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CHAPTER III.

THE CONTRIBUTION OF STATISTICS.[1]

I. The Current Distinction Between Theoretical and Statistical Work.

The review of current theories in Chapter I barely mentions the type of work upon business cycles which is most characteristic of the present and seems most promising for the future—analysis of statistical data. A few of the theorists—notably Henry L. Moore—make skilled and elaborate use of quantitative methods, and almost all cite statistical evidence upon occasion. But most theories of business cycles are still built up by methods which would have seemed familiar to Sismondi and Ricardo.

On the other hand, there has recently appeared a group of business-cycle statisticians who as yet have sought, not to construct general theories, but to establish more precisely the facts concerning cyclical fluctuations in particular economic processes. By their detailed researches, the statistical workers are building up a literature more like the current literature of the natural sciences than like that of economic theory. It contains few treatises, but a multitude of technical papers; it is mathematical in form and empirical in spirit; it deals with restricted problems, lays stress upon measurements, and aspires to prediction.

Between these two groups of workers, the theorists and the statisticians, there has been less communion than their mutual interests require. Many of the statisticians pay little heed to current theories of business cycles, and many of the theorists make little use of statistical methods. A similar divergence of outlook, associated with a similar division of labor, seems not uncommon in modern science. Experimentalists and pure theorists often have difficulty in understanding each other; but in the long run each group provides grist for the other's mill, and scientific progress is a joint product of the

[1] In writing this chapter, I have had generous help from the Staff of the National Bureau, particularly from Dr. Frederick C. Mills, and from the Directors, particularly from Professor Allyn A. Young.

two lines of attack upon the unknown. Such must prove to be the case in work upon business cycles.

Statistical analysis affords the surest means of determining the relations among and the relative importance of the numerous factors stressed by business-cycle theories. In turn, rational hypotheses are the best guides of statistical research, and theoretical significance is the ultimate test of statistical results. Aside from the limitations of investigators or of their resources, the line commonly drawn between statistical and theoretical work has no justification.

II. Development of the Statistical Approach.

1. Why the Early Writers Upon Business Cycles Made Slight Use of Statistics.

A promising beginning of statistical work upon social problems had been made in England by contemporaries of Sir Isaac Newton. The most conspicuous figure in the group, Sir William Petty, dealt with "Political Arithmetick," sought to express himself in "Terms of *Number, Weight,* or *Measure,*" and to "bottom" his discourses upon quantitative "Observations or Positions" which are "either true, or not apparently false . . . and if they are false, not so false as to destroy the Argument they are brought for." [1] But political arithmetic had not prospered greatly. "Observations . . . expressed by *Number, Weight,* or *Measure*" were scarce, and the "Positions," "either true, or not apparently false," upon which Petty's followers bottomed their discourses sometimes led to contradictory conclusions. While Adam Smith was writing the *Wealth of Nations,* Dr. Richard Price was proving that the population of England and Wales had decreased near a quarter since the Revolution, and Arthur Young was proving that the population had increased. It is not surprising that Adam Smith had "no great faith in political arithmetic," and made sparing use of it in expounding "the obvious and simple system of natural liberty." [2]

Yet it is an exaggeration to picture the generation which developed the first theories of business cycles as virtually destitute of

[1] From the preface to *Political Arithmetick,* 1690. See *The Economic Writings of Sir William Petty,* edited by Charles H. Hull. Cambridge, 1899; vol. i, pp. 244, 245.

[2] *The Wealth of Nations,* Cannan's edition, vol. ii. pp. 36 and 184.

significant statistics. One has but to look into such volumes as George Chalmers' *Estimate of the Comparative Strength of Great Britain*, 1782, Sir Frederick M. Eden's *State of the Poor*, 1797, the second edition of Malthus' *Essay on the Principle of Population*, 1803, or Thomas Tooke's *Thoughts and Details on the High and Low Prices of the Last Thirty Years*, 1823, to assure himself that men who had an aptitude for that type of inquiry could gather and use critically a considerable quantity of data covering a considerable range of problems. And the books mentioned are but prominent examples of a type of work which was rapidly increasing in volume, improving in quality, and gaining public support. In 1801 the first census of Great Britain was taken, and in 1832 a Statistical Department was added to the Board of Trade. How voluminous and how varied were the statistical materials which had been quietly accumulating in official sources between these two dates was shown in 1833, when John Marshall published his quarto *Digest of all the Accounts Relating to the Population, Productions, Revenues, Financial Operations, Manufactures, Shipping, Colonies, Commerce, etc., etc., of the United Kingdom, diffused through more than 600 volumes of Journals, Reports, and Papers presented to Parliament during the last Thirty-five Years.*

It is also a mistake to think of the early nineteenth century as altogether lacking in statistical technique. In *Chronicon Preciosum*, published in 1707 and more than once reprinted, Bishop Fleetwood had shown how to treat changes in the purchasing power of money on a quantitative basis. A definite plan for making index numbers of prices had been put before the Royal Society by Sir George Schuckburg-Evelyn in 1798.[3] William Playfair had used graphic methods of presenting time series in the successive editions of his *Commercial and Political Atlas, representing, by Means of Stained Copper-plate Charts, the Progress of the Commerce, Revenues, Expenditures, and Debts of England, during the whole of the Eighteenth Century* (1786, 1787, and 1801). Joseph Fourier had developed harmonic analysis in a memoir crowned by the *Academie des Sciences* in 1812.[4] Most important of all, Laplace had published his *Essai Philosophique sur les Probabilités* in 1814. In this charming essay, the most celebrated mathematician of the age summed up the analytic methods de-

[3] "An Account of Some Endeavors to Ascertain a Standard of Weight and Measure," *Philosophical Transactions of the Royal Society of London*, 1798. Part I, art. viii, pp. 132-186, especially pp. 175, 176.

[4] *Théorie des mouvements de la chaleur dans les corps solides.*

veloped by his predecessors and himself, and proposed their use in dealing with social problems.

> Let us apply to the political and social sciences (he wrote) the method founded upon observations and calculus, a method which has served us so well in the natural sciences.

And to that end, Laplace urged that the compilation of social statistics be made systematic. It is highly important to keep in every branch of

> public administration an accurate record of the effects produced by the various measures taken, which are so many experiments tried on a large scale by governments.[5]

Thus it is an over-simple explanation to ascribe the neglect of measurements by early business-cycle theorists wholly to the lack of pertinent data or to the immaturity of statistical technique. Had they taken the line suggested by Laplace, these writers might have used and added to the available data; they might have learned and developed the analytic methods which had been suggested. But like other economists of the day, most writers upon crises had an easier, a quicker, and, as it seemed to them, a more effective method of working. Observers had no trouble in conceiving plausible explanations of crises, and they could rapidly expand their conjectures into imposing theories by selecting from the facts generally known those which accorded with their leading ideas. In this way they avoided a host of doubts and difficulties which would crop up if they tried to bottom their work upon the stubborn data of statistics. With the contemporary writers of economic treatises, they might admit, when discussing problems of method, that their "deductive" reasoning required "inductive verification"; but they somewhat easily excused themselves from going through the second and more arduous operation.[6] Perhaps still more important is the fact that writers who practiced the "deductive" method were apt to formulate their problems in ways which raised obstacles to "inductive verification,"—obstacles

[5] *Essai Philosophique*, 6th ed., Paris, 1840, p. 135.

[6] Note, for example, the difference between the procedure which John Stuart Mill recommended in his *System of Logic*, 1843, and the procedure which he practiced in *The Principles of Political Economy*, 1848. In but few chapters does Mill really carry out the "concrete-deductive method."

which might have been avoided by a different approach.[7] Statistical method could not develop its full efficiency so long as it was called in only at the end of an inquiry, and asked to answer questions it had no share in framing.

2. The Growth of Statistical Technique.

The mathematical statisticians of the early 19th century did not enter the economic field. The social phenomena to which Laplace applied the theory of probabilities in his *Essai Philosophique* were such matters as the credibility of the evidence given by witnesses, the decisions of assemblies, mortality, and the average duration of marriages. Adolphe Quetelet dealt with physical, intellectual and moral qualities in his two volumes *Sur l'Homme,* 1835. Henry Thomas Buckle sought to introduce statistical method into historical research, holding in his *History of Civilization in England* (1857 and 1861) that human actions are ruled by laws as fixed and regular as those which govern the physical world; but that these laws can be discovered only by a survey of the facts so comprehensive that disturbing factors will cancel one another. Meanwhile, the chief contributions to statistical technique continued to come from mathematicians proper. Gauss published the method of least squares in 1823. In 1837, Poisson made the theory of probability more applicable to social problems by showing how the curve of distribution is modified

[7] Consider Ricardo's contention that it is impossible to determine "the value of a currency" by its "relation, not to one, but to the mass of commodities."

"To suppose that such a test would be of use in practice," Ricardo argued, "arises from a misconception of the difference between price and value.

"The price of a commodity is its exchangeable value in money only.

"The value of a commodity is estimated by the quantity of other things generally for which it will exchange.

"The price of a commodity may rise while its value falls, and *vice versa.* A hat may rise from twenty to thirty shillings in price, but thirty shillings may not procure as much tea, sugar, coffee, and all other things, as twenty shillings did before, consequently a hat cannot procure so much. The hat, then, has fallen in value, though it has increased in price.

"Nothing is so easy to ascertain as a variation of price (*sic*), nothing so difficult as a variation of value; indeed, without an invariable measure of value, and none such exists, it is impossible to ascertain it with any certainty or precision." ("Proposals for an Economical and Secure Currency," 2d ed., 1816. Ricardo's *Works,* edited by J. R. McCulloch, p. 401.)

Nowadays, economists apply to money Ricardo's definition, "The value of a commodity is estimated by the quantity of other things generally for which it will exchange," and proceed to the construction of price indexes. Then they can compare the price fluctuations of single commodities with this general index, and approximate Ricardo's conception of measuring variations in value, without having an "invariable measure of value."

by dropping the assumption of equal *a priori* probabilities at every trial, and directed attention to "the law of great numbers."

The only economist concerned with the theoretical uses of statistics in this period was Augustin Cournot. In considering the old problem of variations in value, he made a casual suggestion that "Here, as in astronomy, it is necessary to recognize *secular* variations, which are independent of *periodic* variations." More important was Cournot's discussion of demand curves. While his own dealings with this subject were confined to mathematical analysis "by means of an indeterminate symbol," he emphasized the need of statistical inquiry into the relations between demand (D) and price (p). In this connection, he suggested the technique by which statistical laws defining the relationships between economic variables may be discovered.

> Since so many moral causes capable of neither enumeration nor measurement affect the law of demand (he wrote), it is plain that we should no more expect the law to be expressible by an algebraic formula than the law of mortality, and all the laws whose determination enters into the field of statistics, or what is called social arithmetic. Observation must therefore be depended on for furnishing the means of drawing up between proper limits a table of the corresponding values of D and p; after which, by the well-known methods of interpolation or by graphic processes, an empiric formula or a curve can be made to represent the function in question; and the solution of problems can be pushed as far as numerical applications.[1]

Yet Cournot made no use of statistical data in his *Theory of Wealth*, and in his *Exposition de la Théorie des Chances et des Probabilités* (published in 1843), he did not apply the theory to a much wider range of social problems than Laplace had considered in 1814. It was left for W. Stanley Jevons to give the first powerful impetus to statistical work in economic theory.[2]

[1] *Researches into the Mathematical Principles of the Theory of Wealth.* Translated by Nathaniel T. Bacon. New York, 1897, pp. 25, 47-49, and 53, 54. Originally published in 1838.

[2] In 1833 the British Association for the Advancement of Science had set up a Statistical Section, and a Statistical Society had been formed in Manchester. But both organizations sought to avoid discussions of the theoretical implications of their data. "Several men of eminence in statistics chafed at being thus relegated to the position of 'hewers and drawers for political economy and philosophy,' so they joined in promoting the Statistical Society of London, now the Royal Statistical Society, with the view of providing therein a wider scope for their inquiries. Their hopes were frustrated, for a time at least, by the same spirit of caution which dictated the limitations imposed upon

While still a student in the University of London, Jevons began studying "periodic commercial fluctuations"—or, as we now say, "seasonal variations." He passed on quickly to an investigation of the changes in the "value of gold" which had followed the Californian and Australian discoveries.

> It has been abundantly shown by Quetelet and others (he remarked), that many subjects of this nature are so hopelessly intricate, that we can only attack them by the use of averages, and by trusting to probabilities.

To ascertain what changes had occurred in the "value of gold," Jevons made index numbers of the wholesale prices of 39 commodities by years from 1845 to 1862. He discussed the best type of average to use; tested his results by taking a larger sample of 118 commodities, and invoked the theory of probabilities to find the cause of the advance in prices, computing that

> the odds are 10,000 to 1 against a series of disconnected and casual circumstances having caused the rise of prices—one in the case of one commodity, another in the case of another—instead of some general cause acting over them all.[3]

And we may regard Jevons' book on *The Coal Question* (1865) as an important contribution to the study of secular trends.

The theory and the use of index numbers made slow progress for a generation after Jevons' pioneer work in 1863. But in 1887 Professor F. Y. Edgeworth began his long series of distinguished contributions to the problem, and about the same time Adolf Soetbeer, Augustus Sauerbeck, and Roland P. Falkner began providing index

the earlier institutions." For their *Journal* expressed its policy by the self-denying motto, *Aliis exterendum*, which was not dropped until about 1857, although the society had re-defined its aims in a wider style by 1840.

See Sir Athelstane Baines, "The History and Development of Statistics in Great Britain and Ireland," in *The History of Statistics*, edited by John Koren (Seventy-fifth anniversary volume of the American Statistical Association), New York, 1918, pp. 385, 386.

