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PREDICTIVE CONTROL OF A STOCHASTIC MODEL OF THE U.K. ECONOMY SIMULATING PRESENT POLICY MAKING PRACTICE BY THE U.K. GOVERNMENT

BY JEREMY BRAY*

Simulations are given of attempts to control a stochastic model of the U.K. economy by the informal predictive control methods actually used by U.K. governments over the past 25 years. The results are similar to those achieved historically, and are offered as a standard of comparison for an optimal stochastic control exercise on the same model. For such an exercise a social welfare function is suggested, the parameters of which are a direct expression of political priorities, representing the range of choice in the U.K. Questions are posed for such an exercise to answer, including how far system behaviour is affected by variation of the political priorities, what are the limitations on the achievable quality of control given the system, its noise characteristics, and parameter indeterminacy and what effects do the rules governing the timing of elections have on policy and hence system behaviour.

1. INTRODUCTION

It is sometimes suggested that while the optimal stochastic control of a national economy is an interesting theoretical problem it is of no practical interest because there is no single authority in control and even if there were that authority would not surrender its policy decisions to a social welfare function, quadratic or otherwise. While there may be a multiplicity of political authorities in the United States (in the President, Congress and the Federal Reserve system) and in some other countries, in the U.K. the government for the time being is firmly in charge, it controls a majority in Parliament, and the Treasury instructs the Bank of England. The general conduct of economic policy in the U.K. has been reviewed by Caves [3] and Cairncross [2].

For 30 years the attempt has been made to regulate the pressure of demand in the U.K. according to Keynesian principles, using short-term forecasts on alternative policy assumptions, with the forecasts being continuously elaborated until now they are made with a formal quarterly econometric model similar to the Wharton and other such models in the U.S. [5]. These forecasts carry great weight.

A forecast is made which will return the economy to a full employment equilibrium growth path within two years. It is almost unknown for a U.K. government to pursue policies which are forecast to deviate from this path by more than 0.5 percent of GDP in the year ahead. Yet the RMS error of year-end ex-ante forecasts of real GDP in the Wharton model of the U.S. for the years 1969–1972 was 0.8 percent, and the errors of U.K. forecasts, which have not been evaluated as systematically, are unlikely to be smaller: the mini-model of the U.K. given below shows standard errors (without residual adjustments on extraneous information) of 1.75 percent of GDP, although these standard errors do not increase greatly for the second and subsequent years.

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The present informal predictive control system with such a high level of system noise seems from a control theory angle a dangerously sub-optimal procedure, liable to generate instabilities of the type which have actually occurred in the U.K. The final requirement for the relevance of a stochastic control exercise is that a social welfare function can be defined which represents, and can be understood by lay politicians to represent, the political priorities of the government. How this requirement can be met is described below.

The poltical and practical circumstances thus exist in the U.K. for an optimal stochastic control approach to the management of the U.K. economy. The interest of such an exercise lies not only in the possibility of improving the quality of economic management in the U.K., which may be slight, but in the identification of the limitations on the quality of control inherent in the uncertainty and lags in the system as described by aggregate national income variables. This paper draws from work in the Programme of Research into Econometric Methods at Queen Mary and Imperial Colleges, London, which is seeking to develop the methodology needed, using a mini-model of the U.K. which has the essential features of a fully operational policy model.

2. The Model

The current version of the model is given in the Appendix. The behavioural equations are linear equations relating rational distributed lags in percent changes, with rational lag error processes. Some of the identities are nonlinear. The uniform use of changes is in accordance with diagnostic work on the time series using Box–Jenkins methods, and the use of percent changes with appropriate normalizing variables preserves commonly assumed structural forms. Details of the estimation methods have been given elsewhere [6].

Rational distributed lags are used first because of their greater economy in the use of parameters in representing underlying economic processes than Almon lags (polynomials in t) or polynomial lags (polynomials in the lag operator B); and second, because they can be directly and economically transformed into the state-vector form useful in control theory [4].

In the first stage of an optimal stochastic control study it is desirable if possible to keep the system linear. For initial control purposes the identities will therefore be linearized about the forecast or control origin, an approximation unlikely to lead to difficulties over two to three years given the methods of linearization used. In the forecasts given in this paper, a compromise is made in that the identities are re-linearized, and the normalizing variables taken at current values, at each forecast step; but each forecast step is a linear process, so direct simultaneous solution of the linear system is sufficient without using Gauss–Seidel methods. Again the approximation errors are small in relation to equation residuals.

The uncertainty of the forecasts is shown by estimates of the standard errors of each forecast variable at different lead times. (Full tables have been given elsewhere [1].) These were calculated by running the forecast many times with the same exogenous variables, with the residual of each forecasting equation supplied by a random number generator, and then averaging in the sense of taking the root mean square of the departures of the simulations from the expected values forecast with zero residuals. The stochastic character of past disturbances were thus reproduced and averaged. Taking *one* standard error means there is a 70 percent probability that the error will be less than that shown, and equally to the point, a 30 percent probability that the error will be greater, positive or negative. Over time, one third of the out turns will lie *outside* these error limits.

The errors may seem large, and indeed they are. The deficit in the balance of trade in 1973 comes out at £660m. ± £373m., or anything between £287m. and £1,033m. Unemployment in the fourth quarter of 1973 is forecast as 436,000 ± 95,000: GDP in 1973 as £36,098m. ± £631m. or ±1.75 percent, and in 1974 as £37,494m. ± £875m. or ±2.3 percent. An interesting observation is that import values and volumes are considerably more uncertain than exports, contrary to the usual assumption. The reason is that export performance is primarily determined by the rest of the world where unpredictable events average out, whereas imports are primarily determined by the home economy which is more variable and less predictable.

