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THE ESTIMATION OF DYNAMIC ECONOMIC RELATIONS FROM A TIME SERIES OF CROSS SECTIONS: A PROGRAMMING MODIFICATION*

BY NEIL W. HENRY, JOHN F. MCDONALD AND HOUSTON H. STOKES

In a recent issue of this journal, Alan Freiden (1) has presented a computer program for the estimation of dynamic economic relations from a time series of cross sections. Freiden employs the error components model developed by Nerlove (2) which is described briefly below. Although Freiden's computer program has wide applicability, his computing procedure will fail to compute the estimates of the model in an important special case: the Freiden program fails if any independent variable is constant over time for all members of the cross section. This problem prevents the use of Freiden's program in many situations, including cases in which individuals are classified with a dummy variable as belonging to one of two mutually exclusive groups (e.g., sex and control or experimental status). The purposes of this note are to demonstrate the existence of the problem and to describe the modifications in the Freiden program which were done to circumvent the problem.

The Nerlove error components model is shown in equation 1 where it is assumed that data exists for

$$y_{it} = \sum_k \beta_k x_{kit} + u_{it} \qquad \begin{matrix}(i = 1 \ldots N)\\(t = 1 \ldots T)\end{matrix} \tag{1}$$

the N individuals for T time periods. The total number of observations pooled over individuals and time periods is $M = N \cdot T$. As shown in equation 2, the total error term for individual i at time t (u_{it}) is assumed to consist of an individual effect which is invariant over time (μ_i) and an effect which varies over time and individuals (ν_{it}).

$$u_{it} = \mu_i + \nu_{it} \tag{2}$$

The intra-class correlation coefficient (ρ) is shown in equation 3, where σ_u^2 is the variance of the error term u_{it}, and σ_μ^2 and σ_ν^2 are the variances of μ_i and ν_{it}, respectively. Given an estimate of ρ, Nerlove (2)

$$\rho = \sigma_\mu^2/(\sigma_\mu^2 + \sigma_\nu^2) = \sigma_\mu^2/\sigma_u^2 \tag{3}$$

shows that the model may be estimated by least squares after the dependent and independent variables are transformed by functions of ρ.

The difficulty with the Freiden procedure is in the estimation of σ_μ^2. Estimates of the β_k's in equation 1 are needed to compute the estimate of σ_μ^2. The Freiden procedure is to compute the $\hat{\beta}_k$'s in a regression of deviations of the dependent

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and independent variables from individual means. However, such a regression is impossible to estimate if one of the independent variables has no variance, a situation which will arise if, for all individuals, one of the independent variables is the same in every time period. Deviations around the mean vanish in this case.

The modification of the program which circumvents this problem involves computing an ordinary least squares regression using the pooled data for the N individuals and T time periods. The estimate of σ_u^2, the total variance, is computed from the residuals of this regression. Next, the estimate of σ_μ^2 is computed by finding the mean residual for each of the N individuals and computing the variance of these mean residuals, weighting each mean residual by T. Equation 4 shows the latter computation.[1]

(4) $$M\hat{\sigma}_\mu^2 = \sum_i T(\bar{y}_i - \bar{x}_i'\hat{\beta})^2$$

where

(5) $$\bar{x}_i'\hat{\beta} = \begin{bmatrix} \bar{x}_{11} & \bar{x}_{12} \ldots & \bar{x}_{1K} \\ \bar{x}_{21} & \ldots & \bar{x}_{2K} \\ \vdots & & \vdots \\ \bar{x}_{N1} & \ldots & \bar{x}_{NK} \end{bmatrix} \begin{bmatrix} \hat{\beta}_1 \\ \vdots \\ \hat{\beta}_K \end{bmatrix}$$

and

$$\bar{y}_i = \begin{bmatrix} \bar{y}_1 \\ \vdots \\ \bar{y}_N \end{bmatrix}$$

The above suggested modification to the Freiden program can be easily made since the individual means for the dependent and independent variables are already stored by the program.

The estimation of ρ via our suggested method substantially simplifies the estimation of the Nerlove model since only two passes are required in comparison to the three pass method originally suggested by Freiden. The suggested procedure for calculating ρ is comparable to the technique used by Nerlove (3) which involved including a dummy variable for each individual in an OLS regression and computing the variance of the estimated N dummies, but is computationally much simpler.

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[1] It is assumed that β_K is the intercept of the OLS regression, so that $\bar{x}_{iK} = 1$ for $i = 1, N$.

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