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STUDIES OF ECONOMIC PROBLEMS

OPTIMAL POLICIES FOR MONETARY CONTROL*

BY ROBERT S. PINDYCK AND STEVEN M. ROBERTS

This paper will present some optimization experiments using a linearized version of the Federal Reserve Board's monthly money market model, which was designed primarily to study the impact of policy instruments on monetary and financial targets. Using linear-quadratic optimal control, we calculated optimal policies for a single instrument, unborrowed reserves, with the objective of forcing monetary aggregates and interest rates to follow desired paths. There is a conflict between the choice of policy target, i.e., there is a trade-off between the control of monetary aggregates and the control of interest rates. By calculating a set of optimal policies using different objectives, that trade-off can be demonstrated. The optimal strategies are also calculated using closed-loop control so as to correct for random disturbances. It is shown how the existence of random disturbances modifies the target trade-offs between monetary aggregates and interest rates, and requires greater flexibility in the movements of the control variable.

1. INTRODUCTION

Recent applications of optimal control theory to economic stabilization policy problems have usually involved calculating time paths for one or more "global" policy variables so as to minimize some macroeconomic cost functional.[1] The aim of these exercises has been to indicate how policy objectives relating to GNP, employment, prices, and the balance of payments might best be attained. The policy variables which can be manipulated might include tax rates, the level of government expenditures, and the money stock. Tax rates and the level of government expenditures are subject to rather direct control. However, the money stock cannot be controlled directly by the Federal Reserve: the Fed can however, manipulate other variables which in turn affect the money stock.[2]

The ultimate concern of monetary policy-makers is with the real economy and how policy involving monetary (e.g., the money stock) and financial (e.g., interest rates) variables can best be used to attain the desired levels of GNP, employment, prices, and the balance of payments. The inability to directly control these policy "instruments" has resulted in a two-stage optimization process in which these instruments are in fact "intermediate" targets and the true policy instruments are those variables over which the Fed has direct control, e.g., required reserve ratios, the discount rate, ceilings on interest payments on bank liabilities, and the use of open market operations to affect either unborrowed reserves or the

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[1] See, for example, recent work by Chow [6], [7], Friedman [10], Livesey [14], Pindyck [16], [17], and Sengupta [18].

[2] During the past few years, there has been a controversy over the ability of the Federal Reserve to control monetary aggregates. For a discussion of some of the issues, see Pierce and Thomson [15], Davis [8], and Andersen [1].

Federal Funds rate.[3] This, in fact, is essentially the way monetary policy is formulated and executed. Several times a year, objectives for GNP, employment, prices, and the balance of payments are specified. Then a menu of possible monetary policy courses and the consequences of each is analyzed and nominal intermediate target paths are chosen for one or two quarters ahead (on a monthly basis). At more frequent intervals, e.g., every three or four weeks, current money market conditions and prospects for economic activity are analyzed at meetings of the Federal Open Market Committee (FOMC). The nominal paths for the control instruments are frequently revised on the basis of this current information, particularly if the monetary and financial target variables have deviated from their nominal trajectories by a significant amount.[4]

This paper will study the problem of how a monetary authority can best manipulate the policy instruments which it can directly control in order to reach its intermediate target objectives. We recognize that in some sense an intermediate target strategy may be less optimal than approaching the problem of economic stabilization directly. However, given that monetary policy is currently formulated using intermediate targets and that more frequent information about real economic variables is needed to solve the stabilization problem directly, we feel justified in exploring ways to improve the intermediate target variable approach. Our aims are rather modest. We seek to examine only two problems which seem to be fundamental to the realization of any optimal monetary policy plan. These are as follows:

First, given a set of chosen intermediate target paths ("optimal" or otherwise) for the money stock and market rates of interest, we would like to indicate how the Federal Reserve might best manipulate those policy instruments which it *can* direct control.[5] In other words, what is the Fed's optimal policy given that it would like the money stock, and other variables, to track as closely as possible some specified time path? This optimal control problems will be treated in a linear quadratic framework, applying the solution derived by Pindyck [16], [17] to a linearized version of a monthly money market model constructed at the Federal Reserve Board. Optimal policies for monetary control will be calculated using several different cost functions, for both deterministic and stochastic cases.

[3] Recent articles by Holbrook and Shapiro [11], Waud [21], and Kareken, Muench, and Wallace [13] discuss the use of intermediate targets in monetary policy formulation. They indicate that if information about movement in targeted variables, i.e., GNP, employment, etc., were available instantaneously it would be more optimal for policymakers to relate the instruments over which they have direct control to their primary targets. They do not claim to know how suboptimal the intermediate target strategy may be. However, given that information about real economic variables is available only quarterly or monthly but monetary and financial data is available weekly, daily, and even hourly, the use of an intermediate target strategy. since it uses all available information, may be better than making policy decisions only when information about real variables becomes available.

[4] For a discussion of monetary policy formulation, see Axilrod [4, 5]. The day-to-day execution of monetary policy is handled primarily by the Open Market Desk, which faces yet another control problem: that of manipulating its portfolio of government securities in a way which will minimize the deviation of the primary control instrument from its specified path. For a discussion of this problem, see Holmes [12].

[5] In the model which we use in this paper, the discount rate and either unborrowed reserves or the Federal Funds Rate are instrument variables. We have chosen unborrowed reserves rather than the Federal Funds Rate as an instrument.

Second, since the set of intermediate targets may not be completely compatible, and therefore, the objectives of the monetary authority may not be mutually obtainable, we would like to know what the trade-offs are between these different targets. This, of course, depends on what the targets are and on the relative importance assigned to each of them. The objective, for example, of controlling *only* the money stock or *only* some short-term market interest rate might be feasible, at least after a few months' lag. The Fed, however, may have more than one intermediate policy objective, e.g., it may wish to reach target values for both the money stock *and* an interest rate simultaneously. This may be impossible. Even in a deterministic world there may be a required trade-off between objectives.

One of the goals of this paper will be to derive a "trade-off curve" which relates the minimum achievable root-mean-square deviations from the target path for the money stock to that for the interest rate. This trade-off curve would depend not only on the dynamic structure of the monetary sector, but also on how "incompatible" the two target paths are which were chosen by the policy makers.

In a stochastic world the trade-off would probably be worsened. Then, even a *single* target would probably not be reachable exactly. We will examine the stochastic case in this paper by calculating optimal monetary policies, and plotting a "trade-off curve" using closed-loop stochastic control under the assumption of certainty equivalence.[6]

In the next section of this paper, we will briefly discuss the monthly money market model developed at the Federal Reserve Board. We will present our linearization of that model, and its re-specification in state-variable form.[7] Next, we will describe the deterministic optimization experiments performed with that model. Optimization experiments were designed to indicate the characteristics of optimal monetary policies, and also to illustrate the inherent trade-off between a monetary and a financial target variable. Stochastic optimization experiments will be presented in the next section. Residuals from an historic simulation are used as random shocks, and optimal policies are calculated by applying the deterministic control law to the model in a closed-loop fashion. Again, a trade-off curve is calculated, and this, as well as some individual optimal instrument paths, are compared to the deterministic case.

2. The Model

The model is a reestimated version of the Federal Reserve Board's monthly model of the U.S. money market.[8] It was designed to provide insight into the short-run behavior of the money market and also to serve as a basis for predicting the consequences of alternative monetary policies. The version presented here has ten estimated equations and eight identities.[9] The main instrument of control in

[6] We consider only additive error terms that are uncorrelated. Under the certainty equivalence theorem (the "separation theorem" in the control literature), the deterministic control law is optimal when used in a closed-loop fashion. See Theil [19], Chow [7], and Athans [2].

[7] For a discussion of the state-variable form of a model, see Athans and Falb [3] or the Appendix of Pindyck [16].

[8] The model is described in detail in its original version in Thomson, Pierce and Parry [20].

[9] The currency equation was dropped from the original version of the model, and currency was made exogenous. This was done because all of the arguments of the currency equation were exogenous anyway.

the model is the level of unborrowed reserves; however, the Fed may also use the discount rate as a policy variable to influence bank borrowing behavior if it so desires.[10]

There are three sectors in the model: the private (non-bank), commercial banks, and the Government. The interaction of these sectors determines values for demand deposits, negotiable certificates of deposit, other time and saving deposits, public and bank holding of Treasury bills, excess reserves, borrowed reserves, and the rates on Federal funds, negotiable certificates of deposit, prime commercial paper, and corporate bonds. It is assumed that the public's demand for money market instruments is constrained by wealth.[11] Banks are constrained by total liabilities, i.e., deposits less required reserves. These constraints make the demand functions homogeneous in dollar values.

A list of the model's variables and their definitions is presented below:

ENDOGENOUS VARIABLES

1. M1—Money Stock (Currency plus Demand Deposits)
2. DDMS—Demand Deposit Component of the Money Stock
3. OTS—Other Time and Savings at Commercial Banks
4. CD—Negotiable Time Certificates of Deposits
5. DEP—Deposits at all Banks less Required Reserves
6. TTSC—Total Time and Savings Deposits at Commercial Banks
7. TTSM—Total Time and Savings Deposits at Member Banks
8. BORR—Member Borrowings from the Federal Reserve
9. EXR—Excess Reserves
10. TR—Total Reserves
11. RR—Required Reserves
12. RTB—Rate on Treasury Bills 90 Days
13. RFF—Rate on Federal Funds
14. RCDP—Primary Rate on Negotiable Certificates of Deposit
15. RBaa—Moody's Baa Corporate Bond Rate
16. RCP—Rate on Prime Commercial Paper
17. QTBP—Quantity of Treasury Bills Held by the Public
18. QTBB—Quantity of Treasury Bills Held by Banks

EXOGENOUS VARIABLES

1. S1—Seasonal Component—DDMS Equation
2. S2—Seasonal Component—OTS Equation
3. S3—Seasonal Component—CD Equation
4. S4—Seasonal Component—RFF Equation
5. S5--Seasonal Component—EXR Equation
6. S6—Seasonal Component—QTBP Equation
7. PI1—Personal Income Almon lag—DDMS Equation
8. PI2—Personal Income Almon lag—OTS Equation

[10] We have chosen to normalize the model so that unborrowed reserves serve as the exogenous control. The model can also be normalized so that the Federal Funds Rate, rather than unborrowed reserves, is the main policy instrument. This is done by using an estimated equation for borrowings and an identity for unborrowed reserves.

[11] A polynomial in personal income is used as a proxy for total wealth since a good measure of wealth is not available monthly.

