Volume Title: New Developments in Productivity Analysis

Volume Author/Editor: Charles R. Hulten, Edwin R. Dean and Michael J. Harper, editors

Volume Publisher: University of Chicago Press

Volume ISBN: 0-226-36062-8

Volume URL: http://www.nber.org/books/hult01-1

Publication Date: January 2001

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Chapter Author: M. Ishaq Nadiri, Ingmar Prucha

Chapter URL: http://www.nber.org/chapters/c10125

Chapter pages in book: (p. 103 - 172)

4 Dynamic Factor Demand Models and Productivity Analysis

M. Ishaq Nadiri and Ingmar R. Prucha

4.1 Introduction

The traditional approach to productivity analysis is to use the Divisia index number methodology. This approach has the advantage of simplicity as well as the benefit of not requiring direct estimation of the underlying technology. Therefore, the often difficult tasks of econometric specification and estimation of structural models can be avoided. However, for the index number approach to provide meaningful estimates of technical change, fairly strong assumptions about the underlying technology and allocation decisions by the firm must be maintained. In particular, it is necessary to assume a constant returns to scale technology, competitive input and output markets, full utilization of all inputs, and instantaneous adjustment of all inputs to their desired demand levels. As a result, the productivity measures based on the index number approach will in general yield biased estimates of technical change, if any of these assumptions are violated.

Technical change is an integral feature of the production process. Changes in variable factor inputs, the accumulation of quasi-fixed factor inputs, and technical change are in general intertwined in that the demand for inputs and the supply of outputs depend on the rate of technical change, while technical change, in turn depends typically on the input and

M. Ishaq Nadiri is the Jay Gould Professor in Economics at New York University and a research associate of the National Bureau of Economic Research.

Ingmar R. Prucha is professor of economics at the University of Maryland, College Park.

We would like to thank, in particular, William Baumol and Dale Jorgenson, and the participants of the CRIW Conference on New Developments in Productivity Analysis for interesting comments. Das Debabrata and Michel Kumhof provided excellent research assistance. We are also grateful for the support received from the C. V. Starr Center for Applied Economics at New York University.

output mix. The traditional measure of total factor productivity (TFP) measures only technical change, but does not explain the complex and simultaneously determined process that governs the evolution of outputs, inputs, and technical change.

A rationale for a general structural econometric modeling approach is that it allows for the careful testing of various features of a postulated model, rather than to simply impose those features a priori. We note that any misspecification of the underlying technology of the firm will typically lead to inconsistent estimates of technical change and the determinants of the investment decisions. A simple illustration of misspecification is the case where the true technology is translog but the hypothesized model is Cobb-Douglas, or the case where the input adjustments involve considerable time lags but are ignored, or where the expectation process is not taken into account or not formulated correctly. In such cases, the estimates of the model parameters, including the estimates of technical change, will be inconsistent. Thus, if the objective is to obtain a consistent estimate of the true model parameters, choosing, for example, a simple model for convenience of presentation and estimation is not admissible empirical practice. The reason for considering a dynamic rather than a static factor demand model is to not impose a priori that all factors are in long-run equilibrium.

A general dynamic factor demand model, as considered in this paper, has a fairly elaborate structure, requires an extensive data set and poses considerable estimation challenges. However, there seem to be two important advantages to this approach: First, the model contains "simpler" models as special cases. In particular, it contains static factor demand models as special cases, but does not impose a priori the premise that all factors are in long-run equilibrium. As in case of static factor demand models, the analysis can be carried out by specifying the technology in terms of a production function, cost/profit function, or restricted cost/profit function, and the model can be estimated from a subset or the complete set of the factor demand equations. Of course, if the model is only estimated from the variable factor demand equations, then we do not have to formulate an intertemporal optimal control problem.

Second, the dynamic factor demand model generates a very rich set of critical information about the structure of production, sources of productivity growth, impact of technical change and effects of policy instruments and expectations on output supply, input demand, direction of technical change and productivity growth. Not only is it possible to calculate the components of traditional productivity measures but also the determinants of employment and investment decisions of the firm simultaneously.

More specifically, the advantages of the estimation of (dynamic) factor demand models—apart from providing for the possibility of testing various modeling hypotheses—may include the following:

1. In estimating the technology we obtain explicit information on the process that transforms inputs into outputs, and on changes of the technological frontier over time. In particular, we obtain estimates of technological characteristics such as, for example, technical change, scale, and scope. We may also gain estimates of the effects of R&D, spillovers, and so on. Furthermore, we can compute marginal products, elasticities of substitution among the inputs, and the like, to describe the underlying structure of production.

2. In estimating the demand for variable and quasi-fixed factors, we gain additional insight into the underlying dynamics of factor allocation and factor accumulation—short-run, intermediate-run, and long-run—as a function of the variables that are exogenous to the firm. The latter variables typically include (expected) factor prices, taxes, exogenous technical change, spillovers, and so on.

3. As a by-product of estimating dynamic factor demand equations, we may gain insight into the expectation formation process and the firms planning horizon, and how this process affects production decisions in general and investment decisions in particular.

4. Furthermore, given that we allow the depreciation rate of capital goods to be endogenously determined, we obtain an economic model for replacement investment and expansion investment. (We note, however, that the modeling framework also covers the case of an exogenous and constant depreciation rate.)

The paper is organized as follows. In section 4.2, we begin by precisely defining input- and output-based technical change on the primal (production) side in the presence of adjustment costs. We then discuss how those measures can be evaluated on the dual (cost) side. We also show how capacity utilization rates can be derived in the context of dynamic factor demand models. Next we discuss the conventional measure of TFP based on the Divisia index, and show how this measure can be biased (as a measure of technical change) if the assumptions underlying its derivation are not satisfied. The biases can, for example, be due to the presence of economies of scale, adjustment costs, and the difference between the shadow prices and long-run rental prices of the quasi-fixed inputs.

In section 4.3, we first specify a general class of dynamic factor demand models, which allows for several nonseparable quasi-fixed factors, for the utilization rate/depreciation rate of some of the quasi-fixed factors to be endogenously determined, and for expectations to be nonstatic. We then discuss the class of linear quadratic dynamic factor demand models in more detail. For this class of models we give explicit analytic expressions for the firm's optimal control solution, that is, for the firm's optimal factor inputs. Those expressions make clear the dependence of the firm's investment decisions on the expectations of future exogenous variables. We also discuss convenient ways of estimating such models based on a reparameterization. Since some of this material may be unfamiliar and technically involved, we have attempted to show step-by-step how the models are derived and estimated. After our discussion of linear quadratic dynamic factor demand models, we discuss several approaches towards the estimation of dynamic factor demand models in general. This includes the estimation of the Euler equations by the generalized method of moments approach, given rational expectations.

Section 4.4 reviews several empirical applications of dynamic factor demand models. Dynamic factor demand models have been widely employed to study the behavior of factor demands including investment and employment decisions, output supply behavior, profitability, nature of technical change, spillover effects of R&D investment, international technology spillovers, role of public investment, taxes and subsidies, and so on. The empirical examples are provided to illustrate the versatility of these models.

In section 4.5 we present briefly the results of a Monte Carlo study that explores the effects of misspecifications. The true data generating process corresponds to a general dynamic factor demand specification with nonseparable quasi-fixed factors, nonconstant returns to scale, and nonstatic expectations. The model and various implied characteristics including technical change are then estimated under the correct specification and under various forms of misspecification. This allows us to assess the degree of bias induced by various forms of misspecifications as when a simple model of the firm's technology is adopted for convenience of presentation and estimation instead of the true model. Concluding comments are given in section 4.6. A longer version of this paper is available as Nadiri and Prucha (1999), which contains many of the underlying mathematical derivations, and a more extensive list of references.

4.2 On the Conventional Approach to Productivity Analysis

As remarked in the Introduction, a focus of this paper is the presentation of recent developments in the dynamic factor demand literature and their application to estimation of technical change and output growth. To set the stage, we first give a brief review of the conventional Divisia index based approach to productivity analysis. To put the discussion on sound footing, we start with a formal definition of technical change.[1]

1. The subsequent discussion makes use of the following notational conventions (unless explicitly indicated otherwise): Let Z_t be some $l \times 1$ vector of goods in period t, then p_t^Z refers to the corresponding $l \times 1$ price vector; Z_{ti} and p_{ti}^Z denote the i-th elements of Z_t and p_t^Z, respectively. Furthermore, in the following we often write $(p_t^Z)'Z_t$ for $\sum_{i=1}^{l} p_{ti}^Z Z_{ti}$ where "′" stands for transpose.

4.2.1 Definition of Technical Change

The conventional Divisia index based measure of total factor productivity growth assumes, in particular: (1) that producers are in long-run equilibrium, (2) that the technology exhibits constant returns to scale, (3) that output and input markets are competitive, and (4) that factors are utilized at a constant rate.[2] The puzzle of the observed slowdown of productivity growth during the 1970s has initiated a critical methodological review of the conventional measure of productivity growth.[3]

The model considered in the following discussion relaxes these assumptions corresponding to the conventional measure of total factor productivity growth. In the following we define, within the context of that model, appropriate measures of technical change. More specifically, in defining technical change we first give such a definition on the (primal) production side. We then show how the measure of technical change so defined can be expressed alternatively on the (dual) cost side. To interpret the expressions on the cost side, we also discuss measures of capacity utilization.

The following discussion allows, in particular, for a technology with multiple outputs, allows for some of the factors to be quasi-fixed (and thus does not assume that the firm is in long-run equilibrium), and allows for nonconstant returns to scale.[4] Now let $Y_t = (Y_{t1}, \ldots, Y_{tk})'$ be the vector of output goods produced by a firm during period t, and let $V_t = (V_{t1}, \ldots, V_{tm})'$ and $X_t = (X_{t1}, \ldots, X_{tn})'$ be the vectors of variable and quasi-fixed inputs utilized during period t, respectively. We then assume that the technology can be represented by the following transformation function

$$F(Y_t, V_t, X_t, \Delta X_t, T_t) = 0, \tag{1}$$

where the vector of first differences ΔX_t represent internal adjustment costs in terms of foregone output due to changes in the quasi-fixed factors, and T_t represents an index of (exogenous) technical change.

In the following it will also be useful to decompose the variable factors into $M_t = V_{t1}$ and $L_t = (V_{t2}, \ldots, V_{tm})'$, and to represent the technology in terms of the following factor requirement function

$$M_t = M(Y_t, L_t, X_t, \Delta X_t, T_t). \tag{2}$$

2. The assumption of constant returns to scale is not necessary in cases where an independent observation of the rental price of capital is available.

3. See, e.g., Berndt and Fuss (1986, 1989), Bernstein and Mohnen (1991), Caves, Christensen, and Swanson (1981), Caves, Christensen, and Diewert (1982), Denny, Fuss, and Waverman (1981a), Hall (1988), Hulten (1986), Mohnen (1992a), Mohnen, Nadiri, and Prucha (1983), Morrison (1986, 1992a,b), Nadiri and Prucha (1986, 1990a,b), Nadiri and Schankerman (1981a,b), and Prucha and Nadiri (1996).

4. Generalizations that allow for variable factor utilization rates will be discussed later.

We can then think of the transformation function to be of the form

$$(3) \qquad F(Y_t,V_t,X_t,\Delta X_t,T_t) = M(Y_t,L_t,X_t,\Delta X_t,T_t) - M_t \equiv 0.$$

For ease of notation, in the following we drop time-subscripts whenever those subscripts are obvious from the context.

Primal Measures of Technical Change

To define technical change formally, assume that the technology index T shifts by, say, δ. Now let $a = a(\delta, Y, V, X, \Delta X, T)$ be the proportionality factor by which all outputs Y can be increased, and let $b = b(\delta, Y, V, X, \Delta X, T)$ be the proportionality factor by which all inputs can be decreased corresponding to this shift in technology when the firm remains on its production surface, that is, $F(aY, V, X, \Delta X, T + \delta) = 0$ and $F(Y, bV, bX, b\Delta X, T + \delta) = 0$. Furthermore, let $c = c(\kappa, Y, V, X, \Delta X, T)$ be the proportionality factor by which all outputs Y can be increased corresponding to an increase in all inputs by a factor κ when the firm remains on its production surface, that is, $F(cY, \kappa V, \kappa X, \kappa\Delta X, T) = 0$. We can then give the following two definitions of technical change, λ^Y and λ^X, and returns to scale, ρ:

$$(4) \qquad \lambda^Y = \left.\frac{\partial a}{\partial \delta}\right|_{\delta=0} = -\frac{\partial F}{\partial T}\bigg/\left[\sum_{i=1}^{k}\frac{\partial F}{\partial Y_i}Y_i\right],$$

$$\lambda^X = -\left.\frac{\partial b}{\partial \delta}\right|_{\delta=0} = \frac{\partial F}{\partial T}\bigg/\left[\sum_{j=1}^{m}\frac{\partial F}{\partial V_j}V_j + \sum_{l=1}^{n}\frac{\partial F}{\partial X_l}X_l + \sum_{l=1}^{n}\frac{\partial F}{\partial \Delta X_l}\Delta X_l\right],$$

$$\rho = \left.\frac{\partial c}{\partial \kappa}\right|_{\kappa=1} = \lambda^Y / \lambda^X,$$

where $F(\cdot)$ is evaluated at $(Y, V, X, \Delta X, T)$. We refer to λ^Y and λ^X, respectively, as the rates of output and input based technical change or productivity growth. For an explicit derivation of the above expressions see appendix A in Nadiri and Prucha (1999). We note that the definitions given above are consistent with those given, for example, in Caves, Christensen, and Swanson (1981) and Caves, Christensen, and Diewert (1982) for the case of technologies without explicit adjustment costs.

In case of a single output good, we can also represent the technology in terms of a production function, say,

$$(5) \qquad Y = f(V,X,\Delta X,T).$$

Input and output based technical change can then also be expressed as usual as[5]

5. See appendix A in Nadiri and Prucha (1999) for a derivation.

(6)
$$\lambda^Y = \frac{\partial f}{\partial T}\Big/ Y,$$

$$\lambda^X = \frac{\partial f}{\partial T}\Bigg/\left[\sum_{j=1}^{m}\frac{\partial f}{\partial V_j}V_j + \sum_{l=1}^{n}\frac{\partial f}{\partial X_l}X_l + \sum_{l=1}^{n}\frac{\partial f}{\partial \Delta X_l}\Delta X_l\right].$$

Dual Measures of Technical Change

We next show how these measures can be evaluated from the cost side, using simple arguments of duality theory. We note that the expressions developed below are given in terms of a restricted or variable cost function. Expressions in terms of the (unrestricted) cost function are contained as a special case, in that for the case where all factors are variable the restricted cost function and the (unrestricted) cost function coincide.

Let p^L denote the price vector for the variable inputs L normalized by the price of the variable input M. The normalized variable cost is then given by $M(Y, L, X, \Delta X, T) + (p^L)'L$. The normalized variable cost function is obtained by minimizing this expression w.r.t. L. Assuming that the factor requirement function $M(\cdot)$ is differentiable and that a unique interior minimum exists, the corresponding first order conditions are given by

(7)
$$\frac{\partial M}{\partial L} + (p^L)' = 0.$$

Let $\hat{L}$ denote the minimizing vector. The normalized variable cost function is then given by

(8)
$$G(p^L, Y, X, \Delta X, T) = \hat{M} + (p^L)'\hat{L}$$

where

$$\hat{M} = M(Y, \hat{L}, X, \Delta X, T).$$

For duality results between factor requirement functions and normalized restricted cost functions $G(\cdot)$, see, for example, Diewert (1982) and Lau (1976). We assume that the function $G(\cdot)$ is twice continuously differentiable in all its arguments, homogeneous of degree zero in p^L, nondecreasing in Y, $|\Delta X|$, and p^L, nonincreasing in X, concave in p^L, and convex in X, and ΔX.

Differentiating equation (3) yields

(9)
$$\frac{\partial F}{\partial Z} = \frac{\partial M}{\partial Z} \text{ for } Z = Y, L, X, \Delta X, T.$$

Differentiating equation (8) and utilizing equation (7) yields

(10)
$$\frac{\partial G}{\partial Z} = \frac{\partial M}{\partial Z} \text{ for } Z = Y, X, \Delta X, T.$$

Consequently, we have

$$\frac{\partial F}{\partial Z} = \frac{\partial G}{\partial Z} \text{ for } Z = Y, X, \Delta X, T. \tag{11}$$

From equation (9) with $Z = L$ and equation (7) we obtain

$$\frac{\partial F}{\partial L} = -(p^L)'. \tag{12}$$

Furthermore, we have from equation (3)

$$\frac{\partial F}{\partial M} = -1. \tag{13}$$

Given the variable inputs $V = [M, L']'$ are chosen optimally, that is, $L = \hat{L}$ and $M = \hat{M}$, it follows from equations (12) and (13), and from equation (8), that

$$G(p^L, Y, X, \Delta X, T) = \hat{M} + (p^L)'\hat{L} = -\sum_{j=1}^{m} \frac{\partial F}{\partial V_j} V_j. \tag{14}$$

Substituting the expressions in equations (11) and (14) into equation (4) yields the following expressions for technical change and returns to scale in terms of the normalized restricted cost function G:

$$\begin{aligned} \lambda^Y &= -\frac{\partial G}{\partial T} \Big/ \left[\sum_{i=1}^{k} \frac{\partial G}{\partial Y_i} Y_i\right], \\ \lambda^X &= -\frac{\partial G}{\partial T} \Big/ \left[G - \sum_{l=1}^{n} \frac{\partial G}{\partial X_l} X_l - \sum_{l=1}^{n} \frac{\partial G}{\partial \Delta X_l} \Delta X_l\right], \\ \rho = \lambda^Y / \lambda^X &= \left[G - \sum_{l=1}^{n} \frac{\partial G}{\partial X_l} X_l - \sum_{l=1}^{n} \frac{\partial G}{\partial \Delta X_l} \Delta X_l\right] \Big/ \left[\sum_{i=1}^{k} \frac{\partial G}{\partial Y_i} Y_i\right]. \end{aligned} \tag{15}$$

The total shadow cost (normalized by the price of the variable factor M) is defined as

$$C = G(p^L, Y, X, \Delta X, T) - \sum_{l=1}^{n} \frac{\partial G}{\partial X_l} X_l - \sum_{l=1}^{n} \frac{\partial G}{\partial \Delta X_l} \Delta X_l, \tag{16}$$

where $-\partial G/\partial X_l$ and $-\partial G/\partial \Delta X_l$ denote the respective "shadow values." The above expressions for output based and input based technical change and returns to scale generalize those given in Caves, Christensen, and Swanson (1981) in that they allow explicitly for adjustment costs.[6]

6. We note that the mathematics used in deriving the expressions in equation (15) is analogous to that used by Caves, Christensen, and Swanson (1981). The expressions also generalize those previously given in Nadiri and Prucha (1990a,b) for single-output technologies with

Observe that substituting equation (16) into equation (15) yields the following expressions for input-based technical change and scale: $\lambda^X = -(\partial G/\partial T)/C$ and $\rho = 1/[\Sigma_{i=1}^k (\partial G/\partial Y_i) Y_i/C]$. In the case where all factors are variable we have $C = G$, and thus in the case of a single output good we have the following simplifications: $\lambda^Y = \rho^{-1}\lambda^X$ with $\lambda^X = -(\partial C/\partial T)/C$ and $\rho = 1/[(\partial C/\partial Y)Y/C]$.

