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# THE INTERTEMPORAL COST OF LIVING INDEX

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*This paper extends the theory of the cost of living index from its traditional one period framework to a multiperiod setting. It discusses both the complete intertemporal index and one period subindexes. It also considers the implications of both "naive" and "rational" habit formation for the construction of these indexes.*

In this paper I extend the theory of the cost of living index from its traditional one period framework to a multiperiod setting. Since we live in a multiperiod world, it might be argued that this is the only appropriate theoretical framework within which to evaluate any calculated index. At the very least, exploring the relationship between the traditional one period index and the intertemporal model may give us some insight into the proper construction and interpretation of the one period index in a multiperiod world. This is especially important in the case of those problems which arise in constructing a cost of living index which do not make sense in a one period model—for example, the treatment of interest rates. An intelligible discussion of such problems requires an explicitly intertemporal framework.

This paper is organized as follows. In Section 1, I define the complete "intertemporal cost of living index." There are two versions of this index, one based on "futures" prices, and the other on "spot" prices and interest rates. The intertemporal cost of living index is a straightforward extension of the traditional theory from its familiar one period setting to an intertemporal one.

In Section 2, I discuss the construction of one period cost of living indexes in a multiperiod world. Since the complete intertemporal cost of living index compares alternative vectors of future prices (or, equivalently, alternative vectors of spot prices and interest rates), a theoretical rationale for comparing alternative one period price vectors must be based on a theory of "subindexes." Section 2 summarizes the theory of subindexes developed in Pollak [1973b] and applies it to the construction of one period indexes. If the intertemporal preference ordering is separable by periods, the "partial" cost of living index is defined in the "natural" way on the basis of a one period preference ordering. Without separability, we can only define "conditional" subindexes, which are based on the conditional preference ordering over the goods in a period when the levels of consumption of all goods in all other periods are fixed at predetermined levels.

In Section 3, I discuss the implications of habit formation for the construction of the complete intertemporal cost of living index and for one period subindexes. The usual discussion of habit formation begins with a short run utility function some of whose parameters depend on past consumption. If we specify

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the consumption levels of all goods in the previous period, we can construct a one period cost of living index on the basis of the preference ordering implied by the specified consumption history. I distinguish between "naive habit formation," in which an individual does not recognize the impact of his present consumption on his future tastes, and "rational habit formation," in which he does. Naive habit formation cannot be integrated into an intertemporal allocation model, and, therefore, does not lead to a complete intertemporal cost of living index. Rational habit formation implies a non-separable intertemporal utility function which can serve as a base for an intertemporal cost of living index. But because the intertemporal preference ordering is not separable by periods, the one period subindex must be a conditional rather than a partial index.

Section 4 summarizes the results of the previous sections and uses them to discuss the treatment of interest rates in the cost of living index.

## 1. The Intertemporal Index

A cost of living index is the ratio of the expenditures required to attain a particular indifference curve under two price regimes. Let $E(P, s)$ denote the minimum expenditure required to attain the indifference curve $s$ of the preference ordering, $R$; the cost of living index, $I(P^a, P^b, s, R)$, is defined by

$$I(P^a, P^b, s, R) = \frac{E(P^a, s)}{E(P^b, s)}.$$

The notation emphasizes that the index depends not only on the comparison prices, $P^a$, and the reference prices, $P^b$, but also on the choice of a base indifference curve, $s$, from that map.

Strictly speaking, the index depends only on the comparison prices, the reference prices, and the base indifference curve. No other indifference curve from the base preference ordering plays a role in constructing the index. Nevertheless, it is useful and realistic to imagine that the base curve is selected by a two-stage procedure: first, a base map is chosen, and then a curve is selected from that map. Treating the base curve as part of an indifference map focuses attention on the sensitivity of the index to the choice of one base curve rather than another. It is well known that the index is independent of the choice of the base curve if and only if the map is homothetic to the origin; see Pollak [1971] for a discussion of the dependence of the index on the choice of the base curve in non-homothetic cases.

The traditional cost of living index is defined in precisely this way. We let $x_i$ denote the individual's consumption of the $i$th good; if there are $n$ goods, the corresponding consumption vector is given by $(x_1, \ldots, x_n)$. The individual's preference ordering, $R$, is defined over these $n$ dimensional consumption vectors, and the reference and comparison price vectors, $P^b$ and $P^a$, are $n$ dimensional vectors of goods prices.

In the intertemporal context, it is useful to introduce a "double subscript" notation for commodities and prices. We let $x_{it}$ denote consumption of the $i$-th good in period $t$, $X_t$ the vector of consumption in period $t$, $X_t = (x_{1t}, \ldots, x_{nt})$, and $X$ the intertemporal consumption vector, $X = (X_1, \ldots, X_T) = (x_{11}, \ldots, x_{nT})$.

"Lifetime consumption paths" are vectors of dimension $nT$ and $R$ denotes a preference ordering over lifetime consumption paths; I shall assume that $R$ can be represented by an "intertemporal utility function," $U(X)$.

We now turn to prices. There are two interpretations of "price" in the intertemporal model, one based on "futures prices" and the other on "spot prices and interest rates," or "spot prices" for short. In the futures markets interpretation, $\hat{p}_{it}$ denotes the amount which must be paid now, at the beginning of period 1, for a contract promising to deliver one unit of good $i$ in period $t$. We let $\hat{P}_t$ and $\hat{P}$ denote the vectors of futures prices corresponding to $X_t$ and $X$. The total "cost" of the lifetime consumption plan $X = (x_{11}, \ldots, x_{nT})$ is given by

$$\sum_{\tau=1}^{T} \sum_{k=1}^{n} \hat{p}_{k\tau} x_{k\tau}.$$

All market transactions are required to take place at the beginning of period 1, and no markets are open thereafter.