Since the model does not use short term leading indicators, such as investment intentions, it is probable that the short term errors are larger than those that would be achievable with a larger model using such indicators, or by purely judgemental forecasting. However the model does treat public expenditure and investment other than plant and machinery as exogenous with zero error. The longer term errors in the second year of the forecast are unlikely to be significantly reduced in other models.

3. Simulations

To show the medium term deterministic behaviour of the model Figure 1 gives history for 1955–1973 and expected values (with zero future residuals) over the period 1974–1978 with policy instruments neutral (i.e. taxes raising a fixed percentage of the corresponding tax base, social expenditure increasing at past average rates, fixed exchange rate, outstanding consumer debt a fixed proportion of disposable income) the behaviour is smooth after the initial few quarters and plausible. With these exogenous control variable values, the forecast was repeated with random number generated residuals (noise), the results of one such forecast run being shown in Figure 2. The smoothness has gone and the stochastic character of the post-1973 simulation is similar to the pre-1973 history. Once clear of the initial conditions in 1973–1974, in the four years 1975–1978, with expected values, GDP growth is 2.86, 3.39, 3.34 and 2.86 percent and with noise 2.56, 4.95, 1.19 and 5.01 percent between fourth quarters. Unemployment, with expected values, varies between 349 and 376 thousand, and with noise, between 240 and 416 thousand. The balance of trade, with expected values, is £362m., 618, 647 and 622; and with noise £462m., 342, 67 and 10. Overall the stochastic simulation seems to have captured the general characteristics of past behaviour.

4. Simulation of the Economic Management Process

U.K. Treasury practice has been to carry out a major forecasting exercise during the first quarter in preparation for the April budget announcing any tax

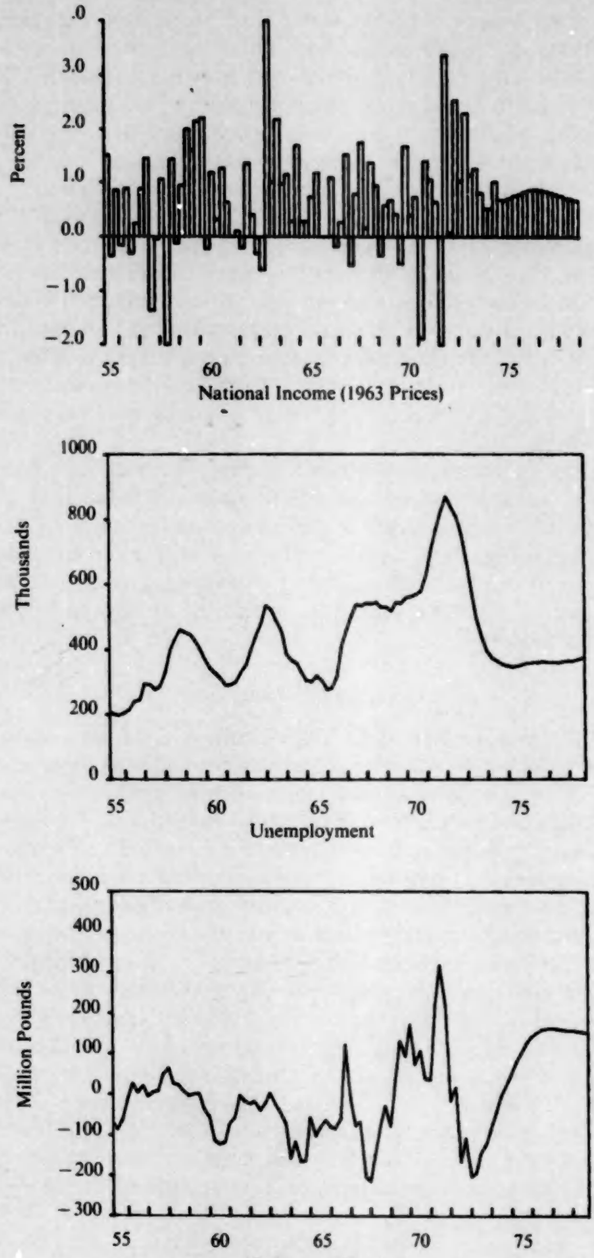
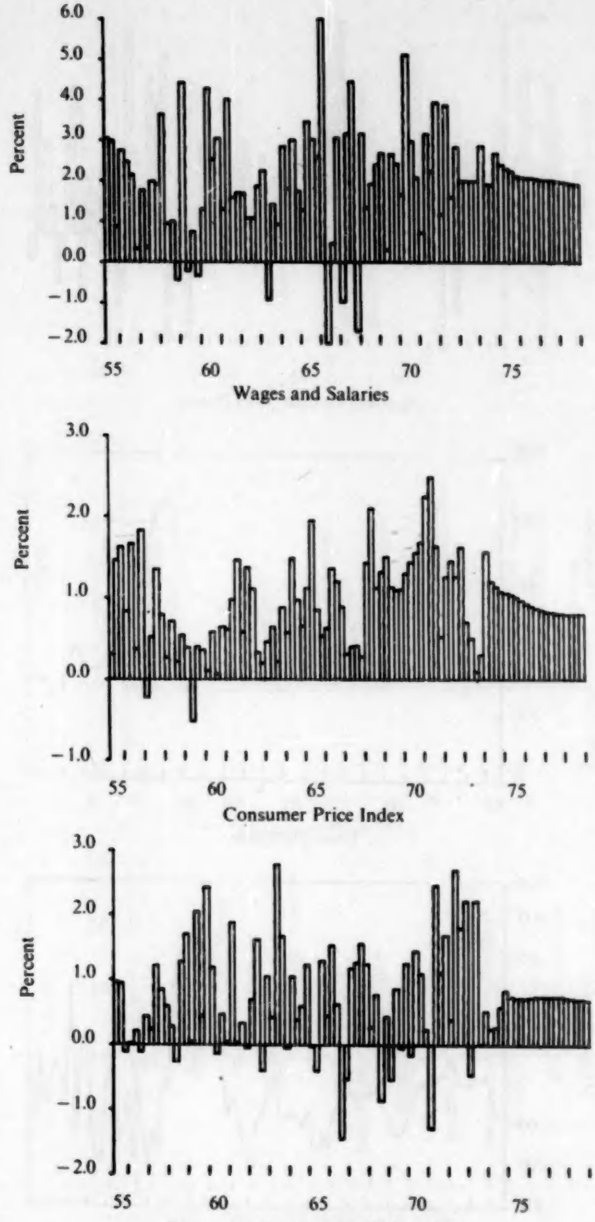


