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THE USE OF INPUT–OUTPUT ANALYSIS IN AN ECONOMETRIC MODEL OF THE MEXICAN ECONOMY*

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The purpose of this paper is to integrate an input–output matrix in a national income determination macroeconometric model. The resulting composite model is used for technological change simulating policies of the Mexican economy. Simulation multipliers are computed and compared for three sectors: agriculture, basic metal industries and transportation. One of the interesting results is that the agricultural (row) multipliers are the highest ones leading to the conclusion that development efforts should give more attention to agriculture.

INTRODUCTION

The present study deals with the linkage of an input–output table to a model of the Mexican economy. Input-output analysis adds a new dimension to models of economic systems since it focuses on the interrelations and flows that occur among sectors of the economy. This, to some extent, is obscured by the use of the national accounts system as a basis for model building.

Following the original ideas developed in the Brookings Econometric Model[1] and the work of R. Preston,[2] we are going to link the 1960 input–output table to a revised version of the DIEMEX-WEFA Forecasting Model of the Mexican Economy.[3] The Input–Output model will be fully integrated into the macro model, so that to solve one model the other will be needed and vice versa. Once this is achieved the complete model will be used to simulate policy measures.

However, in this study attention will be focused on policies involving changes in technology as this is represented in the Input–Output model.

METHODOLOGY AND PROBLEMS OF THE LINKAGE OF INPUT–OUTPUT ANALYSIS TO A MACROECONOMETRIC MODEL

In Figure 1 we have a chart showing the relationships that exist between interindustry accounting and national income accounting from both the expenditure side and the income side.

Before going into the details of how the linkage can be done, let us make explicit the identities involved in Figure 1.

[1] F. M. Fisher, L. R. Klein and Y. Shinkai, "Price and Output Aggregation in the Brookings Econometric Model," *The Brookings Quarterly Econometric Model of the United States*, eds., J. S. Duesenberry *et al.* (Chicago: Rand McNally and Co., 1965).

[2] R. S. Preston, *The Wharton Annual and Industry Forecasting Model*, (Philadelphia, Economic Research Unit, Wharton School, U. of Pa., 1972). Studies in Quantitative Economics, No. 7, pp. 14–20.

[3] See A. Beltran del Rio, *A Macroeconometric Forecasting Model for Mexico: Specification and Simulations*, Unpublished Ph.D. Dissertation, U. of Pa., 1973. Also see R. Montemayor, *An Econometric Model of the Financial Sector: The Case of Mexico*, Unpublished Ph.D. Dissertation, U. of Pa., 1974.

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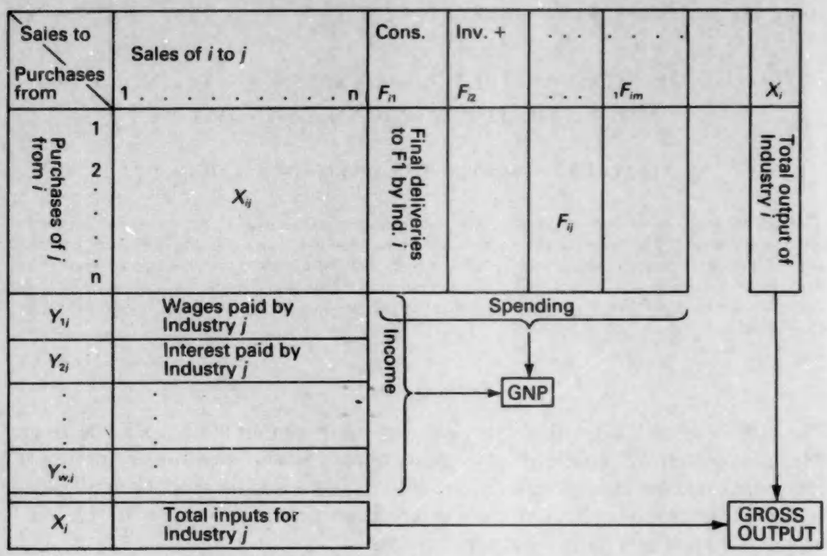

Figure 1 Relationship between interindustry transactions, final demand and factor payments

Looking at the table across rows, the following holds:

$$(1) \qquad \sum_{j=1}^{n} X_{ij} + \sum_{k=1}^{m} F_{ik} = X_i \qquad i = 1, \ldots n$$

That is, the sum of industry i's sales to each of n industries (including itself) (X_{ij}'s) plus the sum of its deliveries to each category of final demand (F_{ik}), consumption, capital formation, etc., will be equal to the gross output of industry i.

If we look, instead, at columns, we have the following:

$$(2) \qquad \sum_{i=1}^{n} X_{ij} + \sum_{k=1}^{W} Y_{kj} = X_j \qquad j = 1, \ldots n.$$

That is, the sum of industry j's purchases from all n industries plus the sum of payments to the w factors of production will equal the total inputs used by industry j (X_j's).

Also, we see that the row totals equal column totals:

$$(3) \qquad \sum_{i=1}^{n} X_i = \sum_{j=1}^{n} X_j = \text{gross output.}$$

Looking at the deliveries made by each industry to each of the m final demands we have:

$$(4) \qquad \sum_{i=1}^{n} F_{ij} = G_j = \text{total final demand } j, \qquad j = 1, \ldots m.$$

And looking at factor payments,

(5) $$\sum_{j=1}^{n} Y_{ij} = Y_i = \text{total payment to factor } i, \qquad i = 1, \ldots w.$$

Also, we know that total spending = total income.

(6) $$\sum_{j=1}^{m} G_j = \sum_{i=1}^{w} Y_i = \text{GNP}.$$

For each industry we have the following identities:

(7) $$\sum_{k=1}^{m} F_{ik} = F_i = \text{total deliveries by industry } i \text{ to final demands} \qquad i = 1, \ldots n$$

(8) $$\sum_{i=1}^{w} Y_{ij} = Y_j = \text{total payments to factors of production or value added by industry } j, \qquad j = 1, \ldots n.$$

If we substitute (7) in (1) for each industry or sector we have:

(9) $$\sum_{j=1}^{n} X_{ij} + F_i = X_i \qquad i = 1, \ldots n.$$

Let us assume that:

(10) $$\frac{X_{ij}}{X_j} = aij.$$

That is, that the output of industry j is proportional to its inputs from industry i. Then, (9) can be re-written as:

(11) $$\sum_{j=1}^{n} aij^{X}j + F_i = X_i \qquad i = 1, \ldots n.$$

Or in matrix form as:

$$\begin{bmatrix} a_{11} \ldots a_{1j} \ldots a_{1n} \\ \vdots \\ a_{n1} \ldots a_{nj} \ldots a_{nn} \end{bmatrix} \begin{bmatrix} X_i \\ \vdots \\ X_j \\ \vdots \\ X_n \end{bmatrix} + \begin{bmatrix} F_i \\ \vdots \\ F_j \\ \vdots \\ F_n \end{bmatrix} = \begin{bmatrix} X_i \\ \vdots \\ X_j \\ \vdots \\ X_n \end{bmatrix}$$

Or compactly as:

(12) $$AX + F = X \text{ or } X = (I - A)^{-1}F.$$

If we know F, we can predict what the gross output vector should be in order to support the given final demand deliveries in vector F.