9. PI3—Personal Income Almong lag—QTBP Equation
10. ROTS—Savings Deposit Rate (7th District) Almon lag—OTS Equation
11. IPI – Industrial Production Index Almon lag—RCP Equation
12. CONS—Constant Term
13. RRND—Reserves Required Against Non Deposit Items—RR Identity
14. QTBT—Quantity of Treasury Bills Total—QTBB Identity
15. CURR—Currency—M1 Identity

POLICY VARIABLES

1. UR—Unborrowed Reserves
2. RDIS—Federal Reserve Discount Rate

Let us present an overview of the model by considering the organization of its three sectors. The public sector of the model could be summarized by the expression given in (1):

$$\text{(1)} \qquad DDMS + CURR + CD + OTS + QTBP + (OAP - BL) = W.$$

Here OAP is other asset holdings of the public and BL is loans from the banking system. Except for currency, which is exogenous, the first five terms in (1) are determined explicitly within the model. Thus, given a proxy for total wealth W, we could solve for the composite asset (OAP – BL).

The banking sector is summarized by the expression given in (2):

$$\text{(2)} \quad RR + EXR + QTBB - BORR - DDMS - CD - OTS = (K - OAB).$$

Required reserves are estimated from an identity which links the public sector to the banking sector through the components of the money stock, CD's, and other time and savings deposits. Excess reserves is determined explicitly in the model and, when added to required reserves, determines the total reserves (TR) held by the banking system. Total reserves less unborrowed reserves, which are determined by the Federal Reserve, yields the amount of borrowings from the Federal Reserve as given by

$$\text{(3)} \qquad BORR = TR - UR.$$

The total quantity of Treasury Bills outsanding (QTBT) is controlled jointly by the Treasury and the Federal Reserve so that the quantity of Treasury Bills held by banks is given by

$$\text{(4)} \qquad QTBB = QTBT - QTBP.$$

Thus, one may calculate the composite item for the banking system (K – OAB) if desired.

2.1. *The Model's Equations*

The estimated equations of the model are presented below, with t-statistics in parentheses. The variables $S1_t$, $S2_t$, etc. refer to seasonal variables. The variable U_{-1} refers to the Cochrane–Orcutt correction term used in the estimation.

Demand Deposit Component of the Money Stock (DDMS)

$$\text{(5)} \qquad \text{DDMS} = \sum_{i=0}^{4} \alpha_i \text{PI}_{t-i} + \sum_{i=0}^{9} \beta_i \text{RTB}_{t-i}\text{PI}_t + \text{S1}_t + 0.9944\text{U}_{-1}.$$

ALMON DISTRIBUTED LAG WEIGHTS

	t	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
PI	0.066924	0.053548	0.040168	0.026784	0.013394	
	(4.013)	(13.81)	(7.230)	(3.106)	(1.966)	
RTB	−0.000059	−0.000208	−0.000323	−0.000403	−0.000449	−0.000460
	(−0.3039)	(−1.497)	(−3.063)	(−4.218)	(−4.491)	(−4.354)

	$t-6$	$t-7$	$t-8$	$t-9$
RTB	−0.000437	−0.000379	−0.000287	−0.000161
	(−4.146)	(−3.959)	(−3.807)	(−3.684)

PI Almon is 2nd degree Constrained to 0 at $t-5$
RTB Almon is 2nd degree Constrained to 0 at $t-10$

$R^2 = 0.9988$ S.E. = 657 M[12] D.W. = 2.307 MEAN DDMS = 177.9 B
S.D. DDMS = 19 B S1_t = Seasonal Coefficient$_t \cdot \text{PI}_t$

Seasonal Coefficients

Feb.	−0.009089 (−28.65)	June	−0.010191 (−18.18)
Mar.	−0.009539 (−21.98)	July	−0.010051 (−17.98)
Apr.	−0.005811 (−11.47)	Aug.	−0.011879 (−21.70)
May	−0.012162 (−22.27)	Sep.	−0.009597 (−18.41)
Oct.	−0.007969 (−16.50)		
Nov.	−0.006350 (−15.17)		
Dec.	−0.001178 (−3.728)		

Other Time and Savings Deposits at Commercial Banks (OTS)[13]

$$\text{(6)} \qquad \text{OTS} = \sum_{i=0}^{4} \alpha_i \text{PI}_{t-i} + \sum_{i=0}^{11} \beta_i \text{RTB}_{t-i} \cdot \text{PI}_t + \sum_{i=0}^{8} \gamma_i \text{ROTS}_{t-i} \cdot \text{PI}_t + \text{S2}_t + 0.9786\text{PI}_t\text{U}_{-1}.$$

ALMON DISTRIBUTED LAG WEIGHTS

	t	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
PI	0.028727	0.046555	0.053039	0.047737	0.030205	
	(1.455)	(2.734)	(8.971)	(3.342)	(1.686)	
RTB	−0.001482	−0.001322	−0.001170	−0.001024	−0.000884	−0.000751
	(−6.111)	(−7.419)	(−8.749)	(−9.080)	(−7.874)	(−6.203)
ROTS	0.000316	0.000916	0.001667	0.002437	0.003092	0.003499
	(0.3089)	(0.6474)	(1.233)	(2.328)	(3.744)	(3.522)

[12] M refers to millions of dollars, and B to billions of dollars.
[13] This equation and equation (8) were originally estimated in ratio form, which is why the coefficients of the Cochrane–Orcutt term are so large.

	$t-6$	$t-7$	$t-8$	$t-9$	$t-10$	$t-11$
RTB	−0.000624	−0.000504	−0.000390	−0.000283	−0.000182	−0.000088
	(−4.834)	(−3.839)	(−3.120)	(−2.588)	(−2.183)	(−1.866)
ROTS	0.003526	0.003041	0.001910			
	(2.735)	(2.230)	(1.922)			

PI	Almon is 3rd degree	Constrained to 0 at $t+1$ and $t-5$
RTB	Almon is 2nd degree	Constrained to 0 at $t-12$
ROTS	Almon is 3rd degree	Constrained to 0 at $t+1$ and $t-9$

$R^2 = 0.9877$ S.E. = 743 M D.W. = 0.9814 MEAN OTS = 170 B
S.D. OTS = 6.6 B $S2_t$ = Seasonal Coefficient$_t \cdot PI_t$.

Seasonal Coefficients

Jan.	0.001357 (2.935)	May	0.002429 (2.929)
Feb.	0.001934 (3.174)	June	0.001217 (1.477)
Mar.	0.002846 (4.001)	July	0.000980 (1.246)
Apr.	0.002827 (3.571)	Aug.	0.000748 (1.016)
Sep.	0.000892 (1.336)		
Oct.	0.001116 (1.954)		
Nov.	−0.000172 (−0.905)		

Negotiable Time Certificates of Deposits (CD)

(7)
$$CD = \underset{(14.61)}{0.72947} CD_{-1} - \underset{(-2.667)}{0.00150} RTB \cdot PI + \underset{(6.903)}{0.00225} RCDP \cdot PI$$
$$\underset{(-2.453)}{0.00128} RBaa \cdot PI + \underset{(2.929)}{0.00154} (RBaa - RCP) \cdot PI + S3_t$$

$R^2 = 0.9995$ S.E. = 582 M D.W. = 1.642 MEAN CD = 21 B
S.D. CD = 6.7 B $S3_t$ = Seasonal Coefficient$_t \cdot PI_t$.

Seasonal Coefficients

Jan.	0.01057 (2.886)	May	0.00952 (2.656)	Sep.	0.01113 (2.986)
Feb.	0.00977 (2.768)	June	0.00971 (2.659)	Oct.	0.01179 (3.167)
Mar.	0.00974 (2.279)	July	0.00163 (3.137)	Nov.	0.01117 (3.016)
Apr.	0.00916 (2.607)	Aug.	0.01208 (3.265)	Dec.	0.01147 (3.086)

Quantity of Treasury Bills Held by the Public (QTBP)

(8)
$$QTBP = \sum_{i=0}^{4} \alpha_i PI_{t-i} + \sum_{i=0}^{4} \beta_i RCP_{t-i} \cdot PI_t + \sum_{i=0}^{4} \gamma_i RTB_{t-i} \cdot PI_t + S6_t$$
$$+ 0.9910 PI_t U_{-1}.$$

ALMON DISTRIBUTED LAG WEIGHTS

	t	$t-1$	$t-2$	$t-3$	$t-4$
PI	0.008315	0.013229	0.014888	0.013199	0.008228
	(0.5220)	(1.029)	(7.759)	(1.043)	(0.5214)
RCP	−0.000545	−0.000719	−0.000751	−0.000642	−0.000392
	(−0.7339)	(−1.614)	(−1.656)	(−1.388)	(−1.213)

RTB	0.001078	0.001436	0.001508	0.001292	0.000790
	(1.646)	(3.043)	(3.030)	(2.685)	(2.436)

PI	Almon is 3rd degree	Constrained to 0 at $t + 1$ and $t - 5$
RCP	Almon is 2nd degree	Constrained to 0 at $t - 5$
RTB	Almon is 2nd degree	Constrained to 0 at $t - 5$

$R^2 = 0.7481$ S.E. = 710 M D.W. = 1.705 MEAN QTBP = 38 B
S.D. QTBP = 3.6 B $S6_t$ = Seasonal Coefficient$_t \cdot PI_t$.

Seasonal Coefficients

Jan.	0.000756 (2.069)	July	−0.002381 (−3.485)
Feb.	0.003407 (6.793)	Aug.	−0.000354 (−5.499)
Mar.	0.003619 (6.149)	Sep.	−0.00841 (−1.451)
Apr.	0.002627 (4.118)	Oct.	−0.00773 (−1.582)
May	0.002817 (4.250)	Nov.	0.00464 (1.320)
June	0.000214 (0.3086)		

Rate on Treasury Bills (RTB)

(9) $$\underset{(8.436)}{\text{RTB} = 1.1608} + \underset{(3.803)}{28.139}\,\text{QTBB/DEP} + \sum_{i=0}^{4} \alpha_i \text{RFF}_{t-i} + \sum_{i=0}^{6} \beta_i \text{RTB}_{t-i} + 0.7684\text{U}_{-1}.$$

ALMON DISTRIBUTED LAG WEIGHTS

	t	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$	$t-6$
RFF	0.36399	0.26614	0.18083	0.10803	0.04715		
	(5.266)	(6.680)	(3.586)	(1.984)	(1.244)		
RTB		−0.05045	−0.08095	−0.09337	−0.08690	−0.07153	−0.04104
		(−0.9505)	(−1.226)	(−1.634)	(−1.888)	(−1.511)	(−1.043)

RFF	Almon is 2nd degree	Constrained to 0 at $t - 5$
RTB	Almon is 3rd degree	Constrained to 0 at $t + 1$ and $t - 7$

$R^2 = 0.6295$ S.E. = 0.2475% D.W. = 1.423 MEAN RTB = 5.254%
S.D. RTB = 1.067%

Excess Reserves (EXR)[14]

(10) $$\text{EXR} = \underset{(16.541)}{0.001433}\,\text{DEP} - \underset{(-12.128)}{0.000764}\,\text{DEP} \cdot \text{K1} + \underset{(-2.010)}{0.096884}\,\Delta\text{UR} - \underset{(-1.775)}{0.090868}\,\Delta\text{RR} + \text{S5}_t + 0.3153\,\text{U}_{-1}$$

$$\text{K1} = \begin{cases} 0 \text{ prior to 1968–10} \\ 1 \text{ after 1968–10} \end{cases}$$

[14] After September 1968, as a result of a change in Regulation D, required reserves are based on deposit levels two weeks earlier. As a result, the dummy variable *K*1 is introduced to capture the effect of this structural change.