Figure 1 Deterministic simulation with neutral policies (1955–1972 is historic data)


6.0
5.0
4.0
3.0
2.0
1.0
0.0
−1.0
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Percent
55
60
65
70
75
Wages and Salaries
3.0
2.0
1.0
0.0
−1.0
Percent
55
60
65
70
75
Consumer Price Index
3.0
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Percent
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75
Consumers Expenditure (1963 Prices)

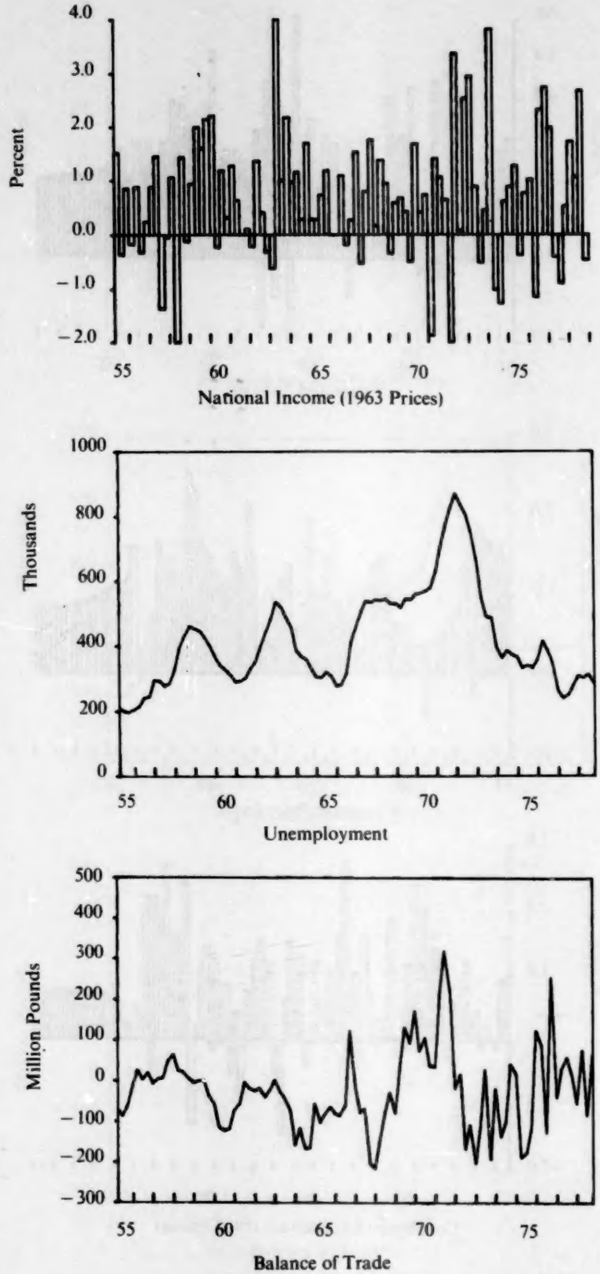
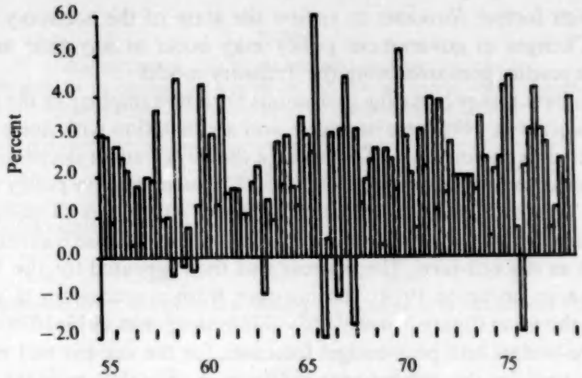
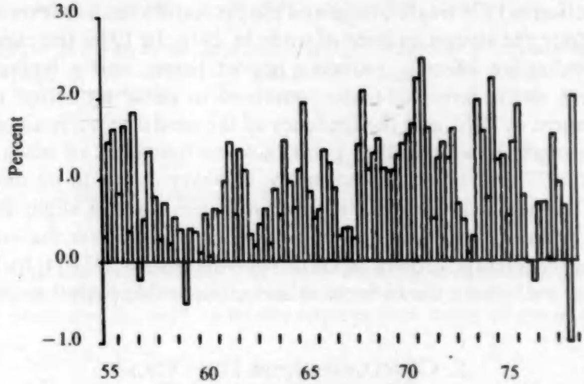


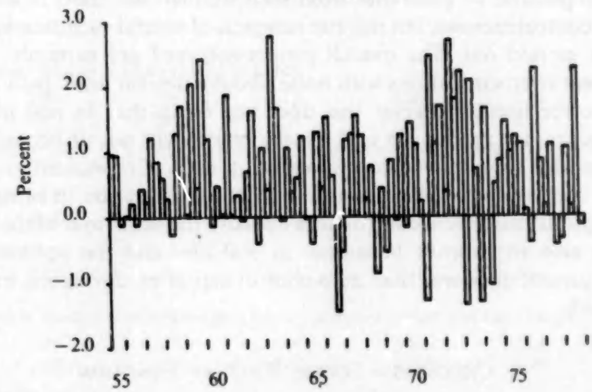
Figure 2 Simulation with neutral policies and noise (1955–1972 is historic data)



Wages and Salaries



Consumer Price Index



Consumers Expenditure (1963 Prices)

changes, with further forecasts to review the state of the economy in July and October. Changes in government policy may occur at any time and forecasts can now be readily prepared using the Treasury model.