The "spot price" interpretation gives a different gloss to the same model. Instead of futures markets, we assume perfect foresight and let $p_{it}$ denote the "spot" price of $x_{it}$: that is, $p_{it}$ is the amount which must be paid in period $t$ for the delivery of one unit of good $i$ in period $t$. We also assume perfect capital markets, so that individuals can borrow or lend without limit at the market rate of interest, and we let $r_t$ denote the interest rate connecting period $t$ with period $t + 1$. There is no period 0, but by convention we let $r_0 = 0$. The present value of the life-time consumption path $X$ is given by

$$\sum_{\tau=1}^{T} \prod_{v=0}^{\tau-1} \left(\frac{1}{1 + r_v}\right) \sum_{k=1}^{n} p_{k\tau} x_{k\tau}.$$

The formal identity of the perfect foresight model and the futures market model becomes apparent when we define "present value prices," $\hat{\hat{p}}_{it}$:

$$\hat{\hat{p}}_{it} = p_{it} \prod_{v=0}^{t-1} \left(\frac{1}{1 + r_v}\right).$$

In terms of present value prices, the present value of the lifetime consumption path is given by

$$\sum_{\tau=1}^{T} \sum_{k=1}^{n} \hat{\hat{p}}_{k\tau} x_{k\tau}.$$

Radner [1970] summarizes both interpretations of the intertemporal allocation model in his review of "Arrow-Debreu theory," emphasizing that in the spot as well as the futures version, "agents have the access to the complete system of prices when choosing their plans." The spot version should not be confused with substantively more complex models involving sequences of markets.

The "futures price intertemporal cost of living index," $\hat{I}(\hat{P}^a, \hat{P}^b, s, R)$, is defined by

$$\hat{I}(\hat{P}^a, \hat{P}^b, s, R) = \frac{E(\hat{P}^a, s)}{E(\hat{P}^b, s)}$$

where $E(\hat{P}, s)$ denotes the expenditure function corresponding to the intertemporal preference ordering $R$ and $\hat{P}^a$ and $\hat{P}^b$ denote comparison and reference vectors of futures prices, respectively. This index like any cost of living index, is the ratio of the expenditure required to attain a particular indifference curve of a particular preference ordering under two price regimes. It differs from the traditional one period cost of living index in that: (a) the preference ordering on which the comparison is based is an intertemporal one which orders lifetime consumption paths; the traditional cost of living index is based on a one period preference ordering which orders consumption patterns for a single period and (b) the two price regimes being compared in the intertemporal index are intertemporal price regimes; the traditional cost of living index compares alternative price regimes for a single period. From a formal standpoint, the intertemporal index appropriate to a world with 4 goods and 3 periods is indistinguishable from a one period index for a world with 12 goods; hence, all of the theorems of the traditional theory hold for the futures price intertemporal index. From a less formal standpoint, it might appear that the one period index is relevant only for an individual who knows that he will die before the beginning of the next period, and that the intertemporal index is the appropriate index for anyone who expects to live into the next period. In Section 2 I argue that there are analogues of the one period index which are useful, interesting and well defined in the intertemporal context under less morbid assumptions.

The "spot price intertemporal cost of living index," $I(P^a, r^a, P^b, r^b, s, R)$, is defined by

$$I(P^a, r^a, P^b, r^b, s, R) = \frac{E(P^a, r^a, s)}{E(P^b, r^b, s)}$$

where $E(P, r, s)$ denotes the minimum value of

$$\sum_{\tau=1}^{T} \prod_{v=0}^{\tau-1} \left(\frac{1}{1+r_v}\right) \sum_{k=1}^{n} p_{k\tau} x_{k\tau}$$

required to attain the indifference curve $s$ of the intertemporal utility function. The difference between the "spot" and the "futures price" versions of the intertemporal cost of living index is one of notation rather than of substance. The "spot" version explicitly identifies the role of interest rates, while their role remains implicit in the "futures price" version.

The effect of a change in an interest rate on the intertemporal cost of living index is easy to analyze. Consider an increase in $r_t$; such an increase will decrease the present value price of every good in every period after period $t$; hence, an increase in $r_t$ causes the intertemporal index to decline.[1]

It is well known that the Arrow–Debreu theory can be reinterpreted to allow for uncertainty about the environment by treating the $x$'s as "contingent commodities." (See Radner [1970] for a summary and references.) Although I shall

[1] This does not imply that every individual is better off with higher interest rates. The cost of living index measures the effect of changes in prices and interest rates on the present cost of attaining a particular indifferences curve but it ignores their effects on an individual's net worth.

not elaborate the details here, it is clear that the "contingent commodity" interpretation leads directly to a theory of the cost of living index under uncertainty.

## 2. Subindexes

The traditional theory of the cost of living index provides a rationale for constructing complete indexes, that is, for constructing one period indexes in a one period world, or, what is formally the same thing, for constructing $T$ period indexes in a $T$ period world; but it offers no guidance for constructing one period indexes in a multiperiod world. We need a theory of "subindexes" of the cost of living index to provide a theoretical rationale for comparing alternative one period price vectors in a multiperiod setting. In this section, I summarize the theory of subindexes developed in Pollak [1973b] and apply it to the construction of one period subindexes.

Although we are interested in applying the theory of subindexes to the construction of one period indexes in a multiperiod setting, I state the formal definitions in more general terms. In Pollak [1973b], I argue that the theory of subindexes is relevant to the construction of indexes for particular groups of goods, such as "clothing," "footwear," or "men's shoes"; to the construction of indexes which ignore the labor-leisure choice and deal only with goods and services; to indexes which ignore the environment, or goods provided by governments; and, of course, to the construction of one period indexes in a multiperiod setting.

To construct any subindex we must specify the two price regimes to be compared, the base preference ordering, and the base indifference curve. Typically, we begin with the two price vectors we wish to compare, and the problem is to select an appropriate base preference ordering, and, from it, a base indifference curve. If the complete utility function is separable, then it is natural to construct a subindex on the basis of a "specific" utility function. We call a subindex based on a specific or "category" utility function a "partial cost of living index." If we are interested in comparing two vectors of clothing prices, the meaning of this assertion is straightforward; if we want to compare alternative one period spot price vectors in a multiperiod world, its meaning requires careful interpretation. The most plausible interpretation is the following: we wish to compare two $n$ dimensional vectors of spot prices, $P^a_*$ and $P^b_*$; these vectors may correspond to actual spot prices in two periods (for example, this period and the previous period), but they may equally well represent hypothetical vectors of spot prices for the same period generated by alternative public policy measures or by alternative assumptions about the behaviour of some exogenous variables such as the weather. The interpretation of the two price regimes as representing hypothetical prices for the same period provides the best starting point for considering one period subindexes, since some special problems which arise when we compare this period's spot prices with last period's spot prices are absent in hypothetical comparisons.

