} = \pi_{c2\phi e1} < 0$	−.769
Workers	$\pi_{c1\phi u} = \pi_{c1\phi e1} = -\Delta_c \cdot \frac{\theta}{1-\theta} < 0$	−.147
Investment	$\pi_{k\phi u} = \pi_{k\phi e1} = -\Delta_k \cdot \frac{\theta}{1-\theta} < 0$	−.265
	D. Shocks to Life Expectancy (future length of life), ϕ_t^e	
Retirees	$\pi_{c2\phi e} = 0$	0
Workers	$\pi_{c1\phi e} = -[1 - \Delta_c \cdot (c^1/A)/y^1] \cdot \gamma_{c2\phi} < 0$	−.288
Investment	$\pi_{k\phi e} = \Delta_k \cdot (c^1/A)/y^1 \cdot \gamma_{c2\phi} > 0$	.865
	E. Changes in Lagged Capital and Productivity, k_{t-1} and a_t	
Retirees	$\pi_{c2k} = -\pi_{c2a} = \gamma_{c2k} > 0$	.333
Workers	$\pi_{c1k} = -\pi_{c1a} = \Delta_c \cdot \alpha > 0$	.278
Investment	$\pi_{kk} = -\pi_{ka} = \Delta_k \cdot \alpha > 0$	.500

Notes: The effect on retirees, on workers, and on investment refers to the effect of the shock(s) named in the panel head on the variables $(\widehat{c^1/A})_t$, $(\widehat{c^2/A})_t$, and $\hat{k}_t$. Since these variables are scaled by the productivity trend A_t, the coefficients for productivity shocks at a_t are negative. The effects of productivity shocks on consumption and investment levels, $1 + \pi_{c2a} > 0$, $1 + \pi_{c1a} > 0$, and $1 + \pi_{ka} > 0$, are nonetheless positive.

The column "Numerical Example" refers to the elasticity values in the numerical example described in the text.

Variables without time subscripts refer to the steady state. The symbols not already defined in the text are as follows:

$\delta^* = \frac{\delta \cdot k/an}{(c^2/A) \cdot \phi/(1+n^w)} \in (0, 1)$, share of old capital in retiree income; $an = (1 + a) \cdot (1 + n^W)$;

$\gamma_{c2k} = (1 - \delta^*) \cdot \alpha + \delta^* \in (0, 1)$, effect of a higher capital-labor ratio on the old;

$\gamma_{c2nw} = 1 - \gamma_{c2k} - (1 - \delta^*) \cdot \frac{\theta \cdot (1-\alpha)}{\alpha + \theta \cdot (1-\alpha)} = \frac{(1-\delta^*) \cdot (1-\sigma) \cdot \alpha \cdot (1-\theta)}{\alpha + \theta \cdot (1-\alpha)} \in (0, 1)$, effect of a higher current labor force on the old;

$\gamma_{c2\phi} = 1 - (1 - \delta^*) \cdot \frac{\theta \cdot (1-\alpha)}{\alpha + \theta \cdot (1-\alpha)} \in (0, 1)$, effect (absolute value) of a longer life span on the old;

$\pi_{Rk} = (1 - (1 - \delta)/R^k) \cdot (1 - \alpha) \in (0, 1)$, effect (absolute value) of a higher capital-labor ratio on the return to capital;

$y^1 = w/A \cdot (1 - \theta)$, income of the young scaled by productivity;

$\Delta_c = \frac{[\gamma_{c2k} + \pi_{Rk}/\eta]}{(c^1/A)/y^1 \cdot (\gamma_{c2k} + \pi_{Rk}/\eta + \gamma_{c2\beta} \cdot \pi_{\beta k}) + k/y^1} > 0$, marginal effect on consumption when the income of the young rises; and

$\Delta_k = \frac{1}{(c^1/A)/y^1 \cdot (\gamma_{c2k} + \pi_{Rk}/\eta + \gamma_{c2\beta} \cdot \pi_{\beta k}) + k/y^1} > 0$, marginal effect on capital investment when the income of the young rises.

direction (the $-\pi_{Rk}/\eta$ term). Finally, expenses for children increase the consumption needs of working-age families (the γ_ρ term with $\gamma_\rho > 0$). Unless the elasticity of intertemporal substitution is high enough to offset both other effects, the net effects are higher consumption ($\pi_{c1b} > 0$) and lower investment ($\pi_{kb} < 0$). In the numerical example, these signs apply even for $\gamma_\rho = 0$; $\pi_{c1b} = 0.08$, and $\pi_{kb} = -0.24$.[21]

Overall, a change in the birthrate triggers changes in consumption and capital investment before it actually affects the labor supply. The effect over time is traced out in figures 6.1 and 6.2. For the figures, I consider a one-time 20 percent *reduction* in the birthrate b_t applied to the elasticities of the numerical example.[22] In period t, retirees (generation $t - 1$) are unaffected. Workers (generation t) realize that the next working-age cohort will be small, which will reduce the return on savings. Assuming that the negative income effect dominates the substitution effect, generation t will reduce their consumption c_t^1 and raise savings k_t. In period $t + 1$, the lower return reduces generation t's consumption despite the increased savings (see fig. 6.1). Generation $t + 1$'s consumption rises, in contrast, because of higher wages. Wages are higher because of the low labor supply and because of the higher capital stock (see fig. 6.2). The increased wage outweighs the increase in tax rates. Since the capital stock rises, subsequent generations are better off, too.

Note that the increased period t savings merely magnify the change in period $t + 1$ wages. A reduction in b_t would make the baby-bust generation better off *even if* the preceding generation did not save more (say, if $1/\eta$ were large enough that $\pi_{kb} = 0$). Increased savings further improve the consumption opportunities of the baby-bust generation and its successors, but this savings response is not crucial.[23]

In terms of the current policy debate, the analysis here suggests that we are perhaps too worried about the baby-bust generation and its ability to pay defined benefits to the baby boomers. Instead, the baby-bust generation can look forward to a substantial growth in wages, whereas the baby-

21. Recall that $\gamma_\rho \in [0, \eta]$. For the upper bound $\gamma_\rho = \eta = 3$, one obtains $\pi_{c1b} = 0.455$ and $\pi_{kb} = -1.365$. Unless otherwise noted, I use $\gamma_\rho = 0$ for the example numbers—for simplicity and to avoid exaggerating the birthrate effects.

22. The 20 percent is somewhat less than both the projected increase in the retiree-worker ratio from 1990 to 2020 (the baby-boom retirement) and the decline in the ratio of the age zero to age twenty-nine population to the age thirty to age fifty-nine population between 1960 and 1990 (the baby bust). The example is indicative of the shape of the impulse-response functions in general, provided $\alpha > \theta/(1 - \theta)$ and $\gamma_{c2nw} + \gamma_\rho/\eta > \pi_{Rk}/\eta$. One exception: for large γ_ρ, the sign of $\hat{c}_{t+1}^2$ and the relative magnitude of $\hat{c}_t^1$ and $\hat{c}_{t+1}^2$ could be reversed, namely, if reduced expenses for children dominate the baby boomers' behavior; but this seems unrealistic.

23. For proof, recall the analysis of μ_{1t} shocks, where anticipation effects did not arise. This point is worth noting because the prediction of higher savings is specific to the overlapping-generations approach. If one assumed Ricardian bequests instead, a fertility decline would likely trigger a slight decline in savings (see Cutler et al. 1990).

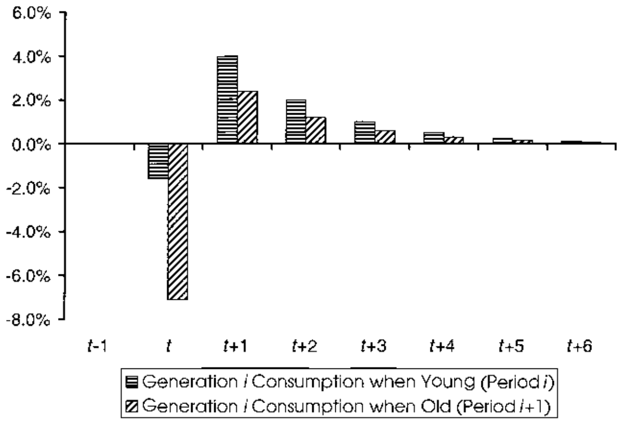

Fig. 6.1 Consumption responses to a birthrate shock
Note: The bars show the percentage deviations of consumption from the steady state in response to a one-time, 20 percent reduction in the birthrate in period t, applied to the parameter values of the numerical example with a defined-benefit social security system. The responses are collected by generation, not by period. The responses under generation $i = t + 2$ refer, e.g., to the changes in c^1_{t+2} (generation $t + 2$ when young) and c^2_{t+3} (generation $t + 2$ when old).

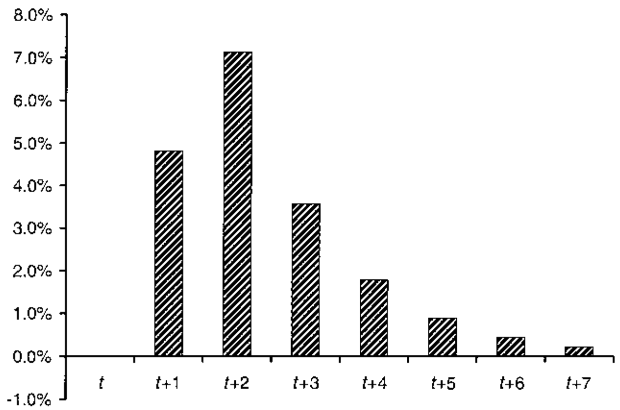

Fig. 6.2 Response of the capital-labor ratio to a birthrate shock
Note: The bars show the percentage deviations of the capital-labor ratio k_t from its steady state in response to a one-time, 20 percent reduction in the birthrate in period t, applied to the parameter values of the numerical example with a defined-benefit social security system.

boom generation may suffer because the small succeeding cohort reduces the return on capital.

