monetary growth cannot be satisfactorily explained by the monetary theory of nominal income. If these periods were cut out of the historical record, my impression is that the model would fit the rest of the record very well—not of course without error but with errors that are on the modest side as aggregate economic hypotheses go.

Periods just after turning points can, I believe, be explained best by incorporating two elements so far omitted. The first is a revision of equation (14) to allow for a difference between actual and desired money balances, as in equation (48), below. The second is a weakening of equation (25) to permit a stronger liquidity effect on interest rates.

12. The Adjustment Process

The key need to remedy the defects common to all the models I have sketched is a theory that will explain (a) the short-run division of a change in nominal income between prices and output, (b) the short-run adjustment of nominal income to a change in autonomous variables, and (c) the transition between this short-run situation and a long-run equilibrium.\(^3\)

In the rest of this paper, the central idea I shall use in sketching the direction in which such a theory might be developed is the distinction between actual and anticipated magnitudes or, to use a terminology that need not be identical but that I shall treat for this purpose as if it is, between measured and permanent magnitudes. At a long-run equilibrium position, all anticipations are realized, so that actual and anticipated magnitudes, or measured and permanent magnitudes, are equal.\(^2\)

I shall regard long-run equilibrium as determined by the earlier quantity-theory model plus the Walrasian equations of general equilibrium. In a full statement, the earlier model should be expanded by including wealth in the consumption and liquidity-preference functions.

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\(^1\) Still other parts of the theoretical framework are developed more fully in the course of the empirical analysis of some of the issues raised in the other chapters of the book from which this paper is abstracted.

\(^2\) Note that the equality of actual and anticipated magnitudes is a necessary but not a sufficient condition for a long-run equilibrium position. In principle, actual and anticipated magnitudes could be equal along an adjustment path between one equilibrium position and another. The corresponding proposition is more complicated for measured and permanent magnitudes and depends on the precise definition of these terms. However, since we shall be considering a special case in which the stated condition is treated as both necessary and sufficient for long-run equilibrium, these complications can be bypassed.
and the capital stock in the investment function, and by allowing for steady growth in output and prices.

I shall regard short-run equilibrium as determined by an adjustment process in which the rate of adjustment in a variable is a function of the discrepancy between the measured and the anticipated value of that variable or its rate of change, as well as, perhaps, of other variables or their rates of change. Finally, I shall let at least some anticipated variables be determined by a feedback process from past observed values.

a) Division of a Change in Nominal Income between Prices and Output

It seems plausible that the division of a change in nominal income between prices and output depends on two major factors: anticipations about the behavior of prices—this is the inertia factor stressed by Keynes—and the current level of output or employment compared with the full-employment (permanent) level of output or employment—this is the supply-demand response stressed by quantity theorists. We can express this in general form as:

$$\frac{dP}{dt} = f \left[ \frac{dY}{dt}, \left( \frac{dP}{dt} \right)^*, \left( \frac{dy}{dt} \right)^*, y, y^* \right],$$

(42)

$$\frac{dy}{dt} = g \left[ \frac{dY}{dt}, \left( \frac{dP}{dt} \right)^*, \left( \frac{dy}{dt} \right)^*, y, y^* \right],$$

(43)

where an asterisk attached to a variable denotes the anticipated value of that variable and where the form of equations (42) and (43) must be consistent with the identity

$$Y = Py,$$

(44)

so that only one of equations (42) and (43) is independent.

To illustrate, a specific linearized version of equations (42) and (43) might be

$$\frac{d \log P}{dt} = \left( \frac{d \log P}{dt} \right)^* + \alpha \left[ \frac{d \log Y}{dt} - \left( \frac{d \log Y}{dt} \right)^* \right] + \gamma \left[ \log y - (\log y)^* \right];$$

(45)

$$\frac{d \log y}{dt} = \left( \frac{d \log y}{dt} \right)^* + (1 - \alpha) \left[ \frac{d \log Y}{dt} - \left( \frac{d \log Y}{dt} \right)^* \right] - \gamma \left[ \log y - (\log y)^* \right].$$

(46)
The sum of these is exactly the logarithm of equation (44), differentiated with respect to time, provided the anticipated variables also satisfy a corresponding identity, so the equations satisfy the specified conditions.