If the intertemporal utility function is separable by periods, then the one period utility functions are the "specific" utility functions on which a subindex might be constructed. To focus on the problem of choosing an appropriate one period preference ordering, consider an individual whose one period preference orderings vary in a definite and predictable pattern over his life cycle, so that his marginal rate of substitution of baby food for foreign vacations is predictably different depending on whether the calculation is based on his one period preferences corresponding to age 20, 25 or 50. The construction of a subindex to compare two alternative hypothetical spot price vectors, $P_*^b$ and $P_*^a$ clearly requires the selection of a base one period preference ordering, just as the construction of the traditional index to compare two price vectors requires the selection of a base preference ordering. The construction of a subindex to compare this period's spot prices with last period's spot prices is essentially the same as the construction of a subindex to compare to hypothetical spot price vectors; there is no presumption in either case that the appropriate one period preference ordering must be the one corresponding to either this period's or last period's tastes.[2]

We first define the relevant notion of separability.

*Definition*: Suppose that the goods are partitioned into two subsets, $\theta$ and $\bar{\theta}$, denote the vectors of goods in $\theta$ and $\bar{\theta}$ by $X_\theta$ and $X_{\bar{\theta}}$, respectively.[3] We say that the goods in $\theta$ are *separable* from those in $\bar{\theta}$ if the utility function can be written as

$$U(X) = U(X_\theta, X_{\bar{\theta}}) = V[V^\theta(X_\theta), X_{\bar{\theta}}].$$

We call $V^\theta(X_\theta)$ the "category utility function" and denote the corresponding preference ordering by $R_\theta$.

When we speak of a preference ordering or a utility function as "separable," we refer to this non-symmetric form of separability rather than the more familiar notion of "weak separability." If the utility function is "weakly separable,"

$$U(X) = V[V^1(X_1), \ldots, V^m(X_m)]$$

then the goods in any category are separable from the remaining goods. But the assumption that the goods in $\theta$ are separable from those in $\bar{\theta}$ is less restritive than weak separability, since separability does not require the goods in $\bar{\theta}$ or its subsets to be separable from those in $\theta$. The earliest papers on separability, Leontief [1947a], [1947b], and Sono [1961] emphasized this non-symmetric form of separability, but later work such as Strotz [1957] and Goldman and Uzawa [1964] emphasized symmetric versions. The non-symmetric version is now undergoing a renaissance as "recursive separability." A good summary and references can be found in Blackorby, Primont and Russell [1974].

*Definition*: Suppose that the goods are partitioned into two subsets, $\theta$ and $\bar{\theta}$, and that the goods in $\theta$ are separable from those in $\bar{\theta}$. The *partial expenditure function* for category $\theta$, $E^\theta(P_\theta, s_\theta)$, is defined by

$$E^\theta(P_\theta, s_\theta) = \min \sum_{k \in \theta} p_k x_k$$

subject to $V^\theta(X_\theta) = s_\theta$ where $V^\theta(X_\theta)$ is the category utility function and $s_\theta$ denotes an indifference curve of $V^\theta(X_\theta)$.

[2] The situation is similar in international price comparisons, where there is no presumption that the appropriate preference ordering on which to base a comparison of Japanese and French prices must be either Japanese or French tastes; indeed, if the U.S. government is trying to set cost of living differentials for its diplomats in Paris and Tokyo, it would be appropriate to base the comparison on U.S. tastes.

[3] Hereafter "goods" or "commodities" will be used interchangeably to refer to the arguments of the complete utility function; the intertemporal model involves $nT$ "goods".

That is, the partial expenditure function shows the minimum expenditure required to attain the indifference curve $s_\theta$ of the category utility function $V^\theta(X_\theta)$.

We now define the "partial cost of living index."

*Definition*: Suppose that the goods are partitioned into two subsets, $\theta$ and $\bar{\theta}$, and that the goods in $\theta$ are separable from those in $\bar{\theta}$. The *partial cost of living index* for category $\theta$, $I^\theta(P_\theta^a, P_\theta^b, s_\theta, R^\theta)$, is defined by

$$I^\theta(P_\theta^a, P_\theta^b, s_\theta, R^\theta) = \frac{E^\theta(P_\theta^a, s_\theta)}{E^\theta(P_\theta^b, s_\theta)}.$$

The partial cost of living index differs from the complete index in that: (a) the preference ordering on which the comparison is based is a category preference ordering rather than the complete preference ordering, and (b) the two price regimes being compared are partial price regimes and, hence, are represented by price vectors of lower dimensionality than the complete price vector. If preferences are separable, then the partial index is a "natural" subindex. Its principal limitation is that it is defined only when preferences are separable.

The subscript $\theta$ in the comparison and reference price vectors is somewhat misleading. The base preference ordering is identified by $R^\theta$, and $s_\theta$ denotes an indifference curve from that preference ordering. The comparison and reference price vectors, $P_\theta^a$ and $P_\theta^b$, must be dimensionally consistent with $R$. That is, if $\theta$ identifies "food" and there are 7 goods in the food category, the reference and comparison price vectors must each have 7 elements. If $P_*^a$ and $P_*^b$ are any two vectors dimensionally consistent with $R$, then we can calculate the cost of living index $I(P_*^a, P_*^b, s, R)$. We interpret the index by treating $P_*^a$ and $P_*^b$ as if they were alternative vectors of "food" prices. That is, we treat the comparisons between $P_*^a$ and $P_*^b$ as one between $P_\theta^a$ and $P_\theta^b$ where $P_\theta^a = P_*^a$ and $P_\theta^b = P_*^b$. In the "food" context, this is of no importance whatever; even if there happen to be the same number of clothing goods as food goods, no one would form an index to compare a vector of food prices with a vector of clothing prices. But in the intertemporal context, the most natural comparison is between vectors of spot prices corresponding to different periods.[4]