[3] *On the Study of Periodic Commercial Fluctuations*, 1862; *A Serious Fall in the Value of Gold Ascertained*, 1863; *The Depreciation of Gold*, 1869. Reprinted in *Investigations in Currency and Finance*, by W. Stanley Jevons, London, 1884. The quotations are from the paper of 1869, pp. 155-157 of the *Investigations*.

Later writers have questioned the applicability of the theory of probability to index numbers. For citations and discussion of the problem, see Professor F. Y. Edgeworth, "The Element of Probability in Index Numbers," *Journal of the Royal Statistical Society*, July, 1925, Vol. 88, pp. 557-575.

numbers for England, Germany and the United States.[4] Thereafter this bit of statistical technique rapidly became current among economists, though skepticism concerning the trustworthiness of the results lingered long in certain quarters.

As slow in winning general recognition was the contribution of another economist—Wilhelm Lexis. Approaching the theory of probability from the statistical side, Lexis showed in a series of investigations that the birth rates of different populations are not distributed around their mean values in accordance with the so-called "normal curve," and developed a mathematical-statistical explanation of the divergencies. Thus the way was opened for the empirical study of actual distributions—a type of work which spread gradually from vital statistics into other fields.[5]

The theoretical justification for applying the theory of probability to economic data at large was worked out mainly by Professor Edgeworth. In 1885 he directed attention to the fact that

> the distribution of averages will be approximately normal even though the distribution of the items composing the averages deviate considerably from normal.

Later he showed that the observations subjected to the probability analysis

> need not be perfectly independent of each other, "it suffices that there should be a considerable amount of independence"; that they need not be of the same order of magnitude, "it suffices that no two or three preponderate"; that the condition for the absence of systematic errors is not necessary, "it suffices that the center of gravity for the series of observations

[4] Edgeworth's early contributions were published in the *Reports of the British Association for the Advancement of Science*, 1887, 1888, 1889 and 1890 (reprinted in *Papers Relating to Political Economy*, by F. Y. Edgeworth, London, 1925, vol. i, pp. 195-343). Soetbeer's index numbers appeared first in 1885, Sauerbeck's first in 1886, and Falkner's in 1893. For more explicit references, see "Index Numbers of Wholesale Prices in the United States and Foreign Countries," *Bulletin of the United States Bureau of Labor Statistics*, No. 284, October, 1921.

[5] See Wilhelm Lexis, *Die französischen Ausfuhrprämien in Zusammenhange mit der Tarifgeschichte und der Handelsentwicklung Frankreichs seit der Restauration*, Bonn, 1870; *Zur Theorie der Massenerscheinungen in der menschlichen Gesellschaft*, Freiburg, 1877; *Abhandlungen zur Theorie der Bevölkerungs- und Moralstatistik*, Jena, 1903. Eugen Altschul gives an interesting account of Lexis' later views concerning the application of statistical method to social problems in a paper on "Konjunkturtheorie und Konjunkturstatistik," *Archiv für Sozialwissenschaft und Sozialpolitik*, January, 1926, vol. lv, pp. 77-82.

indefinitely prolonged should coincide with the true point which forms the quaesitum." [6]

Another technical contribution for which the economists presently found use was the theory of correlation. Invented by Sir Francis Galton as a method of studying the inheritance of characteristics, it was developed in the early 1890's by Professors Karl Pearson, F. Y. Edgeworth and G. Udny Yule, and later applied to measuring the relationship between paired items in time series.[7]

3. The Accumulation of Statistical Data.

The cumulative growth of statistical technique adapted to the treatment of economic problems in the second half of the 19th century was paralleled by a cumulative growth of statistical data. The impetus toward the collection of statistics came from practical activities rather than from scientific inquiries. Most of the ever-shifting issues in the political life of modern nations have had their economic aspects. Those who urged or those who opposed "reforms," and often both parties, sought to strengthen their cases by instituting special inquiries to show the extent of the evils to be remedied, or of the evils which the proposed remedies would produce. And as changes were made in public policy, administrative agencies were set up which had to keep continuous records not unlike those for which Laplace had called in 1814.

Thus the history of statistics in every country bears the impress of its social struggles. The United States owes its relatively abundant statistics of money and banking to the currency problems which the country has faced in different forms from the days of Alexander Hamilton to the days of the Federal Reserve Board. The relative backwardness of vital statistics in the United States arises from the fact that the population problem has been less pressing in this country than elsewhere. Our statistics of immigration and emigration remained meager until the country became exercised about the millions of newcomers from Southern Europe. So, too, our statistics of income

[6] Warren M. Persons, "Statistics and Economic Theory," *Review of Economic Statistics*, July, 1925, vol. vii, pp. 185-186.
This paper contains the best account known to me of the development of statistical methods now utilized by economists. I have drawn upon it freely.

[7] See Galton's *Natural Inheritance*, 1889, and, for later contributions, Karl Pearson, "Notes on the History of Correlation," *Biometrica*, October, 1920.

were vastly improved by the adoption of the federal income tax. Doubtless one might cite similar examples for any other country.

While Governments were being forced by practical exigencies to increase the scope of their statistical work decade after decade, private business was expanding its quota of statistics. On this side, also, practical needs dominated. The closer integration of business activities, the increasing dependence of every section and industry upon other sections and industries, created a widespread interest in business news. Reports of transactions and prices upon the stock and produce exchanges, reports of money-market conditions, bank clearings, security issues, bankruptcies and the like were wanted by a wide public, and numerous agencies collaborated to meet the demand. Trade journals found that statistical data of technical interest to their clientele attracted subscribers. Later, many trade associations began offering statistical service to their members.

In the various branches of economic activity, the progress toward making an adequate statistical record was influenced by the relative ease of collecting data. Highly organized central markets, like the produce and stock exchanges, made the statistician's task relatively simple. In the commodity field, price quotations were easier to get than production statistics, wholesale prices were easier to get than retail prices, and market prices were easier to get than contract prices. On the side of production, it was easier to compile reports for the highly standardized raw materials like coal, petroleum, and agricultural produce, or for partially fabricated materials like pig iron, spelter and cotton sheetings, than for the vast variety of finished goods like machinery, clothing and household supplies. In comparison with production, transportation presents a comparatively simple problem. On the contrary, statistics of stocks on hand, of orders booked, of mercantile operations, of costs and of profits have been very difficult to obtain.

Though the obstacles in the way of collecting adequate data remain formidable, economic evolution is facilitating the task. The organization of workers in trade unions gave the first opening for collecting significant and regular statistics of unemployment, while the concentration of an increasing proportion of employees in large establishments facilitated the collection of the more significant data concerning numbers on payrolls. The rise of department and chain stores made it feasible to gather the first reliable data concerning retail sales. Obviously, the trend toward standardization of technical processes and of products favors the statistician. So does the trend toward the standardization of accounting methods, a trend which is most marked in such fields as banking, railway transportation, and public utilities, where financial reports must be submitted on official forms to government bureaus. The trend toward publicity of corporate accounts, observable in business circles, promises to give us in the future more accurate knowledge of costs and profits. In the not distant future we may know more about the arcana of business than Sismondi's generation knew about market prices. A policy which combines standardization with publicity is favored by modern methods of producing by automatic machinery, selling by national advertising, and financing by appeal to a large circle of investors. Standardization and publicity give the statistician what he wants.[1]

4. The Present Situation.

Thus it happened that by the time writers upon business cycles began to make systematic use of statistics—say in the decade beginning in 1900—they could utilize many methods already developed by mathematicians, anthropometrists, biologists and economists, and many data already collected by public and private agencies. As their work progressed, these men encountered problems which required special adaptations of the methods used in other fields and problems which required data not yet collected. Yet so well had the way been paved for them that they could make rapid progress toward establishing the study of business cycles upon a quantitative basis.

To analyze time series was the central problem. First, the recurrent cyclical fluctuations had to be isolated so far as possible from the other fluctuations to which time series are subject. Second, the relationships among the cyclical fluctuations of many different series had to be ascertained.

Jevons had made a beginning upon the first task by his studies of what are now called seasonal variations, a beginning which George Clare had continued in his brief treatise upon the London money

[1] The details concerning the increase of statistical data since the opening of the 19th century are far too intricate to sketch. Perhaps the best general view of developments on this side is given by the volume published by the American Statistical Association in commemoration of its 75th Anniversary: *The History of Statistics, Their Development and Progress in Many Countries,* Collected and edited by John Koren, New York, 1918.

A more detailed view of the British, French, German and American data relevant to business cycles is afforded by the collection of statistics which the National Bureau of Economic Research has made in preparation for the present book and which the Bureau hopes to publish.

market.[1] Another step was taken by J. H. Poynting and R. H. Hooker, who attacked the problem of determining secular trends, using for that purpose moving averages.[2] The second task, finding the relationships among quantities varying in time, was undertaken by Professor G. Udny Yule in 1899 and Mr. Hooker in 1901, both of whom applied Pearson's methods of correlation to economic data.[3]

In 1902, Dr. John Pease Norton combined and improved upon these various methods in his *Statistical Studies in the New York Money Market.*[4] Norton measured secular trends by fitting exponential curves to his data; he considered the dispersion as well as the averages of seasonal variations, taken as percentages of his trends; he used lines of regression as well as coefficients of correlation in examining the relations among his variables. Another notable step was taken by Professor Henry L. Moore, who in 1914 applied harmonic analysis to time series.[5] The following year, Professor Warren M. Persons made the first of his business barometers,[6] and in 1917 the Harvard Committee on Economic Research enabled him to begin a more elaborate analysis of economic statistics than had been possible for any of his predecessors. By establishing the *Review of Economic Statistics* in January, 1919, this committee provided an organ devoted primarily to the quantitative study of business cycles. The models which Persons and his colleagues have set in this journal have been studied and imitated widely, not only in the United States, but also in Europe.

This list of men who shared in adapting statistical methods to the analysis of economic time series is far from complete; it mentions only a few of the most noteworthy contributors. Still less justice can be done to recent work. Of late the number of economic statisti-

[1] *A Money-Market Primer and Key to the Exchanges,* London, 1891; 2d ed., 1903.

[2] See J. H. Poynting, "A Comparison of the Fluctuations in the Price of Wheat and in the Cotton and Silk Imports into Great Britain," *Journal of the Royal Statistical Society,* 1884, vol. xlvii, pp. 34-64; R. H. Hooker, "On the Correlation of the Marriage-rate with Trade," the same, 1901, vol. lxiv, pp. 485-492. A sketch of the progressive improvements in ascertaining secular trends will be given in Dr. Simon S. Kuznets' forthcoming monograph on that subject.

[3] Hooker as cited in preceding note; Yule, "An Investigation into the Causes of Changes in Pauperism in England," *Journal of the Royal Statistical Society,* 1899, vol. lxii, p. 249 *f.*

[4] Published for the Department of Social Sciences, Yale University; New York, 1902.

[5] *Economic Cycles: Their Law and Cause,* New York, 1914.

[6] "Construction of a Business Barometer Based upon Annual Data" (presented in part to a meeting of the American Statistical Association, August 11, 1915), *American Economic Review,* December, 1916, vol. vi, pp. 739-769. See a supplemental paper "On the Variate Difference Correlation Method and Curve-fitting." *Publications of the American Statistical Association,* June, 1917, vol. xv, pp. 602-642.

cians has grown rapidly. Efforts to improve the technique in detail, and efforts to win fresh knowledge by more intensive and more extensive analysis are being made constantly. In later sections I must attempt a critical summary of the constructive achievements of the whole campaign. Here it suffices to remark that recent contributions are to be found in the *Journal of the American Statistical Association*, the *Review of Economic Statistics*, the *Harvard Business Review*, the *Journal of the Royal Statistical Society*, and less frequently in the journals devoted to economics at large.

On the side of data, the Department of Commerce has rendered valuable service by extending the collection of statistics into fresh fields, and by establishing the *Survey of Current Business* in 1921. In this source are assembled most of the current series, old and new, collected by public or private agencies, which are useful to students of business cycles. Of scarcely less interest are the *Federal Reserve Bulletin* and the bulletins issued by the several Reserve Banks. Many of the most significant series, or indexes made from them, are regularly published in "adjusted" form by the *Review of Economic Statistics.*

As the preceding references indicate, the statistical study of business cycles has had its headquarters in the United States. This country continued to suffer from severe crises for a generation after they had been transformed into mild recessions in Europe and Canada. Thus the problem was especially intriguing to Americans. Further, Americans had rather fuller statistics to work with than were available in any other country, partly because of the prominence of economic issues in American politics, partly because of the highly standardized character of American products. The development was much stimulated and colored by a widespread demand for business forecasts. To improve the statistical technique of forecasting was Professor Persons' chief aim, and the hope of finding trustworthy indexes to the future has animated and secured financial support for many ingenious investigations.

Since the war, interest both scientific and practical in this type of work has been spreading to other countries. Since 1921, the *Economic Bulletin of the Conjuncture Institute,* edited by Professor N. D. Kondratieff, has been compiling, analyzing and interpreting Russian data. In Great Britain, the London and Cambridge Economic Service, cooperating with the Harvard University Committee on Economic Research, began the publication of a *Monthly Bulletin* in 1923. The Institute of Statistics of the University of Paris has a similar plan of coöperation with the Harvard group, and publishes *Indices du Mouvement Général des Affaires en France et en Divers Pays.* Germany has an Institut für Konjunkturforschung under the direction of Dr. Ernst Wagemann, which began publishing *Vierteljahrshefte zur Konjunkturforschung* in January, 1926. Two Institutes of Statistics connected with the Universities of Padua and of Rome have joined forces to publish *Indici del Movimento Economico Italiano* under the direction of a Committee presided over by Professor Corrado Gini. The League of Nations has appointed a "Committee of Experts on Economic Barometers," Professor A. W. Flux, chairman, which held its first meeting at Geneva in December, 1926. Finally, in January, 1927, an Oesterreichisches Institut für Konjunkturforschung was organized in Vienna, with a scientific staff directed by Dr. F. A. von Hayek.

