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MACROECONOMICS

DISCRETION IN THE CHOICE OF MACROECONOMIC POLICIES

BY KENNETH GARBADE*

This paper explores the quantitative implications for aggregate economic performance and stability of conducting a discretionary policy developed from the theory of feedback control of stochastic systems. The control scheme applied here partitions the policy problem into a deterministic planning problem and a stochastic stabilization problem. The results indicate that significant gains are available from a discretionary policy over a non-discretionary policy of fixed instrument choices.

Whether macroeconomic policy for the United States should admit an element of discretion has been an issue among economists for over a decade, and is recognized as one of the principal elements of the monetarist-fiscalist debate. The question has typically been addressed by characterizing the dynamic aspects of the American economy and then asking whether the performance of an economy with such characteristics could be improved by allowing discretionary changes in policy choices from time to time. For example, in his recent review article Leonall Andersen (1973) notes the fiscalist view that exogenous disturbances of the economy "lead necessarily to recurring fluctuations in output and prices which are of a cyclical nature," and the fiscalist belief that "there does not exist a self-correction mechanism" for those fluctuations. As a consequence of these views and beliefs, Andersen observes, fiscalists "have advocated very active stabilization actions in the short run. Even if a disturbance is absorbed, the time interval is considered to be so long that economic welfare will be greatly reduced if short-run stabilization actions are not taken." On the other hand, Andersen continues, "monetarists contend that our economic system is such that disturbing forces ... are rather rapidly absorbed and that output will naturally revert to its long-run growth path following a disturbance," and they believe "that the economy is inherently stable, thereby requiring no off-setting actions."

This paper presents quantitative results on the merit of a discretionary policy relative to one that sticks to pre-selected instrument choices regardless of the evolution of the economy. Rather than follow the historic line of debate and examine the dynamic characteristics of a model of the economy we choose instead to examine the consequences of applying a specific discretionary policy. The set of policy tools and the type of discretion we consider are both limited. Only a few well-known and easily quantified instruments, including government expenditures and a tax surcharge variable, are treated. We do not address the problem of choosing policies of a microeconomic nature, e.g., anti-trust policy or wage and price controls. The discretion we permit is limited to a functional relationship

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between the magnitude of policy instruments in the current period and the state of the economy in past periods. Once these relationships have been defined the element of choice disappears.

The first section of the paper relates the method of analysis, placing it in the context of previous work by economists and control engineers on the feedback control of dynamic systems. Here we develop the relationship between instrument choices and past states of the economy characterizing our discretionary policy. The second section briefly describes an econometric model of the post-World War Two United States economy and the criterion function which we use to measure macroeconomic performance. The third section presents the optimal instrument values for the model with respect to the criterion function when uncertainty is ignored. The last section presents results from simulation of the model in the presence of uncertainty when policy choices are first kept constant and then permitted to vary in response to past states of the economy. The results focus on the contribution of a discretionary policy to stabilization of economic activity and to the improvement of average economic performance as measured by the criterion function.

1. Method of Analysis

Consider an economy described by the reduced-form dynamic model:

(1a) $$x_t = f_t(x_{t-1}, u_t, \zeta_t) \qquad t = 1, 2, \ldots$$

(1b) $$x_0 = \bar{x}_0$$

(1c) $$\zeta_t \sim p(\zeta_t)$$

where x_t is the n-dimensional vector representing the state of economic activity in time t, u_t is the m-dimensional vector of choices for the policy instruments and ζ_t is a vector of random exogenous disturbances with density function $p(\zeta_t)$. We assume ζ_t and ζ_s are statistically independent for $t \neq s$. The econometric model presented in Section 2 can be considered to be in the form of equation (1). Unless the dynamic structure of the system is trivial, e.g., $x_t = f_t(u_t, \zeta_t)$, realizations of ζ_t affect the state of the economy in periods after time t as well as in time t. Whether such exogenous effects are persistent or dissipate rapidly can only be discovered by inspection of the actual model, so the general form of (1) does not pre-judge either the monetarist or fiscalist positions on the matter of persistence of exogenous shocks.

In order to select values for the policy instruments we require a criterion that indicates whether a particular policy strategy is better or worse than another strategy. Our criterion is a serially additive loss function on a state trajectory of finite duration:

(2) $$L(X) = \sum_{t=1}^{T} f_t^0(x_t) \qquad X = (x_0, x_1, \ldots, x_T)$$

where f_t^0 is a scalar-valued function of the state of the economy in period t. We exhibit the actual loss function used in this paper in Section 2.

The optimal policy strategy is obtained by solving the problem of choosing feedback functions:

(3) $$u_t = g_t(x_{t-1}) \qquad t = 1, \ldots, T$$

to minimize the expectation of loss subject to the constraint of the model. This problem can, in principle, be solved by application of the principle of dynamic programming (Bellman, 1957). If no restrictions are placed on the class of admissible feedback functions the optimal functions will generally not be constant with respect to state. Thus optimal discretion in the choice of policies will improve the performance of the economy as measured by the expectation of loss, and restriction of the policy strategy to state-invariant policies will result in at least as large an expected loss. However, in the extreme case of a trivial dynamic model, $x_t = f_t(u_t, \zeta_t)$, where persistence is absent, the optimal feedback functions are in fact constant with respect to x_{t-1}.

As is well-known (Astrom 1970, Chow 1972a, for example), the special case of a linear model and quadratic loss function leads to analytic expressions for the feedback functions. Quantitative aspects of that problem in a macroeconomic context have been investigated by Chow (1972b). As a practical matter for most other models or loss functions the feedback functions of equation (3) may be impossible to obtain analytically. Since the model we use here is non-linear, and the loss function is non-quadratic, an approach other than direct application of dynamic programming is required.

The mean disturbance method, well known to control engineers and summerized by Athans (1972), is one possible alternative. The method begins by asking for that policy sequence, $U = (u_1, \ldots, u_T)$, which solves the deterministic problem:

(4a) $$\min_U \sum_{t=1}^{T} f_t^0(x_t)$$

subject to:

(4b) $$x_t = f_t(x_{t-1}, u_t, E(\zeta_t))$$

(4c) $$x_0 = \bar{x}_0$$

derived from the original problem by replacing the random vector ζ_t with its expected value. This replacement is admittedly ad hoc, and the resulting deterministic model may not exhibit any particularly desirable properties. For example, as Howrey and Kelejian (1971) have noted, unless the model is linear it may not follow that:

(5) $$E(f_t(x_{t-1}, u_t, \zeta_t) | x_{t-1}, u_t) = f_t(x_{t-1}, u_t, E(\zeta_t))$$

but we hope the true property is not too different from (5). In replacing the disturbance with its expected value some comfort is derived from the observation that most simulations of macro-econometric models are presented for model (4b), derived from (1a) in the manner prescribed. [Nagar (1969) is an exception.] Computation of the solution to problem (4), while perhaps difficult, is not impossible, since it requires minimization over a finite number of parameters (Canon, Cullum and Polak 1970, Polak 1971 and Himmelblau 1972).

