Volume Title: Annals of Economic and Social Measurement, Volume 4, number 2

Volume Author/Editor: NBER

Volume Publisher: NBER

Volume URL: http://www.nber.org/books/aesm75-2

Publication Date: April 1975

Chapter Title: Stochastic Control of Environmental Externalities

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Chapter URL: http://www.nber.org/chapters/c10398

Chapter pages in book: (p. 271 - 292)

Annals of Economic and Social Measurement, 4/2, 1975

MICROECONOMICS

STOCHASTIC CONTROL OF ENVIRONMENTAL EXTERNALITIES*

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The nature of externalities implies that they have no discernable market signals. Hence, in an operational setting, various internalization schemes must explicitly recognize that externality outputs are not directly accessible to the policy maker and that an observation system must be employed. The precision of this observation or measurement system along with conventional tax, standard, or "pollution right" instruments are control variables available to public agencies. In this paper, both firm and public agency behavior are analyzed under tax internalization schemes, stochastic externality measurement, and a legal system which resolves conflicts. It is shown, under the conditions specified, that optimal public agency actions involve the derivation of tax controls, measurement controls, and the sequential estimation of inaccessible state variables by a linear Kalman filter. The two sets of controls are found to be separable and thus the optimal conditions may be stated in terms of two problems; the first is concerned with the optimal tax controls and the second with the optimal monitoring controls.

1. INTRODUCTION

As the clamor for instant environmental solutions diminishes, the task of rationally allocating the finite capacity of environment disposal resources will be increasingly viewed in its correct perspective, namely as an economic problem of resource use over time. This problem is complicated by the absence of price signals, often coupled with indeterminant property rights. These characteristics classify the problem as one involving externalities, which in an environmental context are invariably negative.

Much of the recent literature on externalities has investigated the properties of policy instruments imposed through government regulation. These instruments are presented as a means for altering the impact of external diseconomies. Unfortunately, virtually all of this literature treats externality control in the context of zero transaction costs,[1] perfect information, abrupt and instantaneous policy changes, and no uncontrollable exogeneous influences. Hence, it is not surprising that policy makers, faced with uncertain effects of externality controls, inaccuracies, and substantial costs in measuring and monitoring systems, have made scant use of conclusions drawn from these economic models.

The very nature of externalities implies that they have no directly discernable market signals in value or physical terms.[2] In other words, externality

* Giannini Foundation Research Paper No. 387 and Journal Paper No. J-8197 of the Iowa Agriculture and Home Economics Experiment Station, Ames, Iowa. The authors are grateful to Gregory C. Chow and an anonymous referee for helpful comments and suggestions. Project 2061.

[1] Notable exceptions to this observation are Baumol and Oates [1971], Calebresi [1968], and Dales [1968] who have considered problems of transaction costs and information availability for the deterministic case.

[2] In fact, as Starrett [1974, p. 2] recently argued, "... one can think of externalities as synonymous with nonexistence of markets, and define an externality to occur whenever the private economy does not have sufficient incentives to create a potential market." The usual definition of an externality, viz., a decision variable of one economic agent which enters into the production function (or utility function) of some other agent, is far too broad; it defines all commodities in a barter economy as externalities.

outputs are not directly accessible to the policy maker and thus an observation system must be employed. Choices on the type and precision of the observation and measurement system are also control variables available to policy makers.

With respect to measurement costs, the usual assumption that it is possible to determine at no cost precisely what firms release into the environment is untenable. Surely, while a firm may record its financial transactions and its normal outputs, permitting regulation of financial variables, it has no incentive to record its waste discharges, e.g., pollution emissions. The measurement of these discharges involves costs which are clearly uncertain from the standpoint of environmental control agencies.

The above discussion implicitly assumes the need for public control of environmental externalities. This requires some justification. As often recognized, under the usual perfectly competitive assumptions and externalities, decentralized actions by economic agents will not lead to a Pareto optimum. Of course, when the number of agents are few, Coase [1960] has shown that direct negotiation between affected parties may result in an efficient solution. The Coase treatment has been the basis for many expositions of two- or three-party models with fixed joint production possibilities in which environmental externalities are specified to have the consumption characteristics of normal private goods. These specifications do much injustice to most environmental externality problems which correspond more closely to collectively consumed public bads.[3] As Kneese [1971] has noted, most environmental externalities are collectively consumed within some relevant delineation, such as an airshed or watershed. In these situations, exclusion problems arise and property rights are difficult, if not impossible, to define.[4] Furthermore, given the external effect of an environmental bad has public properties over a sufficient number of individuals, there obviously exists an incentive for individuals to misrepresent their true preferences.[5] Since game theory informs us that this sufficient number is small, there are few environmental externalities that fail to satisfy the characteristics of a (quasi) local public good.

[3] The theoretical considerations of collectively consumed externality policy in the absence of explicit transaction costs has been briefly examined by Baumol [1972], Buchanan and Stubblebine [1962], Mohring and Boyd [1971], and in more detail by Whitcomb [1972]. For the properties of a continuum on types of goods between the polar public and private cases, see Kamien *et al.* [1973].

[4] Property rights cannot generally be defined whenever exclusion is costly or impossible; that is, whenever what is available to one agent does not alter what is available to another. Exclusion is clearly costly in case of air, water, or noise pollution and impossible, as Starrett [1974, p. 6] points out, "... by definition when we are dealing with commodities such as national defense, open rangeland, or public parks." Exclusion in the theory of externalities has been explored by Davis and Whinston [1967], Kamien *et al.* [1973], and Turvey [1963], among others.

[5] To isolate the incentives for individuals to misrepresent their preferences for external goods which are collectively consumed, simply derive the society's optimal conditions and the potential gains from individual internalization bribes. Under a zero liability rule, rational parties affected by the externality will not offer an internalization bribe but will remain as "free rider" on those who do offer bribes. The inclusion of the inevitable transaction costs only exacerbates the problem. The particular situation where an incentive for individual internalization action exists is the rare case of full liability on the waste discharge firms for damages and transaction costs.

In general, given numerous parties, private transactions costs[6] will typically be so large, relative to the social cost of the environmental externality, that negotiation among all parties is not a feasible means of reaching an efficient solution. Under the common situation with absence of full liability, significant transaction costs, and in addition locally collective consumption of externalities, decentralized internalization will not take place. However, the public good nature of environmental quality and associated exclusion problems suggests that potential gains exist from "governmental internalization" of the external effect. Governmental internalization involves the establishment of a central controlling agency since enforcement of liability rules by itself, say via a legal system, is not sufficient for optimal internalization. This internalization will, of course, have transaction costs associated with it. These costs include the often neglected measurement costs previously mentioned, other information costs, enforcement costs, and administrative costs. Under the criterion of Pareto efficiency, there is no qualitative reason to expect the minimization of these transaction costs to be less important than the gains emanating from the internalization process.

On the basis of the above arguments, this paper presumes that the establishment of a centralized control agency is desirable. This agency, whether of a local or national character, treats environmental quality as a public good and attempts to determine and regulate its supply. A number of institutional structures for determining and regulating this supply have been offered in the literature (Rausser and Fishelson [1974]). This paper addresses a class of these internalization policies, viz., Pigouvian taxes and taxes advanced to achieve predetermined standards. The emphasis is on evaluating information problems associated with this class of internalization schemes. In particular, transaction costs emanating from a measurement system on the externality states along with control implementation and enforcement costs are analyzed.

The organization of the paper is as follows. In Section 2, the system representing firm behavior which the public agency attempts to influence is specified and briefly interpreted. Section 3 examines the components of public agency control. A stochastic externality control framework is presented in Section 4. One of the special features of this framework is that control variables influence not only the state dynamics but also the stochastic measurement system. That is, the stochastic externality states are specified to be accessible only through a stochastic measurement system, and costly controls are explicitly recognized both for the fundamental process and the measurement system. The optimal performance is derived by the separation of controls into two problems; the first is concerned with the optimal behavioral controls and the second with the optimal measurement or monitoring controls. This separation between the behavioral and measurement controls, under specified conditions, is shown to be optimal. In Section 5, the results of the separable control formulation are interpreted and in Section 6 an empirical application of the method is briefly noted.