The overlapping-generations model produces strikingly different results than one would obtain in a partial equilibrium analysis (say, a trend extrapolation of the type used by the Social Security Administration). This is due to the endogenous factor prices. If one took wages and interest rates as given, a small workforce would leave retirees unaffected, it would make workers worse off because of higher taxes, and, since workers would save less, it would make future generations worse off. If one accounts for factor-price effects, however, the partial equilibrium results are reversed. The effect of factor-price movements dominates the fiscal effect of labor force changes.

The latter finding relies, of course, on the general equilibrium properties of this particular two-period overlapping-generations model. Perhaps most significantly, the factor-price effects would be smaller if the elasticity of factor substitution were higher, for example, with constant elasticity of substitution technology. This and other robustness issues are examined in section 6.5 below.[24]

Third, returning to table 6.1 (panel C), consider a shock to the number of retirees, $\hat{\phi}_t = \hat{\phi}^u_t$. A large number of retirees directly reduces retiree consumption because the old have to spread their capital income over a longer period (or, in case of annuitized savings, over more people). Capital investment and worker consumption are also reduced to the extent that an increased retiree-worker ratio triggers higher payroll taxes. Thus, defined-benefit social security helps share the risk of shocks to the length of life across cohorts.

Fourth, consider a current shock to ϕ^e_t, the expected length of life ("life expectancy") in period $t + 1$. Table 6.1, panel D, shows that current life expectancy has an effect on the young, who will experience a longer life, but no effect on the old ($\pi_{c2\phi e} = 0$, as in the case of b_t shocks). Looking forward, a lagged length-of-life shock matters through its effect on the actual number of retirees (ϕ_{t+1}), like the unexpected shock ϕ^u_{t+1}. The young have an incentive to increase their savings and to reduce their current consumption ($\pi_{k\phi e} > 0$, $\pi_{c1\phi e} < 0$).[25] This risk is not shared with the old.

Finally, consider the capital and productivity coefficients in table 6.1, panel E. Not surprisingly, a high capital-labor ratio raises capital and labor incomes, hence consumption and savings. This makes k_t autocorrelated and propagates shocks. Productivity shocks have a negative effect on consumption and capital when scaled by productivity (c^1_t/A_t, c^2_t/A_t, and k_t) because a rise in A_t raises output less than one for one. In level terms, however, a positive shock to a_t raises consumption (c^1_t, c^2_t) and per capita savings $k_t \cdot A_t$.

Since a shock to productivity affects the capital-labor ratio like an unexpected shock to the workforce, one may wonder to what extent the μ_{1t} and

24. To avoid clutter, I proceed with the basic model and defer all extensions and empirical issues.

25. The overall effects of increased life expectancy over time could be traced out as in fig. 6.1 above, but the results would just confirm the increase in savings and the reduction in per capita consumption.

a_t shocks have similar effects. If social security is small ($\theta = 0$), positive shocks to a_t and μ_{1t} will indeed increase retiree consumption by the same amount ($1 + \pi_{c2a} = \pi_{c2\mu 1}$ for $\theta = 0$). They have very different effects on current workers, however, since an increase in A_t raises the wage while a rise in N_t^W reduces the wage rate. For $\theta > 0$, a_t and μ_{1t} shocks also have different effects on retirees because they have different distributional effects through social security.

6.2.2 Variable Benefits

The analysis so far has shown that most shocks affect different generations differently or even in opposite directions. This suggests some scope for improved risk sharing. The section examines how the allocation of risk is modified by policies with variable social security benefits.

Alternative policies are defined by their elasticity coefficients $\pi_{\beta z}$, that is, by how the replacement rate β responds to different shocks. Table 6.2 shows how the equilibrium dynamics of consumption and capital investment are affected in general by alternative $\pi_{\beta z}$ values. To help interpret the general results, table 6.3 displays the elasticity coefficients corresponding to the four main policy alternatives—the DB, DC, privatized, and conditionally prefunded social security systems—in the numerical example.[26]

In general, the elasticity formulas in table 6.2 include the same elements as the corresponding formulas in table 6.1 above, but there are additional terms that capture the effects of a changing replacement rate. The policy coefficients are generally weighted by the size of government transfers relative to the cohort's income, which is $\gamma_{c2\beta}$ for retirees and $-\theta/(1 - \theta)$ for workers. For workers, the effect is then divided between consumption and savings in proportions $\Delta_c : \Delta_k$.

Any policy that reduces prospective benefits when the birthrate declines and/or life expectancy rises is characterized by policy coefficients $\pi_{\beta b1} > 0$ and/or $\pi_{\beta\phi e1} < 0$. A pure defined-contribution system would have $\pi_{\beta\mu 1} = \pi_{b1} = 1$ and $\pi_{\beta\phi u} = \pi_{\beta\phi e1} = -1$. Since U.S. retirees have generally been protected against unexpected shocks, the U.S. system seems to maintain defined benefits with respect to unexpected changes ($\pi_{\beta\phi u} = \pi_{\beta\mu 1} = 0$) but allow benefits to change after a phase-in, suggesting $\pi_{\beta b1} \neq 0$ and $\pi_{\beta\phi e1} \neq 0$. The tax increases and the trust fund buildup since 1983 suggest that the U.S. system is somewhere between a DC and a DB system with respect to anticipated changes, that is, $0 < \pi_{\beta b1} < 1$ and $0 > \pi_{\beta\phi e1} > -1$. These stylized facts are captured by the conditionally prefunded system ("prefunded" in table 6.3). For the numerical illustration of this system, I assume that $\pi_{\beta b1} = 0.5$ and that $\pi_{\beta\phi e1} = -0.5$.

In the case of shocks to the workforce, table 6.3 (panel A) shows that defined contributions and privatized systems magnify the *negative* expo-

26. The numerical example is broadly indicative of how the elasticities compare in general.

Table 6.2 **Dynamics with Variable Social Security Benefits**

Effect on:	Elasticity Coefficients	
	A. Shocks to the Current Workforce, μ_{1t} and b_{t-1}	
Retirees	$\pi_{c2\mu1} = \gamma_{c2nw} + \gamma_{c2\beta} \cdot \pi_{\beta\mu1},$	$\pi_{c2b1} = \gamma_{c2nw} + \gamma_{c2\beta} \cdot \pi_{\beta b1}$
Workers	$\pi_{c1\mu1} = -\Delta_c \cdot \left(\alpha - \frac{\theta}{1-\theta}\right) - \Delta_c \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta\mu1},$	$\pi_{c1b1} = -\Delta_c \cdot \left(\alpha - \frac{\theta}{1-\theta}\right) - \Delta_c \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta b1}$
Investment	$\pi_{k\mu1} = -\Delta_k \cdot \left(\alpha - \frac{\theta}{1-\theta}\right) -\Delta_k \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta\mu1},$	$\pi_{kb1} = -\Delta_k \cdot \left(\alpha - \frac{\theta}{1-\theta}\right) - \Delta_k \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta b1}$
	B. Shocks to the Current Birthrate, b_t	
Retirees	$\pi_{c2b} = \gamma_{c2\beta} \cdot \pi_{\beta b}$	
Workers	$\pi_{c1b} = [1 - \Delta_c \cdot (c^1/A)/y^1] \cdot [\gamma_{c2nw} - \pi_{Rk}/\eta + \gamma_\rho/\eta] - \Delta_c \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta b} + (1 - \Delta_c \cdot (c^1/A)/y^1) \cdot \gamma_{c2\beta} \cdot \pi_{\beta b1}$	
Investment	$\pi_{kb} = -\Delta_k \cdot (c^1/A)/y^1 \cdot [\gamma_{c2nw} - \pi_{Rk}/\eta + \gamma_\rho/\eta] - \Delta_k \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta b} - \Delta_k \cdot (c^1/A)/y^1 \cdot \gamma_{c2\beta} \cdot \pi_{\beta b1}$	
	C. Shocks to the Current Length of Life, ϕ_t^u and ϕ_{t-1}^e	
Retirees	$\pi_{c2\phi u} = -\gamma_{c2\phi} + \gamma_{c2\beta} \cdot \pi_{\beta\phi u},$	$\pi_{c2\phi e1} = -\gamma_{c2\phi} + \gamma_{c2\beta} \cdot \pi_{\beta b1}$
Workers	$\pi_{c1\phi u} = -\Delta_c \cdot \frac{\theta}{1-\theta} - \Delta_c \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta\phi u},$	$\pi_{c1\phi e1} = -\Delta_c \cdot \frac{\theta}{1-\theta} - \Delta_c \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta\phi e1}$
Investment	$\pi_{k\phi u} = -\Delta_k \cdot \frac{\theta}{1-\theta} - \Delta_k \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta\phi u},$	$\pi_{k\phi e1} = -\Delta_k \cdot \frac{\theta}{1-\theta} - \Delta_k \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta\phi e1}$