Let $\bar{U}$ be the solution to problem (4), which we will call the "nominal" policy sequence, and let $\bar{X}$ be the nominal state trajectory associated with $\bar{U}$ by model (4b, c). In Section 3 we present the values of $\bar{U}$ and $\bar{X}$ for the model and loss function described in Section 2. If we invariantly apply policy $\bar{u}_t$ in period t the economy will evolve according to the model:

(6a) $$x_t = f_t(x_{t-1}, \bar{u}_t, \zeta_t) \qquad t = 1, \ldots, T$$

(6b) $$x_0 = \bar{x}_0.$$

For the purposes of this paper we identify equation (6) with an economy operating under a non-discretionary policy, recognizing, however, that there are other methods of computing fixed policy strategies which yield sequences different from $\bar{U}$, e.g., open-loop optimal control of stochastic systems. In Section 4 we present the estimated expected loss (as an objective measure of performance) and the standard deviations of selected components of state (as a subjective measure of stability) derived from Monte Carlo simulation of model (6).

Since state depends on the realization of the random vectors, as well as on the choice of policies, we do not anticipate that state trajectory $\bar{X}$ will occur in any given simulation, but rather anticipate that the proximity of the economy to the contemporaneous nominal state will become increasingly uncertain through time. Since $\bar{X}$ was the optimal state trajectory subject to the constraint of the deterministic model, a reasonable, albeit heuristic, discretionary strategy might be to stabilize the state of economic activity around the trajectory $\bar{X}$. As described by Athans (1972) the second stage of the mean disturbance approach employs a first-order expansion of the original state model about the point $[\bar{x}_{t-1}, \bar{u}_t, E(\zeta_t)]$ to model the propagation of deviations in state from the nominal trajectory:

(7) $$\Delta x_t = A_t \Delta x_{t-1} + B_t \Delta u_t + e_t$$

where $\Delta x_t = x_t - \bar{x}_t$, $\Delta u_t = u_t - \bar{u}_t$, A_t is the Jacobian of f_t with respect to the state vector, B_t is the Jacobian of f_t with respect to the policy vector, both evaluated at $[\bar{x}_{t-1}, \bar{u}_t, E(\zeta_t)]$, and e_t is an n-dimensional random vector representing both the high-order terms in the expansion and the first-order contribution of the original ζ_t vector. We seek to keep Δx_t small, subject to the model of equation (7), and represent this objective as the linear/quadratic stabilization problem:

(8a) $$\min E\left(\sum_{t=1}^{T} \tfrac{1}{2} \Delta x_t' K_t \Delta x_t\right)$$

subject to:

(8b) $$\Delta x_t = A_t \Delta x_{t-1} + B_t \Delta u_t + e_t$$

(8c) $$\Delta x_0 = 0$$

where K_t is a positive-semi-definite matrix. For this paper we define K_t as the Hessian of f_t^0 evaluated at $\bar{x}_t$. Problem (8) is well-known with exact solution given by:

(9) $$\Delta u_t = G_t \Delta x_{t-1} \qquad t = 1, \ldots, T$$

with feedback matrix:

(10a) $$G_t = -(B_t' H_t B_t)^{-1} B_t' H_t A_t$$

and Ricatti equation:

(10b) $$H_{t-1} = K_{t-1} + (A_t + B_tG_t)'H_t(A_t + B_tG_t) \qquad t = T, \ldots, 2$$
$$H_T = K_T.$$

The feedback function of (9) tells us how to alter policy in period t away from the nominal policy in response to a realization of state away from the nominal state. Noting that $\Delta u_t = u_t - \bar{u}_t$, discretion is represented by the feedback function:

(11) $$u_t = \bar{u}_t + G_t(x_{t-1} - \bar{x}_{t-1}).$$

This function is in the general form of equation (3) but is not necessarily optimal since it may not satisfy the necessary conditions derived from dynamic programming. When the feedback function of (11) is incorporated into the original stochastic model the system becomes:

(12a) $$x_t = f_t[x_{t-1}, \bar{u}_t + G_t(x_{t-1} - \bar{x}_{t-1}), \zeta_t] \qquad t = 1, \ldots, T$$

(12b) $$x_0 = \bar{x}_0.$$

We identify equation (12) as representing the economy operating under a discretionary policy. In Section 4 we present the results of Monte Carlo simulation of (12), again exhibiting the estimated value of expected loss and the growth in uncertainty about future states of economic activity.

It is informative to economists to look upon the mean disturbance approach to the problem of feedback control of a stochastic economy as offering a solution in two parts. The first part is the nominal policy sequence, and provides a long-range policy plan to administrators. The second part, consisting of the feedback matrices G_t, provides a response rule for altering planned policy in the face of random and unanticipated changes in the state of economic activity. Future conditions may force change in planned policy, but if the stochastic components of the economy are not large relative to the predictable components, one expects that actual choices will be in a neighborhood of the planned choices. Moreover, if the effects of the random disturbances on the state of the economy do exhibit persistence, a scheme which takes timely action to offset the disturbances may contribute significantly to stabilizing economic activity. In so doing the scheme may forestall development of a situation where a major shift in policy is necessitated in order to cut off an extended boom or to pull the economy out of a recession. It remains to be established, however, whether the mean disturbance approach will actually dominate the non-discretionary policy. Since the feedback function of equation (11) is likely sub-optimal it is not obvious that the non-discretionary policy will actually result in poorer economic performance compared to the performance of the discretionary policy.

2. An Econometric Model and Loss Function

The model used in this study is organized about a market for aggregate product, a labor sector and a financial sector. It is a quarterly model estimated on data from 1947/I–1969/IV and based on the National Income and Product

Accounts (NIA). The model provides the linkage between choices for a set of familiar policy instruments and the behavior of some principal measures of aggregate activity like unemployment and inflation. Detailed equations of the model are listed in the appendix. We present here a brief description of those equations.

The market for gross private product is patterned on an income-expenditure structure, summing up the components of demand in the NIA product account and then working down the income side to arrive at disposable income. Demand for privately produced goods and services comes from four sectors. Households spend their disposable income on consumer services (A1), non-durable goods (A2) and durable goods (A3). Businesses invest in plant and equipment to maintain a constant capital/output ratio (A4), and in inventories as a function of current and lagged sales and the lagged stock of inventories (A5). Investment in residential structures follows from contemporaneous and lagged housing starts and the lagged stock of residential structures (A6). Imports adjust to a long-run level specified as a linear function of the rate of production (A7) while exports follow a simple autoregressive structure about a time trend (A8). Government purchases of privately produced goods and services are a policy instrument of the model. Gross private product is the sum of demand from the four sectors (A9), and gross national product is obtained by inflating private product and adding compensation of government employees (A10). The latter is an instrument of policy in the model.

On the income side gross corporate earnings are a function of current and past private production (A11). Dividends follow a lagged adjustment process on earnings (A12). Federal and state and local indirect business taxes are a function of current consumption expenditures (A13 and A14). The function for Federal taxes uses a dummy variable to split the sample period as a result of the Excise Tax Reduction Act of 1965, while the function for state and local items uses a linear time trend on the coefficient to model secularly changing schedules. Federal personal taxes are a function of personal income plus contributions for social insurance less government transfer payments less state and local personal taxes (A15). The sample period is split by a dummy variable to account for the reduction in tax rates in 1964. A scaling factor for the federal liability schedule, *S*, is a policy instrument of the model. This factor was unity over the sample period except during 1968/III–1969/IV when it was 1.1, corresponding to the 10 percent surcharge of that period. State and local personal taxes are a function of personal income plus contributions for social insurance less government transfer payments (A16). Contributions for social insurance are a function of the collection rates for the OASDHI program and for the federal unemployment insurance program and of personal income plus contributions less transfer payments (A17). Transfer payments are a function of the population over age 65, a benefit schedule factor for old age and survivors insurance, and the number of workers unemployed (A18). A dummy variable is used to account for the substantial increase in transfers which occurred when the Medicare program was introduced in 1965. Miscellaneous items in the income account are summarized by an autoregressive process about a time varying average (A19). Disposable income is the difference between gross national product and intermediate items in the income account (A20).

