These anticipation equations are in one sense very general, in another, very special. They require that anticipations be determined entirely by the past history of the particular variable in question, not by other past history or other currently observed phenomena. These equations deny any "autonomous" role to anticipations. These equations, or preferable alternatives to them, are not directly related to the monetary issues that are the main concern of this paper, which is why I have treated them so summarily. Their only function here is to close the system.

One subtle problem in this kind of a structure, in which we have identified the absence of a discrepancy between actual and anticipated values as defining long-period equilibrium, is to assure that the feedback relations defined by equations (53)–(56), as well as the other functions, are consistent with the expanded system of Walrasian equations which specify the long-term equilibrium values. At least some values are implicitly determined in two ways: by a feedback relation such as equations (53) and (56), and by the system of long-run equilibrium equations. The problem is to assure that at long-run equilibrium these two determinations do not conflict.

In our empirical work, we have generally used a particular form of anticipated function, namely, one which defines the anticipated values as a declining weighted average of past observed values. For example, a specific form of equation (55) is

$$y^*(t) = \beta \int_{T}^{T-\omega} y(T) dT,$$

where $\alpha$ and $\beta$ are parameters, $\alpha$ defining the long-term rate of growth, and $\beta$, the speed of adjustment of anticipations to experiences (Friedman 1957, pp. 142–47).

13. An Illustration

It may help to clarify the general nature of this theoretical approach if we apply it to a hypothetical monetary disturbance.

Let us start with a situation of full equilibrium with stable prices and full employment and with output growing at, say, 3 percent per year. For simplicity, assume that the income elasticity of demand for money is unity, so that the quantity of money is also growing at the rate of 3 percent per year. Assume also that money is wholly non-interest-bearing fiat money and that its quantity can be taken as autonomous.
Assume that there is a shift at time $t = t_0$ in the rate of growth of the quantity of money from 3 percent per year to, say, 8 percent per year and that this new rate of growth is maintained indefinitely. Figure 1 shows the time path of the money stock before and after time $t_0$. These figures are not drawn strictly to scale. For emphasis, they exaggerate the difference in the slopes of the lines before and after $t_0$.

a) Long-Run Equilibrium

Let us first ask what the long-run equilibrium solution will be. Clearly, after full adjustment, nominal income will be rising at 8 percent per year. If, for the moment, we neglect any effect of this monetary change on real output and the rate of growth of output, this means that prices would be rising at 5 percent per year. It might therefore seem as if the equilibrium path of nominal income would duplicate that of the quantity of money in figure 1 (redrawn as the solid plus dashed line in fig. 2). But this is not the case. With prices rising at the rate of 5 percent per year and, at equilibrium, with this price rise fully anticipated by everyone, it is now more costly to hold money. As a result, equation (7) would...

$$\log M$$

\[ \text{Time Path of Money Stock Before and After Time } t_0 \]

\[ \text{FIGURE 1} \]

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would indicate a decline in the real quantity of money demanded relative to income, that is, a rise in desired velocity. This rise would be achieved by a rise in nominal income over and above that required to match the rise in the nominal quantity of money. The equilibrium path of nominal income would be like the solid line in figure 2 rather than the dashed line.

If equilibrium real output and the rate of growth of real output were unaffected by the monetary change, as I have so far assumed, the equilibrium path of prices would be the same as that of nominal income, except that it would have a slope of 3 percent per year less, to allow for the growth in real income. However, equilibrium real output will not be unaffected by this monetary change. The exact effect depends on just how real output is measured, in particular whether it includes or excludes the nonpecuniary services of money. If it includes them, as in principle it should, then the level of real output will be lower after the monetary change than before. It will be lower for two reasons: first, the higher cost of holding cash balances will lead producers to substitute other resources for cash, which will lower productive efficiency; second,
the flow of nonpecuniary services from money will be reduced (Fried-}
man 1969, pp. 14—15). For both reasons, the price level of output will
have to rise more than nominal income—a solid line and a dashed line
like those for nominal income in figure 2 would be farther apart verti-
cally for prices of final products than for nominal income.

It is harder to be precise about the equilibrium rate of growth, since
that depends on the particular growth model. What is clear is that the
aggregate stock of nonhuman capital, including money, will be lower
relative to human capital, but that the aggregate stock of physical (non-
money) capital will be higher, so that the real yield on capital (essen-
tially our _r_ of equation [7]) will be lower. The nominal interest rate
(the _r_ of equation [7]) will equal this real yield plus the rate of change
in prices, so it will be higher. If these changes have any effect on the
rate of growth of real output, they will tend to reduce it, so that the
equilibrium price level of final products will not only be higher relative
to its initial value than the equilibrium level of nominal income; it may
also rise more rapidly (Stein 1966; Johnson 1967a, 1967b; Marty
1968). For simplicity, I shall neglect this possibility and assume
that the equilibrium rate of rise in prices is 5 percent per year.

b) The Adjustment Process

So much for the equilibrium position. What of the adjustment process?

This description of the equilibrium position already tells us one
thing about the adjustment process. In order to produce the shift in the
equilibrium path of nominal income from the dashed to the solid line,
nominal income and prices must rise over some period at a faster rate
than the final equilibrium rate—at a faster rate than 8 percent per
year for nominal income and 5 percent per year for prices. There must,
that is, be a cyclical reaction, an overshooting, in the rate of change in
nominal income and prices, though not necessarily in their levels.