[6] The significance of transaction costs in the context of externalities was first emphasized by Coase [1960] and has subsequently been analyzed by Calebresi [1968], and Demsetz [1964], among others. In general, transactions can be incorporated in a multiproduct formulation by specifying it as an alternative output which is jointly produced along with a unit reduction of the externality. Obviously, the theoretical optimal internalization differs from the societal Pareto optimal solution by the amount of the transaction costs.

2. Firm Behavior

Public control of environmental externalities involves an attempt to influence the behavior of firms, individuals, and households who emit wastes or byproducts in their pursuit of other activities. For the sake of exposition, we shall deal only with production externalities in the paper.[7] Firms which generate environmental wastes as a result of their production processes will be referred to as emitter firms. Within a particular airshed or watershed, these firms will be assumed to have certain knowledge of perfectly competitive output prices, market input prices, and the production processes for normal goods as well as externality goods. Furthermore, we presume that each emitter firm desires to maximize expected profits over some planning horizon of specified length. While these assumptions simplify the actual situation facing most emitter firms, they do allow the construction of a model which provides much insight into the public control of environmental externalities.

The underlying production process for each firm is characterized as one involving generalized joint production. The production function of the j-th firm for L normal outputs may be represented as $F_j(q_{jt}, x_{qjt}, x_{jt}^*) = 0$, $j = 1, \ldots, J$, while the production function for each of K externality outputs may be represented as $F_{kj}(w_{kjt}, x_{kjt}, x_{jt}^*) = 0$, $k = 1, \ldots, K$, where q_j denotes an L component vector of saleable outputs; x_{qj} denotes a vector of ordinary inputs employed in the production of saleable outputs; x_j^* denotes a vector of joint inputs which are employed in the production of saleable outputs but also influence the level of the externality produced;[8] x_{kj} denotes a vector of ordinary inputs employed to control the amount of the k-th externality, e.g., emission control devices; w_{kj} denotes the kth externality (e.g., sulfur emissions); and t denotes time, i.e., $t = 0, 1, \ldots, T$. The production structure implies that the transformation function between any saleable output and an externality is a single point; given fixed amounts of all inputs, the firm cannot vary the amounts of saleable and externality outputs.[9]

Given the usual convexity assumption on $F_j(\cdot)$ and $F_{kj}(\cdot)$ for all k and j, the cost function $C_j(q_{jt}, w_{jt})$ can be derived whose properties over the relevant range (Rausser and Zerbe [1974]) are

$$(2.1)\qquad \frac{\partial C_j(\cdot)}{\partial q_{jt}} > 0, \qquad \frac{\partial^2 C_j(\cdot)}{\partial q_{jt}^2} > 0, \qquad \frac{\partial C_j(\cdot)}{\partial w_{jt}} < 0, \qquad \frac{\partial^2 C_j(\cdot)}{\partial w_{jt}^2} > 0.$$

[7] The distinction between production and consumption externalities is nicely drawn in Kamien *et al.* [1973].

[8] For example, fuel inputs employed to produce electricity output are also partially responsible for the byproduct smoke, an externality.

[9] This specification generalizes the usual fixed proportion model of externalities, i.e., once the level of saleable output is set, the externality output is automatically determined no matter what the rates of input use. It is also a more appropriate specification that the multiproduct formulation involving a single relationship $F_j(q_{jt}, w_{jt}, x_{jt}, x_{jt}^*) = 0$ where w_{jt} is a K component vector and $x_{jt} = (x_{qjt}, x_{kjt})$. This joint product model, found in most intermediate economic texts, is not generally applicable to the case of externalities. Such a formulation would imply that, given amounts of all inputs, more saleable output can be produced by altering the amount of externality output. This is clearly incorrect; the externality output can only be varied by changing the joint inputs (e.g., type of fuel used) or the amount of fixed or other variable inputs (x_{kjt}).

Employing this cost function, the j-th firm optimization problem prior to any internalization scheme may be represented as

$$\max_{q_j, w_j} V_{j0} = \sum_{t=0}^{T} \beta_j^t [p_{jt} q_{jt} - C_j(q_{jt}, w_{jt})] \tag{2.2}$$

where $\beta_j = 1/(1 + r)^j$, r being a subjective positive discount rate, p_{jt} denotes an L component of saleable output prices at time t, and $w_{jt}' = [w_{1jt}, \ldots, w_{Kjt}]$.

The above problem is, of course, altered by various internalization schemes. These schemes depend upon (1) the controls available to the public agency, (2) the measurement of waste or emissions, and (3) legal recourses allowed a firm which finds its measured emission level objectionable. These factors are examined in the following subsections. When combined, they result in an internalization function (2.7) composed of a stochastic tax bill, monitoring costs, and firm legal expenditures. Introducing this function into (2.2), firm decision rules and behavioral equations are derived. The latter equations state firm saleable outputs (2.12), externality outputs (2.13), and legal inputs (2.14) in terms of output prices and a vector of per unit tax rates.

2.1 *Tax Internalization Schemes*

Two schemes, both leading to per unit taxes imposed upon the emitter firms, will be examined. The first tax internalization system is Pigouvian [1932] in nature, while the second approach is described by Baumol and Oates [1971]. Despite variations, a Pigouvian tax is based on the marginal damages currently caused by the environmental wastes emanating from the production process of each emitter firm. The formal derivation of the basic Pigouvian tax follows directly from the definition of the Pareto optimal transformation for an externality commodity. Since the marginal private product achieved by an emitter firm under perfect competition does not equal the marginal social product of the commodity, a corrective tax is imposed.[10] In principle, this tax provides an incentive for emitter firms to produce socially optimal output levels. Moreover, the unilateral imposition of a Pigouvian tax on emitter firms by a central authority is believed to have lower transaction costs, and therefore greater potential internalization than private individual negotiations among firms effected by the externality (receptor firms) and emitter firms.[11]

The second tax internalization approach focuses on one of the principal limitations of the Pigouvian approach, viz., the marginal damage functions

[10] For a survey of the criticisms and an attempt to reconstruct the Pigouvian approach, see Baumol [1972] or Whitcomb [1972].

[11] Where transactions between receptors and emitters are possible, even though the unilateral Pigouvian tax leads to a Pareto optimum, there may be incentives for the receptor and emitter firms to bargain away from this optimum. Schemes to prevent Pareto suboptimal transactions include bilateral taxes (Rausser and Zerbe [1974]) and compensation paid by the emitter to the receptor firms (Whitcomb [1972]). However, as assumed in Section 1, if transaction costs of decentralized individual action are sufficiently large to justify formation of a central authority, the incentive for "second round" bargaining between receptors and emitters is insignificant and thus can be neglected. Under these circumstances, a unilateral Pigouvian tax adjusted for transaction costs can be implemented to approximate Pareto efficient conditions. The degree of approximation is, of course, a direct function of the transaction costs.

associated with the various receptors or victims of the externality. These functions in some situations are difficult, if not impossible to determine and in most situations contain substantial uncertainty. Faced with such limited information, Baumol [1972] has argued that public agencies should act on the basis of a set of minimum standards of acceptability. These standards are presumed operational since policy makers quite naturally think in terms of minimum acceptability standards. Hence, unlike the Pigouvian approach, this formulation assumes that an aggregate physical standard of acceptable waste levels is forthcoming from an informed political process. Given this standard, the public agency seeks to determine a fixed per unit charge (tax) on environmental wastes capable of achieving the predetermined standard.

An obvious advantage of this approach is simply that it requires little public agency information on receptors for its implementation. To be sure, it does not dispose of difficulties involved in capturing a true optimum.[12] Only if the predetermined standards happen by chance to equal the Pareto optimum levels will this approach lead to the same set of taxes as the Pigouvian approach. In any event, if the taxes are equal to the aggregate shadow prices of environmental wastes at the standard levels, the prespecified standards will be achieved by all firms who employ their available resources rationally.[13] A significant result of this approach is that predetermined standards, at least in principle, will be achieved at minimum cost to society.