If the intertemporal utility function is weakly separable by periods

$$U(X) = W[V^1(X_1), \ldots, V^T(X_T)]$$

the partial cost of living index for period $t$, $I^t(P_*^a, P_*^b, s_t, R^t)$, is given by

$$I^t(P_*^a, P_*^b, s_t, R^t) = \frac{E^t(P_*^a, s_t)}{E^t(P_*^b, s_t)}$$

where $P_*^a$ and $P_*^b$ are alternative one period price vectors and $E^t(P_*, s_t)$ is the cost function corresponding to the one period utility function $V^t(X_t)$ and $s_t$ denotes an indifference curve of $V^t(X_t)$. The partial cost of living index for period $t$ is based on the preference ordering for period $t$, $R^t$, but it can be used to compare price vectors from any periods. For example, the index $I(P_1^a, P_2^b, s_t, R^t)$ compares spot prices from period 1 with spot prices from period 2 on the basis of the preferences of

[4] It would violate no formal rule of analysis to use the partial index to compare vectors of futures prices corresponding to different periods, but the spot price interpretation seems to be a more "natural" one for one period subindexes.

period $t$. We can view this as a comparison of two hypothetical vectors of spot prices from period $t$, $P_t^a$ and $P_t^b$, where $P_t^a = P_1^a$ and $P_t^b = P_2^b$.

If the intertemporal utility function is weakly separable by periods, we can construct a partial cost of living index on the basis of any one of the $T$ one period utility functions $\{V^1, \ldots, V^T\}$. The choice of an appropriate base preference ordering is not a matter for technical economic analysis, although Fisher and Shell [1968] argue convincingly that the most interesting base preference ordering is likely to be the one reflecting the tastes of the current period.

Thus far we have identified the indifference curve to be attained in the expenditure function and the cost of living index by the value of the utility function, $s$. In discussing the relationship between subindexes, it is more convenient to specify the base indifference curve by means of a "base" commodity bundle, $X^0$, than by the value of the utility function. We write the expenditure function as $E(P, X^0)$ and the cost of living index as $I(P^a, P^b, X^0, R)$ instead of $E(P, s)$ and $I(P^a, P^b, s, R)$. The notation is slightly sloppy, since the same symbol is used to denote expenditure as a function of the $2n$ variables $(P, X^0)$ and the $n + 1$ variables $(P, s)$, but the meaning is unambiguous. In the new notation, the partial expenditure function becomes $E^\theta(P_\theta, X^0)$ and the partial cost of living index $I^\theta(P_\theta^a, P_\theta^b, X_\theta^0, R^\theta)$.

We now define the "conditional expenditure function."

*Definition*: Suppose that the goods are partitioned into two subsets, $\theta$ and $\bar{\theta}$, and let $X^0 = (X_\theta^0, X_{\bar{\theta}}^0)$ be the base commodity bundle. The *conditional expenditure function* for category $\theta$, $E^\theta(P_\theta, X^0)$, is given by

$$E^\theta(P_\theta, X^0) = \min_{k\in\theta} \sum p_k x_k$$

subject to $U(X_\theta, X_{\bar{\theta}}) = U(X^0)$ and $X_{\bar{\theta}} = X_{\bar{\theta}}^0$.

That is, the conditional expenditure function shows the expenditure on the goods in $\theta$ required to attain the indifference curve of $X^0$ when the goods in $\bar{\theta}$ are fixed at the levels $X_{\bar{\theta}}^0$. The conditional expenditure function is analogous to a "short run" variable cost function in production theory, when the goods in $\theta$ and $\bar{\theta}$ correspond to variable and fixed inputs, respectively. It is also closely related to the conditional compensated demand functions introduced in Pollak [1969].

*Definition*: Let $\{\theta, \bar{\theta}\}$ be a partition of the set of all goods. The conditional cost of living index for category $\theta$, $L^\theta(P_\theta^a, P_\theta^b, X^0, R)$, is given by

$$L^\theta(P_\theta^a, P_\theta^b, X^0, R) = \frac{E^\theta(P_\theta^a, X^0)}{E^\theta(P_\theta^b, X^0)}.$$

The conditional cost of living index has all the properties of a traditional cost of living index. To prove this, we have only to verify that it corresponds to a "well behaved" preference ordering, namely, the conditional preference ordering over $X_\theta$ given $X_{\bar{\theta}}^0$; the base indifference curve is specified by the requirement that $X_\theta$ must satisfy $U[X_\theta, X_{\bar{\theta}}^0] = U[X_\theta^0, X_{\bar{\theta}}^0]$.[5]

If the goods in $\theta$ are separable from those in $\bar{\theta}$

$$U(X_\theta, X_{\bar{\theta}}) = W[V^\theta(X_\theta), X_{\bar{\theta}}]$$

[5] By a "well behaved" preference ordering we mean one which can be represented by a continuous utility function which is quasi-concave and nondecreasing in its arguments.

then the conditional expenditure function for the goods in $\theta$ is independent of $X_{\bar{\theta}}^{0}$. This follows immediately from the definition of the conditional expenditure function as the minimum expenditure on the goods in $\theta$ subject to

$$U(X_{\theta}, X_{\bar{\theta}}^{0}) = U(X_{\theta}^{0}, X_{\bar{\theta}}^{0})$$

since this constraint becomes

$$V^{\theta}(X_{\theta}) = V^{\theta}(X_{\theta}^{0})$$

and is independent of $X_{\bar{\theta}}^{0}$. Hence, if the goods in $\theta$ are separable from those in $\bar{\theta}$, the conditional cost of living index is independent of $X_{\bar{\theta}}^{0}$.

Conversely, the separable case is the only one in which the conditional cost of living index is independent of $X_{\bar{\theta}}^{0}$. That is, the conditional index is independent of the goods in $\bar{\theta}$ if and only if the goods in $\theta$ rae separable from those in $\bar{\theta}$. Thus, the separable case is the only one in which the goods in $\bar{\theta}$ drop out and play no role in the conditional cost of living index. Furthermore, if the goods in $\theta$ are separable from those in $\bar{\theta}$, then the conditional index coincides with the partial index. These results are proved in Pollak [1973b].