The present chapter aims neither to give a full exposition of the statistical methods which have been applied to the study of business cycles—that would fill a volume and exceed my competence,— nor to show what the statisticians have contributed toward our knowledge of cyclical fluctuations—that will be attempted in a second book. It aims rather to show what problems the statistical workers have attacked, how far they have pushed their researches, and what light they shed upon the character of business cycles.

III. The Analysis of Time Series.

1. The Quality and Quantity of Economic Statistics.

A biologist or anthropologist working upon statistical problems is often able to collect his own data by measuring material in his laboratory. Such measurements can be made to fit the requirements of the problem, and their accuracy can be controlled within assignable limits. A meteorologist is dependent upon data collected mainly by other observers; but these observers are men with at least a modicum of training, using scientific apparatus, and working under scientific direction. The quantitative worker upon economic problems is less fortunate in respect to his raw materials. Seldom can he make in his statistical laboratory a significant collection of measurements. He deals not with "material," but with the behavior of men, and that behavior must be observed and recorded "in the field." Because his phenomena are highly variable, he usually needs a large array of cases, more than he can collect by himself or through the trained assistants at his disposal. Forced to rely upon observations made by others, he must often adapt his problem to the data, when he wishes to adapt his data to the problem.

Economic statistics are usually a by-product of governmental or business administration, collected in a form, at intervals, and by methods determined by some statute, official ruling, or business practice. Even when a public bureau plans a statistical inquiry with an eye to the scientific use of the results—and that happens with increasing frequency—it is usually necessary to fit the results for a variety of uses, and hence not feasible to adapt them precisely to any specific use. Finally, the accuracy attainable in most economic measurements leaves much to be desired. Often the data consist of estimates or rough approximations, and when precision is attainable with reference to the items counted there may be grave uncertainty regarding the representative character of the sample.

It is sometimes suggested that what economic statistics lack in quality they make up in quantity. There are grounds for arguing that a large number of observations compensates for lack of precision in each single observation, and the volume of economic statistics is certainly imposing—it is even intimidating at first sight. But on closer inspection the mass proves to consist less of a multiplication of independent observations upon particular phenomena, than of observations upon a vast variety of phenomena, and of the infinite detail in which certain processes must be observed.

To take the last point first: A student of business cycles is concerned largely with aggregates, for example, bank clearings in the United States, the wheat crops of the world, pig-iron output. On each of these topics and a hundred like them he can find a huge quantity of figures. But the mass is not for all his purposes a large number of observations upon one variable; it consists of many separate items which must be added to get one series of totals. The investigator is glad to have the figures in detail, for he may find occasion to examine the fluctuations (say) of wheat yields in different countries. But he has only one or two estimates for each country each year. Another type of problem is exemplified by efforts to measure changes in the "level" of wholesale or retail prices. Again a huge quantity of raw data is available—the prices of particular commodities, in particular markets, at particular dates. In Professor Edgeworth's phrase, the task is

> to extricate from fallible observations a mean apt to represent the "general trend of prices." [1]

But the investigator has to take the changes between every pair of dates in every country or smaller area as a separate problem, and for no such problem is his number of "fallible observations" very large. The War Industries Board's index number of wholesale prices in 1913-18 included nearly 1500 commodities; but that collection was a *tour de force,* and the largest of the currently published series cover only some 400 commodities, while in many cases the commodities quoted number less than 50.[2]

Hardly less responsible for the bulkiness of economic statistics is the variety of activities covered. Every new attempt to systematize business operations—and such attempts seem endless—involves the making of new records, more and more of which emerge from bookkeeping into statistics. Only a tiny fraction of the records kept appears in published tables; yet in a country like the United States that fraction spreads over a bewildering diversity of processes. Nor can students of business cycles be sure that they are safe in ignoring any section of the record. Certainly they are concerned with the production, exchange, transportation, and distribution of commodities; with wages, rents, interest rates, profits, bond yields, and dividends; with family incomes and expenditures; with prices of all sorts, financial operations, savings, orders, bankruptcies, the launching of new enterprises, patents, construction work, banking in its various aspects, unemployment, migration, imports, exports, and tax receipts. In many cases they need detailed data for particular industries or localities. And they cannot properly neglect pertinent figures from any country which collects them. Even the fact that certain series show slight traces of cyclical fluctuations hardly lessens their task; for that fact may be highly significant in a general view of the problem. They

[1] F. Y. Edgeworth, *Papers Relating to Political Economy,* London, 1925, vol. i, p. 405.

[2] See *History of Prices during the War, Summary,* by Wesley C. Mitchell, (War Industries Board Price Bulletin, No. 1) Washington, 1919, p. 5. For the number of commodities contained in the leading wholesale price indexes of various countries at present, see *Institut International de Statistique, Bulletin Mensuel.* The issue for October, 1925, for example, pp. 76, 77, credits the United States Bureau of Labor Statistics with quoting 404 commodities, and the Canadian Department of Labour with quoting 271. Most of the series for the 31 countries represented are made from quotations for less than 100 commodities.

It may be noted that the question, "What shall be counted a distinct commodity in a price index?" is difficult to answer.

cannot even confine their researches within the field of economic statistics. Hypotheses concerning the causes of business cycles carry them back into meteorological data, morbidity rates, and what not, while interest in the consequences of business cycles carries them forward into the fields of vital statistics, criminal statistics, statistics of dependency, philanthropy, poor relief. And the more they learn about the problem, the wider grows the range of data which they deem it pertinent to study.

As an investigator gets deeper into a quantitative analysis of business cycles, his first impression that the statistical data to be dealt with are embarrassingly abundant turns into a conviction that they are painfully inadequate. Each business cycle becomes to him one case, one chance for observation. His method is to make measured observations upon as many cases as he can, and to draw such generalizations from the array of observations as his skill permits. For a very few of the recent cycles in the United States he finds data which allow him to observe with varying degrees of precision a good many of the processes involved. But seldom do the observations of a given process show marked uniformity from case to case. Hence the investigator becomes eager to increase the number of his cases. But as he passes from recent cycles in the United States to earlier cycles, or as he passes from American to European cycles, he finds the supply of statistics rapidly diminishing, and his doubts increasing about the trustworthiness of what statistics there are. In the end he must content himself with such data as practical needs or accident have preserved, make the best use of what he finds, and hope that his efforts may at least help toward getting for his successors better figures than are available to him.

If such an investigator does not give up the effort to frame a general theory of business cycles, he at least gives up the effort to base his theory wholly upon measurements. What statistical results he does attain, he presents as tentative. These results he prefaces, strings together, interprets, and supplements with analysis based upon non-quantitative observations.

2. Time Series in Their Raw State.

Since business cycles follow one another in time, the statistical data of chief concern to us are time series—data which show the value of some variable at successive points, or during successive periods, of time. Occasionally surveys showing the magnitude of some factor, or the distribution of certain phenomena at a given moment are drawn upon, but the analysis of time series remains the outstanding task.

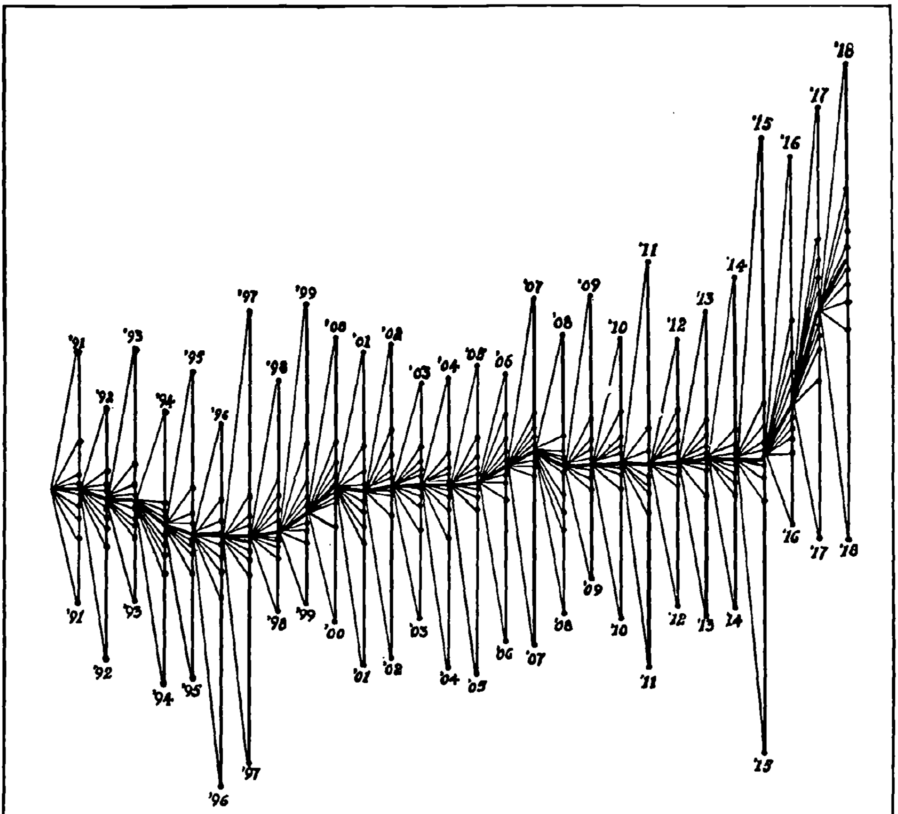
These time series have many forms:—prices, aggregate money values, physical quantities (in units of number, weight, length, area, volume, energy), percentages of a total which changes with time, percentages of a total at some past date, ratios to some other quantity, and so on. The intervals vary from the day, week or month to the year or the decade. Many such series cover but a year or two, some have been kept in fairly uniform fashion for a generation, a very few quasi-continuous series run back or can be pieced back, for a century, two centuries, or even more.[1] Some series are available in comparable form for several different countries—for example, wheat production, coal and iron output, discount rates;—some are available only for one country—for example, the valuable German series showing receipts from the tax on bills of exchange, the American data concerning retail sales, the British statistics of outdoor relief. It is indeed a most miscellaneous collection which the investigator of business cycles must use as his raw materials.

A few of the series which such an investigator uses report the variations in factors which are indivisible units in the business situation. For example, the official minimum discount rate of the Bank of England is a single figure, known with precision for every week through long years. Many different matters have been weighed by the bank's directors in deciding upon the rate announced each Thursday; but once announced there is no analyzing the rate into constituent parts. Most time series, however, are aggregates, or averages, which the investigator can, and frequently should, analyze. For example, if bank clearings in the United States fall five per cent between July and August, it may be that in a majority of the clearing houses transactions increased; and in the minority of towns where transactions shrank, the declines may have varied from a fraction of one per cent to half the July volume. Similarly, an index number of

[1] For example, Sir William H. Beveridge has compiled an index number of "Wheat Prices in Western and Central Europe" by years for the whole period, 1500-1869. See his paper on "Weather and Harvest Cycles," *Economic Journal*, December, 1921, vol. xxxi, pp. 449-451. I understand that he has now secured data which will enable him to carry this table much further back than 1500.

wholesale prices shows for each date merely the net resultant of most diverse changes in the prices of individual commodities—changes which nearly always run the gamut from a considerable decline to a considerable rise.[2] In thinking graphically of price changes we should

[2] By way of illustration consider the following charts of price changes. The first chart (adapted from the *Bulletin of the U. S. Bureau of Labor Statistics*, No. 284) shows the mixture of diversity and concentration in the fluctuations of the wholesale prices of the commodities included in the B.L.S. index number, by years, from 1891 to 1919. Each year the fluctuations are ranked in order of their magnitude, and then divided into ten groups containing equal numbers of commodities. The dividing points, called decils, and the extreme fluctuations of each year are represented on the chart by dots. By an arbitrary convention the 11 dots for each year are connected by slanting lines with the middle point of the distribution of the preceding year. The vertical scale is logarithmic. While the chart gives a lively impression of the variety of price changes



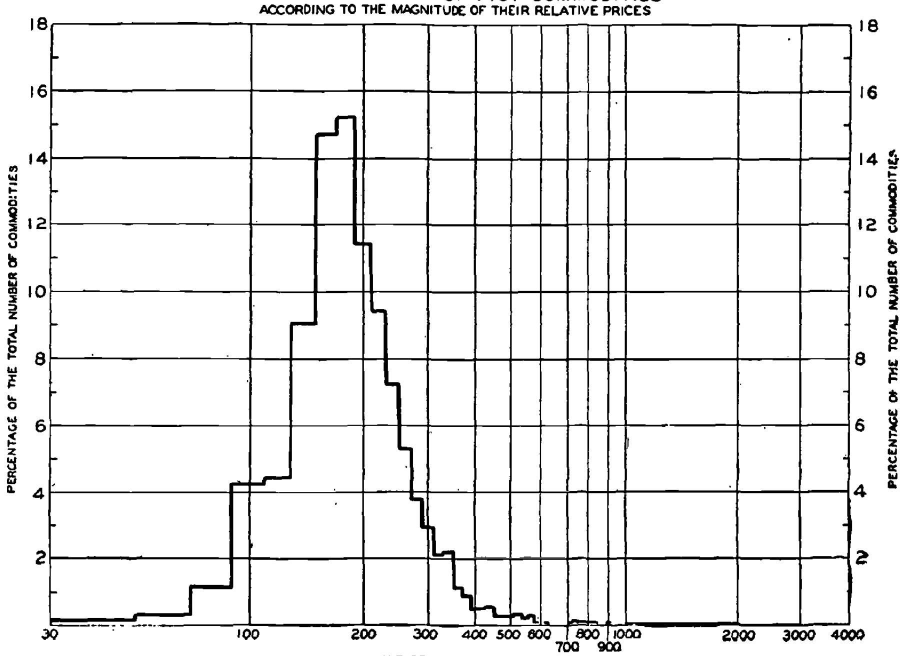
occurring every year, it over-simplifies the situation by picturing the annual changes as all starting from the same point. A chart which did not over-simplify some aspect of the changes would be a hopeless tangle of lines.