Capacity Utilization and Technical Change

The issue of a proper measure of technical change, given the firm is in short-run or temporary equilibrium, but not in long-run equilibrium, has also been discussed, among others, by Berndt and Fuss (1986, 1989), Berndt and Morrison (1981), Hulten (1986), and Morrison (1986). Those authors discuss proper measures of technical change in terms of adjustments of traditional technical change measures by utilization rate measures. Berndt and Fuss (1986) and Hulten (1986) consider single output technologies with constant returns to scale. Morrison also considers single output technologies, but allows for (possibly) nonconstant returns to scale and explicitly takes into account adjustment costs. Berndt and Fuss (1989) consider multiple output technologies with (possibly) nonconstant returns to scale, but do not explicitly consider adjustment costs.

We now show that the measures for λ^Y and λ^X are consistent with the technical change measures of Berndt, Fuss, Hulten and Morrison by demonstrating that λ^Y and λ^X can also be viewed as having been obtained via a capacity utilization adjustment of conventional (long-run) measures of technical change. For this purpose, consider the following restricted total cost function (normalized by the price of the variable factor M):

$$C^+ = \hat{M} + (p^L)'\hat{L} + (c^X)'X = G(p^L, Y, X, \Delta X, T) + (c^X)'X, \tag{17}$$

where c^X denote the vector of rental prices for the quasi-fixed factors X (normalized by the price of the variable factor M). Recall that in long-run equilibrium, or in the case where all factors are variable, we have shown above that input based technical change equals $-(\partial C/\partial T)/C$. Now suppose we attempt to measure technical change in terms of the total restricted cost function C^+ analogously by

$$\lambda_+^X = -(\partial C^+/\partial T)/C^+. \tag{18}$$

Observing that $\partial C^+/\partial T = \partial G/\partial T$ it follows immediately from equation (15) and equation (16) that

adjustment costs. A generalization of those expressions for multiple-output technologies with adjustment costs and variable factor utilization rates is given in Prucha and Nadiri (1990, 1996), and will be discussed shortly.

$$\lambda^Y = \rho\lambda_+^X(C^+/C), \tag{19}$$
$$\lambda^X = \lambda_+^X(C^+/C).$$

Analogously to Berndt, Fuss, Hulten, and Morrison we can interpret

$$CU = C/C^+ \tag{20}$$

as a measure of capacity utilization and we can therefore interpret our input- and output-based measures for technical change as being derived from λ_+^X via an adjustment in terms of a capacity utilization measure to account for temporary equilibrium. Clearly, in long-run equilibrium C^+ equals C and hence in the long run λ_+^X equals λ^X. In general, however, λ_+^X differs from λ^X by the factor C^+/C.

4.2.2 Divisia Index Approach

In the productivity literature, technical change is often estimated as the difference between the growth rate of a measure of aggregate output minus the growth rate of a measure of aggregate input. This approach to estimate technical change in terms of a residual dates back to Solow (1957). In computing aggregate output and input, one of the most widely used methods of aggregation is Divisia aggregation. The conceptual justifications for Divisia aggregation were developed by Jorgenson and Griliches (1967), Richter (1966), Hulten (1973), and Diewert (1976), among others.[7] In the following we first define the conventional measure of total factor productivity based on the Divisia index formula. As remarked, the Divisia index approach is based on a set of particular assumptions concerning the technology and the inputs and output markets. If any one of those assumptions is violated, the measure of total factor productivity based on the Divisia index formula will in general yield biased estimates of technical change in that it may then include, for example, effects of scale economies or temporary equilibrium in addition to shifts in the production frontier.[8] In the following, we first develop a growth accounting equation for technical change. We then compare this expression with that for the conventional measure of total factor productivity, and based on this comparison discuss potential sources of bias in the latter measure. The subsequent discussion builds on Denny, Fuss, and Waverman (1981a), who consider a model where all factors are variable, but where scale is allowed to differ from unity. In the following discussion we take $T = t$.

The Conventional Measure of Total Factor Productivity

For ease of presentation, we start our discussion in continuous time. Recall that $V = [M, L']'$ denotes the vector of all variable factors, and let

7. See also the article by Hulten, chapter 1 in this volume.
8. The TFP measure based on the Divisia index may, however, be of interest.

$p^V = [1, p^{L\prime}]'$ denote the corresponding price vector (normalized by the price of M). Furthermore, let p^Y denote the vector of output prices, and let c^X denote the vector of rental prices for the quasi-fixed factors X (normalized by the price of M). The Divisia index for aggregate output, say Y^a, is now defined by

$$\frac{\dot{Y}^a}{Y^a} = \sum_{i=1}^{k} s_i^Y \frac{\dot{Y}_i}{Y_i} \tag{21}$$

where the s_i^Y's denote output shares in total revenue $R^+ = \Sigma_{i=1}^k p_i^Y Y_i$, i.e.,

$$s_i^Y = \frac{p_i^Y Y_i}{R^+}, \tag{22}$$

and where dots over variables denote derivatives w.r.t. time t. The Divisia index for aggregate input, say F^a, is analogously defined by

$$\frac{\dot{F}^a}{F^a} = \sum_{j=1}^{m} s_j^V \frac{\dot{V}_j}{V_j} + \sum_{l=1}^{n} s_l^X \frac{\dot{X}_l}{X_l} \tag{23}$$

where the s_j^V's and s_l^X's denote input shares in total cost $C^+ = \Sigma_{j=1}^m p_j^V V_j + \Sigma_{l=1}^n c_l^X X_l$; that is,

$$s_j^V = \frac{p_j^V V_j}{C^+}, \quad s_l^X = \frac{c_l^X X_l}{C^+}. \tag{24}$$

The conventional measure of total factor productivity, say, TFP, is now defined as the ratio of the Divisia index of aggregate output over the Divisia index of aggregate input, that is, TFP $= Y^a/F^a$, and thus

$$\begin{aligned} \frac{\dot{\mathrm{TFP}}}{\mathrm{TFP}} &= \frac{\dot{Y}^a}{Y^a} - \frac{\dot{F}^a}{F^a} \\ &= \sum_{i=1}^{k} s_i^Y \frac{\dot{Y}_i}{Y_i} - \sum_{j=1}^{m} s_j^V \frac{\dot{V}_j}{V_j} - \sum_{l=1}^{n} s_l^X \frac{\dot{X}_l}{X_l}. \end{aligned} \tag{25}$$

The above Divisia index based definition of total factor productivity growth is given in continuous time. Empirical data typically refer to discrete time points. For discrete data, the above formulae for the growth rates of aggregate output, aggregate input, and total factor productivity are typically approximated by the following Törnqvist approximations, where Δ denotes the first difference operator:

$$\Delta \ln Y_t^a = \frac{1}{2} \sum_{i=1}^{k} [s_{ti}^Y + s_{t-1,i}^Y] \Delta \ln(Y_{ti}), \tag{26}$$

$$(27) \qquad \Delta \ln F_t^a = \frac{1}{2}\sum_{j=1}^{m}[s_{tj}^Y + s_{t-1,j}^V]\Delta \ln(V_{tj}) + \frac{1}{2}\sum_{l=1}^{n}[s_{tl}^X + s_{t-1,l}^X]\Delta \ln(X_{tl}),$$

and

$$(28) \quad \Delta \ln \mathrm{TFP}_t = \Delta \ln Y_t^a - \Delta \ln F_t^a = \frac{1}{2}\sum_{i=1}^{k}[s_{ti}^Y + s_{t-1,i}^Y]\Delta \ln(Y_{ti}) - \frac{1}{2}\sum_{j=1}^{m}[s_{tj}^V + s_{t-1,j}^V]\Delta \ln(V_{tj}) - \frac{1}{2}\sum_{l=1}^{n}[s_{tl}^X + s_{t-1,l}^X]\Delta \ln(X_{tl}).$$

Diewert (1976) has shown that the Törnqvist index is in fact exact if the underlying potential function has a translog form. We note further that a primary feature of the Divisia/Törnqvist index approach is that it can be implemented even if the number of inputs and outputs is large; see Diewert (1980).

Growth Accounting Equation for Technical Change

For ease of presentation we again start our discussion in continuous time. Consider the continuous time analog of equation (8),

$$(29) \qquad G(p^L, Y, X, \dot{X}, t) = \hat{M} + (p^L)'\hat{L}.$$

Differentiation of the above equation w.r.t. t and observing that $L = \hat{L}$ and $M = \hat{M}$ yields

$$(30) \qquad \left[\frac{\partial G}{\partial p^L}\right]\dot{p}^L + \left[\frac{\partial G}{\partial Y}\right]\dot{Y} + \left[\frac{\partial G}{\partial X}\right]\dot{X} + \left[\frac{\partial G}{\partial \dot{X}}\right]\ddot{X} + \left[\frac{\partial G}{\partial t}\right] = \dot{M} + (p^L)'\dot{L} + (L)'\dot{p}^L.$$

By Shephard's lemma $L = (\partial G/\partial p^L)'$. Upon substitution of this expression into the above equation it is easily seen that

$$(31) \qquad -\frac{\partial G}{\partial t} = \left[\frac{\partial G}{\partial Y}\right]\dot{Y} - \dot{M} - (p^L)'\dot{L} - \left[\frac{-\partial G}{\partial X}\right]\dot{X} - \left[\frac{-\partial G}{\partial \dot{X}}\right]\ddot{X}.$$

As implied by equation (15), input-based technical change is now obtained by dividing the above equation by the continuous time analog of the restricted total shadow cost defined in equation (16), that is, $C = G(p^L, Y, X, \dot{X}, t) - \Sigma_l(\partial G/\partial X_l)X_l - \Sigma_l(\partial G/\partial \dot{X}_l)\dot{X}_l$. Some simple algebra—and recalling that $V = [M, L']'$ and $p^V = [1, p^{L\prime}]'$—then yields the following expression for input-based technical change:

(32)
$$\lambda^X = -\frac{1}{C}\frac{\partial G}{\partial t}$$
$$= \sum_{i=1}^{k} \bar{g}_i \frac{\dot{Y}_i}{Y_i} - \sum_{j=1}^{m} \bar{s}_j^V \frac{\dot{V}_j}{V_j} - \sum_{l=1}^{n} \bar{s}_l^X \frac{\dot{X}_l}{X_l} - \sum_{l=1}^{n} \bar{s}_l^{\dot{X}} \frac{\ddot{X}_l}{X_l}$$

with

$$\bar{g}_i = \frac{\partial G}{\partial Y_i}\frac{Y_i}{C}, \quad \bar{s}_j^V = \frac{p_j^V V_j}{C},$$
$$\bar{s}_l^X = \frac{-\partial G}{\partial X_l}\frac{X_l}{C}, \quad \bar{s}_l^{\dot{X}} = \frac{-\partial G}{\partial \dot{X}_l}\frac{\dot{X}_l}{C}.$$

For given "shadow values" $-\partial G/\partial X_l$ and $-\partial G/\partial \dot{X}_l$ we have $\partial G/\partial Y_i = \partial C/\partial Y_i$, and the $\bar{g}_i$s can be interpreted as the elasticities of the restricted total shadow cost C with respect to the output Y_i. Furthermore, $\bar{s}_j^V$, $\bar{s}_l^X$ and $\bar{s}_l^{\dot{X}}$ represent the input cost shares for, respectively, V_j, X_l and $\dot{X}_l$ in the restricted total shadow cost. An analogous expression to (32) for single output technologies is, for example, given in Morrison (1986, 1992a). Analogous expressions for models without explicit adjustment costs are given in, for example, Denny, Fuss, and Waverman (1981a) and Berndt and Fuss (1989). Generalizations that allow for endogenous factor utilization are given in Prucha and Nadiri (1990, 1996), and will be discussed in section 4.3.

For purposes of interpreting equation (32), observe that in light of equation (15), $\Sigma_{i=1}^{k} \bar{g}_i = 1/\rho$, where ρ denotes the scale elasticity. In case of single output good, the above expression for input based technical change simplifies to

(33)
$$\lambda^X = \rho^{-1}\frac{\dot{Y}}{Y} - \sum_{j=1}^{m} \bar{s}_j^V \frac{\dot{V}_j}{V_j} - \sum_{l=1}^{n} \bar{s}_l^X \frac{\dot{X}_l}{X_l} - \sum_{l=1}^{n} \bar{s}_l^{\dot{X}} \frac{\ddot{X}_l}{\dot{X}_l}.$$

From this expression we see that in calculating input based technical change in case of increasing (decreasing) returns to scale, output growth is diminished (enhanced) before subtracting the growth in aggregate inputs. In case of a single output good, constant returns to scale, and in case all factors are variable, the growth accounting equation for technical change simplifies further to

(34)
$$\lambda^X = \lambda^Y = \frac{\dot{Y}}{Y} - \sum_{j=1}^{m} \bar{s}_j^V \frac{\dot{V}_j}{V_j},$$

which corresponds to the expression developed by Solow (1957).

The above expressions for technical change were derived in continuous time. In appendix A in Nadiri and Prucha (1999), we derive the following discrete time approximation of equation (32):

$$\text{(35)}\qquad \frac{1}{2}(\lambda_t^X + \lambda_{t-1}^X) = \frac{1}{2}\left(-\frac{1}{C_t}\frac{\partial G_t}{\partial t} - \frac{1}{C_{t-1}}\frac{\partial G_{t-1}}{\partial(t-1)}\right)$$
$$= \frac{1}{2}\sum_{i=1}^{k}[\bar{g}_{ti} + \bar{g}_{t-1,i}]\Delta\ln(Y_{ti})$$
$$- \frac{1}{2}\sum_{j=1}^{m}[\bar{s}_{tj}^V + \bar{s}_{t-1,j}^V]\Delta\ln(V_{tj})$$
$$- \frac{1}{2}\sum_{l=1}^{n}[\bar{s}_{tl}^X + \bar{s}_{t-1,l}^X]\Delta\ln(X_{tl})$$
$$- \frac{1}{2}\sum_{l=1}^{n}[\bar{s}_{tl}^{\Delta X} + \bar{s}_{t-1,l}^{\Delta X}]\Delta\ln(\Delta X_{tl})$$

with

$$\bar{g}_{ti} = \frac{\partial G_t}{\partial Y_{ti}}\frac{Y_{ti}}{C_t}, \quad \bar{s}_{tj}^V = \frac{p_{tj}^V V_{tj}}{C_t},$$
$$\bar{s}_{lt}^X = -\frac{\partial G_t}{\partial X_{lt}}\frac{X_{tl}}{C_t}, \quad \bar{s}_{lt}^{\Delta X} = -\frac{\partial G_t}{\partial X_{lt}}\frac{\Delta X_{tl}}{C_t}.$$

Sources of Bias in the Conventional Measure of Total Factor Productivity

We now compare the growth accounting expression for technical change with the conventional measure of TFP and explore sources of potential bias in the latter measure. For ease of presentation, we again start the discussion in continuous time. Consider the following alternative index for aggregate output, say Y^b, defined by

$$\text{(36)}\qquad \frac{\dot{Y}^b}{Y^b} = \sum_{i=1}^{k}\bar{s}_t^Y\frac{\dot{Y}_i}{Y_i}$$

where

$$\bar{s}_i^Y = \frac{\bar{g}_i}{\sum_{i=1}^{k}\bar{g}_i} = \frac{(\partial G/\partial Y_i)Y_i}{R}$$

with $R = \Sigma_{i=1}^{k}(\partial G/\partial Y_i)Y_i$. Furthermore, consider the following index for aggregate input, say F^b, defined by

$$\text{(37)}\qquad \frac{\dot{F}^b}{F^b} = \sum_{j=1}^{m}\bar{s}_j^V\frac{\dot{V}_j}{V_j} + \sum_{l=1}^{n}\bar{s}_l^X\frac{\dot{X}_l}{X_l} + \sum_{l=1}^{n}\bar{s}_l^{\dot{X}}\frac{\ddot{X}_l}{\dot{X}_l}.$$

Recalling that in light of equation (15), $\Sigma_{i=1}^{k} \bar{g}_i = 1/\rho$, where ρ denotes the scale elasticity, we can now write the growth accounting equation (32) for input-based technical change as

$$\lambda^X = \rho^{-1}\frac{\dot{Y}^b}{Y^b} - \frac{\dot{F}^b}{F^b}. \tag{38}$$

As demonstrated in appendix A in Nadiri and Prucha (1999), comparing equations (25) and (38) yields the following decomposition of the conventional measure of total factor productivity growth in continuous time.

$$\frac{\dot{\mathrm{TFP}}}{\mathrm{TFP}} = \lambda^X + (1 - 1/\rho)\frac{\dot{Y}^b}{Y^b} + \left(\frac{\dot{Y}^a}{Y^a} - \frac{\dot{Y}^b}{Y^b}\right) + \left(\frac{\dot{F}^b}{F^b} - \frac{\dot{F}^a}{F^a}\right) \tag{39}$$

with

$$\frac{\dot{Y}^a}{Y^a} - \frac{\dot{Y}^b}{Y^b} = \sum_{i=1}^{k}\left(\frac{(p_i^Y - \partial G/\partial Y_i)Y_i}{R}\right)\left(\frac{\dot{Y}_i}{Y_i} - \frac{\dot{Y}^a}{Y^a}\right), \tag{40}$$

$$\frac{\dot{F}^b}{F^b} - \frac{\dot{F}^a}{F^a} = \sum_{l=1}^{n}\left(\frac{(-\partial G/\partial X_l - c_l^X)X_l}{C}\right)\left(\frac{\dot{X}_l}{X_l} - \frac{\dot{F}^a}{F^a}\right)$$
$$+ \sum_{l=1}^{n}\left(\frac{(-\partial G/\partial \dot{X}_l)\dot{X}_l}{C}\right)\left(\frac{\ddot{X}_l}{\dot{X}_l} - \frac{\dot{F}^a}{F^a}\right).$$

The first term in the above decomposition of $\dot{\mathrm{TFP}}/\mathrm{TFP}$ corresponds to actual (input-based) technical change. The remaining terms decompose the difference between $\dot{\mathrm{TFP}}/\mathrm{TFP}$ and technical change; that is, they reflect sources of potential bias of $\dot{\mathrm{TFP}}/\mathrm{TFP}$ as a measure of technical change. More specifically, the second term reflects scale effects. We note that under increasing returns to scale and positive output growth $\dot{\mathrm{TFP}}/\mathrm{TFP}$ will overestimate technical change. The third term reflects the effects of deviations from marginal cost pricing. The fourth term is due to the presence of adjustment costs. It consists of two effects: One effect stems from the difference in the marginal conditions for the quasi-fixed factors between short- and long-run equilibrium due to adjustment cost, that is, the difference between the shadow price and (long-run) rental price.[9] The other effect reflects the direct effect of adjustment costs in the sense that due to the presence of $\dot{X}$ in the transformation function the growth rates of those terms also enter the decomposition of the output growth rate.

Empirical data typically refer to discrete time points. Equations (26)–

9. Suppose the shadow price for a particular quasi-fixed factor exceeds the long-run price used in the computation of $\dot{\mathrm{TFP}}/\mathrm{TFP}$. In this case $\dot{\mathrm{TFP}}/\mathrm{TFP}$ will, ceteris paribus, overestimate the technical change effects given the growth rate of the quasi-fixed input exceeds that of the aggregate input index.