2.2. *Externality Measurement*

A major difficulty confronted in attempting to apply either of the above schemes is that they both assume externality outputs are directly accessible to the public control agency. In an operational context, as noted in our introductory comments, this assumption is untenable. That is, these internalization policies should not be stated in terms of w_{jt}, an inaccessible vector of externality outputs from the policy maker standpoint, but instead in terms of say w_{jt}^m, a stochastic measurement vector of the externality outputs w_{jt}. Once this distinction is recognized, the j-th firm's optimization problem after internalization becomes

$$\text{(2.3)} \qquad \max V_{j0}^* = E\left\{\sum_{t=0}^{T} \beta_j^t[p_{jt}q_{jt} - C_j(q_{jt}, w_{jt}) - \tau_{jt}(w_{jt}^m, \ldots)]\right\}$$

i.e., maximize the expected present discounted value of net profits after internalization, where E denotes the expected value operator (conditional on information available at $t = 0$).[14]

From the standpoint of the firm, w_{jt} is deterministic while its monitored or measured value w_{jt}^m is stochastic. The relationship between these variables will be represented as

$$\text{(2.4)} \qquad w_{jt}^m = H_w(n_{jt}, g_{jt})w_{jt} + v_{jt}$$

[12] As Baumol [1972, p. 320] points out it sweeps all of these difficulties under the rug.

[13] Note that there is no need to assume that the firms are perfect competitors or that they maximize any particular target variable. In fact, all this approach requires is that firms produce whatever output they select at minimum cost.

[14] Note that we implicitly assume each firm's utility is a linear function of profits and thus that each firm is risk neutral. As before, this assumption is advanced to simplify the exposition while maintaining an empirically useful formulation.

where $H_w(\cdot)$ is a known deterministic $K \times K$ diagonal matrix and v_{jt} is K component stochastic vector, composed of continuous random variables, with mean vector zero and a stationary, scalar covariance matrix. Furthermore, each component of v_{jt} is assumed to be distributed independently over time. The matrix $H_w(\cdot)$ is conditioned upon n_{jt}, the number of observations made by the public control agency during period t, and g_{jt}, the requirements set by the control agency for certification of the firm's control device effectiveness. The former variable might be expanded to include the frequency, accuracy, and form of inspection and monitoring actions by the control agency. The g_{jt} variable might be interpreted as the "set up" components of the monitoring system or simply the factors associated with compliance testing and certification.

The matrix $H_w(\cdot)$ will be specified as the sum of two components, an identity matrix, and a "small sample bias" matrix. That is,

$$H_w(n_{jt}, g_{jt}) = I + \tilde{H}_w(n_{jt}|g_{jt}) \tag{2.5}$$

where $\lim_{n_{jt}\to\infty} \tilde{H}_w(n_{jt}|g_{jt}) = 0$. In other words, the monitoring system for a given g_{jt} is assumed to be based on a sampling procedure which is asymptotically unbiased. What this all implies is that while the first two terms, $p_{jt}q_{jt}$ and $C_j(\cdot)$, appearing on the right-hand side of (2.3) are deterministic, the third or internalization term is stochastic. Hence, the expectation operator need only apply to $\tau_{jt}(\cdot)$.

2.3. *Firm Legal Recourse*

To provide a realistic specification on the additive tax internalization component $\tau_{jt}(\cdot)$, the monitoring and taxing authority of the public control agency will be separated from a court or settlement system which resolves conflicts between the public agency and emitter firms. In particular, emitter firms may object to public agency measurements and seek the assistance of the court system to reduce these levels.[15] Such conflicts between firms and the public agency may be resolved by settlement with or without court trial; the threat of a court trial, of course, provides the basic incentive for an out of court settlement. To simplify the following exposition, no distinction will be made between court litigation and out of court settlements.[16]

The perceptions of the j-th firm with respect to court resolution of conflicts on w_{jt} will be specified as

$$w^l_{jt} = w^m_{jt} + W^l(l_{jt}, l_{cjt}, n_{jt}) \tag{2.6}$$

where w^l_{jt} denotes the court determined level of wastes, l_{jt} denotes the legal efforts incurred by the firm to defend itself against the control agency, l_{cjt} denotes the legal prosecution efforts of the public agency, and w^m_{jt} and n_{jt} are as previously defined. Furthermore, the stochastic internalization function for the tax schemes and a court system to resolve conflicts may be stated as

$$\tau_{jt}(\cdot) = u_{jt}w^l_{jt} + C_{mj}(g_{jt}, w_{jt}) + C_{lj}(l_{jt}) \tag{2.7}$$

[15] This structure is one of a number of possible institutional structures that might be considered. Other structures include firm reporting of externality wastes and public agency determination of the accuracy of these declarations by their monitoring measurements; public agency measurements and no court or settlement system; and firm reporting but no public agency measurements (Rausser [1975]).

[16] For a treatment of this distinction, see Gould [1973] and Posner [1972, 1973].

where u_{jt} is a K component row vector of constant per unit taxes at time t; $C_{mj}(\cdot)$ represents the monitoring "set up" and reporting costs imposed upon the firm; $C_{lj}(\cdot)$ is the cost of legal services; and l_{jt} is the amount of legal services purchased by the j-th firm. The tax vector, u_{jt}, is set by the public control agency in either a Pigouvian or Baumol–Oates fashion.

Employing (2.4) and (2.6), the expected value of the firm internalization cost (2.7) is

$$E\{\tau_{jt}(\cdot)\} = u_{jt}H_w(\cdot)w_{jt} + u_{jt}W^l(\cdot) + C_{mj}(\cdot) + C_{lj}(l_{jt}). \tag{2.8}$$

The four terms of the expected internalized costs (2.8) may be given specific interpretations. For the j-th firm, the first term is the total expected tax bill, given the firm accepts the measured emissions of the public control agency. If it does not accept these measurements, this total expected tax bill is reduced by the second term, the tax savings resulting from a court trial or settlement.[17] The term $C_{mj}(\cdot)$ is total monitoring and reporting costs borne by the firm, and $C_{lt}(\cdot)$ is its total legal expenditure.

2.4 *Firm Decision Rules and Behavioral Equations*

Substituting (2.8) into (2.3) and assuming the usual differentiable and continuity properties of the functions $C_j(\cdot)$, $W^l(\cdot)$, $C_{mj}(\cdot)$, and $C_{lj}(\cdot)$, the first-order conditions for a firm optimum may be represented as

$$p_{jt} - \frac{\partial C_j(\cdot)}{\partial q_{jt}} = 0 \tag{2.9}$$

$$-\frac{\partial C_j(\cdot)}{\partial w_{jt}} - u_{jt}H_w(\cdot) - \frac{\partial C_{mj}(\cdot)}{\partial w_{jt}} = 0 \tag{2.10}$$

and

$$-u_{jt}\frac{\partial W^l(\cdot)}{\partial l_{jt}} - \frac{\partial C_{lj}(\cdot)}{\partial l_{jt}} = 0. \tag{2.11}$$

The first condition (2.9) is the usual firm decision rule for saleable outputs, viz., equate the price of output to associated marginal cost for each saleable output. In the case of externality outputs, condition (2.10) deviates from that found in the economic literature on environmental externalities. More specifically, instead of equating firm marginal control costs $(-\partial C_j(\cdot)/\partial w_{jt})$ to the per unit tax rate, condition (2.10) suggests that the rational firm in the context of (2.3) will equate its expected per unit tax rate $(u_{jt}H_w(\cdot))$ to its marginal control costs plus the marginal enforcement costs borne by the firm $(-\partial C_{mj}(\cdot)/\partial w_{jt})$ and resulting from society's attempt to control environmental wastes. Finally, condition (2.11) suggests that the firm will purchase legal inputs up to the point where the expected marginal revenue product is equal to the price of legal inputs $(p_{lt} = \partial C_{lt}(\cdot)/\partial l_{jt})$.

[17] Note that, in general, $W^l_{jt} \leq W^m_{jt}$, $\partial W^l(\cdot)/\partial l_{jt} \leq 0$, and $W^l(0, l_{cjt}, n_{jt}) = 0$.