One of the first aspects that has to be faced in the linking of demand and production using an interindustry flow model is the level of aggregation of both demand and production[4] so as to have a proper transmission from one side to the

[4] See R. Preston, *op. cit.*, p. 14.

other. In our case we have a forty-five sector breakdown of industrial production but only six final demand categories: thus we had to shrink the table to fifteen sectors to use it with the six-way breakdown of final demand.

The fifteen-sector breakdown of the economy is as follows:

Sector 1: Agriculture, livestock, fishing, and forestry.
Sector 2: Mining.
Sector 3: Crude oil and refinery.
Sector 4: Food, beverages, and tobacco.
Sector 5: Textiles and apparel.
Sector 6: Wood products, furniture, and editorial.
Sector 7: Chemicals, rubber, and plastics.
Sector 8: Nonferrous mineral products fabrication.
Sector 9: Basic metal industries.
Sector 10: Fabricated metal products and repairs.
Sector 11: Construction.
Sector 12: Electricity.
Sector 13: Commerce.
Sector 14: Transporation.
Sector 15: Services.

And the six-way classification of final demand is:

(1) Private consumption.
(2) Public consumption.
(3) Tourist consumption.
(4) Exports.
(5) Total fixed investment.
(6) Inventory change.

Let us look now into the details of the linkage of I–O accounting to a macroeconometric model.

There are three kinds of problems that arise when we attempt to go from national income accounting (which is the basis for macroeconometric models) to interindustry transactions accounting. First, we do not have time series observations on final demand deliveries by each industry: what we have are the GNP final demand components, i.e., consumption, capital formation, etc.

However, for the year for which a direct requirement matrix is available we do have this information. By making the assumptions of proportionality and constancy made before, we can transform GNP components into final demand deliveries by industry.

(13) $$\frac{F_{ij}}{G_j} = h_{ij} \qquad \begin{matrix} i = 1, \dots n \\ j = 1, \dots m. \end{matrix}$$

That is, the amount of output that industry i sells to final demand category j, F_{ij}, is a constant proportion (h_{ij}) of final demand j (G_j).

More compactly:

(14) $$F = HG$$

where

$F = n \times 1$ vector of final demand deliveries by industries.

$H = n \times m$ matrix of industrial or sectoral distribution of final demand categories coefficients. (This is obtained for the year that an I–O table is constructed.)

$G = m \times 1$ vector of final demand by spending categories.

The second problem we face is the following: In national accounts we deal with value added concepts (Y's), whereas in input–output accounting we deal with gross output concepts. The difference between the two are intermediate transactions.

We can easily establish a relation between the two concepts. Value added = gross output − purchases of intermediate goods. Using our symbols:

(15)
$$Y_j = X_j - a_{1j}X_j - a_{2j}X_j - \ldots, - a_{nj}X_j.$$

Or

(16)
$$Y_j = \left(1 - \sum_{i=1}^{n} a_{ij}\right) X_j \qquad j = 1, \ldots n.$$

Let

$$1 - \sum_{i=1}^{n} a_{ij} = b_{jj}.$$

Then (16) can be expressed in matrix form as

(17)
$$Y = BX.$$

Where B is a diagonal matrix with its diagonal elements b_{jj} defined as before and off diagonal elements equal to zero.

Substituting (12) and (14) into (17) we have:

(18)
$$Y = B(I - A)^{-1}HG.$$

Let $B(I - A)^{-1}H = C$. This is a matrix with as many rows as sectors in the direct requirement (a) matrix and as many columns as GNP spending component categories.

In our case we have 15 sectors and 6 final spending categories. Writing out (18) we have:

(19)
$$\begin{aligned} Y_1 &= c_{11}G_1 + c_{12}G_2 + \ldots + c_{1m}G_m \\ &\vdots \\ Y_n &= c_{n1}G_1 + c_{n2}G_2 + \ldots + c_{nm}G_m. \end{aligned}$$

However, the C matrix, given the properties of the A and H matrices, has the property that $\sum_{i=1}^{n} C_{ij} = 1$.[5]

This property implies that:

(20)
$$\sum_{i=1}^{n} Y_i = \sum_{i=1}^{m} G_i = \text{GNP}.$$

[5] See R. S. Preston, *op. cit.*, pp. 16–18.

Thus, given the technical coefficients matrix A and the matrix of industrial distribution of final demands H, we can establish a link between final demand spending categories (G's) and output originating or value added by sector (Y's)—a link that takes into account the structure of industrial interdependence in the economy.

A third problem, however, remains. We have implicitly assumed that the A and H matrices are constant over time. Yet, this is not so in actual life. Technology and tastes change. It is only reasonable to expect the a_{ij} and h_{ij} to change over time even though such changes may be slow and gradual. This, coupled with the fact that such matrices are constructed only every 10 years or so, if at all, will cause our projections made from given A and H matrices to have an error element attached to them.

There are several ways in which we could handle the problem. One could be to try to model each of the elements of the C matrix. However, this is not practical given the present availability of data, especially data referring to interindustry transactions.

A second and more practical way is as follows: Using time series data for the G vector and given the C matrix, a series of $\hat{Y}$ vectors can be estimated from (18), that is:

$$\hat{Y}_{t+i} = CG_{t+i} \qquad i = 1, \ldots T. \tag{21}$$

These $\hat{Y}_{t+i}$ can then be compared with the actual y vectors for the same period, and a series of residual vectors can be constructed.

$$Y_{t+i} - \hat{Y}_{t+i} = U_{t+i} \qquad i = 1, \ldots T \tag{22}$$

The factors that make for changes in C are the same that give rise to the observed errors U_{t+i}; thus, we can attempt to model these errors. There are n equations to be estimated, which is considerably less than the number of equations we would have had we tried to model each of the elements of the C matrix. In this case our projections will be made according to the following formula:

$$\hat{Y}_F = \hat{Y}_F + U_F. \tag{23}$$

Where Y_F will be based on the projections of the G vector that come from the macro model according to (21); U_F will be the projections of the errors based on the model that is developed for the errors observed in the past.

How can we model the vector U? There are different ways in which this can be done. One way, following the approach of Fisher, Klein, and Shinkai,[6] is to use autoregressive models. R. Preston[7] uses two such models:

1. $U_{it} = f(U_{it-1}, t) + e_{it} \qquad i = 1, \ldots n$
2. $U_{it} = f(U_{it-1}, U_{it-2}, t) + e_{it} \qquad i = 1, \ldots n.$

However, if we want to preserve identity (20) the same regressor must be used for each error equation. Otherwise, a method to allocate final discrepancy must be used.