The second chart (adapted from *History of Prices during the War, Summary*, War think, not of the movements of a single curve, such as represents adequately the Bank of England rate, but of a broad, irregular band, within which many lines are moving, some up, some down, some horizontally—lines which are rather far apart near the edges of the band but thickly congregated near the middle, and which keep crossing each other as they shift their relative positions.

It is true that an investigator often writes of series like bank clearings or price indexes as if they represented magnitudes not less definite than the Bank of England rate. Doubtless there are problems which justify the practice—problems in which the one matter of significance is the net resultant of a complicated mass of movements. Yet such problems are rare, and it is always wise to ask explicitly whether the hypothesis in use does not require that notice be taken of the diversity

Industries Board Price Bulletin, No. 1) is another device for illustrating the diversity and concentration of price changes. It shows the distribution of the relative prices of 1437 commodities in 1918, computed on the base average prices, July, 1913, to June, 1914 equal 100. Here the horizontal scale is logarithmic. The chart shows, for example, that one commodity fell 64 per cent in price from the pre-war level to 1918, another commodity rose 2,991 per cent, but that nearly 600 commodities (over two-fifths) were concentrated in the range of 50 to 109 per cent advance.

1918 DISTRIBUTION OF 1437 COMMODITIES
ACCORDING TO THE MAGNITUDE OF THEIR RELATIVE PRICES



of movements which are hidden in the simple-seeming aggregates or averages. Certainly we do not get the full benefit of the statistical approach to the study of business cycles, unless we keep in mind the range and variety of the changes which most of our time series represent.

The first step toward using any series is usually to represent the figures by a line drawn on a chart. The invention of this device in 1786 is claimed by William Playfair, who set forth its advantages as follows:

> As the eye is the best judge of proportion, being able to estimate it with more quickness and accuracy than any other of our organs, it follows, that wherever *relative quantities* are in question, a gradual increase or decrease of any . . . value is to be stated, this mode of representing it is peculiarly applicable; it gives a simple, accurate, and permanent idea, by giving form and shape to a number of separate ideas, which are otherwise abstract and unconnected.[3]

A variant of Playfair's method of charting time series was introduced by Jevons in 1863-65—the use of a logarithmic vertical scale for showing degree of variation, combined with an arithmetical horizontal scale for showing intervals of time.[4] By this device equal percentage changes occuring at equal intervals in a time series are represented by lines of the same slope. Further, two or more different series, expressed in non-comparable units, when plotted upon a ratio chart, will have equal percentage changes in the several series represented by lines of the same slope. Since students of business cycles are usually interested in the relative changes which economic processes undergo recurrently, rather than in the absolute quantities involved in the changes, the "ratio chart" is particularly useful to them.

[3] *The Commercial and Political Atlas,* 3d ed., London, 1801, p. x.
Playfair's claim to be "actually the first who applied the principles of geometry to matters of Finance" is made on pp. viii and ix.

[4] *A Serious Fall in the Value of Gold Ascertained* (1863), and *The Variation of Prices, and the Value of Currency since 1782* (1865). Reprinted in *Investigations in Currency and Finance,* by W. Stanley Jevons, London, 1884. See especially pp. 53, 128, 150 and the charts which follow pp. 56 and 150.
This method of making charts did not come into common use among economists until its merits had been explained by Professors Irving Fisher, "The 'Ratio' Chart for Plotting Statistics," *Publications of the American Statistical Association,* June, 1917, vol. xv, pp. 577-601, and James A. Field, "Some Advantages of the Logarithmic Scale," *Journal of Political Economy,* October, 1917, vol. xxv, pp. 805-841.

To illustrate the materials and problems before us, samples of the time series most useful in studying business cycles are given in Chart 1.[5] These series run in units of long tons of pig iron, pounds of copper, changing percentages of unemployment among members of British trade unions, bushels of wheat, miles of railway track, percentages of the average prices of commodities in 1867-77, billions of dollars, and percentages which the interest received forms of the changing purchase price of bonds. The vertical logarithmic scale of the chart makes comparable the percentage fluctuations in all these different quantities. The only bar to close comparison is that some of the series give data only by years, while others give data by quarters or months.

Looked at broadly, the curves show striking differences. American pig-iron output, copper output and wheat crops present one type of changes. The characteristic of this type is its upward trend more rapid in the two metal curves than in the other, and frequently broken in all three curves by brief reactions. Bank clearings give a similar picture from 1875 to 1914; but the great inflation of the war period introduced a sudden change of trend less marked in the series representing physical production unmixed with prices. American railway building presents a second type of changes. After rising to a peak in the 1880's, it declined unsteadily to a level comparable with that of the 1850's. A third type, characterized by several changes in the direction of movement, is presented by British prices at wholesale. They rise from the 1850's to 1873, fall from 1873 to 1896, rise from 1896 to 1913 at a rate then thought rapid, accelerate their rise in 1914 to 1920, and later fall precipitously. The bond-yield curve also shows several changes in trend; but changes less marked than those of the price curve. Finally, the British unemployment percentages fluctuate about the same general level through the whole 70 years for which they have been compiled.

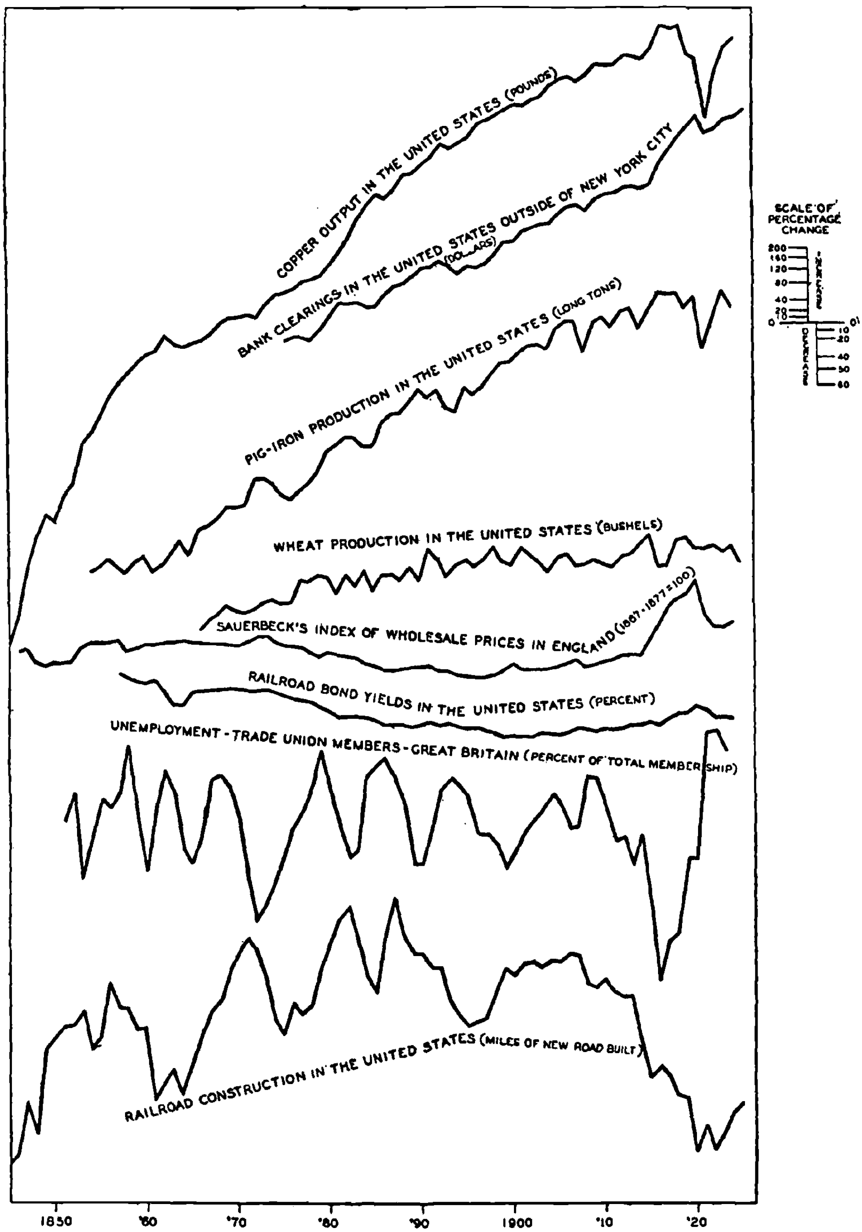
Except in the unemployment series and perhaps that showing railway construction, these long-period shiftings of level, or "secular trends" as they are technically called, are the most conspicuous features of the chart. But the conspicuous feature of the unemployment

[5] The numerical data used in drawing the charts in this volume are taken, with a few exceptions noted in the proper connection, from the collection of statistical materials which the National Bureau of Economic Research is making and hopes to publish in the near future. Since that collection will have a full index, and will state the original sources from which all the series are obtained as well as the methods of compilation, it seems needless to publish the tables here.

CHART 1.

Sample Time Series Plotted on a Logarithmic Vertical Scale.

Only the Relative Slope, Not the Vertical Position, of these Curves Is Significant.



curve is its wavelike movement about a nearly constant level. Of course, these wave-like movements, or "cyclical fluctuations," are the center of our interest. Are they present in the other cases? Traces of them may be seen in all the curves, except perhaps the curve for wheat crops, where the fluctuations seem capricious. But in all the curves, including that for unemployment, the cyclical fluctuations are obscured by intermixture with changes of other sorts. Accordingly, the first problem faced by the student of business cycles in the statistical analysis of time series is whether he can develop a technique which will enable him to isolate the cyclical fluctuations for intensive observation and measurement.

The clarity of the cyclical fluctuations in the unemployment curve suggests the idea of seeking to reduce the secular trends in the other curves to horizontal lines. Can that be done?

Were that accomplished, would the cyclical fluctuations be isolated? Closer examination of the unemployment curve itself raises doubt. The percentage of men out of work in England seems to be greater almost every year in winter than in summer, whether times are good or bad, though that characteristic is not revealed by the annual figures used in Chart 1. Indications of such "seasonal variations" can be traced in a monthly plot of many time series. Can the presence or absence of seasonal variations in a time series be determined, and can they be separated from the cyclical fluctuations?

Finally, the unemployment curve shows sudden breaks now and then, which appear as interruptions in the course which the combined cyclical-seasonal changes seem to be following. Can these "irregular fluctuations" be identified? Can they be measured and separated from the cyclical fluctuations?

Such is the conventional list of problems which statistical investigators of business cycles have attacked. These workers have devised many ingenious methods of measuring and eliminating from time series secular trends and seasonal variations. In dealing with irregular perturbations they have made less progress. One group has sought to find periodic fluctuations in the data by applying harmonic analysis. Much of the work is highly technical. Consequently the following survey of what has been done can be neither brief nor easy.

3. The Problem of Secular Trends

Secular trends of time series have been computed mainly by men who were concerned to get rid of them. Just as economic theorists have paid slight attention to the "other things" in their problems which they suppose to "remain the same," so the economic statisticians have paid slight attention to their trends beyond converting them into horizontal lines. Hence little is yet known about the trends themselves, their characteristics, similarities, and differences. Even their relations to cyclical fluctuations have been little considered. Here lies in obscurity a heap of problems, waiting for properly equipped investigators to exploit.[1]

(1) The Empirical Approach to the Problem.

The procedure adopted in ascertaining secular trends is usually empirical in high degree. Starting with a time series plotted to a convenient scale on a chart, the statistician seeks to find for that one series, within the period covered by his data, the line which best represents the "long-time tendencies" shown by the plotted curve. In cases like that presented by British wholesale prices in Chart 1, he splits the whole period into sections, and seeks to find lines of secular trend for each section separately, or else to find a function which changes direction in the manner of the data.

The technical process usually consists in (1) fitting a "mathematical curve" (for example, a straight line or a third-degree parabola) to the data, or to the logarithms of the data, by the method of least squares or of moments; (2) computing moving or progressive arithmetic means or moving medians, including in the averages whatever number of items seems to give satisfactory results; (3) first computing moving averages and then fitting trend lines to the results; (4) drawing a free-hand curve through the data representing the investigator's impression, formed from careful study, of the long-time tendency; or (5) using ratios between the paired items of series which are believed to have substantially the same secular trends.[1]

[1] So far as I know, the only one working upon secular trends as a problem in its own right is Dr. Simon S. Kuznets, one of the Research Fellows of the Social Science Research Council. Dr. Kuznets has generously allowed me to profit by his results in advance of publication.

[1] Technical directions for computing secular trends by the commoner methods are given in most of the recent textbooks of economic statistics. For example, see Frederick C. Mills, *Statistical Methods,* New York, 1924, chap. vii, W. L. Crum and A. C. Patton, *Economic Statistics,* Chicago, 1925, chap. xx, or Edmund E. Day, *Statistical Analysis,* New York, 1925, chap. xvii.

Among the recent contributions to the subject are Warren M. Persons, "Indices of Business Conditions," *Review of Economic Statistics,* January, 1919, Preliminary vol. i, pp. 8-18: and the following papers in the *Journal of the American Statistical Associa-*

When a secular trend has been fitted by any of these methods or their variants, how can the agreement between the line of trend and the plotted data be determined?

