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Volume Title: R & D, Patents, and Productivity

Volume Author/Editor: Zvi Griliches, ed.

Volume Publisher: University of Chicago Press

Volume ISBN: 0-226-30884-7

Volume URL: http://www.nber.org/books/gril84-1

Publication Date: 1984

Chapter Title: R&D and the Market Value of the Firm: A Note

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Chapter URL: http://www.nber.org/chapters/c10054

Chapter pages in book: (p. 261 - 270)

13 R & D and the Market Value of the Firm: A Note

Andrew B. Abel

In his paper, Ariel Pakes (1981) uses stock market valuation data to study the relation between patents and the economic value of inventive activity. The econometric technique is based on dynamic factor analysis (Geweke 1977) and index models (Sargent and Sims 1977). Although these techniques often involve the frequency domain, Pakes's model can be estimated in the time domain and thus leads to more intuitively interpretable results.

Pakes's model is based on an intertemporally optimizing firm which chooses a research and development program to maximize the expected present value of its net cash flow. One approach to the problem would be to specify the net cash flow function and solve for the optimal decision rules. Pakes shuns this approach because it does not relate changes in research activity to the expected present value of cash flow. Instead, he uses the stock market to evaluate the expected present value of cash flow and associates changes in a firm's market value with the value of new research. This choice between explicitly solving a dynamic optimization problem, on the one hand, and using stock market valuation to determine the level of investment in new capital, or R & D, on the other hand, reflects a dichotomy in the investment literature between structural models and market valuation models based on Tobin's q (1969). Recent research has established a link between these approaches, and, after

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The author thanks Zvi Griliches and Ariel Pakes for helpful comments on an earlier draft.

This chapter originated as a comment on the paper presented by Ariel Pakes at the conference. References to footnotes and equation numbers are to those in Pakes (1981) which was presented at the conference.

commenting briefly on Pakes's paper, I will present a simple model of R & D illustrating this link.

Pakes defines q_t to be the excess one-period rate of return on a firm's equity and relates q_t to R & D activity. One must proceed with caution in associating changes in the value of a firm with the value of R & D activity. The value of the firm can change for a variety of reasons other than successful research activity. The stock market values the firm as an ongoing concern and reflects the value of all the firm's capital assets, both tangible and intangible. Even in the absence of any changes in the level or composition of the capital stock, the value of the firm may change because of changes in demand or in the supply of variable factors of production. Even ignoring these sources of variation in the valuation of the firm, the value of the firm may increase by less than the present value of quasi-rents associated with new R & D if the new R & D makes obsolete some previously valued process (see Bailey 1981). It should be pointed out that Pakes is well aware of the implications of these points. Indeed, he calculates that 95 percent of the variance in q_t is unrelated to either R & D or patents.

The formal tests in the paper are based on the assumed stochastic structure of the innovations a_t, b_t, and g_t. Since Pakes assumes a form for the value function in his equation (2) rather than deriving it from an optimization problem, the interpretation of the shock B_t in this equation is unclear. One interpretation of B_t might be that for some reason (either attitude toward risk or systematic bias) the stock market places a value B_t on an uncertain cash flow with expected present value equal to one. In this case B_t should multiply the expected present value of future cash flows but should not multiply the known current cash flow, R_t. If equation (2) is modified as suggested here, then the optimal value of R_t would not be independent of B_t, and this would have implications for the set of testable restrictions.

Since all of the empirical work is based on the time-series behavior of the trivariate process generating (q_t, r_t, p_t), the timing of these variables is extremely important. As explained in Pakes's footnote 13, q_t refers to the stock market rate of return from date $t-1$ to date t, whereas r_t and p_t refer to R & D and patents from date t to date $t+1$. Letting Ω_{t-1} denote the set of information available at date $t-1$, market efficiency implies $E(q_t | \Omega_{t-1}) = 0$. Pakes assumes that r_{t-1} and p_{t-1} are contained in Ω_{t-1} and tests the implication that $E(q_t | r_{t-1}, p_{t-1}) = 0$. However, the assumption that p_{t-1} is in Ω_{t-1} means that investors know at date $t-1$ the number of successful patent applications between date $t-1$ and date t. Since successful patent applications are not a decision variable of the firm (as is r_{t-1}), but are rather the outcome of a process with a variable lag and an uncertain outcome, the assumption that p_{t-1} is in Ω_{t-1} is quite strong.

