Volume Title: Annals of Economic and Social Measurement, Volume 4, number 1

Volume Author/Editor: Sanford V. Berg, editor

Volume Publisher: NBER

Volume URL: http://www.nber.org/books/aesm75-1

Publication Date: 1975

Chapter Title: Estimation of Dynamic Gorman Polar Form Utility Functions

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Chapter URL: http://www.nber.org/chapters/c10219

Chapter pages in book: (p. 103 - 116)

Annals of Economic and Social Measurement, 4/1, 1975

ESTIMATION OF DYNAMIC GORMAN POLAR FORM UTILITY FUNCTIONS

BY RICHARD BOYCE*

The purpose of this study is to compare the habit formation and the state variable approaches to dynamization of demand systems generated by the Gorman polar form. Full information, maximum likelihood, parameter estimates of branch functions of the generalized S-branch system with four alternative dynamic specifications are presented. These estimates are based on time series of per capita consumption of meat in the U.S. The likelihood ratio tests reveal that the linear habit formation specification is the most efficient representation of dynamic preferences for these particular data.

I. INTRODUCTION

Consumer demand systems that exhibit piecewise[1] linearity in income have played an important part in the empirical analysis of theoretically plausible demand functions.[2] While it is well known that Engel curves are not linear for a variety of consumer expenditures, this assumption is warranted theoretically when aggregate data are being utilized.

Thus, Klein and Rubin [1947] proposed the system known as the linear expenditure system (LES) which was subsequently estimated by Stone [1954] and others; Brown and Heien [1972] specified and estimated the *S*-branch utility system which generalized the LES; and Gorman [1953, 1961] characterized the general class of preferences exhibiting piecewise linearity in income. The indirect utility function of the general case has been termed the Gorman polar form (GPF) by Blackorby, Boyce, Nissen and Russell [1973] in a paper which presents estimates of an example of the GPF which generalizes the *S*-branch system.

The preferences corresponding to the LES, or its generalizations, are not necessarily homothetic. Rather, the utility function corresponding to the LES is a Cobb–Douglas function translated from the origin to some other point in consumption space, say $\gamma = [\gamma_1, \ldots, \gamma_n]$, as shown by Samuelson [1947] and Geary [1950]. The S-branch utility function is a two-level constant elasticity of substitution (CES) function similarly translated, but the GPF cannot, in general, be so characterized. In the GPF income consumption curves (ICC's) emanate from points which lie on a reference frontier in consumption space. The points, on this frontier sometimes can be interpreted as "subsistence bundles." Alternatively, they might be interpreted as "habitual," or "committed," consumption bundles.

The incorporation of some structure of the interdependence of demand over time into demand systems generated by the Gorman polar form has been accomplished in a variety of ways. Lluch [1974] has specified the demand system for

* I am indebted to Charles Blackorby, R. Robert Russell and Lester Taylor for their contributions to the research reported here. All errors are mine.

[1] Piecewise linear functions are continuous functions composed of linear segments.

[2] That is, demand functions that can be generated by utility maximization subject to a budget constraint.

durables based on a dynamic linear expenditure system. The demand functions generated are both theoretically plausible and dynamic in the sense that they result from the solution of an intertemporal utility maximization problem and allocate wealth intertemporally. More frequently, however, dynamization of the LES and S-branch systems has concentrated on estimating the functional relation between the reference quantity—which for these systems is the point of homotheticity—and past consumption. This class of dynamic specifications is based on the assumption that there are changes of taste over time and that those changes are embodied in alteration of the reference bundles. If the reference bundle is interpreted as a set of commodities that are necessarily or habitually consumed, then changes in its magnitude and composition may be attributed to habit formation, with habitual consumption in one period being correlated with total consumption in the previous period. Thus, Pollak and Wales [1969] have assumed each γ_i in period t is a linear function of the previous period consumption of the i-th commodity in the "linear habit formation" specification, or γ_i, in period t, is proportional to the previous period consumption in the "proportional habit formation" specification.

A more complex functional relation between the reference bundle and past consumption may be developed through the use of state variables as pioneered by Houthakker and Taylor [1970] and applied to the LES by Phlips [1972] and Taylor and Weiserbs [1972]. In the state variable approach to dynamization of demand equations it is assumed that there exist nonobservable state variables which are a composite of past consumption and may represent inventories of commodities or stocks of habits. The "linear state variable" approach assumes that the relation between γ_i and i-th state variable is linear while the "proportional state variable" approach assumes the relation is proportional.