It might be thought that a trend can be tested by breaking a series into two parts, computing trends for the separate sections and seeing whether they agree. But lack of agreement would not prove that the trend for either section of the data was wrong for the period it covered. Secular trends are "subject to change without notice," and it is a common experience that a line which gives an excellent fit to the data for one period ceases to fit well when carried backward in time, or projected for a few years. Thus the failure of a trend fitted now to mark the line followed by developments in the near future need not mean that to-day's work is wrong. Perhaps our successor who computes a new trend for the longer series of data available to him will not be able to improve upon our fit for the period we analyzed.

Then why not break long series into relatively brief segments and compute the secular trends for each separately? That is a device to which the statistical student of business cycles resorts at need; but to go far in that direction is to give up the problem of secular trends rather than to solve it. Unless it is possible to find trends which are satisfactory throughout long periods—long in comparison with business cycles—the distinction between secular and cyclical fluctuations is blurred and the whole analysis loses its point. Just how far the process of subdividing periods for the computation of trends shall be carried is a question to be decided by the character of each series and the uses to which the results are to be put.

There is, indeed, no single criterion for determining "goodness of fit." A mathematical test can be applied only in certain cases. Provided one is choosing between two lines of trend whose equations contain the same number of constants, one can compare the standard deviations of the actual values from the trend lines. A test of wider application is to consider the "reasonableness" of the values shown by projecting trend lines into the future, and to choose lines which

tion: W. L. Crum, "The Determination of Secular Trend," June, 1922, vol. xviii, pp. 210-215. and "The Least Squares Criterion of Trend Lines," June, 1925, vol. xx, pp. 211-222; Holbrook Working, "The Determination of Secular Trend Reconsidered," December, 1922, vol. xviii, pp. 497-502; Willford I. King, "Principles Underlying the Isolation of Cycles and Trends," December, 1924, vol. xix, pp. 468-475; Lincoln W. Hall, "A Moving Secular Trend and Moving Integration," March, 1925, vol. xx, pp. 13-24; Olin Ingraham, "The Refinement of Time Series," June, 1925, vol. xx, pp. 231-233.

indicate results judged to be probable. In forecasting work this test is important for projections within the period to which the forecast applies. For the rest, statisticians fall back upon a visual comparison between the actual values and the trend lines within the time limits of the data. Their confidence in a fitted curve seems to be greater the simpler is its equation and the longer the period within which it gives a reasonable fit. But published expressions of opinion show that a fit which seems good to one man would be called poor by another. Personal equations play a large rôle in such judgments.

Nor is there any general method of deciding in advance what one among the several ways of determining trends will yield the best fit to a given time series, according to these rather indefinite criteria. In the same piece of work, an investigator may fit a straight line to one series and a parabola to a second, compute three-year moving medians of a third and seven-year moving arithmetic means of a fourth, run a free-hand curve through a fifth, use ratios to some other series for a sixth, and devise some novel method for a seventh. He may even use two or three unlike methods of determining the trend in different sections of the same series. Nor will he hesitate to compare the deviations of the actual data from the trends measured in these different ways, if he believes that each of the trend lines expresses the long-time tendency of its data better than would one of the other devices.

Each method has technical advantages, which should be considered with reference to the problem presented by each investigation. Subsequent uses of the data may make it desirable to have a trend which can be expressed by a simple equation, as can curves of known properties. The purpose in view may, or may not, make it important to reject curves which, though they may fit the data admirably for a long period in the past, would indicate results deemed absurd in the future. Ease of construction counts in favor of free-hand curves, and ratios to other series. Of course, the last device is of limited use, for it can be employed only when two series are found which seem to have nearly the same trend. Moving averages are controlled by the data as free-hand curves are not, and this independence of the investigator's personal equation gives comfort to many minds. In business-cycle work, moving averages will yield a satisfactory line of trend, if the trend is linear, if the period of the average corresponds to the duration of the cycles, and if the cycles are regular both in duration and intensity. These conditions are seldom strictly satisfied. If the true trend is a convex curve, a moving average lies above the curve, and so produces errors in the cyclical deviations, the magnitude of which increases with the convexity of the curve and the period of the moving average. If the true trend is concave, errors of opposite sign result. Finally, moving averages seldom yield trends that look satisfactory unless they are centered, and such averages cannot be kept up to date except by the hazardous practice of estimating figures for future years.[2] The advantage claimed for combining moving averages with curve fitting is that it minimizes the influence of the extreme years, which "may represent either the accident of the particular phase of the business cycle with which the series begins or ends, or a change in the real secular trend." [3] Free-hand curves, drawn with care, are preferred by some statisticians of wide experience, not merely because they are easier to make than any other trend lines, but also because they meet the test of visual comparison with the plotted data quite as well as the more pretentious mathematical constructions.

To illustrate the results obtained by applying different methods, Chart 2 has been made. The first section of the chart shows several different trends fitted to a single series—pig-iron production in the United States; the second section shows the application of the same method to several different series, and the third section shows different methods applied to different series.

Having ascertained the secular trend of a time series, the investigator's next step is "to eliminate the trend" from the original data.[4] To that end he finds the numerical value represented by the trend line at each successive interval of time, through computation or by readings from the scale of his chart, and then determines the plus and minus deviations of the actual data from those values in actual amounts or in percentages.[5] Finally, he draws a new chart in which he represents the secular trend by a horizontal line, and the devia-

[2] For example, the trend figure for 1927 in a seven-years' moving average, centered, is computed by adding actual and estimated data for 1924-30 and dividing by seven.

[3] Olin Ingraham, "The Refinement of Time Series," *Journal of the American Statistical Association*, June, 1925, vol. xx, p. 233.

[4] Frequently this elimination is postponed until the seasonal variations have been computed. But of that later.

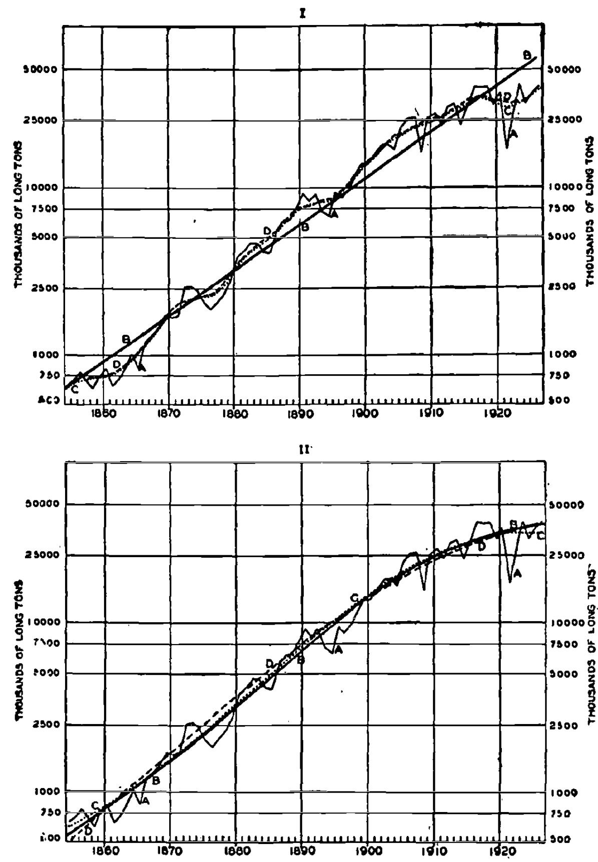
[5] Professor Allyn A. Young has called attention to the fact that the cyclical deviations from a secular trend are likely to be least trustworthy toward the extremities of the period for which the trend is fitted. Further, it is at the extremities that the differences among the various curves which may be employed as trends commonly become most marked. See "An Analysis of Bank Statistics for the United States," *Review of Economic Statistics*, January, 1925, vol. vii, p. 28.

CHART 2.

Examples of Secular Trends of Time Series Fitted by Various Methods.

Section 1.

Different Trends Fitted to the Same Series. Pig-Iron Production in the United States



I A ——— PIG IRON PRODUCTION
B ——— LEAST SQUARES STRAIGHT LINE FITTED TO LOGARITHMS (1854-1926) $Y = 6.7849 + .027542X$ ORIGIN 1890
C ——— TWENTY YEAR BINOMIALLY WEIGHTED MOVING AVERAGES OF LOGARITHMS
D - - - - - - SEVEN YEAR MOVING AVERAGES

II A ——— PIG IRON PRODUCTION
B ——— MODIFIED GOMPERTZ EQUATION FITTED TO LOGARITHMS - SUGGESTED AND FITTED BY F. R. MACAULAY $\text{LOG}\left(Y + \frac{40{,}738{,}000}{199.526}\right) = \text{LOG } 40{,}738{,}000 - (1.025)^{(1.0387)^x} \text{LOG } 199.526$ ORIGIN 1890
C ········· THIRD DEGREE PARABOLA FITTED TO LOGARITHMS $Y = 6.6531 + .032769X - .0001536 4X^2 - .00000654172X^3$ ORIGIN 1890
D - - - - - - LOGISTIC CURVE $Y = \frac{30{,}403}{1 + 10^{1.80987 - .17431X}}$ ORIGIN 1880

CHART 2 (*Continued*).

EXAMPLES OF SECULAR TRENDS OF TIME SERIES FITTED BY VARIOUS METHODS.

Section 2.

The Same Method Applied to Different American Series. Straight-Line Trends from the *Review of Economic Statistics*.

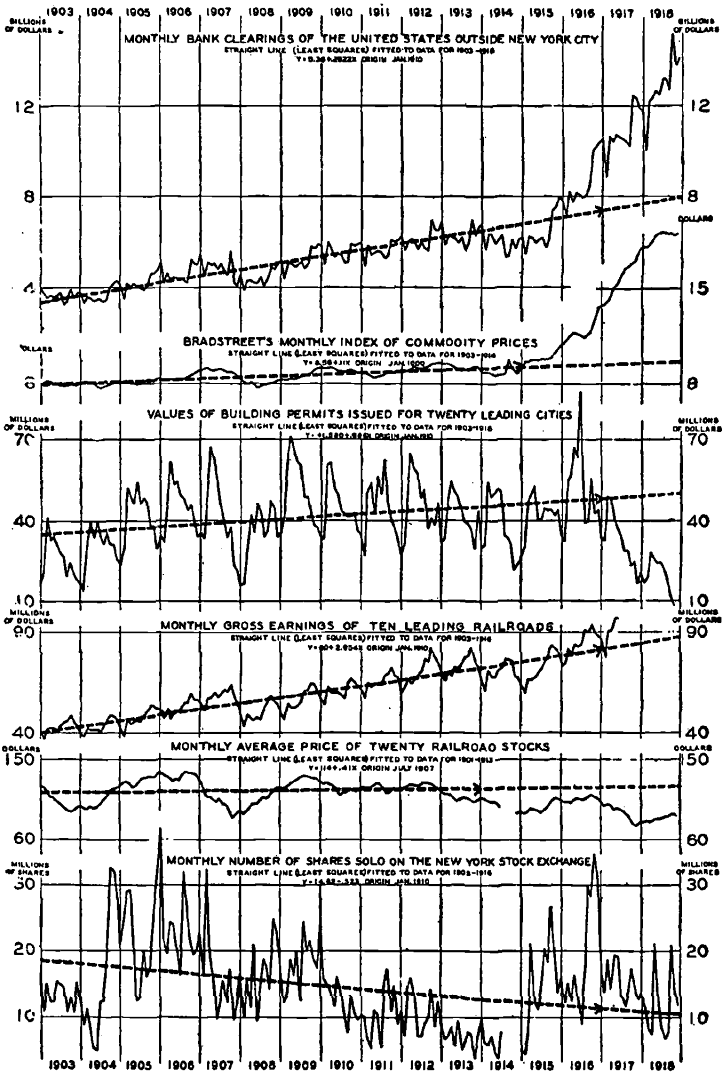


CHART 2 (*Continued*).

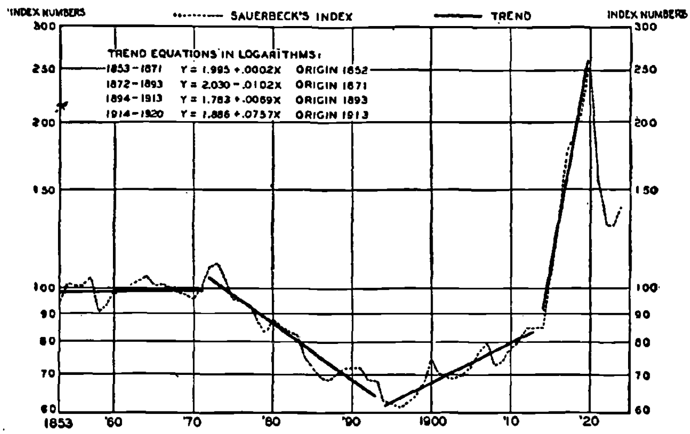
Examples of Secular Trends of Time Series Fitted by Various Methods

Section 3.

Different Methods Applied to Different Series.