(28) provided Törnqvist approximations for the growth rates of the aggregate output Y^a, the aggregate input F^a, and total factor productivity TFP. Analogously, consider the following approximations for the growth rates of the aggregate output Y^b, and of the aggregate input F^b:

(41) $$\Delta \ln Y_t^b = \frac{1}{2}\sum_{i=1}^{k}[\bar{s}_{ti}^Y + \bar{s}_{t-1,i}^Y]\Delta \ln(Y_{ti})$$

where

$$\bar{s}_{ti}^Y = \frac{\bar{g}_{ti}}{\sum_{i=1}^{k}\bar{g}_{ti}} = \frac{(\partial G_t/\partial Y_{ti})Y_{ti}}{R_t}$$

with $R_t = \Sigma_{i=1}^{k} (\partial G_t/\partial Y_{ti}) Y_{ti}$, and

(42) $$\begin{aligned}\Delta \ln F_t^b &= \frac{1}{2}\sum_{j=1}^{m}[\bar{s}_{tj}^V + \bar{s}_{t-1,j}^V]\Delta \ln(V_{tj}) \\ &+ \sum_{l=1}^{n}[\bar{s}_{tl}^X + \bar{s}_{t-1,l}^X]\Delta \ln(X_{tl}) \\ &+ \sum_{l=1}^{n}[\bar{s}_{tl}^{\Delta X} + \bar{s}_{t-1,l}^{\Delta X}]\Delta \ln(\Delta X_{tl}).\end{aligned}$$

As demonstrated in appendix A in Nadiri and Prucha (1999), it is then possible to decompose the Törnqvist index based approximation of the growth rate of the conventional measure of total factor productivity as follows:

(43) $$\begin{aligned}\Delta \ln \mathrm{TFP}_t &= \frac{1}{2}(\lambda_t^X + \lambda_{t-1}^X) + \frac{1}{2}\sum_{\tau=t,t-1}(1 - 1/\rho_\tau)\Delta \ln Y_t^{b,\tau} \\ &+ (\Delta \ln Y_t^a - \Delta \ln Y_t^b) + (\Delta \ln F_t^b - \Delta \ln F_t^a)\end{aligned}$$

with

(44) $$\begin{aligned}&\Delta \ln Y_t^a - \Delta \ln Y_t^b \\ &\quad = \sum_{i=1}^{k}\left[\frac{1}{2}\sum_{\tau=t,t-1}\left(\frac{(p_{\tau i}^Y - \partial G_\tau/\partial Y_{\tau i})Y_{\tau i}}{R_\tau}\right)(\Delta \ln(Y_{ti}) - \Delta \ln Y_t^{a,\tau})\right], \\ &\Delta \ln F_t^b - \Delta \ln F_t^a \\ &\quad = \sum_{l=1}^{n}\left[\frac{1}{2}\sum_{\tau=t,t-1}\left(\frac{(-\partial G_\tau/\partial X_{\tau l} - c_{\tau l}^X)X_{\tau l}}{C_\tau}\right)(\Delta \ln(X_{tl}) - \Delta \ln F_t^{a,\tau})\right] \\ &\quad + \sum_{l=1}^{n}\left[\frac{1}{2}\sum_{\tau=t,t-1}\left(\frac{(-\partial G_\tau/\partial \Delta X_{\tau l})\Delta X_{\tau l}}{C_\tau}\right)(\Delta \ln(\Delta X_{tl}) - \Delta \ln F_t^{a,\tau})\right]\end{aligned}$$

and

$$\Delta \ln Y_t^{a,\tau} = \sum_{i=1}^{k} s_{\tau i}^{Y} \Delta \ln(Y_{ti})$$

$$\Delta \ln F_t^{a,\tau} = \sum_{j=1}^{m} s_{\tau j}^{V} \Delta \ln(V_{tj}) + \sum_{l=1}^{n} s_{\tau l}^{X} \Delta \ln(X_{tl})$$

for $\tau = t, t - 1$. This decomposition and its interpretation is analogous to the continuous time decomposition of TFP growth given in equations (39) and (40). It generalizes analogous expressions given in Denny, Fuss, and Waverman (1981a) for technologies without adjustment costs and in Nadiri and Prucha (1986, 1990a,b) for single output technologies with adjustment costs. Expressions that allow for endogenous factor utilization have been considered in Prucha and Nadiri (1990, 1996), and will be discussed in section 4.3. We note that variations of the decomposition equations (39) or (43) have also appeared in various other studies, including Nadiri and Schankerman (1981b), Bernstein and Mohnen (1991), and Mohnen (1992a).

4.3 Recent Developments in Modeling Dynamic Factor Demand

The recent dynamic factor demand literature rests on the seminal work of several contributors. Four advances in the theory and estimation methodology are of particular importance: The neoclassical theory of investment, the advances in flexible functional forms of the production (cost) functions, the development of duality theory, and the theoretical and empirical developments concerning adjustment costs. It is the confluence of these strands of literature that made the wide empirical applications of factor demand models possible.

First, in a seminal contribution Jorgenson (1963) laid the foundation of the neoclassical model of investment. He developed the concept of the user cost of capital, that included explicitly various taxes and incentives. Also, he modeled the lagged response of investment to changes in demand for capital by generalizing the Koyck (1954) geometric lag distribution to what is called the rational distributed lag; see Jorgenson (1966) and Jorgenson and Stephenson (1967). Many other facets of investment decisions such as the rate of depreciation and the distinction between net and replacement investment were explicitly considered in a series of papers dealing with theory and application of the neoclassical theory of investment; see Jorgenson (1996a,b) for a collection of this important body of work.

Building on the neoclassical model of investment Nadiri and Rosen (1969, 1973) introduced their interrelated disequilibrium model, whereby disequilibrium in one factor market was formally related to the extent of disequilibrium in other factor markets. As a result, short-run overshooting is possible, and the difference between short- and long-run price elasticities for a particular input depends not only on its own partial adjustment parameter, but also on all cross adjustment parameters of other inputs.

A second major advance in the literature has been the formulation of flexible functional forms for the description of the technology. The purpose was to avoid restrictive features inherent in, for example, the Leontief and Cobb-Douglas production functions. Flexible functional forms of cost and production functions have first been introduced in the economics literature in seminal papers by Christensen, Jorgenson, and Lau (1971, 1973) and Diewert (1971). These authors introduced the transcendental logarithmic and the generalized Leontief functional forms, respectively.[10] These functional forms do not impose a priori restrictive constraints such as homotheticity, constancy of elasticity of substitution, additivity, and so on. Another important flexible functional form has been proposed by McFadden (1978) and extended by Diewert and Wales (1987).

The third strand of literature contributing to advances in the theory of production was the development of duality theory. Fundamental contributions include Shephard (1953), Diewert (1971, 1982), Lau (1976), and McFadden (1978).[11] Of course, there was close interaction between the development of flexible functional forms and duality theory. Profit and cost functions (or restricted versions thereof) are widely used in empirical analysis. This may be explained in part by the following observation of McFadden (1978): "In econometric applications, use of the cost function as the starting point of developing models avoids the difficulty of deriving demand systems constructively from production possibilities, while at the same time insuring consistency with the hypothesis of competitive cost minimization" (4).

Fourth, in an effort to construct a dynamic framework capable of yielding a demand for investment Eisner and Strotz (1963) introduced adjustment cost into the neoclassical theory of the firm. Other important contributions include Lucas (1967a,b), Treadway (1969, 1974), and Mortenson (1973). These studies indicated that the multivariate flexible accelerator model can be justified theoretically as a solution of a dynamic optimization problem that incorporates adjustment cost for the quasi-fixed factors. The adjustment cost incurred in order to change the level of the quasi-fixed factors can take two forms. The first type is internal and reflects the fact that firms may have to make trade-offs between producing current

10. The transcendental logarithmic form has been used by Jorgenson with different associates to study the properties of the production structure and productivity analysis in a number of sectors in the U.S. and Japanese economies and to compare productivity growth among different countries; see Jorgenson (1995a,b).

11. For a detailed review of the literature and a collection of various other important contributions see Fuss and McFadden (1978).

output and diverting some of the resources from current production to accumulate capital for future production (e.g., Treadway 1974). The second type is external: As the firm adjusts its quasi-fixed factors it may face either a higher purchase price for these factors (e.g., Lucas 1967a,b) or a higher financing cost for the accumulation of these inputs (e.g., Steigum 1983).

Based on these theoretical development on cost of adjustments a number of dynamic factor demand models referred to as the "third generation models" have been estimated. For comprehensive reviews of this influential literature see Berndt, Morrison, and Watkins (1981) and Watkins (1991). Examples include Denny, Fuss, and Waverman (1981b), Morrison and Berndt (1981), Morrison (1986), and Watkins and Berndt (1992). Several features of the "third generation" dynamic factor demand models are important to note. First, those models are explicitly dynamic and provide the optimal path of investment from temporary to full long-run equilibrium. The dynamic path of adjustment to long-run equilibrium is based on economic optimization at each point in time; thus short-, intermediate-, and long-run are clearly defined. Second, the speed of adjustment of the quasi-fixed factors to their long-run equilibrium levels is allowed to be endogenous and time varying, rather than exogenous and fixed. Third, the short-run demand equations for variable inputs depend on, among other things, prices of variable inputs, output, and stocks of the quasi-fixed inputs. Variable inputs may in the short-run overshoot their long-run equilibrium values to compensate for the partial adjustment of the quasi-fixed factors.

Empirical applications of third generation dynamic factor demand models typically only allowed for one quasi-fixed factor, or, slightly more generally, for several separable quasi-fixed factors. As a consequence of the separability assumption the models did not allow for interactions between the optimal investment paths. The technical reason for maintaining separability between the quasi-fixed factors was that it facilitated a major simplification in the computation of the firm's optimal investment decision. More specifically, separability implies the absence of interaction between the difference equations describing the optimal investment paths of the respective quasi-fixed factors. As a consequence, each of those equations can be solved separately. Technically this entails the solving of a quadratic equation for each of the quasi-fixed factors—which, of course, can readily be done analytically. If, however, separability is not maintained, then rather than having to solve several quadratic scalar equations, one is confronted with a quadratic matrix equation. Analytic expressions for the solution of this quadratic matrix equation, and hence for optimal investment, are then generally not available.

Other characteristics of the empirical implementation of third generation dynamic factor demand models were that the underlying technology was modeled in a linear quadratic fashion, that expectations were typically modeled as static, and that factor utilization rates were assumed to be constant. Recent developments were aimed at a relaxation of those assumptions.

4.3.1 Theoretical Model Specification

Technology and Optimal Control Policy

For the subsequent discussion we generalize the setup of section 4.2, in that we consider a firm that combines the set of variable inputs V_t and the set of quasi-fixed inputs X_t to produce the set of outputs for current sale Y_t, as well as a set of capital inputs for future production. More specifically, in the generalized setup we allow the firm to also choose how much of the beginning-of-period stocks of some (but not necessarily all) of the quasi-fixed capital inputs will be left over at the end of the period. We note that this adopted modeling framework dates back to Hicks (1946), Malinvaud (1953), and Diewert (1980). In the empirical dynamic factor demand literature this framework was first adopted by Epstein and Denny (1980) and Kollintzas and Choi (1985) for the case of a single quasi-fixed factor. Prucha and Nadiri (1990, 1996) generalized the setup by allowing for more than one quasi-fixed factor. They also discuss measures of technical change and capacity utilization for the generalized modeling framework.[12] We note that the generalized modeling framework contains—as discussed in more detail later—the case where a constant fraction of the beginning-of-period stocks is left over at the end of the period as a special case.

In the following we use K_t to denote the vector of the stocks of the quasi-fixed capital inputs at the end of period t for which the firm chooses how much of the beginning-of-period stocks will be left over at the end of the period, and K_t^o to denote the vector of "old" stocks left over at the end of period t from the beginning-of-period stocks K_{t-1}. Of course, being able to choose the level of K_t^o by, for example, choosing appropriate levels of maintenance, is equivalent to being able to choose endogenously the rate of depreciation for those stocks, since we can always write $K_t^o = (1 - \delta_t^K) K_{t-1}$ and interpret δ_t^K as a diagonal matrix of depreciation rates. R_t is the vector of the end-of-period stocks of the quasi-fixed factors, whose depre-

12. On a theoretical level the generalized modeling framework has also been considered by Bernstein and Nadiri (1987). A special case of the model was implemented in Nadiri and Prucha (1996). Bischoff and Kokkelenberg (1987) adopt a related framework in which the depreciation rate is modeled as a function of capacity utilization. For other contributions to the dynamic factor demand literature that allows for the firm to operate at different levels of utilization, but are based on an alternative modeling framework, see, e.g., Abel (1981) and Shapiro (1986).

ciation rates are exogenous to the firm. We assume furthermore that all quasi-fixed factors become productive with a lag.[13] In the notation of section 4.2, we then have $X_t = [K'_{t-1}, R'_{t-1}]'$ and $\Delta X_t = [\Delta K'_t, \Delta R'_t]'$. Furthermore, as in section 4.2, we will decompose the variable inputs as $V_t = [M_t, L'_t]'$.

In more detail, we assume that the firm's technology can be represented by the following factor requirement function:

$$M_t = M(Y_t, L_t, K^o_t, K_{t-1}, R_{t-1}, \Delta K_t, \Delta R_t, T_t). \tag{45}$$

This specification generalizes the factor requirement function considered in equation (2) in that it includes the vector of capital stocks left over at the end of the period K^o_t. The stocks K_t and R_t accumulate according to the following equations:

$$K_t = I^K_t + K^o_t, \quad R_t = I^R_t + (I - \delta^R_t)R_{t-1}, \tag{46}$$

where I^K_t and I^R_t denote the respective vectors of gross investment and δ^R_t denotes the diagonal matrix of exogenous depreciation rates (some of which may be zero).

The firm's cost in period t, normalized by the price of the variable factor M_t, is given by

$$M_t + (p^L_t)'L_t + (q^K_t)'I^K_t + (q^R_t)'I^R_t, \tag{47}$$

where q^K_t and q^R_t denote the prices of new investment goods after taxes, possibly normalized by $1 - u_t$, where u_t denotes the corporate tax rate.[14] We assume that the firm faces perfectly competitive markets with respect to its factor inputs.

Suppose the firm's objective is to minimize the expected present value of its future cost stream.[15] Substitution of equations (45) and (46) into equation (47) then yields the following expression for the firm's objective function:

13. This assumption is made for simplicity of exposition. For a generalization where some of the quasi-fixed factors immediately become productive and some become productive with a lag see Prucha and Nadiri (1990, 1996).

14. As an illustration, suppose K is a scalar and corresponds to the stock of a certain capital good; then q^K may equal $[1 - c - u\,(1 - mc)B]p^{IK}/(1 - u)$, where p^{IK} denotes the price of new investment goods, u denotes the corporate tax rate, c is the rate of the investment tax credit, m is the portion of the investment tax credit which reduces the depreciable base for tax purposes, and B is the present value of depreciation allowances. We note that the appropriate expressions for the price of new investment goods after taxes are actually obtained by explicitly introducing taxes into the firm's objective function. As a result, the price of new investment goods after taxes will in general also depend on expectations on future tax variables. We have not chosen this route for simplicity of presentation.

15. We note that the subsequent theoretical discussion can be readily modified also to apply to the case of a profit maximizing firm.

$$E_t \sum_{\tau=t}^{\infty} \{M(Y_\tau, L_\tau, K^o_\tau, K_{\tau-1}, R_{\tau-1}, \Delta K_\tau, \Delta R_\tau, T_\tau) + (p^L_\tau)' L_\tau - (q^K_\tau)' K^o_\tau + (q^K_\tau)' K_\tau + (q^R_\tau)' [R_\tau - (I - \delta^R_\tau) R_{\tau-1}]\} \prod_{s=t}^{\tau} (1 + r_s)^{-1}, \tag{48}$$

where E_t denotes the expectations operator conditional on the set of information available in period t and r denotes the real discount rate (which may possibly also incorporate variations in the corporate tax rate).

Suppose the firm follows a stochastic closed loop feedback control policy in minimizing the expected present value of its future cost stream defined by equation (48). Then, in period t the firm will choose optimal values for its current inputs L_t, K_t, R_t, and for K^o_t. At the same time the firm will choose a contingency plan for setting L_τ, K_τ, R_τ, and K^o_τ in periods $\tau = t + 1, t + 2, \ldots$ optimally, depending on observed realizations of the exogenous variables and past choices for the quasi-fixed factors. Of course, for given optimal values for L_τ, K_τ, R_τ, and K^o_τ the optimal values for M_τ are implied by equation (45). Prices, output, and the discount rate are assumed to be exogenous to the firm's optimization problem.

Since L_τ and K^o_τ can be changed without adjustment costs the stochastic closed loop feedback control solution can be found conveniently in two steps. In the first step, we minimize the total (normalized) cost in each period $\tau = t, t + 1, \ldots$ with respect to L_τ and K^o_τ for given values of the quasi-fixed factors and the exogenous variables. Substitution of the minimized expressions into equation (48) then leads in the second step to an optimal control problem that only involves the quasi-fixed factors K_τ and R_τ.

The part of total cost that actually depends on L_τ and K^o_τ is given by

$$M(Y_\tau, L_\tau, K^o_\tau, K_{\tau-1}, R_{\tau-1}, \Delta K_\tau, \Delta R_\tau, T_\tau) + (p^L_\tau)' L_\tau - (q^K_\tau)' K^o_\tau, \tag{49}$$

that is, variable cost minus the value of the "old" stocks left over at the end of the period from the beginning of period stocks. Assuming that $M(\cdot)$ is differentiable and that a unique interior minimum of the above expression exists, the first order conditions for that minimum are given by

$$\frac{\partial M_\tau}{\partial L_\tau} + (p^L_\tau)' = 0, \quad \frac{\partial M_\tau}{\partial K^o_\tau} - (q^K_\tau)' = 0. \tag{50}$$

Let $\hat{L}_\tau$ and $\hat{K}^o_\tau$ denote the minimizing vectors, then the minimum of the variable cost minus the value of the "old" stocks is given by

$$\begin{aligned} G_\tau &= G(p^L_\tau, q^K_\tau, Y_\tau, K_{\tau-1}, R_{\tau-1}, \Delta K_\tau, \Delta R_\tau, T_\tau) \\ &= \hat{M}_\tau + (p^L_\tau)' \hat{L}_\tau - (q^K_\tau)' \hat{K}^o_\tau, \end{aligned} \tag{51}$$

with $\hat{M}_\tau = M(Y_\tau, \hat{L}_\tau, \hat{K}^o_\tau, K_{\tau-1}, R_{\tau-1}, \Delta K_\tau, \Delta R_\tau, T_\tau)$. The function $G(\cdot)$ has the interpretation of a normalized variable cost function net of the value of the "old" stocks left over at the end of the period from the beginning of period stocks. Technically it can be viewed as the negative of a normalized restricted profit function. For duality, results between factor requirement functions and normalized variable profit functions see, for example, Diewert (1982) and Lau (1976). We assume that the function $G(\cdot)$ is twice continuously differentiable in all its arguments, homogeneous of degree zero in p^L and q^K, nondecreasing in Y, $|\Delta K|$, $|\Delta R|$ and p^L, nonincreasing in K_{-1}, R_{-1} and q^K, concave in p^L and q^K, and convex in K_{-1}, R_{-1}, ΔK and ΔR.