That is, the restriction $E(q_t | r_{t-1}, p_{t-1}) = 0$ is stronger than implied by market efficiency. Nonetheless, the data fail to reject this restriction.

As a final direct comment on Pakes's paper, I would point out that patents are essentially a sideshow in Pakes's theoretical model. As Pakes observes, patents must have some economic value since resources are expended to acquire them. However, patents do not enter into the value function for the firm. In fairness to Pakes, it must be noted that his research strategy at this stage is merely to determine whether the newly available patent data are related to anything that might be called inventive output. This appears to be a reasonable strategy at this early stage to search the data for correlations and then at a later stage to impose the discipline of a structural model.

A Model of R & D and the Value of the Firm

I will devote the remainder of this chapter to the development of a simple model of the R & D activity and valuation of a firm. Although the model presented below is probably too simple to use directly for empirical work, it does provide an explicit optimizing framework with which to analyze optimal R & D activity and firm value. Furthermore, the stochastic elements are incorporated directly into the firm's optimization problem.

We consider the production and R & D decisions of an intertemporally optimizing, risk-neutral firm. Suppose that the firm uses labor, L_t, and accumulated technology, T_t, to produce output, Q_t, according to the Cobb-Douglas production function $Q_t = L_t^{\alpha} T_t^{1-\alpha}$. Labor is hired in a spot market at a fixed wage rate, w. Technology accumulates over time as a result of the firm's R & D activity, R_t, and can decumulate over time from obsolescence. We specify the technology accumulation equation as

$$T_{t+1} = \eta_{1t+1} R_t + \eta_{2t+1} T_t, \tag{1}$$

where

$$E(\eta_{1t}) = 1 \text{ and } 0 < E(\eta_{2t}) = \delta < 1,$$

where η_{1t} and η_{2t} are each serially uncorrelated random variables. In equation (1) we explicitly recognize that the outcome of R & D activity is uncertain by including the random variable η_{1t}. The assumption that $E(\eta_{1t}) = 1$ is simply a convenient normalization. The random variable η_{2t} reflects the fact that obsolescence also occurs randomly. The assumption that $E(\eta_{2t}) = \delta$ implies that, in the absence of R & D activity, the expected proportional rate of "depreciation" of technology is $1 - \delta$. We assume that η_{2t} takes on only positive values.

We have defined R_t as R & D activity which, according to (1), is the expected gross addition to the stock of technology. We assume that the marginal cost of R_t is an increasing function of R_t. Specifically, it will be convenient to model R & D expenditures as a quadratic function of R & D activity, aR_t^2. Finally, we allow the price of output, p_t, to be random. Specifically, we suppose

$$p_{t+1}/p_t = \epsilon^*_{t+1}, \tag{2}$$

where ϵ^*_{t+1} is serially independent. The random term ϵ^*_{t+1}, which is equal to one plus the rate of inflation, is assumed to take on only positive values.

The firm's decision problem at time t is to maximize the expected present value of its net cash flow. Assuming that the discount rate r is constant, and defining the discount factor $\beta = 1/(1+r)$, we can write the decision problem as

$$\max_{L_t, R_t} E_t \sum_{j=0}^{\infty} \beta^j (p_{t+j} L^a_{t+j} T^{1-a}_{t+j} - wL_{t+j} - aR^2_{t+j}), \tag{3}$$

subject to the technology accumulation equation (1), the price process (2), and the initial condition that T_t is given. Let $V(T_t, p_t)$ denote the maximized value of the present value of expected cash flow in (3), and observe that under risk neutrality $V(T_t, p_t)$ is the value of the firm. The value of the firm is a function of only p_t and T_t, since p_t is a sufficient statistic for the history of price shocks, ϵ^*_{t-i}, and T_t is a sufficient statistic for the history of shocks η_{1t-i} and η_{2t-i}.