I

SAUERBECK'S INDEX OF BRITISH WHOLESALE PRICES (1867-1877=100) WITH TREND
FITTED BY STRAIGHT LINES (LEAST SQUARES) TO LOGARITHMS, BY PERIODS



II

RAILROAD BOND YIELDS, UNITED STATES, WITH TREND
FITTED BY A TWENTY YEAR BINOMIALLY WEIGHTED MOVING AVERAGE
CENTERED OPPOSITE THE ELEVENTH YEAR (F.R.MACAULAY)

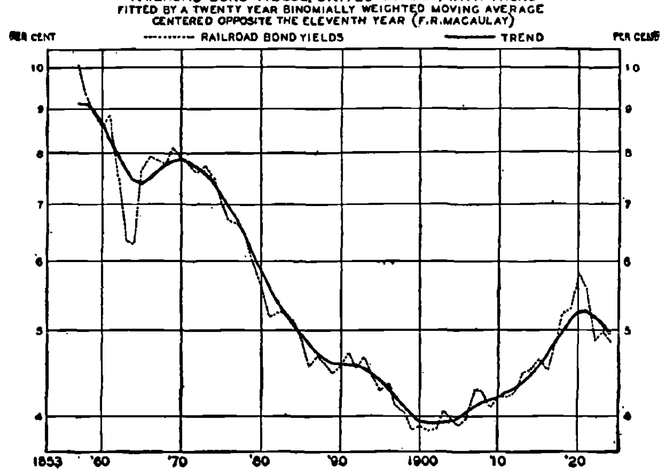


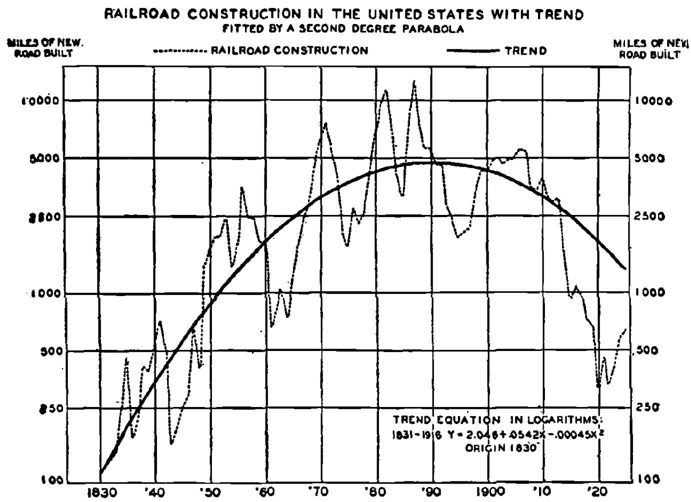
CHART 2 (*Continued*).

Examples of Secular Trends of Time Series Fitted by Various Methods

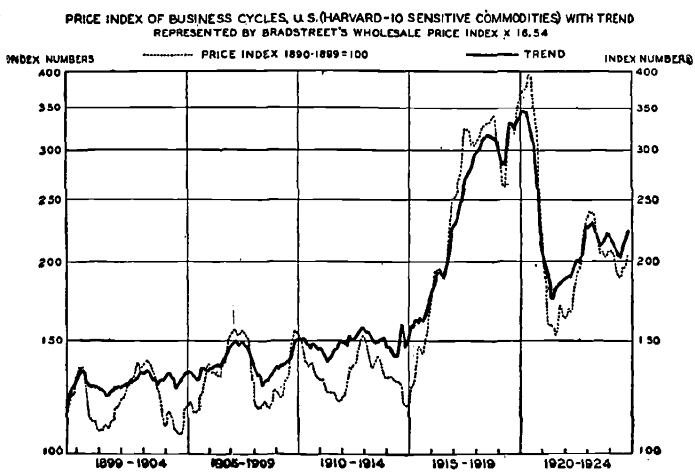
Section 3 (*Continued*).

Different Methods Applied to Different Series.

III



IV



tions by a curve which fluctuates about the horizontal. Several examples of such construction are given in Chart 3.

When a statistical inquirer into business cycles has reached this point in his work, he commonly goes on at once to ascertain the seasonal variations left in his curve of deviations from the trend, or to use the deviations as they stand. But preoccupation with theory requires us to pause here, and to look at the problem from another angle.

(2) The Interpretation of Secular Trends.

What meanings have the secular trends fitted to time series by empirical methods? As Dr. Kuznets remarks, every mathematical curve used as a trend has definite implications, whether the statistician notices them or not. To take the simplest example: a straight line sloping upward implies future increase without limit. Its constant rate of increase per unit of time implies that the size of the variable at one moment does not affect the size of the increment between that moment and the next. Its percentage rate of increase diminishes along a hyperbolic curve. When we find that a straight line trend fits a given series well, do we accept these mathematical implications as characteristic of the economic process represented by the data? Are successful fits of mathematical curves discoveries in economics?

These questions suggest another way of treating the whole problem of secular trends. We form various hypotheses concerning the long-time tendencies of economic developments in population, production, transportation, exchange, and the like. These hypotheses are derived from and linked to causal explanations; but ordinary reasoning does not enable us to test them adequately. Can we choose curves whose mathematical implications correspond to our causal hypotheses, fit them to time series, perhaps modify the hypotheses in the light of the first results, experiment with other curves, and when finally we have secured good fits argue that we have thrown new light upon the characteristics of economic evolution?

A step toward such a conception is represented by the frequent interpretation of certain trend lines as showing the "growth factor." Statisticians dwell with satisfaction upon their demonstrations that certain industries have expanded decade after decade at a substantially uniform rate, or at a rate which has changed in some uniform way. They take almost as much pleasure in contemplating the somewhat similar rates at which different industries have grown in given

CHART 3.

SAMPLES OF TIME SERIES PLOTTED AS DEVIATIONS FROM SECULAR TRENDS REPRESENTED BY HORIZONTAL LINES.

(Selected from Chart 2).

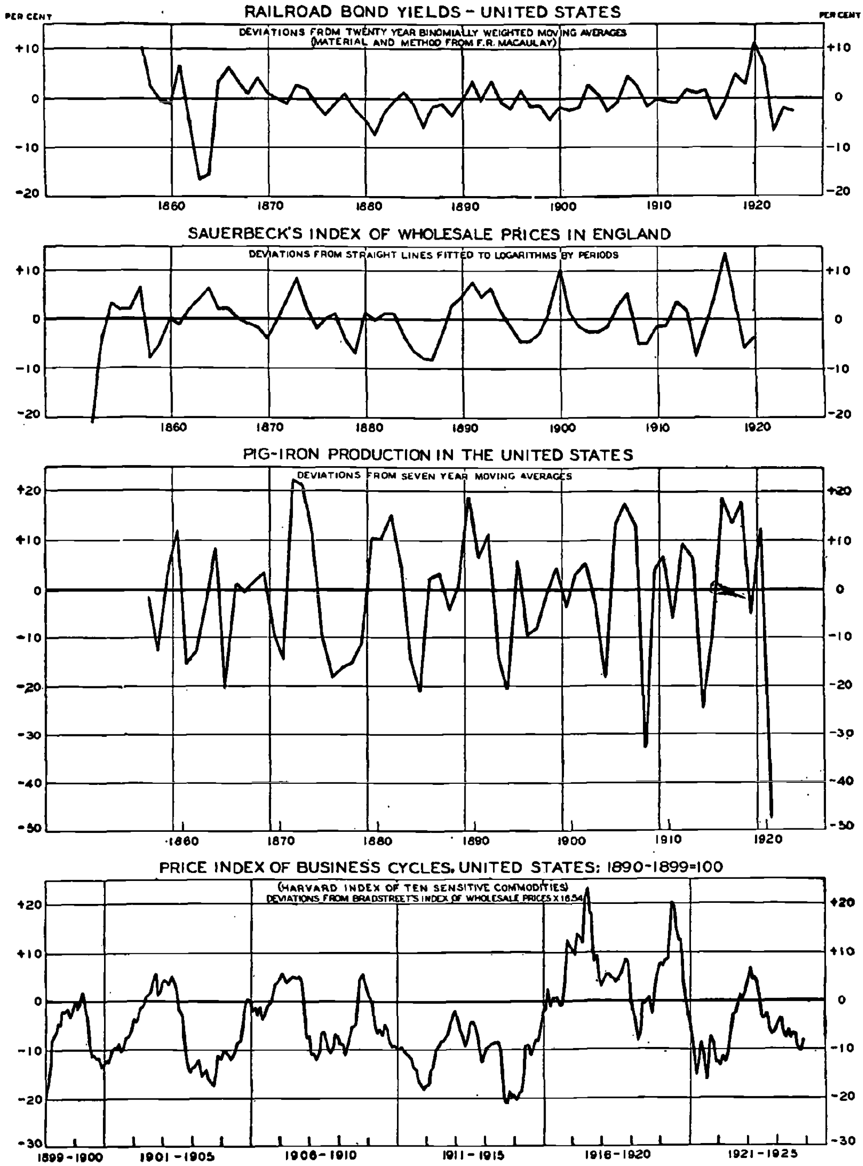
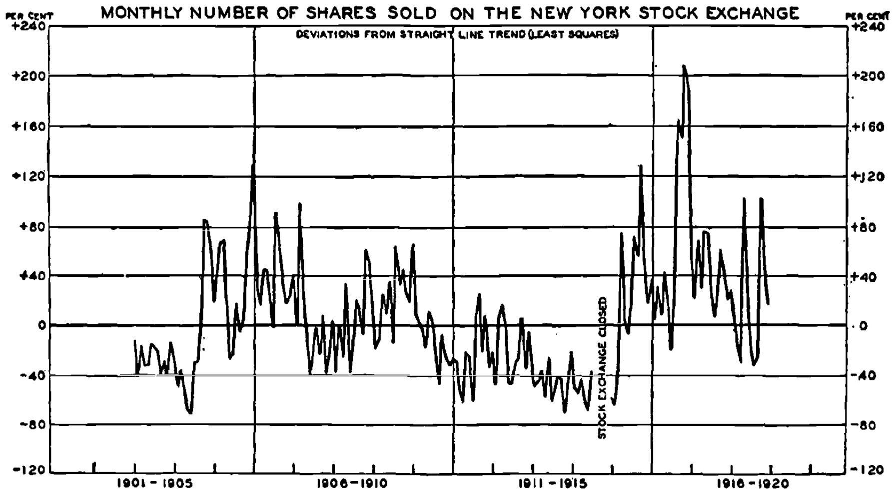
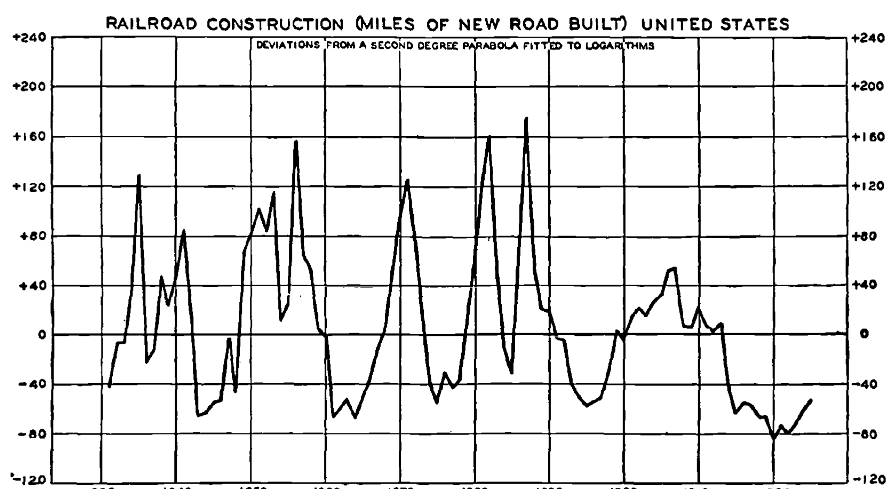


CHART 3 (*Continued*).

EXAMPLES OF TIME SERIES PLOTTED AS DEVIATIONS FROM SECULAR TRENDS REPRESENTED BY HORIZONTAL LINES.

(Vertical Scale—¼ of scale on opposite page).





periods and countries. Nor are they at a loss for explanations of these uniformities. In view of the increase in population characteristic of the great commercial nations, and of the advance in industrial technique, it seems scarcely fanciful to think of modern society as "tending" to produce an ever larger supply of goods for the satisfaction of its wants. On this basis, cyclical fluctuations appear as alternating accelerations and retardations in the pace of a more fundamental process. Secular trends, in short, are taken to measure economic progress generation by generation.

A bold speculation of this sort has been ventured by Mr. Raymond B. Prescott. He suggests that perhaps "all industries, whose growth depends directly or indirectly upon the ability of the people to consume their products," pass through similar phases in the course of their development. Four stages seem to be common:

1. Period of experimentation.
2. Period of growth into the social fabric.
3. Through the point where the growth increases, but at a diminishing rate.
4. Period of stability.

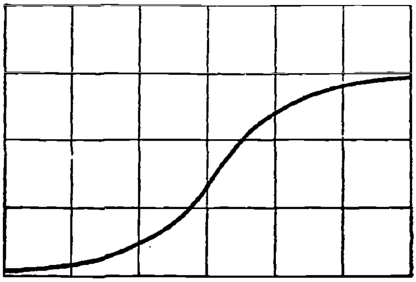
On this basis, Mr. Prescott suggests that the secular trends of all such industries may be represented by a single type of curve—that yielded by the Gompertz equation.

> Every country (he adds) may have a different rate of growth, and so may every industry, because no two industries have the same combination of influences. They will trace the same type of curve, however, even though the rate of growth is different.[1]

[1] "Law of Growth in Forecasting Demand," *Journal of the American Statistical Association*, December, 1922, vol. xviii, pp. 471-479.

To illustrate his suggestion, Mr. Prescott publishes the following copyrighted diagram, of which the caption is significant.

CURVE OF PROGRESS OR LAW OF GROWTH (R. B. PRESCOTT)



Dr. Simon S. Kuznets, who has fitted more Gompertz or "logistic" curves (with three constants) to economic series than anyone else known to me, finds that the Gompertz curve gives good fits to a number of series, but that in more cases he gets good fits by using the three-constant "logistic curve." The important properties of this curve are: (1) Finite limits, zero and an upper value to which the curve is asymptotic. (2) The rate of increase per unit of time is directly proportioned to the size of the variable at a given point, multiplied by the distance between that point and the upper limit. This implies that (say) the output of an industry grows in physical units at a rate which increases from the start to the point of inflection of the curve, and then decreases gradually to zero as the curve approaches its upper asymptote. (3) The percentage rate of increase declines steadily from the beginning. These three characteristics, Dr. Kuznets supposes, appear in the history of the many economic processes, whose long-time statistical record is well described by a logistic curve.