As indicated above, the stochastic closed loop optimal control solution for the quasi-fixed factors can now be found by replacing $M_\tau + (p^L_\tau)'L_\tau - (q^K_\tau)'K^o_\tau$ in (48) by $G(p^L_\tau, q^K_\tau, Y_\tau, K_{\tau-1}, R_{\tau-1}, \Delta K_\tau, \Delta R_\tau, T_\tau)$ defined in equation (51), and then by minimizing

$$(52)\qquad E_t \sum_{\tau=t}^{\infty} \{G(p^L_\tau, q^K_\tau, Y_\tau, K_{\tau-1}, R_{\tau-1}, \Delta K_\tau, \Delta R_\tau, T_\tau) + (q^K_\tau)'K_\tau + (q^R_\tau)'[R_\tau - (I - \delta^R_\tau)R_{\tau-1}]\} \prod_{s=t}^{\tau} (1 + r_s)^{-1}$$

with respect to the quasi-fixed factors $\{K_\tau, R_\tau\}_{\tau=t}^{\infty}$ only. Standard control theory implies that the stochastic closed loop feedback control solution that minimizes (52), say $\{\hat{K}_\tau, \hat{R}_\tau\}_{\tau=t}^{\infty}$, must satisfy the following set of stochastic Euler equations ($\tau = t, t + 1, \ldots$):[16]

$$(53)\qquad -E_\tau \frac{\partial G_{\tau+1}}{\partial K_\tau}(1 + r_{\tau+1})^{-1} = (q^K_\tau)' + \frac{\partial G_\tau}{\partial \Delta K_\tau} - E_\tau \frac{\partial G_{\tau+1}}{\partial \Delta K_{\tau+1}}(1 + r_{\tau+1})^{-1},$$

$$(54)\qquad -E_\tau \frac{\partial G_{\tau+1}}{\partial R_\tau}(1 + r_{\tau+1})^{-1} = (c^R_\tau)' + \frac{\partial G_\tau}{\partial \Delta R_\tau} - E_\tau \frac{\partial G_{\tau+1}}{\partial \Delta R_{\tau+1}}(1 + r_{\tau+1})^{-1},$$

where

$$c^R_\tau = E_\tau[q^R_\tau(1 + r_{\tau+1}) - (I - \delta^R_\tau)q^R_{\tau+1}]/(1 + r_{\tau+1})$$

can be viewed as a vector of rental prices. The firm's optimization decisions with respect to L_τ and K^o_τ are incorporated in the stochastic Euler equations via G_τ. (Recall from equation [51] that G_τ gives the minimal value of the variable cost net of the value of the "old" stocks for given values of the quasi-fixed factors and exogenous variables.) A detailed economic interpretation of the stochastic Euler equations is given in appendix B in Nadiri and Prucha (1999).

The optimal values for L_τ and K^o_τ can be found by differentiating G_τ with

16. Compare, e.g., Stokey, Lucas, and Prescott (1989, ch. 9), for a more detailed list of assumptions and a careful exposition of stochastic control theory, as well as for a discussion of the transversality condition.

respect to p^L_τ and q^K_τ and then making use of equation (50), that is, via Shephard's and Hotelling's lemma:[17]

$$\text{(55)} \qquad \hat{L}_\tau = \left(\frac{\partial G_\tau}{\partial p^L_\tau}\right)', \quad \hat{K}^o_\tau = -\left(\frac{\partial G_\tau}{\partial q^K_\tau}\right)'.$$

The derivatives on the r.h.s. of the above equations need to be evaluated at the optimal control solution for the quasi-fixed factors.

The formulation of a stochastic closed loop control policy generally requires knowledge of the entire distribution of the exogenous variables. Alternatively, one may postulate—as will be the case in the empirical application—that the firm formulates a certainty equivalence feedback control policy, which only requires knowledge of the first moment (mean) of the exogenous variables. In that case, the firm's objective function is given by equations (48) or (52) with the expectations operator moved next to each of the exogenous variables. The firm would now devise in each period t an optimal plan for its inputs in periods t, $t + 1$, . . . such that its objective function in period t is optimized, and then choose its inputs in period t accordingly. In each future period the firm will revise its expectations and optimal plan for its inputs based on new information. In case of a certainty equivalence feedback control policy, the first order conditions for the optimal plan in period t for the quasi-fixed factors would be given by equations (53) and (54) with all exogenous variables replaced by their expected values (conditional on information available at time t and the expectations operator in front of the respective derivatives suppressed). Equation (55) remains the same. If $G(\cdot)$ is linear quadratic, then the well-known certainty equivalence principle implies that the stochastic closed loop and the certainty equivalence feedback control policy are identical.[18]

Generalized Expressions for Technical Change and Total Factor Productivity Decomposition

The discussion in section 4.2 considered the case where the depreciation rates of all of the quasi-fixed factors are exogenously given. In this section, we have allowed the depreciation rate of some of the quasi-fixed factors to be endogenously determined. Analogously to equations (4) and (15) in section 4.2.1, we can define primal and dual measures of input-based technical change λ^X, output-based technical change λ^Y, and scale ρ, and we can define measures of capacity utilization for the generalized technology considered in this section. Those expressions are given in Prucha and Na-

17. In case of a profit maximizing model we have furthermore the following condition for the output vector: $\partial G_\tau / \partial Y_\tau = p^Y_\tau + [\partial p^Y_\tau / \partial Y_\tau] Y_\tau$.

18. For general technologies input decisions corresponding to the latter policy may be viewed as first-order approximations to those of the former policy; see, e.g., Malinvaud (1969) on the principle of certainty and first-order certainty equivalence.

diri (1990, 1996), and are not repeated here in order to conserve space.[19] Analogously to equations (39), (40), (43), and (44), one can also obtain, respectively, a decomposition of the growth rate of total factor productivity and its Törnqvist index based approximation. A generalization of the decomposition in equations (39) and (40) of the growth rate of TFP in continuous time is given in appendix B in Nadiri and Prucha (1999). The generalization of the Törnqvist index–based approximation in equations (43) and (44) is analogous. For the case of a single output good, the latter generalization is also given in Prucha and Nadiri (1990, 1996).

Flexible Functional Forms of Restricted Cost Functions

Empirical specifications of dynamic factor demand models typically model the underlying technology in a "flexible" fashion. As discussed at the beginning of section 4.3, flexible functional forms of cost and production functions have first been introduced by Diewert (1971) and Christensen, Jorgenson, and Lau (1971, 1973). In the dynamic factor demand literature, the technology has often been modeled in terms of a normalized restricted cost function. In the following we discuss some of the functional forms used in the recent literature.[20]

Recall that in our notation K refers to the vector of quasi-fixed factors whose depreciation rate is endogenously determined, and R refers to the vector of quasi-fixed factors whose depreciation rates are exogenous. For ease of presentation, we focus the subsequent discussion on the case where the depreciation rates of all quasi-fixed factors are exogenous to the firm.[21] In this case the normalized restricted cost function equation given in (51) simplifies to

$$G_\tau = G(p^L_\tau, Y_\tau, R_{\tau-1}, \Delta R_\tau, T_\tau) \tag{56}$$

given that we can now suppress K (and thus q^K). Also, we focus the discussion on the case of a single output good Y. Furthermore, for ease of presentation, we drop time subscripts in the following.

Observe that for linear homogeneous technologies we have

$$G(p^L, Y, R_{-1}, \Delta R, T) = g\left(p^L, \frac{R_{-1}}{Y}, \frac{\Delta R}{Y}, T\right)Y. \tag{57}$$

The normalized restricted cost function introduced by Denny, Fuss, and Waverman (1981b) and Morrison and Berndt (1981) is of the form

19. There are some typos in Prucha and Nadiri (1996) in that between equations (3.4) and (3.6) c^K and c^R should read $\underline{c}^K$ and $\underline{c}^R$, and in equation (3.9) C^+ should read C.

20. For general surveys of functional forms in modeling the firm's technology see, e.g., Fuss, McFadden, and Mundlak (1978) and Lau (1986).

21. The discussion can readily be extended to the case where both types of quasi-fixed factors are present.

$$
\begin{aligned}
(58)\quad & G(p^L, Y, R_{-1}, \Delta R, T) \\
&= Y\Bigg[\alpha_0 + (a_L)' p^L + (a_R)' \frac{R_{-1}}{Y} + (a_{\dot{R}})' \frac{\Delta R}{Y} + \alpha_T T \\
&\quad + 0.5 (p^L)' A_{LL} p^L + (p^L)' A_{LR} \frac{R_{-1}}{Y} + (p^L)' A_{L\dot{R}} \frac{\Delta R_{-1}}{Y} + (a_{LT})' p^L T \\
&\quad + 0.5 \frac{R'_{-1}}{Y} A_{RR} \frac{R_{-1}}{Y} + \frac{R'_{-1}}{Y} A_{R\dot{R}} \frac{\Delta R}{Y} + (a_{RT})' \frac{R_{-1}}{Y} T \\
&\quad + 0.5 \frac{\Delta R'}{Y} A_{\dot{R}\dot{R}} \frac{\Delta R}{Y} + (a_{\dot{R}T})' \frac{\Delta R}{Y} T + 0.5 \alpha_{TT} T^2 \Bigg]
\end{aligned}
$$

where α_0, α_T, α_{TT} are (scalar) parameters, a_L, a_R, $a_{\dot{R}}$, a_{LT}, a_{RT}, $a_{\dot{R}T}$ are conformably dimensioned parameter vectors and A_{LL}, A_{LR}, $A_{L\dot{R}}$, A_{RR}, $A_{R\dot{R}}$, $A_{\dot{R}\dot{R}}$ are conformably dimensioned parameter matrices. The normalized restricted cost function given in equation (58) can be viewed as having been obtained from a second order approximation of $g(p^L, R_{-1}/Y, \Delta R/Y, T)$. Following Denny, Fuss, and Waverman (1981b) and Morrison and Berndt (1981), the above normalized restricted cost function can be simplified by imposing parameter restrictions such that the marginal adjustment costs are zero at $\Delta R = 0$, i.e., $a_{\dot{R}} = 0$, $A_{L\dot{R}} = 0$, $A_{R\dot{R}} = 0$, $a_{\dot{R}T} = 0$.

Nadiri and Prucha (1990b) generalize this normalized restricted cost function to cover also homothetic technologies by replacing Y on the r.h.s. of equation (58) by $h(Y) = Y^{\phi_0 + \phi_1 \ln(Y)}$. This generalization is based on the observation that the normalized restricted cost function corresponding to homothetic technologies is of the following general form:

$$
(59)\qquad G(p^L, Y, R_{-1}, \Delta R, T) = g\left(p^L, \frac{R_{-1}}{H(Y)}, \frac{\Delta R}{H(Y)}, T\right) H(Y)
$$

where $H(Y)$ is some function of Y. We note that $h(Y)$ can—apart from a scaling factor—be viewed as a second-order translog approximation of $H(Y)$, assuming the latter function is sufficiently smooth.[22] Utilizing (15) it is readily seen that scale is given by $[(dH/dY)(Y/H)]^{-1}$. In the special case where $H(Y) = Y^{\phi_1}$ scale equals $1/\phi_1$.

A convenient feature of the normalized restricted cost function equation (58) and its generalization is that they allow for closed form solutions for the firm's optimal factor demand. However, the factor demand equations implied by these restricted cost functions are not symmetric in the sense that they are not invariant as to which of the variable factors is chosen as the numeraire. Thus different normalizations represent different specifications of the technology, which may seem arbitrary.

22. Suppose we approximate $H(Y)$ in terms of a second-order expansion in logs, then $\ln H(Y) = const + \phi_0 \ln Y + \phi_1 (\ln Y)^2 = const + \ln h(Y)$, and therefore $H(Y) \alpha Y^{\phi_0 + \phi_1 \ln Y}$.

Recently Mohnen (1992a) introduced a new restricted cost function which treats all factors symmetrically, but also allows for closed form solutions for the firm's optimal factor demand. This cost function generalizes the symmetric Generalized McFadden cost function put forth by Diewert and Wales (1987) through the inclusion of quasi-fixed factors. The manner in which the quasi-fixed factors are introduced is analogous to that in equation (59).

A further restricted cost function which treats all factors symmetrically was suggested by Morrison (1990). This restricted cost function represents an extension of the Generalized Leontief restricted cost function introduced by Diewert (1971). Prucha (1990) points out, however, that Morrison's restricted cost function is not invariant to units of measurement. Thus different choices of the units of measurements represent different specifications of the technology, which may again seem arbitrary. Prucha (1990) suggests a modification of Morrison's restricted cost function such that the resulting function is invariant to units of measurement. Based on the observation in equation (59), he also suggests a generalization to cover homothetic technologies.

For all of the above discussed functional forms, the implied Euler equations form in essence a linear system of difference equations, which can be solved explicitly along the lines discussed next in section 4.3.2. The Euler equation estimation approach discussed in section 4.3.3 does not require an explicit solution of the Euler equations. A functional form that has been used widely in conjunction with this approach is the transcendental logarithmic functional form introduced by Christensen, Jorgenson, and Lau (1971, 1973).

4.3.2 Solution and Estimation of Dynamic Factor Demand Models in Case of Linear Quadratic Technologies

Section 4.3.1 provided a general discussion of recent vintages of dynamic factor demand models. In this section, we consider in more detail dynamic factor demand models in case the firm's optimal control problem is of a "linear quadratic" nature. In this case, it is possible to obtain explicit analytic solutions for the firm's optimal factor inputs. We start the discussion by considering a specific example. We then consider the solution and estimation of a general class of "linear quadratic" dynamic factor demand models. To keep this discussion widely applicable, we only specify the model in terms of a set of first order conditions, rather than in terms of a specific cost or profit maximization problem.

Illustrative Example with Endogenous Depreciation Rate

In this subsection, we illustrate the solution and estimation of dynamic factor demand models by considering in detail a specific example of the model considered in section 4.3.1. As our illustrative model we consider the model employed by Prucha and Nadiri (1990, 1996) in analyzing the production structure, factor demand, and productivity growth in the U.S. electrical machinery industry. More specifically, we consider a model with two variable inputs M_t, and L_t, two quasi-fixed factors K_t and R_t, and one output good Y_t. Following Prucha and Nadiri (1990, 1996), we may assume that M_t and L_t denote, respectively, material input and labor input, and K_t and R_t denote, respectively, the end of period stocks of physical capital and R&D, and Y_t denotes gross output. We allow for the firm to determine the depreciation rate of capital endogenously, in that we allow the firm to choose K_t^o, the level of "old" stocks left over at the end of period t from K_{t-1}. The depreciation rate of R&D δ^R is fixed. With p_t^L we denote the price of labor, and q_t^K and q_t^R denote the after tax acquisition price for capital and R&D normalized by the price of material goods. The real discount rate r is taken to be constant over time.

To model the technology, we specify (dropping subscripts t) the following functional form for the normalized variable cost function net of the value of the "old" stocks as

$$
\begin{aligned}
(60)\quad & G(p^L, q^K, K_{-1}, R_{-1}, \Delta K, \Delta R, Y, T) \\
&= Y^{1/\rho}\left[\alpha_0 + \alpha_L p^L + \alpha_{LT} p^L T + \frac{1}{2}\alpha_{K^oK^o}(q^K)^2\right. \\
&\quad \left. + \alpha_{LK^o} p^L q^K + \frac{1}{2}\alpha_{LL}(p^L)^2\right] \\
&\quad + \alpha_K K_{-1} + \alpha_R R_{-1} + \alpha_{KL} K_{-1} p^L + \alpha_{KK^o} K_{-1} q^K \\
&\quad + \alpha_{RL} R_{-1} p^L + \alpha_{RK^o} R_{-1} q^K + \alpha_{KT} K_{-1} T + \alpha_{RT} R_{-1} T \\
&\quad + Y^{-1/\rho}\left(\frac{1}{2}\alpha_{KK} K_{-1}^2 + \alpha_{KR} K_{-1} R_{-1} + \frac{1}{2}\alpha_{RR} R_{-1}^2\right. \\
&\quad \left. + \frac{1}{2}\alpha_{\dot{K}\dot{K}} \Delta K^2 + \frac{1}{2}\alpha_{\dot{R}\dot{R}} \Delta R^2\right).
\end{aligned}
$$

We note that the adopted functional form is a special case of the linear quadratic restricted cost function specified in equation (58)—abstracting from the fact that for notational simplicity the specification in equation (58) was given only for the case where the depreciation rates for all quasi-fixed factors are exogenously given. In light of the above discussion, we note further that the technology specified by equation (60) is homogeneous of degree ρ. Also recall that by duality theory $G(\cdot)$ is convex in K, R, ΔK, ΔR and concave in p^L and q^K. This implies the following parameter restrictions: $\alpha_{KK} > 0$, $\alpha_{RR} > 0$, $\alpha_{KK}\alpha_{RR} - \alpha_{KR}^2 > 0$, $\alpha_{\dot{K}\dot{K}} > 0$, $\alpha_{\dot{R}\dot{R}} > 0$, $\alpha_{LL} < 0$, $\alpha_{K^oK^o} < 0$, $\alpha_{LL}\alpha_{K^oK^o} - \alpha_{LK^o}^2 > 0$.

Now suppose the firm's objective is to minimize the present value of its future cost stream. Suppose further that the firm determines its inputs according to a certainty equivalence feedback control policy, and holds static expectations on relative prices, output, and the technology. In this case the firm's objective function is given by the certainty equivalence analog of equation (52) with $G(\cdot)$ defined by equation (60). As discussed above, in each period t the firm establishes a plan for periods t, $t + 1, \ldots$ of how to choose its inputs optimally by optimizing this objective function conditional on its expectations, and implements the plan for the current period t (only). The plan is revised every period as new information becomes available. For simplicity, we assume that expected (relative) prices equal current (relative) prices. The certainty equivalence analog of the Euler equations (53) and (54) is then given by ($\tau = 0, 1, \ldots$)

$$
\begin{aligned}
(61)\quad & -\alpha_{\dot{K}\dot{K}}K_{t+\tau+1} + [\alpha_{KK} + (2 + r)\alpha_{\dot{K}\dot{K}}]K_{t+\tau} - (1 + r)\alpha_{\dot{K}\dot{K}}K_{t+\tau-1} + \alpha_{KR}R_{t+\tau} \\
&\qquad = -[\alpha_K + \alpha_{KT}T_t + \alpha_{KL}p_t^L + q_t^K(1 + r + \alpha_{KK^o})]\hat{Y}_t^{1/\rho}, \\
& -\alpha_{\dot{R}\dot{R}}R_{t+\tau+1} + [\alpha_{RR} + (2 + r)\alpha_{\dot{R}\dot{R}}]R_{t+\tau} - (1 + r)\alpha_{\dot{R}\dot{R}}R_{t+\tau-1} + \alpha_{KR}K_{t+\tau} \\
&\qquad = -[\alpha_R + \alpha_{RT}T_t + \alpha_{RL}p_t^L + q_t^R(r + \delta^R)]\hat{Y}_t^{1/\rho}
\end{aligned}
$$

where $\hat{Y}_t$ denotes expected output. In solving equation (61), we restrict the solution space to the class of processes that are of mean exponential order less than $(1 + r)^{1/2}$.[23] This rules out the unstable roots. (Of course, the unstable roots can also be ruled out by imposing the transversality condition.) As demonstrated in appendix B in Nadiri and Prucha (1999), solving equation (61) yields

$$
\begin{aligned}
(62)\qquad \Delta K_t &= m_{KK}(K_t^* - K_{t-1}) + m_{KR}(R_t^* - R_{t-1}), \\
\Delta R_t &= m_{RK}(K_t^* - K_{t-1}) + m_{RR}(R_t^* - R_{t-1}),
\end{aligned}
$$

with

$$
\begin{bmatrix} K_t^* \\ R_t^* \end{bmatrix} = -\begin{bmatrix} \alpha_{KK} & \alpha_{KR} \\ \alpha_{KR} & \alpha_{RR} \end{bmatrix}^{-1}
\times \begin{bmatrix} \alpha_K + \alpha_{KT}T_t + \alpha_{KL}p_t^L + q_t^K(1 + r + \alpha_{KK^o}) \\ \alpha_R + \alpha_{RT}T_t + \alpha_{RL}p_t^L + q_t^R(r + \delta^R) \end{bmatrix}\hat{Y}_t^{1/\rho}.
$$

That is, the optimal quasi-fixed inputs can be described in terms of an accelerator model. The accelerator coefficients $M = (m_{ij})_{i,j=K,R}$ are shown to satisfy the following matrix equation:

23. A vector process, say, η_t is said to be of mean exponential order less than κ if there exist constants c and λ with $0 < \lambda < \kappa$ such that $E_t \| \eta_{t+j} \| \leq c\lambda^{t+j}$ for all t and $j > 0$.

$$BM^2 + (A + rB)M - A = 0 \tag{63}$$

with $A = (\alpha_{ij})_{i,j=K,R}$ and where B is the diagonal matrix with elements $\alpha_{\dot{K}\dot{K}}$ and $\alpha_{\dot{R}\dot{R}}$ in the diagonal. The matrix $C = (c_{ij})_{i,j=K,R} = -BM$ is seen to be symmetric and negative definite.