[6] F. M. Fisher, L. R. Klein, and Y. Shinkai, *op. cit.*

[7] R. S. Preston, *op. cit.*, 19–20.

In the case of Mexico a slightly different model was used. After obtaining the observed errors for the period 1951–1971, principal components were obtained for the 15 sectors' errors. It was observed that the first three principal components accounted for 94.1 percent of the overall variance of the original series; the fourth added only a marginal increment (2.1 percent). The following model was then used:

$$U_{it} = f(\mathrm{PC1}_{t-1}, \mathrm{PC2}_{t-1}, \mathrm{PC3}_{t-1}) + e_{it} \qquad i = 1, \ldots n. \tag{24}$$

Where

$\mathrm{PC1}_{t-1}$ = first principal component lagged one period.

$\mathrm{PC2}_{t-1}$ = second principal component lagged one period.

$\mathrm{PC3}_{t-1}$ = third principal component lagged one period.

The merits of this approach are twofold. On the one hand, the identity of total value added and total final demand (20) is preserved, with the same regressors (the first three components) used for each of the 15 error equations; on the other hand, on the assumption that the principal components are linear combinations of the errors, the use of one period lag greatly facilitates the extrapolation of the principal components and errors into the future.

Thus, the value added equations to be used in the model will be of the following form:

$$\begin{aligned} Y_{it} = {} & C_{i1}G_{1t} + C_{i2}G_{2t} + C_{i3}G_{3t} + C_{i4}G_{4t} + C_{i5}G_5 + C_{i6}G_{6t} \\ & + L_{0i} + L_{1i}\mathrm{PC1}_{t-1} + L_{2i}\mathrm{PC2}_{t-1} + L_{3i}\mathrm{PC3}_{t-1} \qquad i = 1, \ldots 15. \end{aligned} \tag{25}$$

Where

Y_{it} = value added in sector i.

C_{ij} = the ij element of the C Matrix.

G_{it} = the i final demand category at time t.

L_{ki} = regression coefficients of (24).

The equations in (25) will replace the equations for value added in the macro model. However, the model uses only 3 sectors: primary or agricultural sector, industrial sector, and tertiary sector. Therefore, the fifteen value-added equations have to be aggregated into three.

The macro model also has a final demand block.[8] Here there is an equation for each final demand category. Most of them are related directly or indirectly to output—private investment is a modified flexible accelerator equation. Thus, we have come a full circle: output is explained, through the C Matrix, in terms of demand; value added, then feedbacks to labor force requirements, wages, and prices, and demand itself. We need the macro model (G values) to solve the I–O model (Y's) and vice versa. In the solution algorithm we will obtain simultaneously values for G and Y.

[8] For a description of the structure of the model see R. Montemayor, *op. cit.*, Chapter IV. See also A. Beltran del Rio, *op. cit.*, Chapter V.

Below we give the results obtained for the error or residual equations.

TABLE 1
ERRORS EQUATIONS*

	Constant	PC1 − 1	PC2 − 1	PC3 − 1	R^2	DW
ERR1	0.369	3.969	1.636	2.682	0.95	2.94
	(1.085)	(11.828)	(1.622)	(2.509)		
ERR2	−0.056	0.459	−0.078	0.192	0.93	1.80
	(−1.063)	(8.825)	(−0.495)	(1.159)		
ERR3	0.499	−1.178	0.178	−0.204	0.96	2.33
	(4.995)	(−11.936)	(0.599)	(−0.648)		
ERR4	0.288	−0.227	0.975	−0.277	0.74	2.72
	(2.350)	(−1.841)	(2.683)	(−0.718)		
ERR5	−0.045	−0.358	−0.019	0.015	0.89	2.61
	(−0.366)	(−7.069)	(−0.051)	(0.038)		
ERR6	0.192	0.029	0.464	0.161	0.70	1.44
	(1.983)	(1.805)	(1.617)	(0.528)		
ERR7	−0.007	−1.512	−0.063	−0.430	0.98	2.37
	(−0.077)	(17.508)	(−0.244)	(−1.564)		
ERR8	−0.006	−0.166	−0.050	0.024	0.69	2.97
	(−0.141)	(−4.157)	(−0.417)	(0.185)		
ERR9	0.143	−0.225	0.184	0.179	0.86	1.40
	(2.981)	(−4.740)	(1.287)	(1.182)		
ERR10	0.543	−2.063	0.836	0.702	0.93	1.62
	(2.198)	(−8.462)	(1.141)	(1.904)		
ERR11	0.341	0.877	0.216	0.492	0.65	1.98
	(1.457)	(3.803)	(1.313)	(0.670)		
ERR12	0.083	−1.108	−0.203	−0.279	0.98	2.01
	(1.342)	(−18.072)	(−1.105)	(−1.431)		
ERR13	1.790	−2.712	2.540	1.227	0.90	1.33
	(3.557)	(−5.463)	(1.703)	(0.776)		
ERR14	0.058	−0.209	−0.381	−0.093	0.75	2.14
	(1.220)	(−5.656)	(−2.662)	(−0.610)		
ERR15	10.484	−3.568	0.439	1.010	0.97	1.31
	(39.188)	(−13.522)	(0.553)	(1.203)		

* Values in ( ) are t values.

Policy Simulations

Policy implications in econometric models are often studied by incorporating certain policy variables in the equations of the system so that by varying them in a specific way the reaction of the whole system can be observed. Two basic simulations are required: a baseline or control solution and a disturbed solution that is essentially the same control solution plus the change in the policy variable whose effects we are interested in knowing.

Many other kinds of simulation studies could be performed.[9]

We are concerned with the dynamic response of the system to different policy actions. A way to look at these responses is to compute a set of dynamic multipliers

[9] See L. R. Klein, "An Essay on the Theory of Economic Prediction," Markham Publishing Company, Chicago, 1971. Also see G. Fromm and P. Taubman, "Policy Simulations with an Econometric Model," The Brookings Institution, Washington, D.C., 1968, pp. 23–51.

for each policy under consideration. Generally these multipliers are computed as follows:

$$M = \frac{Y_t^d - Y_t^c}{\Delta PV_t}$$

Where

$$Y_t^d = \text{disturbed value of } Y.$$

$$Y_t^c = \text{baseline or control value of } Y.$$

$$\Delta PV_t = \text{change in the policy variable.}$$

That is, the change in the endogenous variables (Y) divided by the amount of the change in the policy variable being considered will give an estimate of the multiplier for the Y variable. However, given the nonlinear nature of the model and the presence of lags, we have to allow for some period to pass so that most of the lag influences have had their major effects in order to get an idea of the "equilibrium" or "long-run" values of the multipliers. However, since we are dealing with a complex, dynamic system of difference equations that is likely to have roots producing fluctuating responses to changes in its driving forces (exogenous), the time path of the multipliers is likely to fluctuate. Nonetheless, these exercises can be helpful in assessing the relative effects of alternative policy actions.

