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MACROECONOMIC POLICY IN A DYNAMIC TWO COUNTRY MODEL†

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We develop a dynamic two country policy model and trace the effects of one country's policy actions as they pass through the world economy. We also attempt to characterize the interdependence of policy making in dynamic context.

As world markets develop, the study of international economic linkages becomes more and more important. We hear unions claim that their government's policies are exporting jobs. One country claims another is exporting inflation or unemployment. The continuing nature of these disputes would seem to indicate a lack of consensus about how international transmission mechanisms work. Closely related is the study of decentralized policy making. The gradual demise of the Bretton Woods system has resulted in attempts to establish a new international regime. Groups of countries have discussed or attempted various types of economic integration. The continuing nature of these discussions suggests that the nature and degree of interdependence in macroeconomic policy making are not well understood either.

While these issues are timely, they are certainly not new; many economists have studied them before us. However, we think that in the past far too much reliance has been placed upon single country models and small country results. This reliance was surely not by choice; it must be due, at least in part, to the lack of techniques capable of handling medium-sized, dynamic models. Here we hope to show that state space techniques, some of which may be unknown to economists, are capable of handling such models. We think that they can be used quite successfully to provide new answers to old questions in the area of international trade.

In this paper, we develop a dynamic two country policy model, and we trace the effects of one country's policy actions as they pass through the world economy. We also attempt to characterize or measure the interdependence of policy making in this dynamic context. The version of the model analyzed here may not provide a very convincing description of international transmission mechanisms or practical solutions to the prob-

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lems of decentralized policy making, but we do hope that we have taken a new step in the right direction.

I. A World Trade Model

Our hypothetical world consists of two countries. Each produces a single good that can be consumed, traded or used as capital. And each supplies two financial assets money and bonds to world markets. To present our new way of carrying out short-run analysis in a simple setting, our model has several simplifying features. For example, we assume that the two countries produce identical outputs and that their bonds are perfect substitutes. This implies that the domestic interest rate, i, must equal the foreign interest rate, i^*, plus the expected rate of the exchange rate change, ρ. Under the additional rationality assumption that the expected rate of the domestic inflation π is related to that of the foreign country by $\pi = \rho + \pi^*$, this means that the real interest rates of the two countries are equal $i - \pi = i^* - \pi^*$. Hence the Keynesian investment functions we use in the model imply that the capital stocks in the two countries are closely related (since the marginal products of capital in the two countries are similar by assumption). There are other simplifying assumptions. They will be discussed as the model is formally presented. The basic dynamics of the model are provided by the two Keynesian investment functions and expectation formation. There are also flow constraints on government policy instruments; these flow constraints require that deficits and foreign reserve acquisitions be financed by increases in government debt.

Our model differs from Mundell's (1968, Chapter 18) familiar small country model (with perfect capital mobility) in two important way. Mundell does not assume all countries produce the same good, so he obtains certain terms of trade effects that are not present in our model. Also, Mundell's basic set of equilibrium conditions includes a "foreign exchange market" equation which states that the balance of payments (or the government's acquisition of foreign reserves) is equal to the trade balance plus net capital inflow. Our model incorporates a stock equilibrium formulation throughout the asset sector. A foreign exchange equation can be derived from our equilibrium conditions, but it will differ from Mundell's in the specification of the determinants of net capital flows.[1] In this and other respects, our model is more in the tradition of the newer "monetary" approach to exchange rate determination.[2] Our model is sim-

[1]N. Wallace (1970) provides a helpful discussion of these issues. It is interesting to note that the foreign exchange equation does not appear among the equilibrium conditions for the two country model Mundell outlines in the appendix of Chapter 18. That model would seem to be at odds with his small country model and more in line with our model.

[2]The *Scandanavian Journal* devoted issue #2, 78, 1976 to this approach.

ilar to Henderson's two country model (1974), but we pay more attention to short-run analysis.

Our analysis is similar to those of Blinder and Solow (1973), Branson (1976) and Turnovsky (1976) in that we pay explicit attention to the dynamic implications of the flow constraints on government instruments. However, our techniques allow us to analyze short-run dynamic effects in addition to comparing long run stationary states.

The model can be described by the following equations:

Equilibrium in the good market:

$$(1) \qquad Y = C + I + \delta K + G + X$$

$$C = C(Y - T);\ I = I(q);\ q = q(Y/K,\ \delta,\ i - \pi)$$

$$(2) \qquad Y^* = C^* + I^* + \delta^* K^* + G^* + X^*$$

$$C^* = C^*(Y^* - T^*);\ I^* = I^*(q^*);\ q^* = q^*(Y^*/K^*,\ \delta^*,\ i^* - \pi^*)$$

$$(3) \qquad Y = S(K,\ P)$$

$$(4) \qquad Y^* = S^*(K^*,\ P^*)$$

$$(5) \qquad X = -X^*$$

$$(6) \qquad P = EP^*$$

Equilibrium in financial markets:[3]

$$(7) \qquad M/P = L(\rho,\ i,\ Y,\ W)$$

[3]In the asset sector, we assume that demands always equal supplies. The demand for domestic money comes from two sources: domestic residents and foreigners. Suppose that the foreign demand for domestic money is given by $M_f/P = \alpha^*(\rho, i, Y^*)(A^*/P^*)$, where A^*/P^* is the real wealth of the foreign country. Then, the domestic demand for domestic money is $(M - M_f)/P = \alpha(\rho, i, Y)(A/P)$, where A/P is domestic real wealth. Aggregating these two sources, we write the demand for domestic money as $M/P = \alpha(\rho,\ i,\ Y,\ Y^*)(A^*/P^* + A/P)$ where α is the weighted sum of α and α^*. We assume that the weights such as $(A^*/P^*)/(A/P + A^*/P^*)$ do not change appreciably in the short-run and take them to be constants.