	D. Shocks to Current Life Expectancy, ϕ_t^e	
Retirees	$\pi_{c2\phi e} = \gamma_{c2\beta} \cdot \pi_{\beta\phi e}$	
Workers	$\pi_{c1\phi e} = -[1 - \Delta_c \cdot (c^1/A)/y^1] \cdot (\gamma_{c2\phi} - \gamma_{c2\beta} \cdot \pi_{\beta\phi e1}) - \Delta_c \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta\phi e}$	
Investment	$\pi_{k\phi e} = \Delta_k \cdot (c^1/A)/y^1 \cdot (\gamma_{c2\phi} - \gamma_{c2\beta} \cdot \pi_{\beta\phi e1}) - \Delta_k \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta\phi e}$	
	E. Changes in Lagged Capital and Productivity, k_{t-1} and a_t	
Retirees	$\pi_{c2k} = \gamma_{c2k} + \gamma_{c2\beta} \cdot \pi_{\beta k}$,	$\pi_{c2a} = -\gamma_{c2k} + \gamma_{c2\beta} \cdot \pi_{\beta a}$
Workers	$\pi_{c1k} = \Delta_c \cdot \alpha - \Delta_c \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta k}$,	$\pi_{c1a} = \Delta_c \cdot \alpha - \Delta_c \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta a}$
Investment	$\pi_{kk} = \Delta_k \cdot \alpha - \Delta_k \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta k}$,	$\pi_{ka} = -\Delta_k \cdot \alpha - \Delta_k \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta a}$

Notes: The notation is as in Table 6.1 above. In addition, define

$$\gamma_{c2\beta} = (1 - \delta^*) \cdot \frac{\theta \cdot (1 - \alpha)}{\alpha + \theta \cdot (1 - \alpha)} > 0.$$

Table 6.3 Alternative Policies in the Numerical Example

A. Shocks to the Workforce, μ_{1t} and b_{t-1}

	Alternative Systems				
	DB (shock to μ_{1t} or b_{t-1})	DC (shock to μ_{1t} or b_{t-1})	Privatized (shock to μ_{1t} or b_{t-1})	Prefunded (shock to μ_{1t})	Prefunded (shock to b_{t-1})
Policy coefficient	0	1.0	N.A.	0	.5
Effect on:					
Retirees	0.436	0.667	0.667[a]	0.436[b]	0.551[c]
Workers	−0.131	−0.278	−0.278[a]	−0.131[b]	−0.204[c]
Investment	−0.235	−0.500	−0.500	−0.235	−0.368

B. Shocks to the Length of Life, ϕ_t^u and ϕ_{t-1}^e

	Alternative Systems				
	DB (shock to ϕ_t^u or ϕ_{t-1}^e)	DC (shock to ϕ_t^u or ϕ_{t-1}^e)	Privatized (shock to ϕ_t^u or ϕ_{t-1}^e)	Prefunded (shock to ϕ_t^u)	Prefunded (shock to ϕ_{t-1}^e)
Policy coefficient	0	−1.0	N.A.	0	−.5
Effect on:					
Retirees	−0.769	−1.0	−1.0[a]	−0.769[b]	−0.885[c]
Workers	−0.147	0.0	0.0[a]	−0.147[b]	−0.074[c]
Investment	−0.265	0.0	0.0	−0.265	−0.111

Note: The notation is as in tables 6.1–6.2. For defined benefits (DB), defined contributions (DC), and privatized social security, μ_{1t} and b_{t-1} have the same effects as ϕ_t^u and ϕ_{t-1}^e. For the conditionally prefunded system ("prefunded" above), the policy coefficients are generally in the range $\pi_{\beta b1} \in (0, +1)$ and $\pi_{\beta\phi e1} \in (-1, 0)$. For the numerical example, I use +0.5 and −0.5, respectively. N.A. = not applicable.

[a]Equal to the DC case.

[b]Equal to the DB case.

[c]In between.

sure of workers to such shocks, as compared to the DB case. They also magnify the *positive* exposure of retirees. Table 6.2 (panel A) shows that this is true in general, whenever $\pi_{\beta\mu1} > 0$ and $\pi_{\beta b1} > 0$. In addition, $\pi_{\beta b1} > 0$ increases workers' instantaneous negative response to birthrate shocks ($\pi_{kb} < 0$ rises in absolute value; see table 6.2, panel B). By making the capital-labor ratio more volatile, $\pi_{\beta b1} > 0$ also exposes future generations to more risk. These observations reinforce the insights from table 6.1. Large cohorts are already demographically disadvantaged at fixed benefits (DB). Hence, a policy of giving them reduced benefits in order to stabilize tax rates is counterproductive.[27]

27. This verdict may raise questions about the welfare criterion. This is addressed below.

In the case of shocks to the current length of life, a system of defined contributions leaves the old more exposed and allocates less risk to the young than a DB system: in table 6.2 (panel C), if $\pi_{\beta\phi e1} < 0$ and/or $\pi_{\beta\phi u} < 0$, then $\pi_{c1\phi u}$, $\pi_{c1\phi e1}$, $\pi_{k\phi u}$, and $\pi_{k\phi e1}$ are all lower in absolute value, whereas $\pi_{c2\phi u}$ and $\pi_{c2\phi e1}$ are increased. With a DC system, length-of-life risk falls entirely on the old. The policy coefficient $\pi_{\beta\phi e1}$ also influences how period t voters respond to news about changes in the future length of life (ϕ^e_t shocks; see table 6.2, panel D). If workers anticipate reduced future benefits, they save more ($\pi_{\beta\phi e1} < 0$ raises $\pi_{k\phi e}$) and consume less ($\pi_{\beta\phi e1} < 0$ reduces $\pi_{c1\phi e}$).

Table 6.2 provides several additional insights. First, the government can influence the propagation of shocks through the capital-labor ratio (π_{kk}) by making benefits a function of k_{t-1} (setting $\pi_{\beta k} \neq 0$; see panel E). Second, the government can influence the incidence of productivity shocks by varying $\pi_{\beta a}$.[28] Third, note that, for $\pi_{\beta b} = \pi_{\beta\phi e} = 0$, only the workers bear the risk of "bad" news about birthrates and life expectancy (see panels B and D). By setting $\pi_{\beta b}$, $\pi_{\beta\phi e} \neq 0$, the government could spread such risks over young and old. This is not done under any of the policies discussed above.

Overall, table 6.3 provides a comparison of the main policy alternatives. Under DC and private savings systems, all length-of-life risk is carried by the old and none by the young. The DB and prefunded systems shift some of these risks to the young. Under DC and private savings systems, birthrate uncertainty and other shocks to the workforce have a positive effect on the old but a negative effect on the young. This negative comovement of worker and retiree consumption is reduced by the DB and prefunded systems, but, provided that $\theta/(1-\theta) < \alpha$, it is not eliminated.

6.3 Missing Annuities and Accidental Bequests

This section examines the ramifications of missing annuities and accidental bequests. Without annuities, some shocks to old-age survival lead to accidental bequests (μ_2 shocks). In addition, the existence of accidental bequests affects the propagation of the shocks examined previously.

The macroeconomic dynamics of the log-linearized model without annuities are summarized in table 6.4. Recall that, in the basic model, ϕ shocks reduced retiree consumption while affecting worker consumption only through a change in taxes. In contrast, if savings are not annuitized, fewer unexpected deaths (higher μ_{2t}^u or μ_{2t-1}^e) have a direct negative effect

28. Here, $\pi_{\beta a} = 0$ holds for all the main policy alternatives (the DB, DC, and conditionally prefunded systems); i.e., their response to productivity shocks is essentially the same. One could consider policies that respond differently (e.g., a DB system promising fixed real benefits instead of a fixed replacement rate), but productivity risk has ramifications that are beyond the scope of this paper (see Bohn 1998, 1999a). Hence, I focus on policies with $\pi_{\beta a} = 0$ and just note that the government has additional degrees of freedom.

Table 6.4 Macroeconomic Dynamics without Annuities Markets

Effect on:	Elasticity Coefficients	Numerical Example
	A. Shocks to Retiree Survival without Annuities, μ^u_{2t} *and* μ^e_{2t-1}	
Retirees	$\pi_{c\mu 2u} = \gamma_{c2\beta} \cdot \pi_{\beta\mu 2u}, \quad \pi_{c2\mu 2e1} = \gamma_{c2\beta} \cdot \pi_{\beta\mu 2e1}, \quad \text{where } \gamma_{c2\beta} = (1-\delta^*) \cdot \frac{\theta/\mu_2 \cdot (1-\alpha)}{\alpha + \theta/\mu_2 \cdot (1-\alpha)} > 0$	0
Workers	$\pi_{c1\mu 2u} = -\Delta_c \cdot \left[(1-q)\cdot(1+\pi_{\beta\mu 2u}) + q \cdot \frac{\mu_2}{1-\mu_2}\right] \cdot \frac{\theta}{1-\theta}, \quad \pi_{c1\mu 2e1} = -\Delta_c \cdot \left[(1-q)\cdot(1+\pi_{\beta\mu 2e1}) + q \cdot \frac{\mu_2}{1-\mu_2}\right] \cdot \frac{\theta}{1-\theta}$	−.147
Investment	$\pi_{k\mu 2u} = -\Delta_k \cdot \left[(1-q)\cdot(1+\pi_{\beta\mu 2u}) + q \cdot \frac{\mu_2}{1-\mu_2}\right] \cdot \frac{\theta}{1-\theta}, \quad \pi_{k\mu 2e1} = -\Delta_k \cdot \left[(1-q)\cdot(1+\pi_{\beta\mu 2e1}) + q \cdot \frac{\mu_2}{1-\mu_2}\right] \cdot \frac{\theta}{1-\theta}$	−.265
	B. Shocks to Future Retiree Survival without Annuities, μ^e_{2t}	
Retirees	$\mu_{c2\mu 2e} = \gamma_{c2\beta} \cdot \pi_{\beta\mu 2e}$	0
Workers	$\pi_{c1\mu 2e} = -[1 - \Delta_c \cdot (c^1/A)/y^1] \cdot (1/\eta - \gamma_{c2\beta} \cdot \pi_{\beta\mu 2e1}) - \Delta^*_c \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta\mu 2e}$	−.125
Investment	$\pi_{k\mu 2e} = \Delta_k \cdot (c^1/A)/y^1 \cdot (1/\eta - \gamma_{c2\beta} \cdot \pi_{\beta\mu 2e1}) - \Delta^*_k \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta\mu 2e}$	.375
	C. Shocks to the Current Workforce, μ_{1t} *and* b_{t-1} [a]	
Retirees	$\pi_{c2\mu 1} = \gamma_{c2nw} + \gamma_{c2\beta} \cdot \pi_{\beta\mu 1}, \quad \pi_{c2b1} = \gamma_{c2nw} + \gamma_{c2\beta} \cdot \pi_{\beta b1}, \quad \text{where } \gamma_{c2nw} = (1-\delta^*) \cdot \left[1 - \alpha - \frac{\theta/\mu_2 \cdot (1-\alpha)}{\alpha + \theta/\mu_2 \cdot (1-\alpha)}\right] > 0$	
Workers	$\pi_{c1\mu 1} = -\Delta_c \cdot \left[(1-q)\cdot\alpha + q \cdot \pi_{Rk} - (1-q)\frac{\theta}{1-\theta}\right] - \Delta^*_c \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta\mu 1},$	