$R^2 = 0.9175$ S.E. = 67 M D.W. = 2.100 MEAN EXR = 285 M
S.D. EXR = 97 M $S5_t$ = Seasonal Coefficient$_t \cdot DEP_t$.

Seasonal Coefficients

Jan.	−0.0001117 (−1.200)	July	−0.0002221 (−2.136)
Feb.	−0.0000577 (−0.497)	Aug.	−0.0001048 (−1.002)
Mar.	−0.0001132 (−1.057)	Sep.	−0.0001380 (−1.372)
Apr.	−0.0002581 (−2.515)	Oct.	−0.0001419 (−1.484)
May	−0.0000647 (−0.6217)	Nov.	−0.0000064 (−0.0732)
June	−0.0001156 (−1.091)		

Rate on Federal Funds (RFF)

$$\text{(11)}\quad RFF = -0.37139 + 239.785\frac{BORR}{DEP} + 0.69062\,RDIS - 18.749\frac{UR}{DEP} + \sum_{i=0}^{2}\alpha_i RTB_{t-i} + S4_t + 0.8646U_{-1}.$$

ALMON DISTRIBUTED LAG WEIGHTS

	t	$t-1$	$t-2$
RTB	0.280090	0.240233	0.146870
	(3.227)	(4.418)	(2.569)

RTB Almon is 2nd degree Constrained to 0 at $t - 3$

$R^2 = 0.7748$ S.E. = 0.2067% D.W. = 1.989 MEAN RFF = 5.469%
S.D. REF = 1.535% $S4_t$ = Seasonal Coefficient.

Seasonal Coefficients

Jan.	0.026738 (0.319)	May	0.339200 (2.256)
Feb.	−0.009530 (−0.086)	June	0.396140 (2.727)
Mar.	−0.069757 (−0.554)	July	0.268329 (1.937)
Apr.	0.207777 (1.501)	Aug.	0.322428 (2.491)
Sep.	0.253465 (2.186)		
Oct.	0.247266 (2.463)		
Nov.	0.169825 (2.255)		

Primary Rate on Negotiable Certificates of Deposit (RCDP)[15]

$$\text{(12)}\quad RCDP = \underset{(20.764)}{0.95390}\,RTB \cdot NORUN + \underset{(3.035)}{0.13632}\,(RBaa\text{-}RTB) \cdot NORUN$$
$$-\underset{(-1.733)}{13.000}\,(CD/DEP) \cdot NORUN + \sum_{i=1}^{11}\alpha_i RCDP_{t-i} \cdot NORUN$$
$$+\underset{(3.616)}{2.192} \cdot NORUN + 1.000\,RQCD \cdot RUN$$

$$NORUN = \begin{cases} 1.0 \text{ if no Run-off occurs} \\ 0.0 \text{ otherwise} \end{cases}$$

$$RUN = 1.0 - NORUN.$$

[15] The variables NORUN and RUN refer to the effect of Regulation Q ceilings on the CD market. If the rate on CD's is driven to the ceiling by market forces, no new CD's will be issued and a run-off occurs. This is explained further when we discuss the linearization of the model.

ALMON DISTRIBUTED LAG WEIGHTS

	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$	$t-6$
RCDP	-0.00120	-0.00373	-0.00713	-0.01093	-0.01466	-0.01786
	(-0.1985)	(-0.4143)	(-0.7416)	(-1.252)	(-1.955)	(-2.443)
	$t-7$	$t-8$	$t-9$	$t-10$	$t-11$	
RCDP	-0.02008	-0.02083	-0.01967	-0.01611	-0.00971	
	(-2.333)	(-2.023)	(-1.764)	(-1.576)	(-1.440)	

RCDP Almon is 3rd degree Constrained to 0 at t and $t-12$

$R^2 = 0.9994$ S.E. $= 0.1523\%$ D.W. $= 1.121$ MEAN RCDP $= 5.697\%$
S.D. RCDP $= 0.732\%$

Rate on Price Commercial Paper (RCP)

$$\text{RCP} = \sum_{i=0}^{8} \alpha_i \text{IPI}_{t-i}/\text{IPI}_t + \sum_{i=0}^{3} \beta_i \text{RTB}_{t-i} + \underset{(2.180)}{0.24097}\,\text{RCDP} + 0.9198\,\text{U}_{-1}. \qquad (13)$$

ALMON DISTRIBUTED LAG WEIGHTS

	t	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
RTB	0.409337	0.235571	0.109426	0.030903		
	(5.404)	(6.443)	(2.770)	(0.9389)		
IPI	1.15744	1.54797	1.33531	0.707351	-0.12391	-0.92230
	(1.695)	(1.987)	(2.033)	(1.379)	(-0.2932)	(-1.835)
	$t-6$	$t-7$	$t-8$			
IPI	-1.42753	-1.35516	-0.39662			
	(-2.006)	(-1.707)	(-0.4702)			

RTB Almon is 2nd degree Constrained to 0 at $t-4$
IPI Almon is 4th degree Constrained to 0 at $t+1$

$R^2 = 0.9270$ S.E. $= 0.1756\%$ D.W. $= 1.611$ MEAN RCP $= 6.156\%$
S.D. RCP $= 1.300\%$.

Long Term Rate—Moody's Baa Corporate Bonds (RBaa)

$$\Delta RBaa = \underset{(4.580)}{0.19631}\,\Delta RTB + \underset{(3.678)}{0.33852}\,\Delta RBaa_{-1} + 0.0783\,\text{U}_{-1} \qquad (14)$$

$R^2 = 0.4079$ S.E. $= 0.1018\%$ D.W. $= 1.985$ MEAN RBaa $= 7.02\%$
S.D. RBaa $= 1.46\%$.

Money Stock (M1)

$$\text{M1} = \text{DDMS} + \text{CURR}. \qquad (15)$$

Required Reserves (RR)[16]

$$\text{RR} = (K_1B_1)\text{M1} + (K_2B_2)\text{M1} + K_3\text{CD} + K_4\text{OTS} + \text{RRND}. \qquad (16)$$

[16] K_1, K_2, K_3, and K_4 are the required reserve ratios against demand deposits at Reserve city and country banks, CD's, and other time and savings respectively. $B1$ and $B2$ are the ratios of demand deposits to $M1$ at Reserve city and country banks respectively.

Total Reserves (TR)

(17) $$TR = EXR + RR.$$

Deposits Less Required Reserves (DEP)

(18) $$DEP = DDMS + TTSC - RR.$$

Total Time and Savings Deposits at All Commercial Banks (TTSC)

(19) $$TTSC = OTS + CD.$$

Total Time and Savings at Member Banks (TTSM)

(20) $$TTSM = 0.7787\,TTSC.$$

Quantity of Treasury Bills Held by Banks (QTBB)

(21) $$QTBB = QTBT - QTBP.$$

Member Borrowings from the Federal Reserve (BORR)

(22) $$BORR = TR - UR.$$

The model imposes market clearing in both the reserves and the bills markets. Interest rates adjust to equilibrate exogenous supplies with quantities demanded. The Treasury bill rate is determined explicitly using the quantity of bills held by banks. The Federal Funds Rate, which clears the reserves market, is estimated to depend upon both the amount of bank borrowings from the Federal Reserve and the amount of unborrowed reserves available. Three additional interest rates are determined endogenously: the primary rate on CD's (RCDP) is estimated as a supply relationship; a reduced form equation is used to determine the rate on prime commercial paper (RCP); and a simple Koyck type term structure equation is used to estimate a long term rate, the rate on Moody's Baa Corporate Bonds (RBaa). The other identities give variables needed to close the model.

2.2. *Strengths and Weaknesses of the Model*

The design of the model, its monthly time frame, and its focus on the U.S. money market, makes it possible to observe and to some extent isolate the sources of fluctuations which influence intermediate monetary control. The use of budget constraints in the public and banking sectors provides some insights into the reaction of the money market to exogenous shocks from the real sectors of the economy. The use of polynomial distributed lags makes it possible to avoid the estimation problems produced by the use of lagged endogenous variables. They also provide information regarding lags in the transmission of monetary policy. Finally, the use of non-seasonally adjusted data avoid the problems of possible bias built in by seasonal factors.

The model does have a number of shortcomings which if corrected would increase its ability to provide insights into the operation of the money market by adding structural information, and, in turn, additional channels for the transmission of monetary policy. The model does not differentiate between the behavior of banks of different sizes which are subject to different reserve requirements against demand deposits. Nor does it have a mechanism for handling cash drains or inflows to the banking system. In fact, except for the quantity of treasury bills held by banks, excess reserves, and required reserves, the asset side of the banking sector's balance sheet is not explored. Thus, banks' portfolio adjustments with respect to loans and long term U.S. Government Securities are not developed; these relationships are not easily identifiable with monthly data.

2.3. *The Linearization*

The model as estimated is almost linear in its original form. Nonlinearities do arise for two reasons. First, the desire to have the model homogeneous in dollar values makes it necessary to impose restrictions through budget constraints, and this implies weighting interest rates and seasonal dummies by either personal income or deposits. Second, the CD market is nonlinear because of the existence of interest rate ceilings imposed by Federal Reserve Regulation Q. The nonlinearity manifests itself in the dummy variable describing the run-off phenomenon in the CD market, as will be described below.