The labor sector traces demand for labor services from man-hours paid for (A21) as a function of the rate of production to private non-farm employment (A22) as a function of manhours paid for. Farm employment is assumed to follow a simple trend model (A23) and government employment is a policy instrument of the model. Summing private non-farm, farm and government employment yields total employment on a jobs filled basis (A24). The labor force employed follows from total employment on an assumption that multiple job holding is sensitive to the opportunity for employment as measured by the unemployment rate (A25). The total labor force (A29) is the sum of the participation of three groups. males age 25 to 54 (A26), other males (A27) and females (A28). The unemployed labor force is the difference between the total labor force and the employed labor force (A30).

The financial sector consists of equations for the corporate bond rate, change in deposits at financial intermediaries and demand for transactions balances (currency and demand deposits). The bond rate is assumed to adjust to an equilibrium level given by the Treasury bill rate and a proxy for the expected rate of inflation (A31). Demand for demand deposits follows an interest-elastic proportional transactions demand model with the bill rate and disposable income as the arguments (A32). Demand for currency follows the same type of model using the bond rate as the interest argument (A33). The change in savings deposits at commercial banks, savings and loan associations and mutual savings banks is a fraction of disposable income not expended on consumer goods and services, with the fraction varying with the spread between the bond and bill rates (A34). The change in deposits at savings and loan associations and mutual savings banks is a simple fraction of the change in savings deposits which varies linearly with time (A35). The model will accept either the bill rate or the money supply (defined as currency plus demand deposits) as an instrument of policy. In this study we use the bill rate as the policy variable, leaving the money supply as an endogenous state variable.

Explanation of housing starts, the proxy for the expected rate of inflation and the level of the price deflator for gross private product completes the behavioral equations of the model. Housing starts are assumed to follow from the flow of funds to the two major suppliers of residential mortgages, savings and loan associations and mutual savings banks, and from the change in Federal Home Loan advances to savings and loan associations (A36). The latter is an instrument of policy in the model. The proxy for the expected rate of inflation is a convex combination of its lagged value and the lagged value of the actual rate of inflation (A37). The current rate of inflation is a function of the proxy for the expected rate and the difference between the actual rate of production and a standard rate of production based on an unemployment rate of four percent (A38). The price deflator follows immediately from the rate of inflation (A39). Four indentities yield the end of quarter stocks of consumer durables (A40), producers plant and equipment (A41), residential structures (A42) and business inventories (A43) as the sum of current gross additions and the undepreciated portion of the previous period stocks.

There are three central elements of the model for policy purposes. The first is the demand for labor services as a function of gross private production coupled with the appearance of inflationary pressure when production rises above the standard rate. These two phenomena define the short-run Phillips relation between unemployment and inflation. Increases in either government purchases of privately produced goods and services or government compensation of its employees adds to demand, the former directly and the latter through the consumption functions. The consequent stimulus to the rate of production both adds to the inflationary pressure on the economy and reduces unemployment. The Phillips curve of the model is horizontal with respect to contemporaneous changes in policy. Over progressively longer runs it grows steadily steeper due to the presence of the proxy for the expected rate of inflation in the equation for the actual rate of inflation.

The second central element of the model is the elasticity of aggregate demand and production with respect to change in the Treasury bill rate. This elasticity is derived by tracing through the effect of the spread between short and long term interest rates on change in savings deposits to change in thrift deposits to housing starts and finally to investment in residential structures. The third central element is the direct effect on the unemployment rate of a change in government employment.

In constructing a loss function on economic performance we were concerned with specifying two objectives. Our major interest with respect to the variables of state was stabilization of the rate of unemployment at four percent and stabilization of the price level. Of subsidiary interest was increasing consumption and stocks of residential structures. The second major objective was stabilization of the change in the policy instruments from quarter to quarter in order to guard against unreasonably large fluctuations in those instruments. We chose as the single-period loss function the form:

$$
\begin{aligned}
(13) \qquad f_t^0 = {} & (0.9925)^t (16.66(Rp)^2 + 33.33\,(Ru - 4.0)^2 \\
& - 20.0[(Es + En + 0.2478\,Kd)/Pt] - 10.0(Kh/Pt) \\
& + [(G - 1.01157\,G_{-1})/2.22]^2 + [(Eg - 1.00930\,Eg_{-1})/0.100]^2 \\
& + [(Rtb - Rtb_{-1})/0.372]^2 + [(S - 1.0)/0.025]^2 \\
& + [(FHL - P\,\widehat{FHL})/0.882]^2 + 100.0[(Yg - P\,\widehat{Wg}\,Eg)/0.770]^2).
\end{aligned}
$$

The first term indicates our preference for a four percent rate of unemployment (*Ru*) and zero inflation (*Rp*). The second and third items account for our preference for greater per capita consumption and residential housing. The next five items serve to stabilize government purchases of privately produced goods and services (*G*), government employment (*Eg*), the Treasury bill rate (*Rtb*), the federal personal tax scaling factor (*S*) and Federal Home Loan advances (FHL), respectively. $\widehat{FHL}$ is a target level of deflated advances constructed from the predictions of a simple time trend on actual deflated advances. The last item ties government compensation of its employees (*Yg*) to the number of employees through a per capita real wage index ($\widehat{Wg}$). The index was constructed from the predictions of a time trend on the actual real wage of employees.

To specify the numerical parameters of the loss function we inspected the post-war behavior of the policy instruments. Estimation, e.g., of the simple quarterly model $G = \beta G_{-1} + \zeta$ reveals that government purchases have grown on average at a rate of 1.157 percent per quarter with a standard error of 2.22 billion dollars at 1958 prices. Arguing that such long-term behavior stems from causes other than management of aggregate activity, for example, meeting demands for public goods, it seems reasonable to penalize short-run policy choices when they deviate from trend behavior. The term $[(G - 1.01157\, G_{-1})/2.22]$ represents a normalized measure of the deviation of current government purchases from the target level of $1.01157\, G_{-1}$. The construction of the quadratic stabilization terms for government employment, the Treasury bill rate, Federal Home Loan advances and government compensation is similarly motivated. The absolute weights on unemployment and inflation are arbitrary but the relative weights were chosen to penalize an increase in the unemployment rate above four percent twice as heavily as an equal increase in inflation above zero. In the absence of any theory on the construction of loss functions defined over alternative states of aggregate activity, particularly in those cases where primary concern is directed towards unemployment and inflation, any parameter choices are somewhat arbitrary. Our approach in developing normalized penalty functions for instrument stabilization was to narrow, however incompletely, the limits of choice. Only after several quantitative studies have been reported, e.g., Pindyck (1973) and Chow (1972b), will we begin to see whether optimal policies are robust with respect to specification of the loss function.

We chose a planning interval of eleven quarters. Earlier work with the model (Garbade 1975) has shown that optimal policy choices exhibit a noticeable influence from the proximity of the planning horizon in the last four quarters, so $T = 11$ gives us seven quarters of meaningful policies. More arbitrarily we choose 1960/I as the initial quarter, and set the initial state vector $\bar{x}_0$ to its historic value in that quarter.