Thus, (7) should have Y^* as an additional argument. However, its absence can be justified in the following way. The asset equations are to be used in deviational or variational forms to perform short-run analysis. Since the deviational output rates of the two countries are related it is not necessary to carry both variables (y and y^* in our notation) in the asset equation. Aggregating demands for bonds similarly, we can use real world wealth as the argument in the asset demand functions. Let $(B_g - R_h - B_g^f + B)/P + \beta(\rho, i, Y)A/P$ where $B = qKP$: domestic demand for domestic bonds. $(B_g^*)^h/P = \gamma(\rho, i, Y) \cdot A/P$: domestic demand for foreign government bonds. The foreign counterparts are similarly defined. Aggregating them, we obtain $(B_g - R + B)/P + (B_g^* - R^* + B^*)/P^* = (\bar{\beta} + \bar{\gamma})(A/P + A^*/P^*)$ where $\bar{\beta} + \bar{\gamma}$ is the weighted average of $\beta + \gamma$ and $\beta^* + \gamma^*$. Here there is additional justification to assume that $\beta + \gamma \approx \beta^* + \gamma^*$ since the two countries have similar financial structures by assumption. Then $\bar{\beta} + \bar{\gamma} \approx \beta + \gamma \approx \beta^* + \gamma^*$ and there will be little aggregation bias. Even if this assumption is not valid, the other two reasons advanced above indicate that the variational expressions obtained using the real world wealth are approximately correct.

$$M^*/P^* = L^*(\rho, i^*, Y^*, W) \tag{8}$$

$$i = i^* + \rho \tag{9}$$

$$\begin{aligned} W = {} & (1/P)M + qK + (1/P)(B_g - R) \\ & + (1/P^*)M^* + q^*K^* + (1/P^*)(B_g^* - R^*) \end{aligned} \tag{10}$$

Capital formation:

$$\dot{K} = I(q) \tag{11}$$

$$\dot{K}^* = I^*(q^*) \tag{12}$$

Constraints on government instruments:[4]

$$\dot{M} + \dot{B}_g = E\dot{R}^* + P(G - T) \tag{13}$$

$$\dot{M}^* + \dot{B}_g^* = \dot{R}/E + P^*(G^* - T^*) \tag{14}$$

$$\Delta M + \Delta B_g = E\Delta R^* \tag{15}$$

$$\Delta M^* + \Delta B_g^* = (1/E)\Delta R \tag{16}$$

[4]There are two additional equations for the current accounts of the two countries. One equates the rate of change of the home private holdings and the home government holding of the foreign government bonds (since we assume that private bonds are not internationally traded) to the current account (interest receipts on the home country's holding of the foreign government bonds plus net trade balance.) There is a similar current account equation for the foreign country's holding of the home government bonds.

Just as (13) can be specialized to discuss full money or full bond financing of the government deficit, the domestic private sector holding of the foreign government bonds can vary with time depending on the degree of sterilization adopted by the home government (in addition to the instruments which can affect the current account itself). If the interest receipts are neglected as a small percentage of the home disposable income, then the current account equals the trade balance, and we can show that it is affected only by the capital stocks and G and G^*.

To examine the dynamics in full generality, we must introduce two new instruments to denote the degree of money financing of the government deficit and the degree of sterilization. In order not to increase the dimension of the dynamics, we can and do examine some special cases, such as the case of fully money financing of the government deficits and full sterilization by both countries.

In this case, the private holding of the foreign government bonds remain constant and does not appear in the variational equation. The variational equation of (13) becomes

$$\delta\dot{M} + \delta\dot{B}_g = E\delta\dot{R}^* + \delta[P(G - T)]$$

where

$$\delta\dot{M} = \delta[P(G - T)]$$

hence $\delta\dot{B}_g = E\delta\dot{R}^* = \delta(PX)$. Here we neglect variation in the domestic holdings of the foreign money. As we mentioned above, X does not depend on R^* and this equation does not enter the state equation and does not increase the dimension of the state vector. It is another target equation. Similarly for the foreign country under full money financing of the deficit and full sterilization.

We, therefore, do not exhibit these equations here. This type of simplification is no longer valid if the output deviations are price elastic. Details of such dynamic complications is set out in a small country framework in Aoki (1977).

Expectation formation:

$$(17) \qquad \pi = \rho + \pi^*$$

$$(18) \qquad \pi \equiv \dot{P}^e/P = \beta(P - P^e) + \text{constant}$$

$$\pi^* \equiv \dot{P}^{*e}/P^* = \beta^*(P^* - P^{*e}) + \text{constant}$$

Y, X and K are the real home country output, trade balance and capital stock; P is the home currency price of output; q is the price of existing capital relative to new capital (i.e., output); i is the interest rate paid on home bonds; G and T are the real rates of home government spending and taxation (net of transfers); M the home money stock; B_g is the nominal value of home government bonds held by the private sector; W is the real wealth of the private sector; the exchange rate E is the home currency price of foreign currency; ρ is the expected rate of depreciation or $\dot{E}/E$, and π is the expected rate of inflation or $\dot{P}/P$. Variables with "*" superscripts denote the foreign counterparts of home variables. R and R^* will be defined below.

Equation (1) describes home demand for output. Home demand consists of consumption demand (which depends upon disposable income), gross investment demand (that is, net investment, I, plus depreciation, δK) and government demand. The excess of home production over home demand is the home trade balance. Equation (2) describes foreign demand for world output. Equations (3) and (4) are the supply curves for world output: in the present version of the model they are price-elastic. Equations (5) and (6) are equilibrium conditions in the goods market. The first requires that world supply of output equal world demand; the second requires that the home price of output equal the foreign price.

Equations (7) through (10) describe the asset sector. There are actually four assets in the model: home money, foreign money, home bonds and foreign bonds. These assets earn 0, ρ, i and $i^* + \rho$ respectively in terms of home currency, or $-\rho$, 0, $i - \rho$ and i^* in terms of foreign currency: the interest rate differentials are, of course, the same from either point of view. Since home and foreign bonds are assumed to be perfect substitutes, their interest rate differential will be zero in equilibrium (equation (9)), and the number of assets is effectively reduced to three: home money, foreign money, and world bonds. The demands for these assets are aggregated over the citizens of both countries. Using the world wealth constraint, it can be shown that all three asset markets must be in equilibrium when any two markets are in equilibrium: this is Walras' law applied to the stock portfolio equilibrium. Equations (7) and (8) are the equilibrium conditions for the home and foreign money markets: they imply equilibrium throughout the whole asset sector. Equation (10) gives the real world wealth of the private sector. Firms are assumed to hold all of the capital stock and to issue bonds (or equity) to finance new investment. The real market value of the existing capital stock is $qK + q^*K^*$ where q, the price of existing capital relative to new capital, depends in a familiar way upon the real rate of return on bonds and the marginal productivity of capital.[5] (Here, Y/K is used as a proxy for the marginal productivity of capital.) The two governments' bonds are assumed to be perfect substitutes for private bonds, and their real value, $(1/P)B_g + (1/P^*)B_g^*$, is a component of wealth.[6]