To simulate this process the exogenous variables implicit in the post-budget Treasury forecast in 1973 were assumed, and a simulation with noise run to 1973 (4) and treated as the out-turn. To simulate the 1974 budget decision forecasts of expected values were prepared for 1974 and 1975 on alternative policy assumption. To help in choosing policies, tables of dynamic multipliers of policy variables were available. Having chosen a policy, a simulation with noise was run to 1974 (4), and treated as the out-turn. The process was then repeated for the 1975 budget decision and so on up to 1978. The out-turn from a succession of such budget decisions is shown in Figure 3, with 1955–1972 history, and 1973–1978 simulations.

The pre-budget and post-budget forecasts, for the current and ensuing year and the out-turn for the current year is shown in the table, with the reasons for the measures taken. The course of the economy is unexpected but not implausible. The revaluation in 1976 was too large and too premature but was reversed promptly in 1977 despite the strong balance of trade in 1976. In 1976, the expenditure tax cut, the revaluation effect in reducing import prices, and a further fortuitous improvement in the terms of trade combined to cause an actual reduction of consumer prices in 1976, and the tendency of the model to maintain current rates of inflation together with further good luck on the terms of trade kept prices constant in 1977 and 1978. The economy, however, is left in an uncomfortable state in 1978 with little growth, low unemployment and a slight deficit in the balance of trade. By maintaining the priority of growth over the exchange rate nevertheless the average growth of GDP over the period 1972 (4) to 1978 (4) was 3.74 percent, well above the average in any comparable period since the war.

5. Conclusion from Simulations

It is impossible to generalize from such simulations, short of a full optimal stochastic control exercise, but this run is typical of several such simulations which have been carried out. The overall results achieved are certainly not a clear improvement over simulations with noise about a deterministic path but without in-course corrections. However this does not mean that in real life in-course corrections are not needed: in real life the model will not fit behaviour exactly whereas in the simulation, stochastically, it does. Consequently, in real life behaviour can wander off further and need more correction to bring it back. If however optimal stochastic control does improve the behaviour of the simulation, it is likely also to improve behaviour in real life: and the optimal stochastic controller cannot do worse than zero control action on deviations from a deterministic path.

6. Choosing a Social Welfare Function

In the process of economic management a judgement of priorities is constantly made as between consumer expenditure, public expenditure, inflation, growth and so on. A politician would not be able to say which path of all possible paths

TABLE

Simulation of the Economic Management Process

All variables are given as % changes 4th quarter on 4th quarter, except unemployment which is 4th quarter level, and balance of trade which is an annual total

	GDP	Unemployment	Consumer Price Index	Wages and Salaries	Consumer Expenditure	Balance of Trade
1974 Pre-budget forecast with no change in policy						
1974	1.46	311	5.63	9.53	2.18	−185
1975	2.50	346	4.45	8.94	2.77	450
Post-budget forecast with cut in income tax yielding 2% of disposable income and increase in social expenditure of additional 2% from 1974 (3) to give faster growth						
1974	2.71	288	5.53	9.74	3.23	−279
1975	2.47	300	4.41	9.36	2.86	205
Outturn with noise						
1974	1.66	344	6.30	8.11	0.63	−560
1975 Pre-budget forecast with no change in policy; also post-budget since no change made in budget						
1975	2.22	372	4.44	10.04	3.33	−131
1976	3.87	378	4.00	8.85	3.09	50
Outturn with noise						
1975	3.10	280	4.72	13.08	3.14	634
1976 Pre-budget forecast with no change in policy						
1976	3.18	289	3.47	9.44	3.72	938
1977	3.39	289	3.02	7.98	3.09	1009
Post-budget forecast with revaluation of 5% in 1976 (2) and 5% in 1976 (3), cut in expenditure taxes raising 3% less of total expenditure, increase in social expenditure of 1% in each quarter from 1976 (3) to 1977 (3), an increase of 5% in all: to transfer resources from strong balance of trade to home consumption.						
1976	3.97	270	−0.75	9.70	5.12	824
1977	2.52	280	1.21	4.41	2.61	−155
Outturn with noise						
1976	5.12	300	−2.27	2.77	5.60	1121
1977 Pre-budget forecast with no change in policy						
1977	2.84	271	−0.18	5.30	2.68	−160
1978	0.85	330	1.87	5.15	2.19	−1051
Post-budget forecast with 10% devaluation in 1977 (1) to anticipate deficit and direct growth possible back into exports						
1977	4.11	250	1.00	5.51	2.55	−46
1978	2.18	259	2.89	7.11	2.50	−284
Outturn with noise						
1977	3.82	248	−0.56	5.71	3.96	838
1978 Pre-budget forecast with no change in policy; also post-budget since no change made in budget						
1978	2.47	237	1.66	6.83	2.44	168
1979	2.02	273	2.64	6.39	2.39	98
Outturn with noise						
1978	0.47	318	0.40	2.05	1.00	−124