These theorems summarize the relationships between the partial and conditional indexes. The partial index embodies our intuitive view that in the separable case we can construct meaningful subindexes on the basis of category utility functions. The conditional index is an extension of the partial index to the general case.

In Pollak [1973b] I also define the "generalized conditional cost of living index," $L^{\theta}(P_{\theta}^{a}, P_{\theta}^{b}, X_{\bar{\theta}}^{*}, X^{0}, R)$. This index is based on the conditional preference ordering corresponding to $X_{\bar{\theta}}^{*}$, which, in the intertemporal context, specifies consumption in "other periods." The base indifference curve is identified by $X^{0}$. In the conditional index, $X^{0}$ plays a double role: it identifies the base indifference curve and it also specifies the levels at which the goods in $\bar{\theta}$ are held fixed. In the generalized conditional index, these roles are separated. If $X_{\bar{\theta}}^{*} = X_{\bar{\theta}}^{0}$, then the generalized conditional index coincides with the conditional index.

Separability is the crucial simplifying condition for the construction of one period indexes, since it allows us to ignore consumption outside the period for which we are constructing the index. If the period for which we are constructing the index is not separable from the rest, then the conditional index for that period depends on consumption in other periods. In the absence of this type of separability, to construct a subindex for period $t$ we must specify a base consumption vector $X^{0}$ which serves a double function: it identifies the conditional preference ordering on which the index is based, and it identifies a base indifference curve. The theory does not dictate the choice of a particular $X^{0}$, and guidance must be sought from the problem at hand.

### 3. Habit Formation

In this section I discuss the implications of habit formation for the construction of the complete intertemporal cost of living index and for one period subindexes. The usual discussion of habit formation begins with a short run utility function some of whose parameters depend on past consumption. Given a specification of the levels of consumption of all goods in the previous period, we can use this preference ordering as a base on which to construct a one period cost of living index. But the dependence of this index on the specified consumption history does present a difficulty unless the problem at hand singles out a particular consumption history as uniquely appropriate. An implication of C. C. von Weizsäcker's analysis of habit formation (von Weizsäcker [1971]) is that one can construct a cost of living index without specifying a consumption history by basing it on the "long run utility function," that is, the utility function which rationalizes the long run or steady state demand functions. I have argued elsewhere (Pollak [1973a]) that von Weizsäcker's analysis is incorrect: except in rare special cases von Weizsäcker's long run utility function does not exist, and even when it does exist, it has no welfare significance; hence, the "long run utility function" does not provide a satisfactory framework for constructing a cost of living index.[5b]

Virtually all specifications of habit formation assume that the individual does not recognize the impact of his present consumption on his future tastes. This assumption, coupled with the assumption that total expenditure in each period is determined exogenously, substantially simplifies the analysis of demand behavior in each period. But it is difficult to integrate a model of allocation within a single period based on these assumptions into the intertemporal allocation framework. I call a specification of habit formation in which an individual does not recognize that his present consumption has an impact on his future tastes "naive habit formation." In contrast, "rational habit formation" refers to a specification in which the individual takes full account of the effect of his current consumption on his future preferences. In a model of rational habit formation an individual maximizes an intertemporal utility function, and this utility function can serve as the base preference ordering for a complete intertemporal cost of living index. But because current consumption influences futures tastes, the intertemporal preference ordering is not separable by periods, so one period subindexes must be conditional rather than partial indexes. Naive habit formation, because it resists incorporation into a model of intertemporal allocation, does not provide a satisfactory starting point for constructing a complete intertemporal cost of living index.

The usual approach to habit formation is to begin with a "short run" utility function, postulate that some of its parameters depend on past consumption, and examine the resulting system of short run demand functions. See, for example, Stone [1966] and Pollak [1970]. Formally, let $V(X_t; X_{t-1})$ denote a short run utility function over $X_t$, given the consumption history $X_{t-1}$. In period $t$ the individual takes $X_{t-1}$ as given and chooses $X_t$ to maximize $V(X_t; X_{t-1})$ subject to the budget constraint $\sum_{k=1}^{n} p_{kt}x_{kt} = \mu_t$, where $\mu_t$ denotes total expenditure. Total expenditure is assumed to be exogenously determined and the focus of the analysis is on the determination of the consumption pattern for a particular period; we denote the short run demand functions by $X_t = h(P_t, \mu_t, X_{t-1})$. This approach appears somewhat *ad hoc* when viewed against the models of intertemporal allocation discussed in Section 1, but this comparison ignores the essentially empirical orientation of the habit model. The habit model is intended to provide

[5b] In a very interesting paper, Peter J. Hammond [1974] investigates the existence of a "long run preference relation" without requiring the relation to be an ordering.

an empirically useful dynamic generalization of the traditional static model of utility maximization, and has been reasonably successful in providing a theoretical foundation for empirical work.

One can certainly base a cost of living index on the short run preference ordering $R_*$ corresponding to the utility function $V(X_t; X_{t-1})$. We first define the expenditure function, $E(P_*, X_*^-, X_*^0)$, as the minimum expenditure required to attain the indifference curve $X_*^0$ at prices $P_*$ when consumption in the previous period was equal to $X_*^-$. In this notation, $X_*^0$ and $X_*^-$ are one period consumption vectors, and $E(P_*, X_*^-, X_*^0) = \min \sum_{k=1}^n p_{k*}x_{k*}$ subject to $V(X_*; X_*^-) = V(X_*^0; X_*^-)$. We define the cost of living index, $I(P_*^a, P_*^b, X_*^-, X_*^0, R_*)$, by

$$I(P_*^a, P_*^b, X_*^-, X_*^0, R_*) = \frac{E(P_*^a, X_*^-, X_*^0)}{E(P_*^b, X_*^-, X_*^0)}.$$

The index compares the cost of attaining the indifference curve of $X_*^0$ in the price situations $P_*^a$ and $P_*^b$.

Since short run preferences depend on past consumption, construction of the short run index requires us to specify the consumption history to identify the base preference ordering. The situation is analogous to the role of consumption in "other periods" in the generalized conditional index. As in that case, specification of an appropriate consumption history must come from the problem being considered.