The purpose of this study is to compare the habit formation and state variable approaches to dynamization of consumer demand systems in the context of the GPF. In Section II, the habit formation and state variable dynamic consumption models are applied to an example of the GPF that generates the branch demand functions of the S-branch system as a special case. In Section III the estimation techniques are discussed and the maximum likelihood parameter estimates of four functional specifications are presented.

II. Dynamization of the Gorman Polar Form

Let the preference structure of the per capita consumer be represented by $U:\Omega^n \to \Omega$, where U is a continuous, non-decreasing, quasiconcave utility function mapping the non-negative Euclidean n-orthant into the non-negative real line. Commodity vectors are represented by elements $x \in \Omega^n$ and prices by $p \in \Omega^n_{++}$, the strictly positive n-orthant. The preference structure may equivalently be represented by the cost function

$$C(\bar{U}, p) = \min_x \{p \cdot x | U(x) \geq \bar{U}\},$$

which is the minimum cost of obtaining utility level $\bar{U}$. The function C is increasing in $\bar{U}$, continuous, concave, and positively, linearly homogeneous (PLH) in p. It possesses second order partial derivatives almost everywhere.

Gorman has shown that, with the assumption of linear Engel curves, the cost function may be written as

$$C(\tilde{U}, p) = \tilde{U} \cdot \Pi(p) + \Lambda(p),$$

where both $\Pi(p)$ and $\Lambda(p)$ are PLH in p. These two functions inherit the rest of the properties of C as well.

The vector of Hicksian compensated demand functions

$$x = \nabla C(\tilde{U}, p) = \tilde{U}\, \nabla\Pi(p) + \nabla\Lambda(p)$$

is the gradient (where it exists) of the cost function with respect to prices. The compensated demand functions are linear in real income (measured in any given normalization) for all price configurations.

The GPF cost function does not generally represent preferences over the entire nonnegative orthant. Preference structures which are represented by a GPF cost function can be described with reference to any continuous, convex function $\theta : R^{n-1} \to R$. Define the zero[3] level set by

$$\{x | x \in \Omega^n, U(x) \geq 0\} = \{x | x \in \Omega^n, x \geq z \text{ for some } z \in g(\theta)\} = B(\theta).$$

The GPF cost function represents preferences over the set $B(\theta)$. The optimal consumption bundle at zero utility level depends upon prices and is given by

$$x = \Delta\Lambda(p)$$

This construction is illustrated in the accompanying figure for the case where $n = 2$. The point $\bar{x}(\bar{p})$ on the graph of $\theta(x_1)$ satisfies $\bar{x} = \nabla\Lambda(\bar{p})$. ICC($\bar{p}$) is the income consumption curve emanating from $\bar{x}$. Notice, however, that at prices $\mathring{p}$, the income consumption curve is only piecewise linear from the origin to $A(\mathring{p})$ and then the line ICC($\mathring{p}$).[4] Evidently if $\bar{x}(\mathring{p})$ lies outside the nonnegative orthant it cannot be interpreted as a subsistence bundle.

The structure of preferences above the base indifference curve is determined by the function $\Pi(p)$ which is interpreted as a price index. It is convenient to choose $\Pi(p)$ as the unit cost function of the preference function.

The static formulation of the model used in this study to compare the habit formation and state variable approaches to dynamization is a branch demand function of the generalized S-branch system (GSBS) investigated by Blackorby, Boyce, Nissen and Russell [1973]. It is specified by a CES unit cost function

$$\Pi(p) = \left(\sum_i \beta_i^\sigma p_i^{1-\sigma} \right)^{1/1-\sigma}, \qquad \sigma > 0, \beta_i > 0, \quad \forall i,$$

and a generalized Leontief reference expenditure function

$$\Lambda(p) = \sum_i \sum_j \gamma_{ij} p_i^{1/2} p_j^{1/2}, \qquad \gamma_{ij} = \gamma_{ji}, \quad \forall ij.$$

[3] This is simply a convenient normalization.

[4] The *estimated* income consumption curve is characterized in terms of $\bar{x}(\mathring{p})$ rather than $A(\mathring{p}) = \nabla\Lambda(\mathring{p})$ for example. Therefore we estimate the parameters of the graph of $\theta(x)$ (the reference frontier), rather than $\nabla\Lambda(p)$.

