(9)' and (11), real income. The price level would then be given by the ratio of the nominal income obtained from equations (31) and (33) to the real income given by equations (9)', (10)', (11), and (34). The two sets of equations combined would be a complete system of seven equations in seven variables determining both real and nominal magnitudes.

Such a combination, if it were acceptable, would be intellectually very appealing. Over a decade ago, during the early stages of our comparison of the predictive accuracy of the quantity theory and the income-expenditure theory, my hopes were aroused that such a combination might correspond with experience. Some of our early results were consistent with the determination of the real variables by the multiplier, and the nominal variables by velocity. However, later results shattered the hope for this outcome (Friedman and Meiselman 1963). The unfavorable empirical findings, moreover, are reinforced by theoretical considerations.

The major theoretical objections are twofold. First, it seems entirely satisfactory to take the anticipated real interest rate (or the difference between the anticipated real interest rate and the secular rate of growth) as fixed for the demand for money. There, the real interest rate is at best a supporting actor. Inflation and deflation are surely center stage. Suppressing the variations in the real interest rate (or the deviations of the measured real rate from the anticipated real rate) is unlikely to introduce serious error. The situation is altogether different for saving and investment. Omitting the real interest rate in that process is to leave out Hamlet. Second, the consumption function (9)' is highly unsatisfactory, especially once we take inflation and deflation into account. Wealth, anticipations of inflation, and the difference between permanent and measured income are too important and too central to be pushed off stage completely.

Hence for both empirical and theoretical reasons, I am inclined to reject this way of marrying the real and the nominal variables and to regard the saving-investment sector as unfinished business, even on the highly abstract general level of this paper.

9. Some Dynamic Implications of the Monetary Theory of Nominal Income

In equation (31), which determines \( r \), we have so far taken \( ((1/Y)(dY/dt))^* \) as a predetermined variable at time \( t \) and not looked closely at its antecedents. It is natural to regard it as determined by past history. If it is, we can write equation (33) as
\[ Y(t) = V[Y(T)] \cdot M(t), \quad T < t, \]  

where \( V \) is now a functional of the past history of income, \( Y(T) \) for \( T < t \). However, the past history of income in its turn is a function of the past history of money, thanks to equation (33) for earlier dates. Hence, we can also write equation (33) as

\[ Y(t) = F[M(T)] \cdot M(t), \quad T < t, \]  

where \( F \) is a functional of the past history of money. There is also imbedded in these equations the value \( k_0 \), i.e., the assumed fixed value of the difference between the anticipated real interest rate and the secular rate of growth of output. So equations (35) and (36) must be interpreted as depicting the movements of nominal income around a long-term trend on which \( k_0 \), and its components, \( \rho^* \) and \( \bar{g}^* \), adjust to more basic long-term forces—fundamentally for both, changes in the quantity of resources available (human and nonhuman) and in technology.

A specific example may help to bring out the dynamic character of this simple model. Take logarithms of both sides of equation (33) and differentiate with respect to time. This gives

\[ \frac{1}{Y} \frac{dY}{dt} = \frac{1}{V} \frac{dV}{dt} + \frac{1}{M} \frac{dM}{dt}. \]  

Replace \((1/V)(dV/dr)\) by \(s\) (to stand for the slope of the regression of log \( V \) on \( r \)), and \( dr/dt \) by the derivative of the right-hand side of equation (31):

\[ \frac{1}{Y} \frac{dY}{dt} = s \cdot \frac{d}{dt} \left[ \frac{1}{Y} \frac{dY}{dr} \right] + \frac{1}{M} \frac{dM}{dt}. \]  

Assume that the anticipated rate of growth of income is determined by a simple adaptive expectations model:

\[ d \left[ \frac{1}{Y} \frac{dY}{dt} \right] = \beta \left[ \frac{1}{Y} \frac{dY}{dt} - \left( \frac{1}{Y} \frac{dY}{dt} \right)^* \right]. \]  