3. The Nominal Policy Sequence and State Trajectory

Table 1 presents the quarterly sequences of each of our six policy instruments which are optimal for problem (4). For comparison we also exhibit in Table 1 the historic choice of policies over the same interval. Note that while optimal government purchases (G) fluctuate over the planning interval, government employment (*Eg*) and hence government compensation (Yg) grow monotonically, albeit at a slightly declining rate through time. With respect to the criteria of unemployment and inflation with which we are primarily concerned, better performance is evidently obtained by government spending on direct employment rather than seeking an indirect stimulus to employment by purchases of privately produced goods and services. The Treasury bill rate is steady, and Federal Home Loan advances grow evenly until the last year of the planning interval. It is interesting to note that the federal personal tax scaling factor hardly varies from its no loss value of unity. The choice of fiscal policy which is optimal for the model and loss function is quite stimulative, and occurs entirely as an increase in expenditures rather than as a decrease in tax rates.

Table 2 shows the nominal and historic development of gross private production (*X*), the rate of inflation (*Rp*) and the labor force unemployed (*Lu*) over the

TABLE 1
Dynamic Characteristics of Policy

	Historic	Nominal	Disc Results Mean	Disc Results Std Dev	Historic	Nominal	Disc Results Mean	Disc Results Std Dev
	Government Purchases (*G*)				**Government Compensation (*Yg*)**			
1960/I	50.6				46.0			
II	50.9	51.7	51.7	0.0	47.0	48.7	48.7	0.0
III	51.6	53.1	53.0	2.07	48.1	51.2	51.1	0.52
IV	52.2	54.2	54.0	2.76	48.8	53.8	53.6	0.94
1961/I	53.4	54.3	54.3	2.84	49.5	56.4	56.2	1.18
II	55.2	53.6	53.5	4.24	50.3	59.0	58.8	1.44
III	57.3	52.2	52.2	4.67	51.2	61.6	61.4	1.96
IV	57.2	51.1	50.4	3.74	52.6	64.3	64.1	2.36
1962/I	59.0	50.5	49.0	3.64	53.8	66.7	66.4	2.67
II	60.8	51.4	49.4	4.89	54.4	69.1	68.6	2.56
III	60.6	53.5	51.7	6.00	54.8	71.3	71.0	2.72
IV	61.6	55.6	53.7	6.48	55.7	73.5	72.9	3.77
	Government Employment (*Eg*)				**Federal Tax Scaling Factor (*S*)**			
1960/I	9.75				1.000			
II	9.81	10.18	10.18	0.0	1.000	1.000	1.000	0.0
III	9.80	10.57	10.55	0.108	1.000	0.999	0.999	0.005
IV	9.85	10.95	10.95	0.186	1.000	0.997	0.998	0.006
1961/I	9.92	11.39	11.35	0.227	1.000	0.998	0.997	0.006
II	9.96	11.79	11.74	0.275	1.000	0.999	0.998	0.007
III	10.07	12.19	12.15	0.367	1.000	1.000	0.999	0.006
IV	10.31	12.56	12.52	0.453	1.000	1.001	1.002	0.008
1962/I	10.52	12.90	12.83	0.505	1.000	1.003	1.005	0.006
II	10.60	13.21	13.11	0.470	1.000	1.003	1.003	0.007
III	10.64	13.48	13.40	0.458	1.000	1.000	0.999	0.005
IV	10.68	13.73	13.63	0.478	1.000	0.996	0.996	0.004
	Treasury Bill Rate (*Rtb*)				**FHL Advances (*FHL*)**			
1960/I	3.94				1.52			
II	3.09	3.92	3.92	0.0	1.77	1.91	1.91	0.0
III	2.39	3.87	3.87	0.060	1.74	1.88	1.81	0.440
IV	2.36	3.82	3.82	0.096	1.98	2.11	2.05	0.534
1961/I	2.38	3.79	3.78	0.081	1.48	2.16	2.18	0.603
II	2.33	3.77	3.76	0.109	1.87	2.25	2.30	0.615
III	2.33	3.77	3.76	0.126	2.12	2.54	2.66	0.617
IV	2.48	3.80	3.80	0.120	2.66	2.79	2.83	0.703
1962/I	2.74	3.83	3.86	0.097	2.15	2.91	2.83	0.532
II	2.72	2.83	3.87	0.108	2.76	2.03	2.01	0.563
III	2.86	3.75	3.78	0.129	3.04	2.32	2.39	0.460
IV	2.80	3.69	3.73	0.143	3.48	3.26	3.23	0.342

planning interval. Since the historic state trajectories reflect the contribution of the historically realized random disturbances as well as the historic choices for policy instruments, comparison of nominal and historic states is not entirely valid. We display both to give the reader a reference benchmark for the level of the nominal states. The stability of nominal inflation at about two percent and nominal unemployment at about 3.4 million workers (corresponding to an

TABLE 2
DYNAMIC CHARACTERISTICS OF STATE

	Historic States	Nominal States	Mean States Non-disc	Mean States Disc	Standard Deviations Non-disc	Standard Deviations Disc
	Gross Private Product (billions of dollars at 1958 prices)					
1960/I	447.0					
II	445.8	450.1	449.2	449.2	3.85	3.85
III	443.5	451.5	449.5	449.5	7.20	5.96
IV	439.5	451.0	448.3	447.9	8.50	6.26
1961/I	438.4	453.9	451.2	450.9	9.52	6.13
II	448.4	456.4	453.8	453.5	11.84	6.41
III	456.6	457.0	454.2	454.0	14.10	7.42
IV	466.0	457.9	456.5	455.8	14.56	8.05
1962/I	473.0	461.3	461.5	459.2	13.79	9.37
II	480.8	464.5	466.9	462.9	12.80	8.68
III	486.3	470.3	476.7	471.4	14.09	8.53
IV	491.3	482.6	492.9	486.2	18.52	9.06
	Rate of Inflation (percent)					
1960/I	1.80					
II	1.70	1.86	2.06	2.06	1.11	1.11
III	0.67	1.88	2.10	2.10	1.03	1.03
IV	1.96	1.87	1.71	1.69	1.38	1.35
1961/I	0.93	1.80	1.74	1.71	1.11	1.04
II	0.12	1.86	1.68	1.65	1.21	1.17
III	−0.08	1.92	2.34	2.31	1.25	1.17
IV	1.97	1.90	1.88	1.86	1.55	1.32
1962/I	1.32	1.89	1.96	1.91	1.41	1.40
II	0.52	1.97	2.20	2.07	1.48	1.43
III	0.83	2.04	2.29	2.08	1.52	1.38
IV	1.14	2.17	2.56	2.28	1.74	1.40
	Labor Force Unemployed (millions of workers)					
1960/I	3.557					
II	3.652	3.492	3.419	3.419	0.338	0.338
III	3.889	3.382	3.274	3.291	0.436	0.383
IV	4.400	3.416	3.284	3.322	0.558	0.423
1961/I	4.785	3.432	3.298	3.339	0.641	0.419
II	4.927	3.429	3.390	3.430	0.843	0.482
III	4.762	3.446	3.362	3.388	0.874	0.515
IV	4.348	3.481	3.329	3.372	0.862	0.479
1962/I	3.958	3.478	3.336	3.447	0.748	0.297
II	3.871	3.460	3.323	3.513	0.680	0.335
III	3.931	3.435	3.178	3.427	0.763	0.363
IV	3.911	3.262	2.932	3.255	0.883	0.490

unemployment rate of 4½ percent) is striking. The rise in production and dip in unemployment in the last quarter (1962/IV) clearly reflects the influence of the proximity of the planning horizon on the nominal policy choices. Table 2 also presents the nominal and historic trajectories for expenditure on consumer durables (*Ed*), investment in plant and equipment (*Ip*) and investment in residential structures (*Ih*). With the exception of the former there appears to be little difference in the nominal and historic patterns. The minimum value of loss attained by the optimal policy sequence and associated state trajectory was 449.12.