In principle, we could change any one of the predetermined variables of the system and calculate multipliers showing its effects on the system. However, we will limit our inquiry to policies affecting the production functions of the economy as they are represented by the I–O table. That is, what will happen if government spending is changed by, say, 1 billion real pesos and the increased spending is directed toward increasing the efficiency of a specific sector of the economy? Not only are the global amounts changing but those changes are bringing about a change in the productive structure of the economy.

Questions of this kind can be at least partially answered through the use of the I–O model.

Essentially, there are two ways we can approach this problem. On the one hand, it can be assumed that as a result of the policy change a given sector has become more efficient (perhaps through the import of better equipment), so that per unit of its own output, less output of the other sectors is required. Alternatively, it can be said that now that same sector is able to deliver inputs of better quality to the other sectors so that, per unit of output, they need less input from that sector.

The first case amounts to a change on the production function of the sector in question. That is, that sector's column in the direct requirement matrix has changed.

In terms of technical coefficients the change means a reduction in the column coefficients that pertain to that sector.

The second case is equivalent to a change in the row coefficients of the "A" matrix, the row being that of the sector delivering the better inputs as a result of the policy. This case can be also viewed as a change in the production functions of all the sectors, a change that pertains only to their use of a specific sector's output.

The Mexican I–O model is composed of 15 sectors.[10] Three of them were chosen to make this type of calculation. One sector from each of the three major

[10] A list of the fifteen sectors can be found on page 534.

sectors of the economy—primary, secondary, and tertiary—was selected.

The sectors chosen were:

Sector 1: Agriculture (primary sector).
Sector 9: Basic Metal Industries (secondary sector).
Sector 14: Transportation (tertiary sector).

They were chosen because of their relevance in developing economies. Basic metal industries which include heavy industry like steel are usually thought of as being a key sector for development. It is common to find unusual efforts on the part of government in developing economies to promote heavy industries, sometimes to the detriment of agriculture.

For each of these sectors two simulations were done. In one it was assumed that the increase in government investment was coupled with a 10 percent reduction in the column coefficient of the sector. The second supposes the same change in government investment, but now the efficiency increase is reflected in the row coefficients. Again a 10 percent reduction in the row coefficients was assumed.

In all six cases, the changes are made in the "A" matrix. This implies that the output conversion matrix "C" has to be recalculated.

Let us recall that:

$$C = B(I - A)^{-1}H.$$

We assume that the industrial distribution of final demand, the H matrix, remains constant. The other two matrices—B and $(I - A)^{-1}$—are changed as a result of the change in A. Once the new C matrix is recalculated the model is simulated for a six-year period, starting in 1961.

A control solution in this case was obtained using the historical values of all exogenous variables. The disturbed solution embodies two changes: the increase of one billion in real government investment and the change in coefficients of the A matrix. Also, a simulation was done to calculate the effects of a change in government spending alone. This we will call Policy 1 (See Table 1).

Tables 2 through 7 show the effects of each case on the fifteen sectoral outputs, the condensed three major sectors and the total (GDPR).

The first point to be observed is that a column change in the sector's production function brings about a substantial increase in the output of that sector. To better appreciate this change, let us compare these results with the ones obtained for an increase in government investment alone (Policy 1). Thus we note that the column change in agriculture coefficients has a multiplier in that sector output that is about 0.32 billion 1950 pesos at the end of five periods, greater than the multiplier of Policy 1 (See Table 1). This difference is almost the same as the Policy 1 multiplier itself, for that sector (0.38).

If the basic metal industries production function is changed, the multiplier for that sector at the end of five periods is more than double the Policy 1 multiplier for the same sector.

For the transport sector column change the multiplier is 0.26 billion 1950 pesos, compared with a multiplier of 0.10 for Policy 1.

In each of the column changes we note that a sector becomes more efficient in the use of other sectors' outputs. Thus, if we compare their effects (multipliers)

TABLE 2

SECTORAL OUTPUT MULTIPLIERS: POLICY 1*

(1950 BILLION PESOS)

	Period					
	1	2	3	4	5	6
Sector 1	0.11	0.21	0.28	0.33	0.38	0.29
Sector 2	0.01	0.00	0.00	0.00	−0.01	−0.04
Sector 3	0.04	0.07	0.09	0.10	0.12	0.09
Sector 4	0.03	0.06	0.09	0.11	0.14	0.11
Sector 5	0.03	0.05	0.07	0.08	0.10	0.08
Sector 6	0.03	0.05	0.06	0.06	0.07	0.06
Sector 7	0.03	0.05	0.06	0.07	0.08	0.06
Sector 8	0.03	0.05	0.06	0.06	0.07	0.05
Sector 9	0.04	0.07	0.08	0.09	0.10	0.07
Sector 10	0.08	0.13	0.16	0.17	0.19	0.15
Sector 11	0.22	0.32	0.40	0.43	0.49	0.38
Sector 12	0.01	0.02	0.03	0.04	0.04	0.03
Sector 13	0.32	0.56	0.77	0.91	1.09	0.93
Sector 14	0.02	0.05	0.07	0.08	0.10	0.09
Sector 15	0.09	0.16	0.24	0.30	0.36	0.30

* One billion increase in real government investment with no change in public financing pattern.

TABLE 3

MULTIPLIERS: ONE BILLION INCREASE IN REAL GOVERNMENT INVESTMENT AND 10 PERCENT EFFICIENCY INCREASE IN AGRICULTURE

(COLUMN CHANGES IN "A" MATRIX)

	Period					
	1	2	3	4	5	6
Sector 1	0.37	0.48	0.57	0.63	0.70	0.66
Sector 2	0.01	0.00	0.00	0.00	−0.01	−0.04
Sector 3	0.00	0.03	0.05	0.06	0.07	0.05
Sector 4	−0.01	0.02	0.04	0.06	0.08	0.05
Sector 5	0.01	0.03	0.05	0.06	0.08	0.05
Sector 6	0.02	0.04	0.05	0.05	0.06	0.05
Sector 7	0.00	0.02	0.03	0.02	0.03	0.02
Sector 8	0.03	0.05	0.06	0.06	0.07	0.05
Sector 9	0.04	0.06	0.07	0.08	0.09	0.06
Sector 10	0.07	0.12	0.15	0.15	0.17	0.13
Sector 11	0.21	0.31	0.39	0.41	0.47	0.36
Sector 12	0.01	0.01	0.02	0.03	0.03	0.02
Sector 13	0.24	0.47	0.67	0.79	0.96	0.79
Sector 14	0.02	0.05	0.06	0.07	0.09	0.08
Sector 15	0.07	0.14	0.22	0.27	0.33	0.26
Primary Sector	0.37	0.48	0.57	0.63	0.70	0.66
Secondary Sector	0.39	0.69	0.91	0.98	1.14	0.80
Tertiary Sector	0.33	0.66	0.95	1.13	1.38	1.13
TOTAL	1.09	1.83	2.43	2.74	3.22	2.59