The simple quantity theory assumption, that all of the change in income is in prices, and that output is always at its permanent level, is obtained by setting \( a = 1 \) and \( \gamma = \infty \). An infinite value of \( \gamma \) corresponds to "perfectly flexible prices" and assures that \( y = y^* \). The unit value of \( a \) assures that prices absorb any change in nominal income, so that real income grows at its long-term rate of growth.

The simple Keynesian assumption, that all of the change in income is in output, so long as there is unemployment, and all in prices, once there is full employment, is obtained by setting \( [(d \log P)/(dt)]^* = 0 \), and \( a = \gamma = 0 \) for \( y < y^* \), and then shifting to the quantity theory specification of \( a = 1 \), \( \gamma = \infty \) for \( y \geq y^* \). The zero value of \( [(d \log P)/(dt)]^* \) assures that anticipations are for stable prices and, combined with the zero values of \( a \) and \( \gamma \), that \( (d \log P)/(dt) = 0 \). It would be somewhat more general, and perhaps more consistent with the spirit rather than the letter of Keynes's analysis, and even more that of his modern followers, to let \( [(d \log P)/(dt)]^* \) differ from zero while keeping \( a = \gamma = 0 \) for \( y < y^* \). This would introduce the kind of price rigidity relevant to Keynes's short-period analysis, yet could be regarded as capturing the phenomenon that his modern followers have emphasized as cost-push inflation.

The simple monetary theory of nominal income is of course consistent with these equations in their general form since it does not specify anything about the division of a change in nominal income between prices and output.

In their general form, equations (45) and (46) do not by themselves specify the path of prices or output beginning with any initial position. In addition, we need to know how anticipated values are formed. Presumably these are affected by the course of events so that, in response to a disturbance which produces a discrepancy between actual and anticipated values of the variables, there is a feedback effect that brings the actual and anticipated variables together again (see

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Footnotes:

33 This also explains why \( [(d \log y)/(dt)]^* \) does not appear explicitly in equation (45), or \( [(d \log P)/(dt)]^* \) in equation (46), as they do in equations (42) and (43). They are implicitly included in \( [(d \log Y)/(dt)]^* \).

34 With \( y = \infty \), and \( \log y = \log y^* \), the final expression in equations (45) and (46) is \( \infty \cdot 0 \), or technically indeterminate. The product can be taken to be zero in general, except possibly for a few isolated points at which \( \log y \) deviates from \( \log y^* \), a deviation closed instantaneously by infinite rates of change in \( \log P \) and \( \log y \).
b) Short-Run Adjustment of Nominal Income

For monetary theory, the key question is the process of adjustment to a discrepancy between the nominal quantity of money demanded and the nominal quantity supplied. Such a discrepancy could arise from either a change in the supply of money (a shift in the supply function) or a change in the demand for money (a shift in the demand function). The key insight of the quantity-theory approach is that such a discrepancy will be manifested primarily in attempted spending, thence in the rate of change in nominal income. Put differently, money holders cannot determine the nominal quantity of money (though their reactions may introduce feedback effects that will affect the nominal quantity of money), but they can make velocity anything they wish.

What, on this view, will cause the rate of change in nominal income to depart from its permanent value? Anything that produces a discrepancy between the nominal quantity of money demanded and the quantity supplied, or between the two rates of change of money demanded and money supplied. In general form

\[
\frac{dY}{dt} = f\left[\left(\frac{dY^*}{dt}\right), \frac{dM^s}{dt}, \frac{dM^D}{dt}, M^*, M^D\right],
\]

where \(M^s\) refers to money supplied, \(M^D\) refers to money demanded, and the two symbols are used to indicate that the two are not necessarily equal. That is, equation (47) replaces the adjustment equation (14), \(M^D = M^s\), common to all the simple models, as well as the special adjustment equation (41) derived from the monetary theory of nominal income.