The firm's demand equations for the variable factors and the firm's optimal choice for the "old" stock (to be left over from the beginning-of-period capital stock) can be derived from equation (60)—using Shephard's and Hotelling's lemma—as $M_t = G_t - p_t^L L_t + q_t^K K_t^o$, $L_t = \partial G_t / \partial p_t^L$, and $K_t^o = -\partial G_t / \partial q_t^K$:

$$\begin{aligned} M_t = & \left[\alpha_0 - \frac{1}{2}\alpha_{K^oK^o}(q_t^K)^2 - \alpha_{LK^o}p_t^L q_t^K - \frac{1}{2}\alpha_{LL}(p_t^L)^2\right]\hat{Y}_t^{1/\rho} \\ & + \alpha_K K_{t-1} + \alpha_R R_{t-1} + \alpha_{KT}K_{t-1}T_t + \alpha_{RT}R_{t-1}T_t \\ & + \left[\frac{1}{2}\alpha_{KK}\Delta K_{t-1}^2 + \alpha_{KR}K_{t-1}R_{t-1} + \frac{1}{2}\alpha_{RR}R_{t-1}^2 + \frac{1}{2}\alpha_{\dot{K}\dot{K}}\Delta K_t^2\right. \\ & \left. + \frac{1}{2}\alpha_{\dot{R}\dot{R}}\Delta R_t^2\right]/\hat{Y}_t^{1/\rho}, \\ L_t = & [\alpha_L + \alpha_{LT}T_t + \alpha_{LK^o}q_t^K + \alpha_{LL}p_t^L]\hat{Y}_t^{1/\rho} + \alpha_{KL}K_{t-1} \\ & + \alpha_{RL}R_{t-1}, \end{aligned} \tag{64}$$

and

$$K_t^o = -[\alpha_{LK^o}p_t^L + \alpha_{K^oK^o}q_t^K]\hat{Y}_t^{1/\rho} - \alpha_{KK^o}K_{t-1} - \alpha_{RK^o}R_{t-1}. \tag{65}$$

Equation (65) provides an economic model for K_t^o and hence for the depreciation rate of capital δ_t^K; recall that the depreciation rate of capital is implicitly defined by $K_t^o = (1 - \delta_t^K)K_{t-1}$. Equation (65) explains K_t^o as a function of relative prices, output, and lagged stocks. The case of a constant and exogenously given depreciation rate is contained as a special case with $\alpha_{LK^o} = \alpha_{K^oK^o} = \alpha_{RK^o} = 0$ and $\alpha_{KK^o} = -(1 - \delta^K)$. We emphasize that by imposing those zero restrictions we can formally test whether or not the depreciation rate is constant.[24]

One difficulty we face in trying to estimate this model is that in general the quadratic matrix equation (63) cannot be solved for M in terms of A and B. The equation can, however, be solved for A in terms of M and B: $A = BM(M + rI)(I - M)^{-1}$. Since the real discount rate r was assumed to be constant, the matrix M is constant over the sample. Hence, instead

24. As discussed in more detail in section 4.4, Prucha and Nadiri (1996) cannot reject the hypothesis of a constant depreciation rate for physical capital in the U.S. electrical machinery industry.

of estimating the elements of A and B, we may estimate those of M and B.[25] To impose the symmetry of C we can also estimate B and C instead of B and M. Observe that $A = C - (1 + r)[B - B(C + B)^{-1}B]$ and hence

$$(66)\qquad \begin{aligned} \alpha_{KK} &= c_{KK} - (1 + r)[\alpha_{\dot{K}\dot{K}} - (\alpha_{\dot{K}\dot{K}})^2(\alpha_{\dot{R}\dot{R}} + c_{RR})/f], \\ \alpha_{RR} &= c_{RR} - (1 + r)[\alpha_{\dot{R}\dot{R}} - (\alpha_{\dot{R}\dot{R}})^2(\alpha_{\dot{K}\dot{K}} + c_{KK})/f], \\ \alpha_{KR} &= c_{KR} - (1 + r)(\alpha_{\dot{K}\dot{K}}\alpha_{\dot{R}\dot{R}}c_{KR})/f, \end{aligned}$$

with

$$f = (\alpha_{\dot{K}\dot{K}} + c_{KK})(\alpha_{\dot{R}\dot{R}} + c_{RR}) - c^2_{KR}.$$

To reparameterize equation (62), it also proves helpful to define $D = (d_{ij})_{i,j=K,R} = -MA^{-1}$. Observe that $D = B^{-1} + (1 + r)(C - rB)^{-1}$. Hence, D is symmetric and its elements are given by

$$(67)\qquad \begin{aligned} d_{KK} &= 1/\alpha_{\dot{K}\dot{K}} + (1 + r)[c_{RR} - r\alpha_{\dot{R}\dot{R}}]/e, \\ d_{RR} &= 1/\alpha_{\dot{R}\dot{R}} + (1 + r)[c_{KK} - r\alpha_{\dot{K}\dot{K}}]/e, \\ d_{KR} &= -(1 + r)c_{KR}/e, \end{aligned}$$

with

$$e = (c_{KK} - r\alpha_{\dot{K}\dot{K}})(c_{RR} - r\alpha_{\dot{R}\dot{R}}) - c^2_{KR}.$$

Given the definition of D we can rewrite equation (62) as

$$(68)\qquad \begin{aligned} \Delta K_t &= d_{KK}[\alpha_K + \alpha_{KT}T_t + \alpha_{KL}p^L_t + q^K_t(1 + r + \alpha_{K^oK^o})]\hat{Y}^{1/\rho}_t \\ &\quad + d_{KR}[\alpha_R + \alpha_{RT}T_t + \alpha_{RL}p^L_t + q^R_t(r + \delta^R)]\hat{Y}^{1/\rho}_t \\ &\quad + [c_{KK}/\alpha_{\dot{K}\dot{K}}]K_{t-1} + [c_{KR}/\alpha_{\dot{K}\dot{K}}]R_{t-1}, \\ \Delta R_t &= d_{KR}[\alpha_K + \alpha_{KT}T_t + \alpha_{KL}p^L_t + q^K_t(1 + r + \alpha_{K^oK^o})]\hat{Y}^{1/\rho}_t \\ &\quad + d_{RR}[\alpha_R + \alpha_{RT}T_t + \alpha_{RL}p^L_t + q^R_t(r + \delta^R)]\hat{Y}^{1/\rho}_t \\ &\quad + [c_{KR}/\alpha_{\dot{R}\dot{R}}]K_{t-1} + [c_{RR}/\alpha_{\dot{R}\dot{R}}]R_{t-1}. \end{aligned}$$

The reparameterized factor demand equations are now given by equations (64), (65), and (68) with α_{KK}, α_{RR}, α_{KR}, d_{KK}, d_{RR}, and d_{KR} defined by equations (66) and (67). Once the model has been estimated in the reparameter-

25. This reparameterization approach was first suggested by Epstein and Yatchew (1985) for a somewhat different model with a similar algebra, and was further generalized by Madan and Prucha (1989). It will be discussed in more detail and within a generalized setting in the next subsection. For additional empirical studies utilizing the reparameterization approach, see, e.g., Mohnen, Nadiri, and Prucha (1986) and Nadiri and Prucha (1990a,b).

ized form, we can obtain estimates for the original model parameters via $A = C - (1 + r)[B - B(C + B)^{-1}B]$.

A further difficulty in estimating the factor demand equations is that

$$K_t = I_t^K + K_t^0 \tag{69}$$

is unobserved, since K_t^o depends on a set of unknown model parameters. (We note that K_t^o is unobserved even in the special case of a constant and exogenously given depreciation rate, that is, even in the case where $\alpha_{LK^o} = \alpha_{K^oK^o} = \alpha_{RK^o} = 0$ and $\alpha_{KK^o} = -(1 - \delta^K)$, as long as δ^K is estimated from the data.) We now assume, analogously to the approach taken by Epstein and Denny (1980), that equation (65) for K_t^o holds exactly. This assumption is clearly strong. However, it facilitates expression of the unobservable stocks K_t and K_t^o, at least in principle, as functions of observable variables and the unknown model parameters. More specifically, by solving equation (65) together with the identity $K_t = I_t^K + K_t^o$ recursively for K_t and K_t^o from some given initial capital stock, say K_0, we can express K_t as a function of I_t^K, I_{t-1}^K, . . . ,K_0, R_{t-1}, R_{t-2}, . . . , the exogenous variables and the model parameters. Consequently, upon replacing K_t and K_{t-1} in the variable factor demand equation (64) and in the quasi-fixed factor demand equation (68) by the expressions so obtained we can, at least in principle, rewrite the system of factor demand equations as a dynamic system of equations that determines I_t^K, R_t, M_t, and L_t, and where in the so obtained system *all variables are observable.* (If the initial stock is unobserved we may treat it as an additional parameter.)

For purposes of estimation, we need to add stochastic disturbance terms to each of the factor demand equations (64) and (68). Those disturbances can be viewed as random errors of optimization, errors in the data, or as stemming from random shocks observed by the firm but not by the researcher; cp., for example, Epstein and Yatchew (1985). Assuming that the disturbances are not correlated with the variables in the firm's information set we can, for example, use those variables (and functions of them) as instruments in estimating the model by the generalized method of moments (GMM) approach. The GMM estimation approach was introduced by Hansen (1982) within the context of stationary data generating processes. To allow for (possibly unknown) correlation over time, we may estimate the variance covariance matrix of the moments with a heteroskedasticity and autocorrelation robust variance covariance matrix estimator. For a general discussion and recent results concerning the asymptotic properties of GMM estimators for (possibly) temporally dependent and nonstationary data generating processes, including a discussion and consistency results of heteroskedasticity and autocorrelation robust variance covariance matrix estimator, see, for example, Gallant and White (1988) and Pötscher and Prucha (1997).

Numerical algorithms for the computation of estimators that are defined as optimizers of some statistical objective function—as, for example, the generalized methods of moments estimator or maximum likelihood estimator—generally require the numerical evaluation of the statistical objective function for different sets of parameter values. We note that for the actual numerical computation of estimators of the model parameters it is not necessary to solve equations (65) and (69) analytically for K_t (and K_t^o). Rather we can first solve, for any given set of parameter values, equations (65) and (69) numerically for K_t (and K_t^o), and then employ the numerical solution for K_t (rather than the analytic solution) in evaluating the statistical objective function. This approach is, however, typically cumbersome in that it requires the programming of the estimation algorithm by the researcher. Recently Prucha (1995, 1997) suggested a more convenient approach based on a reformulation of the analytic solution. This approach can be performed with standard econometric packages such as TSP.

Solution and Estimation of a General Class of Models

The illustrative example presented in the previous subsection can be viewed as a special case of a more general class of models where the firm's optimization problem involves the computation of a stochastic closed loop optimal control solution and where the objective function is "linear quadratic." As discussed above, the stochastic closed loop optimal control solution can always be found in two steps. In the first step we optimize the firm's objective function in each period with respect to the variable factors, for given values of the quasi-fixed factors. Substitution of the optimized values for the variable factors back into the firm's objective function then yields a new optimal control problem that only involves the quasi-fixed factors, which can be solved in a second step. In the following, let X_t denote the, say, $n \times 1$ vector of quasi-fixed factors—that is, the vector of control variables for the second step.

For a wide class of linear quadratic optimal control problems, the optimal control solution will have to satisfy a set of linear second order difference equations (possibly after recasting a higher order difference equation system into a second order one). In particular, assume that the control variables satisfy the following set of difference equations ($\tau = t, t + 1, \ldots$):

$$-\underline{B}E_\tau X_{\tau+1} + \underline{G}X_\tau - (1 + r)\underline{B}'X_{\tau-1} = E_\tau\phi_\tau \tag{70}$$

where $\underline{B}$ and $\underline{G}$ are $n \times n$ matrices, the ϕ_τs represent a set of forcing variables, r is the discount rate, and where the respective expectations are assumed to exist. Since the objective function is linear quadratic, certainty equivalence implies that solving equation (70) is equivalent to solving the difference equations ($\tau = t, t + 1, \ldots$):

$$-\underline{B}X_{\tau+1} + \underline{G}X_\tau - (1 + r)\underline{B}'X_{\tau-1} = (1 + r)\underline{a}_\tau \tag{71}$$

with $\underline{a}_\tau = E_t\phi_\tau/(1 + r)$.

We note that while the methodology discussed here is presented within the context of dynamic factor demand models, it applies more generally to any rational expectations model where the data generating process X_t is determined in the preceding manner.[26] The literature on finding optimal control solutions and solving rational expectations models has a long history.[27] The aim of the methodology outlined below is not only to obtain a solution of equation (71) for the X_t, but to express the solution so that the estimation of the model can be performed by standard econometric packages (such as TSP).

We assume that $\underline{B}$ is nonsingular and restrict the solution space to the class of processes X_t that are of mean exponential order less than $(1 + r)^{1/2}$. The characteristic roots of the difference equation system (71) are defined as solutions of

$$p(\lambda) = \det[-\underline{B}\lambda^2 + \underline{G}\lambda - (1 + r)\underline{B}'] = 0. \tag{72}$$

It is well known and not difficult to show that those characteristic roots come in pairs multiplying to $(1 + r)$. We assume that these roots are distinct. It then follows that there are exactly n roots that are less than $(1 + r)^{1/2}$ in modulus. Let Λ be the $n \times n$ diagonal matrix of these roots, and let V be the $n \times n$ matrix of solution vectors corresponding to those roots, that is,

$$-\underline{B}V\Lambda^2 + \underline{G}V\Lambda - (1 + r)\underline{B}'V = 0. \tag{73}$$

As in Kollintzas (1986) and Madan and Prucha (1989), we assume that V is nonsingular, and define $M = I - V\Lambda V^{-1}$. Given the maintained assumptions, the following theorem follows, for example, from Madan and Prucha (1989):

THEOREM 1. *The solution for X_t of the difference equation system (70) (or, because of certainty equivalence, [71]) is uniquely given by the following accelerator model:*

$$\begin{aligned} X_t &= MX_t^* + (I - M)X_{t-1}, \quad X_t^* = \underline{A}^{-1}J_t, \\ J_t &= D\sum_{\tau=t}^{\infty}(I + D)^{-(\tau-t+1)}\underline{a}_\tau, \\ D &= (1 + r)(I - M')^{-1} - I, \\ \underline{A} &= (I - M')^{-1}(rI + M')\underline{B}M/(1 + r) = D\underline{B}M/(1 + r). \end{aligned} \tag{74}$$

26. We note that the discussion also applies to processes described by a set of higher order difference equations, as long as that system can be rewritten as a second order difference equation system of the above form.

27. See, e.g., Epstein and Yatchew (1985), Hansen and Sargent (1980, 1981), Kokkelenberg and Bischoff (1986), Kollintzas (1985), and Madan and Prucha (1989). See also Binder and Pesaran (1995, 1997) for a recent review of rational expectations models and macroeconometric modelling.

The accelerator matrix M satisfies

$$(75)\qquad -\underline{B}(I - M)^2 + \underline{G}(I - M) - (1 + r)\underline{B}' = 0.$$

Furthermore, $S = \underline{B}(I - M)$ *is symmetric.*

In the case of static expectations on the forcing variables we have $\underline{a}_\tau = \underline{a}_t$. In this case the above solution simplifies in that in this case $J_t = \underline{a}_t$.

Madan and Prucha's proof of the theorem is based on a decomposition of X_τ into a backward component, given by $(I - M)X_{\tau-1}$, and a forward component, given by $g_\tau = X_\tau - (I - M)X_{\tau-1}$, where M is determined by equation (75). This basic approach has recently also been used by Binder and Pesaran (1995, 1997) to solve rational expectations models, where, in our notation, $\underline{B}$ is allowed to be nonsingular. Binder and Pesaran refer to this approach as the quadratic determinantal equation (QDE) method.

The quadratic matrix equation (75) can generally not be solved for M in terms of the original parameter matrices $\underline{B}$ and $\underline{G}$, except in case X_t is a scalar; that is, $n = 1$. However, we can use equation (75) to express $\underline{G}$ in terms of M and $\underline{B}$, that is,

$$(76)\qquad \underline{G} = \underline{B}(I - M) + (1 + r)\underline{B}'(I - M)^{-1}.$$

Thus, we can reparameterize the model in terms of M and $\underline{B}$, and estimate M and $\underline{B}$ rather than the original parameter matrices $\underline{G}$ and $\underline{B}$. As remarked above, this reparameterization approach was first suggested by Epstein and Yatchew (1985) within the context of a symmetric dynamic factor demand model where $\underline{B} = \underline{B}'$ (and $\underline{G} = \underline{G}'$). Madan and Prucha (1989) point out that this symmetry is, for example, typically violated if factors are allowed to become productive at different points in time—for example, if some factors become productive immediately and some with a lag—and/or if we allow for nonseparability between the adjustment cost terms and the inputs. Madan and Prucha (1989) then extend the reparameterization approach to nonsymmetric dynamic factor demand models with $\underline{B} \neq \underline{B}'$ (and $\underline{G} = \underline{G}'$). This approach is presented in more detail in appendix B in Nadiri and Prucha (1999).[28] The discussion in this appendix also considers an explicit specification of the stochastic process governing the forcing variables. In adopting a re-parameterization for the parameters describing that process, it is possible, as also demonstrated in this appendix, to obtain closed form analytic expressions for X_t in terms of the model parameters and the forcing variables. The advantage of the reparameterized model is that it can be estimated with standard econometric packages such as TSP.[29]

28. Both Epstein and Yatchew (1985) and Madan and Prucha (1989) consider matrices G with additional structure, which they utilize during the reparameterization. The discussion in this appendix shows that the reparameterization approach works even without additional structure on G.

29. The above discussion of the solution and estimation of dynamic factor demand models is given in form of a discrete time model. The reason for this is that empirical data typically

4.3.3 Estimation of Dynamic Factor Demand Models for General Technologies

The theoretical model specified in section 4.3.1 is quite general, and allows for the firm's technology and optimal control problem to be "non–linear quadratic." We note that in case the firm's optimal control problem is not of a linear quadratic nature, it is generally not possible to obtain an explicit analytic expression for the firm's stochastic closed loop feedback control solution. In the following, we discuss strategies for estimating non–linear quadratic dynamic factor demand models. Those strategies can, of course, also be applied in estimating linear quadratic dynamic factor demand models.

Before proceeding we reemphasize that while the model specification in section 4.3.1 is quite general, the discussion does not impose this generality. That is, the discussion also covers implicitly less general specifications as special cases. The specification in section 4.3.1 contains in particular the case where all factors are variable—and hence the firm is at each point in time in long-run equilibrium—or the case where the depreciation rates of all quasi-fixed factors are exogenously given as special cases.