TABLE 4

MULTIPLIERS: ONE BILLION INCREASE IN REAL GOVERNMENT INVESTMENT AND 10 PERCENT EFFICIENCY INCREASE IN BASIC METAL INDUSTRIES (COLUMN CHANGES IN "A" MATRIX)

	Period 1	2	3	4	5	6
Sector 1	0.11	0.21	0.28	0.33	0.38	0.29
Sector 2	−0.01	−0.02	−0.02	−0.03	−0.04	−0.08
Sector 3	0.02	0.05	0.07	0.08	0.09	0.06
Sector 4	0.03	0.06	0.09	0.11	0.14	0.11
Sector 5	0.03	0.05	0.07	0.08	0.10	0.08
Sector 6	0.03	0.05	0.06	0.06	0.07	0.06
Sector 7	0.03	0.05	0.06	0.07	0.08	0.06
Sector 8	0.03	0.05	0.06	0.06	0.07	0.05
Sector 9	0.13	0.17	0.20	0.22	0.25	0.24
Sector 10	0.08	0.13	0.16	0.17	0.19	0.15
Sector 11	0.22	0.32	0.40	0.43	0.49	0.38
Sector 12	0.01	0.02	0.03	0.03	0.03	0.02
Sector 13	0.30	0.54	0.75	0.88	1.06	0.90
Sector 14	0.02	0.05	0.07	0.08	0.10	0.09
Sector 15	0.07	0.14	0.22	0.28	0.34	0.28
Primary Sector	0.11	0.21	0.28	0.33	0.38	0.29
Secondary Sector	0.59	0.93	1.18	1.25	1.47	1.13
Tertiary Sector	0.39	0.73	1.04	1.24	1.50	1.27
TOTAL	1.09	1.87	2.50	2.82	3.35	2.69

TABLE 5

MULTIPLIERS: ONE BILLION INCREASE IN REAL GOVERNMENT INVESTMENT AND 10 PERCENT EFFICIENCY INCREASE IN TRANSPORTATION (COLUMN CHANGES IN "A" MATRIX)

	Period 1	2	3	4	5	6
Sector 1	0.11	0.21	0.28	0.33	0.37	0.28
Sector 2	0.01	0.00	0.00	0.00	−0.01	−0.04
Sector 3	−0.01	0.02	0.04	0.04	0.06	0.02
Sector 4	0.03	0.06	0.09	0.11	0.14	0.11
Sector 5	0.03	0.05	0.07	0.08	0.10	0.08
Sector 6	0.03	0.05	0.06	0.06	0.07	0.06
Sector 7	0.02	0.04	0.05	0.06	0.07	0.04
Sector 8	0.03	0.05	0.06	0.06	0.07	0.05
Sector 9	0.04	0.07	0.08	0.09	0.10	0.07
Sector 10	0.08	0.13	0.15	0.16	0.18	0.14
Sector 11	0.22	0.32	0.40	0.42	0.48	0.37
Sector 12	0.01	0.02	0.03	0.04	0.04	0.03
Sector 13	0.29	0.52	0.73	0.86	1.04	0.87
Sector 14	0.14	0.18	0.21	0.23	0.26	0.26
Sector 15	0.07	0.14	0.22	0.28	0.34	0.27
Primary Sector	0.11	0.21	0.28	0.33	0.37	0.28
Secondary Sector	0.49	0.81	1.03	1.12	1.30	0.93
Tertiary Sector	0.50	0.84	1.16	1.37	1.64	1.40
TOTAL	1.10	1.86	2.47	2.82	3.31	2.61

TABLE 6

MULTIPLIERS: ONE BILLION INCREASE IN REAL GOVERNMENT INVESTMENT AND 10 PERCENT EFFICIENCY INCREASE IN AGRICULTURE (ROW CHANGES IN "A" MATRIX)

	Period 1	2	3	4	5	6
Sector 1	−0.30	−0.21	−0.16	−0.14	−0.11	−0.23
Sector 2	0.01	0.00	0.00	0.00	−0.01	−0.04
Sector 3	0.03	0.06	0.09	0.10	0.12	0.09
Sector 4	0.42	0.47	0.52	0.57	0.63	0.64
Sector 5	0.08	0.10	0.13	0.14	0.17	0.15
Sector 6	0.05	0.08	0.09	0.09	0.11	0.10
Sector 7	0.05	0.07	0.08	0.10	0.11	0.10
Sector 8	0.03	0.05	0.06	0.06	0.07	0.05
Sector 9	0.04	0.07	0.08	0.09	0.10	0.07
Sector 10	0.08	0.14	0.17	0.18	0.19	0.16
Sector 11	0.23	0.33	0.41	0.45	0.52	0.44
Sector 12	0.01	0.02	0.03	0.04	0.04	0.04
Sector 13	0.32	0.55	0.77	0.92	1.12	0.99
Sector 14	0.02	0.06	0.07	0.08	0.10	0.10
Sector 15	0.09	0.16	0.25	0.31	0.38	0.35
Primary Sector	−0.30	−0.21	−0.16	−0.14	−0.11	−0.23
Secondary Sector	1.03	1.39	1.66	1.82	2.05	1.80
Tertiary Sector	0.43	0.77	1.09	1.31	1.70	1.44
TOTAL	1.16	1.95	2.59	2.99	3.64	3.01

TABLE 7

MULTIPLIERS: ONE BILLION INCREASE IN REAL GOVERNMENT INVESTMENT AND 10 PERCENT EFFICIENCY INCREASE IN BASIC METAL INDUSTRIES (ROW CHANGE IN "A" MATRIX)

	Period 1	2	3	4	5	6
Sector 1	0.11	0.21	0.28	0.33	0.38	0.29
Sector 2	0.00	−0.01	−0.02	−0.02	−0.03	−0.06
Sector 3	0.03	0.06	0.07	0.08	0.10	0.07
Sector 4	0.04	0.07	0.10	0.12	0.15	0.12
Sector 5	0.03	0.05	0.07	0.08	0.10	0.08
Sector 6	0.03	0.05	0.06	0.06	0.07	0.06
Sector 7	0.03	0.05	0.06	0.07	0.08	0.06
Sector 8	0.03	0.05	0.06	0.06	0.07	0.05
Sector 9	−0.03	−0.01	−0.01	−0.01	−0.01	−0.06
Sector 10	0.14	0.19	0.23	0.25	0.28	0.29
Sector 11	0.27	0.38	0.47	0.52	0.59	0.52
Sector 12	0.01	0.02	0.03	0.04	0.04	0.03
Sector 13	0.31	0.55	0.76	0.89	1.07	0.91
Sector 14	0.02	0.05	0.07	0.08	0.10	0.09
Sector 15	0.08	0.15	0.23	0.29	0.35	0.29
Primary Sector	0.11	0.21	0.28	0.33	0.38	0.29
Secondary Sector	0.58	0.90	1.12	1.25	1.44	1.16
Tertiary Sector	0.41	0.75	1.03	1.26	1.52	1.29
TOTAL	1.10	1.86	2.43	2.84	3.34	2.74

with those of Policy 1, we find that for those sectors the effects of Policy 1 are greater in most cases.