The nonlinearities which arise from the homogeneity of the model were overcome by multiplying the coefficients of endogenously determined independent variables by the mean value of the particular weighting variable calculated over the control period of interest.[17] For example, in the DDMS equation, the Treasury bill rate coefficients are multiplied by the mean of Personal Income calculated over the twelve months of 1971. Thus, in that equation the linearization results in

$$\sum_{i=0}^{9} \beta_i \mathrm{RTB}_{t-i}\mathrm{PI}_t = \sum_{i=0}^{9} (\beta_i \overline{\mathrm{PI}})\mathrm{RTB}_{t-i} \tag{23}$$

The seasonal variables are handled somewhat differently. For example, since PI is exogenous, we can form a series for the seasonal variable from the following relationship:

$$\mathrm{S1}_t = \text{Seasonal Coefficient}_t \cdot \mathrm{PI}_t \tag{24}$$

which is an entirely exogenous series. Calculation of the linearized exogenous variables are shown in Table 1.

The nonlinearity in the CD market is shown explicitly in the equation for the CD primary rate. When the equation was estimated, a test for the occurrence of a CD run-off was made and if no run-off occurred we set RUN = 0 and NORUN = 1.[18] In 1971, the period which we will be using for the control experiments, we know that no run-off occurred. Therefore, we set NORUN = 1 and do not include the dummy variable which pertains to run-off periods.

In order to evaluate the performance of the linearized form of the model, we ran a twelve-period simulation of both the linear and nonlinear forms of the model. The root mean squared errors (RMSE) for the 10 stochastic equations used in

[17] An alternative method is to allow the coefficients to change in each period, i.e., the coefficients would be multiplied by the actual value of the weight variable in each period rather than the mean. This would involve specifying A_{0t}, A_{1t}, B_{1t}, and C_{1t}, $t = 1, \ldots, T$.

[18] The test compares the secondary CD rate with the exogenous Q ceiling for CD's. If the secondary rate is higher than the Q ceiling, it is assumed that a run-off is occurring. For an explicit description of how the CD market works, see Farr, Roberts, and Thomson [9].

TABLE 1

Linearized Exogenous Variables

Variable	Equation
1. $S1_t = SEASONAL_t \cdot PI_t$	(DDMS)
2. $S2_t = SEASONAL_t \cdot PI_t$	(OTS)
3. $S3_t = SEASONAL_t \cdot PI_t$	(CD)
4. $S4_t = SEASONAL_t$	(RFF)
5. $S5_t = SEASONAL_t \cdot DEP_t$	(EXR)
6. $S6_t = SEASONAL_t \cdot PI_t$	(QTBP)
7. $PI1_t = \sum_{i=0}^{4} \alpha_i PI_{t-i}$	(DDMS)
8. $PI2_t = \sum_{i=0}^{4} \alpha_i PI_{t-i}$	(OTS)
9. $PI3_t = \sum_{i=0}^{4} \alpha_i PI_{t-i}$	(QTBP)
10. $ROTS_t = \sum_{i=0}^{8} \alpha_i ROTS_{t-i} \cdot PI_t$	(OTS)
11. $IPI = \sum_{i=0}^{8} \alpha_i IPI_{t-i}/IPI_t$	(RCP)

TABLE 2

Model Error Analysis—1971

Variable	(1) Standard Error	(2) Linearized Model RMSE	(3) Non-Linear Model RMSE	(4) Ratio of Column 2 to 3
DDMS	657	3,130	2,284	1.37
OTS	743	5,291	3,644	1.45
CD	582	1,012	887	1.14
QTBP	710	1,357	1,647	0.81
EXR	67	35	39	0.90
RFF	0.207	0.453	0.421	1.07
RTB	0.248	0.481	0.460	1.05
RCDP	0.152	0.475	0.467	1.02
RCP	0.176	0.471	0.453	1.04
RBaa	0.102	0.397	0.406	0.98

the model and the estimated standard errors are shown in Table 2. In simulation, the forecasting performance of the model depends on the initial conditions, so that the results shown would be different if a different starting point were chosen. A twelve-period simulation was chosen because in the control experiments we are interested in the 12 months of 1971. If a shorter time frame were used in the simulation, the RMSE would probably be lower. This is especially true of the linearized version of the model which uses the mean levels of personal income and deposits as weights in some of the equations.

In three of the ten equations (the quantity of bills held by the public, excess reserves, and the rate on Baa bonds), the linear version has lower RMSE than the nonlinear version. In five of the other equations, the differences in RMSE are less than 15%, the nonlinear version having lower RMSE's. Only two equations, the demand deposit component of the money stock, and other time and savings deposits at all commercial banks, have an RMSE substantially larger in the linear version of the model. This result is due to the high elasticity of DDMS and OTS with respect to personal income. Consequently, the weighting of coefficients by mean personal income causes large RMSE's in these equations.

2.4. *State-Variable Form of the Model*

Before optimization experiments can be performed, the model must be put in the state-variable form:

$$x_{i+1} - x_i = Ax_i + Bu_i + Cz_i \tag{25}$$

with known initial condition

$$x_0 = \xi. \tag{26}$$

x_i is a vector of endogenous variables, u_i a vector of control variables, and z_i a vector of uncontrollable exogenous variables. New state variables must be defined to replace those variables that appear in the model with lags greater than one period. The definitional equations of these variables are then appended to the model.

We will assume that the actual values of the control variables RDIS and UR are the results of, and equal to, the desired levels that were specified by decision makers in the previous period. This will make them true control variables. The control variables as they appear in the Federal Funds Rate and excess reserves equations and in the borrowings identity are lagged by one month.

Another problem which we recognize but shall not deal with at this point is that of the Cochrane–Orcutt serial correlation adjustments which were employed in the estimation. These terms will be omitted in the present formulation of the model since their basic function is to give more efficient estimates of the coefficients in the estimated equations. However, in simulation they are quite important as a mechanism for keeping the equations on track. At a future time, we will experiment with incorporating them into the model.

The state variable form is completed by adding 28 new state variables and their definitional equations to the model. We define the following variables:

$$\begin{aligned}
\mathrm{RTB1} &= \mathrm{RTB}_{-1} & \mathrm{RCDP1} &= \mathrm{RCDP}_{-1}\\
\mathrm{RTB2} &= \mathrm{RTB}_{-2} & \mathrm{RCDP2} &= \mathrm{RCDP}_{-2}\\
\mathrm{RTB}_3 &= \mathrm{RTB}_{-3} & \mathrm{RCDP3} &= \mathrm{RCDP}_{-3}\\
\mathrm{RTB4} &= \mathrm{RTB}_{-4} & \mathrm{RCDP4} &= \mathrm{RCDP}_{-4}\\
\mathrm{RTB5} &= \mathrm{RTB}_{-5} & \mathrm{RCDP5} &= \mathrm{RCDP}_{-5}\\
\mathrm{RTB6} &= \mathrm{RTB}_{-6} & \mathrm{RCDP6} &= \mathrm{RCDP}_{-6}\\
\mathrm{RTB7} &= \mathrm{RTB}_{-7} & \mathrm{RCDP7} &= \mathrm{RCDP}_{-7}
\end{aligned}$$

$$\begin{array}{ll} RTB8 = RTB_{-8} & RCDP8 = RCDP_{-8} \\ RTB9 = RTB_{-9} & RCDP9 = RCDP_{-9} \\ RTB10 = RTB_{-10} & RCDP10 = RCDP_{-10} \\ RFF1 = RFF_{-1} & RCP1 = RCP_{-1} \\ RFF2 = RFF_{-2} & RCP2 = RCP_{-2} \\ RFF3 = RFF_{-3} & RCP3 = RCP_{-3} \\ RBaa1 = RBaa_{-1} & UR1 = UR_{-1} \end{array}$$

The new model is now in this form:

$$x_t = A_0 x_t + A_1 x_{t-1} + B_1 u_{t-1} + C_1 z_{t-1}. \tag{27}$$

There are a total of 46 state variables (18 endogenous variables and 28 new state variables), two control variables, and 15 exogenous and uncontrollable variables.

3. Deterministic Optimization Experiments

3.1. *Formulation of the Problem*

The linear-quadratic tracking problem involves the minimization of the cost functional:

$$J = \tfrac{1}{2} \sum_{i=0}^{N} \{(x_i - \hat{x}_i)' Q (x_i - \hat{x}_i) + (u_i - \hat{u}_i)' R (u_i - \hat{u}_i)\} \tag{28}$$

subject to the constraints of the economic system

$$x_{i+1} - x_i = Ax_i + Bu_i + Cz_i \tag{29}$$

with initial condition $x_0 = \xi$. Equation (29) is just the state-variable form of the econometric model; x_i is the vector of state variables, u_i the vector of control (policy) variables, and z_i a vector of uncontrollable exogenous variables. Equation (27) can be expressed in the form of equation (29) by setting its coefficient matrices equal to:

$$\begin{aligned} I + A &= (I - A_0)^{-1} A_1 \\ B &= (I - A_0)^{-1} B_1 \\ C &= (I - A_0)^{-1} C_1. \end{aligned} \tag{30}$$

In order to conserve space, the A_0, A_1, B_1 and C_1 matrices of the model are not presented here but are available on request.

The vectors $\hat{x}_i$ and $\hat{u}_i$ represent the nominal (ideal) state and control vectors that we would like to track as closely as possible, and we assume that they have been specified for the entire planning period. The matrices Q and R determine the relative penalties for deviations of the target and control variables respectively from their nominal paths. Typically, Q and R are diagonal matrices, although this is not necessary. Varying the weights on the diagonal of Q allows us to place more or less emphasis on monetary versus financial variables.

The nominal trajectories used in the experiments were chosen to reflect a smoothing of the growth paths that actually occurred for monetary aggregates over 1971 with interest rates, excess reserves, and borrowings held constant. This condition was set forth because it was felt that policy makers would like smooth long term growth in the aggregates, and stationary interest rates. Over 1971 the money stock grew at approximately 6 percent, a figure which has generally been interpreted as a long-run target of FOMC policy.

It should be noted that the growth rates which policymakers talk about are for seasonally adjusted data. The model which we use is structured in terms of non-seasonally adjusted data so that the nominal trajectories will not look smooth, although the trajectories for the underlying seasonally adjusted data are constructed assuming smooth growth paths. The nominal trajectories are presented in Table 3. The demand deposit component of the money stock, seasonally adjusted, expands at a 6 percent annual rate compounded monthly. This and all other series are transformed to non-seasonally-adjusted levels using the ratio of non-seasonally adjusted (N.S.A.) to seasonally adjusted (S.A.) data. M1 is formed by adding actual N.S.A. currency to the N.S.A. DDMS nominal path. The nominal paths for other time and savings deposits at commercial banks and negotiable certificates of deposits grow at seasonally adjusted annual rates of 17.5 and 25 percent respectively. These growth rates are close to the actual rates of growth over the historic 12-month period, and are assumed to be compatible with the 6 percent growth in DDMS. TTSC's nominal path is the sum of OTS and CD. The nominal path for total reserves is based upon a growth rate for seasonally adjusted data of 8.25 percent.