In this model, we assume that the home government holds foreign bonds, R^*, as a foreign reserve asset. It is more conventional to model currency as the foreign reserve asset; however, we think that our assumption is more realistic. The foreign government holds home bonds, R, as its foreign reserve asset. So government holdings deplete the supply of real wealth available to the private sector by the amount $(1/P)R + (1/P^*)R^*$; the wealth equation (10) reflects this fact.[7]

Equations (1) through (10) determine the static or instantaneous equilibrium values of Y, Y^*, P, P^*, i, i^*, X, X^*, E and W. The dynamic equations, (11) through (14) and (18), propell the instantaneous equilibrium through time. Equations (11) and (12) are "Keynesian" investment functions. The constraints, (13) and (14), require governments to pay for their budget deficits and foreign reserve acquisitions by issuing money or bonds.[8] Equation (18) states that expectations are formed adaptively; the

[5] More specifically, $q = (MPK - \delta)/(i - \pi)$ where MFK is the marginal product of capital. See Tobin (1969) or Sargent and Wallace (1971). This is the simplest way to rationalize models incorporating Keynesian investment functions and having no direct link between interest rates and the marginal productivity of capital.

[6] Government bonds are modeled as call loans. Like savings accounts, they pay the going rate of interest while their nominal value remains fixed.

[7] Conventional definitions of policy regimes must be modified in an obvious way. A "floating rate" regime is one in which R and R^* are held constant, and so on.

[8] We can derive the exact expression for the world disposable income and indicate the nature of approximation we employ. In the process we can also demonstrate how the government budget flow constraint equations (13) and (14) are employed. Differentiate (10)

$$\begin{aligned}\dot{W} &= (\dot{M} + \dot{B}g - \dot{R})/P - [(M + Bg - R)/P]\pi \\ &\quad + (\dot{q}K + q\dot{K}) + (\dot{M}^* + \dot{B}_g^* - \dot{R}^*)/P^* \\ &\quad - [(M^* + Bg^* - R^*)/P^*]\pi^* + (\dot{q}^*K^* + q^*\dot{K}^*).\end{aligned}$$

Since $P = EP^*$, we have $\dot{R}^*/P^* = E\dot{R}^*/P$ and $\dot{R}/P = (\dot{R}/E)/P^*$. We can thus regroup the first and the fourth terms respectively as $(\dot{M} + \dot{B}g - E\dot{R}^*)/P$ and $(\dot{M}^* + \dot{B}g^* - \dot{R}/E)/P^*$. Replace them by (13) and (14). Next using the GNP identities (1) and (2) (setting $\delta = \delta^* = 0$), we obtain assuming $\dot{q} = \dot{q}^* = 0$,

$$\begin{aligned}\dot{W} &= (Y - T) + (Y^* - T^*) - (C + C^*) + (q - 1)\dot{K} + (q^* - 1)\dot{K}^* \\ &\quad - (M + Bg - R)\pi/P - [(M^* + BG^* - R^*)/P^*]\pi^*.\end{aligned}$$

If we ignore the loss due to inflation and assume $(q - 1)\dot{K}$ and $(q^* - 1)\dot{K}^*$ are small, then $(Y - T) + (Y^* - T^*)$ is the world disposable income, which is the amount that can be consumed while leaving real world wealth intact.

cases of static expectations and perfect foresight correspond to the special cases where $\beta \to 0$ and $\beta \to \infty$. (The latter may require a special definition of derivatives and a terminal condition ruling out "speculative bubbles" or instability; see Sargent (1973), Sargent and Wallace (1973) and Kouri (1976).) Equation (17) is a rationality constraint on expectation formation. In a perfect foresight model, it would follow directly from (6). While we do not assume perfect foresight, this constraint does seem natural in a one good model.

A "$\cdot$" denotes a time derivative, while a "Δ" denotes a change at a given moment in time. Equations (15) and (16) are the stock constraints corresponding to the flow constraints (13) and (14). The home government chooses time paths for G, T, M, B_g and R^* subject to (13) and (15) and the foreign government chooses time paths for G^*, T^*, M^*, B_g^*, and R subject to (14) and (16); then the model determines the time paths of the remaining variables.

II. State Space Representations

We begin by expressing the model in a standardized form called a state space representation. This will enable us to make use of several well known procedures that have been developed in terms of that form.[9] A state space representation consists of two matrix equations, the state equation and the target equation:

$$\dot{z}(t) = Az(t) + B_1 v_1(t) + B_2 v_2(t) \tag{19}$$

$$1(t) = Cz(t) + D_1 v_1(t) + D_2 v_2(t) \tag{20}$$

The state equation describes the dynamics in the model. The dimension of the state vector z is, roughly speaking, the dynamic dimension of the model. The instruments being studied comprise the elements of vector v_1; the rest of the instruments and the exogenous variables (including intercept terms) are put in v_2. The target equation is a reduced form equation describing the instantaneous equilibrium at a given moment in time. Any subset of the endogenous variables can make up the elements of the target vector 1. The state vector z pushes the instantaneous equilibrium and the target vector through time. In this section, we show how the world trade model can be represented in this form.

We have already noted that in this model the real rates of interest must equalize; that is, equations (9) and (17) imply $i - \pi = i^* - \pi^*$. This is an immediate result of the assumptions that the two countries' bonds are perfect substitutes and that their products are identical. In what fol-

[9]Aoki (1976) presents a comprehensive introduction to the state space approach for economists. A detailed discussion of most of the techniques we use in this paper may be found in Aoki's book.

lows we will make four more simplifying assumptions: (i) The output supply curves are price inelastic. (ii) Investment is just a function of the real interest rate. (iii) There is no depreciation (i.e. $\delta = 0 = \delta^*$). (iv) Expectations are static (i.e. π and π^* are fixed and equation (18) is ignored).

These additional assumptions are not particularly appealing, and in principle there is no reason that they have to be made. The state space techniques described here are capable of handling the model as described in section I. However, it turns out that in most cases we would require numerical estimates of various parameters in order to obtain conclusive results. Lacking these numerical estimates, we prefer to simplify the model to the point where we can obtain interesting results based simply upon sign restrictions. We leave it to future research to determine how robust these results will be. We will be able to make some observations about adaptive expectation formation and perfect foresight in this model.