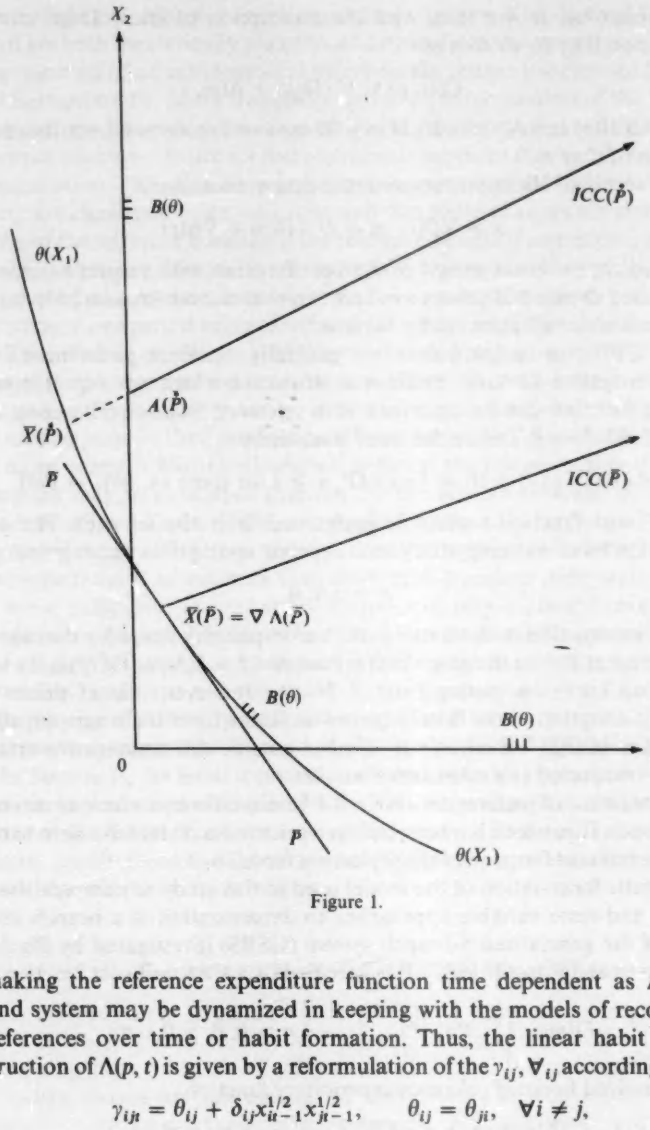

Figure 1.

By making the reference expenditure function time dependent as $\Lambda(p, t)$, the demand system may be dynamized in keeping with the models of reconstitution of preferences over time or habit formation. Thus, the linear habit formation construction of $\Lambda(p, t)$ is given by a reformulation of the γ_{ij}, $\forall_{ij}$ according to

$$\gamma_{ijt} = \theta_{ij} + \delta_{ij} x_{it-1}^{1/2} x_{jt-1}^{1/2}, \qquad \theta_{ij} = \theta_{ji}, \quad \forall i \neq j,$$
$$\delta_{ij} = \delta_{ji}, \quad \forall i \neq j.$$

If all off diagonal θ_{ij}'s and δ_{ij}'s equal zero, i.e., $\theta_{ij} = 0\ \forall i \neq j$, $\delta_{ij} = 0$, $\forall i \neq j$, this construction collapses to the linear habit formation specification of $\Lambda(p, t)$ in the LES as given by Pollak and Wales [1969]. That is,

$$\Lambda(p, t) = \sum_i \gamma_{it} p_{it}$$

where

$$\gamma_{it} = \theta_i + \delta_i x_{it-1} \quad \forall i, t.$$

Whereas the γ_{it}, $i = 1, \ldots, n$ are the elements of the reference bundle in period t (and the point of homotheticity) in the LES and S-branch systems, the γ_{ijt} in the GSBS system define the base indifference curve or reference frontier in period t and define the substitution characteristics along that frontier. However, they are not quantities. Rather, the reference bundle is given by

$$\nabla\Lambda(p, t) = \left[\sum_j \theta_{ij} p_{it}^{-1/2} p_{jt}^{1/2} + \sum_j \delta_{ij} x_{it-1}^{1/2} x_{jt-1}^{1/2} \times p_{it}^{-1/2} p_{jt}^{1/2}, i = 1, \ldots, n\right].$$

The complex interrelations of past consumption and present prices in determining $\nabla\Lambda(p, t)$ for the generalized Leontief $\Lambda(p, t)$ point toward the interpretation of the habit formation hypothesis in the context of general GPF preferences. That is, the base utility contour of the per capita consumer is altered in response to previous consumption with the partial effect on the reference consumption of the i-th commodity being related, not only to consumption of the i-th commodity in the last period, but to consumption of all commodities in the last period and, of course, present period prices. If we take the subsistence consumption interpretation of the base utility contour seriously, this more complex structure has particular appeal as we imagine the consumer adjusting his strategy for achieving minimal nutritional requirements by evaluating all elements of last period's food consumption simultaneously and then selecting from multiple elements of a revised strategy on the basis of present period relative prices.

