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Chapter Author: W. A. Jayatissa, R.W. Farebrother

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A PREDICTIVE TEST FOR THE REDUCED FORM MODEL

BY W. A. JAYATISSA AND R. W. FAREBROTHER

In this paper we establish a test of whether two sets of observations come from the same reduced form model.

1. INTRODUCTION

In a recent issue of the *Annals of Economic and Social Measurement* Dhrymes *et al.* [2] extended the single equation stability testing procedure to the reduced form of the linear simultaneous equations model. However, as Jayatissa [4] has pointed out, their result is not correct when there are two or more additional observations. The purpose of the present paper is to obtain a result which is generally valid.

2. THEORY

Consider the reduced form model

$$(1) \quad Y = X\Pi + V$$

where Y is an $N \times G$ matrix of observations on the G endogenous variables of the model, X is an $N \times K$ matrix of observations on the K exogenous variables of the model, Π is a $K \times G$ matrix of unknown parameters and V is an $N \times G$ matrix of disturbances. We assume that X has rank K and that the rows of V are independently and identically normally distributed with zero mean and variance Σ , that is

$$(2) \quad EV_{.i} = 0 \quad EV'_{.i} V_{.j} = \delta_{ij} \Sigma$$

or

$$(3) \quad EV_{.i} = 0 \quad EV_{.i} V'_{.j} = \sigma_{ij} I_N$$

where δ_{ij} is the Kronecker delta.

Let Z be an $N \times (N - K)$ matrix satisfying

$$(4) \quad Z'X = 0 \quad \text{and} \quad Z'Z = I_{N-K}$$

and let $U = Z'Y = Z'V$, then

$$(5) \quad EU_{.i} = 0 \quad EU_{.i} U'_{.j} = \sigma_{ij} I_{N-K}$$

whence

$$(6) \quad EU_{.i} = 0 \quad EU'_{.i} U_{.j} = \delta_{ij} \Sigma$$

that is, the rows of U are independently and identically normally distributed with zero mean and variance Σ .

Let A be an $(N - n) \times m$ matrix satisfying $A'A = I_m$ where $n < N$ and $m \leq N - n$, let

$$(7) \quad w' = [U_1, U_2, \dots, U_{n-K}]$$

and let

$$(8) \quad u' = [U_{n-K+1}, \dots, U_{N-K}](A \otimes I_G)$$

Then

$$(9) \quad u \sim N(0, I_m \otimes \Sigma)$$

and

$$(10) \quad u'(I_m \otimes \Sigma^{-1})u \sim \chi^2(mG)$$

In practice Σ is not known and this statistic is not operational. However we may obtain an estimate of $I_m \otimes \Sigma$ from

$$(11) \quad w \sim N(0, I_{n-K} \otimes \Sigma)$$

Let r denote the largest integer less than, or equal to, $(n - K)/m$. Partition w into r groups of mG elements and let w_j denote the j th group. Then we have for $j = 1, 2, \dots, r$

$$(12) \quad Ew_j = 0 \quad Ew_j w_j' = I_m \otimes \Sigma$$

Applying the result of Anderson [1, p. 106] we have

$$(13) \quad \frac{u' \Lambda^{-1} u}{r}, \frac{r - mG + 1}{mG} \sim F(mG, r - mG + 1)$$

provided that $r \geq mG$, where

$$(14) \quad \Lambda = \frac{1}{r} \sum_{j=1}^r w_j w_j'$$

3. APPLICATION

Let Y , X and V be partitioned by their first n rows and the remaining $N - n$ rows

$$(15) \quad Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}, \quad X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where X_1 has full column rank, K . And let Z be partitioned as

$$(16) \quad Z = \begin{bmatrix} Z_{11} & Z_{12} \\ 0 & Z_{22} \end{bmatrix}$$

where Z_{11} , Z_{12} and Z_{22} are $n \times (n - K)$, $n \times (N - n)$ and $(N - n) \times (N - n)$ matrices respectively. Then

$$(17) \quad U' = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} Z'_{11} Y_1 \\ Z'_{12} Y_1 + Z'_{22} Y_2 \end{bmatrix}$$

where Z_{11} is defined by

$$(18) \quad Z'_{11} X_1 = 0 \quad \text{and} \quad Z'_{11} Z_{11} = I_{n-K}$$

Thus U_1 , w and Λ depend only on X_1 and Y_1 and (13) may be used as a test of whether the observations X_2 and Y_2 were generated by the same model as generated X_1 and Y_1 . Large values of the statistic leading to rejection of the null hypothesis. Ideally we would like to choose $A = I_{N-n}$ but for this we require $n - K \geq G(N - n)^2$.

The remaining problem of obtaining a matrix Z satisfying (16) is most easily resolved by applying Givens' method to the matrix $[X' Y]$. This procedure yields the matrix $U' = Z'Y$ each of whose columns is a set of "recursive residuals" (see [3] and [5] for details).

4. RELATION

We now indicate the relation between our test and that of Dhrymes *et al.* [2, eq. (12)] when $m = N - n$ and $A = I_m$. Let us rearrange the elements of

$$(19) \quad u' = [U_1^{(2)} U_2^{(2)} \dots U_m^{(2)}]$$

as

$$(20) \quad u'_* = \text{coi} \{U_1^{(2)} U_2^{(2)} \dots U_G^{(2)}\}$$

then

$$(21) \quad u'_*(\Sigma^{-1} \otimes I_m)u'_* \sim \chi^2(MG)$$

since the left side of (21) is the same as the left side of (10). Let $M = I_N - X(X'X)^{-1}X'$, then we have from (4) that $M = ZZ'$, whence

$$(22) \quad \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ 0 & Z_{22} \end{bmatrix} \begin{bmatrix} Z'_{11} & 0 \\ Z'_{12} & Z'_{22} \end{bmatrix}$$

and

$$(23) \quad U^{(2)} = Z'_2 Y = Z_{22}^{-1} M_2 Y = Z_{22}^{-1} P$$

where M is partitioned conformably with ZZ' and $P = M_{22}^{-1} M_2 Y = Y_2 - X_2(X'_1 X_1)^{-1} X'_1 Y_1$. From (23) we have

$$(24) \quad u_* = (I_G \otimes Z_{22}^{-1})e$$

where

$$(25) \quad e = \text{col}\{P_{.1} P_{.2} \dots P_{.G}\}$$

Thus equation (21) may be rewritten

$$(26) \quad e'(\Sigma^{-1} \otimes M_{22})e \sim \chi^2(mG)$$

where

$$(27) \quad M_{22} = Z_{22} Z'_{22} = I_m - X_2(X'_1 X_1)^{-1} X'_2$$

and

$$(28) \quad M_{22}^{-1} = I_m + X_2(X'_1 X_1)^{-1} X'_2$$

Finally if $m = 1$ we have from (14)

$$(29) \quad \begin{aligned} (n - K)\Lambda &= \sum_{j=1}^{n-K} U'_j U_j \\ &= U'_1 U_1 \\ &= Y'_1 Z_{11} Z'_{11} Y_1 \\ &= Y'_1 [I_n - X_1(X'_1 X_1)^{-1} X'_1] Y_1 \end{aligned}$$

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University of Manchester

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