Volume Title: A Theoretical Framework for Monetary Analysis

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Volume Publisher: NBER

Volume ISBN: 0-87014-233-X

Volume URL: http://www.nber.org/books/frie71-1

Publication Date: 1971

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Chapter URL: http://www.nber.org/chapters/c0917

Chapter pages in book: (p. 31 - 34)

Equation (13) is the supply function of nominal money. To be consistent with the literature, the interest rate enters as a variable. However, no purpose for which we shall use the model would be affected in any way by treating M^S as simply an exogenous variable, determined, say, by the monetary authorities.[17]

Equation (14) is the counterpart of equation (11), a market-clearing or adjustment equation specifying that money demanded shall equal money supplied.

These six equations would be accepted alike by adherents of the quantity theory and of the income-expenditure theory. On this level of abstraction, there is no difference between them. However, while there are six equations, there are seven unknowns: C, I, Y, r, P, M^D, M^S. There is a missing equation. Some one of these variables must be determined by relationships outside this system.[18]

7. The Missing Equation: Three Approaches

The difference between the quantity theory and the income-expenditure theory is the condition that is added to make the equations determinate.

The simple income-expenditure theory adds the missing equation in one form. Different versions of the quantity theory add it in two other forms. Of these, the missing equation that has been generally regarded in the literature as defining the simple quantity theory is discussed in this section. The missing equation supplied by an alternative version of the quantity theory that is implicit in much recent literature but has not heretofore been made explicit is discussed in the following section. I shall designate the alternative version of the quantity theory as the monetary theory of nominal income.

The simple quantity theory adds the equation

$$\frac{Y}{P} = y = y_0; \tag{15}$$

that is, real income is determined outside the system. In effect, it appends to this system the Walrasian equations of general equilibrium, regards

[17] This would be consistent with Cagan's findings about the absence of any significant effect of changes in the interest rate on the supply of money. However, to be consistent with his findings, income or some other indicator of business cycles would have to be included as a variable, as has been done in some empirical studies of the supply of money. See Cagan (1965, pp. 150, 228–32) and Hendershott (1968).

[18] Of course, this is speaking figuratively. It is not necessary that a single variable be so determined. What is required is an independent relation connecting some subset of the seven endogenous variables with exogenous variables, and that subset could in principle consist of all seven variables.

them as independent of these equations defining the aggregates, and as giving the value of Y/P, and thereby reduces this system to one of six equations determining six unknowns.[19]

The simple income-expenditure theory adds the equation[20]

$$P = P_0; \tag{16}$$

that is, the price level is determined outside the system, which again reduces the system to one of six equations in six unknowns. It appends to this system a historical set of prices and an institutional structure that is assumed either to keep prices rigid or to determine changes in prices on the basis of "bargaining power" or some similar set of forces. Initially, the set of forces determining prices was treated as not being incorporated in any formal body of economic analysis. More recently, the developments symbolized by the "Phillips curve" reflect attempts to bring the determination of prices back into the body of economic analysis, to establish a link between real magnitudes and the rate at which prices change from their initial historically determined level (Phillips 1958).

For the quantity theory specialization, given that $Y/P = y_0$, equations (9), (10), and (11) become a self-contained set of three equation in three unknowns: C/P, I/P, and r. Substituting (9) and (10) into (11), we have

$$y_0 - f(y_0, r) = g(r), \tag{17}$$

or a single equation which determines r. Let r_0 be this value of r. From equation (13), this determines the value of M, say M_0 which, using equation (14), converts equation (12) into

$$M_0 = P \cdot l(y_0, r_0), \tag{18}$$

which now determines P.

[19] This is the essence of what has been called the classical dichotomy. Strictly speaking, the division between consumption and investment and the rates of exchange between current and future goods or services (the set of "real" or "own" interest rates) are also determined in a Walrasian "real" system, one which admits of growth, which is why quantity theorists have tended to concentrate only on equations (12), (13), and (14). On this view, equations (9), (10), and (11) are a summarization or aggregation or subset of the Walrasian system.