The classical supply assumptions, when combined with the fact that the real interest rates must equalize, have two important implications: First, the instantaneous equilibrium dichotomizes in a classical manner. The real interest rate is determined in the market for world output while price levels and the exchange rate are determined in financial markets. Equation (5) may be used to eliminate X^* in (1) and (2), and then these equations may be solved for X and $i - \pi (= i^* - \pi^*)$. Since world output is price-inelastic, and since the capital stocks are fixed at a given moment in time, the real interest rate is the only variable left to equilibrate the world output market. Prices and the exchange rate are determined by the two money market equations and the law of one price. Second, the two countries' capital stocks and outputs are closely tied. Log-linearizing the capital formation equations (11) and (12), we have $\dot{k} = j_0 - j_1(i - \pi)$ and $\dot{k}^* = j_0^* - j_1^*(i^* - \pi^*) = j_0^* - j_1^*(i - \pi)$ where k and k^* are the logs of the domestic and foreign capital stocks, and j_1 and j_1^* are positive constants.[10] Thus

$$(1/j_1^*)\dot{k}^* = [(j_0^*/j_1^*) - (j_0/j_1)] + (1/j_1)\dot{k}$$

and integrating, we obtain equation (30) below where c is a constant depending upon initial values of the home and foreign capital stocks.[11] The classical supply assumptions then imply a log-linear relationship between the two countries' outputs, equation (22)′ below.

Making use of these preliminary remarks, we can express the world model in the following log-linear form: (Two notational conventions are observed below. A "~" below a letter denotes a vector of domestic and

[10] The reader will note that $\dot{k} = \dot{K}/K$ that is we are actually assuming that the percentage increase in the capital stock is a function of $i - \pi$.

[11] For mathematical convenience, we have assumed that $j_0/j_1 = j_0^*/j_1^*$; the results that follow can be modified in an obvious way if this assumption is not justified.

foreign variables: for example. $p = [p p^*]$. Generally, small letters denotes the logs of the corresponding capital letters. The exceptions to this rule are i and i^*, which represent actual interest rates and not their logs. So, for example, $i - \dot{p}^e = i - \dot{P}^e/P$ is the expected real rate of interest.)

(21) $$y = s_0 + s_1 k$$

(22) $$y^* = s_0^* + s_1^* k^*$$

or

(22)′ $$y^* = (s_1^*/s_1)(j_1^*/j_1)y + \text{constant}$$

(23) $$i - \pi = n_0 - n_1 y - n_1^* y^* + n_2 g + n_2^* g^* = \dot{p}^* - \pi^*$$

(24) $$x = h_0 + h_1 y - h_1^* y^* - h_2 g + h_2^* g^*$$

(25) $$p = e + p^*$$

(26) $$m - p = m_0 - m_1 \rho - m_2 i + m_3 y + m_4 w$$

(27) $$m^* - p^* = m_0^* + m_1^* \rho - m_2^* i^* + m_3^* y^* + m_4^* w$$

(28) $$i = i^* + \rho$$

(29) $$w = w_0 + w_1 k + w_1^* k^* + w_2 m + w_2^* m^* + w_3 b_g + w_3^* b_g^* - w_4 p - w_4^* p^*,\ w_4 = w_2 + w_3,\ w_4^* = w_2^* + w_3^*$$

(30) $$k^* = c + (j_1^*/j_1)k$$

(31) $$\dot{k} = j_0 - j_1(i - \pi)$$

(32) $$\alpha_1 \dot{m} + \alpha_2 \dot{b}_g = \alpha_0 + \alpha_4 p + \alpha_5 g$$

(33) $$\alpha_1^* \dot{m}^* + \alpha_2^* \dot{b}_g^* = \alpha_0^* + \alpha_4^* p^* + \alpha_5^* g^*$$

(34) $$\alpha_1 \Delta m + \alpha_2 \Delta b_g = 0$$

(35) $$\alpha_1^* \Delta m^* + \alpha_2^* \Delta b_g^* = 0$$

(36) $$\pi = \rho + \pi^*$$

(37) $$\underline{\pi} \equiv \dot{\underline{p}}^e = \beta(\underline{p} - \underline{p}^e)$$

where all of the coefficients are positive.

Equations (21) and (22) are log-linear versions of the supply curves (3) and (4). Equations (23) and (24) are the log-linear solutions of (1) and (2). Equations (25), . . . , (28) correspond to equations (6), . . . , (9). Equation (29) is a log-linearization of the wealth equation (10) where capital gains and losses have been ignored: $w_1, w_1^*, \ldots, w_3, w_3^*$ are the relative sizes of the various components of wealth. Equations (31), . . . , (37) correspond to equations (11), (13), . . . , (18). α_1 and α_2 are the relative sizes of the components in the sum $M + B_g$: so, for example, $\alpha_1/\alpha_2 = w_2/w_3$.

$(=M/B_g)$ and $\alpha_1^*/\alpha_2^* = w_2^*/w_1^*(=M^*/B_g^*)$. Taxes and foreign reserves will be held constant in the exercises that follow: so T, T^*, R and R^* have been subsumed in the intercept terms.

We begin by deriving the target equations, or the reduced form equations for the instantaneous equilibrium. Prices are determined in financial markets: using (21), (22), (23), (36), (29) and (30) to eliminate $\underset{\sim}{y}, \underset{\sim}{i}, \rho, k^*$ and w, we can solve (26) and (27) for

$$(38) \quad \underset{\sim}{p} = N_6^{-1}(-N_1 k + N_2 \underset{\sim}{g} + N_3 \underset{\sim}{m} - N_4 \underset{\sim}{b}_g + N_5 \underset{\sim}{\pi}) + \text{constant}$$

$$N_1 = \begin{pmatrix} m_2(n_1 s_1 + n_1^* s_1^* j_1^*/j_1) + m_3 s_1 + m_4 w_1 \\ m_2^*(n_1 s_1 + n_1^* s_1^* j_1^*/j_1 + m_3 s_1^* j_1^*/j_1 + m_4^* w_1^* \end{pmatrix}$$

$$N_2 = \begin{pmatrix} m_2 n_2 & m_2 n_2^* \\ m_2^* n_2 & m_2^* n_2^* \end{pmatrix} \qquad N_3 = \begin{pmatrix} 1 - m_4 w_2 & 0 \\ 0 & 1 - m_4^* w_2^* \end{pmatrix}$$