How will this adjustment process be reflected in my theoretical sketch
of the adjustment process? The shift in \((d \log M^e)/(dt)\) at time _t_ from
3 percent to 8 percent introduces a discrepancy of positive sign into the
second term in parentheses of equation (48), while initially leaving the
third term in parentheses unchanged. As a result, \((d \log Y)/(dt)\) will
increase, exceeding \([(d \log Y)/(dt)]^*\), which, viewed in this transitional
process as an anticipated value rather than as a long-run equilibrium
value, is unchanged from the prior long-run equilibrium value. How
rapidly the rate of growth of nominal income rises depends partly on the
value of _ψ_, the coefficient indicating speed of adjustment, and partly on
the demand function for money. If the latter depends only on anticipated values (that is, if all the variables in equation (7) have asterisks), 
\((d \text{ log } M^0)/(dt)\) will initially be unchanged, so everything will depend on \(\psi\), which might have any value, from zero, meaning no adjustment, to a value higher than unity, meaning that nominal income would rise initially by more than 5 percent per year.36

Whatever the rate of rise in nominal income, it will be divided into a rise in prices and in output, in accordance with equations (45) and (46). If \(\alpha\) is less than unity, both real output and prices will start rising, their relative rates depending on the size of \(\alpha\).

The rising prices and nominal income will start affecting anticipated rates of change, through equations (53) and (56), feeding back into (48) and (45) and (46).

All of this is so at time \(t_0\), with no effect on the levels of any of the variables. However, as the process continues, the levels start being affected. In equation (48), \(\log M^3\) comes to exceed \(\log M^0\), so the second term of equation (48) adds to the upward pressure on \((d \text{ log } Y)/(dt)\), making for a speeding up in the expansion of nominal income. In equations (45) and (46), \(\log y\) comes to exceed \(\log y^*\), thus increasing the fraction of income increase absorbed by prices and reducing the fraction absorbed by output. The changed levels of \(y\) and \(P\) feed into equations (55) and (56) and so start altering \(y^*\) and \(P^*\).

The changes in all of the variables now start affecting the demand functions for money, both directly, as these variables enter the demand functions, and indirectly, as they affect other variables, such as interest rates, which in turn enter the demand functions. As a result, \((d \log M^0)/(dt)\) and \(M^0\) in equation (48) will start to change. The process will, of course, finally be completed when the relevant measured variables are all equal to their permanent counterparts and these equal the long-run equilibrium values discussed above.

It is impossible to carry much farther this verbal statement of the solution of an incompletely specified system of simultaneous differential equations. The precise adjustment path depends on how the missing elements of the system are specified and on the numerical values of the parameters, but perhaps this much is enough to give the flavor of the kind of adjustment process they generate, and to indicate why this process is necessarily cyclical.

What is the reflection in these equations of the point made in the second paragraph of this section, namely, that \((d \text{ log } Y)/(dt)\) and \((d
log \( P/(dt) \) must, during the transition, average higher than their final long-term equilibrium values? Consider equation (48). Suppose that over a period the average value of \( (d \log Y)/(dt) \) and \( (d \log P)/(dt) \) had been 8 percent per year and 5 percent per year, respectively. Suppose the anticipations functions (53) and (56) were such that this was fully reflected in anticipated values. Then, as we have seen, although \( M^s \) would have risen at the rate of 5 percent per year, \( M^d \) would not have; so the final term in equation (48) would not be zero, even though the middle term on the right-hand side might be. Hence, \( (d \log Y)/(dt) \) would exceed \( [(d \log Y)/(dt)]^* \), which by assumption is at its long-run equilibrium value; so full equilibrium would not have been attained.

Figure 3 summarizes various possible adjustment paths of \( (d \log Y)/(dt) \) consistent with the theory sketched. The one common feature of all of them is that the area above the 8 percent line must exceed the

**FIGURE 3**

*Possible Adjustment Paths of Rate of Change in Nominal Income*
area below. In principle, of course, still other paths are possible. For example, it is conceptually possible for the adjustment to be explosive rather than damped. Restricting ourselves to damped paths is an empirical judgment.

14. Conclusion

In concluding this discussion of a theoretical framework, it may be worth stating that it is not a framework special to me or to those economists who view the operation of the economy in terms of the quantity theory either in its simple form or in the form of the monetary theory of nominal income. No doubt other economists would expand the framework differently, stress different parts of it, elaborate points I have skimmed over, and skim over points I have elaborated. But almost all economists would accept the framework, and this is true even, I believe, of the least thorough part, the sketch of the adjustment process in the preceding two sections.

One purpose of setting forth this framework is to document my belief that the basic differences among economists are empirical, not theoretical: How important are changes in the supply of money compared with changes in the demand for money? Are transactions variables or asset variables most important in determining the demand for money? How elastic is the demand for money with respect to interest rates? With respect to the rate of change in prices? When changes in demand or supply occur that produce discrepancies between the quantity of money that the public holds and the quantity it desires to hold, how rapidly do these discrepancies tend to be eliminated? Does the adjustment impinge mostly on prices or mostly on quantities? Is the adjustment process cyclical or asymptotic? Is the adjustment to sharp changes over short periods different in kind or only in degree from the adjustment to slower changes over longer periods? How long does it take for people to alter their anticipations in light of experience?

Much of the controversy that has swirled about the role of money in economic affairs reflects, in my opinion, different implicit or explicit answers to these empirical questions. The reason such differences have been able to persist is, I believe, that full adjustment to monetary disturbances takes a very long time and affects many economic magnitudes. If adjustment were swift, immediate, and mechanical, as some earlier quantity theorists may have believed, or, more likely, as was attributed to them by their critics, the role of money would be clearly and sharply etched even in the imperfect figures that have been avail-