The short run demand functions of the habit model imply a system of long run or steady state demand functions. Formally, the long run demand functions are defined as the steady state solutions to the system of short run demand functions: $X_* = h(P_*, \mu_*, X_*)$. We denote the long run demand functions by $X_* = H(P_*, \mu_*)$. C. C. von Weizsäcker [1971] claims that the long run demand functions of the habit formation model can be rationalized by a utility function, $V(X_*)$, and argues that this utility function is an appropriate indicator of welfare. If von Weizsäcker is correct, then we can use the long run utility function as a base for one period cost of living index and avoid the problem of specifying a consumption history. But we can only do this if the long run utility function exists. In Pollak [1973] I show that von Weizsäcker is incorrect about the existence of the long run utility function: the long run demand functions can be rationalized by a utility function only in very special cases. The demonstration is long and tedious, and I shall not attempt to summarize it here. I also argue in Pollak [1973] that even when the long run utility function exists, it has no welfare significance. That is, the long run utility function is the same type of construct as a community indifference map which rationalizes market demand functions; if it exists, it is a convenient device for coding all of the information about demand behavior, but this is all. (See Samuelson [1956]). In general, market demand functions cannot be rationalized by a "market utility function." In those special cases when they can be, the utility function must be scrupulously interpreted in terms of positive economics; it has no normative or welfare significance.

The question of the welfare interpretation of the long run utility functions reduces to the following: suppose that there exists a sequence of consumption bundles which enable an individual to go from an initial consumption situation $X_*^0$ to a terminal situation $\overline{X}_*$ in a finite number of steps, feeling that he is better off at each step than at the one before. Does this imply that he is better off—in terms of his own preferences—at $\bar{X}_*$ than at $X_*^0$? To quote from Pollak [1973a]: "I interpret an individual's willingness to move from $X_*^0$ to $\bar{X}_*$ in a sequence of small steps when he is unwilling to do so in a single large step as indicative of his failure to understand the habit formation mechanism, and not of the underlying superiority of $\bar{X}_*$." Consider, for example, the process by which a non-eater of artichokes develops a taste for artichokes by gradually increasing his consumption of them; consider the same process for cigarettes. This is precisely what von Weizsäcker has in mind when he speaks of long run preferences, but the interpretation of such sequences in terms of "long run preferences" is misleading. The relevant notion of preference must surely be an intertemporal one, not one which depends crucially on the individual's failure to understand the dynamics of his own tastes.

One might think that the utility function $V^*(X_*) = V(X_*; X_*)$ would rationalize the steady state demand functions. But even when the long run demand functions can be rationalized by a utility function, that utility function does not coincide with $V^*(X_*)$. In the habit model, the individual makes a sequence of short run decisions and always treats his own past consumption as fixed. Maximization of $V^*(X_*)$ implies maximization with respect to both current and past values and hence is not consistent with the habit model.[6]

Since neither the long run utility function nor an approach based on $V^*(X_*)$ provide a satisfactory cost of living index, the construction of a one period index in the habit model requires the specification of a consumption history, and the resulting index reflects not only the prices being compared, but also the particular consumption history specified.

The habit model of Pollak [1970] is a modification of the static one period approach of traditional demand theory. Tastes in each period depend on consumption in the previous period, and perhaps also on consumption in the more distant past. However, the model requires that the individual make current consumption decisions in a one period framework without recognizing that these decisions will affect his future tastes. I say that an individual who fails to take account of the impact of his current consumption on his future tastes exhibits "naive habit formation."

In the naive habit model of Pollak [1970] total expenditure in each period is taken as given. There are two ways to embed this model in a more general one which explains the determination of total expenditure in each period. The first is to assume that savings decisions reflect some rule of thumb, rather than maximization of an intertemporal utility function. This approach precludes construction of a complete intertemporal cost of living index, since such an index is based on an intertemporal utility function. The second way to explain the determination of total expenditure in each period is to integrate naive habit formation into a model of intertemporal allocation. There are several ways to do this, none of them

[6] Furthermore, the utility function $V^*(X_*)$ depends not only on the conditional preference ordering over $X_t$ given $X_*^-$, but also on the cardinal properties of the short run utility function $V(X_t; X_{t-1})$ used to represent that conditional preference ordering. For example, the short run utility functions $V(X_t; X_{t-1})$ and $V(X_t; X_{t-1}) + \phi(X_{t-1})$ represent the same conditional preference ordering over $X_t$ given $X_{t-1}$; but the utility functions $V(X_*; X_*)$ and $V(X_*; X_*) + \phi(X_*)$ imply different preference orderings over $X_*$.

very satisfactory. If we are not concerned with the individual's allocation of expenditure within any period except period 1, then the naive habit model is consistent with an intertemporal utility function of the form

$$U(X) = W[V(X_1; X_0), X_2, \ldots, X_T; X_0].$$

This implies that allocation within period 1 is made without reference to the future; it does not, however, imply that the allocation within period 2 can be made without reference to the future. If we require that allocation in each period be independent of the future, we are led to an intertemporal utility function of the form

$$U(X) = W[V(X_1, X_0), V(X_2, X_0), \ldots, V(X_T, X_0)].$$

This implies that the individual behaves as if his current one period preferences will persist in all future periods. The persistent preference solution permits the construction of a complete intertemporal cost of living index, but the implied index is based on an intertemporal preference ordering which embodies a false and repeatedly falsified assumption about preferences.

The naive habit model is susceptible to two serious criticisms. First, it does not deal with the determination of total expenditure in each period, and it cannot easily be modified to do so in an acceptable way: neither the rule of thumb nor the persistent preference solution is appealing. Second, the naive model does not even produce a satisfactory account of the allocation of expenditure within a single period. The basic assumption of the naive habit model—an assumption carried over from traditional demand theory—is that total expenditure can be allocated among the goods available in each period without considering the future. But an individual whose current tastes depend on his past consumption might be expected to realize that his future tastes will depend on his current consumption; and once he realizes this, his choice of a current consumption pattern will take account of its impact on his future tastes. The hallmark of naive habit formation is that the individual does not allow for the impact of his current consumption on his future tastes.