	$\pi_{c1b1} = -\Delta_c \cdot \left[(1-q)\cdot\alpha + q\cdot\pi_{Rk} - (1-q)\frac{\theta}{1-\theta}\right] - \Delta_c^* \cdot \frac{\theta}{1-\theta}\cdot\pi_{\beta b1}$
Investment	$\pi_{k\mu 1} = -\Delta_k \cdot \left[(1-q)\cdot\alpha + q\cdot\pi_{Rk} - (1-q)\frac{\theta}{1-\theta}\right] - \Delta_k^* \cdot \frac{\theta}{1-\theta}\cdot\pi_{\beta\mu 1},$
	$\pi_{kb1} = -\Delta_k \cdot \left[(1-q)\cdot\alpha + q\cdot\pi_{Rk} - (1-q)\frac{\theta}{1-\theta}\right] - \Delta_k^* \cdot \frac{\theta}{1-\theta}\cdot\pi_{\beta b1}$
	D. Shocks to the Current Length of Life, ϕ_t^u and ϕ_{t-1}^e [a]
Retirees	$\pi_{c2\phi u} = -\gamma_{c2\phi} + \gamma_{c2\beta}\cdot\pi_{\beta\phi u}, \quad \pi_{c2\phi e1} = -\gamma_{c2\phi} + \gamma_{c2\beta}\cdot\pi_{\beta b1}, \quad \text{where } \gamma_{c2\phi} = 1-(1-\delta^*)\cdot\frac{\theta/\mu_2\cdot(1-\alpha)}{\alpha+\theta/\mu_2\cdot(1-\alpha)} > 0$
Workers	$\pi_{c1\phi u} = -\Delta_c^*\cdot\frac{\theta}{1-\theta}\cdot(\pi_{\beta\phi u}+1), \quad \pi_{c1\phi e1} = -\Delta_c^*\cdot\frac{\theta}{1-\theta}\cdot(\pi_{\beta\phi e1}+1)$
Investment	$\pi_{k\phi u} = -\Delta_k^*\cdot\frac{\theta}{1-\theta}\cdot(\pi_{\beta\phi u}+1), \quad \pi_{k\phi e1} = -\Delta_k^*\cdot\frac{\theta}{1-\theta}\cdot(\pi_{\beta\phi e1}+1)$
	E. Shocks to the Current Birthrate, b_t [a]
Retirees	$\pi_{c2b} = \gamma_{c2\beta}\cdot\pi_{\beta b}$
Workers	$\pi_{c1b} = [1-\Delta_c\cdot(c^1/A)/y^1]\cdot(\gamma_{c2nw} - \pi_{Rk}/\eta + \gamma_\rho/\eta + \gamma_{c2\beta}\cdot\pi_{\beta b1}) - \Delta_c^*\cdot\frac{\theta}{1-\theta}\cdot\pi_{\beta b}$
Investment	$\pi_{kb} = -\Delta_k\cdot(c^1/A)/y^1\cdot(\gamma_{c2nw} - \pi_{Rk}/\eta + \gamma_\rho/\eta + \gamma_{c2\beta}\cdot\pi_{\beta b1}) - \Delta_k^*\cdot\frac{\theta}{1-\theta}\cdot\pi_{\beta b}$

(*continued*)

Table 6.4 (continued)

Effect on:	Elasticity Coefficients	Numerical Example
	F. Shocks to the Future Length of Life, ϕ_t^e [a]	
Retirees	$\pi_{c2\phi e} = \gamma_{c2\beta} \cdot \pi_{\beta\phi e}$	
Workers	$\pi_{c1\phi e} = -[1 - \Delta_c \cdot (c^1/A)/y^1] \cdot (\gamma_{c2\phi} - \gamma_{c2\beta} \cdot \pi_{\beta\phi e1}) - \Delta_c^* \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta\phi e}$	
Investment	$\pi_{k\phi e} = \Delta_k \cdot (c^1/A)/y^1 \cdot (\gamma_{c2\phi} - \gamma_{c2\beta} \cdot \pi_{\beta\phi e1}) - \Delta_k^* \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta\phi e}$	
	G. Changes in Lagged Capital and Productivity, k_{t-1} and a_t [a]	
Retirees	$\pi_{c2k} = \gamma_{c2k} + \gamma_{c2\beta} \cdot \pi_{\beta k}, \quad \pi_{c2a} = -\gamma_{c2k} + \gamma_{c2\beta} \cdot \pi_{\beta a}$	
Workers	$\pi_{c1k} = \Delta_c \cdot [(1-q) \cdot \alpha + q \cdot \pi_{Rk}] - \Delta_c^* \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta k}, \quad \pi_{c2a} = -\Delta_c \cdot [(1-q) \cdot \alpha + q \cdot \pi_{Rk}] - \Delta_c^* \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta a}$	
Investment	$\pi_{kk} = \Delta_k \cdot [(1-q) \cdot \alpha + q \cdot \pi_{Rk}] - \Delta_k^* \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta k}, \quad \pi_{ka} = -\Delta_k \cdot [(1-q] \cdot \alpha + q \cdot \pi_{Rk}] - \Delta_k^* \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta a}$	

Note: The notation is as in tables 6.1–6.2 above, except for the following symbols:

$q = Q^1/[w \cdot (1-\theta) + Q^1]$ = share of bequests in worker's income;

$\Delta_c^* = \Delta_c \cdot (1-q), \quad \Delta_k^* = \Delta_k \cdot (1-q).$

[a] As in table 6.2 above, but with modified coefficients if $q \neq 0$ or $\mu_2 \neq 0$.

on the young because of reduced bequests, while the old are affected only through changes in benefits (see panel A). If benefits are held constant, the consumption of the young is further reduced because of higher taxes.

Table 6.4, panel A, also provides numerical values for the limiting case of $q \approx 0$ and a DB social security system. For $q \approx 0$ and DB, survival shocks affect the worker exactly like a length-of-life shock (see table 6.1, panel C, above). The key difference is that retirees are unaffected. Hence, for dealing with μ_2-type shocks, a movement toward defined contributions or privatization looks much more promising than it does for ϕ-type shocks.

Table 6.4, panel B, illustrates how an increase in the expected future probability of survival (μ_2^e) increases workers' incentives to save. Panels C–G show how accidental bequests modify the other policy coefficients as compared to table 6.2 above. The modifications are proportional to the ratio of accidental bequests to bequests plus wage income (q). If this ratio is small, as one might expect in practice, the previous results remain virtually unchanged. For this reason, no new illustrative values are provided.

6.4 Efficient Risk Sharing

If there is scope for risk sharing, what exactly should be done? This section derives a simple efficiency benchmark and explores its policy implications. In general, the set of efficient (ex ante Pareto-optimal) allocations can be obtained by maximizing a welfare function

$$W = E\left\{\sum_{t=-1}^{\infty} \Omega_{t-1} \cdot N_{t-1} \cdot U_t\right\} \tag{10}$$

with welfare weights $\Omega_{t-1} > 0$, subject to the feasibility constraints (1)–(4) and given K_0.[29] The efficiency conditions are

$$\Lambda_t \cdot N_t^W = \Omega_{t-1} \cdot N_{t-1} \cdot \mu_{1t} \cdot \frac{dE_t U_t}{dc_t^1}, \tag{11}$$

$$\Lambda_t \cdot N_t^R = \Omega_{t-2} \cdot N_{t-2} \cdot \mu_{1t-1} \cdot \mu_{2t} \cdot \frac{dU_{t-1}}{dc_t^2},$$

$$\Lambda_t = E_t[\Lambda_{t+1} \cdot R_{t+1}^k],$$

29. The definition of efficiency is nontrivial because one might instead consider a welfare function with state-contingent weights. In a model without a childhood period, Peled (1982) has shown that the market allocation without government is Pareto efficient if one interprets generation t individuals born in different states of nature as different individuals and applies state-contingent weights. With a childhood period, the market allocation is inefficient even with state-contingent weights. Moreover, Peled's definition is too weak here because it would rationalize any shift of risk from current to unborn generations as efficient (under some state-contingent welfare weights) and therefore make the policy analysis vacuous. Readers who object on philosophical grounds to the notion of unborn individuals may instead inter-

where Λ_t is the shadow value of the resource constraint (4). Equivalently,

$$\rho_1 \cdot (c_t^1)^{-\eta} = E_t[R_{t+1}^k \cdot \rho_2 \cdot (c_{t+1}^2)^{-\eta}], \tag{12a}$$

$$\rho_1(b_t) \cdot (c_t^1)^{-\eta} = \frac{\Omega_{t-2}}{\Omega_{t-1}} \cdot \rho_2 \cdot (c_t^2)^{-\eta}, \tag{12b}$$

define the efficient linkages of consumption over time and across generations. Note that equation (12a) is identical to the individual optimality condition (7a) for generation *t*'s savings with annuities. The fundamentally new equation is (12b). It links period *t* worker and retiree consumption, and it depends only on population growth and the welfare weights.