We will solve the maximization problem in (3) using stochastic dynamic programming. The equation of optimality, known as the Bellman equation, is

$$V(T_t, p_t) = \max_{L_t, R_t} E_t [p_t L_t^a T_t^{1-a} - wL_t - aR_t^2 + \beta V(T_{t+1}, p_{t+1})]. \tag{4}$$

The Bellman equation merely states that the value of the firm is the maximized sum of the current cash flow plus the expected present value of the firm next period.

Since the labor input, L_t, affects only current cash flow, we can easily "maximize it out" of the equation (4). Recognizing that the optimal value of L_t equates the marginal revenue product of labor, $ap_t(L_t/T_t)^{a-1}$, with the wage rate, w, we obtain

$$\max_{L_t} (p_t L_t^a T_t^{1-a} - wL_t) = h p_t^{\frac{1}{1-a}} T_t, \tag{5}$$

where

$$h = (1-a)\left(\frac{a}{w}\right)^{\frac{a}{1-a}}$$

Substituting (2), (3), and (5) into (4), we obtain

$$(6)\quad V(T_t,p_t) = \max_{R_t} E_t\left[hp_t^{\frac{1}{1-a}}T_t - aR_t^2 + \beta V(\eta_{1t+1}R_t + \eta_{2t+1}T_t, p_t\epsilon^*_{t+1})\right].$$

Equation (6) is a functional equation which must be solved for the value function $V(T, p)$. The solution to this functional equation is

$$(7)\qquad V(T_t,p_t) = \gamma_1 p_t^{\frac{1}{1-a}}T_t + \gamma_2 p_t^{\frac{2}{1-a}},$$

where

$$(7a)\qquad \gamma_1 = \frac{h}{1-\beta(\delta\bar{\epsilon} + \sigma_{\epsilon\eta_2})},$$

and

$$(7b)\qquad \gamma_2 = \frac{1}{4a(1-\beta\overline{\epsilon^2})}[\beta\gamma_1(\bar{\epsilon} + \sigma_{\epsilon\eta_1})]^2,$$

and where we have defined $\epsilon_t \equiv \epsilon_t^{*\frac{1}{1-a}}$, $E(\epsilon_t) = \bar{\epsilon}$, $E(\epsilon_t^2) = \overline{\epsilon^2}$, $\text{cov}(\epsilon_t, \eta_{1t}) = \sigma_{\epsilon\eta}$, and $\text{cov}(\epsilon_t, \eta_{2t}) = \sigma_{\epsilon\eta_2}$. We assume that the random vector $(\eta_{1t}, \eta_{2t}, \epsilon_t)$ is serially uncorrelated. This solution can be obtained using the method of undetermined coefficients by assuming that the value function has the form in (7) and then solving for the coefficients γ_1 and γ_2. Rather than derive the solution here, we will verify that the function in (7)–(7b) is the solution to the Bellman equation.

To show that (7) is indeed the solution to the Bellman equation, we must calculate $E_tV(T_{t+1},p_{t+1})$. Substituting (1) and (2) into (7) and calculating expected values, we obtain

$$(8)\quad E_tV(T_{t+1},p_{t+1}) = \gamma_1 p_t^{\frac{1}{1-a}}R_t(\bar{\epsilon} + \sigma_{\epsilon\eta_1}) + \gamma_1 p_t^{\frac{1}{1-a}}T_t(\bar{\epsilon}\delta + \sigma_{\epsilon\eta_2}) + \gamma_2 p_t^{\frac{2}{1-a}}\overline{\epsilon^2}.$$

From equation (4) it is clear that the optimal value of R_t is such that the marginal cost, $2aR_t$, is equal to $\beta E_t\{[\partial V(T_{t+1},p_{t+1})]/\partial R_t\}$. Using (8) to calculate $\beta E_t\{[\partial V(T_{t+1},p_{t+1})]/\partial R_t\}$, the optimal value of R_t is

$$(9)\qquad R_t = \frac{1}{2a}\beta\gamma_1(\bar{\epsilon} + \sigma_{\epsilon\eta_1})p_t^{\frac{1}{1-a}}.$$

Note that the optimal rate of R & D activity is independent of the level of accumulated technology, T_t. We will discuss further properties of the optimal R & D activity later.