$$N_4 = \begin{pmatrix} m_4 w_3 & 0 \\ 0 & m_4^* w_3^* \end{pmatrix} \qquad N_5 = \begin{pmatrix} m_1 + m_2 & -m_1 \\ -m_1^* & m_1^* + m_2^* \end{pmatrix}$$

$$N_6 = \begin{pmatrix} 1 - m_4 w_4 & 0 \\ 0 & 1 - m_4^* w_4^* \end{pmatrix}$$

(The off-diagonal elements in N_3, N_4 and N_6 are neglected as small compared with the respective diagonal elements.) Then the exchange rate is given by the law of one price, equation (25). Home and foreign output are given by (21) and (22), and the home trade balance is given by

$$(39) \qquad x = (h_1 s_1 - h_1^* s_1^* j_1^*/j_1)k - [h_2 - h_2^*]\underset{\sim}{g} + \text{constant}.$$

The target equation follows immediately from these equations.

Using (21), (22)′ and (23), the capital formation equation becomes

$$(40) \qquad \dot{k} = j_1 n_4 k - [j_1 n_2 \quad j_1 n_2^*]\underset{\sim}{g} + \text{constant}$$

where

$$n_4 = n_1 s_1 + n_1^* s_1^* j_1^*/j_1$$

The state equation follows immediately from (40), (32), (33) and (38).

III. The Effects of Macroeconomic Policy

In this section, we analyze the effects of monetary and fiscal policy in the simplified version of the model outlined in the last section. The techniques we use can be described in terms of the state and target equations, (19) and (20).

"Impact" effects are changes in the instantaneous equilibrium brought about by instantaneous changes in the instrument vector. They can be calculated from the target equation: that is,

$$\Delta \mathbf{l}(t) = D_1 \Delta v(t)$$

None of the dynamic aspects of the model are involved here; the variables in the state vector are held constant.

We also wish to analyze the on-going dynamic effects of macroeconomic policy; that is, we want to incorporate the dynamic elements described by the state equation. There are several ways of doing this. The approach we use here is often called a "perturbation" analysis. This approach is not familiar to most economists, but it is worthy of their note since it can be used to analyze dynamic effects in unstable (as well as stable) models.

In a perturbation analysis we first define reference time paths for all of the instruments and all of the exogenous variables. Let $v_1(t)$ and $v_2(t)$ be these reference paths; then the state and output equations determine a reference path $\mathbf{l}(t)$ for the target vector. Now consider an alternate path $\hat{v}_1(t)$ for the instruments, and let this alternate path be defined by

$$\hat{v}_1(t) \equiv \begin{cases} v_1(t) \text{ for } t \text{ not in } J = [\tau, \tau + \epsilon\Delta\tau] \\ v_1(t) + \Delta v_1 \text{ for } t \text{ in } J = [\tau, \tau + \epsilon\Delta\tau] \end{cases}$$

where J is a time interval that can be made arbitrarily small by making ϵ small. Corresponding to the new instrument path, there will be a new path $\hat{\mathbf{l}}(t)$ for the target vector, and we can calculate the deviation

$$\Delta \mathbf{l}(t) \equiv \hat{\mathbf{l}}(t) - \mathbf{l}(t)$$

of the new target vector from its old reference path for all $t > \tau$. $\Delta \mathbf{l}(t)$ describes both the instantaneous and the dynamic effects of the perturbation in the path of the instrument vector.

It turns out that

$$\Delta \mathbf{l}(t) = Ce^{A(t-\tau)}B\epsilon\Delta\tau\Delta v_1 + D_1\delta_J(t)\Delta v_1 + o(\epsilon)$$

where

$$\delta_J(t) \equiv \begin{cases} 1 \text{ if } t \text{ is in } J \\ 0 \text{ if } t \text{ is not in } J \end{cases}$$

$$o(\epsilon)/\epsilon \to 0 \text{ as } \epsilon \to 0$$

and the matrix e^{At} is defined by

$$e^{At} = I + tA + (t^2/2)A^2 + (t^3/3!)A^3 + \cdots$$

Pontryagin (1962, Chapter II) discusses the derivation of this equation, and Aoki (1976: Appendix A) shows how the Cayley-Hamilton theorem can be used to calculate e^{At} in a convenient way. The term $Ce^{A(t-\tau)}B\epsilon\Delta_\tau$ is a dynamic multiplier: it multiplies the perturbation in the instrument vector to give the perturbation in the target vector $t - \tau$ ago. The second term is just the impact effect; it disappears after time $\tau + \epsilon\Delta\tau$.

It is important to understand what is (and what is not) assumed to be "small" here. No restriction is placed upon the size of the perturbation (Δv_1) or on the length of time considered ($t - \tau$). It is the length of the perturbation in the instrument vector ($\epsilon\Delta\tau$) that must be "small."

Impact Effects:

It is not necessary to derive the target equation explicitly. In this simple version of the model, the results are immediately apparent from equations (38), (39), (21), (22) and (25).

An Open Market Operation

Suppose the home government buys back some of its bonds; that is,

$$\Delta m = -(\alpha_2/\alpha_1)\Delta b_g > 0$$

From equation (38), we have

$$\Delta \underset{\sim}{p} = \begin{bmatrix} [1 - m_4 w_2 + (\alpha_1/\alpha_2) m_4 w_3]/(1 - m_4 w_4) \\ 0 \end{bmatrix} \Delta m$$

$$= \begin{bmatrix} 1/(1 - m_4 w_4) \\ 0 \end{bmatrix} \Delta m$$

where the second equality is due to the fact that $\alpha_1/\alpha_2 = w_2/w_3$. From (25), (39), (21) and (22), we have

$$\Delta e = (1 - m_4 w_4)^{-1} \Delta m \quad \text{and} \quad \Delta x = \Delta y = \Delta y^* = 0$$

Since m_4 and w_4 are both less one, we know that $(1 - m_4 w_4)$ is between zero and one. So the home price level (and the exchange rate) rises, and the increase is more than proportional to the increase in the money supply.