For risk-sharing issues, it is again useful to distinguish the economy's perfect-foresight path (obtained by setting all shocks to zero) from the stochastic fluctuation around this path. For the log deviations from the perfect-foresight path, equation (12b) implies

$$\hat{c}_t^1 = \hat{c}_t^2 + \gamma_\rho/\eta \cdot \hat{b}_t. \tag{13}$$

This is a strong restriction on the comovements of worker and retiree consumption: in any efficient allocation, both generations' consumption must respond in equal proportions to *all* unexpected disturbances, except to the extent that parents' consumption needs vary with the number of children (b_t).

The key underlying assumption is CRRA utility, which assigns an equal relative risk aversion to both generations. For utility functions with age-dependent risk aversion, Bohn (1998) has shown that macroeconomic risks would be shared in inverse proportion to the relative risk aversions. The same would be true here, but age-dependent risk aversion would unnecessarily complicate the analysis. Age-dependent risk aversion would not, in any case, overturn the basic point that all risks should be shared across generations.

In addition to sharing risks between living generations, government policy has the ability to reallocate risks between current and future generations by imposing history-dependent policies. This is generally necessary to obtain a first-best allocation, and it typically involves making policies a function of the capital-labor ratio k_{t-1} (see Bohn 1998). For the analysis here, making β_t a function of k_{t-1} would be a distraction. Instead, I focus on the necessary efficiency condition (13) when comparing alternative social security systems. Its key implication for the elasticity coefficients is

pret the state-independent weights as an assumption of "distributional neutrality," meaning that we are looking for allocations in which the government does not arbitrarily value individuals born in one state of nature more highly than individuals with equal consumption born in another state.

that, for all shocks, the consumption coefficients for workers and retirees should be equal. The only exceptions are the b_t coefficients to the extent that expenses for children matter.

Applied to the different demographic shocks, the optimality condition (13) yields a set of optimal policy coefficients $\pi^*_{\beta z}$ that are displayed in table 6.5. For shocks to the actual workforce (μ_{1t}, b_{t-1}), the optimal policy coefficients $\pi^*_{\beta\mu 1}$ and $\pi^*_{\beta b1}$ are clearly negative for reasonable α and θ values. This is true, not only for $\alpha > \theta/(1-\theta)$, but even for higher θ values, provided that

(14) $$\alpha + (\gamma_{c2nw} + \Delta_c \cdot q \cdot \pi_{Rk}) / \Delta^*_c > \theta / (1 - \theta).$$

Since the bracketed term is positive, this strengthens the previous observation that large cohorts are worse off than small cohorts even with PAYGO-DB. Intuitively, the bracketed term captures the effect of interest rate movements that favor small cohorts. In the numerical example, $\pi^*_{\beta\mu 1} = \pi^*_{\beta b1} = -1.5$ are far below zero. Applied to the current baby-boom/bust situation, this implies that benefits should be increased as the baby-boom

Table 6.5 **Optimal Policy Responses to Demographic Shocks**

Policy response to changes in the current workforce:

$$\pi^*_{\beta\mu 1} = \pi^*_{\beta b1} = -\frac{\gamma_{c2nw} + \Delta_c \cdot q \cdot \pi_{Rk} + \Delta^*_c \cdot [\alpha - \theta/(1-\theta)]}{\gamma_{c2\beta} + \Delta^*_c \cdot \theta/(1-\theta)}$$

Policy response to changes in the current birthrate:

$$\pi^*_{\beta b} = \left[1 - \Delta_c \cdot \frac{(c^1/A)}{y^1}\right] \cdot \frac{\gamma_{c2nw} - \pi_{Rk}/\eta + \gamma_{c2\beta} \cdot \pi_{\beta b1}}{\gamma_{c2\beta} + \Delta^*_c \cdot \theta/(1-\theta)} - \Delta_c \cdot \frac{(c^1/A)}{y^1} \cdot \frac{\gamma_\rho/\eta}{\gamma_{c2\beta} + \Delta^*_c \cdot \theta/(1-\theta)}$$

Policy response to changes in the current length of life:

$$\pi^*_{\beta\phi u} = \pi^*_{\beta\phi e1} = \frac{\gamma_{c2\phi} - \Delta^*_c \cdot \theta/(1-\theta)}{\gamma_{c2\beta} + \Delta^*_c \cdot \theta/(1-\theta)}$$

Policy response to changes in current retiree survival without annuities:

$$\pi^*_{\beta\mu 2u} = \pi^*_{\beta\mu 2e1} = -\frac{\Delta_c \cdot [1 - q + q \cdot \mu_2/(1-\mu_2)] \cdot \theta/(1-\theta)}{\gamma_{c2\beta} + \Delta^*_c \cdot \theta/(1-\theta)}$$

Policy response to changes in the future length of life:

$$\pi^*_{\beta\phi e} = -\left[1 - \Delta_c \cdot \frac{(c^1/A)}{y^1}\right] \cdot \frac{\gamma_{c2\phi} - \gamma_{c2\beta} \cdot \pi_{\beta\phi e1}}{\gamma_{c2\beta} + \Delta^*_c \cdot \theta/(1-\theta)}$$

Policy response to changes in future retiree survival without annuities:

$$\pi^*_{\beta\mu 2e} = -\left[1 - \Delta_c \cdot \frac{(c^1/A)}{y^1}\right] \cdot \frac{1/\eta - \gamma_{c2\beta} \cdot \pi_{\beta\mu 2e1}}{\gamma_{c2\beta} + \Delta^*_c \cdot \theta/(1-\theta)}$$

Note: The notation is as in tables 6.1–6.2 and 6.4. The asterisks denote efficient values.

cohort retires. This is contrary to most proposals in the current policy debate.

The optimal response to a current birthrate shock (b_t) is somewhat more complicated. In the formula for $\pi^*_{\beta b}$ in table 6.5, if $\gamma_{c2nw} - \pi_{Rk}/\eta > 0$, the positive income effect of higher future returns on capital exceeds the substitution effect and tends to increase worker consumption. Efficiency would call for this "windfall" to be shared with the old through higher benefits. On the other hand, if $\pi^*_{\beta b1} = \pi^*_{\beta b1} < 0$ takes its optimal negative value, worker income is reduced, which would call for a benefit reduction. The γ_ρ term reflects the cost of children. If workers have higher expenses for more children, a reduction in social security benefits would be efficient. The sum of these effects has an ambiguous sign.

In the numerical example, $\pi^*_{\beta b} = 0.212$ is positive if $\pi_{\beta b1} = 0$ (e.g., with DB), $\pi^*_{\beta b} = 0.441$ is even higher if $\pi_{\beta b1} = 1$ (e.g., with DC), but $\pi^*_{\beta b} = -0.131$ takes a negative value if $\pi_{\beta b1} = \pi^*_{\beta b1} = -1.5$ is set optimally. Intuitively, the lagged policy response $\pi_{\beta b1}$ matters because workers' period t decisions depend on how they expect to be treated by the government as retirees. If a rise in the birthrate signals no change in future benefits (with DB) or increased retirement benefits (with DC), workers expect to be very well off as retirees and increase their current consumption. The optimality condition (13) implies that the good fortune should be shared with current retirees. A reduced birthrate—the current U.S. scenario—would then call for an immediate benefit cut. If future benefits are set optimally, on the other hand, a rise in the birthrate signals a benefit cut, and workers will reduce their consumption. Then the optimal current policy response has the reverse sign.

In any case, efficiency calls for current retirees to share the effect of birthrate shocks. And, unless the baby boomers are confident that future policy makers will follow the advice of this paper (that $\pi^*_{\beta b1} < 0$) rather than the thrust of the current social security debate (moving toward $\pi_{\beta b1} > 0$), they are well advised to reduce current consumption and to save more.

Next, consider length-of-life shocks without effect on accidental bequests (ϕ^u_t, ϕ^e_{t-1}). Recall that, in a DB system, both generations' consumption falls in response to an increase in the length of life. The optimal policy response therefore depends on the relative effect. For reasonably small θ values, the old are more affected than the young (recall table 6.1, panel C, above). Then the benefits to the old should be increased in response to longer life expectancy, that is, $\pi^*_{\beta\phi u} = \pi^*_{\phi e1} > 0$. In the numerical example, $\pi^*_{\beta\phi u} = \pi^*_{\phi e1} = 1.647$ is indeed far above zero.

Without annuities, the results are different. With defined benefits, only the young would bear the cost of survival shocks (μ^u_{2t}, μ^e_{2t-1}). A benefit reduction, $\pi^*_{\beta\mu 2u} = \pi^*_{\mu 2e1} < 0$, is therefore efficient. Provided that μ_2 and q are small enough that $\gamma_{c2\beta} > q \cdot \mu_2/(1 - \mu_2) \cdot \theta/(1 - \theta)$, the optimal policy is in the range $-1 < \pi^*_{\beta\mu 2u} = \pi^*_{\mu 2e1} < 0$, and efficiency therefore calls at most for a partial movement to DC. In the numerical example, one finds that $\pi^*_{\beta\mu 2u} = \pi^*_{\mu 2e1} = -0.389$.

Overall, if one asks the broad question of how social security should respond to lower mortality per se, the right answer is that it depends on the type of shock. If the type is unknown, the large positive π^* coefficient for ϕ shocks in the numerical example as compared to the small negative coefficient for μ_2 shocks suggests that there is no strong case for a benefit reduction.