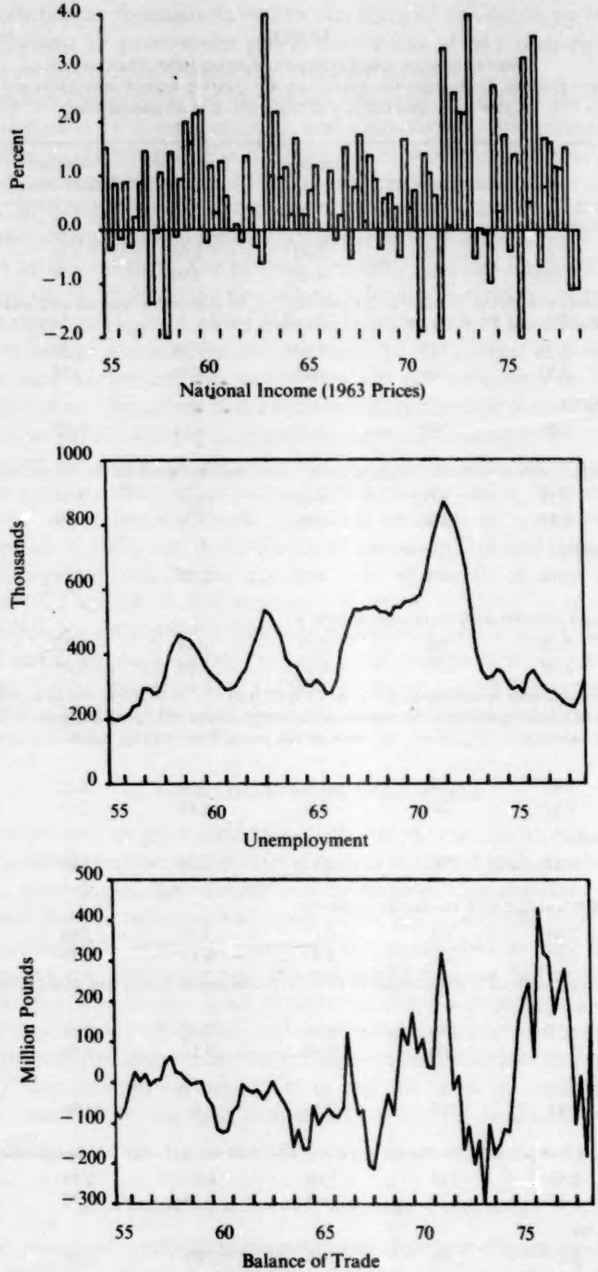
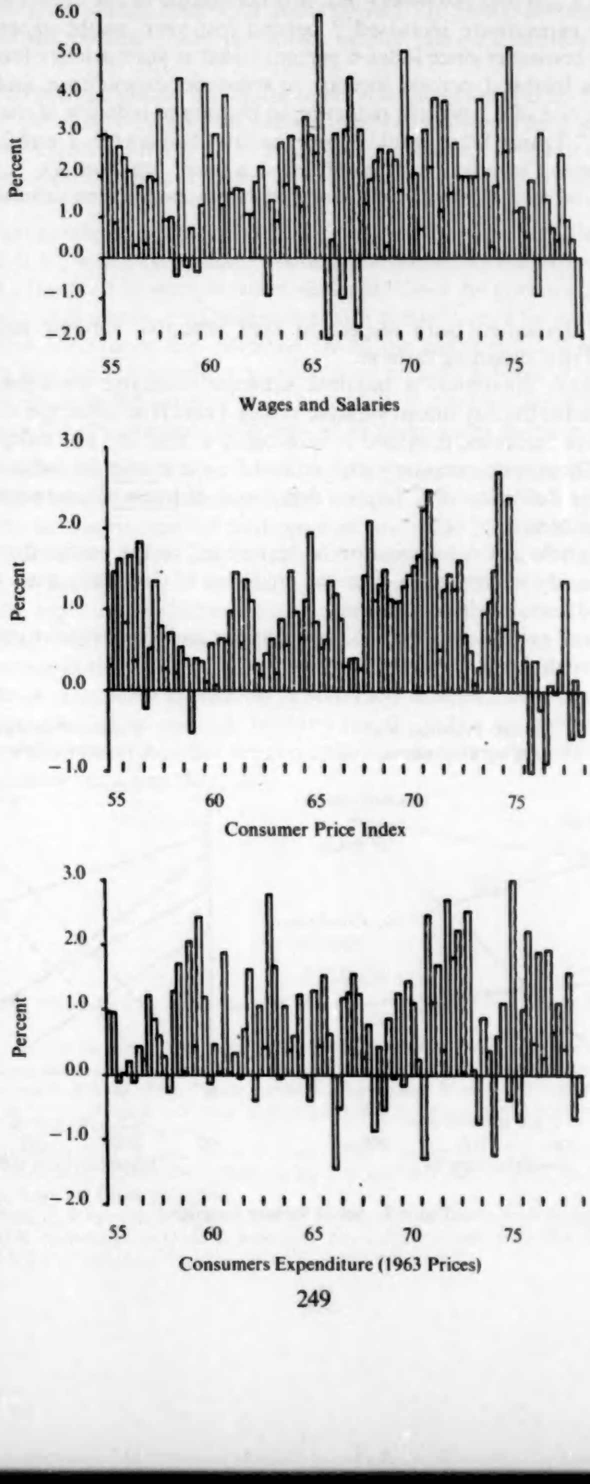


Figure 3 Budget-making simulation with noise (1955–1972 is historic data)

Figure 3. Budget-making simulation with noise (cont.)
(1955-1972 is historie data)



he would most prefer, if only because the set of possible paths is not readily definable in a comprehensive way. But it is reasonable to ask questions like this: "Consumer expenditure increased 7 percent last year, public expenditure 3.5 percent, the consumer price index 6 percent; what is your priority (on a scale of 0 to 10) of a further 1 percent increase in consumer expenditure, and in public expenditure, and of a 1 percent reduction in the rate of inflation of the consumer price index?" Then, "What would it be if the rates had been 3, 7, and 2 percent?" By this means a "priority" can be defined as a linear function $f_i(x_i - g_i)$ of each objective variable x_i. A quadratic social welfare function is then defined as

$$C = -\sum_i \tfrac{1}{2} f_i (x_i - g_i)^2$$

with partial derivatives with respect to each objective variable equal to the "priority" of that objective variable.