[20] Keynes distinguished between the price level of products and the wage rate and allowed for a change in the ratio of the one to the other as output changed, even before the point of full employment. However, this change in relative prices plays no important role in the aspects of his theory that are relevant to our purpose, so I have simplified the model by taking prices rather than wages as rigid—a simplification that has been widely used. However, explicit reference to this simplification should have been made in an earlier paper (Friedman 1970). I am indebted to an unpublished paper by Paul Davidson for recognition that the earlier exposition on this point may have been misleading.

Equation (18) is simply the classical quantity equation, as can be seen by multiplying and dividing the right-hand side by y_0 and replacing $l(y_0,r_0)/y_0$ by its equivalent, $1/V$. If we drop the subscripts, this gives,

$$M = \frac{Py}{V}, \tag{19}$$

or

$$P = \frac{MV}{y}. \tag{20}$$

For the income-expenditure specialization, setting $P = P_0$ does not in general permit of a sequential solution. Substituting equations (9) and (10) into equation (11) gives

$$\frac{Y}{P_0} - f\left(\frac{Y}{P_0}, r\right) = g(r), \tag{21}$$

an equation in two variables, Y and r. This is the *IS* curve of Hicks's famous *IS–LM* analysis (Hicks 1937). Substituting equations (12) and (13) into equation (14) gives

$$h(r) = P_0 \cdot l\left(\frac{Y}{P_0}, r\right), \tag{22}$$

a second equation in the same two variables, Y and r. This is Hicks's *LM* curve. The simultaneous solution of the two determines r and Y.

Alternatively, solve equation (21) for Y as a function of r, and substitute in equation (22). This gives a single equation which determines r as a function of the demand for and supply of money. This can be regarded as the Keynesian parallel to equation (18), which determines P as a function of the demand for and supply of money.

A simpler sequential analysis, faithful to many textbook versions of the analysis and to Keynes's own simplified model, is obtained by supposing either that Y/P is not an argument in the right-hand side of equation (12) or that absolute liquidity preference holds so that equation (12) takes the special form:

$$\begin{aligned} M^D &= 0 \text{ if } r > r_0 \\ M^D &= \infty \text{ if } r < r_0. \end{aligned} \tag{12a}$$

In either of these cases, equations (12) or (12a), (13), and (14) determine the interest rate, $r = r_0$ (just as in the simple quantity approach, equations [9], [10], and [11] do); substituting the interest rate in equation (10) determines investment, say at $I = I_0$ and in equation (9) makes consumption a function solely of income, so that real income must then be determined by the requirement that it equate saving with investment.

If we approximate the function $f(Y/P, r_0)$ by a linear form, say,

$$\frac{C}{P} = C_0 + C_1 \frac{Y}{P}, \tag{23}$$

substitute equation (23) in equation (11), and solve for Y/P, we get

$$\frac{Y}{P} = \frac{C_0 + I_0}{1 - C_1}, \tag{24}$$

or the simple Keynesian multiplier equation, with $C_0 + I_0$ equalling autonomous expenditure and $1/(1 - C_1)$ equalling the multiplier.

8. The Missing Equation: The Third Approach Examined

A third form of the missing equation involves bypassing the breakdown of nominal income between real income and prices and using the quantity theory to derive a theory of nominal income rather than a theory of either prices or real income.

a) Demand for Money

As a first step, assume that the elasticity of the demand for money with respect to real income is unity. We can then write (12) in the equivalent form:

$$M^D = Y \cdot l(r), \tag{12b}$$

where the same symbol l is used to designate a different functional form. This enables us to eliminate prices and real income separately from the equations of the monetary sector.

This assumption cannot, so far as I am aware, be justified on theoretical grounds. There is no reason why the elasticity of demand for money with respect to per capita real income should not be either less than one or greater than one at any particular level of income, or why it should be the same at all levels of real income. However, much empirical evidence indicates that the income elasticity is not very different from unity. The empirical evidence seems to me to indicate that the elasticity is generally larger than unity, perhaps in the neighborhood of 1.5 to 2.0 for economies in a period of rapid economic development, and of 1.0 to 1.5 for other circumstances. Other scholars would perhaps set it lower. More important, the present theory is for short-