A column change in agricultural production functions causes all sectors except the mining and nonferrous metals sectors to have smaller multipliers than if only government investment were increased (Policy 1). The above can be observed in Table 2. There it is shown that a column change in agriculture produces a smaller multiplier than Policy 1 for the secondary and tertiary sectors. The increased efficiency in the agriculture sector is in effect comparatively reducing the demand for the output of its main suppliers. Thus, to get a given final demand, less output from those sectors is needed, given the increase in efficiency. In a way, we see that some resources are being freed by the change, resources that could be used somewhere else, thus permitting a further increase in activity.

Basic metal industries have a less widespread effect. Only Sectors 2, 3, 13, and 15 are affected by its change in production function.

When the transportation sector column is changed its effects are felt mainly by Sectors 3, 7, 10, 13, and 15. None of those sectors has a noticeable effect on agriculture. Transport's column change has only a slight effect on agriculture.

In sum, we note that a change in a sector's production function tends to change the composition of output, with relative gains to itself and relative losses to its main suppliers.

The second set of simulations assuming a reduction of the row coefficient of the A matrix are presented in Tables 6 to 8. This case—as was mentioned before—is equivalent to a change in the production functions of the sectors that use as input the output of the sector whose row has been changed.

TABLE 8

MULTIPLIERS: ONE BILLION INCREASE IN REAL GOVERNMENT INVESTMENT AND 10 PERCENT EFFICIENCY INCREASE IN TRANSPORTATION (ROW CHANGES IN "A" MATRIX)

	Period					
	1	2	3	4	5	6
Sector 1	0.11	0.21	0.28	0.33	0.38	0.29
Sector 2	0.01	0.00	0.00	0.00	−0.01	−0.04
Sector 3	0.02	0.05	0.07	0.08	0.10	0.07
Sector 4	0.03	0.06	0.09	0.11	0.14	0.11
Sector 5	0.03	0.05	0.07	0.08	0.10	0.08
Sector 6	0.03	0.05	0.06	0.06	0.07	0.06
Sector 7	0.03	0.05	0.06	0.07	0.08	0.06
Sector 8	0.03	0.05	0.06	0.06	0.07	0.05
Sector 9	0.04	0.07	0.08	0.09	0.10	0.07
Sector 10	0.08	0.13	0.16	0.17	0.19	0.16
Sector 11	0.22	0.32	0.40	0.43	0.49	0.38
Sector 12	0.01	0.02	0.03	0.04	0.04	0.03
Sector 13	0.45	0.70	0.92	1.07	1.26	1.16
Sector 14	−0.09	−0.07	−0.06	−0.06	−0.05	−0.06
Sector 15	0.09	0.16	0.24	0.30	0.36	0.30
Primary Sector	0.11	0.21	0.28	0.33	0.38	0.29
Secondary Sector	0.52	0.85	1.08	1.19	1.37	1.03
Tertiary Sector	0.45	0.79	1.10	1.31	1.57	1.40
TOTAL	1.08	1.85	2.46	2.83	3.32	2.72

Table 6 shows that when we assume an increase in agricultural efficiency (reflected now in row coefficients' change), the output of agriculture itself is decreased substantially. The production functions of the other sectors are such now that per unit of output they need less of agriculture's output. The same holds true when the basic metal industries' row and transportation's row are reduced. The striking fact is that a change in the agricultural row produces the greatest multipliers for the secondary and tertiary sectors (see Table 9), and despite the reduction in agricultural output, the greatest stimulus to total activity. It has a greater impact on the secondary or manufacturing sector than a column change in the basic metals industries.

TABLE 9

I–O SIMULATION MULTIPLIERS COMPARED TO POLICY 1 (SELECTED INDICATORS)*

	I	II	III	Total
Government Investment Alone (Policy 1)	0.38	1.36	1.53	3.26
Row Change in Agriculture	− 0.11	2.05	1.70	3.64
Row Change in Basic Metal Industries	0.38	1.44	1.52	3.34
Row Change in Transportation	0.38	1.37	1.57	3.32
Column Change in Agriculture	0.70	1.14	1.38	3.22
Column Change in Basic Metal Industries	0.38	1.47	1.50	3.35
Column Change in Transportation	0.37	1.30	1.64	3.31

* The numbers refer in each case to the period in which a peak was achieved. I = Primary Sector; II = Secondary Sector; III = Tertiary Sector.

The above simulations tend to indicate that development efforts may do well by giving more attention to agriculture than is sometimes the case.

Usually national governments tend to give more emphasis to developing a heavy industrial sector. This, given the limited resources available, means that the agricultural sector is neglected. At the same time, we see that it would be a greater stimulus to industrial development if agriculture were able to increase its productivity (while maintaining relatively low agricultural prices) than the situation where agriculture is neglected and becomes a bottleneck.

The above results are subject to one limitation. Recalling that:

(1) $$Y_i = CG + U_i \qquad i = 1, \ldots 15$$

and

(2) $$U_i = f(\mathrm{PC1}_{-1}, \mathrm{PC2}_{-1}, \mathrm{PC3}_{-1}) \qquad i = 1, \ldots 15$$

We see that the error models that try to make up for changes in the C matrix are functions of the lagged first three principal components. These, in turn, are a linear combination of the errors. Thus, if we are introducing a policy change in period t, the $\mathrm{PC1}_{-1}$, $\mathrm{PC2}_{-1}$, $\mathrm{PC3}_{-1}$ will be determined by the errors in $t - 1$. This means that the second part of the set of equations (1) will be independent of the policy change. A way to get around this problem would be to develop equations for the principal components. That is, to make each principal component used in (2) a function of some variable that is determined by the model. In Appendix I, the graphs for each principal component are presented. Looking at them, we see that the first component represents a trend factor, the second, a standard business cycle, and the third, a shorter-run cycle. This suggests a possible way to tackle the above problem.[11]

A second problem with the above simulations is that it has been assumed that the "H" matrix of industrial distribution of final demand remains unchanged. A change in this matrix can be viewed as a change in the composition of a certain category of final demand. For example, as time goes by, there can be a change in the composition of consumption. That is, total consumption expenditures could have a greater proportion of durable goods and less food components. In terms of the "H" matrix the above means that the deliveries of agriculture and food-producing industries would represent a smaller proportion of total consumption deliveries while the sectors that produce durable goods like automobiles, refrigerators, and so on would deliver a greater proportion. One such calculation was made. The results are shown in Table 10. It was assumed that there would be a 10 percent increase in the proportion of durable goods in total consumption. Accordingly, there was a similar reduction in food. To achieve this, the coefficient of agriculture and food industry deliveries to consumption was decreased and that of the fabricated metal products and repairs sector was increased. This is reflected in Table 10. Sectors 1 and 4 (agriculture and food, respectively) suffer a decline in output due to the change in demand. Sector 10 (fabricated metal products and repairs) shows a substantial increment in output. The overall effect is a rise in total production—the changed demand pattern produces a positive effect on total output. Even greater is the effect on the composition of output, with the secondary sector becoming relatively more important and the primary sector declining.