The nominal paths for BORR, EXR, RTB, RFF, and RBaa are constant as mentioned above.[19] A constant level of BORR given the nominal path for total reserves yields the nominal path for unborrowed reserves, the major control instrument. The level of nominal borrowings and nominal excess reserves are set near the actual averages for the period. The nominal paths for the interest rate variables which the policy makers are most concerned with are kept stationary because it is felt that in our experiments such an "ideal" strategy would be neutral in its effect on the money market. The same is true for the discount rate, the minor control instrument.[20] The nominal discount rate is above the nominal short term rates, and the Treasury bill rate is set below the Federal Funds Rate. This ordering makes the discount rate a true penalty cost, the discount window a true lender of last resort, and Federal Funds an attractive alternative to Treasury bills.

Although we have specified nominal paths for 11 endogenous variables only a subset of those will have non-zero weights specified in the Q matrix. When a zero weight is assigned to a variable in the Q matrix it does not enter into the objective function (equation (28)). All of the nominal paths are presented here for completeness.

[19] During 1971 none of these variables exhibited a definite trend, therefore, their nominal paths were set at their means.

[20] In the experiments that follow, the discount rate is forced to follow its nominal path. This is done by assigning a very high weight to the corresponding coefficient in the R matrix. We did not make the discount rate an uncontrollable exogenous variable because in some experiments (which are not reported here) a lower weight was assigned to it, allowing it to deviate from its nominal path.

TABLE 3

NOMINAL PATHS

		DDMS		M1	OTS		CD		TTSC
		S.A.	N.S.A.	N.S.A.	S.A.	N.S.A.	S.A.	N.S.A.	N.S.A.
1971	1	166,000	172,304	221,442	207,800	206,883	26,600	26,932	233,815
	2	166,808	165,319	214,419	210,611	210,116	27,099	27,001	237,117
	3	167,620	165,941	215,441	213,461	214,246	27,608	27,510	241,756
	4	168,436	170,018	220,118	216,349	217,429	28,126	27,418	244,847
	5	169,256	165,734	216,234	219,276	220,260	28,654	27,749	248,010
	6	170,080	168,326	219,326	222,243	222,638	29,192	28,199	250,837
	7	170,907	169,254	221,154	225,250	225,350	29,740	28,859	254,209
	8	171,739	168,524	220,424	228,297	228,701	30,298	30,691	259,392
	9	173,575	171,999	223,899	231,386	231,488	30.867	31,355	262,843
	10	173,415	173,217	225,412	234,517	234,415	31,446	32,311	266,726
	11	174,259	175,649	228,449	237,690	236,364	32,036	33,528	269,893
	12	175,108	180,888	234,388	240,906	239,581	32,637	33,126	272,707
1972	1	175,960	182,658	235,258	244,165	242,950	32,350	33,751	276,701
	2	176,816	175,227	227,827	247,468	246,861	33,874	33,674	280,534
	3	177,677	175,899	229,099	250,817	251,834	34,510	34,407	286,241
	4	178,542	180,220	233,820	254,210	255,440	35,158	34,246	289,685
	5	179,411	175,646	229,646	257,650	258,883	35,817	34,633	293,516
	6	180,284	178,500	233,100	261,135	261,547	36,490	35,211	296,758

		TR								
		S.A.	N.S.A.	BORR	EXR	RTB	RFF	RBaa	RDIS	UR
1971	1	29,390	29.488	370	279	4.505	4.253	8.74	4.875	29,958
	2	29,585	29.865	450	200	4.375	4.750	8.50	4.875	29,415
	3	29,781	29.688	450	200	4.375	4.750	8.50	4.875	29,238
	4	29,978	29,872	450	200	4.375	4.750	8.50	4.875	29,422
	5	30,177	30,268	450	200	4.375	4.750	8.50	4.875	29,818
	6	30,377	29,875	450	200	4.375	4.750	8.50	4.875	29,425
	7	30,578	30,486	450	200	4.375	4.750	8.50	4.875	30,036
	8	30,781	30,493	450	200	4.375	4.750	8.50	4.875	30,043
	9	30,985	30,715	450	200	4.375	4.750	8.50	4.875	30,265
	10	31,190	31,168	450	200	4.375	4.750	8.50	4.875	30,718
	11	31,397	31,380	450	200	4.375	4.750	8.50	4.875	30,930
	12	31,605	31,689	450	200	4.375	4.750	8.50	4.875	31,239
1972	1	31,815	32,909	450	200	4.375	4.750	8.50	4.875	32,459
	2	32,026	32,336	450	200	4.375	4.750	8.50	4.875	31,886
	3	32,238	32,126	450	200	4.375	4.750	8.50	4.875	31.676
	4	32,451	32,359	450	200	4.375	4.750	8.50	4.875	31,909
	5	32,667	32.649	450	200	4.375	4.750	8.50	4.875	32,199
	6	32,883	32,366	450	200	4.375	4.750	8.50	4.875	31,916

3.2. *Deterministic Policy Experiments*

The Federal Reserve Board may, as part of its objectives, try to reach target values for both the money stock and some interest rate simultaneously. This may be impossible even in a deterministic world, and in fact a trade-off curve could be derived which relates the minimum achievable root-mean-square deviation from the target path for the money stock with that for the interest rate. In the first set of experiments, a trade-off curve will be derived for the objectives of controlling the money stock (M1) and the Treasury bill rate (RTB). The trade-off will be measured using the root-mean-square deviations as defined in equations (31) and (32) below, where a star refers to an optimal path.

$$\text{(31)} \qquad \mathrm{RMSD_{M1}} = \left(\frac{1}{N}\sum_{t=0}^{N} [\mathrm{M1}_t^* - \widehat{\mathrm{M1}_t}]^2\right)^{1/2}$$

$$\text{(32)} \qquad \mathrm{RMSD_{RTB}} = \left(\frac{1}{N}\sum_{t=0}^{N} [\mathrm{RTB}_t^* - \widehat{\mathrm{RTB}_t}]^2\right)^{1/2}.$$

We will calculate these root-mean-square deviations only over the second six months of the planning period. There are two reasons for this. First, we would like to allow six months for the target variables to get "on track," because of the lags inherent in the transmission of monetary policy. Second, even though we allow the optimal control program to run for 18 months, we ignore the last six months of results because of possible end-point problems that are inherent in a finite horizon optimization problem.

A single trade-off curve is obtained by performing several optimization experiments in which different weights are placed on the Q matrix coefficients for the money stock and the interest rate. All of the other coefficients in the Q matrix are set to zero. In the R matrix, a very high cost is associated with the discount rate, but almost no cost is attached to the level of unborrowed reserves, so that this variable is allowed to move freely. For any particular combination of weights on M1 and RTB, the optimal solution will give us one point on the trade-off curve.

The trade-off curve for the first set of experiments is shown graphically in Figure 1. The corresponding results are presented in Table 4. Let us examine some of the more obvious aspects of these results. First, note that it is very difficult to come close to the nominal path for the money stock; however, it is not so difficult to hit the interest rate exactly. This can be seen in experiments A and F respectively. In experiment A, a very high cost is attached to the money stock, and no cost to the interest rate. Nonetheless, the root-mean-square deviation for the money stock is 713 million dollars. In experiment F, however, where a high cost is attached to the interest rate and no cost is attached to the money stock, we find that the root-mean-square deviation for the interest rate is less than two basis points.

Second, note that when a high cost is attached to the money stock, the trajectories for variables other than M1 behave wildly. Interest rates, borrowings, and unborrowed reserves all oscillate between extreme values that are sometimes even negative. This may in part be a limitation of the linearized model, but it seems to indicate that it is rather difficult to force the money stock to follow its nominal path exactly, at least on a month-to-month basis. All of this seems to be a preliminary indication that (within the context of this model) it might be preferable for the monetary authority to focus more attention on interest rates rather than on the money stock.

The results that occurred when large relative costs were attached to the money stock seemed to us to be unreasonable. Therefore, we ran a second set of experiments in which some penalty is imposed when borrowings deviates from its nominal path. It was felt that this modification would make the results more

TABLE 4

RESULTS OF TRADE-OFF BETWEEN M1 AND RTB

Experiment	Q(M1)	Q(RTB)	Root-Mean-Square Deviation				
			M1	RTB	BORR	UR	RDIS
A	1×10^6	0	713	86.934	523,228	623,991	0.017
B	2×10^4	1×10^8	1,494	10.815	62,019	72,215	0.001
C	2×10^3	1×10^8	1,958	5.076	12,488	15,706	0
D	200	1×10^8	2,478	3.115	5,796	6,968	0
E	20	1×10^8	2,848	1.654	2,921	3,660	0
F	0	1×10^8	4,501	0.017	467	430	0

Note: In each experiment R(UR) = 1.0 and R(RDIS) = 1×10^{11}. Root-Mean-square deviations for M1, BORR, and UR are in millions of dollars, while those for RTB and RDIS are in percent per year. In experiment D, M1 and RTB are weighted equally (after adjusting for their mean values).

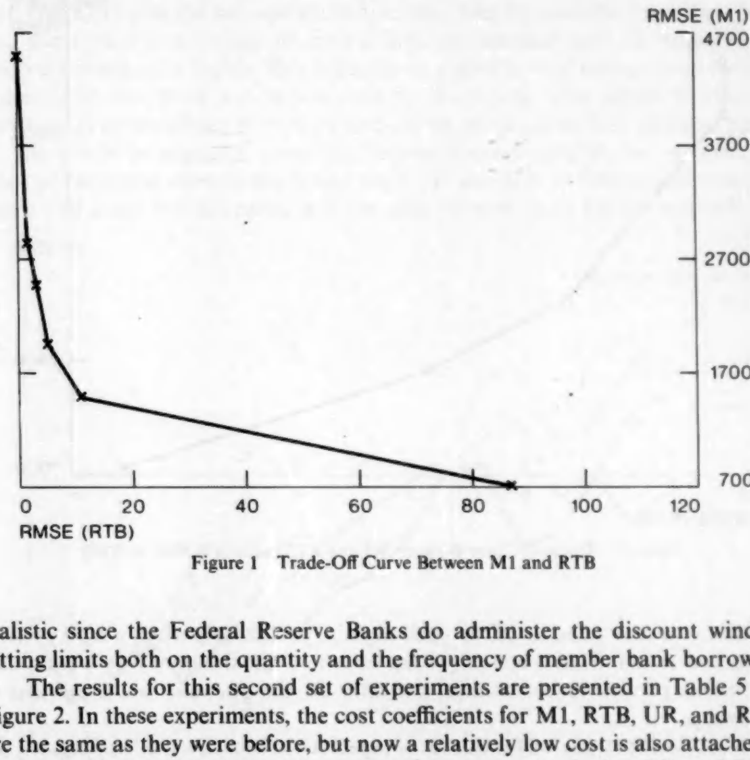

Figure 1 Trade-Off Curve Between M1 and RTB

realistic since the Federal Reserve Banks do administer the discount window, setting limits both on the quantity and the frequency of member bank borrowing.