Estimation of Variable Factor Demand Equations

In estimating a factor demand model we can, in principle, always attempt to estimate the unknown model parameters from only a subset rather than the entire set of factor demand equations. Statistically there are pros and cons for such a strategy: If the model is correctly specified, we will generally obtain more efficient estimates by utilizing the entire set of factor demand equations rather than a subset. However, if one or a subset of the factor demand equations is misspecified, then not only the parameters appearing in the misspecified equations, but in general all model parameters will be estimated inconsistently.

As is evident from the discussion in section 4.3.1, certain aspects of the model specification such as the nature of the optimal control policy and the expectation formation process only enter into the specification of the demand equations for the quasi-fixed factors. Consequently, in this sense the demand equations for the quasi-fixed factors are more susceptible to potential misspecification than the demand equations for the variable factors. In cases where the determinants of the demand for the quasi-fixed factors are not of real interest, but where one is especially concerned about the possibility of misspecification of the quasi-fixed factor demand equations, it may be prudent only to estimate the variable factor demand equa-

refer to discrete time points. Potential pitfalls in using formulas for the optimal factor inputs derived from a continuous time model for estimation from discrete data are considered in Prucha and Nadiri (1991). For a more detailed discussion see Nadiri and Prucha (1999).

tions. By estimating only the variable factor demand equations, we are typically also faced with a less complex estimation problem.

The variable factor demand systems can take various forms depending on the specification of the technology. For example, in case the technology is specified in terms of a translog restricted cost function the variable factor demand system is typically given by a system of share equations.[30] The model specified in section 4.3.1 allows depreciation rates of some of the quasi-fixed factors to be determined endogenously and to be modeled as a function of unknown parameters. As remarked above, as a result the stocks of those quasi-fixed factors are then unobserved. To estimate the system of variable factors, we may proceed analogously as outlined at the end of section 4.3.2, subsection "Illustrative Example with Endogenous Depreciation Rate"; for empirical applications see, for example, Epstein and Denny (1980) and Nadiri and Prucha (1996).

Euler Equation Estimation Approach

In section 4.3.1 we derived a general set of stochastic Euler equations that need to be satisfied by the stochastic closed loop feedback optimal control solution for the quasi-fixed factors without restricting the technology to be linear quadratic. Those stochastic Euler equations are given by equations (53) and (54). In section 4.3.2 we solved those equations explicitly for the case where the technology is, indeed, linear quadratic. In case the technology is non–linear quadratic, such an explicit solution is generally not available. In this case we may then adopt an alternative estimation approach due to Kennan (1979), Hansen (1982), Hansen and Sargent (1982), and Hansen and Singleton (1982).[31] In this approach all expectations of future variables are replaced by their observed values in future periods. More specifically, in this approach we would rewrite the stochastic Euler equations (53) and (54) as

$$(77)\qquad -\frac{\partial G_{t+1}}{\partial K_t}\Big/(1+r_{t+1}) - (q_t^K)' - \frac{\partial G_t}{\partial \Delta K_t} + \frac{\partial G_{t+1}}{\partial \Delta K_{t+1}}\Big/(1+r_{t+1}) = v_t^K,$$

$$(78)\qquad -\frac{\partial G_{t+1}}{\partial R_t}\Big/(1+r_{t+1}) - (c_t^R)' - \frac{\partial G_t}{\partial \Delta R_t} + \frac{\partial G_{t+1}}{\partial \Delta R_{t+1}}\Big/(1+r_{t+1}) = v_t^R.$$

30. The variable factor demand equations typically form a triangular structural system. Lahiri and Schmidt (1978) point out that the full information maximum likelihood (FIML) estimator and the iterative seemingly unrelated regressions (SUR) estimator are identical for triangular structural systems. This identity might be thought to imply that for such systems the variance covariance matrix estimator typically associated with the SUR estimator is a consistent estimator for the asymptotic variance covariance matrix. However, Prucha (1987) points out that this is generally not the case.

31. The approach has been used widely in empirical work. Early empirical implementations include Pindyck and Rotemberg (1983) and Shapiro (1986).

If expectations are truly formed rationally, we have $E_t v_t^K = 0$ and $E_t v_t^R = 0$, and equations (77) and (78) can then be estimated consistently by the GMM estimation approach.[32] Of course, the stochastic Euler equations (77) and (78) can be augmented by the demand equations for the variable factors. Recall that K_t denotes the vector of quasi-fixed factors for which the depreciation rates are determined endogenously and are modeled as a function of unknown parameters. Thus, as remarked, K_t is unobserved. In estimating the demand equations, we may again proceed analogously as outlined at the end of the "Illustrative Example" subsection.

The Euler equation estimation approach allows considerable flexibility in the choice of the functional form for the technology. Also, it does not require an explicit specification of the process that generates the variables exogenous to the firm's decision process or specific assumptions concerning the firm's planning horizon. However, it is generally not fully efficient in that it neglects information from the entire set of Euler equations (and, e.g., the transversality condition), which only comes into play by actually solving the Euler equations. In their comparison of alternative methods for estimating dynamic factor demand models, Prucha and Nadiri (1986) report that small sample biases and efficiency losses seem especially pronounced for parameters that determine the dynamics of the demand for the quasi-fixed factors. We note further that, although the Euler equation estimation approach does not require either an analytic or numerical solution for the firm's optimal demand for the quasi-fixed factors, such a solution—or some approximation to it—will be needed, for example, for tax simulations.[33]

4.3.4 Further Developments

There have been other important developments in addition to those described above. In particular, since the late 1970s there was a process of convergence between the investment literature based on Tobin's (1969) q and the investment literature with explicit adjustment costs. In Tobin's investment model the rate of investment is a function of q, defined as the ratio of the market value of capital to its replacement cost. Hayashi (1982) shows the equivalence of the two investment theories for a general class of models; see also Mussa (1977) and Abel (1983). The literature distinguishes between average q, defined as the ratio of the market value of existing capital to its replacement cost, and marginal q, defined as the ratio of the market value of an additional unit of capital to its replacement cost.

32. For references concerning the asymptotic properties of GMM estimators see the discussion at the end of section 4.3.2.

33. A further approach for modeling and estimating dynamic factor demand models that allows for "non–linear quadratic" technologies and nonstatic expectations, and is based on a finite horizon specification, was suggested in several papers by Prucha and Nadiri (1982, 1986, 1988, 1991); recent contributions include Gordon (1996) and Steigerwald and Stuart (1997). A more detailed discussion is given in Nadiri and Prucha (1999).

While average q is observable, marginal q, which is the quantity relevant for the firm's investment decision, is not observable. However, Hayashi (1982) also derives an exact relationship between average q and marginal q, which is important for a proper empirical implementation of the q theory of investment.

Another important development is an expanding literature that considers the effects of irreversibility combined with uncertainty and timing flexibility on the firm's investment decision. Irreversibility is another avenue that introduces a dynamic element into the investment decisions. The literature on irreversible investment dates back to Arrow (1968). The more recent literature utilizes option pricing techniques to determine the firm's optimal investment pattern under irreversibility. A survey of this literature and exposition of those techniques is given in Dixit and Pindyck (1994). One way to incorporate irreversibility into an adjustment cost model is to assume infinitely large adjustment costs for negative investment. This approach was, for example, taken by Caballero (1991). Another approach was explored by Abel and Eberly (1994). Their model incorporates Arrow's observation that the resale price of capital may be less than the purchase price of new capital, which includes the case where the resale of capital is impossible, corresponding to the extreme case of a resale price of zero. Additionally, their model includes adjustment costs as well as fixed costs and thus provides for an interesting integration of the irreversible investment and adjustment cost literature.

4.4 Applications

There are numerous applications of the factor demand models using different sets of data and answering important questions of theoretical, empirical, and policy interest. There is a vast literature showing the widespread use of factor demand models for empirical analysis.[34] The class of dynamic factor demand models considered in section 4.3 has been used to study a variety of subjects ranging from the analysis of the production structure of various industries, the rate of technical change, the impact of R&D investment and R&D spillovers, the convergence of productivity levels, the effect of public infrastructure on the private sector productivity, the impact of financial variables on production decisions, the cyclical behavior of utilization and markup of prices over costs, and so on. Here we will provide only a brief description of a few applications of dynamic factor demand models for illustrative purposes. To save space, we do not report on the formal structure of the models used in the studies.

As remarked above, besides analyzing productivity behavior, the dynamic factor demand methodology also addresses issues concerning the

34. See, e.g., Berndt, Morrison, and Watkins (1981), Jorgenson (1986, 1995a,b, 1996a,b), Watkins (1991), and Good, Nadiri, and Sickles (1997) for partial references.

structure of production such as substitution among factors of production in response to changes in relative prices; technological change; changes in public capital; international, interindustry, or interfirm spillovers due to R&D investment; and so on. The time path of the adjustment of different types of capital and the linkages between short-, intermediate-, and long-run behavior are explicitly modeled and estimated. Changes in capacity utilization rates and depreciation rates of different types of capital and their effects on the demand for other inputs can be estimated. Given estimates of the depreciation rates it is possible to decompose gross investment into replacement and net investments, and generate consistent measures of capital stocks within the framework of the dynamic factor demand model.

4.4.1 Tax Incentives, Financing, and Technical Change

The effect of taxes and other incentives on factor demand and output growth has been of a long and ongoing interest in the literature. The role of taxes as a component of the user cost of capital was made clear in seminal papers by Jorgenson (1963) and Hall and Jorgenson (1967, 1971). Jorgenson and his associates have examined the impact of tax incentives for business investment in the United States in a series of papers; see Jorgenson (1996b) for more detailed references. Hall and Jorgenson (1967, 1971) modeled the accelerated depreciation of the 1954 tax law and guidelines for asset lifetimes, the investment tax credit introduced in 1962, the reduction in corporate tax rate in 1964 and the suspension of the investment tax credit in 1966. These tax law changes were incorporated as elements of the user cost of capital. The general conclusion of this body of work was that investment incentives exert a considerable long-run effect on the rate of capital accumulation. Each major change in investment incentives was followed by an investment boom which in turn led to increases in the level of economic activity that induced further increases in investment. However, the lag between changes in investment incentives and investment expenditure was found to be fairly long.

As discussed in section 4.3, given the underlying intertemporal optimization framework, the notion of a user cost of capital or the after tax acquisition price is also present within the framework of the dynamic factor demand models reviewed in this paper. A general discussion of the effect of taxes and incentives within the context of dynamic factor demand models is, for example, provided by Bernstein and Nadiri (1987). We emphasize that in general the effect of any tax changes designed to affect a particular factor of production will influence also the demand for other inputs. This arises from the interrelatedness of factors. Early generations of dynamic factor demand models assumed separability between the quasi-fixed factors and hence could not fully capture such effects.

Tax policy operates through factor prices. The effect of changes in tax policy depends in general on the degree of substitutability or complementarity among factors of production. As discussed in section 4.2, technological change, if not neutral, depends on relative prices (as is, e.g., evident from its definition on the cost side). Hence, tax policy can also affect technological change. We note that in the short-run the effects of changes in tax policy on factor demands may be quite different from their effects in the long-run in that in the short-run the firm may find it advantageous to over-adjust some of its variable factors to lessen the effects of adjustment costs. The framework of a dynamic factor demand model also provides a natural setting for analyzing the effects of expectations about future tax policies.

As an illustration of the application of the dynamic factor demand models for tax analysis, consider the recent study by Bernstein (1994). The tax instruments considered are the corporate income tax (CIT), the investment tax allowances (ITA), and capital consumption allowance (CCA). A normalized variable profit function with quadratic adjustment costs for capital stock is formulated and the model is estimated using data for the Turkish electrical machinery, non-electrical machinery, and transportation equipment industries. The empirical results suggest the following findings:

1. The adjustment cost parameter estimates suggest that these industries are not in long-run equilibrium. The mean value of the speed of adjustment ranged between 0.33 to 0.36 for each of the industries, implying that about 35 percent of the capital stock adjustment occurs within the first year of capital accumulation.

2. The effects of taxes and incentives on production and investment decisions are transmitted though changes in the rental price of capital. The magnitudes of the input elasticities to changes in tax instruments differ in short-, intermediate-, and long-run due to the presence of adjustment costs. They also differ with respect to the various tax policy instruments, as well as across industries. The long-run elasticities of output and the inputs are quite small, but larger than the short- and intermediate-run elasticities. Another important point is that, because of the interdependence among production decisions embedded in the dynamic factor demand models, taxes and incentives targeted toward a particular input also have effects on the other inputs.

3. Productivity growth can be affected by the tax policies. This arises since productivity growth depends on the growth of output and inputs, which are affected by changes in factor prices. As noted above, the latter are in turn affected by changes in tax instruments.

As illustrated by the above discussion, dynamic factor demand models provide a powerful framework to trace the effects of various policy decisions, such as tax and incentive policies and financial decisions. It is possible to examine the impact of these decisions in the short, intermediate, and long run on the production decisions and productivity performance. Also, the effect of expectations can be examined in this framework.

4.4.2 R&D Investment, Production Structure, and TFP Decomposition

The role of R&D and the behavior of other factors of production in the United States, Japanese, and German manufacturing industries was explored by Mohnen, Nadiri, and Prucha (1983, 1986) based on a special case of the dynamic factor demand model considered in section 4.3. One of the results of the study was that the average net rates of return were similar for both R&D and capital in the manufacturing sectors of the three countries. However, the rate of return on R&D was greater than that on capital in each sector. A further finding was that it takes a considerably longer time for the R&D stock to adjust to its optimum value than for the physical capital stock. The average lag for capital was approximately three years in the three countries, while the average lag for R&D was about five years in the United States, eight years in Japan, and ten years in Germany. The patterns of own- and cross-price elasticities of the inputs varied considerably among countries. The own-price elasticities were generally higher than the cross-price elasticities. There was mostly a substitutional relationship between the inputs. The output elasticities of the inputs in the short and intermediate runs differed from each other and across countries. The materials input overshot in the short run its long-run equilibrium value to compensate for the sluggish adjustments of the two quasi-fixed inputs, capital, and R&D; the output elasticities of the capital stock were larger than those of R&D in the short and intermediate runs; also, there was evidence of short-run increasing returns to labor. The Japanese manufacturing sector seems to have higher elasticities than the U.S. manufacturing sector and to display more flexibility.

Nadiri and Prucha (1990b) explore the production structure of the U.S. Bell System before its divestiture.[35] They consider a model with two variable factors, labor and materials, and two quasi-fixed factors, physical and R&D capital. The technology is not assumed to be linear homogeneous, but is allowed to be homothetic of a general form. The estimated degree of scale was about 1.6. As a consequence, in decomposing the traditional TFP measure they find that almost 80 percent of the growth of TFP is attributable to scale. The conventional TFP measure, if it is considered as a measure of technical change, was thus seriously biased upwards. The estimated rate of technical change was only about 10–15 percent of the measured TFP. The most significant source of output growth was the growth of capital with a contribution of over 50 percent, while labor and material contributed about 15 percent, and the contribution of technical change

35. For an exploration of Bell Canada see, e.g., Bernstein (1989b) and Fuss (1994).

was about half as much. The growth of R&D contributed about 2 percent, which, given its small share in the production is fairly substantial. The rate of return on R&D, however, was much greater than that on plant and equipment investment. The net rate of return for R&D investment is about 20 percent in comparison to the net rate of return of about 7 percent for investment in physical capital.

The study by Nadiri and Prucha also considers alternative specifications of the length of the planning horizon and the expectation formation process. They find that the optimal plans for the finite horizon model converge rapidly to those of the infinite horizon model as the planning horizon extends. This observation suggests that additional planning costs will quickly exceed additional gains from extending the planning horizon, which may provide a rationale for why many firms plan only for short periods into the future. Parameter estimates differ in their sensitivity to alternative specifications of the expectation formation process. Estimates of parameters determining the adjustment path of capital and R&D turned out to be sensitive. On the other hand, estimates of other characteristics of the underlying technology such as scale seem to be insensitive to the specification of the expectation formation process.

Recently the dynamic factor demand framework has been used to explore the role of high-tech capital and information technology equipment, as well as human capital, on the production structure and productivity growth in U.S. manufacturing; see Morrison (1997) and Morrison and Siegel (1997). One finding is that high-tech capital expansion increases demand for most capital and non-capital inputs overall, but saves on material inputs.

4.4.3 Technological Spillovers and Productivity Growth

An important feature of R&D investment that distinguishes it from other forms of investment is that firms which undertake R&D investment are often not able to exclude others from freely obtaining some of the benefits; that is, the benefits from R&D investment spill over to other firms in the economy, and the recipient firms do not have to pay for the use of knowledge generated by the investing firms' R&D activity. R&D spillovers may affect the production structure and factor demand in several ways. In particular, R&D investment may shift the production function up (or the cost function downward). This is the direct productivity effect. Also, changes in the R&D spillover may cause factor substitution. In the language of the technological change literature, changes in R&D spillovers may cause factor biases, which may be either factor using or factor saving. Changes in the R&D spillovers may also affect the adjustment process of the quasi-fixed factors.

There are a number of empirical studies using the dynamic factor demand framework to measure the impact of technology spillover.[36] As an illustration, consider the Bernstein and Nadiri (1989) study which provides an example of intraindustry spillover effects among the U.S. instruments, machinery, petroleum, and chemical industries. Several interesting results are reported:

1. The adjustment process of the two quasi-fixed inputs were shown to be interdependent; that is, as the physical and R&D capitals adjust toward their equilibrium levels, the speed of adjustment of one is affected by the adjustment of the other. The estimates indicate that about 33–42 percent of the adjustment of the physical capital stock occurred within a single year. R&D capital adjustment is lower than that of physical capital; the estimates show that about 22–30 percent of the adjustment of R&D capital occurred in one year. The adjustment processes vary across the industries.
2. There are a number of effects associated with the intraindustry R&D spillover. First, costs decline as knowledge expands for the externality-receiving firms. Second, production structures are affected, as factor demands change in response to the spillover. Third, the rates of both physical and knowledge capital accumulation are affected by the R&D spillover. The results indicate that the short-run demand for R&D and physical capital decreased in response to an increase in the intraindustry spillovers. Both the variable and average costs for each industry declined in response to the intra-industry spillovers. Spillover-receiving firms gained a 0.05 percent, 0.08 percent, 0.11 percent, and 0.13 percent average cost reduction, respectively, in the instruments, machinery, petroleum, and chemical industries as a result of a 1 percent increase in the intraindustry spillover. Not surprisingly, the effect of spillovers on the factor inputs and cost was larger in the long run than in the short run.
3. The results also indicate that for all four industries the net social rate of return greatly exceeded the net private rate of return. However, there was significant variation across industries in the differential between the returns. For chemicals and instruments, the social rate of return exceeded the private rate of return by 67 and 90 percent, respectively. Machinery exhibited the smallest differential of about 30 percent, and the petroleum industry exhibited the greatest differential as the social rate exceeded the private rate by 123 percent.

36. In recent years there has been a considerable effort to model and estimate the role of R&D spillover. There are a number of different approaches that have been taken to specify and measure technical spillover effects. Recent studies on R&D spillovers within the framework of dynamic factor demand models other than those discussed in this section include papers by Bernstein (1989a), Bernstein and Nadiri (1988), Goto and Suzuki (1989), Mohnen (1992a), Mohnen and Lépine (1991), and Srinivasan (1995).

Bernstein's (1988) study of Canadian industries shows that spillovers occur between rival firms within the same industry and between firms operating in different industries. These spillovers, specially those associated with interindustry spillovers, caused unit costs to decline and the structure of production of the receiving industries to change as the spillovers induced factor substitution. The productivity effect of the spillovers and the gap between the private and social rates of return to R&D varied among the industries.