TABLE 2
DYNAMIC CHARACTERISTICS OF STATE (continued)

	Historic States	Nominal States	Mean States Non-disc	Disc	Standard Deviations Non-disc	Disc
	Expenditures on Consumer Durables (billions of dollars at 1958 prices)					
1960/I	45.4					
II	45.6	46.6	46.1	46.1	1.89	1.89
III	45.0	47.4	46.8	46.8	2.55	2.35
IV	43.5	47.9	47.3	47.2	2.75	2.63
1961/I	41.7	48.8	48.1	48.0	3.02	2.86
II	43.2	49.6	48.7	48.6	3.52	2.95
III	44.5	50.1	49.4	49.3	3.36	2.58
IV	46.3	50.6	50.1	50.0	4.29	3.40
1962/I	48.1	51.0	51.0	50.7	4.02	3.51
II	48.1	51.8	52.2	51.6	4.00	3.39
III	49.7	52.9	54.1	53.3	4.90	4.14
IV	50.8	54.9	56.9	55.9	5.10	3.99
	Investment in Plant and Equipment (billions of dollars at 1958 prices)					
1960/I	46.6					
II	47.6	48.0	47.9	47.9	1.00	1.00
III	47.0	48.3	48.4	48.4	1.75	1.57
IV	47.0	47.8	47.9	47.8	2.13	1.59
1961/I	44.9	47.5	47.7	47.6	2.61	1.85
II	44.6	47.3	47.3	47.2	3.69	2.54
III	45.7	47.0	46.6	46.5	4.59	2.87
IV	46.6	46.6	46.6	46.4	4.69	2.64
1962/I	47.6	46.6	47.0	46.6	4.36	2.26
II	49.3	46.8	47.7	46.9	3.91	2.14
III	51.1	47.5	49.5	48.3	4.17	2.55
IV	50.7	49.4	52.2	50.5	5.17	3.16
	Investment in Residential Structures (billions of dollars at 1958 prices)					
1960/I	23.7					
II	22.0	22.5	22.6	22.6	0.79	0.79
III	21.0	22.2	22.1	22.1	1.15	1.15
IV	20.7	20.9	20.8	20.7	1.59	1.79
1961/I	20.9	22.5	22.3	22.4	1.66	2.07
II	21.1	23.1	22.9	23.0	1.45	1.71
III	21.6	22.0	22.1	22.2	1.46	1.67
IV	22.6	21.4	21.4	21.4	1.34	1.52
1962/I	23.1	22.8	22.6	22.4	1.32	1.60
II	23.8	22.1	22.1	21.9	1.66	2.01
III	24.2	21.2	21.4	21.4	1.46	1.50
IV	23.8	22.8	23.4	23.1	1.56	1.53

4. SIMULATION OF POLICY STRATEGIES FOR A STOCHASTIC ECONOMY

In this section we introduce uncertainty into our description of the economy and study the impact on economic stability and performance of discretionary change in the nominal policy sequence. In all cases 30 stochastic simulations of the model provides the sample set for statistical estimation.

The expected value of the loss function is perhaps the most comprehensive single measure of economic performance. From the Monte Carlo simulations we obtained:

Non-discretionary Policy (the model of equation (6))

Estimated Expected Loss	10,532.
Estimated Standard Deviation of Loss	16,637.

Discretionary Policy (the model of equation (12))

Estimated Expected Loss	5,298.
Estimated Standard Deviation of Loss	4,698.

These results may be compared with the value of loss for the model simulated in the deterministic mode of equation (4b, c) of 449.12. The consequence for loss of a stochastic economy is clearly substantial. Equally obvious is the important contribution of the discretionary policy in mitigating the effects of random shocks, with expected loss reduced by 50 percent when discretion is permitted. This estimate of a 100 percent increase in loss when policy makers follow a fixed sequence of policy choices accords with the results of Chow (1972b) and adds to the accumulating evidence of the sacrifice in economic performance implicit in the recommendation of a non-discretionary policy when a valid representation of the economy is available.

Looking at the contribution of discretion to stabilization of economic activity, Table 2 presents the estimated mean trajectories and the estimated standard deviation about those trajectories of selected components of state for both non-discretionary and discretionary policy strategies. In the mean disturbance approach applied policy is altered away from the nominal policy only when the realized state in the previous period differs from the nominal state. Since there is no difference in the realized and nominal states in the initial period, the choice of policy in 1960/II is identical in both regimes (cnf. equation (8c) and the resulting implication from equation (11) that $u_1 = \bar{u}_1$) and the means and standard deviations of state are similarly identical. After 1960/II the policy choices generated by a discretionary strategy can vary away from the nominal choice and we observe a difference in the development of the means and standard deviations. One obvious difference is the greater stability of economic activity, i.e., smaller standard deviations, when discretion in the policy process is permitted. Uncertainty in projected levels of private production and the unemployed labor force is reduced about 50 percent by discretionary change in planned policies. The effectiveness of discretion in stabilizing the rate of inflation is not as great, but it is still positive.

Uncertainty in household expenditures on durable goods and in business investment in plant and equipment is also reduced by a discretionary strategy. Expenditures on consumer durables depend in part on the change in disposable income (equation A3), which is stabilized by the contemporaneous effect of automatic stabilizers (unemployment insurance, positive marginal tax rates, etc.) in the structure of the economy. Investment in plant and equipment, on the other hand, depends on the change in production (equation A4), so it is not surprising that the stabilizing effect of a discretionary policy is relatively greater for plant and equipment. For two decades economists have commented on the important contribution tax and transfer programs have made to economic stability (Lewis, 1962 for example) because of their stabilizing effect on disposable income.

Residential construction alone among the sectors of the economy displays less stability when discretion is permitted. Table 2 indicates that the degradation is not substantial, the standard deviation increasing about 20 percent with discretion, but this result does demonstrate the peculiar position of housing in bearing a disproportionate share of the bundens of short-run macroeconomic policy actions. The existence of this problem has received attention in the literature (Federal Reserve Staff Study: *Ways to Moderate Fluctuations in Housing Construction*, 1972). It would be informative to explore the costs in terms of inflation and unemployment of requiring greater stability in home building by adding terms to the K_t matrices of equation (8a) penalizing variation in investment in residential structures away from the nominal level of investment. Policy actions directed towards stabilizing home building activity have been undertaken from time to time by the Federal Home Bank System, and there is some question whether such actions have been consistent with the larger goals of macroeconomic policy.

Under the discretionary policy the chosen values of the instruments, depending as they do on realized states of economic activity, are random variables after 1960/II. (They equal the nominal values in that first quarter.) We may, consequently, inquire as to their estimated mean values and standard deviations. Table 1 summarizes this information.