Substitute equation (39) in equation (38) and solve for \((1/Y)(dY/dt)\). The result is

\[ \frac{1}{Y} \frac{dY}{dt} = \left( \frac{1}{Y} \frac{dY}{dt} \right)^* + \frac{1}{1 - \beta s} \left[ \frac{1}{M} \frac{dM}{dt} - \left( \frac{1}{Y} \frac{dY}{dt} \right)^* \right]. \]
Subtract \((1/M)(dM/dt)\) from both sides, and equation (40) can also be written:

\[
\frac{1}{V} \frac{dV}{dt} = \frac{\beta s}{1 - \beta s} \left[ \frac{1}{M} \frac{dM}{dt} - \frac{1}{Y^*} \frac{dY^*}{dt} \right].
\]  

(41)

Assume that \(0 < \beta s < 1\). Equations (40) and (41) give a very simple and appealing result. If the rate of change of money equals the anticipated rate of change of nominal income, then nominal income changes at the same rate as money—we are in the simple quantity equation world. If the rate of change of money exceeds the anticipated rate of change of nominal income, so will the actual rate of change of nominal income, which will also exceed the rate of change of money—velocity is increasing in a "boom." Conversely for a "contraction" or "recession," interpreted as a slower rate of growth in the actual than in the anticipated rate of growth of income.

Note that this way of introducing a procyclical movement in velocity is an alternative or complement to the approach I suggested in an earlier article (Friedman 1959 and 1969). There the procyclical movement of velocity was explained by the difference between measured and permanent income. The two approaches are not mutually exclusive—as I indicated in my earlier article, when I left room for interest rate effects on velocity (Friedman 1969, pp. 130–136). In the present context, the simplest way to introduce both effects would be to rewrite (12b) as

\[
M^P = Y^* l(r),
\]

(12c)

where \(Y^*\) is permanent nominal income. To complete the system, equation (14) must be replaced with a more sophisticated adjustment mechanism involving \(Y\)—otherwise the system, with \(Y^*\) treated as determined by the past history of \(Y\), would be overdetermined. Such a more sophisticated mechanism is discussed in section 12 below.

In summary, the key elements of the monetary theory of nominal income are:

(a) A unit elasticity of the demand for money with respect to real income.
(b) A nominal market interest rate equal to the anticipated real rate plus the anticipated rate of change of prices, kept at that level by speculators with firmly held anticipations.
(c) A difference between the anticipated real interest rate and the real secular rate of growth determined outside the system.

This is the condition for dynamic stability of the system. See Cagan 1956.
(d) Full and instantaneous adjustment of the amount of money demanded to the amount supplied.

These elements are borrowed mostly from Irving Fisher and John Maynard Keynes. Together they yield a simple two-equation system that determines the time path of nominal income but has nothing to say directly about the division of changes in nominal income between prices and quantity.

This simple model for analyzing short-term economic fluctuations seems to me more satisfactory than either the simple quantity theory which takes real output as determined outside the system and regards economic fluctuations as a mirror image of changes in the quantity of money or the simple Keynesian income-expenditure theory which takes prices as determined outside the system and regards economic fluctuations as a mirror image of changes in autonomous expenditures.

10. Comparison of the Three Approaches

None of the three simple theories—the simple quantity theory, the simple income-expenditure theory, the simple monetary theory of nominal income—professes to be a complete, fully worked out analysis of short-term fluctuations in aggregate economic magnitudes. All are to be interpreted rather as frameworks for such analyses, establishing the broad categories within which further elaborations will proceed.

The simple quantity theory puts in center stage the relation at each point in time between a particular flow—the flow of spending or income—and a particular stock—the quantity of money. The simple income-expenditure theory emphasizes the relation at each point in time between two components of the flow of income—autonomous and induced spending. The simple monetary theory of nominal income emphasizes the relation between the flow of income at each point in time and the past history of the quantity of money.

The simple quantity theory and the simple income-expenditure theory have six common elements, in addition to sharing the same six-equation model, that deserve emphasis because they indicate what are the main unresolved problems.

1. Both analyze short-run adjustments in terms of shifts from one static equilibrium position to another.

2. Both implicitly regard each equilibrium position as characterized by a stable level of prices or output. Neither explicitly introduces changing prices or changing output into the formal theoretical analysis.