The linear habit formation specification generates the proportional habit formation specification when the θ_{ij} are assumed to be identically zero. That is, when $\theta_{ij} = 0, \forall i, j,$

$$\Lambda(p, t) = \sum_i \sum_j \delta_{ij} x_{it-1}^{1/2} x_{jt-1}^{1/2} p_{it}^{1/2} p_{jt}^{1/2}, \qquad \delta_{ij} = \delta_{ji} \quad \forall i, j.$$

The reference bundle for the proportional habit model in period t is then

$$\nabla\Lambda(p, t) = \left[\sum_j \delta_{ij} x_{it-1}^{1/2} x_{jt-1}^{1/2} p_{it}^{-1/2} p_{jt}^{1/2}, i = 1, \ldots, n\right].$$

An alternative class of dynamic specifications may be formulated in terms of state variables. As proposed by Houthakker and Taylor [1970] and utilized by Phlips [1972] and Taylor and Weiserbs [1972], the nonobservable state variables embody the effect of past consumption according to the continuous relations

$$\dot{s}_i = x_i - \delta_i s_i \qquad i = 1, \ldots, n,$$

such that there exists one state variable associated with each commodity, and the value of the i-th state variable is given by the depreciated purchases of the i-th commodity. Thus, δ_i is a constant rate of depreciation associated with the i-th commodity.

The discrete analogs to these functions applied in this work are given by

$$s_{it} = (1 - \delta)s_{it-1} + x_{it-1}, \qquad s_{it} \geq 0, \quad \forall i, t.$$

These discrete relations are not identical to the finite approximations to the continuous relations applied by Houthakker and Taylor, Phlips and Taylor and Weiserbs. Two fundamental differences are that the depreciation rate is assumed to be the same for all commodities and, while the stock of the commodity or habit existing last period is depreciated, the purchases last period are not. This second restriction permits the state variable formulation to generate linear and proportional habit formation specifications as special cases when $\delta = 1$, and thus facilitate empirical comparison of the two approaches.

The "linear state variable" specification for the reference expenditure function relates γ_{ijt} to past consumption through the state variables according to the relation

$$\gamma_{ijt} = \theta_{ij} + \alpha_{ij} s_{it}^{1/2} s_{jt}^{1/2}, \qquad \theta_{ij} = \theta_{ji}, \quad \forall i, j, \qquad \alpha_{ij} = \alpha_{ji}, \quad \forall i, j,$$

and the reference expenditure function is then

$$\Lambda(p, t) = \sum_i \sum_j \theta_{ij} p_{it}^{1/2} p_{jt}^{1/2} + \sum_i \sum_j \alpha_{ij} s_{it}^{1/2} s_{jt}^{1/2} p_{it}^{1/2} p_{jt}^{1/2}.$$

The reference bundle for the linear state variable model in period t is

$$\nabla\Lambda(p, t) = \left[\sum_j \theta_{ij} p_{it}^{-1/2} p_{jt}^{1/2} + \sum_j \alpha_{ij} s_{it}^{1/2} s_{jt}^{1/2} p_{it}^{-1/2} p_{jt}^{1/2}, i = 1, \ldots, n \right].$$

An alternative functional specification for $\Lambda(p, t)$ in the state variable approach is given by

$$\gamma_{ijt} = \alpha_{ij} s_{it}^{1/2} s_{jt}^{1/2}, \qquad \alpha_{ij} = \alpha_{ji}, \quad \forall i, j.$$

With regard to this "proportional state variable" specification, it should be noted that satisfaction of the long run equilibrium conditions $s_{it} = s_{it-1}$, $\forall i, t$, implies homothetic long run preferences.

III. Estimation

In this section, the four dynamic specifications discussed are applied to the demand for fish, poultry, pork and beef in the United States from 1946–1968. The data used are annual time series on quantities consumed per capita converted from U.S. Department of Agriculture sources and prices taken from Bureau of Labor Statistics retail price series. These same data were used by Brown and Heien [1972] for the estimation of the meat branch of the S-branch system.