Finally, for shocks to current life expectancy (ϕ^e_t and μ^e_{2t}), recall that both shocks reduce the consumption of the young without directly affecting the old (see table 6.1, panel D, and table 6.4, panel B). Hence, the optimal policy response is to reduce the benefits to the old, $\pi^*_{\beta\phi e} < 0$ and $\pi^*_{\beta\mu 2e} < 0$.[30] Intuitively, increased life expectancy requires resources in the future so that the young need to save more. For the old to share the burden, current social security benefits should be reduced immediately. This conclusion applies regardless of the state of annuity markets.

In the current reform debate, many proposals call for a reduction in benefits as mortality declines, for example, by increasing the retirement age. The analysis here suggests that the efficiency of such benefit cuts depends importantly on their timing. Cuts are efficient if they are imposed quickly (at time t, $\pi^*_{\beta\phi e} < 0$) but not if they are imposed so late that they fall on the longer-lived cohort itself (at time $t + 1$, $\pi^*_{\beta\phi e1} > 0$). None of the systems discussed in the current reform debate is efficient in this sense, nor is the current policy debate moving in the direction of cutting benefits to current retirees.

6.5 Extensions and Empirical Issues

The magnitude of factor-price movements in response to demographic shocks was a key issue in the analysis presented above. Is the model consistent with the empirical evidence? Are there natural extensions of the model that would yield different results? To address these concerns, this section comments on the empirical evidence and on some extensions of the model.

6.5.1 Empirical Evidence

The most direct way to settle questions about the factor-price effects of demographic change would be to refer to empirical evidence—if convincing evidence were available. This is not the case, however. The main prob-

30. In the numerical example, one finds $\pi^*_{\beta\phi e} = -0.76$ if $\pi_{\beta\phi e1} = 0$ and $\pi^*_{\beta\phi e} = -1.14$ if $\pi_{\beta\phi e1} = \pi^*_{\beta\phi e1} = 1.647$. Without annuities, $\pi^*_{\beta\mu 2e} = -0.057$ if $\pi_{\beta\mu 2e1} = 0$ and $\pi^*_{\beta\mu 2e} = -0.146$ if $\pi_{\beta\mu 2e1} = \pi^*_{\beta\mu 2e1} = -0.389$. The $\pi_{\beta\phi e1}$ and $\pi_{\beta\mu 2e1}$ coefficients matter because workers take the expected future policy response to any shock to life expectancy into account when they decide about their consumption (as explained in the case of b_t shocks).

lem is that, for generational issues, a single observation takes twenty to thirty years of data. In terms of generational time units, we have only two to three observations for the U.S. economy with social security, perhaps four to five for countries like Germany. Even the idea of retirement—that it is normal for nondisabled adults to stop working just because of their age—is fairly novel. Hence, there are no time-series data of sufficient length and stationarity (without serious structural breaks) to allow credible statistical inferences.[31]

There is, however, some indirect evidence about the effect of demographic changes on wages. First, there is a large literature on cross-country growth that suggests a negative correlation between population growth (or fertility) and per capita income (notably Mankiw, Romer, and Weil 1992; see also Cutler et al. 1990). Assuming near-constant labor shares (Cobb-Douglas production), this suggests a negative correlation between population growth and wages.[32]

Second, there is a labor economics literature examining linkages between demographics and *relative* wages (e.g., Welch 1979; Berger 1985; Easterlin 1987; Murphy and Welch 1992; Macunovich 1998).[33] Easterlin (1987) and Macunovich (1998) focus almost exclusively on demographics and argue that the effects are large. Welch (1979) and Berger (1985) find significant negative effects of cohort size on cohort wages, although they disagree about persistence over a worker's career. Murphy and Welch (1992) argue that demographic variables are only a minor determinant of relative wages, but even they find nontrivial cohort effects.

To be conservative, I focus on Welch (1979) and Murphy and Welch (1992). Welch's (1979) elasticity estimates for the "persistent" effect of cohort size (narrowly defined as a five-year age window) on annual wage income are around -0.20, with some variation across education categories. Murphy and Welch's (1992, 324) simulations imply that a 20 percent increase in the number of young workers reduces their wages by 6–15 percent, suggesting an elasticity of *relative* wages in the range of from -0.30 to -0.75.

For comparison, the overlapping-generations model assumes an elasticity of wages with respect to the *aggregate* workforce of $-\alpha$ or about -0.33, a value well within the range of elasticities given above. Moreover, if capital owners have some ability to substitute labor across narrowly defined age cohorts, the elasticity of wages with respect to the aggregate

31. Poterba (1998) makes similar arguments.

32. There is some debate about the strength of this relation (see Barro and Sala-i-Martin 1995; and Temple 1998). While cross-sectional evidence is attractive to circumvent the lack of multigeneration time series, it also raises new concerns about causality and control variables. Hence, the evidence should be interpreted cautiously.

33. This literature should also be interpreted cautiously. Despite the richness of panel data, the data provide aggregate information about only one to two generations.

workforce should be at least as high as the relative-supply elasticities. Thus, the assumptions of the overlapping-generations model are not inconsistent with the labor economics evidence.

Finally, I should comment on the relation between demographics and the return on capital. The recent review by Poterba (1998) finds little evidence of a systematic relation. Poterba suggests that this may be due to the small number of generational degrees of freedom. Theoretical considerations suggest an additional rationalization: if old capital is a large share of the total return (if $[1 - \delta]/R^k$ is near one), then the elasticity of R^k with respect to the capital-labor ratio is small and may be difficult to detect empirically.[34] Thus, the inability to find an empirical link between demographics and stock returns is not inconsistent with the model.

6.5.2 CES Production

From a theoretical perspective, the magnitude of factor-price movements depends importantly on the elasticity of factor substitution. By assuming Cobb-Douglas technology, the analysis presented above implicitly assumes a unit elasticity. An elasticity of factor substitution above 1.0 will imply smaller factor-price changes than with Cobb-Douglas and, hence, a different allocation of risk. To examine the importance of this issue, this section replaces Cobb-Douglas with a constant elasticity of substitution (CES) production function.

For this section only, let output be produced with a CES technology, $Y_t = [\alpha_\varphi \cdot K_t^{1/(1-\varphi)} + (1 - \alpha_\varphi) \cdot (A_t \cdot N_t^W)^{1/(1-\varphi)}]^{1-\varphi}$, where φ is the elasticity of substitution between capital and labor, and $0 < \alpha_\varphi < 1$. Cobb-Douglas technology is covered as the limiting case $\varphi \to 1$. Leaving all other assumptions unchanged (and setting $\mu_2 = 1$ for simplicity), the economy is still a Markov process with unchanged state variables but with modified dynamics.

Table 6.6 summarizes the consumption and investment dynamics with CES production. The key difference from table 6.2 above is that the elasticities of the wage and the return on capital with respect to movements in the capital-labor ratio are scaled down by a factor φ.[35] In the young generation's response to birthrate shocks, α is replaced by α/φ, and, in π_{Rk}, $(1 - \alpha)/\varphi$ replaces $(1 - \alpha)$, where α is now the steady-state capital share.

The effect of birthrate and other workforce shocks on the fortunes of differently sized cohorts now depends on the relation between α/φ and

34. For annual data, Bohn (1999a) suggests $(1 - \delta)/R^k \approx 85$ percent so that $\pi_{Rk} \approx 0.10$. (In the numerical example, the role of δ was ignored for simplicity.) The same argument suggests that the transmission of demographics to the stock market may occur in part through variations in the value of old capital (say, if $1 - \delta$ is stochastic) and not only through the production function. This is an open question left for future research.

35. A variable factor share also complicates the calculation of the old generation's income and alters the propagation of shocks.

Table 6.6 **Macroeconomic Dynamics with CES Production**

Effect on:	Elasticity Coefficients
	A. Shocks to the Current Workforce, μ_{1t} and b_{t-1}
Retirees	$\pi_{c2\mu1} = \gamma_{c2mw} + \gamma_{c2\beta} \cdot \pi_{\beta\mu1}, \quad \pi_{c2b1} = \gamma_{c2mw} + \gamma_{c2\beta} \cdot \pi_{\beta b1}$
Workers	$\pi_{c1\mu1} = -\Delta_c \cdot \left(\frac{\alpha}{\varphi} - \frac{\theta}{1-\theta}\right) - \Delta_c \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta\mu1}, \quad \pi_{c1b1} = -\Delta_c \cdot \left(\frac{\alpha}{\varphi} - \frac{\theta}{1-\theta}\right) - \Delta_c \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta b1}$
Investment	$\pi_{k\mu1} = -\Delta_k \cdot \left(\frac{\alpha}{\varphi} - \frac{\theta}{1-\theta}\right) - \Delta_k \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta\mu1}, \quad \pi_{kb1} = -\Delta_k \cdot \left(\frac{\alpha}{\varphi} - \frac{\theta}{1-\theta}\right) - \Delta_k \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta b1}$
	B. Changes in Lagged Capital and Productivity, k_{t-1} and a_t
Retirees	$\pi_{c2k} = \gamma_{c2k} + \gamma_{c2\beta} \cdot \pi_{\beta k}, \quad \pi_{c2a} = -\gamma_{c2k} + \gamma_{c2\beta} \cdot \pi_{\beta a}$
Workers	$\pi_{c1k} = \Delta_c \cdot \alpha/\varphi - \Delta_c \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta k}, \quad \pi_{c1a} = -\Delta_c \cdot \alpha/\varphi - \Delta_c \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta a}$
Investment	$\pi_{kk} = \Delta_k \cdot \alpha/\varphi - \Delta_k \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta k}, \quad \pi_{ka} = -\Delta_k \cdot \alpha/\varphi - \Delta_k \cdot \frac{\theta}{1-\theta} \cdot \pi_{\beta a}$

Note: The notation is as in tables 6.1–6.2, except for the following symbols:

$\alpha = \frac{\alpha_\varphi \cdot (k/an)^\varphi}{\alpha_\varphi \cdot (k/an)^\varphi + 1 - \alpha_\varphi}$ = average capital share:

$\beta^* = \frac{\beta \cdot (w/A)}{(c^2/A)} / (1 - \delta^*)$ = share of old income that is wage indexed;

$\gamma_{c2k} = (1 - \delta^*) \cdot [\alpha + (1 - \alpha - \beta^*) \cdot (\varphi - 1)/\varphi] + \delta^*$;

$\gamma_{c2nw} = (1 - \delta^*) \cdot \left[1 - \alpha - (1 - \alpha - \beta^*) \cdot (\varphi - 1)/\varphi - \frac{\theta \cdot (1 - \alpha)}{\alpha + \theta \cdot (1 - \alpha)}\right]$; and

$\pi_{Rk} = (1 - (1 - \delta)/R^k) \cdot (1 - \alpha)/\varphi$.

