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SEARCHING FOR THE LOWEST PRICE WHEN THE DISTRIBUTION OF PRICES IS UNKNOWN: A SUMMARY

BY MICHAEL ROTHSCHILD

Economists have devoted increasing attention recently to variants of the following problem: A man is considering purchase of some good which is sold at different prices at different stores. He does not know which stores have the lowest prices but can elicit price quotations from the various stores by paying a fee. What search strategy would he follow? The problem can be set up so as to be identical to the following standard stopping rule problem. A man is sampling, without recall, from some distribution of prices $F(\cdot)$. For each observation he pays a fixed cost c. His total loss is c times the number of observations plus the last price he draws from $F(\cdot)$ (that is, the price the searcher buys at). When the distribution $F(\cdot)$ is assumed known, the optimal rule has the simple form: accept any price less than a reservation price R, otherwise keep on searching. The reservation price is the solution to the following equation:

$$c = \int_0^R F(p)\,dp. \tag{1}$$

This result is quite pleasing to economists for it yields behavior on the part of consumers which is both plausible and easy to put into economic models. In particular, in a market whose consumers search according to the rule (1):

A. Each store faces a well-defined demand function in the sense that expected sales are a decreasing function of its price.

B. As costs decrease, consumers intensify their search by lowering their reservation price.

C. As the distribution of prices becomes more dispersed, expected total costs incurred by the searcher decrease.

D. As the distribution of prices becomes more dispersed, customers intensify their search activity by lowering their reservation price.[1]

These results all depend on the assumption that the searcher knows the distribution of prices $F(\cdot)$. In any economic context this is a very poor assumption. Very little is known about price distributions by economists beyond the fact that their characteristics are quite volatile across markets. It seems absurd to suppose that consumers know the characteristics of the price distributions they face or that they behave as if they do. Gastwirth (1971) has shown that the optimality properties of the rule given by (1) are not robust to misspecifications of the distribution $F(\cdot)$. It thus seems natural to ask what the optimal search rule should be when the distribution of prices is unknown and whether or not properties A through D hold when searchers follow this rule. This paper is devoted to precisely that question. I consider the special case where it is known that prices belong to some finite set so that the price distribution is known to be a multinomial distribution. However the parameters of that distribution are unknown. The searcher is assumed to have a prior distribution over these parameters which he updates as he observes more prices according to Bayes rule. The searcher's problem is a standard Bayesian decision problem and it is straightforward to use standard dynamic programming techniques to show that an optimal strategy exists.

The optimal strategy is more complicated than the simple rule of (1). Searchers' rules are always characterized by an acceptance set. That is, there is a set of prices which the searcher will accept and a set which if he receives, he will choose to continue to sample rather than buy at that price.

Acceptance sets change as searchers accumulate more information. In general they become larger. As customers search and do not accept because the prices they observe are too high they become discouraged and more willing to accept such prices. There is a finite t such that whatever his experience, a customer has either accepted a price and stopped searching before visiting t stores, or he has become so discouraged that he will accept any price offered to him on his t-th search. Thus, there will be at most t searches. This means that computing the optimal strategy is a finite problem and can in principle be done on a computer.

Despite the added complexity of searchers' rules it is still possible to prove analogues of **A** through **D** for some cases of search from unknown distributions.

A′. If the acceptance set is such that if a searcher will accept a price p he will accept any price lower than p then we shall say it has the reservation price property. If the acceptance sets of all searchers have the reservation price property then stores will face well defined demand functions in the same sense as was discussed in **A** above. In general, customers' acceptance sets do not always have the reservation price property. It is possible to give conditions which guarantee that acceptance sets have the reservation price property. In particular, this will be true if searchers' prior distributions are Dirichlet. (The Dirichlet is the conjugate prior to the multinomial).

B′. As costs decrease, customers intensify their search by narrowing their acceptance sets, whatever their prior beliefs.

C′. If the prior distribution is Dirichlet, if its mean becomes more dispersed (which means that customers expect that prices are more dispersed) expected total costs to the searcher decrease.

D′. If the prior distribution is a Dirichlet and its mean becomes more dispersed, then customers intensify their search activity by lowering their reservation price.

It is not clear how general these results are for the Dirichlet is quite a special case and **A′**, **C′**, and **D′** are restricted to the Dirichlet case. The nature of the proofs suggests both that the results hold more generally and that it will be very difficult to find other examples of any generality about which anything can be proved.

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[1] The first two results are obvious; the last two are due to Kohn and Shavell (1974).