The results for this second set of experiments are presented in Table 5 and Figure 2. In these experiments, the cost coefficients for M1, RTB, UR, and RDIS are the same as they were before, but now a relatively low cost is also attached to the level of borrowings (about 5 percent of the costs attached to M1 and RTB).

In examining these results, we first note that attaching a cost to borrowings seems to, at least in part, clear up some of the strange results that occurred in experiments A, B, and C before. Overall, the controllability of M1 does decrease somewhat, but this is expected. The interest rate, borrowings, and unborrowed reserves all follow their nominal paths much more closely than they did before.

TABLE 5

RESULTS OF TRADE-OFF BETWEEN M1 AND RTB (WITH Q(BORR) = 5,000)

Experiment	Q(M1)	Q(RTB)	Root-Mean-Square Deviation				
			M1	RTB	BORR	UR	RDIS
A	1×10^6	0	1,967	5.533	12,162	11,321	0.629
B	2×10^4	1×10^8	2,826	2.828	4,942	6,105	0.253
C	2×10^3	1×10^8	3,453	1.597	2,538	3,141	0.132
D	200	1×10^8	5,067	0.210	474	424	0.025
E	20	1×10^8	5,514	0.192	62	62	0.003
F	0	1×10^8	5,572	0.222	11	229	0.000

Note: R(UR) = 1.0 and R(RDIS) = 1×10^{11}.

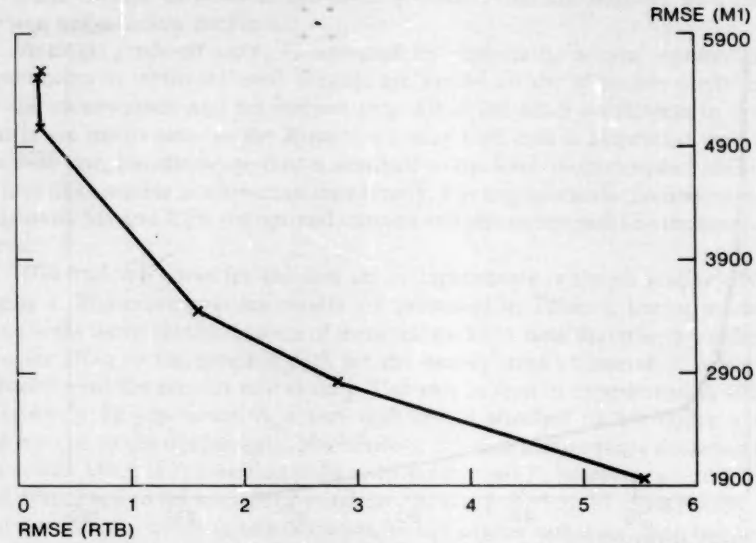

Figure 2 Trade-off Curve Between M1 and RTB (with Q(BORR) = 5000)

The discount rate wanders off its path slightly, particularly when a high penalty is attached to M1. This is not surprising since now the discount rate is penalized less heavily relative to borrowings, and after all, the discount rate is the cost of borrowing.

Note that the trade-off for this set of experiments is backward-bending, i.e., the root-mean-square deviation for the bill rate does not decrease monotonically as we increase its relative cost. The same behavior is also true of unborrowed reserves, whose root-mean-square deviation also does not decline monotonically. The reason for this is that, as we decrease the cost on M1 to zero, the effective relative cost on borrowings increases. Thus, when in experiment F we insist on a level of borrowings that stays close to its nominal path, we are in effect requiring that the level of unborrowed reserves be used through its influence on other markets to make this possible. We note also that the discount rate follows its nominal path exactly in this experiment, so that the only instrument that can be used to control borrowings is the level of unborrowed reserves.

Again, the results seem to indicate that the interest rate might be a better target variable than the money stock. We can see from the results that the loss of controllability of M1 (as we decrease its relative cost) is more than offset by an increase in controllability of RTB, and furthermore that this increase in controllability of RTB is accompanied by more reasonable behavior in the levels of borrowings and unborrowed reserves.

In Figures 3 through 10, on the following pages, we have plotted the results for experiment E both when there is zero weight on borrowings and a non-zero weight on borrowings. In particular, we look at the endogenous variables M1, RTB, RFF, OTS, CD, BORR, and RBaa, as well as the policy variable UR. In each graph, we plot the two optimal trajectories and the nominal trajectory. Note that when there is a weight on borrowings, the optimal path for unborrowed reserves is somewhat higher. This is because a higher level of unborrowed reserves is needed so that there will be less need for borrowing. This higher level of unborrowed reserves allows both OTS and CD to get closer to their nominal paths.

As would be expected, when the Federal Reserve supplies less reserves, i.e., when unborrowed reserves are lower, banks, in an effort to meet reserve requirements and other commitments, will not only borrow more heavily but will also

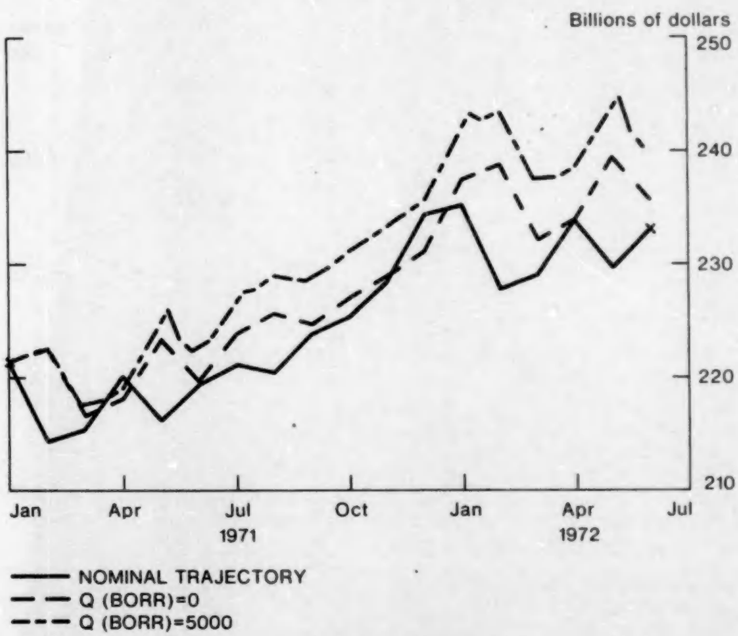

Figure 3 Money stock (M1)

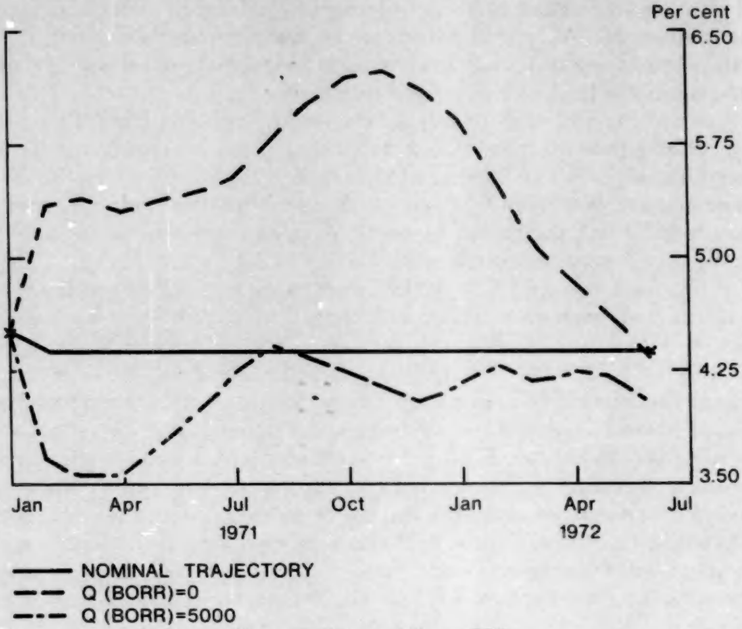

Figure 4 Treasury bill rate (RTB)

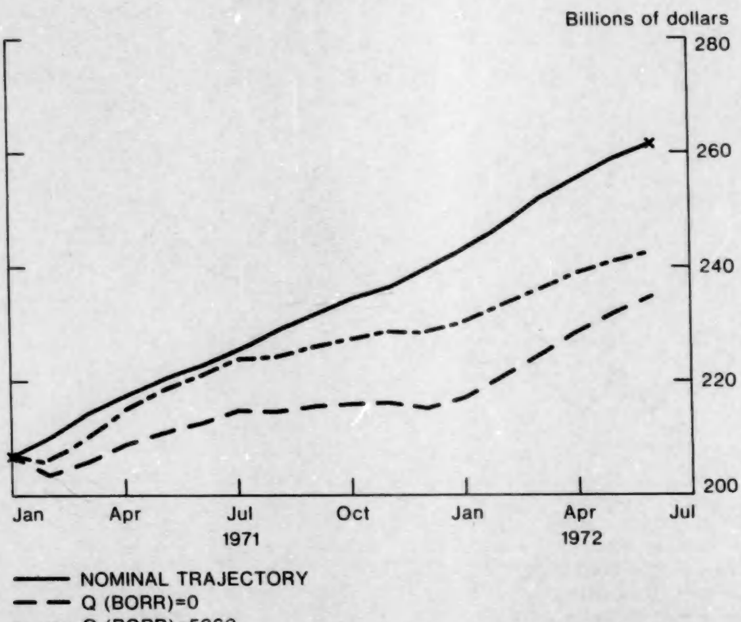

Figure 5 Other time and savings deposits (OTS)