The open market operation substitutes money for bonds in private portfolios (leaving total wealth initially unchanged), and it creates an excess supply of money equal to the increase in the money supply. If expectations are static, the result is quite simple. The interest rate is determined in the world output market, so prices must move to equilibrate the home money market. The home price increase would be proportional to the increase in the money supply were it not for the wealth effect. Rising prices lower real wealth and money demand, thus increasing the excess supply of money. The foreign price level, the world output market and the trade balance are not disturbed.[12]

If expectations were adaptive the result would be more complicated. In this case, the rising home price level would create the expectation of a domestic inflation and a depreciation of the home currency. This would shift demand from home assets to foreign assets, creating an excess supply of home money and an excess demand for foreign money. The home price level would rise more and the foreign price level would actually fall in reponse to these expectations effects.[13]

If, however, expectations are very sensitive to current prediction errors (i.e. β is large), then the whole process can invert. The open market operation causes an excess supply of home money, but the home price falls, creating a large expected appreciation of the home currency. This shifts demand from foreign assets to home assets, eliminating the excess supply of home money and causing an excess supply of foreign money and a rise in the foreign price level. In this pathological case, the home open market operation would be deflationary at home, inflationary aboard and the exchange rate would appreciate.[14]

Our results are similar in some respects to those obtained by Mundell (1968) in his small country model. Mundell also concluded that increasing the money supply would be inflationary; however, his reasoning was different from ours. An increase in the money supply was thought to stimulate output and exert downward pressure on the domestic interest rate (as would be explained by the familiar IS-LM curve analysis). But if domestic and foreign bond-equity are perfect substitutes, any fall in the domestic interest rate causes an immediate "capital outflow," increasing the demand for foreign exchange and depreciating the exchange rate. This in turn stimulates export demand and leads ultimately to a new equilibrium with higher output and a depreciated domestic currency. The differences between his reasoning and ours come from the stock-flow distinctions mentioned earlier and from his assumption that home and

[12]Clearly, this classical result depends upon the price inelasticity of the output supply functions.

[13]These results may be verified by substituting (37) into (38) and recalculating Δp for the case of adaptive expectations. If β is sufficiently small, the results described in this paragraph will be obtained.

[14]Using Samuelson's terminology, there appears to be a "correspondence" between stability conditions and impact effects in this model. We have been able to demonstrate in a single country version of the model that stability rules out pathological impact effects similar to those described above. The present version of the model is, however, unstable for all values of β (because of growth in the capital stocks); this will be demonstrated below. An obvious extension of the present work would be to redefine the model in terms of deviations about some growth trend (presumably tied to the capital stocks). We speculate that Samuelson's correspondence would yield interesting conclusions in such a model, but this remains to be seen.

foreign outputs are not identical. His reasoning depends heavily upon terms of trade effects that are not present in our model.

An Increase in Government Spending

Suppose home government spending is increased. From (38), we have

$$\Delta p = \begin{pmatrix} m_2 n_2(1 - m_4 w_4)^{-1} \\ m_2^* n_2(1 - m_4^* w_4^*)^{-1} \end{pmatrix} \Delta g$$

Then (25), (21), (22) and (39) give

$$\Delta e = n_2[m_2(1 - m_4 w_4)^{-1} - m_2^*(1 - m_4^* w_4^*)^{-1}]\Delta g$$

$$\Delta y = \Delta y^* = 0 \quad \text{and} \quad \Delta x = -h_2 \Delta g$$

Increasing home government spending increases demand for world output. The real interest rate must rise to restore equilibrium in this market. This increases demand for bonds and decreases demand for both currencies. Both price levels rise to restore equilibrium in financial markets. If $m_2 > m_2^*$, the rising interest rate lowers home money demand more than foreign money demand. In this case, the home price level rises more than the foreign price level and the home currency depreciates.[15] If $m_2^* > m_2$, home currency appreciates. Notice that increasing foreign government spending produces analogous results. Either increases world demand for output, raises the real interest rate, and increases both price levels. No matter which government's spending is increased, the inflationary effect will be greater in the country with the larger interest elasticity of demand for money (unless wealth effects dominate), and that country's currency will depreciate. If expectations are adaptive the outcome is again more complicated, and a large β can produce pathalogical results.

The analysis is not complete until we explain how the new government spending is financed. Money or bond financing will not alter the results above. The rate of growth of either form of government debt does not affect the instantaneous equilibrium. If, however, the new spending is financed by an increase in taxes, the familiar reasoning behind the "balanced budget" multiplier leads us to the conclusion that the impact effects will be moderated, but qualitatively the same.

In Mundell's (1968) small country model, expansionary fiscal policy simply crowds out the trade balance, and this seems to be the conventional wisdom about fiscal policy in open economies with perfect capital mobility and flexible exchange rates. However, we see that this result is

[15]This discussion assumes that home and foreign wealth effects are roughly equivalent; that is, $1 - m_4 w_4 \sim 1 - w_4^* w_4^*$. A relatively strong foreign wealth effect could then turn the result around.

essentially due to the partial equilibrium assumptions associated with the small country model. When the real interest is free to move, an increase in government spending is indeed inflationary.

Dynamic Effects

In the last section, we discussed the impact or instantaneous effects of an increase in home government spending. We noted that debt financing of the new government spending produced no impact effects; however, it does produce effects that accrue over time. Changing the rate of growth of the money supply or the bond supply will effect financial markets over time, and it is an analysis of these dynamic effects to which we now turn.

Consider the following experiment: First, we choose reference time paths for home government spending and all of the exogenous variables in the model; then the model determines reference time paths for the endogenous variables. Now suppose instead that government spending is Δg above its reference path in the "short" time interval $J = [\tau, \tau + \epsilon\Delta\tau]$; outside the interval J, government spending is as before. How will this perturbation in the time path of g effect the time paths of y, p and x? We can answer this question by calculating the differences between the new paths and the reference paths; that is, we can calculate $\Delta y(t)$, $\Delta p(t)$ and $\Delta x(t)$ for all $t > \tau$.