The demand relations are fitted in expenditure form by a technique that yields maximum likelihood estimates under the assumed error structure. The errors are assumed to be additive, jointly normally distributed, with zero means, constant over time with unknown variances–covariances. The covariances of errors in different time periods are assumed to be zero. This assumption, while hardly tenable in view of the dynamic specifications, is predicated by an inability to estimate the autoregressive structure on the errors that likely exists. One equation is deleted for estimation to avoid the singularity of the variance–covariance matrix of residuals implied by the budget constraint. The highly nonlinear concentrated likelihood function associated with the errors on the remaining $n - 1$ functions is maximized using the Bard [1967] version of the Gauss–Newton algorithm.[5]

The following tables present the estimation results. Table 1 contains the estimated maximum values of the log of the likelihood function (minus a constant) and the number of estimated parameters in each system. The linear state model generates the linear habit, static, proportional state and proportional habit models as special cases. The linear habit model generates the static and proportional habit models but not the proportional state model as a special case. The proportional state model generates only the proportional habit model as a special case.

Statistical tests of significance between the estimates of the nested structures are based on the asymptotic Chi-square distribution of $-2 \log \lambda$ where λ is the ratio of the maximum of the likelihood function for the constrained system over the unconstrained system. These tests reveal that while each linear specification is superior to its proportional counterpart and the static model, the linear state specification is not superior to the linear habit specification.

The ranking of the estimated systems indicated by values of the likelihood function is reinforced by the estimated parameter values given in Tables 2–5. Table 2 contains the estimated parameter values for the linear habit specification underscored by their standard errors. There are many significant δ_{ij} parameters which indicates the validity of a dynamic component to consumer preferences. The greater generality of the CES specification for the Π function over a Cobb–Douglas form is indicated by the estimate of σ. The parameter σ, which is the

TABLE 1

ESTIMATED LIKELIHOOD VALUES

	log likelihood	number of estimated parameters
Linear State	−18.1	29
Linear Habit	−18.1	24
Static	−47.5	14
Proportional State	−54.9	19
Proportional Habit	−57.2	14

[5] The computer processing was done on the IBM 360–75 at the University of California, Santa Barbara. Convergence to a maximum occurred in about 180 seconds for all specifications. For the habit formation specifications the parameter estimates were independent of the initial guesses; whereas for the state variable models, the parameter estimates were highly sensitive to the initial guesses.

TABLE 2

LINEAR HABIT FORMATION PARAMETER ESTIMATES

σ		Meat		
		1.16		
		(0.031)		
β_i	Fish	Poultry	Pork	Beef
	0.048	0.205	0.204	1
	(0.038)	(0.035)	(0.044)	(—)
θ_{ij}	Fish	Poultry	Pork	Beef
Fish	−13.4	4.87	6.14	29.2
	(14.2)	(4.30)	(4.83)	(22.1)
Poultry		−64.5	48.2	130.0
		(6.95)	(6.65)	(17.4)
Pork			−72.3	157.0
			(9.35)	(22.1
Beef				305.0
				(89.6)
δ_{ij}	Fish	Poultry	Pork	Beef
Fish	3.71	−0.643	−0.575	−1.96
	(2.86)	(0.639)	(0.388)	(1.59)
Poultry		7.13	−2.00	−6.90
		(0.659)	(0.399)	(0.888)
Pork			3.06	−4.51
			(0.309)	(0.630)
Beef				−7.43
				(2.26) .

elasticity coefficient for supernumerary consumption,[6] is seen to be significantly different from one.

Table 3 presents parameter estimates for the linear state variable approach to dynamizing the GPF. Recall that δ is the depreciation rate for the inventories of the "stocks" of habits in the relation

$$s_{it} = (1 - \delta)s_{it-1} + x_{it-1}.$$

When $\delta = 1$ the state variable specifications collapse to the corresponding habit formation models. As reported in Table 3, $\delta = 1.0$ to 3 significant places and the estimated parameter values for α_{ij} and θ_{ij} are nearly identical to those of the linear habit formation model. These parameter estimates resulted by using as initial guesses the reported linear habit formation parameter estimates for σ, β_i, θ_{ij}, α_{ij}; with δ set close to 1, and s_{i0}, $i = 1, 4$, the 0-th period values of the stock set equal to the 0-th period bundle; i.e., $s_{i0} = x_{i0}$, $i = 1, 4$. The estimation procedure converged after 20 iterations with most of the adjustment in parameters taking place in the s_{i0}, as can be seen by comparing Tables 2 and 3. The resulting estimates for $s_{i0}, i = 1, 4$ are not significant—indeed, they generate t-statistics of the order 10^{-5}.