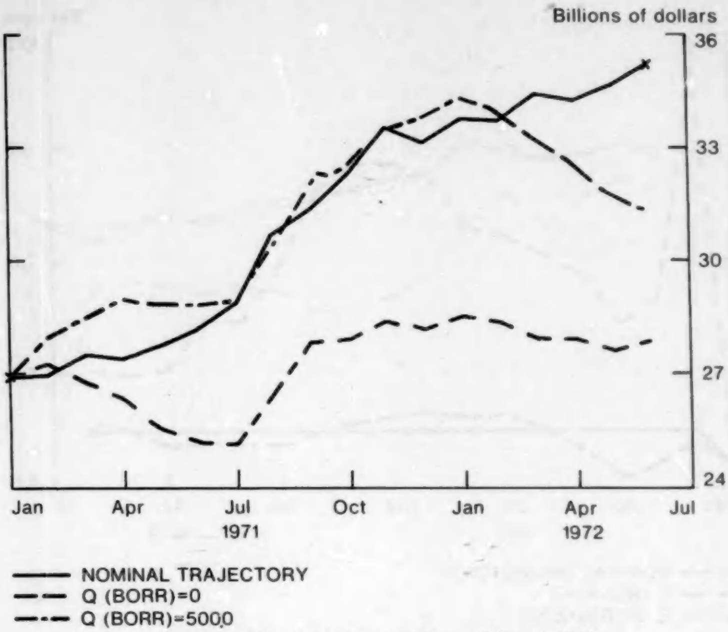

Figure 6 Negotiable time certificates of deposit (CD)

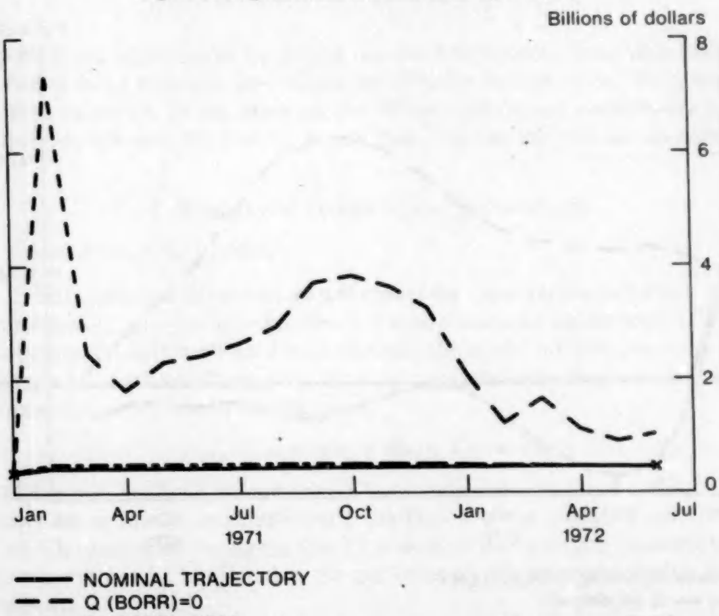

Figure 7 Member bank borrowing from Federal Reserve banks (BORR)

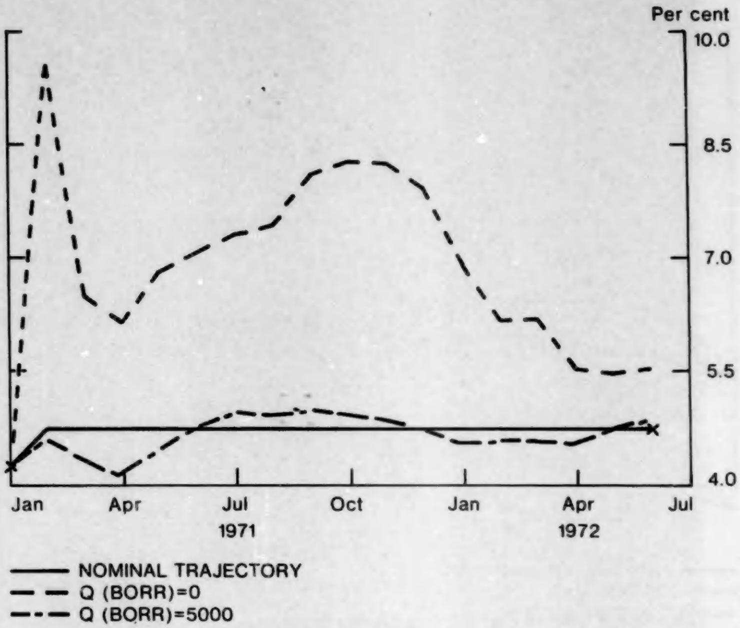

Figure 8 Federal funds rate (RFF)

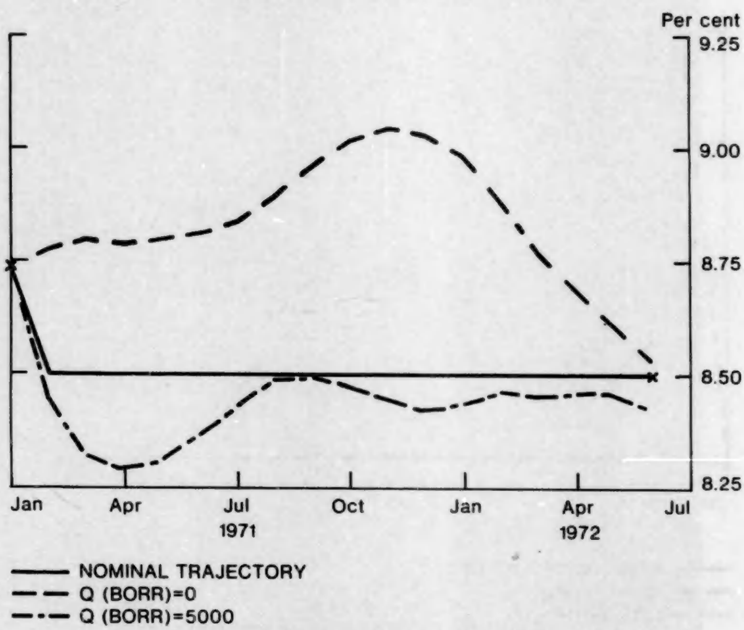

Figure 9 Baa corporate bond rate (RBaa)

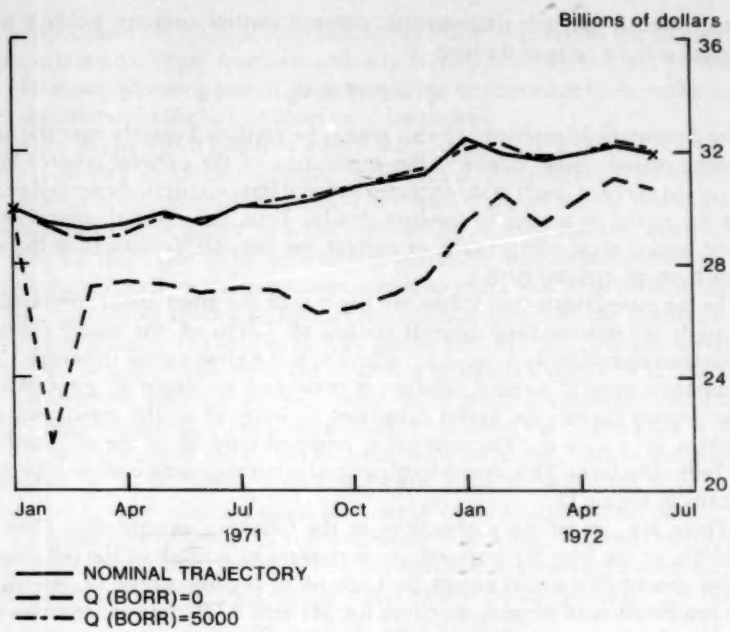

Figure 10 Unborrowed reserves (UR)

make portfolio adjustments by selling interest-bearing securities, thus lowering the price of those securities and raising the effective interest rates. We indeed see this effect uniformly in the experiments. When unborrowed reserves are lower, the Treasury bill rate, the Federal Funds Rate, and the Baa rate are all higher.

4. Stochastic Optimization Experiments

4.1. *Formulation of the Problem*

In this section of the paper, we will repeat the experiments performed earlier, but now taking into account the effects of random shocks on the model. We will assume that the only random shocks affecting the model are additive noise terms which are not autocorrelated, thus allowing certainty equivalence to be invoked. Our model is now given by equation (33).

$$x_{i+1} - x_i = Ax_i + Bu_i + Cz_i + D\varepsilon_i. \tag{33}$$

The error vectors ε_i in equation (33) are generated by either adding or subtracting the residuals obtained from a simulation of the model. These residuals will only be generated during the first 12 months of the 18 month planning period, since we will not be interested in the performance of the model during the last six months.

The optimal solution to this stochastic control problem is obtained by applying the deterministic optimal control solution to the model in a closed-loop manner. Recall that the deterministic optimal control solution yields a linear feedback rule, i.e., it is of the form:[21]

$$u_i^* = F_i x_i^* + G_i. \tag{34}$$

In the deterministic problem, x_i^* can always be predicted exactly over the entire planning period. Now, however, the application of the optimal control in the first period may not result in the expected optimal state vector in the second period, since the model is subject to random shocks. Thus, the optimal control in the second period must compensate, or correct, for possible deviations in the state vector from its optimal path.

In the experiments that follow, we begin with the given initial condition x_0, and apply the deterministic optimal control u_0^*. Given u_0^*, the model generates x_1 and then computes $\tilde{x}_1 = x_1 + \varepsilon_1$, where ε_1 is the noise vector in period 1. The deterministic optimal control solution is then used to obtain u_1^* *given this* $\tilde{x}_1$. In the second period, the model calculates x_2 using u_1^* as the input, and then computes $\tilde{x}_2 = x_2 + \varepsilon_2$. This process is repeated until all of the u_i^*'s and $\tilde{x}_i$'s have been calculated. This closed-loop optimal control process is shown diagrammatically in Figure 11.

There are two primary objectives in the following experiments. First, we would like to see how the trade-off curve changes as a result of the influence of random shocks. We would expect the trade-off to become worse, i.e., no matter what combination of weights we chose for M1 and RTB, the root-mean-square deviations for both would be larger. The question, however, is how much worse? As we will see, if the monetary authority allows itself more flexibility with respect to movements in the control instrument, i.e., in unborrowed reserves, then the trade-off curve is not very much worse at all. What we want to demonstrate as the second objective of these experiments is exactly this point, i.e., that the monetary authority must allow itself greater flexibility with respect to the movements of its instrument variables.