In the previous section we commented that optimal use of government expenditure policy seems to apply the purchases component as a flexible instrument while compensation follows a secular schedule. Table 1 reinforces this observation. The standard deviation of purchases under the discretionary policy is anywhere from 50 percent to 100 percent greater than that of compensation. This result on the relative variation of the two expenditure tools accords with our intuitive observation that it is easier and more satisfactory to alter a purchasing pattern for goods than to hire and fire public employees.

Not only does the nominal Treasury bill rate remain reasonably steady (the result of the previous section), the chosen bill rate is also not subject to substantial uncertainty. The bill rate is simply not an especially active instrument in either the short or intermediate runs for the model and loss function we are considering. The summary statistics for the federal personal tax scaling factor shown in Table 1 demonstrate that the planned variation in an instrument can be exceeded by its unplanned variation in response to realized economic activity. The standard deviation of the scaling factor in every quarter but the last exceeds the difference between the expected value of the factor and the no loss value of unity. The personal tax scaling factor emerges as a policy instrument more relevant for short-run stabilization than for meeting intermediate-run goals. This accords with the observations of politicians as well as economists that proportional alteration of tax schedules is one of the least costly methods of implementing a discretionary policy. Further experimentation might include reducing the penalty attached to variations in the scaling factor in equation (13) to investigate the effects of allowing a more flexible tax policy than that permitted in the present study. In particular, it would be of interest to determine whether a flexible tax policy could replace a substantial part of the stabilization activity currently supported by variation in government expenditures.

One further observation which can be drawn from Table 1 is the evidence of bias in the discretionary policy. Were the model linear and the loss function quadratic the expected value of state under both the discretionary and non-discretionary policies would equal the corresponding nominal state, and the expected value of policy under the discretionary strategy would equal the nominal policy. The statistical evidence indicates that, for government purchases especially, these equalities fail a significant number of times, and suggests a modification to what we have called the program plan in the discretionary case. Let u_t^e be the expected choice of policy in period t and let x_t^e be the expected state. Since $u_t = \bar{u}_t + G_t(x_{t-1} - \bar{x}_{t-1})$ by definition it follows that $u_t^e = \bar{u}_t + G_t(x_{t-1}^e - \bar{x}_{t-1})$ and hence that:

$$u_t = u_t^e + G_t(x_{t-1} - x_{t-1}^e). \tag{14}$$

Since it is important to anticipate as much as possible future policy choices, simulation of the model can generate unbiased estimates of the expected choices and expected states of economic activity. The estimated expected policy sequence would replace the nominal sequence as the program plan for administrators. As economic activity unfolds through time, applied policies would be altered away from the expected policy in response to realizations of state away from the expected state, according to equation (14). The numerical values of the applied policies would be the same under this feedback function as under that of equation (11), and the economy will still evolve according to equation (12). The only difference is that the planned policies would be unbiased estimates of the policies to be applied.

5. Conclusions

The results of this paper indicate that the recommendation of a non-discretionary macroeconomic policy may in some cases increase the expected value of a loss function on economic performance by as much as 100 percent. While the structure of any particular function might not command widespread acceptance, our results concur closely with the independently derived results of Chow (1972b). There also appear to be substantial gains available from a discretionary policy in reduction of uncertainty about future levels of private production and unemployment. Less satisfactory implications for stabilization of the price level were obtained.

This study also pointed out the contribution of a discretionary policy towards stabilizing investment in plant and equipment and the relatively smaller reduction of uncertainty in expenditures on consumer durables. The least satisfactory, but none the less enlightening, result was the demonstration that, without special attention, the residential housing industry may be vulnerable to short-run stabilization policies.

Continued research by economists will certainly investigate the effects of relaxing some of the assumptions of the present study, especially that the behavioral parameters of the model are non-random and equal to their estimated values. (Tse (1974), Chow (1974) and Abel (1974) have already started in this direction in economic contexts.) This study has shown that substantial benefits may accrue to discretionary amendment of policy choices when the economy has been adequately modeled. There is, of course, considerable debate on the structure as well as on the parameter values of an adequate model. The policy program and feedback functions developed for this study displayed desirable features when applied to the model of the study, they may not appear so desirable when applied to an alternative model. Thus, for economists, describing how policy choices affect economic activity is still a fundamental problem.

Even were the question of the model resolved, however, the question of to what extent macroeconomic policy instruments, especially expenditure items, are susceptible to short-run amendment would remain. Lewis (1962) has addressed this question in an ex-post framework and Friedlaender (1968) has considered some of the institutional reasons for suspecting that such instruments are not all that flexible. Pierce (1974) has pointed out that the Federal Reserve System, alone among the institutions of government, has an ongoing policy review and revision program. Federal tax policy, potentially the easiest instrument to change on short notice, is not now suitable for economic stabilization. Congress has consistently rejected Presidential requests for stand-by authority to impose proportional changes in liability schedules. Economists might approach this question of instrument flexibility from two directions. First, within the current institutional framework they can develop models of the degree of flexibility as a function of time. While government expenditure policy next quarter is likely highly inflexible, expenditure policy six quarters ahead is much less so. This would lead to a generalization of the usual exogenous/pre-determined dichotomy to a more continuous scale of controllability for policy instruments. This issue as it relates to control of monetary aggregates has received attention in a conference of the Federal Reserve Bank of Boston (1972) and Pindyck and Roberts (1974). Second, economists can investigate the degradation in economic performance occasioned by instrument inflexibility. For example, how much is lost by restricting discretion to only monetary instruments, and how much could be gained by opening tax policy to short-run discretionary amendment?

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Appendix: Alphabetical List of Symbols

Curr	currency in the hands of the public, NSA
DD	demand deposits held by the public, NSA
Ed	household expenditures on durable goods, 1958 prices
Ef	farm employment
*Eg**	government employment
En	household expenditures on non-durable goods, 1958 prices
Enf	private non-farm employment
Es	household expenditure on services, 1958 prices
Et	total employment
*FHL**	Federal Home Loan advances to savings and loan associations
Fi	imports, 1958 prices
Fx	exports, 1958 prices
*G**	government purchases of privately produced goods and services, 1958 prices
Gp	government transfer payments to persons

HS	housing starts, NSA
Ih	investment in residential structures, 1958 prices
Ip	investment in plant and equipment, 1958 prices
Iv	investment in inventories, 1958 prices
Kd	stock of consumer durables
Kh	stock of residential housing
Kp	stock of plant and equipment
Kv	stock of inventories
Le	labor force employed
Lf	females participating in the labor force
*Lm*13	males 16–24 and over 55 participating in the labor force
*Lm*2	males 25–54 participating in the labor force
Lt	total labor force
Lu	labor force unemployed
M	manhours paid for
M_1^{**}	money supply, NSA, (= *Curr* + *DD*)
Other	other items in the income account
P	implicit price deflator for gross private product
Rcb	*Aaa* corporate bond rate
Rp	rate of inflation
R_p^n	proxy for the expected rate of inflation
*Rtb***	90 day treasury bill rate
Ru	unemployment rate, (= 100 *Lu*/(*Lm*13 + *Lm*2 + *Lf*))
*S**	federal personal tax scaling factor
Δ *Sav*	change in savings deposits, NSA
Tc	contributions for social insurance
Δ *Thr*	change in thrift deposits, NSA
Tibf	federal indirect business tax and non-tax liabilities
Tibs	state and local indirect business tax and non-tax liabilities
Tpf	federal personal tax and non-tax payments
Tps	state and local personal tax and non-tax payments
u	residual of a stochastic equation displaying serial correlation
X	gross private product, 1958 prices
Y	gross national product
Ycor	corporate earnings
Yd	disposable income
Ydv	corporate dividend disbursements
*Yg**	government compensation of its employees

NSA: not seasonally adjusted
* policy instrument
** either *Rtb* or M_1 may be taken as an instrument of monetary policy

Appendix: Equations of the Model

A1. Consumer Services

$$Es = -0.01115\, Pt + 0.02519\, Yd/P + 0.945\, Es_{-1}\, Pt/Pt_{-1} + 0.001\, Pt\, \zeta$$

s.e. = 4.270.