Given the optimal rate of R & D activity in (9), we can now express $E_tV(T_{t+1}, p_{t+1})$ as a function of the state variables T_t and p_t. Substituting (9) into (8) and using (7b), we obtain

$$(10)\quad E_tV(T_{t+1},p_{t+1}) = \gamma_1 p_t^{\frac{1}{1-a}} T_t(\bar{\epsilon}\delta + \sigma_{\epsilon\eta_2}) + \gamma_2\left(\frac{2}{\beta} - \overline{\epsilon^2}\right)p_t^{\frac{2}{1-a}}.$$

Finally, we can express the current cash flow under optimal behavior, $hp_t^{1/1-a}\, T_t - aR_t^2$, as function of the state variables T_t and p_t by using (9) to substitute for R_t,

$$(11)\qquad hp_t^{\frac{1}{1-a}}T_t - aR_t^2 = hp_t^{\frac{1}{1-a}}T_t - \gamma_2(1 - \beta\overline{\epsilon^2})p_t^{\frac{2}{1-a}}.$$

The value of the firm is the sum of the current cash flow in (11) and the expected present value of the firm's value next period in (10). Thus, the value of the firm is obtained by adding β times the right-hand side of (10) plus the right-hand side of (11). Performing this operation verifies that (7)–(7b) is indeed the solution to the Bellman equation (6).

Observe from (7) that the value function is a linear function of the stock of accumulated technology. The economic intuition underlying the value function is quite straightforward. Let λ_t denote the expected present value of marginal revenue products accruing to technology from time t onward. That is,

$$(12)\qquad \lambda_t = hp_t^{\frac{1}{1-a}} + E_t\left[\sum_{j=1}^{\infty} \beta^j\left(\prod_{i=1}^{j} \eta_{2t+i}\right)hp_{t+j}^{\frac{1}{1-a}}\right].$$

Recognizing that

$$E_t\left(p_{t+j}^{\frac{1}{1-a}}\prod_{i=1}^{j} \eta_{2t+1}\right) = p_t^{\frac{1}{1-a}}E_t\left(\prod_{i=1}^{j} \epsilon_{t+i}\eta_{2t+i}\right)$$
$$= p_t^{\frac{1}{1-a}}(\bar{\epsilon}\delta + \sigma_{\epsilon\eta_2})^j,$$

we can (if $\beta(\bar{\epsilon}\delta + \sigma_{\epsilon\eta_2}) < 1$) rewrite (12) as

$$(13)\quad \lambda_t = hp_t^{\frac{1}{1-a}}\left[1 + \sum_{j=1}^{\infty} \beta^j\left(\bar{\epsilon}\delta + \sigma_{\epsilon\eta_2}\right)^j\right] = \frac{hp_t^{\frac{1}{1-a}}}{1 - \beta(\bar{\epsilon}\delta + \sigma_{\epsilon\eta_2})}.$$

Observe that λ_t is equal to the slope coefficient (with respect to T_t) of the value function (7). Thus, the first term in the value function (7), $\gamma_1 p_t^{1/1-a}\, T_t$, represents the present value of net cash flow accruing to technology existing at time t.

To interpret the second term in the linear function (7), we observe that the quadratic form of the R & D expenditure function implies the existence of inframarginal rents to R & D activity. In figure 13.1 we illustrate the first-order condition for the optimal rate of R & D activity, namely, that marginal R & D expenditure, $2aR_t$, is equal to the expected present value of the marginal contribution of R & D activity. The shaded area in figure 13.1 represents the rents accruing to the inframarginal