[6] That is, expenditure above the "subsistence" level.

TABLE 3

LINEAR STATE VARIABLE PARAMETER ESTIMATES

σ		Meat		
		1.16		
		(0.050)		
β_i	Fish	Poultry	Pork	Beef
	0.048	0.205	0.204	1.0
	(0.035)	(0.065)	(0.058)	(—)
δ		Meat		
		1.00		
		(0.021)		
s_{i0}	Fish	Poultry	Pork	Beef
	34.2	23.8	173.0	0.00
	(10^7)	(10^7)	(10^8)	(10^8)
θ_{ij}	Fish	Poultry	Pork	Beef
Fish	−13.4	4.87	6.14	29.2
	(13.3)	(4.24)	(4.25)	(19.7)
Poultry		−64.5	42.8	131.0
		(8.52)	(9.68)	(18.8)
Pork			−72.3	157.0
			(12.7)	(30.8)
Beef				305.0
				(155.0)
α_{ij}	Fish	Poultry	Pork	Beef
Fish	3.71	−0.643	−0.575	−1.96
	(2.67)	(0.776)	(0.337)	(1.37)
Poultry		7.13	−2.00	−6.90
		(1.22)	(0.577)	(1.42)
Pork			3.06	−4.51
			(0.473)	(0.890)
Beef				−7.43
				(3.43)

TABLE 4

PROPORTIONAL HABIT FORMATION PARAMETER ESTIMATES

σ		Meat		
		1.01		
		(0.023)		
β_i	Fish	Poultry	Pork	Beef
	0.079	0.224	0.332	1.0
	(0.053)	(0.111)	(0.249)	(—)
δ_{ij}	Fish	Poultry	Pork	Beef
Fish	3.52	−0.664	−0.504	−1.50
	(1.85)	(0.493)	(0.516)	(1.05)
Poultry		3.12	−0.681	−2.95
		(1.23)	(0.809)	(1.67)
Pork			2.22	−3.03
			(0.550)	(0.972)
Beef				−0.701
				(2.46)

TABLE 5
Proportional State Variable Parameter Estimates

σ		Meat 1.70 (4.55)		
β_i	Fish 0.187 (0.839)	Poultry 0.139 (0.705)	Pork 0.322 (0.977)	Beef 1.0 (—)
α_{ij}	Fish	Poultry	Pork	Beef
Fish	0.728 (2.06)	−0.362 (0.219)	0.196 (0.404)	0.002 (1.30)
Poultry		0.222 (0.642)	0.473 (0.229)	0.113 (0.647)
Pork			0.503 (0.859)	−0.094 (1.50)
Beef				0.229 (3.18)
δ		Meat 0.978 (0.179)		
s_{i0}	Fish 163.0 (914.0)	Poultry 71.9 (10^4)	Pork 380.0 (10^5)	Beef 1062.0 (10^6)

TABLE 6
Brown–Heien Meat Expenditure Data

		E-Fish	E-Poultry	E-Pork	E-Beef	E-Meat
1	1946	4.794	12.414	25.043	25.386	67.638
2	1947	5.254	11.932	33.431	40.435	91.052
3	1948	6.366	12.992	33.635	44.104	97.097
4	1949	6.476	13.091	31.079	41.301	91.947
5	1950	6.889	13.581	31.362	44.542	96.374
6	1951	7.498	15.059	34.494	45.592	102.643
7	1952	7.247	15.541	34.427	50.085	107.300
8	1953	6.862	15.102	33.599	49.545	105.109
9	1954	6.955	14.389	32.767	49.065	103.176
10	1955	6.366	14.070	31.340	48.884	100.661
11	1956	6.435	13.834	29.963	49.594	99.827
12	1957	6.412	14.345	31.342	51.950	104.050
13	1958	6.949	15.308	32.923	55.969	111.149
14	1959	7.600	14.446	32.889	57.127	112.062
15	1960	7.048	14.231	31.339	57.852	110.470
16	1961	7.501	14.140	31.404	57.800	110.845
17	1962	7.736	14.721	32.682	60.152	115.290
18	1963	7.810	14.717	32.506	62.076	117.109
19	1964	7.454	14.710	32.338	63.749	118.250
20	1965	7.974	16.128	32.995	66.387	123.484
21	1966	8.270	18.249	37.430	72.565	136.514
22	1967	8.550	17.830	37.953	74.058	138.391
23	1968	8.815	18.111	39.295	79.247	145.468

The estimated parameter values for the proportional models are reported in Tables 4 and 5. As measured by frequency of significant parameters, neither specification is very satisfactory. Estimation of the proportional state variable model was hampered by the existence of numerous local maxima as is also the case for the linear state model.