In concluding this section, we must bear in mind that, although the results look definite in the tables, they are subject to a margin of error. We do not have estimates of the forecasting errors of the model; however, the error estimates over the sample period indicate that an error of between ± 5 percent and ± 10 percent must be expected in most cases. Nonetheless, the results can be indicative of the possible effects of different policy actions.

Summary

The purpose of the present study was to show the uses that input–output analysis can have in macro models. This was done through the example of the Mexican case.

First, the algebra of the linkage was developed and the different identities involved in I–O were made explicit. There are some problems in the linkage because of data availability and different concepts used—gross vs. net—in the two accounting systems, but they are easily solved if we make an assumption of proportionality similar to the one that is made for the A matrix. This, in turn, gives rise to the problem of how to deal with the fact that the coefficients do change over time. At this point use was made of principal components to model the errors or residuals produced by using constant coefficients.

[11] Some preliminary work has been done to implement these ideas, but as yet no satisfactory results have been obtained for the third principal component, which represents the short-term cycle.

TABLE 10

Effects of a Change in Consumption Patterns. 10 percent Increase in Durable Goods Proportion. Selected Indicators

	1	2	3	4	5	6
Sector 1	−3.10	−3.20	−3.35	−3.54	−3.75	−4.02
Sector 2	0.13	0.14	0.15	0.17	0.18	0.20
Sector 3	0.04	0.06	0.07	0.08	0.10	0.11
Sector 4	−1.38	−1.43	−1.50	−1.59	−1.68	−1.80
Sector 5	−0.01	0.00	0.01	0.02	0.03	0.04
Sector 6	0.04	0.05	0.06	0.07	0.08	0.09
Sector 7	0.11	0.12	0.14	0.15	0.17	0.19
Sector 8	0.01	0.02	0.02	0.03	0.04	0.04
Sector 9	0.42	0.45	0.48	0.53	0.57	0.61
Sector 10	3.73	3.92	4.16	4.46	4.78	5.17
Sector 11	0.01	0.05	0.10	0.17	0.22	0.23
Sector 12	0.02	0.03	0.03	0.04	0.04	0.05
Sector 13	0.00	0.07	0.16	0.27	0.39	0.48
Sector 14	0.02	0.03	0.04	0.05	0.06	0.07
Sector 15	0.14	0.17	0.21	0.27	0.33	0.40
Primary Sector	−3.10	−3.20	−3.35	−3.54	−3.75	−4.02
Secondary Sector	3.12	3.41	3.72	4.16	4.53	4.93
Tertiary Sector	0.16	0.27	0.41	0.59	0.78	0.95
Total	0.18	0.48	0.78	1.21	1.56	1.86

Once the I–O is linked to the macro model, there is a wealth of policy actions to choose from that can be simulated. How can the structure of the economy be altered? Which sectors to promote to achieve faster growth?, et cetera.

Three sectors were chosen to simulate the effects of government investment. The striking fact that was revealed by the exercise was that investment in agriculture, which tended to increase its efficiency (as measured by the row coefficients), would bring about a greater stimulus to industrial production and, despite the reduction in agriculture output itself, to total activity than new government investment directed to the industrial sector.

The results, as pointed out before, are subject to some limitations. Yet, they are suggestive of new directions for promoting development efforts and of the usefulness of incorporating input–output analysis to econometric models.

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APPENDIX 1: INPUT–OUTPUT AND RELATED DATA

TABLE A1
DIRECT REQUIREMENT MATRIX "A" FOR YEAR 1960

	1	2	3	4	5
Sector 1	0.06310	0.00320	0.00010	0.27330	0.08420
Sector 2	0.0	0.23360	0.01340	0.00080	0.00030
Sector 3	0.01930	0.03570	0.30710	0.01510	0.01430
Sector 4	0.05160	0.00280	0.00110	0.15950	0.00420
Sector 5	0.01620	0.00280	0.00160	0.00320	0.19800
Sector 6	0.00380	0.00810	0.00770	0.01490	0.00770
Sector 7	0.02300	0.02300	0.00560	0.00450	0.05100
Sector 8	0.00010	0.00280	0.00070	0.00860	0.00060
Sector 9	0.00150	0.01580	0.00260	0.00490	0.00450
Sector 10	0.00700	0.01160	0.00500	0.01080	0.00880
Sector 11	0.00440	0.00560	0.00270	0.00150	0.00290
Sector 12	0.00230	0.02660	0.00030	0.00560	0.00940
Sector 13	0.03720	0.03830	0.02080	0.08930	0.11780
Sector 14	0.0	0.00210	0.00130	0.00190	0.00250
Sector 15	0.00490	0.03520	0.02370	0.02310	0.02600

	6	7	8	9	10
Sector 1	0.09300	0.06260	0.00040	0.00040	0.00270
Sector 2	0.00140	0.01060	0.04310	0.09700	0.01360
Sector 3	0.01730	0.01440	0.08620	0.05860	0.01650
Sector 4	0.00590	0.01480	0.00200	0.00450	0.00290
Sector 5	0.00680	0.01070	0.00470	0.00530	0.00770
Sector 6	0.18520	0.02800	0.03640	0.00920	0.01730
Sector 7	0.02390	0.11590	0.01940	0.00720	0.04090
Sector 8	0.00230	0.00640	0.07120	0.01340	0.00490
Sector 9	0.00700	0.00750	0.01310	0.18610	0.11890
Sector 10	0.00910	0.01080	0.01700	0.01220	0.08610
Sector 11	0.00290	0.00230	0.00990	0.00470	0.00100
Sector 12	0.01110	0.00850	0.02970	0.01320	0.00600
Sector 13	0.08620	0.09900	0.09850	0.06700	0.05180
Sector 14	0.00370	0.00340	0.00320	0.00150	0.00440
Sector 15	0.04300	0.03950	0.03080	0.05100	0.02970

