Volume Title: A Theoretical Framework for Monetary Analysis

Volume Author/Editor: Milton Friedman

Volume Publisher: NBER

Volume ISBN: 0-87014-233-X

Volume URL: http://www.nber.org/books/frie71-1

Publication Date: 1971

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Chapter Author: Milton Friedman

Chapter URL: http://www.nber.org/chapters/c0918

Chapter pages in book: (p. 34 - 40)

come must then be determined by the requirement that it equate saving with investment.

If we approximate the function $f(Y/P,\ r_0)$ by a linear form, say,

$$\frac{C}{P} = C_0 + C_1 \frac{Y}{P}, \tag{23}$$

substitute equation (23) in equation (11), and solve for Y/P, we get

$$\frac{Y}{P} = \frac{C_0 + I_0}{1 - C_1}, \tag{24}$$

or the simple Keynesian multiplier equation, with $C_0 + I_0$ equalling autonomous expenditure and $1/(1 - C_1)$ equalling the multiplier.

8. The Missing Equation: The Third Approach Examined

A third form of the missing equation involves bypassing the breakdown of nominal income between real income and prices and using the quantity theory to derive a theory of nominal income rather than a theory of either prices or real income.

a) Demand for Money

As a first step, assume that the elasticity of the demand for money with respect to real income is unity. We can then write (12) in the equivalent form:

$$M^D = Y \cdot l(r), \tag{12b}$$

where the same symbol l is used to designate a different functional form. This enables us to eliminate prices and real income separately from the equations of the monetary sector.

This assumption cannot, so far as I am aware, be justified on theoretical grounds. There is no reason why the elasticity of demand for money with respect to per capita real income should not be either less than one or greater than one at any particular level of income, or why it should be the same at all levels of real income. However, much empirical evidence indicates that the income elasticity is not very different from unity. The empirical evidence seems to me to indicate that the elasticity is generally larger than unity, perhaps in the neighborhood of 1.5 to 2.0 for economies in a period of rapid economic development, and of 1.0 to 1.5 for other circumstances. Other scholars would perhaps set it lower. More important, the present theory is for shortterm fluctuations during which the variation in per capita real income is fairly small. Given that the elasticity is unlikely to exceed 2.0, no great error can be introduced for such moderate variations in income by approximating it by unity.[21]

b) *Savings and Investment Functions*

As a second step, it is tempting to make a similar assumption for the savings and investment functions, i.e., to write:

$$C = Y \cdot f(r), \tag{9a}$$

or,

$$C = Y \cdot f(r, Y), \tag{9b}$$

and

$$I = Y \cdot g(r), \tag{10a}$$

which would eliminate any separate influence of prices and real income from the savings-investment sector also. However, this is an unattractive simplification on both theoretical and empirical grounds. Theoretically, it dismisses Keynes' central point: the distinction between expenditures that are independent of current income (autonomous expenditures) and expenditures dependent on current income (induced expenditures). Empirically, much evidence suggests that the ratio of consumption to income over short periods is not independent of the level of measured income [equation (9a)], or of the division of a change in income between prices and output [equation (9b)]. The extensive literature on the consumption function rests on this evidence.

c) *Interest Rates*

A more promising route is to combine a key idea of Keynes' with a key idea of Irving Fisher's.

The idea that we take over from Keynes is that the current market interest rate (r) is largely determined by the rate that is expected to prevail over a longer period (r^*) (see section 5c above) [Leijonhufvud 1968, pp. 158, 405, 411].

Carrying this idea to its limit gives

$$r = r^*. \tag{25}$$

[21] Of course, considerations such as these can at most be suggestive. The real test of the usefulness of this, and the later assumptions, is in the success of the resulting theory in predicting the behavior of nominal income.

The idea that we take over from Fisher is the distinction between the nominal and the real rate of interest:

$$r = \rho + \left(\frac{1}{P}\frac{dP}{dt}\right), \tag{26}$$

where ρ is the real rate of interest and $(1/P)(dP/dt)$ is the percentage change in the price level. If the terms r and $(1/P)(dP/dt)$ refer to the observed nominal interest rate and observed rate of price change, ρ is the realized real interest rate. If they refer to "permanent" or "anticipated" values, which we shall designate by attaching an asterisk to them, then ρ^* is likewise the "permanent" or "anticipated" real rate.