If a unique maximum is required sufficient objective variables must be included to define the instrument variable values. There is no objection to including more objective variables, provided it is recognized they are not independent of each other. There is no necessary requirement for the maximum value of C to be zero, since the definition of C implies definitions of trade-offs between different objective variables.

On the whole this representation of "priorities" as the partial derivatives of C is more readily interpretable than the contours of C, usually used to discuss trade-offs and social welfare functions.

The overall priorities in the U.K. of a Labour and Conservative government respectively might be as shown in Figure 4.

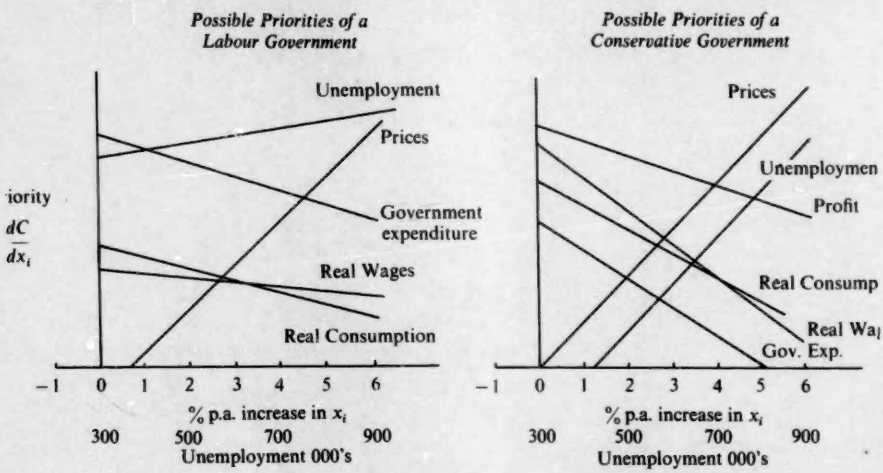


Figure 4 Social Welfare Functions

7. Questions to be Explored

1. How far is system behaviour under optimal stochastic control affected by variation of the parameters in the social welfare function, expressing political priorities?

2. What are the limitations on the achievable quality of control of the system in terms of the value of the social welfare function, and the expected values and variances of variables, for any given social welfare function? What is the trade-off between the performance of different variables under different social welfare functions?

3. What are the effects of introducing finite time horizons at the latest election date in the U.K., where the timing of elections is chosen by the Prime Minister in office with a limit of five years between elections? Does the system exhibit cyclical tendencies, or different cyclical tendencies, with and without a finite time horizon?

4. What effect does the introduction of the indeterminacy of the model parameters into the optimization criterion have on system behaviour?

5. Finally, with the knowledge of the system thus gained, what is the optimal policy, given the governments priorities, at any particular point in time?

The exploration of such questions should lay the ground work for the preparation of a fully operational policy model.

It is by no means obvious that optimal stochastic control at national level by a single authority necessarily secures the "best" behaviour of the economy in any sense. A multiplicity of competing or complementary authorities, divided functionally, regionally or hierarchically, may produce better results, given all the limitations of uncertainties and lags in information and response. Indeed a major question arises in the U.K. if it is found that the achievable quality of control at national level of national aggregates is politically unacceptable. The search must then continue for better systems. In this sense the policy problem in the U.S. is of a more difficult and possibly more fruitful variety, than the policy problem in a highly unitary state like the U.K.

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London

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APPENDIX

MODEL PREM 1

The model was estimated by Dr. Kent Wall and Mrs. Philipa Carling of the Programme of Research into Econometric Methods of Queen Mary College, London to act as a system on which to develop the application of optimal stochastic control theory methods to the management of an economy. An important part of the problem turned out to be the development of suitable methods for estimating a model in a form appropriate to the application of control theory methods. Box–Jenkins methods were used consistently to estimate the behavioural equations which follow.

The variables are defined as follows:

$Y_i(t)$ is the value of the variable i in quarter t.

NORM. METHOD OF NORMALIZING FIRST DIFFERENCE AS % CHANGE

0 $$y_i(t) = 100 \times \frac{(Y_i(t) - Y_i(t-1))}{Y_i(t-1)}$$

1(j) $$y_i(t) = 100 \times \frac{(Y_i(t) - Y_i(t-1))}{Y_j(t-1)}$$

2 $$y_i(t) = Y_i(t) - Y_i(t-1)$$

3(j) $$y_i(t) = 100 \times \frac{Y_i(t)}{Y_j(t)}.$$

TYPE — TYPE OF EQUATION
- C — CONTROL VARIABLE
- B — DEPENDENT VARIABLE IN BEHAVIOURAL EQUATION
- S — DEPENDENT VARIABLE IN AN IDENTITY WHICH IS A LINEAR SUM OF OTHER VARIABLES
- P — DEPENDENT VARIABLE IN AN IDENTITY WHICH IS A PRODUCT OF OTHER VARIABLES

SOURCE
- BANK Q — BANK OF ENGLAND QUARTERLY BULLETIN
- IN.REV — CALCULATED BY INLAND REVENUE
- L.B.S. — CALCULATED BY LONDON BUSINESS SCHOOL
- NIER — NATIONAL INSTITUTE ECONOMIC REVIEW
- TRENDS — ECONOMIC TRENDS