TABLE 7

LINEAR HABIT FORMATION REFERENCE EXPENDITURES

		E-Fish	E-Poultry	E-Pork	E-Beef	E-Meat
1	1946	2.305	−0.250	9.047	−58.578	−47.476
2	1947	1.725	−6.228	14.337	−84.651	−74.816
3	1948	0.847	−14.678	4.624	−143.375	−152.582
4	1949	3.038	−4.878	13.196	−79.173	−67.817
5	1950	2.420	−6.923	10.075	−91.620	−86.048
6	1951	2.918	−8.815	8.753	−103.150	−100.293
7	1952	4.767	2.069	19.062	−36.031	−10.131
8	1953	3.013	−4.352	15.121	−83.215	−69.433
9	1954	−0.056	−20.628	−3.275	−182.947	−206.908
10	1955	−0.405	−22.679	−7.157	−195.723	−225.965
11	1956	−0.558	−23.995	−8.666	−195.210	−228.431
12	1957	−1.879	−30.729	−13.994	−237.296	−283.899
13	1958	−1.633	−32.450	−13.585	−247.261	−294.930
14	1959	−0.140	−27.641	−8.630	−203.344	−239.756
15	1960	−0.482	−29.193	−11.378	−209.987	−251.042
16	1961	−0.697	−31.055	−12.307	−220.538	−264.598
17	1962	−1.138	−35.107	−15.614	−245.177	−297.037
18	1963	−1.182	−35.322	−16.270	−245.502	−298.278
19	1964	−2.207	−39.518	−20.917	−271.479	−334.124
20	1965	−3.811	−48.246	−28.070	−328.429	−408.557
21	1966	−3.652	−49.370	−27.211	−338.299	−418.533
22	1967	−4.409	−56.057	−32.642	−371.555	−464.665
23	1968	−5.693	−63.820	−40.285	−414.156	−523.956

The dynamic specifications discussed above all relate current preferences to past consumption through changes in the reference frontier in the context of the Gorman polar form. A comparison of the simulated values of the reference points based on the several specifications reveals few differences between the linear habit, linear state and proportional habit specifications but greater differences between these points and the simulated values for the proportional state values. The actual values of expenditures are given in Table 6 and the simulated values for the reference expenditures are given in Tables 7–10 for each of the four specifications. In the first three cases, the reference expenditures move into the negative orthant. Recall that the interpretation of negative values as subsistence expenditures is not tenable. In the case of the proportional state model though, the frontier moves relatively little and in no single direction.

TABLE 8

LINEAR STATE VARIABLE REFERENCE EXPENDITURES

		E-Fish	E-Poultry	E-Pork	E-Beef	E-Meat
1	1946	2.304	−0.258	9.039	−58.624	−47.538
2	1947	1.724	−6.238	14.328	−84.712	−74.899
3	1948	0.845	−14.691	4.613	−143.447	−152.681
4	1949	3.037	−4.891	13.184	−79.244	−67.914
5	1950	2.419	−6.935	10.063	−91.690	−86.144
6	1951	2.916	−8.829	8.740	−103.231	−100.404
7	1952	4.766	2.056	19.049	−36.111	−10.241
8	1953	3.011	−4.363	15.111	−83.277	−69.518
9	1954	−0.058	−20.639	−3.286	−183.013	−206.997
10	1955	−0.407	−22.692	−7.170	−195.797	−226.066
11	1956	−0.561	−24.007	−8.679	−195.283	−228.530
12	1957	−1.882	−30.742	−14.007	−237.375	−284.006
13	1958	−1.636	−32.466	−13.600	−247.353	−295.056
14	1959	−0.143	−27.656	−8.645	−203.434	−239.878
15	1960	−0.485	−29.207	−11.392	−210.071	−251.156
16	1961	−0.700	−31.069	−12.321	−220.622	−264.711
17	1962	−1.141	−35.122	−15.629	−245.265	−297.156
18	1963	−1.185	−35.338	−16.285	−245.590	−298.398
19	1964	−2.210	−39.533	−20.932	−271.565	−334.241
20	1965	−3.814	−48.262	−28.087	−328.523	−408.686
21	1966	−3.655	−49.388	−27.229	−338.4[illegible]4	−418.676
22	1967	−4.413	−56.074	−32.659	−371.658	−464.805
23	1968	−5.697	−63.838	−40.304	−414.266	−524.105