Combine equation (25) and the version of (26) that has asterisks attached to the variables. This gives:

$$r = \rho^* + \left(\frac{1}{P}\frac{dP}{dt}\right)^*, \tag{27}$$

which can be written as:

$$r = \rho^* + \left(\frac{1}{Y}\frac{dY}{dt}\right)^* - \left(\frac{1}{y}\frac{dy}{dt}\right)^* = \rho^* - g^* + \left(\frac{1}{Y}\frac{dY}{dt}\right)^* \tag{28}$$

where $g^* = [(1/y)(dy/dt)]^*$ = "permanent" or "anticipated" rate of growth of real income, i.e., the secular or trend rate of growth.

Let us now assume that

$$\rho^* - g^* = k_o, \tag{29}$$

i.e., that the difference between the anticipated real interest rate and the anticipated rate of real growth is determined outside the system. This equation is the counterpart of the full employment and rigid price assumptions [equations (15) and (16)] of the simple quantity theory and the simple Keynesian income-expenditure theory.

There are two ways that assumption (29) can be rationalized: (1) that over a time interval relevant for the analysis of short-period fluctuations, ρ^* and g^* can separately be regarded as constant; (2) that the two can be regarded as moving together, so the difference will vary less than either. Of course, in both cases, what is relevant is not absolute constancy, but changes in $\rho^* - g^*$ that are small compared to changes in $[(1/P)(dP/dt)]^*$, and hence in r.

(1) The stock of physical capital, the stock of human capital, and the body of technological knowledge are all extremely large compared to annual additions. Physical capital is, say, of the order of three to five years' national income; annual net investment is of the order of

$\frac{1}{10}$ to $\frac{1}{5}$ of national income or 2 to 8 per cent of the capital stock. Let the capital *stock* be subject even to very rapidly diminishing returns and the real yield will not be much affected in a few years time. Similar considerations apply to human capital and technology.

If we interpret g^* as referring to growth potential, then a roughly constant yield on capital, human and nonhuman, and a slowly changing stock of capital imply a slowly changing value of g^* as well.

Empirically, a number of pieces of evidence fit in with these assumptions. We have interest rate data over very long periods of time, and these indicate that rates are very similar at distant times, if the times compared have similar price behavior (Gupta 1964). More recently, the Federal Reserve Bank of St. Louis has been estimating the "real rate," and their estimates are remarkably stable despite very large changes in nominal rates.

Similarly, average real growth has differed considerably at any one time for different countries—compare Japan in recent decades with Great Britain—but for each country has been rather constant over considerable periods of time.

(2) Let $s^* =$ the fraction of permanent income which is invested. Then the permanent rate of growth of income as a result of this investment alone will be equal to $s^*\rho^*$. Empirically, the actual rate of growth tends to be larger than this product, if s^* refers only to what is recorded as capital formation in the national income accounts. One explanation, frequently suggested, is that recorded capital formation neglects most investment in human capital and in improving technology and that allowance for these would make the relevant s^* much higher than the 10 or 20 per cent that is the fraction estimated in national income accounts, both because it would increase the numerator of the fraction (investment) and decrease the denominator (income) by requiring much of what is commonly treated as income to be treated as expenses of maintaining human capital and the stock of technology. In the limit, as s^* approaches unity, ρ^* approaches g^*, so $\rho^* - g^* = 0$.[22] Without going to this extreme,

$$\rho^* - g^* = (1 - s^*)\rho^*. \tag{30}$$

The preceding argument suggests that ρ^* is fairly constant, and subtracting g^* decreases the error even further.

[22] An argument justifying this equality on a purely theoretical level has been developed ingeniously and perceptively by Stephen Friedberg in some unpublished papers that take Frank H. Knight's capital theory as their starting point. This equality is also a key implication of Von Neumann's general equilibrium model (Von Neumann 1945, p. 7).

Empirically, it does seem to be the case that ρ^* and g^* tend to vary together, though in the present state of evidence, this is hardly more than a rough conjecture.

(*d*) *The Alternative Model*

If we substitute equation (12b) for equation (12), keep the original equations (13) and (14), and substitute equation (29) in equation (28) to replace the remaining equations of the initial simple model, we have the following system of four equations:

$$M^D = Y \cdot l(r) \tag{12b}$$

$$M^S = h(r) \tag{13}$$

$$M^D = M^S \tag{14}$$

$$r = k_o + \left(\frac{1}{Y}\frac{dY}{dt}\right)^*. \tag{31}$$

At any point of time, $[(1/Y)\ (dY/dt)]^*$, the "permanent" or "anticipated" rate of growth of nominal income is a predetermined variable, presumably based partly on past experience, partly on considerations outside our model. As a result, this is a system of four equations in the four unknowns, M^D, M^S, Y, and r.