For the effect of shocks not listed here, the formulas in table 6.2 apply with the modified symbols defined here.

$\theta/(1 - \theta)$. Given a defined-benefit social security system, unexpected shocks to the labor force are beneficial to a small cohort if and only if

$$\alpha/\varphi > \theta/(1 - \theta). \tag{15}$$

For elasticity values $\varphi < 1$, this inequality is satisfied even more clearly than for Cobb-Douglas. To overturn (15), one would have to argue that the capital-labor elasticity is far above one. In the numerical example with $\theta = 15$ percent and $\alpha = 1/3$, one would need an elasticity above 1.88. The empirical production literature suggests, however, that the elasticity is probably below rather than above one (e.g., Lucas 1969). Hence, it is difficult to question (15) on the basis of production theory.

Outside the model, one might think of international capital and labor movements as factors that could weaken the link between U.S. factor supplies and factor prices. If one interprets $1/\varphi$ more broadly as parameterizing the magnitude of factor-price movements in response to demographic change, increased openness might be interpreted as an increased φ value. Feldstein and Horioka (1980) have documented, however, that international savings-investment linkages have historically been unimportant, justifying a closed-economy analysis.[36]

Thus, concerns that the Cobb-Douglas assumption might overemphasize factor-price movements are probably unwarranted. Based on production-function estimates, Cobb-Douglas might even understate the factor-price movements, which would give small cohorts an even better starting position.

6.5.3 Elastic Labor Supply

Elastic labor supply is another consideration that could change the effect of demographics. The most serious concern is that, if small cohorts supplied more labor, birthrate changes would have a reduced effect on the capital-labor ratio and on factor prices.

A complete model with endogenous labor supply would complicate the analysis too much to fit into this already long paper. Some results can be obtained quite easily, however. Assume DB social security and Cobb-Douglas technology. Then, at any level of per capita labor supply, a large cohort will face a lower after-tax wage than a smaller cohort if and only if the inequality $\alpha > \theta/(1 - \theta)$ is satisfied. Thus, large cohorts face a relatively reduced opportunity set. This shows that labor supply considerations cannot overturn the basic qualitative finding that large cohorts are demographically disadvantaged for $\alpha > \theta/(1 - \theta)$.

Quantitatively, the implications of a variable labor supply depend on a

36. Also, openness would presumably matter most if demographic change abroad were orthogonal to that in the United States. But many other countries are undergoing demographic transitions similar to that in the United States.

trade-off between income and substitution effects. The negative income effect of a low wage may induce a large cohort to work more, while the negative substitution effect would encourage taking leisure. If the substitution effect is weak, a variable labor supply might even magnify movements in the effective capital-labor ratio.

6.5.4 Time Aggregation

Factor-price changes and cohort welfare may also be affected by time aggregation. If one used a more elaborate model of the life cycle with multiple working-age periods, large and small cohorts might overlap in the labor force, leading to reduced fluctuations in the labor force and in the retiree-worker ratio. In addition, "middle-aged" workers might supply both capital and labor, which would reduce the welfare effect of factor-price changes.[37] Are such extensions likely to overturn the results obtained here?

A more disaggregate approach would clearly yield different quantitative implications, but it is doubtful that these modifications will overturn any important results. To see why, first consider labor supply. Suppose one started out with, say, cohorts defined by year of birth. Then the significance of being in a small or a large birth cohort depends on the persistence of birthrate shocks and on the substitutability of wages across birth cohorts. If workers of different ages are close substitutes, wage movements are small unless the aggregate labor force varies significantly. And, if shocks are temporary, they would have little effect on the labor force. The baby-boom/bust phenomena suggest, however, that demographic shocks have enough persistence to matter at generational frequencies. And the labor literature (see above) suggests that substitution across cohorts is not perfect.

To sidestep any controversy about relative wage effects, assume for the sake of argument that all workers are perfect substitutes.[38] If small and large cohorts overlap in the labor force, it is true that the magnitude of wage fluctuations would be less than in a crude model that abstracts from such overlap. However, the same overlap would also reduce the fluctuations in the PAYGO tax rate and by the same percentage. Provided that $\alpha > \theta/(1 - \theta)$, changes in the workforce still affect wages more than taxes. Thus, an overlap of large and small cohorts in the workforce is unlikely to affect the *relative* importance of fiscal versus factor-price effects.

Second, consider the issue of middle-aged workers receiving both capital and labor income. This issue is not about the size of factor-price changes but about their welfare effect. Members of a large cohort are less worse off than in the basic model if they receive some of the high capital

37. I thank Kevin Murphy, the discussant, for raising this issue. Murphy also raised the issue of retirees receiving labor income, but I doubt that this is quantitatively as significant.

38. Otherwise, even changes in narrowly defined cohorts would have factor-price effects.

incomes generated by their own large cohort size. Note, however, that demographically driven changes in the return to capital were only one of several "transmission mechanisms" in the analysis presented above. Smaller cohorts would be better off than large ones even if the return on capital were held constant. To make large cohorts better off, the demographic effects through the return to capital would have to outweigh the effects through the after-tax wage. Empirically, most of the gross return on aggregate capital on an annual basis is due to the value of old capital (see above). The "within-a-generation" elasticity of R^k with respect to the capital-labor ratio is therefore likely small. In addition, households tend to accumulate financial assets fairly late in their careers (Poterba 1998). Hence, the receipt of capital income by worker households is unlikely to overturn the results from the basic overlapping-generations model.

6.6 Conclusions

The paper examines demographic uncertainty in a neoclassical growth model with overlapping generations. I compare the allocation of risk implied by alternative social security policies to the ex ante efficient allocation. The policy answers depend significantly on how strongly factor prices respond to demographic change. For plausible tax rates and elasticities of factor substitution, small cohorts are actually better off than large cohorts even in a defined-benefit social security system. This is because small cohorts enjoy favorable wage and interest-rate movements. Benefit cuts and/or prefunding in response to an unexpected decline in the birthrate would be inefficient.

The efficient responses to changes in life expectancy depend significantly on the type of change. If individuals know that they will live longer, or if fair annuities are available to diversify the risk of unexpected deaths, a longer life expectancy should trigger an increase in retirement benefits to those who live longer but a benefit reduction to the previous cohort. Reduced benefits to those who expect to live longer are efficient only if increased old-age survival leads to reduced accidental bequests to the next generation.

Overall, the efficiency analysis yields policy conclusions that differ significantly from the proposals in the current reform debate. Notably, the efficient response to a baby boom is to increase the retirement benefits of the baby boomers, even at the cost of tax increases to the baby-bust generation, and the efficient response to news about increased future life expectancy is to cut benefits to current retirees.

With regard to birthrate shocks, I obtain conclusions that differ from the conventional wisdom because my analysis includes endogenous factor-price movements. Factor-price effects are largely ignored in the current policy debate. The Social Security Administration, for example, makes long-run projections of future wages and interest rates by extrapolating past trends. The analysis presented in this paper suggests that the omission of endogenous factor-price movements is seriously misleading under empirically realistic parametric assumptions.

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Comment Kevin M. Murphy

Henning Bohn's paper makes several important points that sometimes get lost in the debate on the effect of demographic changes on social insurance programs (like the impending retirement of the baby-boom cohorts in the United States). The most important of these points is that it is not the existence of pay-as-you-go social insurance programs that makes demographic changes important or of interest to economists. Demographic changes can have significant effects even without such programs or without government programs of any kind. Neither do the effects of such changes hinge critically on market failures. Changes in demographics, like changes in technology, represent changes in the fundamentals of the economy and as such have real effects on individual outcomes, like the realized levels of wages, interest rates, and consumption. While few would argue with this point, it is also missed or ignored in most analyses.

Bohn also analyzes a somewhat different aspect of risk than that addressed by many others looking at the risk of investment-based social insurance systems. The risk examined by Bohn is not the risk induced by the selection of an investment portfolio (although that may be one way to implement some of the contingent contracting that he advocates in the paper) but the risk induced by the demographic changes themselves. He begins by recognizing that, in a defined-benefit pay-as-you-go system (where benefits for those retired at date t are indexed to wages for workers at date t), small birth cohorts face an added fiscal burden in that they must pay higher social insurance tax rates to finance the benefits of a relatively larger retiree cohort. Such increases in the dependency ratio are

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the source of much of the doom and gloom over the future of social security in the United States and similar programs in other countries. While Medicare suffers from this as well, other problems (like changes in the consumption of medical care) add to the rising burden there. However, this is not the whole story. While small cohorts are hurt by the social insurance burden, small cohorts may gain for other reasons. Small cohorts may earn higher wages as the labor-capital ratio falls during their working years and relatively high returns on investment as they save for retirement when capital stocks are relatively small owing to a decline in per capita life-cycle savings. Bohn's essential argument is that these other advantages from being in a small birth cohort may more than outweigh the disadvantages induced by a defined-benefit social insurance program.