The above conditions lead to the following behavioral equations for firm actions on q_{jt}, w_{jt}, and l_{jt}. These equations may be represented as

$$q_{jt} = Q_{jt}(p_{jt}, u_{jt}) \tag{2.12}$$

$$w_{jt} = W_{jt}(p_{jt}, u_{jt}) \tag{2.13}$$

and

$$l_{jt} = L_{jt}(p_{jt}, u_{jt}) \tag{2.14}$$

where it is assumed that each firm takes n_{jt}, g_{jt}, l_{cjt}, and all its input prices as given.

3. Components of the Public Control Agency

The immediate concern of the public agency is to influence the behavior of w_{jt} by its setting of taxes, u_{jt}. These actions, for the framework advanced in Section 2, also influence the behavior of q_{jt} and l_{jt}. The criteria by which the public agency makes these decisions must be based, in part, upon firm emission devices, monitoring, and legal costs along with the public agency monitoring, control implementation, and legal costs. In addition, the social costs of reductions in saleable outputs as well as the social benefits of reductions in damages resulting from public agency decisions should be taken into account.

For most empirical situations, damages emanating from environmental externalities occur at receptor locations which differ from the emitter locations. Hence, externality concentration states at the receptor locations, their measurements, and the dispersion relationships between these states and the emission outputs (w_{jt}) are required. This component along with transaction costs composed of information, monitoring, and enforcement and the public agency criterion function are the topics of this section.[18]

3.1. *Information and Monitoring*

Externality policy, in a stochastic context, requires two principal types of information, viz., initial estimation and monitoring. The former is composed of information on initial levels of the state variables, their transformation functions over time, and the measurement system equations. The latter equations extend over the control horizon and provide a basis for estimating the state variables which are inaccessible to the public control agency.

Monitoring of externalities can assume many forms and take place in many locations (Rausser and Fishelson [1974]). In our treatment, monitoring will be performed to identify the emission measurement stations (point sources or representative locations), estimate the levels of the externality outputs and the concentration of environmental wastes at various receptor locations. The principal monitoring methods available include estimating the externality states by process definition or equipment specification; by periodic sampling at random times; and by continuous monitoring.[19] The first method is the least expensive and also the least

[18] For a more detailed analysis of these components in the context of a particular environmental externality, see Rausser and Fishelson [1974].

[19] Strictly speaking, without a dispersion specification for each emitter firm, only the second and third methods are possible for monitoring at receptor locations.

precise. The last approach is the most precise and expensive surveillance method. Unfortunately, available technology is not sufficiently advanced to provide accurate measurements by use of this method. Thus, we shall only be concerned here with the statistical sampling method of monitoring. This method may include self declarations of emissions by individual firms with monitoring employed to determine the accuracy of the declarations.

The use of statistical monitoring to measure environmental externalities differs from the usual measurement system described in the control theory literature (Aoki and Li [1969] and Kushner [1969]). As typically specified, a single measurement unit is employed which is either "on" or "off" during a particular time period. In this situation, the variance of the measurement observation is either finite or infinite. The environmental monitoring system for a given region, however, invariably consists of several measurement points that can be operated separately or simultaneously during a time period. All sources may be measured randomly with the same frequency (uniform sampling) or in a responsive or sequential fashion where the frequency of measurements is conditioned upon measured emissions. The framework advanced in Section 4 will admit the latter type of monitoring but will not explicitly treat the spacing or scheduling problem.

The monitoring system at the emission sites is reflected in the specification of firm behavior by the variable w_{jt}^m and at the receptor sites by y_{jt}^m. As in the case of (2.3), monitored receptor concentrations of environmental externalities will be represented by

$$y_{st}^m = H_y(n_{st}, g_{st})y_{st} + v_{st} \tag{3.1}$$

where the $K \times K$ known matrix $H_y(\cdot)$ is specified as

$$H_y(n_{st}, g_{st}) = I + \tilde{H}_y(n_{st} | g_{st}); \tag{3.2}$$

$\lim_{n_{st} \to \infty} \tilde{H}_y(n_{st} | g_{st}) = 0$; $s = 1, \ldots, S$ denotes the receptor site at which monitoring takes place; n_{st} denotes the number of observations at site s during period t; g_{st} denotes the initial "set up" factors associated with system at site s; and v_{st} is a K component stochastic vector, composed of continuous random variables, with mean vector zero and a stationary, scalar covariance matrix. Each component of v_{st} is assumed to be distributed independently over time but not necessarily independently of contemporaneous components in measurement errors at the emission sites, (v_{jt}). In our treatment, the initial "set up" components, g_{jt} and g_{st}, will be taken as given and thus the precision of the state variable estimates, w_{jt}^m and y_{st}^m, obtained by monitoring will be stated in terms of n_t where $n_t' = [n_{wt}, n_{yt}]$, $n_{wt}' = [n_{1t} \ldots n_{Jt}]$, and $n_{yt}' = [n_{1t} \ldots n_{St}]$. Hence, public agency variable costs associated with monitoring, including administration, during period t will be represented as $C_{mt}(n_t)$.

3.2. *Enforcement*

Monitoring measurements at both emitter and receptor locations represent an enforcement activity. If firms do not report emission outputs, measurements must be performed by the public agency before tax controls can be applied. Moreover, if firms object to public agency measurements, legal settlements or court determination of emission outputs will be required. In this instance, legal costs will be incurred by the public agency. These costs during period t will be represented as $C_{lt}(l_{ct})$ where $l'_{ct} = [l_{c1t} \dots l_{cJt}]$. In the determination of l_{cjt}, $j = 1, \dots, J$, the public agency is constrained by court behavior; in particular, court determination of w^l_{jt}. Although the public agency perception of this court (or settlement) determined component may differ from the firm, it will be assumed equivalent to (2.6).

3.3. *Dispersion and Damages*

To implement the Pigouvian tax scheme, we require both global damage and dispersion relationships. For the Baumol–Oates tax scheme, "localized damage" and dispersion measures are needed. For this scheme, since taxes are employed to achieve predetermined targets or standards, only localized measures of damages incurred by deviating from standards are required. The dispersion relationships for both schemes are necessary since damages occur at receptor locations which differ from emission sites. Moreover, externality states at the receptor locations are usually stated in terms of concentrations (e.g., parts per million) while externality states at the emission sites are expressed on a weight per unit time basis.

In most empirical situations, estimation of individual receptor dispersion and damage functions required for a Pareto optimum are simply impractical. Assuming a few relevant receptor locations can be identified,[20] the required dispersion functions summarize relationships between average concentration at each of these locations (which are S in number) and externality output rates at each of the J emission sources. These relationships depend upon climatic conditions, geography, and chemical reactions. As noted in Tietenberg [1974], they involve four main phases—transport, dilution, depletion, and reaction. These phases will be subsumed in the following specification

$$y_{t+1} = y_t + f(w_t, y_t, e_t) \tag{3.3}$$

where $y'_t = [y_{1t} \dots y_{st}]$ denotes a vector of externality concentrations at representative receptor locations during period t; $f(\cdot)$ denotes the steady state dispersion function, ($\partial f_t/\partial w_t > 0$, $\partial f_t/\partial y_t < 0$, and $\partial f_t/\partial e_t \gtreqless 0$); $w'_t = [w_{1t} \dots w_{Jt}]$; and e_t denotes a vector of uncontrollable exogenous factors, e.g., weather conditions. Although this specification simplifies the actual process, it is nevertheless more complex than those which have been previously employed (Tietenberg [1974]).

3.4. *Criterion Function*

To evaluate alternative controls, a criterion function for the tax internalization schemes must be specified. On efficiency grounds, this function should reflect the damages resulting from environmental externalities and the costs of controlling these externalities. In Section 4, damages will be quadratic in the externality concentration states; the control device, monitoring, and enforcement costs borne by the firm will be quadratic in the externality output states; social costs of reductions in saleable outputs will be quadratic in the normal output states; public

[20] Factors affecting the selection of receptor locations include (i) the degree of physical homogeneity of the externality airshed, watershed, or region, (ii) the effects of exogenous influences such as weather, and (iii) the degree of homogeneity over receptor preferences.

agency administrative costs will be quadratic in the behavioral controls; public agency legal enforcement will be linear and separable across behavioral and measurement controls; and public agency measurement costs will be an additive, nonlinear function of measurement controls. The criterion function will incorporate all six of these components, and the objective is to minimize its expected value over the public agency planning horizon.