To illustrate, a particular linearized version of equation (47) would be

\[
\frac{d \log Y}{dt} = \left(\frac{d \log Y}{dt}\right)^* + \psi \left(\frac{d \log M^s}{dt} - \frac{d \log M^D}{dt}\right) + \phi (\log M^s - \log M^D).
\]

Unlike equations (45) and (46), the two final adjustment terms on the right-hand side do not explicitly include any asterisked magnitudes. But implicitly they do. The amount of money demanded will depend on anticipated permanent income and prices as well as on the anticipated rate of change in prices.
The three simple models considered earlier all require setting \( \Phi = \infty \) in equation (48) to assure that \( M^S = M^D \). However, once this is done, the rest of the equation provides no information on the adjustment process, since the final term, which is of the form \( \infty \cdot 0 \) is indeterminate. Hence, even though \( M = M^I \) implies that
\[
\frac{d \log M}{dt} = \frac{d \log M}{dt},
\]
so that the second term on the right-hand side of equation (48) is zero for any finite value of \( \Phi \), it does not follow that
\[
\frac{d \log Y}{dt} = \left( \frac{d \log Y}{dt} \right)^*.
\]
The requirement (49) leads to the equation
\[
\frac{d \log Y}{dt} = \frac{d \log M}{dt}
\]
for the simple quantity theory since, with real income and the interest rate fixed, the quantity of money demanded is proportional to prices and hence to nominal income. This equation says that a change in money supply is reflected immediately and proportionately in nominal income.

For the simple Keynesian theory, equation (49) leads, from equation (22), to
\[
\frac{d \log M}{dt} = \left[ \frac{\partial \log I}{d \log Y} + \frac{\partial \log I}{d r} \frac{dr}{d \log Y} \right] \frac{d \log Y}{dt}
\]
where \( dr/d \log Y \) is to be calculated from equation (21), the IS curve. In the special case of absolute liquidity preference \( \partial \log I/\partial r = \infty \); in the special case of completely inelastic investment and saving functions, \( dr/d \log Y = \infty \). In either of these cases, equation (52) implies that, for \( d \log M/dt \) finite, \( d \log Y/dt \approx 0 \); i.e., a change in the supply of money has no influence on income. In the more general case, equation (52) says that a change in money supply is reflected immediately, but not necessarily proportionately, in nominal income.

For the monetary theory of nominal income, equation (49) implies, as we have seen earlier, equation (41), which allows for a delayed adjustment of permanent income to measured income, but not for any discrepancy between \( M^S \) and \( M^D \).

In its general form, equation (48) allows for changes in both supply of money and demand for money. It also implicitly allows for the forces
emphasized by Keynes, shifts in investment or other autonomous expenditures, through the effect of such changes on \( M^d \) and \( M^s \). For example, an autonomous rise in investment demand will tend to raise interest rates. The rise in interest rates will tend to reduce \( M^d \), introducing a discrepancy in one or both of the bracketed expressions on the right-hand side of equation (48), which will cause \((d \log Y)/(dt)\) to exceed \((d \log Y)/(dt)^*\).

c) Money Demand and Supply Functions

As this comment indicates, in order to complete the theory of the adjustment process, it is necessary to specify the functions connecting \( M^d \) and \( M^s \) with other variables in the system, and also to provide relations determining any additional variables—such as interest rates—entering into these functions. Sections 3 and 4 above discuss the demand and supply functions for money that we regard as relevant for this purpose, so only a few brief supplementary comments are required for present purposes.

First, in much of our empirical work we have taken \( M^s \) itself as an autonomous variable and have not incorporated in the analysis any feedback from other adjustments. A major reason that we have done so is our judgment that the supply function has varied greatly from time to time.

Second, in the notation we have been using in this section, the variables \( y \) and \((1/P)(dP/dt)\) in equation (7) should have asterisks attached to them.