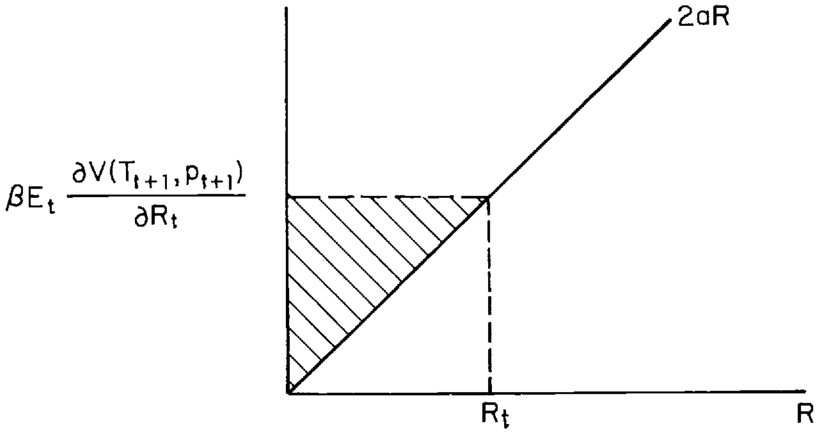

Fig. 13.1

R & D activity. The area of the shaded region is equal to aR_t^2, which, using (9), is equal to

$$\frac{1}{4a}\beta^2\gamma_1^2\left(\bar{\epsilon} + \sigma_{\epsilon\eta_1}\right)^2 p_t^{\frac{2}{1-a}}.$$

The expected present value of these rents over the entire future is

$$(14) \qquad E_t \sum_{j=0}^{\infty} \beta^j \frac{\beta^2\gamma_1^2(\bar{\epsilon} + \sigma_{\epsilon\eta_1})^2}{4a} p_{t+j}^{\frac{2}{1-a}}$$

$$= \frac{\beta^2\gamma_1^2(\bar{\epsilon} + \sigma_{\epsilon\eta_1})^2 p_t^{\frac{2}{1-a}}}{4a}\left[1 + \sum_{j=1}^{\infty} \beta^j E_t\left(\prod_{i=1}^{j} \epsilon_{t+i}^2\right)\right].$$

Observe that $E_t(\Pi_{i=1}^{j}\, \epsilon_{t+i}^2) = (\overline{\epsilon^2})^j$. Therefore, we obtain (if $\beta\overline{\epsilon^2} < 1$)

$$(15) \qquad 1 + \sum_{j=1}^{\infty} \beta^j E_t\left(\prod_{i=1}^{j} \epsilon_{t+i}^2\right) = \frac{1}{1 - \beta\overline{\epsilon^2}}$$

Substituting (15) into (14), the expected present value of rents to inframarginal R & D activity is

$$(16) \qquad \frac{\beta^2\gamma_1^2(\bar{\epsilon} + \sigma_{\epsilon\eta_1})^2}{4a(1 - \beta\overline{\epsilon^2})} p_t^{\frac{2}{1-a}}.$$

Note that the expression in (16) is equal to the second term in the value function (7).

To summarize our description of the value function, we have shown that it is a linear function of the stock of accumulated technology, T_t. The term which is proportional to T_t represents the expected present value of net revenues accruing to the existing stock T_t over the remaining lifetime of this technology. The constant term in the linear value function represents the expected present value of inframarginal rents to present and future R & D activity.

The Effects of Uncertainty

The model developed above explicitly incorporates three channels for uncertainty: the price of output (ϵ_t), the contribution of R & D activity to the gross increase in the stock of technology (η_{1t}), and the rate of obsolescence (η_{2t}). Note from equations (7)–(7b) and (9) that increased uncertainty in either of the two shocks to technology accumulation (η_{1t} and η_{2t}) will affect optimal R & D activity and the value of the firm only if these shocks are correlated with the price shock $\epsilon_t \equiv \epsilon_t^{*1/(1-a)}$. If $\sigma_{\epsilon\eta_1}$ and $\sigma_{\epsilon\eta_2}$ are zero, then only the expected values, but not the variances, of η_{1t} and η_{2t} are relevant.

The optimal rate of R & D activity is an increasing function of both $\sigma_{\epsilon\eta_1}$ and $\sigma_{\epsilon\eta_2}$. A higher value of $\sigma_{\epsilon\eta_1}$ increases $E_t(hp_{t+1}^{1/(1-a)}\eta_{1t+1}R_t)$, the expected value of the next period's net revenue accruing to current R & D activity. Therefore, the optimal level of R & D activity is an increasing function of $\sigma_{\epsilon\eta_1}$. A higher value of $\sigma_{\epsilon\eta_2}$ increases $E_t[hp_{t+j}^{1/(1-a)}(\Pi_{i=2}^{j}\eta_{2t+i})\eta_{1t+1}R_t]$, $j = 2, 3, \ldots$, which is the expected net revenue in period $t+j$ accruing to R & D activity undertaken in period t.