A2. **Consumer Non-durables**

$$En = 0.19142\,Pt + 0.05258\,Yd/P + 0.58957\,En_{-1}\,Pt/Pt_{-1}$$
$$+ 0.16750\,Es_{-1}\,Pt/Pt_{-1} + 0.001\,Pt\,\zeta$$

s.e. = 9.476.

A3. **Consumer Durables**

$$Ed = -0.30825\,(Pt - \gamma d\,Pt_{-1}) - 0.001683\,[Pt(Rcb + \delta_d^a - Rp)$$
$$- \gamma d\,Pt_{-1}\,(Rcb_{-1} + \delta_d^a - Rp_{-1})] + 0.30476\,(Yd/P - \gamma d\,Yd_{-1}/P_{-1})$$
$$+ 0.90307\,Ed_{-1}\,Pt/Pt_{-1} + 0.004\,Pt\,\zeta$$

s.e. = 3.130.

A4. **Investment in Plant and Equipment**

$$Ip = -34.28\,(1.0 - \gamma p) + 0.34324\,(X - \gamma p\,X_{-1}) + 0.91648\,Ip_{-1}$$
$$- 1.07712\,[(M^* - M_{-1}) - \gamma p(M^*_{-1} - M_{-2})]$$
$$- 1.8801\,[(Rcb - R_p^n)_{-1} - \gamma p\,(Rcb - R_p^n)_{-2}] + 4\,\zeta$$

s.e. = 0.2378 $\quad M^* = 22.405 + 0.76007\,X\,\exp(-0.02174\ \text{time})$.

A5. **Investment in Business Inventories**

$$Iv = -65.408 + 0.39895\,FS_{-1} - 0.82856\,Kv_{-1} - 0.39516\,\Delta FS$$
$$+ 0.79286\,Iv_{-1} + 0.39442\,u_{-1} + \zeta$$

s.e. = 2.734 $\quad FS = En + Ed + Fx - Fi + G$.

A6. **Investment in Residential Structures**

$$Ih = 16.568\,HS\,\exp(-0.01675\ \text{time}) + 16.033\,HS_{-1}\,\exp(-0.01675\ \text{time})$$
$$+ 7.097\,DI\,\exp(-0.01675\ \text{time}) - 6.805\,DIII\,\exp(-0.01675\ \text{time})$$
$$+ 0.01651\,Kh_{-1} + \zeta$$

s.e. = 0.467.

A7. **Imports**

$$Fi = -3.929 + 0.02025\,X + 0.79715\,Fi_{-1} + 0.41130\,u_{-1} + \zeta$$

s.e. = 0.682.

A8. **Exports**

$$Fx = -50.845 + 1.031\ \text{time} + 0.57420\,Fx_{-1} + \zeta$$

s.e. = 0.762.

A9. **Gross Private Product**

$$X = Es + En + Ed + Ip + Iv + Ih + Fx - Fi + G.$$

A10. Gross National Product

$$Y = P\,X + Yg.$$

A11. Gross Corporate Earnings

$$Ycor = 0.704 + 0.01138\,P\,X + 0.91509\,Ycor_{-1} + 0.38142\,P\,\Delta X + \zeta$$

s.e. = 1.174.

A12. Dividends

$$Ydv = 0.03300\,Ycor + 0.82634\,Ydv_{-1} + \zeta$$

s.e. = 0.355.

A.13. Federal Indirect Business Taxes

$$Tibf = Dibfa\,[16.096 - 0.38548\text{ time} + 0.06287\,P\,(Es + En + Ed)] + Dibfb\,[5.076 + 0.02364\,P\,(Es + En + Ed)] + 0.62210\,u_{-1} + \zeta$$

s.e. = 0.234.

A14. State and Local Indirect Business Taxes

$$Tibs = (-0.02636 + 0.00204\text{ time})\,P\,(Es + En + Ed) + 0.87825\,u_{-1} + \zeta$$

s.e. = 0.266.

A15. Federal Personal Taxes

$$Tpf = Pt\,(Dpfa\,S\,0.14656\,[(Ygr/Pt) - 0.767] + Dpfb\,S\,[0.09774 + 0.00766\,(Ygr/Pt)]\,[(Ygr/Pt) - 0.767) + 0.77301\,u_{-1} + \zeta]$$

s.e. = 0.00460 $\quad Ygr = Yd + Tpf + Tc - Gp.$

A16. State and Local Personal Taxes

$$Tps = Pt(0.3321 - 0.00515\text{ time} - (0.14267 - 0.00255\text{ time})(Ygr/Pt) + 0.89438\,u_{-1} + \zeta)$$

s.e. = 0.00118 $\quad Ygr = Yd + Tpf + Tps + Tc - Gp.$

A17. Contributions for Social Insurance

$$Tc = Dca\,[0.78284 - 0.08687\,(Ygr/Pt)]\,Rs\,Ygr + 0.49656\,Dcb\,Rs\,Ygr - (0.68703 - 0.02163\text{ time})\,Ru\,Ygr + 0.83111\,u_{-1} + \zeta$$

s.e. = 0.199 $\quad Ygr = Yd + Tpf + Tps + Tc - Gp.$

A18. Government Transfer Payments

$$Gp = (-2.6059 + 0.06011\text{ time})\,B\,Page - (8.8679 - - 0.13620\text{ time})\,Dmed\,Page + 0.02831\text{ time}\,Lu + 0.47606\,u_{-1} + \zeta$$

s.e. = 0.744.

A19. **Other Items in the Income Account**

$$\text{Other} = 8.805 - 0.14035\ \text{time} + 0.80711\ \text{Other}_{-1} + \zeta$$

s.e. = 1.175.

A20. **Disposable Income**

$$Yd = Y - Ycor + Ydv - Tibf - Tibs - Tpf - Tps - Tc + Gp - \text{Other}.$$

A21. **Private Non-farm Manhours Paid for**

$$M = 11.017 + 0.37375\ X \exp(-0.02175\ \text{time}) + 0.50827\ M_{-1} + 0.65574\ u_{-1} + \zeta$$

s.e. = 0.577.

A22. **Private Non-farm Employment**

$$Enf = 0.29162\ M \exp(0.00339\ \text{time}) + 0.27045\ Enf_{-1} + 0.79596\ u_{-1} + \zeta$$

s.e. = 0.120.

A23. **Farm Employment**

$$Ef = 4.740 - 0.05601\ \text{time} + 0.73875\ Ef_{-1} + \zeta$$

s.e. = 0.172.

A24. **Total Employment**

$$Et = Enf + Ef + Eg.$$

A25. **Labor Force Employed**

$$Le = (0.68004 + 0.27134\ Lu_{-1}/Lt_{-1})\ Et + 0.27250\ Le_{-1} + 0.73997\ u_{-1} + \zeta$$

s.e. = 0.179.