If deficits are money financed, the government constraint becomes

$$\dot{m} = (\alpha_4/\alpha_1)p + (\alpha_5/\alpha_1)g + \text{constant}.$$

Assuming expectations are fixed, we can use (38) to eliminate p, and recalling the capital formation equation (40), the state equation becomes

$$\begin{pmatrix} \dot{k} \\ \dot{m} \end{pmatrix} = \begin{pmatrix} j_1 n_4 & 0 \\ -a\alpha_4/\alpha_1 & c_1\alpha_4/\alpha_1 \end{pmatrix} \begin{pmatrix} k \\ m \end{pmatrix} + \begin{pmatrix} -n_2 j_1 \\ (\alpha_5 + c_2\alpha_4)/\alpha_1 \end{pmatrix} g + \text{constant}$$

where $a \equiv [m_2(n_1 s_1 + n_1^* s_1^* j_1^*/j_1) + m_3 s_1 + m_4 w_1]/(1 - m_4 w_4) > 0$

$c_1 \equiv (1 - m_4 w_2)/(1 - m_4 w_4) > 0$

$c_2 \equiv m_2 n_2/(1 - m_4 w_4) > 0.$

Note that the eigenvalues ($j_1 n_4$ and $c_1\alpha_4/\alpha_1$) are both positive; this equation is unstable, as may have been expected. From equations (38) and (21), we get the target equation

$$\begin{pmatrix} p \\ y \\ p^* \end{pmatrix} = \begin{pmatrix} -a & c_1 \\ s_1 & 0 \\ -a^* & 0 \end{pmatrix} \begin{pmatrix} k \\ m \end{pmatrix} + \begin{pmatrix} c_2 \\ 0 \\ c_2^* \end{pmatrix} g + \text{constant}$$

$$\text{where}\quad a^* = [m_2^*(n_1 s_1 + n_1^* s_1^* j_1^*/j_1) + m_3^* s_1^* j_1^*/j_1 + m_4^* w_1^*]/(1 - m_4^* w_4^*) > 0$$

$$c_2^* = m_2^* n_2/(1 - m_4^* w_4^*)$$

It is sufficient to consider this three dimensional target vector since the deviations in y^*, e and x follow immediately from those in y and p.

The matrix e^{At} can be shown to be[16]

$$e^{At} = \begin{pmatrix} e^{j_1 n_4 t} & 0 \\ \gamma_1(e^{(a\alpha_4/\alpha_1)t} - e^{j_1 n_4 t}) & e^{(c_1\alpha_4/\alpha_1)t} \end{pmatrix}$$

where $\gamma_1 = (a\alpha_4/\alpha_1)(j_1 n_4 - c_1\alpha_4/\alpha_1)^{-1}$

So the dynamic effects[17]

$$\Delta 1(t) = Ce^{A(t-\tau)}B\epsilon\Delta\tau\Delta z_1$$

become

$$\begin{aligned}\Delta p(t) &= [aj_1 n_2 e^{j_1 n_4(t-\tau)} + [(\alpha_5 + a_4 c_2)c_1/\alpha_1]e^{(t-\tau)c_1\alpha_4/\alpha_1} \\ &\quad + \gamma_1 c_1(e^{j_1 n_4(t-\tau)} - j_1 n_2 e^{(t-\tau)c_1\alpha_4/\alpha_1})]\epsilon\Delta\tau\Delta g \\ \Delta y(t) &= -s_1 j_1 n_2 e^{j_1 n_4(t-\tau)}\epsilon\Delta\tau\Delta g \\ \Delta p^*(t) &= a^* j_1 n_2 e^{j_1 n_4(t-\tau)}\epsilon\Delta\tau\Delta g\end{aligned}$$

Two immediate effects of an increase in government spending are an increase in the real rate of interest and an increase in the deficit. These two impact effects set off two dynamic processes that produce the dynamic effects just calculated. The increase in the interest rate retards capital formation, and this implies smaller output and higher prices, both domestic and foreign. A smaller capital stock also implies a higher real interest rate, so this effect feeds back and perpetuates itself over time. The "$n_2 j_1$" exponential terms above are attributable to this initial interest rate effect. It is important to note that these inflationary effects are due to supply constraints (rather than excessive demand) and that they have nothing to do with how the deficit is financed.

Another immediate effect of an increase in government spending is a higher domestic price level. This, combined with the higher real rate of government spending, implies an immediate increase in the deficit, setting off the second dynamic process. The rate of growth of the money supply increases, and this increases domestic prices.[18] The higher prices imply

[16] Again, the reader is referred to Aoki (1976, Appendix A). When the eigenvalues are known, the e^{At} matrix can be calculated in a relatively straightforward manner.

[17] Here we have omitted the impact term $D_1\Delta z_1\delta_I(t)$ for notational simplicity. This effect is only present in the interval I; it is the impact effect discussed earlier.

[18] These price effects differ from the open market operation discussed in the last section because there is no offsetting decrease in the supply of bonds that keeps private wealth constant.

larger deficits (even after g returns to its reference path), so this effect also feeds back and perpetuates itself. The second exponential term in $\Delta p(t)$ is attributable to this financing effect.

The third term in $\Delta p(t)$ is due to a cross feedback effect. From the state equation, we see that a lower capital stock feeds back into faster growth in the money supply. The lower capital stock produces supply constraints, higher domestic prices and thus larger deficits to be financed. We would expect the term to be positive, and this must eventually be so.[19] From equations (22), (25), and (39), we have

$$\Delta y^*(t) = (s_1^*/s_1)(j_1^*/j_1)\Delta y(t)$$

$$\Delta e(t) = \Delta p(t) - \Delta p^*(t)$$

$$\Delta x(t) = h_1[1 - (h_1^*/h_1)(s_1^*/s_1)(j_1^*/j_1)]\Delta y(t)$$

The most interesting question appears to be what happens to the exchange rate. The first term in $\Delta p(t)$ grows at the same rate as does $\Delta p^*(t)$. The second term adds to the growth in $\Delta p(t)$, but the third term may initially be negative. Thus the exchange rate could conceivably appreciate in the short tun, but it will eventually depreciate.