The model developed above can be used to study the stochastic behavior of the value of the firm and of R & D activity. For instance, recall that Pakes analyzes the stochastic properties of the one-period excess rate of return, q_t. Pakes calculated q_t empirically as

$$q_t = \frac{V(T_t,p_t) - V(T_{t-1},p_{t-1}) + (1+r)D_{t-1}}{V(T_{t-1},p_{t-1})} - r, \tag{17}$$

where $D_{t-1} = p_{t-1}L_{t-1}^{a}T_{t-1}^{1-a} - wL_{t-1} - aR_{t-1}^{2}$ is the dividend earned at time $t-1$. In words, q_t is simply the excess of the sum of the dividend and capital gain over the rate of interest. (As noted by Pakes, since dividends are paid at the beginning of the period, we must include within-period interest earnings on the dividend. Empirically, Pakes found that inclusion of rD_{t-1} made no substantial difference.) We can rewrite (17) using the Bellman equation (4) to obtain

$$q_t = \frac{V(T_t,p_t) - E_{t-1}V(T_t,p_t)}{V(T_{t-1},p_{t-1})}. \tag{18}$$

From (18) it is clear that q_t is completely unforecastable as of time $t-1$. This finding is simply the well-known implication of efficient markets theory which states excess returns are uncorrelated with any past information.

Recalling that R & D expenditures are equal to aR_t^2, we can easily calculate optimal R & D expenditures using (9),

$$aR_t^2 = \frac{1}{4a}[\beta\gamma_1(\bar{\epsilon} + \sigma_{\epsilon\eta_1})]^2 p_t^{\frac{2}{1-a}} \tag{19a}$$

$$= (aR_{t-1}^2)\epsilon_t^2. \tag{19b}$$

Thus, current R & D expenditures, aR_t^2, depend only on the previous period's R & D expenditure, aR_{t-1}^2, and the contemporaneous shock to the price of output, ϵ_t. Of course, this simple relation in (19b) is a consequence of the simple structure of the model. However, even this simple model illustrates that much of the variance in q_t can be unrelated to R & D expenditures. Recall that the variation in q_t is from ϵ_t, η_{1t}, and η_{2t}, whereas the variation in R & D expenditures is from ϵ_t^2. To the extent that η_{1t} and η_{2t} account for much of the variation in q_t and to the extent that they are uncorrelated with ϵ_t^2, we would expect only a small part of the variation in q_t to be related to R & D activity. This situation is consistent with Pakes's finding that only 5 percent of the variation in q_t is related to either R & D or patents.

The model presented above provides a useful stochastic framework for analyzing the value of the firm and R & D activity. However, this model, like the model in Pakes's paper, fails to account explicitly for patents. Although it would be straightforward to model the cost of obtaining a patent, modeling the benefits accruing to a patent requires further work.

References

Bailey, Martin N. 1981. Productivity and the services of capital and labor. *Brookings Papers on Economic Activity*, 1:1–50.

Geweke, John. 1977. The dynamic factor analysis of economic time-series models. In *Latent variables in socio-economic models*, ed. D. J. Aigner and A. S. Goldberger, 365–83. Amsterdam: North-Holland.

Pakes, Ariel. 1981. Patents, R & D, and the stock market rate of return. National Bureau of Economic Research Working Paper no. 786. Cambridge, Mass.: NBER.

Sargent, Thomas J., and Christopher A. Sims. 1977. Business cycle modeling without pretending to have too much a priori economic theory. In *New methods in business cycle research: Proceedings from a conference*, ed. Christopher A. Sims, 45–109. Minneapolis: Federal Reserve Bank of Minneapolis.

Tobin, James. 1969. A general equilibrium approach to monetary theory. *Journal of Money, Credit and Banking* (February): 15–29.