The quadratic form of the criterion function is both analytically tractable and adaptable to alternative internalization schemes. Moreover, it is well suited for externality policy problems. The symmetric property of this form reflects the social losses from either insufficient or excessive internalization which are, for many operational problems, equally costly to society. It also allows possible risk aversion, a property commonly observed in public agency behavior.

4. Stochastic Control of Externalities

The problem of public control of externalities emitted by decentralized firms is expressed here as a discrete linear quadratic Gaussian control problem. To obtain a tractable solution which can be easily applied, we assume that the firms take the public agency measurement controls as given while public agency takes firm legal efforts as given. Under these assumptions, the controls are those that act on the behavioral system of the decentralized firms and those that affect the outcome of the monitoring system. The behavioral controls are u_t while the latter controls are n_t and l_{ct}. Using the notion of sufficient statistics and Bellman's [1961] principle of optimality, the model is shown to be separable into three distinct phases: the derivation of the optimal deterministic behavioral controls; derivation of the optimal monitoring controls; and the sequential estimation of inaccessible state variables by a linear Kalman filter.[21]

4.1. *Specification of Policy Problem*

The cost of the state variables in time t will be represented as $2a_t'z_t + z_t'A_tz_t$ where deleting the t subscript for the sake of convenience

$$(4.1)\qquad z = \begin{bmatrix} w \\ q \\ y \end{bmatrix}, \quad A = \begin{bmatrix} A_{ww} & A_{wq} & 0 \\ A_{qw} & A_{qq} & 0 \\ 0 & 0 & A_{yy} \end{bmatrix}, \quad a' = \begin{bmatrix} a_w \\ 1/2p \\ a_y \end{bmatrix}.$$

In terms of the firm behavior, A_{ww} and a_w denote the current additive coefficient effect of changes in w_t while A_{ww} and A_{wq} denote the current interaction coefficient effect of changes in w_t and q_t on firm control and monitoring costs; A_{qq} denotes the current additive coefficient effect of changes in q_t and A_{qw} denotes the current interaction coefficient effect of w_t on firm saleable output costs; and p_t denotes the saleable output price vector. The submatrix A_{yy} of A and a_y denote the current coefficient effect of changes in y_t, the SK component vector of externality concentrations.

[21] For derivation and explanation of the linear Kalman filter, see Kalman [1960].

The implementation and administrative costs of the behavioral controls to the agency will be represented as $2b_t'u_t + u_t'B_tu_t$, the monitoring costs as $C_t^m(n_t)$, and the agency legal costs as $C_{ct}^l(l_{ct})$. Given these definitions, the public criterion function for a planning horizon of length T may be expressed as

$$(4.2)\qquad V = E\left\{\sum_{t=0}^{T-1} [2a_t'z_t + 2b_t'u_t + z_t'A_tz_t + u_t'B_tu_t + C_t^m(n_t) + C_{ct}^l(l_{ct})] + 2a_Tz_T + z_TA_Tz_T\right\}.$$

The matrices and vectors a_t, b_t, A_t, and B_t are expressed in present value terms, i.e., the coefficients incorporate the public discount rate.

The constraints for the externality state variables are derived from the firm behavior equations (2.12), (2.13), and the dispersion relationships (3.3). If the firm functions $C_j(\cdot)$, $W^l(\cdot)$, $C_{mj}(\cdot)$, and $C_{lj}(\cdot)$ are quadratic or if they can be reasonably approximated by no more than a second-order Taylor series expansion, the firm behavioral equations will be linear. Furthermore, if emitter firms form expectations on output prices, externality taxes, etc., adaptively, the firm behavioral system can be represented as a set of first-order difference equations. Additive stochastic disturbances should also be incorporated to reflect unpredictable variations in firm activities (2.12) and (2.13) from the public agency standpoint. When these equations are combined with (3.3), we have a block recursive system in the current state variable vector z_t. Assuming $f(\cdot)$ in (3.3) is linear, this system can be cast into its reduced form which will be represented as

$$(4.3)\qquad z_{t+1} = \phi_tz_t + \psi_tu_t + \xi_t, \qquad t = 0, \ldots, T.$$

Depending upon the actual empirical situation, (4.3) may be a simple first order or a "compact" first order, i.e., y_t may include current and lagged values of itself as well as current and possibly lagged control variables. Note that ξ_t incorporates both uncontrollable exogenous variables and their effects on z_t, and the stochastic disturbances entering the various equations.

The monitoring system on the inaccessible state variables may be stated as

$$(4.4)\qquad z_t^m = Z_t^l(l_{ct}, n_t|l_t) + H_t(n_t|g_t)z_t + v_t, \qquad t = 0, \ldots, T$$

where

$$(4.5)\qquad z_t^m = \begin{bmatrix} w_t^l \\ q_t^m \\ y_t^m \end{bmatrix}, \quad Z_t^l(\cdot) = \begin{bmatrix} W_t^l(l_{ct}, n_{wt}/l_t) \\ 0 \\ 0 \end{bmatrix}, \quad l_t = \begin{bmatrix} l_{1t} \\ \vdots \\ l_{Jt} \end{bmatrix}$$

$$H_t(\cdot) = \begin{bmatrix} H_{wt}(n_{wt}/g_{wt}) & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & H_{yt}(n_{yt}/g_{yt}) \end{bmatrix}, \quad v_t = \begin{bmatrix} v_{wt} \\ v_{qt} \\ v_{yt} \end{bmatrix}$$

In other words, the only inaccessible states of importance are those associated with firm emissions (w_t) and receptor concentrations of externalities (y_t). Note that although y_t^m refers to the effective measures of the externality states at the receptor locations, w_t^m represented by (2.4) is not an effective measurement vector. Instead, the effective measurement vector of the externality states at the emission sources is w_t^l, the court determined levels of w_t. These determined levels depend upon the public agency measurements at firm sites (w_t^m); more specifically, the subvector w_t^l of z_t^m is simply a condensed version of (2.4) and (2.6) for all emission sources.

The stochastic components of the above model have the Gaussian distributions:

$$
\begin{aligned}
p(z_0) &= \delta_1 \exp[z_0 - \bar{z}_0)'(Q_0)^{-1}(z_0 - \bar{z}_0)] \\
p(\xi_t) &= \delta_2 \exp[\xi_t' Q_t^{-1} \xi_t] \qquad (4.6) \\
p(v_t) &= \delta_3 \exp[v_t' R_t^{-1}(n_t, l_{ct}) v_t]
\end{aligned}
$$

where δ_1, δ_2, and δ_3 are appropriate constants; Q_t and R_t are the covariance matrices of disturbance terms ξ_t and v_t; Q_0 is the covariance of the initial period state estimates; and $\bar{z}_0$ is the initial state estimate. Note that monitoring precision is reflected by $R_t^{-1}(n_t, l_{ct})$.

The behavioral and monitoring controls are constrained by their respective admissibility sets:

$$
u_t \in U, \qquad n_t \in N. \tag{4.7}
$$

For the behavioral controls, the set represents the limits of politically and legally acceptable controls. The monitoring control set is constrained by physical feasibility which is defined in terms of the monitoring "capital complex."