The use of logistic and Gompertz curves is not limited to describing the secular trends of growing industries. They can be adapted also to the conditions of industries with a shrinking output. Such industries are not unimportant, even in a highly prosperous country like the United States. Conspicuous examples are canal traffic, gold mining, shipbuilding, bicycle manufacturing, and lumbering. Indeed, a declining phase may be anticipated for most industries, if we define an industry somewhat strictly in terms of its specific output and its geographical location. The cause of the decline may be competition from some other industry (for example, canals *versus* railways, bicycles *versus* automobiles); the depletion of natural resources (for example, lumbering and gold mining), or the approximate satisfaction, at least for a time, of a non-recurrent need (for example, railway building). In adapting his analysis to such cases, Dr. Kuznets takes the maximum output reached by the industry as the upper limit of the variable, and supposes a decline toward zero along an inverted logistic curve. By introducing an additional constant into his equations at need, he is able to secure satisfactory fits.

There remain numerous time series to which the idea of growth and decay is not applicable. Such is the case with all series of prices whether of commodities or other goods, and with many series of ratios, such as interest rates, bond yields, percentages of bank reserves to deposits, marriage rates, and credit ratios. Some of these series are highly important in the study of business cycles. It is a simple task to find empirical trend lines which fit the data reasonably well. For example, in Chart 1 the British unemployment series suggests the use of a horizontal line, and the British index of wholesale prices suggests a series of oblique lines which meet each other at certain well-known dates. But it is exceedingly difficult to rationalize the procedure, as one can in devising trends for quantities which grow or decay. Yet it may be that certain recent investigations are laying foundations on which can be built a rational analysis even of changing price trends.

(3) The Hypotheses of "Secondary Trends" and of "Long Waves"

In studying the relations between the data points of his industrial series, and the logistic or Gompertz trend lines he had fitted, Dr. Kuznets observed that the annual deviations tended to have positive values during the periods when wholesale-price indexes followed rising trends, and to have negative values when the trend of the price indexes was declining. Of course, this observation seems to be a statistical version of the familiar statement that a long-period rise of wholesale prices stimulates, and that a long-period fall of wholesale prices retards, industrial progress. But Kuznets went further, extending his analysis to the wholesale prices of various great staples. In dealing with these series he had to use empirical trends. Again he found that the annual deviations swerved upward from their long-period trends when the general level of wholesale prices was rising, and swerved downward when the general level of prices was sagging.

To test these observations, Kuznets removed the cyclical fluctuations of the annual deviations from the primary trends so far as possible by taking moving averages with periods slightly longer than business cycles, and measured the duration of the swerves, which he called "secondary trends." He found that the swerves averaged a little shorter in the production series than in the corresponding price series—about 11 years in the one case and about 12 years in the other. Doubling these periods to get the full period from crest to crest, or trough to trough, of these fluctuations, he concluded that the average duration since say the middle of the 19th century has been a little less than 25 years.

Dr. Kuznets inclines to treat these secondary trends as a distinct species of business fluctuations, intermediate between the much longer primary trends and the much shorter business cycles. To account for them he thinks economists must develop a special theory. His tentative explanation combines emphasis upon factors of a non-business origin with emphasis upon the fact that certain cyclical developments in business activities have effects which persist from one cycle to the next.

In 1913, a Dutch economist, J. van Gelderen, called attention to what he named "large cycles" in economic development, covering about 60 years. A compatriot, S. de Wolff, confirmed van Gelderen's results in 1924 by the use of more technical statistical analysis. Meanwhile the Russian investigator, N. D. Kondratieff, had developed the same idea independently in 1922. Not content with his first results, Kondratieff collected and analyzed all the time series he could find which covered long periods. The results of his work, which agree substantially with those of van Gelderen and de Wolff, were published in Russian in 1925 and summarized in a German article of last December.[1]

Kondratieff starts with the "long waves" of British wholesale price indexes; the rise from 1789 to 1814, the fall to 1849, the rise to 1873, the fall to 1896, the rise to 1920. Similar waves appear in the interest yields upon French *rentes* and British consols; also in French and English wages. Turning to series which show aggregate values or physical quantities, Kondratieff adjusted them to his needs by reducing the original data to a per-capita basis, fitting mathematical trend lines, computing deviations from the trends, and smoothing the deviations with 9-year moving averages. Such smoothed deviations show "long waves" in French imports, exports, and total foreign trade; British exports; French savings-bank balances; the portfolio of the Bank of France; coal production (or consumption) in France, England, the United States, Germany, and the world; iron production in England, the United States, and the world; lead production in England; and the area planted to oats in France and to cotton in the United States. On the other hand, "long waves" are not found in

[1] See Van Gelderen (using the pen-name "J. Fedder"), "Springvloed Beschouwingen over industrielle ontwikkeling en prijsbeweging," *De Nieuwe Tijd*, 1913, pp. 253-277, 369-384, 445-464; de Wolff, "Prosperitäts- und Depressionsperioden," in *Der lebendige Marxismus, Festgabe zum 70. Geburtstage von Karl Kautsky*, Jena, 1924, pp. 13-43; Kondratieff, *The World Economy and its Conjunctures during and after the War* (in Russian), Moscow, 1922; "The Problem of Economic Conditions," *Monthly Economic Bulletin of the Conjuncture Institute*, 1925, Supplement, pp. 28-79 (in Russian); "Die langen Wellen der Konjunktur," *Archiv für Sozialwissenschaft und Sozialpolitik*. December, 1926, vol. lvi, pp. 573-609.

French cotton consumption, American wool and sugar production, or "in the movement of certain other elements."

Surveying the whole body of his results, Kondratieff concludes that the western world has seen two-and-a-half "long waves" since the closing years of the 18th century.[2] The turning points are as follows:

	Trough of the wave	Crest of the wave	Trough of the wave	Approximate duration
1st long wave.......	Late 1780's or early 1790's	1810–17	1844–51	50–60 years
2nd long wave.......	1844–51	1870–75	1890–96	40–50 years
3rd long wave.......	1890–96	1914–20	.....	....

While Kondratieff believes that his statistical results make the existence of "long waves" highly probable, he offers no hypothesis to account for them. But he does regard these waves as cyclical phenomena, and believes that they arise from causes inherent in "capitalistic economy"—not from accident. Like van Gelderen and de Wolff, he rejects the easy explanation that the "long waves" in prices, and hence in other processes, are due to accidental discoveries of gold deposits combined with improvements in mining and metallurgical methods. To these economists it seems more probable that the business conditions characteristic of the ebbing of a "long wave" offer strong incentives to prospecting for new gold deposits and to improving the technical processes of exploiting known deposits. Thus the ebbing of a "long wave" tends to produce effects which favor a rise in prices and a mounting wave. *Vice versa,* the business conditions which characterize the rising of a "long wave" tend to make gold production unprofitable, hence to check the output, to stop the rise of prices, and so to reverse the direction of the whole movement once more. In other words, they treat the long-period swings in gold output as themselves an organic part of the 40-50-60-year cycles. But

[2] De Wolff suggests that each half of the long waves is composed of 2½ smaller cycles, and that the latter cycles are growing briefer according to this schedule:

10 years	10 years	10 years
9 "	9 "	9 "
8 "	8 "	8 "
7 "		

Beginning with 1825 (de Wolff's starting point), this scheme works out as follows:

		Beginning	Duration in years	Ending
1st cycle	Declining phase..........	1825	10+10+5=25	1850
2nd "	Ascending "	1850	5+ 9+9=23	1873
	Declining "	1873	9+ 8+4=21	1894
3rd "	Ascending "	1894	4+ 8+7=19	1913

See *Der lebendige Marxismus,* pp. 37, 38.

Kondratieff, at least, does not regard this suggestion as an adequate theory of the mechanism by which long cycles are alternately initiated and ended.

Until some adequate reason has been shown why we should expect more or less regular recurrences of "long waves" in economic activities, we shall have nothing beyond empirical evidence concerning their existence. We may admit the probable validity of Kondratieff's statistical argument that two-and-a-half "long waves" have occurred in various economic processes since the end of the 18th century, and yet hold open the question whether the series will be continued. Two-and-a-half recurrences do not suffice to establish empirically a presumption that any feature of modern history will repeat itself.

Another uncertainty is presented by the difference in order of magnitude between the duration of the "long waves" found by van Gelderen, de Wolff and Kondratieff, and the duration of the "secondary trends" found by Kuznets. It may be that the difference in the results is due to differences in methods of fitting trends, and computing and smoothing deviations. Or perhaps the European investigators, looking for replicas of the major swings of wholesale prices, paid slight attention to swerves which attract notice in a less preoccupied mind. Or it may be that there really are two sets of long-term fluctuations in economic activities, one of which averages double the duration of the second. Further research is needed to settle these issues.

But whether there be two sets of long-term fluctuations or only one set, whether (if there be but one set) the typical length is about a quarter or about half a century, and whether these fluctuations are merely historical episodes of considerable interest or an inherent characteristic of "capitalist economy," the investigations of van Gelderen, de Wolff, Kondratieff and Kuznets open an alluring perspective of future work. Starting with the study of commercial crises, the realistic students of economic activities have discovered successively several types of fluctuations which, at least for more than a century, appear to have been recurrent—the seasonal variations to which Jevons directed attention in his youth, the inter-crisis cycles of Juglar and others, the shorter business cycles of later writers, the secular trends of empirical statisticians, the 22-24-year "secondary trends" of Kuznets, and "long waves." Of course, some of these discoveries may prove to be invalid; but, on the other hand, recurrences of other periods may be revealed Sismondi's problem was merely to explain crises: now the problem is to ascertain how many types of fluctuations are intermixed in economic experience, to differentiate these types from each other, to measure the wave-length of each type and to ascertain its regularity of recurrence, finally to construct a theory which will account adequately for all the types of fluctuations and make clear their relations to each other. As treatises upon crises, or crises and depressions, are giving place to treatises upon business cycles, so the latter may in turn give place to treatises upon economic oscillations.

While the time for undertakings of such scope has not quite come, we should keep constantly in mind the probable interconnection between business cycles and the less-studied fluctuations of shorter or longer duration. One connection is clear. Our writers on secular trends confirm an old contention when they point out that during the ebbing phase of a "long wave" years of depression predominate in the inter-crisis cycles, while during the mounting phase of a "long wave" years of prosperity predominate.[3]

We stand to learn more about economic oscillations at large and about business cycles in particular, if we approach the problem of trends as theorists, than if we confine ourselves to strictly empirical work. The trends which promise the most important additions to our knowledge are those which correspond to rational hypotheses, although they may not "fit the data" so well as empirical constructions which are difficult to interpret. For it may prove possible to integrate the rational hypotheses which yield instructive trends with the theory of business cycles.

(4) Conclusion.

The upshot of this discussion is that lines of secular trend show the effects of causes which, though subject to change at any moment, have influenced an economic process in some regular, or regularly changing, way through periods of time long in comparison with business cycles. What these causes have been, and whether they are still in operation, are matters for further inquiry. The empirical inquirer measures something which he knows merely as secular trend; that something is a set of net resultants; he may or we may not try to find out to what that something is due.

If we embark upon a search for causes of secular trends, we must

[3] See, for example, Kondratieff's article cited above, *Archiv für Sozialwissenschaft und Sozialpolitik,* December, 1926, vol. lvi, p. 591.

expect to find not one cause peculiar to each series, but a peculiar combination of a multitude of interrelated causes. These causes may be classified as (1) causes related to changes in the number of population, (2) causes related to the economic efficiency of the population—its age, constitution, health, education, technical knowledge and equipment, methods of coöperation, methods of settling conflicts of interest, and many other matters; (3) causes related to the quantity and quality of the natural resources exploited by the population.

Not only the second, but also the first and the third of these categories consists of a complex of factors which sustain all sorts of relations to each other. Even kinds of causes which the classification separates are interacting. The growth of population is influenced by changes in industrial technique and in natural resources; the growth of population also influences the development of technique and of resources; finally, changes in technique produce changes in resources (the iron-ore ranges and water powers of North America did not exist for the red men), and changes in resources are ever stimulating changes in technique. All of which means that we must think of every type of economic activity in a country as conditioned by a concert of fundamental factors, albeit a concert in which the ranking of the factors differs endlessly from case to case.

Another approach may make the conclusion clearer and more significant. There are secular causes which affect many economic activities in much the same way, for example, changes in gold output, depletion of soil fertility. There are secular causes which affect economic activities in different ways: the extension of the railway net over the United States checked canal building, diminished river traffic, and led to the abandonment of many eastern farms; but it stimulated the settlement of western lands, built up interior cities, and fostered the expansion of the coal and steel trades. There are secular causes whose direct effects are confined mainly to some single line of activity: —a series of inventions which cheapens the cost of producing some article of secondary importance is an example. Those secular causes which influence many activities in much the same way produce a measure of likeness among the secular trends of different time series. Those secular causes which influence various activities in opposite ways, and those secular causes whose perceptible influence is limited in scope introduce diversities in secular trends. There can scarcely be any time series whose trend is not a joint product of factors which tend toward uniformity and factors which tend toward diversity.