UNITS
- LM.CON — MILLIONS OF POUNDS AT 1963 PRICES
- LM.CUR — MILLIONS OF POUNDS AT CURRENT PRICES

ALL DATA IS SEASONALLY ADJUSTED

NO.	LABEL	DESCRIPTION	NORM.	TYPE	SOURCE	UNITS
0	G.D.P.	G.D.P. AT FACTOR COST (AV.EST.) 67.2025*INDEX	0	S	TRENDS	LM.CON
1	UNEMPL	UNEMPLOYMENT (WHOLLY, EX.SC.LEAVERS)	0	B	TRENDS	THOUS.
2	PL&MAC	INVESTMENT IN PLANT AND MACHINERY	0	B	TRENDS	LM.CON
3	STOCKS	VALUE PHYSICAL INCREASE IN STOCKS	3(0)	B	TRENDS	LM.CON
4	CONSUM	CONSUMERS' EXPENDITURE	0	B	TRENDS	LM.CON
5	PRICES	CONSUMERS' PRICE INDEX 100*Y51/Y4	0	P	TRENDS	1963=100
6	WAGES	PERSONAL INCOMES LESS PROFITS Y42−Y7	0	B		LM.CUR

NO.	LABEL	DESCRIPTION	NORM.	TYPE	SOURCE	UNITS
7	PROFIT	GROSS TRADING PROFITS OF COMPANIES	0	B	TRENDS	LM.CON
8	EXPORT	EXPORTS OF GOODS AND SERVICES	0	B	TRENDS	LM.CUR
9	IMPORT	IMPORTS OF GOODS AND SERVICES	0	B	TRENDS	LM.CUR
10	EXP.PR	EXPORTS PRICE INDEX 100*Y8/Y14	0	B		1963=100
11	IMP.PR	IMPORTS PRICE INDEX 100*Y9/Y15	0	B		1963=100
12	WORLDP	WORLD INDUSTRIAL PRODUCTION INDEX	0	B	NIER	1963=100
14	EXCO.P	EXPORTS OF GOODS AND SERVICES	0	B	TRENDS	LM.CON
15	IMCO.P	IMPORTS OF GOODS AND SERVICES	0	B	TRENDS	LM.CON
16	TX%PRO	TAXES ON COMPANIES AS % OF PROFITS 100*Y34/Y7	2	C		%
17	DIS.IN	PERSONAL DISPOSABLE INCOME Y42−Y31	0	S	TRENDS	LM.CUR
18	R.D.IN	REAL PERSONAL DISPOSABLE INCOME 100*Y17/Y5	0	P	TRENDS	LM.CON
19	BOCO.P	BORROWING BY HOUSEHOLDS 100*Y33/Y5	1(18)	P		LM.CON
20	TX%EXP	TAXES ON EXPENDITURE AS % OF G.D.P. 10000*Y32/(Y0*Y5)	2	C		%
21	PRE-EXT	CONSUMER FACTOR COST INDEX (100−Y20)*Y5/100	0	B		1963=87.1
22	LACOST	LABOUR COST INDEX 100*Y6/Y0	0	P		
23	F.C.A.	FACTOR COST ADJUSTMENT	0	B		LM.CON
24	GDPERR	G.D.P. RESIDUAL ERROR Y0−Y2−Y3−Y4−Y14+Y15−Y55+Y23	1(0)	B		LM.CON
25	BAL.TR	BALANCE OF TRADE Y8−Y9	1(8)	S		LM.CUR
28	SAVING	SAVINGS Y18−Y4	1(18)	S		LM.CON
29	SAVRTO	SAVINGS RATIO 100*Y28/Y18	2	P		%
31	INCTAX	TAXES ON INC,NI CONS,T'FERS ABROAD	0	P	TRENDS	LM.CUR
32	EXDTAX	TAXES ON EXPENDITURE	0	P	TRENDS	LM.CUR
33	BORROW	BORROWING BY HOUSEHOLDS	1(42)	C	BANK Q	LM.CUR
34	COMTAX	TAXES ON COMPANIES	0	P	TRENDS	LM.CUR
35	INCENT	COST OF L100 MACHINE LESS PRESENT VALUE OF GRANTS AND ALLOWANCES	0	C	IN.REV	%
36	EXRATE	EXCHANGE RATE INDEX	0	C	L.B.S.	1963=100
40	TOFIEX	TOTAL FINAL EXPENDITURE EX. STOCKS Y2+Y4+Y14+Y55	0	S		LM.CON
41	TX%INC	TAXES ON INC. ETC. AS % PERS.INC. 100*Y31/Y42	2	C		
42	INCOME	PERSONAL INCOME BEFORE TAX	0	S	TRENDS	LM.CUR
50	GDPIND	GDP INDEX (AVERAGE ESTIMATE)	0		TRENDS	1963=100
51	COCU.P	CONSUMERS' EXPENDITURE (CURR.PR.)	0	P	TRENDS	LM.CUR
55	SOC.EX	INVESTMENT OTHER THAN PLANT & MAC. PLUS PUBLIC CURRENT EXPENDITURE	0	C	TRENDS	LM.CON

B is the backward displacement operator giving $By(t) = y(t - 1)$. $e_i(t)$ is an uncorrelated random variable with zero mean and unit variance.