Prices and quantity do not enter separately, so the set of equations constitutes a model of nominal income.

It will help to clarify the essence of this third approach to simplify it still further by assuming that the nominal money supply can be regarded as completely exogenous, rather than a function of the interest rate,[23] and to introduce time explicitly in the system. Let $M(t)$ be the exogenously determined supply of money. We then have from equations (12b), (13), and (14)

$$Y(t) = \frac{M(t)}{l(r)}, \tag{32}$$

or

$$Y(t) = V(r) \cdot M(t), \tag{33}$$

where V stands for velocity of circulation. This puts the equation in standard quantity theory terms, except that it does not try to go behind

[23] Alternatively, we could write equation (13) as

$$M^S = H \cdot m(r),$$

where H is high-powered money and $m(r)$ is the money multiplier.

nominal income to prices and quantities. Equations (31) and (33) then constitute a two-equation system for determining the level of nominal income at any point in time. To determine the path of nominal income over time, there is needed in addition some way to determine the anticipated rate of change of nominal income. I shall return to this below.

Although the symbolism in the demand equation for money [(12b) or (33)] is the same as in the two other specializations of the general model, there is an important difference in substance. Both the simple quantity theory and the income-expenditure theory implicitly define equilibrium in terms of a stable price level, hence real and nominal interest rates are the same. The third approach, based on a synthesis of Keynes and Fisher, abandons this limitation. The equations encompass "equilibrium" situations in which prices may be rising or falling. The interest rate that enters into the demand schedule for money is the nominal interest rate. So long as we stick to a single interest rate, that rate takes full account of the effect of rising or falling prices on the demand for money.

(*e*) *The Saving-Investment Sector*

What about equations (9) to (11), which we have so far completely bypassed? Here the interest rate that is relevant, if a single rate is used, is clearly the real not the nominal rate. If we replace r by ρ, these equations become

$$\frac{C}{P} = f\left(\frac{Y}{P}, \rho\right) \tag{9$'$}$$

$$\frac{I}{P} = g(\rho) \tag{10$'$}$$

$$\frac{Y}{P} = \frac{C}{P} + \frac{I}{P}. \tag{11}$$

If we were to accept a more restricted counterpart of equations (25) and (29), namely

$$\rho = \rho^* = \rho_0, \tag{34}$$

i.e., the realized real rate of interest is a constant, then these equations would be a self-contained consistent set of five equations in the five variables, C/P, I/P, Y/P, ρ, ρ^*. Equations (34) would give the real interest rate. Equation (10)$'$ would give real investment and equations

(9)′ and (11), real income. The price level would then be given by the ratio of the nominal income obtained from equations (31) and (33) to the real income given by equations (9)′, (10)′, (11), and (34). The two sets of equations combined would be a complete system of seven equations in seven variables determining both real and nominal magnitudes.

Such a combination, if it were acceptable, would be intellectually very appealing. Over a decade ago, during the early stages of our comparison of the predictive accuracy of the quantity theory and the income-expenditure theory, my hopes were aroused that such a combination might correspond with experience. Some of our early results were consistent with the determination of the real variables by the multiplier, and the nominal variables by velocity. However, later results shattered the hope for this outcome (Friedman and Meiselman 1963). The unfavorable empirical findings, moreover, are reinforced by theoretical considerations.

The major theoretical objections are twofold. First, it seems entirely satisfactory to take the anticipated real interest rate (or the difference between the anticipated real interest rate and the secular rate of growth) as fixed for the demand for money. There, the real interest rate is at best a supporting actor. Inflation and deflation are surely center stage. Suppressing the variations in the real interest rate (or the deviations of the measured real rate from the anticipated real rate) is unlikely to introduce serious error. The situation is altogether different for saving and investment. Omitting the real interest rate in that process is to leave out Hamlet. Second, the consumption function (9)′ is highly unsatisfactory, especially once we take inflation and deflation into account. Wealth, anticipations of inflation, and the difference between permanent and measured income are too important and too central to be pushed off stage completely.

Hence for both empirical and theoretical reasons, I am inclined to reject this way of marrying the real and the nominal variables and to regard the saving-investment sector as unfinished business, even on the highly abstract general level of this paper.

9. Some Dynamic Implications of the Monetary Theory of Nominal Income

In equation (31), which determines r, we have so far taken $[(1/Y)(dY/dt)]^*$ as a predetermined variable at time t and not looked closely at its antecedents. It is natural to regard it as determined by past history. If it is, we can write equation (33) as