A26. **Prime Age Males Participating in the Labor Force**

$$Lm2 = Pm2\,(0.82601 - 0.00037\ \text{time} - 0.06267\,(Et/Pt)_{-1} + 0.19985\,(Lm2/Pm2)_{-1} + \zeta)$$

s.e. = 0.00154.

A27. **Other Males Participating in the Labor Force**

$$Lm13 = Pm13\,(0.19867 - 0.00201\ \text{time} + 0.29002\,(Et/Pt)_{-1} + 0.61997\,(Lm13/Pm13)_{-1} + \zeta)$$

s.e. = 0.00372.

A28. **Females Participating in the Labor Force**

$$Lf = Pf\,[-0.02068 + 0.00105\ \text{time} + 0.07356\,(Et/Pt)_{-1} + 0.76931\,(Lf/Pf)_{-1} + \zeta]$$

s.e. = 0.00308.

A29. **Total Labor Force**

$$Lt = Lm2 + Lm13 + Lf.$$

A30. **Unemployed Labor Force**

$$Lu = Lt - Le.$$

A31. **Corporate Bond Rate**

$$Rcb = 0.062 + 0.25965\,Rtb - 0.20874\,Rtb_{-1} + 0.06763\,R_p^n + 0.93237\,Rcb_{-1} + \zeta$$

s.e. = 0.100.

A32. **Demand for Demand Deposits**

$$DD = (Rtb)^{-0.02430}(Yd)^{0.07947}(P\,DD_{-1}/P)^{0.92053}\exp(-0.06313 - 0.03400\,\mathscr{S}\text{I} - 0.00514\,\mathscr{S}\text{II} + 0.00348\,\mathscr{S}\text{III} + 0.22363\,u_{-1} + \zeta)$$

s.e. = 0.00692.

A33. **Demand for Currency**

$$Curr = (Rcb)^{-0.03098}(Yd)^{0.10427}(P\,Curr_{-1}/P_{-1})^{0.89573}\exp(-0.21330 - 0.02944\,\mathscr{S}\text{I} + 0.00621\,\mathscr{S}\text{II} + 0.00359\,\mathscr{S}\text{III} + 0.23274\,u_{-1} + \zeta)$$

s.e. = 0.00418.

A34. **Change in Savings Deposits**

$$\Delta Sav = -1.268\,\mathscr{S}\text{I} + 1.673\,\mathscr{S}\text{II} + [0.06145 + 0.08112\,\mathscr{S}\text{I} - 0.05235\,\mathscr{S}\text{II} - 0.03394\,\mathscr{S}\text{III} + 0.01557\,Rcb\,(Rcb - Rtb)][Yd - P(Es + En + Ed)] + 0.84227\,u_{-1} + \zeta$$

s.e. = 0.852.

A35. **Change in Thrift Deposits**

$$\Delta Thr = (1.23620 - 0.01177\ \text{time} - 0.10470\,\mathscr{S}\text{I} - 0.08007\,\mathscr{S}\text{III})\,\Delta Sav + 0.49018\,u_{-1} + \zeta$$

s.e. = 0.457.

A36. **Housing Starts**

$$HS = 1.10944 + 0.03477\,\Delta Thr/[P\exp(-0.01675\,\text{time})] + 0.06817\,(FHL - FHL_{-1})/[P\exp(-0.01675\,\text{time})] - 0.1717\,\mathscr{S}\text{I} + 0.2125\,\mathscr{S}\text{II} + 0.1875\,\mathscr{S}\text{III} + 0.67439\,u_{-1} + \zeta$$

s.e. = 0.0847.

A37. **Proxy for the Expected Rate of Inflation**

$$Rp^{n} = 0.1\,Rp_{-1} + 0.9\,Rp^{n}_{-1}.$$

A38. **Rate of Inflation**

$$Rp = Rp^{n} + 0.43080 - 0.03912(\bar{X}_{-1} - X_{-1}) + 0.03329(Rp - Rp^{n})_{-1} + \zeta$$

s.e. = 1.231

$$\bar{X}_{-1} = 1.31567\,(\bar{M}_{-1} - 22.405)\exp(0.02175\,\text{time})$$

$$\bar{M}_{-1} = 2.503\,\overline{Enf}_{-1}\exp(-0.00339\,\text{time})$$

$$\overline{Enf}_{-1} = 1.011\,Lt_{-1} - Ef_{-1} - Eg_{-1}.$$

A39. **Implicit Deflator for Gross Private Product**

$$P = P_{-1}(1.0 + Rp/100)^{0.25}.$$

A40. **Stock of Consumer Durables**

$$Kd = 0.25\,Ed + \gamma d\,Kd_{-1}.$$

A41. **Stock of Producers Plant and Equipment**

$$Kp = 0.25\,Ip + \gamma p\,Kp_{-1}.$$

A42. **Stock of Residential Structures**

$$Kh = 0.25\,Ih + \gamma h\,Kh_{-1}.$$

A43. **Stock of Business Inventories**

$$Kv = 0.25\,Iv + Kv_{-1}.$$

Appendix: Parameters of the Model

Time-invariant Parameters **(other than estimated behavioral parameters)**

- γd — unity minus the quarterly rate of depreciation of consumer durable goods (= 0.93129)
- γh — unity minus the quarterly rate of depreciation of residential structures (= 0.99317)
- γp — unity minus the quarterly rate of depreciation of plant and equipment (= 0.97180)
- δ^{a}_{d} — compounded annual rate of depreciation of consumer durable goods (= 24.78 %)

Time-varying Parameters

B	compounded benefit index for OASDHI benefits, 1954/IV = 1.0
*D*I	seasonal dummy for the first quarter of every year, *D*II, *D*III and *D*IV similarly
Page	population age 65 and over, millions
Pf	female population age 16 and over, millions
*Pm*13	male population age 16–24 and 55 and over, millions
*Pm*2	male population age 25–54, millions
Pt	total population over age 16 (= *Pm*13 + *Pm*2 + *Pf*), millions
Rs	contribution rate for OASDHI program, fraction
Ru	contribution rate for Federal Unemployment Insurance, fraction
$\mathscr{S}$I	independent seasonal dummy for first quarter of every year, (= *D*I − *D*IV), $\mathscr{S}$II and $\mathscr{S}$III similarly defined
time	clock time index (= 47.00 in 1947/I, = 47.25 in 1947/II, etc.)

Schedule Shift Parameters (a special class of time-varying parameter)

Contributions for Social Insurance:

$$Dca = \begin{cases} 1. & 1955/\text{I}–1965/\text{IV} \\ 0. & 1966/\text{I}–1969/\text{IV} \end{cases}$$

$$Dcb = 1.0 - Dca$$

Federal Indirect Business Tax and Non-tax Payments:

$$Dibfa = \begin{cases} 1.0 & 1954/\text{I}–1965/\text{I} \\ 0.5 & 1965/\text{II} \\ 0.0 & 1965/\text{III}–1969/\text{IV} \end{cases}$$

$$Dpfb \quad 1.0 - Dibfa$$

Federal Personal Tax Payments:

$$Dpfa = \begin{cases} 1.0 & 1954/\text{I}–1964/\text{I} \\ 0.0 & 1964/\text{II}–1969/\text{IV} \end{cases}$$

$$Dpfb = 1.0 - Dpfa.$$

Government Transfer Payments:

$$Dmed = \begin{cases} 1.0 & \text{after } 1966/\text{I} \\ 0.0 & \text{otherwise} \end{cases}$$

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