Behavioural Equations Estimated on data for 1955 (1) to 1972 (2)

1. *Unemployment* Y_1 depends on GDP Y_0 and *plant and machinery* Y_2

$$y_1(t) = -\frac{\overset{(\pm 0.59)}{3.6383}}{1 - \underset{(\pm 0.06)}{0.7703}B} y_0(t) + \frac{\overset{(\pm 0.21)}{0.3988}}{1 - \underset{(\pm 0.11)}{0.8674}B} y_2(t-3) + 8.6498 + 5.92 e_1(t).$$

2. *Plant and machinery* Y_2 depends on GDP Y_0, *company taxes* Y_{16} and *investment incentives* Y_{35}

$$y_2(t) = \overset{(\pm 0.28)}{1.3522} y_0(t) - \frac{\overset{(\pm 0.084)}{0.2169}}{1 - \underset{(\pm 0.21)}{0.6613}B} y_{16}(t-4) - \overset{(\pm 0.10)}{0.1854} y_{35}(t-6) + 0.38751$$
$$+ 2.425 e_2(t).$$

3. *Stockbuilding* Y_3 depends on GDP Y_0

$$y_3(t) = \frac{\overset{(\pm 0.054)}{0.1723}}{1 - \underset{(\pm 0.11)}{1.5783}\mathrm{B} + \underset{(\pm 0.11)}{0.6929}B^2} y_0(t) - 0.006 + \frac{0.729}{1 - \underset{(\pm 0.12)}{0.2073}B} e_3(t).$$

4. *Consumer expenditure* Y_4 depends on *real disposable income* Y_{18} and *consumer borrowing* Y_{19}

$$y_4(t) = \overset{(\pm 0.068)}{0.3257} y_{18}(t) + (\overset{(\pm 0.0925)}{0.1578} + \overset{(\pm 0.0954)}{0.3361}B) y_{19}(t-1) + 0.3704$$
$$+ (1 - \underset{(\pm 0.1149)}{0.5001}B) 0.7278 e_4(t).$$

5. *Consumer's factor cost index* Y_{21} depends on *import prices* Y_{11} and *unit labour cost* Y_{22}

$$y_{21}(t) = \frac{\overset{(\pm 0.038)}{0.1015}}{1 - \underset{(\pm 0.096)}{0.8308}B} y_{11}(t-1) + \frac{\overset{(\pm 0.048)}{0.0901}}{1 - \underset{(\pm 0.034)}{0.4978}B} y_{22}(t-1) + 0.2993$$
$$+ (1 - 0.29\mathrm{B}) 0.7689 e_{21}(t).$$

6. *Personal income less company profits* Y_6 depend on *unemployment* Y_1 and *consumer prices* Y_5

$$y_6(t) = -\overset{(\pm 0.018)}{0.0532} y_1(t) + \overset{(\pm 0.25)}{0.7588} y_5(t-3) + 1.3722 + (1 - \overset{(\pm 0.12)}{0.3761}B) 1.485 e_6(t).$$

7. *Company profits* Y_7 depend on GDP Y_0, *consumer prices* Y_5 and "*wages*" Y_6

$$y_7(t) = \overset{(\pm 0.45)}{2.8075} y_0(t) + \frac{\overset{(\pm 0.19)}{0.3232}}{1 - \underset{(\pm 0.083)}{0.9029}B} y_5(t) - \overset{(\pm 0.30)}{1.6106} y_6(t) - 0.4299$$
$$\pm (1 - \overset{(\pm 0.12)}{0.3400}\mathrm{B}) 3.718 e_7(t).$$

8. *Exports* Y_8 depend on *export prices* Y_{10}, *world industrial production* Y_{12}, and *exchange rate* Y_{36}

$$y_8(t) = \underset{(\pm 0.16)}{0.9802}y_{10}(t) + \frac{\overset{(\pm 0.083)}{0.2624}}{1 - \underset{(\pm 0.11)}{0.7261}B}y_{12}(t) - \frac{\overset{(\pm 0.043)}{0.1866}}{1 - \underset{(\pm 0.048)}{0.8675}B}y_{36}(t) - 0.3509$$
$$+ (1 - \overset{(\pm 0.096)}{0.6748}B)1.956e_8(t).$$

9. *Imports* Y_9 depend on GDP Y_0, *import prices* Y_{11} and *exchange rate* Y_{36}

$$y_9(t) = \overset{(\pm 0.28)}{0.7004}y_0(t) + \overset{(\pm 0.21)}{1.008}y_{11}(t) - \overset{(\pm 0.19)}{0.1859}y_{36}(t) + 0.6740$$
$$+ (1 - \overset{(\pm 0.1240)}{0.3751}B)\,2.359e_9(t).$$

10. *Export prices* Y_{10} depend on *import prices* Y_{11}, *unit labour cost* Y_{22}, and *exchange rate* Y_{36}

$$y_{10}(t) = \overset{(\pm 0.082)}{0.4117}y_{11}(t-1) + \frac{\overset{(\pm 0.043)}{0.1440}}{1 - 0.7531B}y_{22}(t) - \overset{(\pm 0.077)}{0.2179}y_{36}(t) - 0.3138$$
$$+ (1 - \overset{(\pm 0.13)}{0.066}B - \overset{(\pm 0.13)}{0.3074}B^2)\,0.9011e_{10}(t).$$

11. *Import Prices* Y_{11} depend on *exchange rate* Y_{36}

$$y_{11}(t) = (-\overset{(\pm 0.12)}{0.3078} - \overset{(\pm 0.12)}{0.2387}B)y_{36}(t) + 0.43966 + 1.221e_{11}(t).$$

12. *World production index* Y_{12} autoregressive only

$$y_{12}(t) = 1.2732 + \frac{1.240}{1 - 0.2206B + 0.041B^2 - 0.1243B^3 + 0.244B^4}e_{12}(t).$$

13. *Factor cost adjustment* Y_{23} depends on *consumers' expenditure* Y_4, *real exports* Y_{14}, and *social expenditure* Y_{55}

$$y_{23}(t) = \overset{(\pm 0.24)}{1.4911}y_4(t) + \overset{(\pm 0.09)}{0.1091}y_{14}(t) + \overset{(\pm 0.13)}{0.3266}y_{55}(t) + 0.3882$$
$$+ (1 - \overset{(\pm 0.11)}{0.5198}B)\,1.796e_{23}(t).$$

14. GDP *residual error* Y_{24} autoregressive only

$$y_{24}(t) = 0.024725 + (1 - \overset{(\pm 0.066)}{0.80744}B)\,0.632e_{24}(t).$$