4.2. *Separation of Controls*

From (4.3) and (4.4), the state variables in any period t are functions of agency controls u_{t-1}, n_t, l_{ct}, and all previous values of controls and monitoring observations Z^m. All of this information may be summarized by the information state Ξ_t which is defined as

$$
\Xi_t \equiv h(z_t | Z_t^m, U_{t-1}, N_t, L_t, n_0, l_{c0}) \tag{4.8a}
$$

where $Z_t^m \equiv (z_0^m \ldots z_t^m)$, $U_{t-1} = (u_0 \ldots u_{t-1})$, $N_t = (n_1 \ldots n_t)$, and $L_t = (l_{c1} \ldots l_{ct})$. A recursive equation for the information state, i.e.,

$$
\Xi_{t+1} = F_t(\Xi_t, z_{t+1}^m, u_t, n_{t+1}, l_{ct+1}), \qquad t = 0, \ldots, T-1 \tag{4.8b}
$$

may be found by application of Bayes' rule.[22] Using Bellman's [1961] principle of optimality, the recursive relation for the criterion function can be stated in terms of Ξ as

$$
J_t(\Xi_t) = \underset{u_t, n_{t+1}, l_{ct+1}}{\mathrm{Min}} \left(V_t(\Xi_t, u_t, n_{t+1}, l_{ct+1}) + E\{J_{t+1}[F_t(\Xi_t, u_t, n_{t+1}, l_{ct+1}, z_{t+1}^m)]\}\right) \tag{4.9}
$$

subject to (4.7) where the expectation E on the second component is taken with respect to z_{t+1}^m. Since the behavioral equations and the measurement system are specified as linear with Gaussian error terms, the conditional update process of

[22] Our treatment is similar to that found in Meier, L., *et al.* [1967] who examined physically constrained measurements in the context of radar systems.

the information state Ξ_t (4.8b) is most efficiently performed by the Kalman filter. It follows that the information state can be specified by the sufficient statistics from the Kalman filter, viz., $\hat{z}_{t/t}$ the mean updated estimate of z_t, and the covariance update matrix $\hat{P}_{t/t}$. Thus, $\Xi_t = (\hat{z}_{t/t}, \hat{P}_{t/t})$.

Employing $\hat{z}_{t/t}$ and $\hat{P}_{t/t}$ in the first term of (4.9) after taking expectations and neglecting the uncontrollable exogenous variables entering ξ_{1t}, we have

$$V_t(\Xi_t, u_t, n_{t+1}, l_{ct+1}) = 2a_t'\hat{z}_{t/t} + 2b_t'u_t + \hat{z}_{t/t}'A_t\hat{z}_{t/t} + u_t'B_tu_t + C_{t+1}^m(n_{t+1}) + C_{ct+1}^l(l_{ct+1}) + \mathrm{tr}\,[\hat{P}_{t/t}A_t], \quad t = 0, \ldots, T-1. \tag{4.10}$$

From standard results on the deterministic linear control model,[23] the second component of (4.9) may be expressed as

$$J_{t+1}(\Xi_{t+1}) = \hat{z}_{t+1/t+1}'P_{t+1}\hat{z}_{t+1/t+1} + 2\rho_{t+1}\hat{z}_{t+1/t+1} + \mathrm{tr}\,[P_{t+1}\hat{P}_{t+1/t+1}] + J_{t+1}^m(\hat{P}_{t+1/t+1}) + \eta_{t+1} \tag{4.11}$$

where $J_{t+1}^m(\cdot)$ is the value function for the measurement and agency legal system, and the term η_{t+1} is independent of u_t, n_{t+1}, and l_{ct+1}. The symbols P_t and ρ_t refer to the recursive cost matrix and vector, respectively, which are derived as

$$P_t = A_t + \phi_t'P_{t+1}\phi_t - P_{t+1}^* \tag{4.12a}$$

where

$$P_{t+1}^* = \phi_t'P_{t+1}\psi_t(\psi_t'P_{t+1}\psi_t + B_t)^{-1}\psi_t'P_{t+1}\phi_t \tag{4.12b}$$

and

$$\rho_t = a_t' + \rho_{t+1}\phi_{t+1} - \rho_{t+1}^* \tag{4.13a}$$

where

$$\rho_{t+1}^* = (\rho_{t+1}\psi_t + b_t')[\psi_t'P_{t+1}\psi_t + B_t]^{-1}\psi_tP_{t+1}\phi_t. \tag{4.13b}$$

Calculating $\hat{z}_{t+1/t+1}$ by using its sufficient statistics in terms of the available estimate $\hat{z}_{t/t}$ yields

$$E\{\hat{z}_{t+1/t+1} \mid \hat{z}_{t/t}, \hat{P}_{t/t}\} = \phi_t\hat{z}_{t/t} + \psi_tu_t \tag{4.14}$$

where, for sake of simplicity, $E(\varepsilon_t)$ is assumed to be zero. Furthermore,

$$\begin{aligned} &E\{[z_{t+1}^m - H_{t+1}(\phi_t\hat{z}_{t/t} + \psi_tu_t)][z_{t+1}^m - H_{t+1}(\phi_t\hat{z}_{t/t} + \psi_tu_t)]'\} \\ &\quad = E\{[v_{t+1} + H_{t+1}(z_{t+1} - \phi_t\hat{z}_{t/t} - \psi_tu_t)] \\ &\qquad \cdot [v_{t+1} + H_{t+1}(z_{t+1} - \phi_t\hat{z}_{t/t} - \psi_tu_t)]'\} \\ &\quad = R_{t+1} + H_{t+1}(\phi_t\hat{P}_{t/t}\phi_t' + \hat{Q}_t)H_{t+1}'. \end{aligned} \tag{4.15}$$

That is, the prediction error covariance of $\hat{z}_{t+1/t+1}$ is composed of the monitoring system error covariance in $t+1$ and the filter mean prediction error covariance,

[23] For derivation and proof of the deterministic control model and its recursive cost matrices P_t and P_{t+1}^*, see Joseph and Tou [1961].

which is itself a function of the covariance update in t and the state transition equation covariance in time t.

To manipulate (4.11) in terms of $\hat{z}_{t/t}$, we require the following results from the Kalman filter:

the filter gain matrix

$$\hat{K}_{t+1} \equiv \hat{P}_{t+1/t}H'_{t+1}[H_{t+1}\hat{P}_{t+1/t}H'_{t+1} + R_{t+1}(n_{t+1}l_{ct+1})]^{-1}; \tag{4.16}$$

the covariance prediction equation

$$\hat{P}_{t+1/t} \equiv \hat{Q}_t + \phi_t\hat{P}_{t/t}\phi'_t; \tag{4.17}$$

the mean update equation

$$\hat{z}_{t+1/t+1} \equiv \phi_t\hat{z}_{t/t} + \psi_t u_t + \hat{K}_{t+1}[z^m_{t+1} - H_{t+1}(\phi_t\hat{z}_{t/t} + \psi_t u_t)]; \tag{4.18}$$

and the covariance update equation

$$\hat{P}_{t+1/t+1} = \hat{P}_{t+1/t} - \hat{K}_{t+1}H_{t+1}\hat{P}_{t+1/t}. \tag{4.19}$$

Proceeding by employing (4.14), we have for the first term of (4.11)

$$\begin{aligned} &E\{\hat{z}'_{t+1/t+1}P_{t+1}\hat{z}_{t+1/t+1}|\hat{z}_{t/t}\hat{P}_{t/t}\} \\ &= (\phi_t\hat{z}_{t/t} + \psi_t u_t)'P_{t+1}(\phi_t\hat{z}_{t/t} + \psi_t u_t) \\ &\quad + \operatorname{tr}\{P_{t+1}\hat{K}_{t+1}(R_{t+1} + H_{t+1}\hat{P}_{t+1/t}H'_{t+1})\hat{K}'_{t+1}\}. \end{aligned} \tag{4.20}$$

Defining the last term of (4.20) as $\operatorname{tr}\Lambda_{t+1}$ and using (4.16), we obtain

$$\operatorname{tr}\Lambda_{t+1} = \operatorname{tr}P_{t+1}\hat{P}_{t+1/t}H'_{t+1}\hat{K}'_{t+1}. \tag{4.21}$$

This expression can be restated by employing (4.19) and (4.12) as

$$\operatorname{tr}\Lambda_{t+1} = \operatorname{tr}(P^*_{t+1} + P_t - A_t)\hat{P}_{t/t} + P_{t+1}(\hat{Q}_t - \hat{P}_{t+1/t+1}). \tag{4.21a}$$

The second term of (4.11) can be expressed likewise as:

$$2\rho_{t+1}\hat{z}_{t+1/t+1} = 2\rho_{t+1}(\phi_t\hat{z}_{t/t} + \psi_t u_t). \tag{4.22}$$