Bond financing of deficits can be analyzed in an analygeous manner. In this case the state equation is

$$\begin{pmatrix} \dot{k} \\ \dot{b}_g \end{pmatrix} = \begin{pmatrix} j_1 n_4 & 0 \\ -a\alpha_4/\alpha_2 & -d_1\alpha_4/\alpha_2 \end{pmatrix} \begin{pmatrix} k \\ b_g \end{pmatrix} + \begin{pmatrix} -n_2 j_1 \\ (\alpha_5 + d_2\alpha_4)/\alpha_2 \end{pmatrix} g + \text{constant}$$

where $d_1 \equiv m_4 w_3/(1 - m_4 w_4) > 0$ and $d_2 \equiv c_2$. The eigenvalues are $j_1 n_4$ and $-d_1\alpha_4/\alpha_2$, so the state equation is again unstable. In this case, the dynamic effects, $Ce^{A(t-\tau)}B\epsilon\Delta\tau\Delta g$, are

$$\Delta p(t) = [aj_1 n_2 e^{j_1 n_4(t-\tau)} - [(\alpha_5 + \alpha_4 d_2)d_1/\alpha_2]e^{-(t-\tau)d_1\alpha_4/\alpha_2}$$
$$- \gamma_2 d_1(e^{j_1 n_4(t-\tau)} - j_1 n_2 e^{-(t-\tau)d_1\alpha_4/\alpha_2})]\epsilon\Delta\tau\Delta g$$

$$\Delta y(t) = [-s_1 j_1 n_2 e^{j_1 n_4(t-\tau)}]\epsilon\Delta\tau\Delta g$$

$$\gamma_2 \equiv (a\alpha_4/\alpha_2)(j_1 n_4 + d_1\alpha_4/\alpha_2)^{-1} > 0$$

$$\Delta p^*(t) = [a^* j_1 n_2 e^{j_1 n_4(t-\tau)}]\epsilon\Delta\tau\Delta g$$

Again, the "$j_1 n_2$" exponential term is due to the higher real rate of interest and its effect on capital formation. The second term in $\Delta p(t)$ is the financing effect, and the third term is a cross feedback effect from the capital stock to the government flow constraint.

Bond financing of deficits increases the demand for home money

[19] If $\gamma_1 > 0$, then the first exponential term grown more quickly than the second; if $\gamma_1 < 0$, then the second exponential term growl more quickly than the first. In either case, the third term will eventually be positive.

(through the wealth effect), thereby causing lower home prices. The lower prices reduce the deficits that have to be financed, so this effect dies out over time. (The second term in $\Delta p(t)$ approaches zero as $t - \tau$ becomes large.) The cross feedback term in $\Delta p(t)$ has two components. The positive component dies out over time, so the negative component will (eventually) dominate. Thus, a bond financed increase in government spending is definitely inflationary abroad, but its effects on the domestic price level and the exchange rate appear to be indeterminant without numerical estimates of various parameters within the model.

A final word of caution may be appropriate. Throughout this analysis we have assumed that all of the exogenous variables except government spending remain on their reference paths. If the foreign government responds to the home government's change of policy, the analysis will have to be modified.

IV. The Interdependence of Policy Making

Returning to the general state space representation equations (19) and (20) we might ask whether there exists a time path for the instrument vector v_1 that will guide the target vector 1 along any arbitrarily selected target path. If so, the target vector is said to be "perfectly controllable." It can be shown that the target vector is perfectly controllable if the rank of the matrix

$$[D_1 \vdots CB_1]$$

is equal to the dimension of the target output vector 1.[20]

It turns out that the home monetary and fiscal policy instruments are capable of perfectly controlling home output, the home price level, and the exchange rate.[21] More specifically, it can be shown that given any time paths for the exogenous variables there exists a path for the instrument vector $[m\,g]'$ that will guide the target vector $[p\,y\,p^*]'$ along any arbitrarily selected target path. It might be noted that to control both the home price level and the exchange rate, the home government must control the foreign price level, and the home policy instruments are capable of doing just that.

Why is the supply curve, (21), not a price-output trade-off with which the home policy authorities must be content? In the present context, policy making is not confined to the instantaneous equilibrium; the dy-

[20]Aoki (1976) provides a proof, or see Aoki and Canzoneri (1976) for a more detailed discussion.

[21]Form a three dimensional state equation from (31), (32) and (33), letting k, $z \equiv \alpha_1 m + \alpha_2 b_g$ and $z^* \equiv \alpha_1^* m^* + \alpha_2^* b_g^*$ be the state variables and letting g and m be the instrument variables in v_1. Form a target equation with y, p and p^* (or e) as target variables. The rank of D_1 will be seen to be only two, but the rank of $[D_1 \vdots CB_1]$ is three.

namics of capital formation can also be exploited. By controlling the interest rate and the capital stock, the home policy authorities can shift both supply and demand for output over time.

How can two policy instruments control three endogenous variables? Again, they can do this because they are able to exploit dynamic elements within the model. The familiar comparative statics rules about the number of instruments equaling the number of targets do not apply in this dynamic framework.

The real significance of this controllability result is perhaps more subtle than a straightforward reading would seem to indicate. It is quite possible that a set of variables is "perfectly controllable," and at the same time, the policy authority has little chance of actually achieving its goals. This may be seen by noting that the two country model is perfectly symmetric; the foreign monetary and fiscal policy instruments are also capable of controlling the output vector $[p\, y^*\, p]'$. If, for example, the two governments are pursuing different exchange rate targets, at least one of them must be frustrated. It might also be recalled that the perfect substitutability assumptions combine with the classical supply assumptions to imply a log-linear relationship between the two countries' outputs; if the governments pursue independent employment goals, at least one must be frustrated.

How are these observations consistent with the controllability results? Statements about controllability are essentially statements about the *existence* of policies with certain desirable outcomes. Understanding this is fundamental, for it is at the heart of what is implied by controllability and what is not. To actually calculate the monetary and fiscal policies that would force the vector $[y\, p\, e]'$ along some target path, the home government must know the time paths of all of the exogenous variables in the model. If, for example, the home government does not know what the foreign government will do with its monetary and fiscal instruments, then it cannot solve for the appropriate monetary and fiscal policies even though they are known to exist.

We think that the real implication of these controllability results is that the world model is too controllable or too intertwined for autocratic forms of decentralized policy making. Both countries policy authorities are likely to be frustrated.

V. Conclusion

In this paper we presented a dynamic two country policy model, and we used state space techniques to discuss dynamic policy effects and the interdependence of policy making. We analyzed a simplified version of the model since we were only able to place sign restrictions upon parameters in the model. An obvious extension of the present work would be to apply the same techniques to an econometric model.

Three other extensions may also prove interesting. First, the relationship between stability and impact effects may deserve more attention. This extension was discussed in a footnote in section III. Second, the perfect foresight case deserves some attention in a policy evaluation model. State space representations can be useful in this context also; see Aoki and Canzoneri (1977). Finally, the concepts of "controllability" and "decoupling" can be used to identify dynamic policy trade-offs and dynamic instrument assignments, and to design workable decentralized policy regimes; see Aoki and Canzoneri (1976).

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