TABLE 9

PROPORTIONAL HABIT FORMATION REFERENCE EXPENDITURES

		E-Fish	E-Poultry	E-Pork	E-Beef	E-Meat
1	1946	−10.379	−29.399	−39.260	−159.749	−238.788
2	1947	−14.529	−43.832	−50.695	−220.406	−329.463
3	1948	−17.726	−54.348	−69.503	−259.674	−401.251
4	1949	−14.811	−47.266	−59.536	−227.948	−349.562
5	1950	−16.087	−50.669	−65.237	−240.973	−372.966
6	1951	−17.828	−57.789	−74.929	−273.143	−423.688
7	1952	−16.481	−52.493	−66.465	−255.184	−390.622
8	1953	−15.399	−48.180	−57.173	−242.604	−363.356
9	1954	−17.154	−54.007	−68.605	−259.079	−398.844
10	1955	−16.987	−53.605	−70.285	−253.050	−393.927
11	1956	−16.667	−52.395	−68.372	−246.557	−383.991
12	1957	−18.777	−57.961	−75.358	−274.189	−426.185
13	1958	−20.576	−63.593	−84.208	−295.522	−463.899
14	1959	−19.237	−60.577	−80.275	−278.888	−438.977
15	1960	−19.053	−60.560	−79.327	−283.539	−442.480
16	1961	−19.146	−60.140	−79.582	−281.006	−439.873
17	1962	−20.070	−63.759	−84.302	−295.132	−463.163
18	1963	−19.865	−63.094	−83.426	−291.974	−458.358
19	1964	−20.173	−63.695	−84.309	−295.005	−463.183
20	1965	−22.582	−69.941	−92.836	−325.019	−510.379
21	1966	−23.641	−73.964	−97.800	−342.821	−538.226
22	1967	−24.043	−75.528	−100.367	−344.685	−544.623
23	1968	−25.781	−80.201	−106.495	−367.980	−580.457

TABLE 10

PROPORTIONAL STATE VARIABLE REFERENCE EXPENDITURES

		E-Fish	E-Poultry	E-Pork	E-Beef	E-Meat
1	1946	4.019	12.083	23.078	8.452	47.633
2	1947	3.854	12.935	28.728	7.581	53.100
3	1948	4.130	12.971	27.072	10.461	54.636
4	1949	4.339	11.548	24.867	8.843	49.599
5	1950	4.258	11.763	24.457	9.848	50.327
6	1951	5.211	12.956	26.848	11.371	56.387
7	1952	4.754	13.525	27.898	9.902	56.081
8	1953	4.698	14.096	30.896	8.346	58.036
9	1954	4.685	12.962	27.604	10.190	55.442
10	1955	4.150	12.705	23.699	10.949	51.505
11	1956	4.248	11.590	23.659	10.486	49.985
12	1957	4.425	13.089	27.054	11.467	56.036
13	1958	4.535	13.470	26.477	13.258	57.742
14	1959	4.5[illegible]	12.525	23.778	12.882	53.700
15	1960	4.940	13.290	26.407	12.288	56.926
16	1961	4.930	12.408	25.765	12.326	55.430
17	1962	5.021	13.387	25.936	13.311	57.656
18	1963	5.006	13.116	25.607	13.188	56.919
19	1964	4.985	13.205	25.962	13.295	57.449
20	1965	5.136	14.535	28.969	14.581	63.223
21	1966	5.510	15.638	30.369	15.448	66.966
22	1967	5.378	15.179	28.013	16.397	64.968
23	1968	5.480	16.597	30.733	17.241	70.052

While these experiments provide some basis for eschewing proportional dynamic specifications in favor of the more complex linear forms, the choice between habit formation and state variable approaches is not clearcut. The estimated value for δ, the depreciation coefficient, was close to one which is reasonable for annual time series for a type of food. Much of the difficulty in estimating the state variable functional form might be removed if it were applied to data where $\delta \neq 1$, as in the case of durable goods.

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