Mohnen (1992b) explored the question of possible cross-country R&D spillovers among the manufacturing sectors of the United States, Japan, France, and the United Kingdom. He used a cost function with quasi-fixed factors and adjustment cost based on the symmetric generalized McFadden functional form. The results indicated that foreign R&D yields greater cost reduction than own R&D, own R&D and foreign R&D are complementary and foreign R&D can explain part of the productivity convergence among the manufacturing sectors of the leading industrial countries. In the case of the Canadian manufacturing sector, Mohnen (1992a) reports surprisingly weak spillover effects for R&D undertaken in other major industrialized countries. Bernstein and Mohnen (1998) have developed a bilateral model of production between U.S. and Japanese economies and trace the effects of international R&D spillovers on production cost, traditional factor demands, the demand for R&D capital, and productivity growth in each country. Their results show that international spillovers increased U.S. productivity growth by about 15 percent, while productivity growth of the Japanese economy is increased by 52 percent. The R&D spillovers affect the structure of production in both countries, particularly the demand for labor in Japan.

4.4.4 Capital Utilization, Depreciation Rates, and Replacement Investment

In general, productivity growth may, at least in the short run, be influenced by whether various factors of production are fully utilized. Most of the studies of firm demand for factors of production assume a constant rate of utilization of inputs and ignore the fact that the firm can choose simultaneously the level and rate of utilization of its inputs. As discussed above, a model which allows for the capital utilization and depreciation rate to be determined endogenously, along the lines of the dynamic factor demand model considered in section 4.3, was first implemented empirically by Epstein and Denny (1980). Using only the demand equations for the variable factors, their model was estimated from U.S. manufacturing data, based on a data set developed by Berndt and Wood. Epstein and Denny report an average rate of depreciation of 0.126 for physical capital. The estimated depreciation rates vary between 0.11 and 0.145 over the sample. The model generates a capital stock series which is quite different from that implied by the Berndt and Wood data. Also, their model indicates substantial cross-price elasticities, showing the interrelated nature of the choice about capital usage and other inputs and outputs which would be ignored if a simpler framework is used to describe the firm's technology. Kollintzas and Choi (1985) and Bischoff and Kokkelenberg (1987) report estimates of 0.126 and 0.106 on average for the rate of depreciation of physical capital in the U.S. manufacturing sector.

Morrison (1992a) reports that a significant portion of cost declines in the U.S., Canadian, and Japanese manufacturing industries, resulting from fluctuations in capacity utilization and scale economies, has been erroneously attributed to technical change. Morrison (1992b) finds furthermore that the markups of prices over costs are significant and influence measured productivity. Galeotti and Schiantarelli (1998) have examined the counter-cyclical behavior of markups in U.S. two-digit manufacturing industries in the context of a dynamic optimization model. Their results show that markups are affected by both the level and growth of the demand facing an industry in the presence of cost of adjustment.

As discussed above, Prucha and Nadiri (1990, 1996) apply the dynamic factor demand model specified in section 4.3.2 to data for the U.S. electrical machinery industry for the period 1960–80. This study builds on an earlier study by Nadiri and Prucha (1990a) that is based on capital stock data from the Office of Business Analysis (OBA). They estimate two versions of the model. In the more general version of the model, K_t^o, the stock of capital left over at the end of the period from the beginning of period stock, or equivalently the depreciation rate of capital, is permitted to be determined as a function of output and relative prices; see equation (65). In the other version of the model the depreciation rate of capital is taken to be constant but unknown by imposing the parameter restrictions $\alpha_{LK^o} = \alpha_{K^oK^o} = \alpha_{RK^o} = 0$. We note that for both models the depreciation rate is estimated and the respective capital stocks are generated internally during estimation in a theoretically consistent fashion. The paper reports the following findings:

1. The depreciation rate of capital is estimated to be 0.038 as compared to 0.055 for the OBA capital stock series. This translates into a difference of 16 percent in magnitude between the implied capital stock series and the OBA capital stock series at the end of the sample period.

2. Based on their tests Prucha and Nadiri accept the model corresponding to a constant depreciation rate. This finding is interesting, since the assumption of a constant depreciation rate has a long history, but has also been the subject of considerable debate. The assumption of a constant depreciation rate was challenged by, among others, Feldstein and Foot (1971), Eisner (1972), Eisner and Nadiri (1968, 1970), and Feldstein

(1974). It was forcefully defended by Jorgenson (1974).[37] Among other things he pointed out that some of the earlier studies on replacement investment were not fully consistent, in that they employed capital stock data that were generated under a different set of assumptions than those maintained in those studies. Within the modeling framework discussed here the capital stocks are generated in an internally consistent fashion from gross investment data. Thus, as a by-product, a consistent decomposition of gross investment into replacement investment and net investment can be obtained. In particular, replacement investment I_t^{KR} is defined as the difference between the beginning of period stocks and what is left over from these stocks at the end of the period, that is, $I_t^{KR} = K_{t-1} - K_t^o$. Net investment I_t^{KE} is defined as the difference between gross investment and replacement investment, that is, $I_t^{KE} = I_t^K - I_t^{KR} = K_t - K_t^o$. For the entire sample period, net investment as a percent of gross investment was about 60 percent. As expected, this ratio exhibited cyclical patterns with a low of 41 percent in 1975. The ratio of net investment to gross investment based on the estimated model is much higher than the rates implied by the OBA capital stock series.

3. The study finds significant adjustment costs. The own accelerator coefficient for physical capital is approximately 0.20, while that for the R&D capital is 0.15. The cross accelerator coefficients are small (about 0.02). The total adjustment costs are about 15 percent of total gross investments for each of these two types of capital.

4. The pattern of output elasticities reveals that the variable factors of production, labor, and materials, respond strongly in the short run to changes in output; in fact, they overshoot their long-run equilibrium values in the short run. The output elasticities of the quasi-fixed factors, capital, and R&D, are small in the short run but increase over time. The long-run output elasticities suggest an estimate of economics of scale of approximately 1.2. The own price elasticities are, as expected, all negative. The results also suggest that the cross price elasticities of labor and capital may be sensitive to whether or not the rate of depreciation is endogenous.

5. The study also provides a decomposition of the sources of TFP growth. This decomposition is reproduced in table 4.1 for both versions of the model. It shows that the estimate of productivity growth based on the traditional TFP measure is approximately three times larger than the estimate of pure technical change generated by the econometric model. The main source of the difference is the scale effect which represents about 46 percent of the growth in the traditional TFP measure. The remainder of the difference is mainly due to the presence of adjustment costs, which accounts for almost 21 percent of total factor productivity growth. The

37. The validity of a constant depreciation rate has also been tested in several papers by Hulten and Wykoff; see, e.g., Hulten and Wykoff (1981a,b).

Table 4.1 **Decomposition of TFP Growth in the U.S. Electrical Machinery Industry in Percentages, 1960–80**

	Model (estimated capital stock)	
	Constant Depreciation Rate	Variable Depreciation Rate
Technical change	0.69	0.66
Scale effect	0.83	0.93
Adjustment cost effects		
Temporary equilibrium effect	0.42	0.39
Direct adjustment cost effect	0.02	0.02
Variable depreciation effect	0.00	0.02
Unexplained residual	0.03	−0.03
Total factor productivity	1.99	1.99

estimated pure technical change exhibits a very smooth pattern and increases over time.

Nadiri and Prucha (1996) employ a special case of the model in section 4.3—where the depreciation rates are modeled as constant but unknown—to estimate the depreciation rates of both physical and R&D capital for the U.S. total manufacturing sector. The depreciation rate of R&D capital was, in particular, estimated to be about 0.12, which is quite similar to the ad hoc assumption of the R&D depreciation rate used in many studies that use the R&D capital stock as an input in the production function.[38] Given estimates for the depreciation rates, gross investment can again be decomposed into net and replacement investment. For the entire sample period, net investment in R&D in the U.S. total manufacturing as a percent of gross investment was 16 percent. However, during the 1970s, this percentage declined to 5 percent, reflecting the near collapse of R&D investment in that period.

4.5 Effects of Misspecification: A Monte Carlo Study

In this section we briefly explore, by means of a Monte Carlo study, the effects of model misspecification on the estimation of important characteristics of the production process such as technical change, scale, and adjustment speed. The "true" model from which the data for the Monte Carlo study are generated has the same basic structure as the model for the U.S. electrical machinery sector considered in Prucha and Nadiri (1996), but

38. See, e.g., Griliches (1980), Bernstein and Nadiri (1988), and Mamuneas and Nadiri (1996).

differs in terms of the specification of the restricted cost function and in terms of the assumed expectation formation. The model considered by Prucha and Nadiri is discussed in detail in section 4.3.2. The restricted cost function for that model is given by equation (60), which is a linear-quadratic function in p^L, q^K, q^R, K_{-1}/Y^p, R_{-1}/Y^p, $\Delta K/Y^p$, $\Delta R/Y^p$, T, multiplied by Y^p, where we have maintained the notation of section 4.3.2. In contrast, the restricted cost function of the true model underlying this Monte Carlo study is linear quadratic in p^L, q^K, q^R, K_{-1}, R_{-1}, ΔK, ΔR, Y, T, which allows for an explicit analytic solution even under non-static expectations. In the following, we use the abbreviations LQR (short for "linear quadratic in ratios") and LQ (short for "linear quadratic") to denote the former and latter restricted cost function. The true model assumes that prices and output are generated by simple first order autoregressive processes and takes expectation to be rational (and thus nonstatic). Explicit expressions for the demand equations for the labor, materials, capital, and R&D of the true model and the equations for the forcing variables are given in appendix C in Nadiri and Prucha (1999).

The selection of the true model parameters for the Monte Carlo study was guided by fitting a static version of the model to the U.S. electrical machinery data used in the Prucha and Nadiri study, and by estimating first order autoregressive processes for the forcing variables from those data. The aim was to select the true model parameters such that the generated data exhibited properties consistent with those found in the study by Prucha and Nadiri. The selection of the variance and covariances of the disturbance processes was also guided by those empirical results, as well as by computational considerations to keep the computing time within practical limits. In analogy to the study by Prucha and Nadiri, the data were generated for the period 1960 to 1980, with the initial values taken from the data set for that study. Each Monte Carlo experiment consisted of 100 trials.[39]

The R^2 values (calculated as the squared correlation coefficient between the actual variables and their fitted values calculated from the reduced form based on true parameter values) for the factor demand equations were approximately 0.98; those for the forcing variables ranged from 0.93 to 0.85. Output-based technical change, λ_Y, and scale, ρ, are computed from equation (15). Their values depend on the input and output mix. The median value of λ_Y, computed from the Monte Carlo sample, corresponding to the true parameter values decreased in a smooth pattern from 1.53 in 1961 to 1.00 in 1976 and 0.94 in 1980. The median value of ρ corresponding to the true parameter values was 1.09 in 1961, 1.11 in 1976, and 1.12 in 1980. The true accelerator coefficients m_{KK}, m_{KR}, m_{RK}, and m_{RR} take

39. The number of Monte Carlo trials is small, but is reflective of the considerable computational complexities underlying this study.

Table 4.2 Description of Monte Carlo Experiments

	Basic Characteristics of Estimated Model			
Number	Cost Function	Expectation	Returns to Scale	Adjustment Costs
1A, 1B	LQ	Rational	Nonconstant	Nonzero
2A, 2B	LQ	Static	Nonconstant	Nonzero
3A, 3B	LQ	Static[a]	Nonconstant	Zero
4A, 4B	LQR	Static	Nonconstant	Nonzero
5A, 5B	LQR	Static	Constant	Nonzero

[a] Expectations do not come into play, since the adjustment costs are zero.

the values 0.22, −0.02, −0.01, 0.15. The true depreciation rate δ^K was for simplicity taken to be constant and assumed to be 0.038.

Table 4.2 gives a description of the respective Monte Carlo experiments. The first experiment reestimates the true model from the generated data. As discussed above, the true model is based on the LQ restricted cost function, takes expectations to be rational, allows for nonconstant returns to scale and for nonzero adjustment costs; the equations for the true model are given in appendix C in Nadiri and Prucha (1999). In our second experiment, we estimate the same model, except that expectations are misspecified in that they are taken to be static. The third experiment estimates again the same model, but imposes zero adjustment costs (and thus imposes incorrectly $m_{KK} = m_{RR} = 1$). Of course, with zero adjustment costs expectations do not come into play. In experiment four, we then misspecify the functional form of the restricted cost function. More specifically, we estimate the model discussed in section 4.3.2 based on the restricted cost function LQR. We also take expectations to be static. Experiment five is as experiment four, except that here also scale is incorrectly assumed to be equal to unity. For each of the experiments we run two variants. Variant "A" takes the stock of capital (or equivalently, the depreciation rate of capital) as observed. Variant "B" takes the stock of capital as unobserved and estimates the (constant) depreciation rate of capital δ^K jointly with the other model parameters. As an estimation procedure, we use 3SLS with lagged inputs, output, prices, and squares of those lagged values as instruments. The sample period is 1961 to 1980. The study was performed using TSP 4.4.

In tables 4.3 and 4.4 we report, respectively, on the estimation results obtained from the Monte Carlo experiments corresponding to variants A and B of the experiments. Rather than to report on all parameter estimates, we focus on estimates of the adjustment coefficients, and the parameters determining those coefficients, and on estimates of technical change λ_Y and scale ρ. As in the Prucha and Nadiri study, we report estimates for λ_Y and ρ in 1976. The estimated values in tables 4.3 and 4.4 are Monte

Table 4.3 **Estimates of Model Parameters, Technical Change, and Scale (capital stock observed)**

		Estimates for Monte Carlo Experiment Number				
Parameter	True Value	1A	2A	3A	4A	5A
α_{KK}	0.420	0.472	0.314	0.598	1.151	2.356
		(0.214)	(0.298)	(1.703)	(1.457)	(5.941)
α_{KR}	−0.066	−0.096	−0.310	−0.336	−0.442	−1.232
		(0.097)	(0.091)	(1.314)	(0.839)	(3.338)
α_{RR}	0.376	0.391	0.338	0.585	0.357	0.921
		(0.138)	(0.177)	(1.534)	(0.632)	(1.987)
$\alpha_{\dot{K}\dot{K}}$	5.616	5.487	4.337	0.0	8.655	8.134
		(2.323)	(2.978)		(4.842)	(4.573)
$\alpha_{\dot{R}\dot{R}}$	10.98	12.52	13.05	0.0	8.645	10.27
		(6.487)	(4.870)		(7.204)	(10.37)
m_{KK}	0.217	0.226	0.217	1.0	0.268	0.332
		(0.050)	(0.058)		(0.093)	(0.218)
m_{KR}	−0.021	−0.029	−0.012	0.0	−0.060	−0.183
		(0.031)	(0.043)		(0.078)	(0.161)
m_{RK}	−0.011	−0.013	−0.004	0.0	−0.066	−0.131
		(0.013)	(0.012)		(0.164)	(0.180)
m_{RR}	0.147	0.144	0.125	1.0	0.134	0.119
		(0.024)	(0.034)		(0.177)	(0.199)
λ_Y	1.000	0.968	0.604	1.296	0.503	1.216
	(0.167)	(0.287)	(0.490)	(0.217)	(0.454)	(0.323)
Scale	1.110	1.136	1.212	1.025	1.341	1.0
	(0.084)	(0.121)	(0.194)	(0.115)	(0.152)	

Carlo medians. The second column contains the true values for comparison. As a measure of spread of the respective estimates, we report in parenthesis their interquantile ranges.[40] In table 4.4 we also present estimates for the depreciation rate of capital δ^K.

The estimates based on the true model, which are reported under experiments 1A and 1B in tables 4.3 and 4.4 are, in general, close to the true values. We note that the interquantile ranges of the estimates are generally smaller for experiment 1A than for experiment 1B, reflecting the fact that in the latter experiment also δ^K is being estimated in addition to the other model parameters. It is also interesting to note that the interquantile ranges for the estimates of the adjustment cost coefficients $\alpha_{\dot{K}\dot{K}}$ and $\alpha_{\dot{R}\dot{R}}$ are comparatively large. This observation is consistent with a similar finding

40. Since λ_Y and ρ depend on the input and output mix, their values vary in respective Monte Carlo trials even if evaluated at the true parameter values. It is for that reason that we also report an inter-quantile range for the true values of λ_Y and ρ. The variability of the parameter estimates reflects the small sample size and the assumptions on the variances of the disturbance processes.

Table 4.4 Estimates of Model Parameters, Technical Change, and Scale (capital stock unobserved; capital depreciation rate estimated)

Parameter	True Value	Estimates for Monte Carlo Experiment Number				
		1B	2B	3B	4B	5B
α_{KK}	0.419	0.469	0.218	1.350	0.414	0.774
		(0.422)	(0.466)	(5.003)	(1.151)	(1.129)
α_{KR}	−0.066	−0.077	0.020	−0.035	−0.194	−1.596
		(0.149)	(0.134)	(0.955)	(0.587)	(0.898)
α_{RR}	0.376	0.414	0.362	0.375	0.408	0.477
		(0.178)	(0.214)	(0.614)	(0.719)	(1.001)
$\alpha_{\dot{K}\dot{K}}$	5.616	5.403	4.862	0.0	9.581	6.465
		(3.710)	(2.923)		(9.299)	(5.594)
$\alpha_{\dot{R}\dot{R}}$	10.98	12.84	13.68	0.0	12.26	13.09
		(7.048)	(5.796)		(5.541)	(7.689)
m_{KK}	0.217	0.224	0.194	1.0	0.165	0.246
		(0.110)	(0.171)		(0.249)	(0.157)
m_{KR}	−0.021	−0.024	0.010	0.0	−0.051	−0.164
		(0.038)	(0.058)		(0.087)	(0.147)
m_{RK}	−0.011	−0.010	0.004	0.0	−0.037	−0.067
		(0.018)	(0.020)		(0.078)	(0.092)
m_{RR}	0.147	0.141	0.121	1.0	0.119	0.090
		(0.023)	(0.043)		(0.127)	(0.111)
λ_Y	1.000	0.960	0.552	1.664	−0.003	0.816
	(0.167)	(0.405)	(0.688)	(0.823)	(1.872)	(0.982)
Scale	1.110	1.128	1.213	0.825	1.390	1.0
	(0.084)	(0.161)	(0.244)	(1.037)	(0.266)	
δ_K	0.038	0.036	0.033	0.117	0.004	0.015
		(0.018)	(0.039)	(0.070)	(0.058)	(0.037)

in an earlier Monte Carlo study by Prucha and Nadiri (1986). In experiments 2A and 2B expectations are misspecified as being static. The effect of this misspecification is to substantially decrease the estimates of technical change to 0.60 and 0.55, respectively, as compared to a true value of 1.00, and to increase the estimates of scale to 1.21, as compared to a true value of 1.11. If the model is further misspecified by assuming that adjustment costs are zero, the estimates for technical change increase to 1.30 and 1.66, as reported under experiments 3A and 3B. Scale falls to 1.02 and 0.82, respectively. This type of misspecification also has a considerable effect on the estimate of δ^K. The median estimate is 0.11 as compared to a true value of 0.038.

Misspecifying the functional form of the restricted cost function in terms of equation (60), and assuming static expectations, results in estimates of technical change of 0.5 and 0, as reported under experiments 4A and 4B, respectively. The estimate for scale increases to 1.30 and 1.39, respectively. The estimates of the accelerator coefficients and the depreciation rate of capital are also fairly sensitive to this form of misspecification. Imposing constant returns to scale, as in experiments 5A and 5B, results in less bias in the technical change estimates, and in estimates of m_{KK} and m_{RR} that are higher and lower than the true values. There is also substantial downward bias in the estimates of the depreciation rate of capital. These Monte Carlo results suggest that the estimates of model parameters and model characteristics may be quite sensitive to misspecification of the functional form, especially since the functional form misspecification imposed in this study may be considered as modest in that equation (60) can be viewed as a second-order approximation of the true restricted cost function.