Now by successive substitution of (4.21a) into (4.20); and (4.20), (4.22) into (4.11); (4.11) and (4.10) into (4.9); the value of the criterion function in t can be expressed in terms of the Kalman filter condition estimate in $t(\hat{z}_{t/t})$, i.e.,

$$\begin{aligned} J_t(\psi_t) = \min_{u_t, n_{t+1}, l_{ct+1}} &\{2a'_t\hat{z}_{t/t} + 2b'_t u_t + \hat{z}'_{t/t}A_t\hat{z}_{t/t} + u'_t B_t u_t + C^m_{t+1}(n_{t+1}) \\ &+ C^l_{ct+1}(l_{ct+1}) + \operatorname{tr}[\hat{P}_{t/t}A_t] + 2\rho_{t+1}(\phi_t\hat{z}_{t/t} + \psi_t u_t) \\ &+ (\phi_t\hat{z}_{t/t} + \psi_t u_t)'P_{t+1}(\phi_t\hat{z}_{t/t} + \psi_t u_t) \\ &+ \operatorname{tr}[(P^*_{t+1} + P_t - A_t)\hat{P}_{t/t} + P_{t+1}(\hat{Q}_t - \hat{P}_{t+1/t+1})] \\ &+ \operatorname{tr}[P_{t+1}\hat{P}_{t+1/t+1}] + J^m_{t+1}(\hat{P}_{t+1/t+1}) + \eta_{t+1}\}. \end{aligned} \tag{4.23}$$

After some simplifications, this control optimization can be separated into terms involving either the behavioral controls or the monitor and legal controls as arguments, but not both and thus can be separately optimized. That is,

$$
\begin{aligned}
(4.24)\qquad J_t(\psi_t) = \min_{u_t} \{ & 2a_t'\hat{z}_{t|t} + 2b_t'u_t + \hat{z}_{t|t}'A_t\hat{z}_{t|t} + u_t'B_tu_t \\
& + (\phi_t\hat{z}_{t|t} + \psi_tu_t)'P_{t+1}(\phi_t\hat{z}_{t|t} + \psi_tu_t) + 2\rho_{t+1}(\phi_t\hat{z}_{t|t} + \psi_tu_t) \\
& + \operatorname{tr}[P_t\hat{P}_{t|t}]\} + \min_{n_{t+1}, l_{ct+1}} \{C_{t+1}^m(n_{t+1}) + C_{ct+1}^l(l_{ct+1}) \\
& + \operatorname{tr}[P_{t+1}^*\hat{P}_{t|t}] + J_{t+1}^m(\hat{P}_{t+1|t+1})\} + \operatorname{tr}[P_{t+1}\hat{Q}_t] + \eta_{t+1}.
\end{aligned}
$$

4.3. *Behavioral Controls*

From that part of the criterion function containing the behavioral controls, it is clear that its form is the same as the familiar linear quadratic Gaussian (L.Q.G.) control model.[24] The separation properties of the L.Q.G. model allow the optimal controls to be derived separately from the derivation of the conditional estimate $\hat{z}_{t|t}$. The optimal behavioral controls are

$$(4.25)\qquad u_t = G_t\hat{z}_{t|t} + g_t$$

where the control gain matrix b_t is defined as

$$(4.26a)\qquad G_t = -(\psi_t'P_{t+1}\psi_t + B_t)^{-1}(\psi_t'P_{t+1}\phi_t)$$

$$(4.26b)\qquad g_t = -(\psi_t'P_{t+1}\psi_t + B_t)^{-1}(\psi_t'\rho_{t+1} + b_t)$$

and P_t is given by (4.12), ρ_t by (4.13), and $\hat{z}_{t|t}$ by (4.18). The significance of this result is that the optimal behavioral controls u_t are expressed in terms of G_t, g_t, P_t, P_t^*, ρ_t, ρ_t^* which are independent of the matrices R_t and H_t, and thus can be derived independently of n_{t+1} and l_{ct+1}.

4.4. *Monitoring and Legal Controls*

If the terms in (4.24) that are independent of u_t, n_{t+1}, and l_{ct+1} are specified as additive over time, then b_t is defined

$$
(4.27)\qquad
\begin{aligned}
b_t &= \operatorname{tr}[P_{t+1}\hat{Q}_t] + b_{t+1} \qquad t = 0, \ldots, T-1 \\
b_T &= \operatorname{tr}[P_{T+1}\hat{Q}_T].
\end{aligned}
$$

The optimal measurement and legal controls may therefore be obtained from the following nonlinear deterministic control problem

$$(4.28)\qquad \min_{n_t, l_{ct}} J = \sum_{t=0}^{T} \{C_t^m(n_t) + C_{ct}^l(l_{ct}) + \operatorname{tr}[P_{t+1}^*, \hat{P}_{t|t}]\}$$

subject to (4.19) and the admissibility constraints on n_t. For this problem, the Kalman covariance update function ($\hat{P}_{t|t}$) acts as the state constraint equations. Due to the nonlinearity, there is no exact analytical derivation for the optimal measurement controls. However, gradient procedures can be employed to solve this problem.

[24] For a survey of the linear quadratic Gaussian model. see Athans [1972].

4.5. *Combined Systems Control*

Examination of the separated optimal monitoring and legal control problem (4.28) shows that the optimal controls are obtainable *a priori*. The cost matrix P_t^* is obtained *a priori* from the solution of the deterministic linear control problem. Likewise, the covariance update matrix $\hat{P}_{t/t}$ is available. Thus, (4.28) can be solved for the optimal n_t and $l_{c\beta}$ for $t = 1, \ldots, T$. The solution dictates that the marginal legal and monitoring cost in a time period be equated with the imputed value of a "smaller" state covariance estimate to the public agency.

The overall solution procedure involves four principle steps. First, using the prior estimates of $\bar{z}_0$ and Q_0, derive the trajectory of G_t, P_t, P_t^*, ρ_t, ρ_t^* matrices. Second, combining the results of step one with the prior knowledge of the monitor error covariance $R(\cdot)$, derive the trajectory of optimal measurement controls and $\hat{P}_{t/t}$ over the complete planning horizon. Third, observe the monitor records for time period t, z_t^m, and using $\hat{P}_{t/t}$ from step two, calculate with the Kalman filter the conditional estimate of z_t, $\hat{z}_{t/t}$. Fourth, using $\hat{z}_{t/t}$ and the control gain matrix for the behavioral controls calculated in step one, derive the optimal behavioral controls u_t for time t given z_t^m. Steps three and four are repeated for all time periods in the horizon and all observations z_t^m. The resulting overall optimal criterion function for the problem may be stated as

$$J^* = 2\bar{z}_0'\rho_0 + \bar{z}_0'P_0\bar{z}_0 + \operatorname{tr}[P_0Q_0] + \sum_{t=0}^{T} \{\operatorname{tr}[P_{t+1}\hat{Q}_t + P_{t+1}^*\hat{P}_{t/t}] + C_t^m(n_t^*) + C_{ct}^l(l_{ct}^*)\} \tag{4.29}$$

where n_t^* and l_{ct}^* are the optimal measurement and legal controls at time t.

5. Economic Interpretations

Each of the seven terms entering the optimal loss function (4.29) have a precise economic interpretation. The first two terms, $\bar{z}_0'P_0\bar{z}_0$ and $2\bar{z}_0'\rho_0$, result from the linear decision rule which obtains by minimizing the costs of resource misallocation due to the externality and the behavioral controls as specified in the criterion function. Under the assumptions imposed, this cost is equal to the "certainty equivalent" cost. Clearly, the recursive specification of P_0 and ρ_0, i.e., (4.12) and (4.13), implies the optimality of behavioral controls and externality states over all time periods. In addition, the derivation of P_0 demonstrates that it is additive in four cost components. These components are: the cost of externalities in the current period; the cost of the present externality states in future time periods; the cost of changes in present behavioral controls in terms of future externality levels; and the administrative cost of implementing the behavioral controls. Likewise, ρ_0 is based on the same four cost components in linear form.