I should say at this point that Bohn's analysis is significantly broader than I have laid out here. He examines not only fluctuations in birthrates but also fluctuations in technology and changes in both pre- and postretirement life expectancy as well as the effects of alternative annuity market structures. But the essential message is similar: Before we think about how such changes affect our social insurance system, we should examine what broader changes they induce through the market itself. I think that this is indeed an important message.

Bohn presents his analysis using an overlapping-generations model with three generations: children, workers, and retirees. The basic policy role induced in the model is one of ex ante insurance (and to a lesser extent an effort to overcome inefficiencies in the private insurance market). The basic idea is that ex ante contracting between generations can spread the risk of demographic and other shocks and improve expected utility for all generations. Viewed in this way, the social insurance system represents an opportunity for intertemporal risk sharing across generations and is not necessarily a source of additional risk. Our goal, then, he argues, should be to set up a system that counterbalances the risk inherent in the market outcomes. Of course, if private individuals could engage in such trading through trading securities or other assets that embed such risks, then such an undertaking would not be necessary. I think that he should do more on this dimension to convince us that such private mechanisms are not available since that is a key aspect of his analysis.

For now, I continue to work within his framework and assume that neither the dynastic family nor private markets can address these issues. Under these conditions, the essential question is to determine what kind of generation-specific risks are induced by the private market solution. As I mentioned above, Bohn's analysis is based on an extended two-generation overlapping-generations model with the usual model of a working period and a retirement period augmented by a period of childhood, which serves to give the economy advance warning about the size of the coming generation of workers. This formulation serves to generate significant risks. First, the stark separation between workers as the suppliers of labor and the earners of labor income and retirees as holders of capital and earners of capital income makes fluctuations in cohort size translate directly into fluctuations in factor supplies and fluctuations in factor prices translate directly into fluctuations in relative incomes. In a world in which differences in the ownership of capital and labor were less discrete, we would see smaller effects as cohort-size changes would be muted in terms of both how they affect factor supplies and how factor prices feed back on incomes. The incomplete nature of markets inherent in the overlapping-generations structure also limits the ability of private markets to achieve efficient allocations of risk.

To make a long story short, under this structure, the factor-price changes induced by birthrate and other changes more than offset the effects of a social insurance system the size of the U.S. social security and Medicare systems. This means that, from an intertemporal insurance perspective, even with defined benefits small cohorts may still be better off owing to the factor-price effects. Switching to a defined-contribution pay-as-you-go system (where we cut benefits rather than raise taxes as the baby boom retires) would only make the insurance worse. I think that this analysis adds a different perspective than most of us have taken on this issue.

But should we believe it? That to me is a tougher question. First, while I follow Bohn's analysis, I am not sure that such factor-price effects are as large as the model makes out. If we augment the model to have the working-age population supply capital (i.e., we have more than just life-cycle savings) or to allow for outside sources of capital or trade in labor-intensive and capital-intensive goods, then such factor-price effects of cohort size will be reduced. The effects of changing the assumptions can be significant. For example, if over the working life individuals supply 1 unit of labor and 0.5 units of capital and the reverse is true at retirement, then the effects of cohort-size changes are cut by almost 90 percent compared to the case where the young provide 1.5 units of labor and the old provide 1.5 units of capital. This happens since *both* the effects of cohort size on factor supplies and the effects of factor supplies on relative incomes are cut by two-thirds. Hence, it would seem to me that that stark contrast induced by the overlapping-generations structure works to make these effects large. In addition, the overlapping-generations structure limits the ability of private markets to provide such insurance relative to a dynastic family approach or some other framework with effectively longer-lived agents. It need not be that generations are perfectly linked through altruism since I could set up contracts that lead to mutual gains to trade between my descendants and those alive today or the descendants of others with even modest amounts of altruism.

Second, as someone who has looked at the data to try to find the empirical effects of cohort size on wages, I will say that the effects that I have seen are not overwhelming. Whether they are big enough to offset the anticipated social insurance effects is unclear. Indeed, I think that we always tend to underestimate the degree of substitutability that will occur, particularly for such long-run changes. Whether it is through the factor-price equalization of trade, induced technological progress, the effects described above, or other forces, I think that the structure that Bohn lays out is likely to overstate the actual factor-price effects. As a result, I think that we need to examine a wider class of models to see how well this result generalizes to other market and production structures. While Bohn's work provides us with an important data point in this regard, I think that we need to examine things more closely.

Finally, I would like to say that I am not sure that intertemporal insurance is the most important aspect of designing a social insurance system. The taxes used to finance such systems induce significant excess burden, and the benefits structures significantly distort retirement incentives. In my opinion, these effects are likely to be more important than the insurance effects of improved intergenerational insurance. Indeed, the convex nature of the excess burden provides a strong rationale against raising the tax rate in response to having a large retirement-age cohort and toward cutting benefits. It would really help in this regard if the paper provided some estimates of the magnitude of the gains associated with the improvement in insurance that could be compared to the estimates of the deadweight burden from taxation.

Discussion Summary

David Backus noted a caveat in the paper. The capital-labor ratio might behave differently in open economies than in closed economies. In particular, international capital flows could change some of the predictions of the model that are driven by factor-price movements induced by the dynamics of the capital-labor ratio.

Andrew Samwick concurred with Backus's comment.

Stephen Ross noted that the model treats population growth as exogenous. It might be important for analyzing the issues discussed in the paper to acknowledge that population growth is endogenous. Furthermore, he observed that Bohn does not embrace the Ricardian view but instead considered the other extreme, namely, a Rawlsian perspective where one attaches substantial weight to generations to be born in the remote future.

Zvi Bodie stated first that he liked the paper because he believes that it frames the issues in the right way. The actual techniques used are of course debatable. In this context, Bodie noted that, although an abstract neoclassical model looking behind the veil of institutions has many advantages, it is not useful for the study of the optimal institutional mechanism needed for the implementation of the efficient risk-sharing arrangement derived by Bohn. This is nevertheless an important question, studied, for instance, by Martin Feldstein and Andrew Samwick (1998). Finally, Bodie noted that he is not convinced that the current social security system is, or is perceived to be, a true defined-benefit system. It is perhaps better described as a defined-contribution plan.

James Poterba remarked that there is substantial disagreement on the relation between population size, age structure, and factor prices. For instance, larger cohorts may generate more ideas and therefore spur productivity growth more quickly according to some versions of endogenous-growth theory. These issues are very complex and suggest at least that the Easterlin hypothesis has many plausible alternatives. Poterba concluded that Bohn should acknowledge these alternative hypotheses and discuss their implications for the results obtained in the paper.

Directly related to Poterba's comment, *Robert King* noted that it would be interesting to have some—even crude—evidence about the linkages between wages rates and population size. Economic historians have examined this, particularly with respect to immigration flows at various points in time, and have found that the effects are surprisingly small. Some additional discussion that would help one evaluate the magnitude of these effects would be useful.

With respect to the previous comments, *David Cutler* noted that some of the empirical evidence supports the predictions of the model. He further noted that the paper does not consider uncertainty about productivity while at work. In particular, there is no uncertainty about the length of the period during which a young worker is able to work. Integrating this into the model is an important extension. Finally, Cutler remarked that it would be interesting to study which system is best at sharing different types of risks. One may want to consider hybrid systems with different risk-sharing rules, depending on the sort of risk to be insured.

Antonio Rangel noted that the Cobb-Douglas specification for the production function is crucial. In particular, it predicts perfect correlation between wages and interest rates and might therefore be responsible for the similarity between defined-contribution and prefunded (privatized) systems in terms of risk-sharing properties. He also remarked that the paper allows for saving only in the form of physical capital, not in the form of financial assets.

Amir Yaron noted that a defined-contribution system differs from a prefunded system once one considers heterogeneity and liquidity constraints. He also commented that the paper compares steady states and thereby ignores what happens along transition paths. Finally, Yaron wondered what happened to accidental bequests in the model.

Andrew Abel suggested enriching the model by allowing for labor supply elasticity. He argued that an endogenous labor supply is important to consider in a model where the size of the workforce matters so much. First, the fact that retirees do not share in demographic risk would be tempered by this extension. Second, the factor-price movements predicted in the paper would also be smaller when labor is supplied elastically. Finally, labor supply elasticity would endogenize the length of the working life along the lines suggested by Cutler.

Richard Zeckhauser noted that, since the paper is theoretical, its results should not depend on empirical measures. The results should instead be presented in their most general form. With respect to possible empirical exercises to test some of the model's predictions, he cautioned against the use of cross-sectional data: different countries have different social security arrangements, and this might obscure the empirical analysis.

Henning Bohn responded to these comments as follows. First, with respect to the labor supply elasticity, he noted that there would be a substitution and an income effect. It seems that this extension would preserve the main results of the paper, at least for plausible parameter values. In particular, he remarked that the prediction involving the condition on the sign of $\alpha - \theta/(1 - \theta)$ would still obtain. The length of the working life, suggested by Cutler, however, is another issue. This modification of the model could potentially change some of the results. With respect to the comments of Backus and Samwick on the importance of the closed-economy assumption for the factor-price movements derived, Bohn responded that international capital flows would not be sufficiently strong to overturn the results, unless the Feldstein-Horioka puzzle disappears altogether.

Bohn agreed that the Cobb-Douglas assumption was important for the finding that defined-contribution and prefunded systems have identical risk-sharing properties. He added that a paper by Mankiw, Romer, and Weil (1992) shows that the Cobb-Douglas assumption is not rejected by the data. Also, the paper considers an extension to a constant elasticity of substitution production function and shows that, unless the elasticity of factor substitution is above unity, the factor-price movements and other predictions still obtain. Finally, he concluded that an empirical analysis of the link between cohort size and factor-price movements was indeed interesting but beyond the scope of this paper.

References

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