4.6 Concluding Remarks

In this paper we have discussed some recent advances in modeling and in the estimation of dynamic factor demand, and have argued that this approach provides a powerful framework to analyze the determinants of the production structure, factor demand, and technical change. The basic message of this paper can be summarized briefly. The conventional index number approach will measure the rate of technical change correctly if certain assumptions about the underlying technology of the firm and output and input markets hold. Furthermore, the conventional index number approach is appealing in that it can be easily implemented. However, if the underlying assumptions do not hold, then the conventional index number approach will, in general, yield biased estimates of technical change.

The index number approach also does not provide detailed insight into the dynamics of the production process and the determinants of factor demand and factor accumulation. The dynamic factor demand modeling approach reviewed in this paper provides a general framework to estimate the structure of the underlying technology and to relate the investment decisions and variation of technical change. Of course, in this approach there is, as in any other econometric investigation, the danger of misspecification. However, the basic appeal of this modeling strategy is its flexibility, that enables it to incorporate and analyze in a consistent framework both theoretical considerations and institutional factors that influence technical change, and to test various hypothesis concerning the specification of the technology and the optimizing behavior of the firm.

The dynamic factor demand modeling framework described in this paper enables us to examine a number of issues of both basic research and policy interest. Using the model it is possible to identify the possible biases in the conventional measure of total factor productivity growth. These biases can result from scale effects, the difference between marginal products and long-run factor rental prices in temporary equilibrium due to adjustment costs, the direct effect of adjustment costs as they influence output growth, and the selection of the depreciation rate by the firm. The model presented in this paper can be used to estimate the structure of the underlying technology and to specify the magnitudes of these biases if they are present. If the biases are not isolated, relying on the conventional TFP measure will, for example, overestimate technical change in the presence of increasing returns to scale and positive output growth.

The model also provides an analytical framework for estimating the response of input demands to changes in relative prices, exogenous technical change, and other exogenous variables that may shift the production or cost function. Since a clear distinction is drawn between variable and quasi-fixed inputs due to the presence of adjustment costs, the short-, intermediate-, and long-run responses of output, factors of production, and productivity growth can be estimated. (Of course, the approach does not impose the existence of adjustment costs and quasi-fixity, but rather leaves that to be determined empirically.) The class of models reviewed also allows for non-static expectations and nonseparability among the quasi-fixed factors of production. It is therefore possible to estimate possible substitution or complementaries in the short, intermediate, and long runs among various types of capital such as physical, R&D, and human capital.

It is also possible to formulate and estimate an appropriate measure of capacity utilization consistent with the underlying production technology. Moreover, the model allows for the decision on depreciation rates of various quasi-fixed factors of production such as physical and R&D capital to be endogenous. We note, however, that models in which the depreciation rate is constant are included as special cases. The framework thus allows for the econometric testing of the constancy hypothesis. Estimating the depreciation rates permits generating consistent capital stock series which may differ from the official estimates. It also allows the decomposition of gross investments of various types of capital into the net and replacement investments. The time profiles of these two types of investment have important analytical and policy implications.

To illustrate the workings of the model, we have discussed briefly some empirical results from several studies based on the dynamic factor demand model. These examples indicate how it is possible to account for the influences of scale, relative price movements, the rate of innovation due to R&D efforts and R&D spillovers, and financial decisions concerning the level of debt and dividend payouts on the production structure and technical change. The overall conclusion reached from these examples is that the econometric modeling approach allows us to identify the contribution of a complex and often competing set of forces that shape productivity growth, and to test their significance statistically. Generally speaking, the empirical results based on dynamic factor demand models suggest that the estimated rate of technological change is often much smaller and smoother than the conventionally measured total factor productivity growth. Also, there is evidence of substantial degree of interrelatedness embedded in the production process of the firm that could not be captured using simple formulations of the firm's technology. The evidence from several studies suggests that some factors of production such as physical and R&D capital are quasi-fixed in the short run. Also, there is evidence that economies of scale characterizes the production process in some industries and that the elasticity of factor substitution is often much smaller than unity. Investment in R&D is an integral part of the production structure and often significantly contributes to a reduction in cost. In addition, R&D spillovers among firms, industries, and economies often reduce the cost of production of the recipient. Dynamic factor demand models also permit studying the production and financial decisions of the firm in a consistent framework and to analyze the effect of taxes and other exogenous policy instruments on these decisions.

To illustrate how estimates of important characteristics of the production process can be affected by various forms of misspecification, a Monte Carlo study was undertaken. The results suggest, in particular, that estimates of the rate of technical change are sensitive to misspecification of the expectation formation process, to misspecification regarding whether or not the firms is in temporary or long-run equilibrium, and to misspecifications of the functional form of the cost/production function including scale. The exhibited sensitivity of technical change (and other model characteristics) to misspecification suggests that adopting simple specifications for reasons of convenience may result in serious estimation biases. This points to the importance of specification testing in the estimation of the cost/production functions and derived factor demands. Dynamic factor demand models provide a general framework for carrying out specification tests, and yield important insights in the complexity of the production decisions.

However, estimation of dynamic factor demand models is often challenging. These models are often complex and the estimation of these models requires considerable effort. Nonetheless, in order to measure technical change properly and to capture the dynamics of the adjustment of factor demands, and to analyze effects of relative prices and other exogenous variables such as taxes, subsidies, R&D spillovers, and so on, on factor demand and productivity growth, the dynamic factor demand modeling framework presented in this paper is an important tool of analysis.

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Comment Dale W. Jorgenson

This very ambitious and stimulating paper is organized around the concept of dynamic factor demand models. Quasi-fixed factors are characterized by internal costs of adjustment. The production possibility frontier depends on outputs and inputs, technology, economies of scale, and rates of change of the quasi-fixed factors.

Section 4.2 of the paper connects most directly with the topic of the conference, new developments in productivity analysis. The key result is contained in section 4.2.3. This productivity measure is given in continuous time in equation (39) and discrete time in equation (43).

Equation (39) decomposes the growth rate of productivity in a model that does not maintain the standard assumptions. These are constant returns to scale, competitive markets for inputs and outputs, and no internal costs of adjustment. Terms in the decomposition correspond to scale effects, deviations from marginal cost pricing, adjustment costs, and the effects of changes in the quasi-fixed factors.

The empirical issue is how to measure the effects of departures from the standard assumptions. The authors' proposal is to specify and fit a dy-

Dale W. Jorgenson is the Frederic E. Abbe Professor of Economics at Harvard University.

namic factor demand model. It is significant that there is no empirical example of the full implementation of this proposal. Nonetheless, it is very valuable to have a well-specified alternative to the production model that underlies the productivity measures generated by official statistical programs.

The production model that underlies the Törnqvist index of productivity used in official productivity measurements also gives rise to an econometric model of factor demand. As Nadiri and Prucha point out, this model is much more restrictive and could be tested within their dynamic factor demand model. However, there is an important issue of research strategy here. Is it best to relax all the assumptions of the standard model at once, while limiting the empirical analysis to time series data on outputs and inputs and their prices?

Let me discuss one example: namely, modeling economies of scale. This is one of the most fruitful areas for econometric modeling of production. The most satisfactory approach has been to study economies of scale in isolation from other departures from the standard assumptions. Cross-sectional and panel data, especially for regulated industries, have been modeled extensively and reported in the econometric literature.

Intercity telecommunications and electricity generation industries are characterized by increasing returns to scale, whereas transportation industries are characterized by constant returns. As a consequence of these findings, transportation has been largely deregulated. Regulatory reform in electricity generation and telecommunications has limited regulation to areas where economies of scale are significant.

Section 4.3 is the core of the paper and presents a framework that encompasses all the features of production enumerated in section 4.2. Section 4.3.1 considers minimization of the firm's expected present value of cost, subject to the production possibility frontier. This leads to an Euler equation for modeling the dynamics of factor demand.

A more restrictive dynamic factor demand model, based on a linear quadratic specification, is given in equation (27). This model is characterized by certainty equivalence and takes the form of a system of linear difference equations. This idea has been present in the economic literature for several decades and in the engineering literature for even longer.

Section 4.5 of the paper presents a Monte Carlo study of the effects of misspecification on measures of productivity and economies of scale. This compares the linear quadratic model shown in equation (16) and employed by Prucha and Nadiri (1996) with a "true" linear quadratic model that can be solved analytically under rational expectations. Table 4.3 shows that the departures from the assumptions of the true model have sizable impacts on measured TFP and economies of scale.

Section 4.4 of the paper summarizes empirical applications of dynamic factor demand models. The description of these models is necessarily brief, but I was unable to find evidence of a successful empirical application of the rational expectations approach featured in section 4.3. Nonetheless, the list of topics covered is impressive:

1. Tax incentives, financing, and technical change (section 4.4.1)
2. R&D investment, production structure, and TFP decomposition (section 4.4.2)
3. Technological spillovers and productivity growth (section 4.4.3)
4. Capacity utilization, depreciation rates, and replacement investment (section 4.4.4)

The conclusions are worth summarizing:

First, when taxes are evaluated in terms of their effectiveness, measured as investment expenditures per government revenue loss, specific tax instruments are substantially more effective than is the corporate income tax rate. This conclusion is familiar to readers of the literature on tax policy and investment behavior. However, the methodology of choice in this area of policy analysis is general equilibrium modeling, which leads to measures of the impact of policy changes on consumer welfare.

Second, emphasizing economies of scale, Nadiri and Prucha have modeled production in the U.S. Bell System. They find that economies of scale accounted for more than 80 percent of the impressive productivity growth in the Bell System, suggesting a serious bias in conventional measures of TFP. This was a part of the unsuccessful defense of the Bell System against the breakup that resulted from the Department of Justice antitrust case.

Third, studies of R&D investment have produced evidence of spillovers, defined as effects of investment by one firm, industry, or country on the productivity of other firms, industries, or countries. This literature is surveyed by Griliches and Nadiri. The predominant methodology has been the estimation of cross-sectional production functions, which was pioneered by Griliches and is the subject of its own very substantial literature.

Fourth, the final applications are to measures of depreciation rates from investment in fixed assets and R&D. This has been far less influential than research focusing on used asset prices. The studies of Hulten and Wykoff have provided the basis for the recent revision of the capital accounts that underly the U.S. income and product accounts. This is described by Fraumeni (1997).

What can one claim for the empirical literature on dynamic factor demand modeling? This methodology has generated an impressive literature on a wide range of issues. However, in each of the areas reviewed in section 4.4, competing methodologies isolate one of the issues. This has been a more successful research strategy and one that has had a major impact on official productivity measures and the underlying national accounting magnitudes, as well as on economic policy.

In concluding, it is important to emphasize that Nadiri and Prucha have very forcefully reminded us how much remains to be done. For example, an important challenge for empirical economists is to develop satisfactory measures of R&D outputs as opposed to the inputs. These will be required for any definitive assessment of the prospects for endogenizing productivity growth through R&D investment and spillovers across producing units, as Nadiri and Prucha have suggested.

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Reply M. Ishaq Nadiri and Ingmar R. Prucha

We thank Professor Jorgenson for his stimulating comments. In his own work Jorgenson has evidently not embraced dynamic factor demand models, which are the subject of our review. This sets the stage for a discussion of important issues of research strategy. Not surprisingly there is some disagreement. To respond to some of the questions raised by Jorgenson's comments, and to reemphasize the contributions of the vast dynamic factor demand literature surveyed in our paper, we focus our reply on three sets of issues related to (a) the treatment of the quasi fixity of some inputs, (b) estimation strategies, and (c) comments on specific empirical studies.

First, as documented in our paper, the dynamic factor demand modeling approach has attracted many eminent scholars and has generated a voluminous literature in the past three decades. We emphasize that the approach builds on seminal contributions by other eminent economists. In particular, the dynamic factor demand literature builds on Jorgenson's neoclassical theory of investment and production. By introducing (internal or external) adjustment costs explicitly into the firm's decision-making process, dynamic factor demand models yield optimal factor demands not only in the long run, but also in the short and intermediate run. The introduction of adjustment costs is seen by many as a natural extension of the neoclassical theory of investment and production that permits a consistent modeling framework for *both* temporary and long-run equilibrium. As such, dynamic factor demand models provide a formal framework for tracing the evolution of investment and productivity growth over the short, intermediate, and long run.

The major methodological difference between the modeling approach favored by Jorgenson and the dynamic factor demand modeling approach is the latter's incorporation of adjustment costs to explicitly account—within the firm's decision-making process—for the widely documented quasi fixity of some inputs, such as the physical capital stock. The quasi fixity of capital inputs was in fact recognized in Jorgenson's own empirical study of investment expenditures some thirty years ago via the specification of an accelerator model. We note that dynamic factor demand models provide a formal economic justification for accelerator models of investment. Apart from the treatment of adjustment costs, both approaches are similar: In empirical applications both have specified the production technology in a similar general fashion using flexible functional forms, and both have considered data sets of similar levels of aggregation. As is discussed in our paper, the use of flexible functional forms was pioneered by Diewert, Jorgenson, and Lau in the early seventies.

It is also worth pointing out that the dynamic factor demand literature has adopted various modeling approaches ranging from linear quadratic specifications with an explicit solution for variable and quasi-fixed factor demands, to quadratic and nonlinear quadratic specifications in which the demand for the quasi-fixed factors is only described in terms of the Euler equations, to specifications in which only the variable factor demand equations are used for estimation. Static equilibrium models are, of course, contained as a special case. In developing methodologies that cover both complex and simple specifications, the dynamic factor demand literature presents a menu of flexible modeling options to the empirical researcher. The development of methodologies for complex specifications should be interpreted not as a prescription but as an option that can be selected when such a choice is indicated empirically. Concerning the question whether complex specifications have been implemented successfully, we point out, as an example, the decomposition of the conventional measure of TFP in the U.S. electrical machinery industry given in table 4.1. This decomposition reports on all forms of possible biases considered in equations (39) and (43).

Second, Jorgenson also raises an important issue of research strategy, which is closely related to the points made above. In particular, he questions whether it is best to relax all the assumptions of the standard model at once. As remarked above, the dynamic factor demand literature considers both simple and complex specifications, and we agree that a full-fledged dynamic factor demand model may not be suitable in all situations. However, we do advocate that specification tests are necessary prior to imposing potentially restrictive assumptions such as constant returns to scale, zero adjustment costs, competitive input and output markets, and so on. Assuming away the complexity of the underlying production process may result—as is shown by many studies and illustrated by the Monte Carlo study in our paper—in substantial mismeasurement of the determinants of factor demand, technical change, and other characteristics of the production process. This mismeasurement is then likely to affect significantly the research findings in several areas of economic studies; for example, industrial organization, income distribution, business cycle analysis, and growth modeling. For example, in the recent endogenous growth theory, the form of the production function is typically assumed to be Cobb-Douglas or AK. If the underlying production function is more complex, the results of these models are likely to be quite different. In such cases, the potential loss due to misspecifications may substantially exceed the costs arising from the complexity of the analysis. Flexible functional forms, such as the translog production and cost functions developed by Christensen, Jorgenson, and Lau provide important protections against potential misspecifications. We believe that for analogous reasons it is also important to allow for the possibility of quasi fixity in some of the inputs.

As remarked earlier, one way of taking into account the quasi fixity of some inputs is to base the estimation only on the demand equations for the variable factors (derived, e.g., from a restricted cost or profit function). This approach allows for the consistent estimation of the "technology" parameters but does not provide full insight into the dynamics of the production process. Whether this approach is appropriate—abstracting from questions of efficiency—depends on whether the dynamics of the production process is a focus of the investigation.

We also note that dynamic factor demand models can be estimated from panel data subject to the usual cautions in the pooling of data. Of course, those cautions also apply to static factor demand models.

Third, Jorgenson also makes several comments on specific empirical studies. He points to alternative approaches toward the measurement of depreciation rates, to evaluate and measure the effects of tax policy, spillovers, and economies of scale. We disagree that those alternative approaches represent more successful research strategies. In a nutshell, we see the various methodologies mentioned by Jorgenson and those reviewed in our paper as complementary approaches and not as substitutes. For example:

(a) We fully agree that Hulten and Wykoff have made seminal contributions to the measurement of depreciation rates using prices of used assets. Unfortunately, however, these types of data are not available for many sectors of the economy and are limited in coverage. We hence believe there is room for various approaches. In fact, one can view Hulten and Wykoff's approach as modeling the demand side of used assets, whereas the approach discussed in our review, which dates back to Hicks, Malinvaud, and Diewert, models the supply side. It seems interesting to try to combine the two approaches in future research. Also, the dynamic factor demand literature has generated an extensive literature on capital and capacity utilization, which are allowed to affect the depreciation rate of capital. Thus, this literature speaks not only to the magnitude of the depreciation rate, but also to wider related issues. Also the dynamic factor demand models have been used to estimate the depreciation rate of R&D, which was otherwise typically assumed as given, for example, in the literature referred to in the comments.

(b) General equilibrium models are certainly useful for evaluating the effects of changes in tax policies. Obviously, one of the points of contention in putting together a general equilibrium model for tax simulations is the choice of appropriate parameter values. (One of the major sessions of the recent world congress of the econometric society was devoted to the contentious issue of calibration versus classical estimation. In one view calibration is an estimation method, though typically with unknown statistical properties.) In estimating a dynamic factor demand model we obtain estimates of important parameters using classical econometric techniques with known statistical properties.

(c) Empirical estimation of the degree of scale at the firm, industry, and even aggregate economy level is critical for an understanding of economic growth. A number of models have successfully examined the degree of scale in various sectors and industries. The focus of our study of the Bell system was to understand the dynamics of the capital and R&D investment process as well as the nature of the production frontier faced by the Bell system, including the magnitudes of returns to scale and exogenous technical change. The study was undertaken after the Bell breakup. It was undertaken to study "what is," and not to argue either on the side of the government or on that of the Bell system. Moreover, that dynamic factor demand models have generally been estimated using time series data at the industry level is due to the paucity of specific output and factor prices at the firm level.

(d) Dynamic factor demand models provide a general dynamic framework for studying the contributions of physical capital and R&D and the spillover effects of R&D among sectors and economies. This literature has provided explicit estimates of the magnitude of the R&D spillovers and their contribution to the growth of output and productivity; the convergence of growth rates among industries and economies; and the evolution of productivity growth over the short, intermediate, and long run. It is true, as Jorgenson has pointed out, that modeling spillover effects poses a substantial challenge. We believe that the dynamic factor demand modeling framework can serve as a starting point to understand this dynamic phenomenon.

In conclusion we note again that the dynamic factor demand literature builds, in particular, on Jorgenson's seminal contributions to the neoclassical theory of investment and production. The static equilibrium model, which is the underlying foundation of the conventional measure of TFP, is a special case of the dynamic factor demand model. We argue that imposing a priori restriction on the production structure for the sake of simplicity can seriously bias estimates of productivity growth and can lead to a misdiagnosis of the sources of economic growth, among other problems. The Monte Carlo results reported in our paper clearly substantiate this phenomenon. Hence, specification tests are essential as a justification for imposing potentially restrictive assumptions on the form of the production structure and its dynamic evolution. Dynamic factor demand models provide a framework for such tests and furthermore serve as the basis of a general approach to estimating the rate of technical changes and the dynamics of productivity growth.