We now consider a version of habit formation in which the individual takes full account of the impact of current consumption on future preferences.[7] Consider the weakly separable intertemporal utility function

$$U(X) = W[V(X_1), \ldots, V(X_T)].$$

Now replace each of the one period utility functions by a "short run utility function" which depends on both current consumption and consumption in the previous period. This yields

$$U(X) = W[V(X_1, X_0), V(X_2, X_1), \ldots, V(X_T, X_{T-1})].$$

This intertemporal utility function is the basis for the model of *rational* habit formation. Since the new utility function is not separable by periods, it is not correct to call $V(X_t, X_{t-1})$ a "one period utility function" except in a metaphoric sense. From the standpoint of empirical analysis of the allocation of expenditure

[7] See Lluch [1974] for a treatment of consumption patterns and saving behavior in a model of rational habit formation.

within each period, rational habit formation provides a much less tractable model than naive habit formation. Because the intertemporal utility function is not separable, the allocation of expenditure within each period cannot be understood without reference to behavior in future periods. We can define an intertemporal cost of living index on the basis of the intertemporal utility function; but since the intertemporal preference ordering is not separable by periods, the appropriate subindex is the conditional rather than the partial cost of living index.[8]

## 4. Summary and Implications

In this section I summarize the discussion of the last three sections and develop its implications for the treatment of interest rates.

There are two ways in which the theory of the cost of living index can be extended from its traditional one period framework to yield a complete index in a multiperiod setting. The first relies on futures prices and yields an index which has all the properties of the traditional cost of living index, differing from it only in that it is based on the intertemporal preference ordering and the prices it compares are futures prices. The second assumes perfect foresight and perfect capital markets, and uses spot prices and interest rates to construct the complete intertemporal index; the resulting index is formally identical to one based on futures prices with "present value prices" playing their role. These intertemporal cost of living indexes provide a theoretically satisfying solution of the problem of constructing a complete cost of living index in a multiperiod framework.

The trouble with these intertemporal cost of living indexes is a practical one. Futures markets do not exist for most commodities, expectations are not held with certainty, and capital markets are not perfect. The gap between theory and practice appears greater for the complete intertemporal index than for the usual one period index. The difficulties which stand in the way of constructing the complete intertemporal index provide one motivation for focusing on subindexes which compare alternative one period spot price vectors. Such subindexes would be of interest even if we could construct the complete intertemporal index, and since we cannot, they are the best we can hope to do. In Pollak [1973b] I develop a theory of subindexes. If preferences are separable, I define the "partial" cost of living index in the "natural" way on the basis of the category utility function. If preferences are not separable, I define the "conditional" index. The conditional index is based on the conditional preference ordering; the consumption of other goods is held fixed at predetermined levels. I show that, when the group of goods for which we are constructing the subindex is separable from the rest, then the conditional index is independent of the predetermined levels of the remaining goods and coincides with the partial index. Furthermore, this is the only case in which the conditional index is independent of the other goods.

The theory of subindexes applies directly to the construction of one period indexes in a multiperiod world. If the period whose preferences we are using as the base for constructing the subindex is separable from the rest, then the partial

[8] I am grateful to Steven M. Goldman for helpful comments on this and related issues. See Goldman [1974] for an analysis of when it is possible to construct a conditional cost of living index which depends only on past levels of consumption.

index is the "natural" one period index. If it is not, we must turn to the conditional index. To specify the base preference ordering for the conditional index we must specify the level of consumption of every good in every other period. This specification must come from the particular problem which the index is intended to resolve, not from abstract theoretical arguments.

If the intertemporal utility function is weakly separable by periods

$$U(X) = W[V^1(X_1), \ldots, V^T(X_T)]$$

we can base a comparison of one period price vectors on any of the $T$ one period preference orderings $\{V^1, \ldots, V^T\}$. Again, the problem at hand must dictate the choice of a particular base preference ordering, and, once it has been selected, the choice of a base indifference curve.

Habit formation presents a new set of conceptual difficulties. Because it was introduced into demand analysis as a dynamic generalization of the one period static models of traditional demand theory—the usual approach is to begin with a short-run utility function and assume that some of its parameters depend on past consumption—habit formation is difficult to integrate into a model of intertemporal allocation. It is straightforward to construct a one period cost of living index in this model by specifying consumption levels for all goods in the previous period and basing the index on the short run preference ordering implied by that consumption history. The problem at hand must determine the choice of an appropriate consumption history on which to base the index, and it is incorrect to view the process of selecting a particular consumption history as one which imparts an element of arbitrariness to the index.

Since the habit model leads to long run or steady state demand behavior which is independent of past consumption, it might be thought that we could circumvent the problem of specifying a consumption history by following C. C. von Weizsäcker's procedure and basing the cost of living index on the utility function which rationalizes the long run demand functions. This is incorrect. First, except in certain special cases, the long run demand functions cannot be rationalized by a long run utility function. Second, even when such a utility function exists, it has no welfare significance. There is no justification for using the long run utility function as a base for a cost of living index.

To construct a complete intertemporal cost of living index in a habit formation model it is first necessary to integrate habit formation into an intertemporal allocation model. This requires us to distinguish between naive and rational habit formation. An individual who fails to take account of the impact of his current consumption on his future tastes exhibits naive habit formation; it does not seem to be possible to integrate naive habit formation into a multiperiod model in a way which yields a plausible intertemporal cost of living index. This is unfortunate since most habit models considered in demand analysis are of this type.

An individual who recognizes the impact of his current consumption on his future tastes exhibits "rational habit formation." Since every consumption decision always allows for its impact on "future tastes," rational habit formation can and must be treated in a multiperiod model, and the complete intertemporal cost of living index is defined in a straightforward way. Since the intertemporal utility function implied by rational habit formation is not separable, the appropriate one period subindex is the conditional rather than the partial index.

The implications of this analysis for the treatment of interest rates are straightforward. We first consider their treatment in the complete intertemporal index, and then in one period subindexes.

We defined two complete intertemporal cost of living indexes in Section 1, the "futures" index, $\hat{I}(\hat{P}^a, \hat{P}^b, s, R)$, which does not explicitly depend on interest rates, and the "spot" index, $I(P^a, r^a, P^b, r^b, s, R)$, which does. As we saw in Section 1, an increase in any interest rate will cause the spot index to decline. If we regard futures prices as functions of spot prices and interest rates, that is, as present value prices, then the futures index will reflect changes in interest rates through the implied changes in present value prices. If, on the other hand, we regard futures prices themselves as fundamental, then the question of the role of interest rates seems ill-posed, since the question presupposes that it is appropriate to treat interest rates as independent variables.