4.2. *Stochastic Policy Experiments*

In our experiments we will use as values for the error vectors ε_i in equation (33) the residuals generated from a simulation of the model.[22] Two experiments

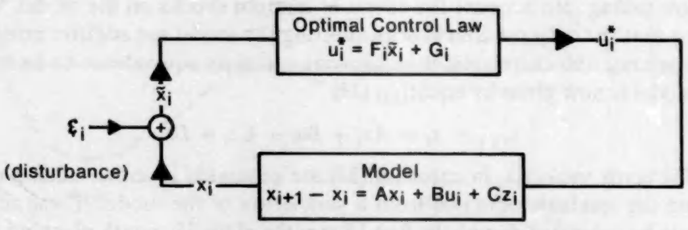

Figure 11 Closed loop control

[21] See Pindyck [16].

[22] This method was an expedient alternative to performing a set of Monte Carlo experiments. It should also be pointed out that one can analytically obtain the expected sum of squares of deviations of the variables from their target paths, as shown by Chow [7].

will be performed; in the first the residuals will be added and in the second they will be subtracted. These residuals will only be generated for the first 12 months of the 18-month planning period, since we will not be interested in the performance of the model during the last six months of the period.

In the results which follow, a weight was attached to the level of borrowings, as in the second set of deterministic optimization experiments. This should be kept in mind when comparing results. The trade-off curves for the two stochastic cases are presented together with the trade-off curve from the deterministic case in Figure 12. Note that all of the trade-off curves are very close together, so that the presence of random shocks does not seem to result in a large deterioration of the optimal control results, as long as the optimal solutions are calculated in a closed loop manner. The reason for this can be seen by looking at the movement in the level of unborrowed reserves, as shown in Tables 6 and 7. What we find is that unborrowed reserves generally must move more dramatically in order to attain policy objectives. This is particularly true when more emphasis is placed on M1. This is another indication that following an interest rate target might be preferable for the monetary authority. When a heavier emphasis is placed on interest rates, unborrowed reserves stays much closer to its nominal path.

To summarize these results, it is interesting to note that the closed loop control is self-correcting, so that the trade-off is not substantially worsened as long as new observations are used in making the next optimal policy decision. The Federal

TABLE 6

RESULTS OF TRADE-OFF BETWEEN M1 AND RTB WITH Q(BORR) = 5,000 AND DISTURBANCES ADDED

			Root-Mean-Square Deviations				
Experiment	Q(M1)	Q(RTB)	M1	RTB	BORR	UR	RDIS
A	1×10^6	0	1,865	6.131	13,679	14,992	0.710
B	2×10^4	1×10^8	2,734	2.744	5,129	6,567	0.263
C	2×10^3	1×10^8	3,189	1.558	2,477	3,431	0.129
D	200	1×10^8	4,582	0.341	465	774	0.024
E	20	1×10^8	4,999	0.305	60	242	0.003
F	0	1×10^8	5.054	0.319	10	180	0.000

Note: R(UR) = 1.0 and R(RDIS) = 1×10^{11}.

TABLE 7

RESULTS OF TRADE-OFF BETWEEN M1 AND RTB WITH Q(BORR) = 5,000 AND DISTURBANCES SUBTRACTED

			Root-Mean-Square Deviations				
Experiment	Q(M1)	Q(RTB)	M1	RTB	BORR	UR	RDIS
A	1×10^6	0	2,073	4.968	10,718	11,722	0.554
B	2×10^4	1×10^8	2.945	2.921	4,835	5,688	0.248
C	2×10^3	1×10^8	3,761	1.648	2,616	2,862	0.136
D	200	1×10^8	5,570	0.171	498	111	0.026
E	20	1×10^8	6.044	0.207	64	523	0.003
F	0	1×10^8	6,105	0.242	10	594	0.000

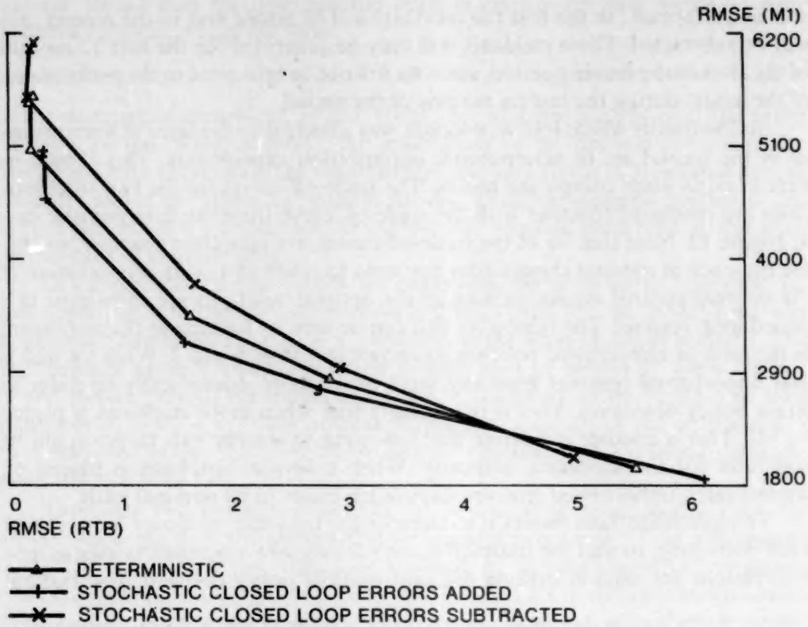

Figure 12 Trade-off curves, stochastic experiments

Reserve Board now is operating in a very different way but may be arriving at much the same result. As is evidenced by the published record of policy actions of the FOMC (starting with the meeting of February 15, 1972), the existence of random shocks in the economy has resulted in the specification of a range of acceptable values for targeted variables.[23] The Fed's staff transforms the specified ranges on target variables into an appropriate range within which the policy instruments may fluctuate in order to meet policy objectives. The range on acceptable values for policy variables indicates that current policy is predicted on the assumption that policy instruments must be allowed to fluctuate so as to compensate for random disturbances in the economy. This same necessary condition holds if one formulates policy using optimal control. The root-mean-square deviations for unborrowed reserves are in general larger for the stochastic experiments than they are for the deterministic ones. This can be seen by looking at Figure 13, which shows that the optimal paths for unborrowed reserves in the two stochastic experiments (E) bound the optimal path for the corresponding deterministic experiment.

5. Summary and Conclusions

Let us summarize our results and their possible implications for policy making. First, we have observed that the deterministic closed loop control law adequately corrects for random shocks, although more freedom of movement is

[23] The record of policy actions is published periodically in the *Federal Reserve Bulletin.*

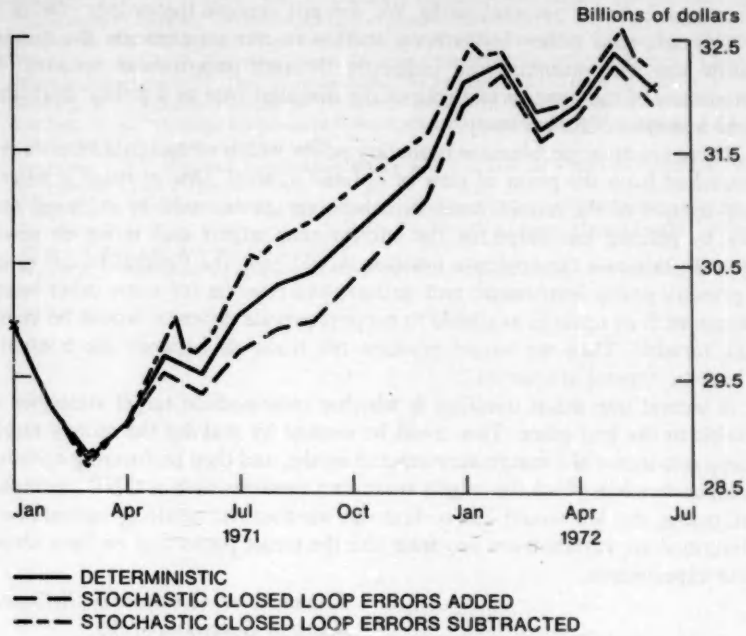

Figure 13 Comparison of UR paths stochastic and deterministic experiments

required with respect to the policy instrument.[24] We did not, of course, take into account our imperfect knowledge of the true values of the model's coefficients when obtaining our stochastic control solutions. If the estimated value of critical coefficients have large standard errors, this could decrease the precision of our control. Also, we have not explored fully the limitations inherent in the model's linearity. Our results might be less meaningful if the economy were experiencing rapid structural change.

In both the deterministic and stochastic cases, we find that the monetary aggregate M1 can indeed be closely controlled, but only at the great expense of considerable fluctuations in other variables. The problem does not occur when attention is focused primarily on interest rates as the policy objective. Interest rates can be controlled very closely without much loss in the control of other variables. Note that we are *not* saying that it is best to focus on interest rates from the point of view of overall stabilization policy. If, however, interest rates are the intermediate targets of the monetary authority, then precise control becomes easier to attain.

We also found that monetary control is best achieved by administering the discount window to some extent (i.e., by placing some cost on the deviations of

[24] The effects of random shocks are much more serious if the deterministic control law is applied in an open loop manner, i.e., without observing the state-vector each period. We ran one set of stochastic experiments using open loop control, and found the root-mean-square deviations for most variables to be 50% to 100% larger than in the closed loop case. This was consistent with Chow's findings [7] using a small macro-econometric model.

borrowings from its nominal path). We did not explore the proper role of the discount rate as a policy instrument, so that in our experiments the discount window was only administered indirectly through unborrowed reserves. The examination of the appropriate role of the discount rate as a policy instrument will be a subject of future study.

There are other problems in monetary policy which we feel could fruitfully be approached from the point of view of optimal control. One of these is whether closer control of the money stock or other aggregates could be achieved more easily by placing less emphasis on interest rate targets and more on reserve targets. In this case the objective function would have the Federal Funds rate as the primary policy instrument, and unborrowed reserves (or some other reserve measure, such as reserves available to support private deposits) would be made a target variable. Then we would examine the trade-off between the control of M1 and the control of reserves.

A second important question is whether intermediate target strategies are desirable in the first place. This could be studied by making the money market model a sub-sector of a macro-econometric model, and then performing optimization experiments in which the targets are macro variables such as GNP, unemployment, prices, etc. We would like to find out whether the resulting optimal paths for intermediate variables are anything like the target paths that we have chosen for our experiments.

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