Third, the function specifying \( M^d \) might in principle include a transitory component. That is, there is nothing inconsistent with the theory here sketched and distinguishing between a short-run and long-run demand for money, as some writers have done (Heller 1965; Chow 1966; Konig 1968).

d) Determination of Interest Rates

Given that interest rates enter into the demand function for money (equation [7]) and also, presumably, into the supply function, a complete model must specify the factors determining them. Our long-run model determines their permanent values. So what is needed is an analysis of the adjustment process for interest rates comparable with that for prices and nominal income discussed above—provided, as seems reasonable, that measured as well as permanent values of interest rates enter into the money demand and supply functions.
The monetary theory of nominal income incorporates one possible adjustment process—via the anticipated rate of price change. We have not worked out the formal theory of a more sophisticated adjustment process in any detail. The one aspect we have considered is the effect of changes in $M^8$ on interest rates. In that analysis, we have in effect regarded interest rates as adjusting very rapidly to clear the market for loanable funds, the supply of loanable funds as being possibly linked to changes in $M^8$, and the demand for and supply of loanable funds expressed as a function of the nominal interest rate as depending on $Y$ and $((1/P)(dp/dt))^*$ along with other variables.

In some of our empirical work, we have treated interest rates as exogenous.

**e) Determination of Anticipated Values**

The transition between the short-run adjustment process and long-run equilibrium is produced by an adjustment of anticipated values to measured values in such a way that, for a stable system, a single disturbance will set up discrepancies that will in the course of time be eliminated. To put this in general terms, we must have

\[
\left[ \frac{d \log P}{dt} (t) \right]^* = f \left[ \frac{d \log P}{dT} (T) \right],
\]

(53)

\[
\left[ \frac{d \log Y}{dt} (t) \right]^* = g \left[ \frac{d \log Y}{dT} (T) \right],
\]

(54)

\[
y^*(t) = h[y(T)],
\]

(55)

\[
P^*(t) = j[P(T)],
\]

(56)

where $t$ stands for a particular point in time and $T$ for a vector of all dates prior to $t$.

A disturbance of long-term equilibrium, let us say, introduces discrepancies in the two final terms in parentheses on the right-hand side of equation (48). This will cause the rate of change in nominal income to deviate from its permanent value, which through equations (45) and (46) produces deviations in the rate of price change and output change from their permanent values. These may in turn re-enter equation (48) but whether they do or not, they will, through equations (53)–(56), produce changes in the anticipated values that will, sooner or later and perhaps after a cyclical reaction process, eliminate the discrepancies between measured and permanent values.

See chapter 7 in the forthcoming book from which this paper is abstracted.
These anticipation equations are in one sense very general, in another, very special. They require that anticipations be determined entirely by the past history of the particular variable in question, not by other past history or other currently observed phenomena. These equations deny any “autonomous” role to anticipations. These equations, or preferable alternatives to them, are not directly related to the monetary issues that are the main concern of this paper, which is why I have treated them so summarily. Their only function here is to close the system.

One subtle problem in this kind of a structure, in which we have identified the absence of a discrepancy between actual and anticipated values as defining long-period equilibrium, is to assure that the feedback relations defined by equations (53)—(56), as well as the other functions, are consistent with the expanded system of Walrasian equations which specify the long-term equilibrium values. At least some values are implicitly determined in two ways: by a feedback relation such as equations (53) and (56), and by the system of long-run equilibrium equations. The problem is to assure that at long-run equilibrium these two determinations do not conflict.

In our empirical work, we have generally used a particular form of anticipated function, namely, one which defines the anticipated values as a declining weighted average of past observed values. For example, a specific form of equation (55) is

\[ y^* (t) = \beta \int_{T-\infty}^{T} e^{(\sigma-\alpha)(t-T)}y(T) dT, \]  

(57)

where \( \alpha \) and \( \beta \) are parameters, \( \alpha \) defining the long-term rate of growth, and \( \beta \), the speed of adjustment of anticipations to experiences (Friedman 1957, pp. 142—47).

13. An Illustration

It may help to clarify the general nature of this theoretical approach if we apply it to a hypothetical monetary disturbance.

Let us start with a situation of full equilibrium with stable prices and full employment and with output growing at, say, 3 percent per year. For simplicity, assume that the income elasticity of demand for money is unity, so that the quantity of money is also growing at the rate of 3 percent per year. Assume also that money is wholly non-interest-bearing fiat money and that its quantity can be taken as autonomous.