It might be thought that interest rates would also affect the complete intertemporal index through their effects on the prices of the services of consumer durables. For definiteness, we discuss this problem in terms of the spot index, but essentially the same analysis applies to the futures index. If some of the $x$'s represent the services of consumer durables, then the corresponding $p$'s are the spot prices of these services (that is, the one period rentals), there is no difficulty treating a change in interest rates, whether or not it is accompanied by changes in these rental prices (it need not be, if there are offsetting changes elsewhere, say, in factor prices). Changes in spot prices caused by changes in interest rates have the same effect on the index as the same changes in spot prices caused by changes in raw materials prices; the fact that the underlying change was in interest rates does not imply that special treatment is called for. Since we do not usually observe rental prices for the services of consumer durables we must compute implicit rentals for them from observable prices and interest rates. In the absence of transactions costs and various other "frictions," it is possible to calculate an implicit price for the services of a consumer durable in period $t$ using its purchase price in period $t$, its expected second hand price in $t + 1$, and the interest rate, $r_t$. If we use this procedure, then changes in interest rates will cause changes in the implicit prices of the services of consumer durables. But it is useful to keep separate the direct effects of changes in interest rates and the indirect effects which operate through the implicit prices of the services of consumer durables, just as it is useful to keep separate the direct effect of higher gasoline prices and the indirect effects which operate through higher explicit prices of goods shipped by truck.

Since the purpose of a one period subindex is to focus on a particular period and to isolate it from intertemporal complications, one would not expect such a subindex to involve the interest rate directly, but only indirectly, through its effect on the implicit prices of the services of consumer durables. An examination of the definition of the conditional index bears out this expectation. To construct a conditional subindex for period $t$, we hold fixed the levels of consumption of all goods in all other periods and calculate the ratio of the expenditures on the goods in period $t$ required to attain a particular indifference curve. These expenditures depend on the vectors of spot prices being compared, but are independent of interest rates and of prices in other periods. Hence, interest rates play no direct role in one period subindexes, but when prices of the services of consumer durables are not observable, interest rates play an indirect role through their effect on the implicit rentals of the services of consumer durables.

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# COMMENT

BY STEVEN MARC GOLDMAN

In his paper "The Intertemporal Cost of Living Index," Professor Robert A. Pollak examines the increased expenditure necessary, in any single period, to compensate an individual for price changes in that same period. He demonstrates that the construction of such a cost of living index which is to be independent of past and future consumption is meaningful only for the case where the intertemporal utility function

$$U(X_0, \ldots, X_T) \tag{1}$$

is weakly separable by periods. For this situation, there is a preference ordering over the commodity subspace for each period which is independent of earlier or later consumption bundles. The needed change in expenditure is simply that needed to return the consumer to his old sub-indifference surface for that period. In the absence of such separability, preferences over each subspace depend on consumption in all periods and only a *conditional* cost of living index can be constructed.

Professor Pollak then explores a model of habit formation and argues that such conditions for separability will not, *in general*, be satisfied and that only a conditional cost of living index, in which past and future levels of consumption are specified, is possible. We shall examine here the conditions under which a model of habit formation permits the construction of a cost of living index requiring the knowledge of only past activities.

A more general statement of Pollak's habit model is that the intertemporal utility function (1) be written in the following form:

$$\tilde{U}[X_0, V^1(X_0, X_1), \ldots, V^{T-1}(X_0, \ldots, X_{T-1}), V^T(X_0, \ldots, X_T)] \tag{2}$$

Each of the functions $V^t(\,.\,)$ may be interpreted as the short run utility from the consumption of $X_t$, which depends upon past consumption levels. The individual's overall satisfaction is then a function of these short run utilities. Thus, the notion of habit formation appears in the conditioning of present preferences upon past behavior. But while, for each $t$, the short run utility function provides an ordering for the $t$-th period consumption possibilities, this ordering will, in general, differ from that implied by the full intertemporal ordering (as described by $U(\,.\,)$) since $X_t$ also enters into the short run utility functions of all later periods. A quick check indicates that the marginal rate of substitution between different commodities in any single period will be sensitive to changes in the quantity of any commodity consumed in other periods—both past and future. Therefore, the intertemporal utility function may not be separable by periods and only a conditional cost of living index is available involving knowledge of future consumption.

Rather than working with the general form (2), Professor Pollak assumes that the short run utility at time $t$ depends only upon the consumption bundle from the immediately preceeding period, or

$$W[V(X_1, X_0), V(X_2, X_1), \ldots, V(X_T, X_{T-1})]$$

but the difficulty indicated above remains.

We shall now seek to determine under what conditions a cost of living index may be constructed which depends only on the knowledge of past levels of consumption. This requires, simply, that for each $t$, it be possible to construct from the intertemporal utility function a preference ordering for that period which is independent of future consumption. A necessary and sufficient condition for this construction is that the marginal rate of substitution between any two commodities at time $t$ be unchanged by variations in consumption beyond that date. These, however, are exactly the conditions of direct weak recursivity[1] and, if fulfilled, require that the intertemporal utility function (1) may be rewritten:

$$U(f^T, f^{T-1}, \ldots, f^1, f^0)$$

where

$$f^t = f^t(X_t, f^{t-1}, f^{t-2}, \ldots, f^1, f^0) \quad \text{and} \quad f^0 = f^0(X_0)$$

The functions $f^t$ are interpretable as short run utility functions and, again, intertemporal satisfaction may be viewed as the result of these short run utilities. Following Professor Pollak's methodology, it is then possible to construct a cost of living index for each period, $t$, once having specified the short run utility levels for that and all prior periods but without any reference to future consumption.

This "gain" is made at the cost of some generality in the notion of habit formation. The ordering of consumption alternatives at time $t$ can no longer depend on the bundles of goods consumed previously but only upon the short run utilities derivable from those bundles.

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[1] See, for example, Blackorby, Primont and Russell, *this issue*.