	11	12	13	14	15
Sector 1	0.00020	0.0	0.00010	0.0	0.00060
Sector 2	0.01550	0.00220	0.00020	0.00090	0.00020
Sector 3	0.00890	0.09790	0.00300	0.12710	0.00300
Sector 4	0.00290	0.00090	0.00100	0.00350	0.00350
Sector 5	0.00270	0.00230	0.00170	0.00150	0.00270
Sector 6	0.04400	0.00590	0.00800	0.00600	0.01150
Sector 7	0.01370	0.00500	0.00150	0.03780	0.01030
Sector 8	0.10250	0.00050	0.00040	0.00020	0.00060
Sector 9	0.08370	0.00590	0.00250	0.00340	0.00480
Sector 10	0.06220	0.02360	0.00380	0.01200	0.01490
Sector 11	0.00430	0.01950	0.00160	0.00900	0.01180
Sector 12	0.00350	0.03630	0.00470	0.00370	0.00500
Sector 13	0.11910	0.04220	0.00650	0.07700	0.04130
Sector 14	0.00140	0.00540	0.05490	0.00300	0.00420
Sector 15	0.02500	0.01950	0.03010	0.03020	0.10780

TABLE A2
First Principal Component

Period	Value
1959	−0.196
1960	−0.145
1961	−0.368
1962	−0.471
1963	−0.759
1964	−1.075
1965	−1.357
1966	−1.613
1967	−1.818
1968	−2.121
1969	−2.510
1970	−2.790
1971	−2.997

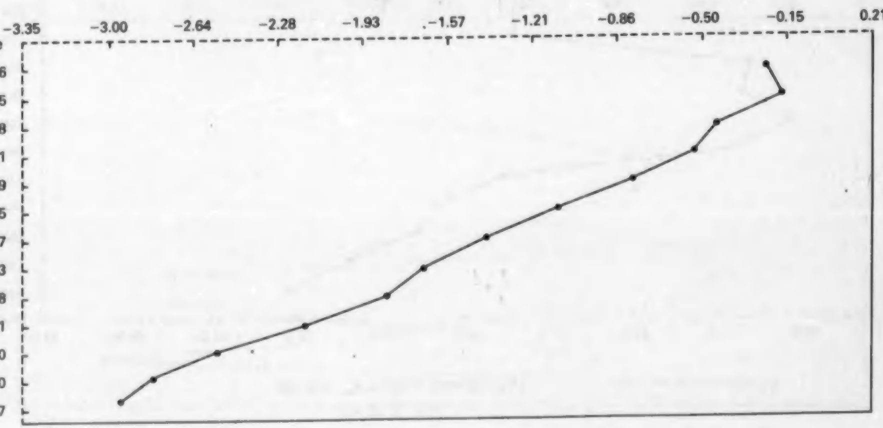

TABLE A3
SECOND PRINCIPAL COMFONENT

Period	Value
1959	−0.015
1960	0.081
1961	0.208
1962	0.244
1963	0.370
1964	0.740
1965	0.863
1966	0.745
1967	0.768
1968	0.389
1969	0.403
1970	0.255
1971	−0.320

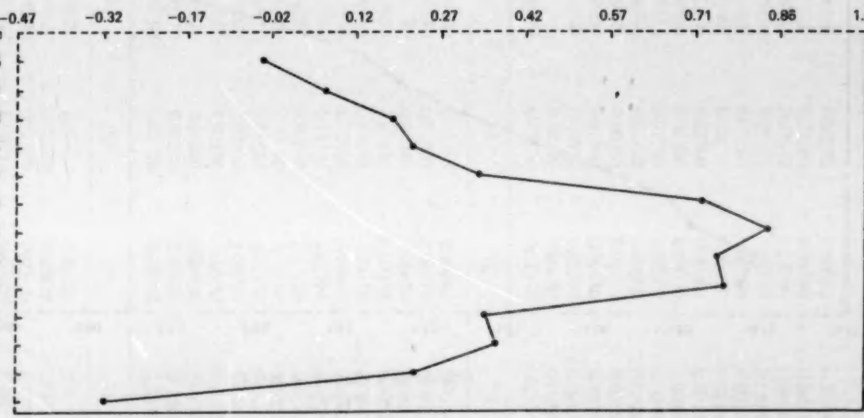

TABLE A4
THIRD PRINCIPAL COMPONENT

Period	Value
1959	0.054
1960	0.070
1961	−0.002
1962	0.303
1963	0.436
1964	0.417
1965	0.307
1966	0.477
1967	−0.107
1968	−0.108
1969	0.209
1970	−0.205
1971	0.590

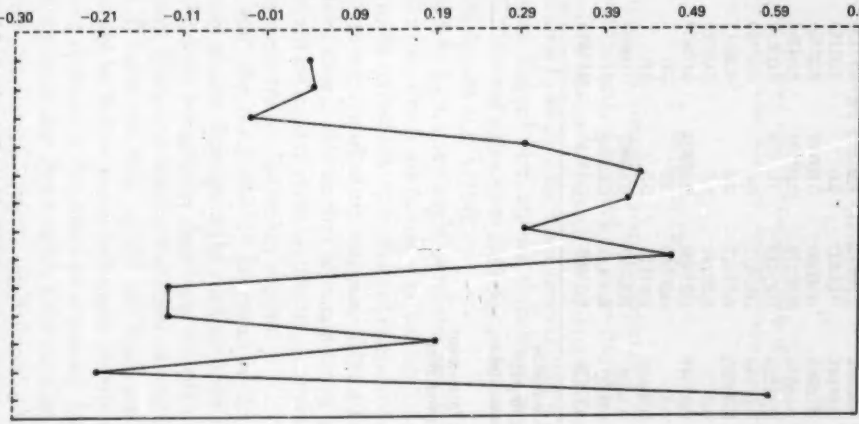

TABLE A5
"H" Matrix
Industrial Distribution of Final Demand for Year 1960

	1	2	3	4	5	6
Sector 1	0.12860	0.00158	0.03496	0.24609	0.03778	0.23416
Sector 2	0.00002	0.00012	0.0	0.17155	0.00096	0.01276
Sector 3	0.01848	0.01407	0.0	0.02188	0.00008	0.04721
Sector 4	0.16983	0.00509	0.03975	0.20549	0.00160	0.08379
Sector 5	0.06914	0.00279	0.09100	0.04308	0.00108	0.11570
Sector 6	0.01106	0.01419	0.01676	0.00812	0.00200	0.04232
Sector 7	0.03149	0.02632	0.0	0.01889	0.00292	0.08060
Sector 8	0.00086	0.01419	0.0	0.00615	0.00104	0.02148
Sector 9	0.0	0.00291	0.0	0.00786	0.00889	0.05572
Sector 10	0.02843	0.03409	0.04502	0.00957	0.10131	0.13101
Sector 11	0.0	0.00340	0.0	0.0	0.51226	0.02786
Sector 12	0.00468	0.02171	0.0	0.0	0.00461	0.0
Sector 13	0.31041	0.02693	0.16092	0.09608	0.14522	0.04551
Sector 14	0.03733	0.02183	0.05316	0.0	0.00349	0.0
Sector 15	0.17826	0.03603	0.55843	0.01855	0.01510	0.0

Col 1 = Private Consumption
Col 2 = Public Consumption
Col 3 = Tourists Consumption
Col 4 = Exports
Col 5 = Total Fixed Investment
Col 6 = Change in Inventories