The third term $\operatorname{tr}[P_0Q_0]$ is the cost of uncertainty associated with the initial estimates of the state variables. The experimental information value of more precise estimates of $\bar{z}_0$ is shown not only through Q_0 but also via the Kalman filter covariances, especially in the initial stages. Reductions in the filter covariances, of course, also lower the cost of the fifth term of (4.29). The fourth term $\sum_{t=0}^{T} \operatorname{tr}(P_{t+1}\hat{Q}_t)$ is the trajectory of costs from uncertain estimates of the state transition equations. Since the covariance $\hat{Q}_t$ also affects $\hat{P}_{t/t}$ via the covariance prediction equation (4.17), returns to investment in passive information in the reduction of Q_t may be derived. Obviously, the investment in experimentation is most valuable if performed before the control program commences. The fifth term, $\sum_{t=0}^{T} \operatorname{tr}[P^*_{t+1}\hat{P}_{t/t}]$, is the cost of inaccurate filter estimates of the current state variables. It is through this term that the benefits (reductions in the measurement covariance R_t) of the measurement controls enter the criterion function. Note that, unless the functional relations of $\hat{Q}$ and R in $\hat{P}_{t/t}$ are linear, a change in the value of $\hat{Q}$ changes the information value from a given reduction in R. Reductions in the measurement covariance R are achieved by both agency measurement controls, n_t and l_{ct}. The cost reductions from agency increased monitoring precision are equated to the returns from agency legal inputs. The latter inputs are employed by the agency to minimize the costs of inaccurate adjustment of the monitored emission levels by court action. Finally, the terms $C^m_T(n_t)$ and $C^l_{ct}(l_{ct})$ are the operating costs borne by the agency of the monitoring and court system.

The separable control results of Section 4.2 and the associated economic interpretations[25] can be extended in a number of directions. Under the assumed structure of Section 4, the introduction of fixed public agency budgets which are binding requires an iterative approach if the separability between the behavioral and measurement control problems is to be maintained. This is simply because binding agency budgets must be allocated to both behavioral control and measurement control costs.

If the assumed institutional structure is modified to include firm reporting, the separability between the behavioral and measurement control problems no longer holds. For this institutional structure, a behavioral component depends upon the measurement component and thus the optimal behavioral and measurement controls must be determined simultaneously. A similar situation exists when the public agency does not take firm legal efforts as given but instead recognizes the behavioral equation (2.14). Of course, if firms do not take the measurement controls of the public agency as given, the separable result of Section 4 again breaks down. In general, if both the firms and public agency have reactions functions on the activities or policies of the other, a game theoretic formulation would be required, and an indeterminant solution would result.

As forcefully argued in a simpler context by Posner [1972] for most empirical problems involving public agency control, it is reasonable to assume that reaction functions exist only for the agency. That is, an asymmetry between the position of the emitter firms and the public control agency is presumed. For this case, emitter firms would take the policy rules on behavioral and measurement controls as given, but the public agency would take explicit account of all its rules upon the emitter firm's decision rules (2.12) through (2.14). Following Lucas [1974], Kydland and Prescott [1973] have referred to this formulation as a hierarchical structure in which the public agency is dominant. Due to space limitations, this and other modifications and extensions noted above will not be treated here;

[25] The detailed properties of the behavior controls (4.26), the measurement controls (4.29), their comparison to existing formulations of environmental externality problems, and conditions under which a stationary state obtains are presented in a technical appendix to this paper. This appendix is available upon request.

instead they will be topics examined in a future paper on environmental externality problems.

6. Empirical Implementation

The model developed in this paper is being applied to the problem of agricultural pesticide externalities. The use of pesticide inputs by the agricultural sector result in occupational injury externalities. These external effects necessitate some minor changes in the model specification advanced in this paper. Although general, it is conceptualized in the context of air, land, or water pollution externalities. Moreover, the empirical model for these externalities pertains to the State of California. In what follows, we briefly review the empirical implementation of a stochastic framework for the control of California pesticide externalities.

Given the physical and institutional setting of the problem, the internalization of pesticide externalities cannot practically be affected by a Pigouvian tax scheme. The transaction costs of identifying the marginal damage functions from point emitter sources would be so great for all but extreme worker symptoms that Pigouvian solutions are unworkable. The current institutions in California, however, readily admit a Baumol–Oates tax internalization scheme.

One departure of the empirical model from the theoretical model is to ignore the legal dimension of the firm and agency decision functions. The reason is the absence of data on legal inputs from the firms, and the very small use currently made of legal inputs and sanctions by the local enforcement agencies in California. If a policy of less bark and more bite in enforcement sanctions is adopted, the costs of legal action will doubtless enter the firm and agency decision process.

The firms using the pesticides and producing the occupational injury externalities are dominantly small family firms. As such, they will approximate the assumptions of perfectly competitive behavior and dominant agency actions of the theoretical model specification. In addition, the institutions of standards and uniform taxes to achieve those standards avoids the need for knowledge of the individual firm's production functions.

The agency controlling pesticide use in California is responsible to State Department of Agriculture. The standards governing use, and the tax rate on pesticides is legislated in the Agricultural Code; but monitoring, inspection and enforcement activities are decentralized to local County Agricultural Commissioners. Under the agricultural code the County Commissioners must be informed by a formal permit of the details of each use of a restricted pesticide. The reports are monitored for violations of application or later field work standards. The Commissioner inspects both the records of pesticide dealers to detect reporting violations and the field operations during and after a proportion of the applications. The enforcement capabilities of the Commissioner extend from formal hearings without sanctions to cancellation of operating permits which involves a pest control operator or grower in substantial costs.[26]

[26] The occupational injuries of the workers are theoretically reported and paid for through the State Workman's Compensation Fund. In practice many of the pesticide related injuries go unreported and often uncompensated, due to the nature of the symptoms that are debilitating rather than acute. Moreover, many workers are often only on daily contracts, have language problems and are ignorant of the Workman's Compensation system.

The principal components of the stochastic control framework are estimated in the following manner.

Behavioral Dispersion and Injury Equations: Three state equations were specified which pertain to firm behavior, (2.12), (2.13), acres of land allocated to agricultural production, saleable output and pesticide externality levels. In addition, dispersion relationships (3.3) are subsumed in the specification of two other state variable equations, viz. pest control worker and field worker injuries. These dynamic relationships are estimated from a time series of cross sections related to incidence rates from public health records, a primary firm worker survey, and pesticide use data. In estimating the behavioral equations, the price elasticity of demand for pesticides is based upon nationwide data.

Externality Measurement Equations: For this problem, it was not possible to estimate (2.4) on the basis of sample data. Hence, subjective estimates pertaining to the precision of pesticide externality measurement were parameterized in the model. Due to the low incidence of enforcement and high frequency of permit monitoring by County Commissioners, the rational firm would report all but the most incriminating information.[27] In the case of (3.1), sample based estimates of worker injury reporting accuracy is available. These estimates are based upon primary survey data collections and official reports for the same point in time and area; knowledge of the Workman's Compensation System by the farm workers in the primary survey; and case studies by California Department of Public Health.

Criterion Function: On the basis of the concern with industrial safety it is deduced that certain levels of occupational injury are merit goods. Thus, that portion of criterion function associated with externality damages is specified to be a quadratic function of the deviation of pesticide related worker injury rates from aggregate industrial injury rates. The weighting coefficients are the costs to the individual of pesticide injury, estimates from public and primary survey data. Firm control, monitoring and enforcement costs are aggregated and specified in the criterion function as the cost (quadratic) of pest control industry safety equipment and industry variable safety inputs. The remaining costs entering the criterion function are as listed in Section 3.4 and are stated in terms of County Commissioner control actions.

Behavioral and Measurement Controls: Using the estimates outlined above, the stochastic controls of Section 4 are presently being derived using the separable results, (4.25) and (4.28). From these control derivations, policy implications will emerge with respect to pesticide externality taxes, measurement control priorities and the value of passive experimental information on the empirical model's parameters. In this empirical setting, the implications of a common agency budget constraint across both behavioral and measurement controls will also be analyzed. To facilitate this analysis, the separability among controls will be maintained and an iterative scheme will be employed to achieve consistency between the two sets of controls and a predetermined public agency budget. This approach will allow us to compare two administrative frameworks in which tax determination and monitoring and enforcement are the responsibility of the same agency or two segregated agencies.

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[27] Examination of the monitored information shows that some gross violations are blithely reported.

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