The most valuable contributions toward an understanding of the trends empirically established by statisticians have been made by the economic historians. These workers have studied in detail such great movements as the Agricultural and the Industrial Revolutions, the evolution of capitalism, the world-spread of the European races, the discovery, utilization and impairment of natural resources. They have sought not merely to record but also to explain these long-period changes in human affairs, using such statistics as they could gather and such hypotheses as their materials suggested. But they have been kept so busy mastering a vast mass of materials that they have not yet begun a systematic attack upon the problem of secular trends. Nor have they equipped themselves with the statistical technique needed for the most effective use of numerical data.

Economic theorists also have shown some interest in secular trends, but their contributions have been speculations concerning the future, rather than analysis of the past. The "pure" economic theory of recent years has dealt mainly with "static" problems, from which secular changes are barred. Economic "dynamics" has been regarded as more treacherous ground, and the mischances of eminent men who have walked thereon by the light of theory have been discouraging. Ricardo, for example, expected that mankind would be forced to resort to ever poorer soils and ever more intensive cultivation to get food for their increasing numbers; that in consequence real wages would remain at best constant, profits would decline unsteadily, and rents rise until "almost the whole produce of the country, after paying the laborers, will be the property of the owners of land and the receivers of tithes and taxes." [1] Of the numerous speculations of this type, those of Karl Marx are of especial interest here, because they include the increasing frequency and increasing severity of business crises among the secular trends which are to usher in the socialistic state.[2]

It is no wonder that a field which requires the fusion of statistical technique with historical learning and theoretical finesse has not yet

[1] *Principles of Political Economy*, ed. by C. K. Gonner, London, 1891, p. 99.

[2] Marx's theory of crises runs through all three volumes of *Capital*, not to speak of his other writings. For a brief statement of the rôle of crises in the scheme of economic revolution, see the preface which he wrote in 1873 for the first volume of *Capital* (English translation by Moore and Aveling, Chicago, 1915, p. 26.)

John R. Commons, H. L. McCracken, and W. B. Zeuch have made a systematic study of the ideas concerning secular trends propounded by economic theorists. They hold that the conceptions of trends and business cycles have been derived from the theories of value entertained by the writers they discuss and from the types of organization these writers had in mind. See "Secular Trends and Business Cycles: a Classification of Theories," *Review of Economic Statistics*, Preliminary vol. iv, pp. 244-263.

been explored. Yet the time may be near when the problem of secular trends will have as definite a standing in economic research, attract as many investigators, and yield as interesting results as the problem of business cycles. In the meanwhile, students of the latter problem suffer the grave disadvantage of having to deal with a factor which is both important and obscure. They cannot anticipate the results of researches not yet made; they cannot make adequate studies of secular trends as an incident in their studies of cyclical fluctuations, and they cannot let the problem alone.

One set of questions is particularly insistent. Is there a definite relation between secular trends and cyclical fluctuations? Are activities characterized by a rapidly rising trend subject to more frequent, or more violent, cycles than activities whose trend is nearer the horizontal? And more at large, can the trends of time series, after they have been measured, be discarded as of no further interest? Or must the trends themselves be brought into the explanations of cyclical fluctuations, as is suggested by those theories which connect business cycles with "progress"? [3] Are the trends themselves generated by cyclical fluctuations, as Mr. Lawrence K. Frank has argued? [4] While these questions arise at this point, they cannot be answered by any process short of considering the pertinent evidence in detail. But the mere fact that such problems must be faced by the business-cycle theorist suffices to show that he cannot imitate the business-cycle statistician in merely eliminating secular trends.

4. The Problem of Seasonal Variations.

Chart 4, drawn on a larger scale than the charts illustrating secular trends, shows how time series differ in respect to seasonal variations. As in Chart 1, the data are plotted in their original form, in order to make clear the varied difficulties which confront the statistician who is trying to isolate cyclical fluctuations.

There are series in the chart which undergo seasonal changes each year of much the same sort, and of an amplitude so large as to obscure the cyclical fluctuations, if not the secular trends. There are series in which the seasonal variations, while fairly regular, are not wide. There are others which suggest the presence of seasonal factors

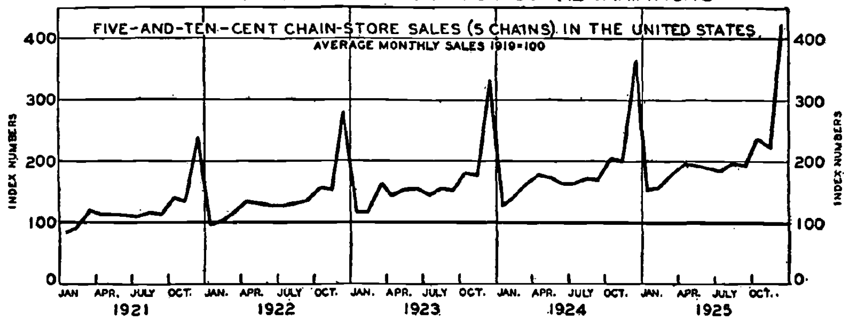
[3] Compare Chapter I, section iv, 4.

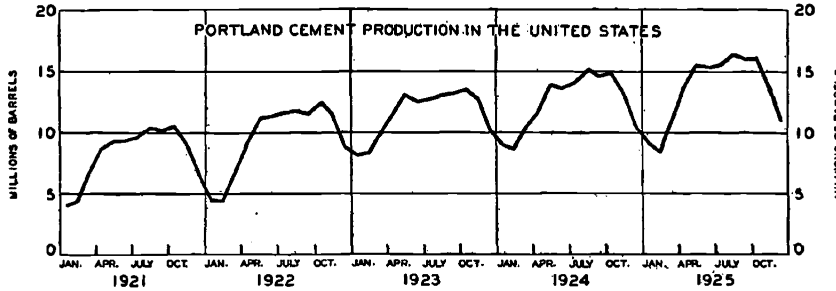
[4] See Mr. Frank's paper, "Long-Term Price Trends," *Journal of the American Statistical Association,* September, 1923, vol. xviii, pp. 904-908.

CHART 4.

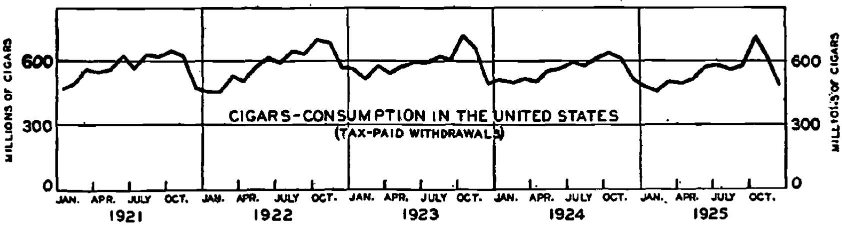
A Collection of Time Series to Illustrate Differences in Seasonal Variations.

GROUP I - LARGE AND REGULAR SEASONAL VARIATIONS





GROUP II - SMALL AND FAIRLY REGULAR SEASONAL VARIATIONS



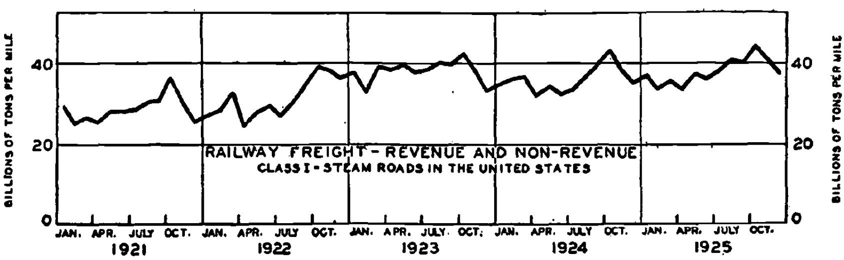
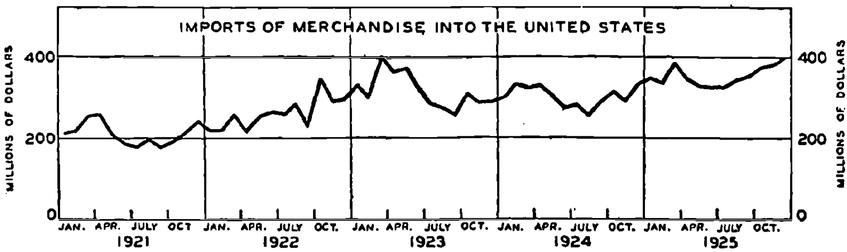
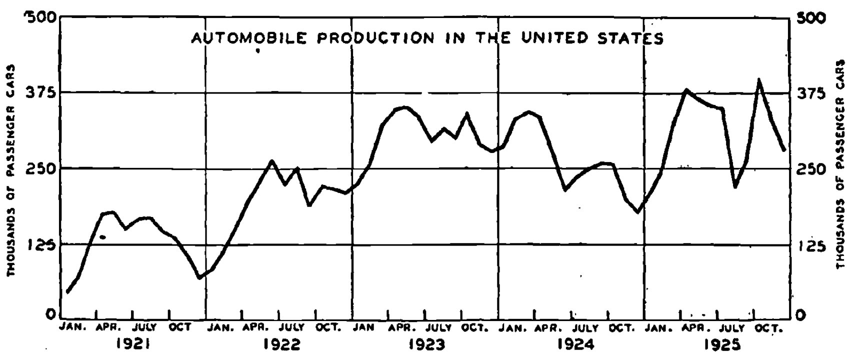


CHART 4 (*Continued*)

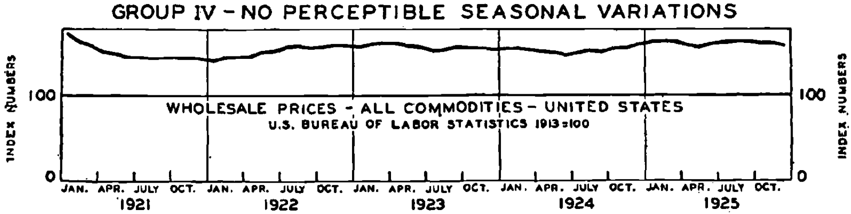
A Collection of Time Series to Illustrate Differences in Seasonal Variations

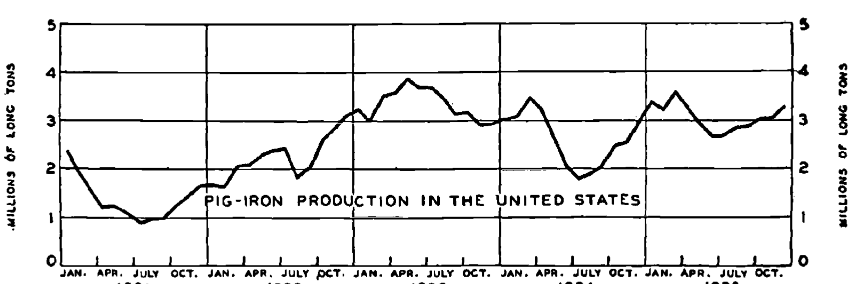
GROUP III – RATHER IRREGULAR SEASONAL VARIATIONS





GROUP IV – NO PERCEPTIBLE SEASONAL VARIATIONS





rather irregular in themselves, or so combined with changing conditions of other types as to produce rather irregular results. Finally, there are series in which it is difficult on inspection, or even on analysis, to detect any semblance of regularity in the month to month movements of different years.

To isolate cyclical fluctuations for close study, we should be able to determine in doubtful cases the presence or absence of seasonal variations, to measure their amplitudes where found, and finally to get rid of them when we wish. How to accomplish all this is a problem which the business-cycle statisticians have attacked with vigor.[1] Business-cycle theorists, on the contrary, have paid little attention to these short-period oscillations—an omission which it is unwise to imitate.

(1) The Causes and Pervasiveness of Seasonal Variations.

Two types of seasons produce annually recurring variations in economic activities—those which are due to climates and those which are due to conventions. Upon some activities these seasons act directly, upon others indirectly.

(1) In the temperate zones at least, where lie the areas of chief concern to us, climatic seasons control the growth of crops and of such animal products as wool. They exercise a marked influence upon the current supply of many other animal products, such as fish, milk, poultry and eggs. In varying degree they affect almost all processes carried on out-of-doors, building, logging, transportation, road making, construction work at large. They are a factor in the efficiency or the cost of many processes carried on indoors. Certainly climatic seasons influence the death rate; doubtless they influence morbidity rates and so affect almost all activities in some measure.[1]

Upon the demand for certain types of goods, the effect of climatic seasons is not less clear than its effects upon certain types of produc-

[1] The earliest studies of seasonal variations known to Jevons were made in 1854 to 1856 by J. W. Gilbert (bank-note circulation, *Statistical Journal*, vol. xvii, pp. 289-321, and vol. xix, pp. 144-168) and Charles Babbage (bank clearings, the same, vol. xix, p. 28). Jevons began his own economic studies with a paper, "On the Study of Periodic Commercial Fluctuations," which he sent to the meeting of the British Association in 1862. In 1866 he read a much more elaborate paper, "On the Frequent Autumnal Pressure in the Money Market," to the Statistical Society of London. See *Investigations in Currency and Finance*, by W. S. Jevons, London, 1884, pp. 1-12, and 160-193.

[1] On the seasonal variations of the death rate, see Maurice B. Hexter, *Social Consequences of Business Cycles*, Boston and New York, 1925, pp. 55-57.

tion. The sort of clothing we wear, our sports, the amount of fuel we use, undergo radical changes from winter to summer.

Market supply is kept far steadier than the rate at which goods subject to strong seasonal influences are produced. This steadiness is a triumph of economic planning. It is won by arranging compensatory seasonal variations in the activities which intervene between producing and consuming. The processes of preserving, storing, transporting and distributing goods are purposely made to vary in such a way that a highly unstable rate of production is converted into a fairly even rate of market supply, or into a rate of market supply which varies with seasonal changes in demand. Some efforts are made to counteract also the variations in demand produced by climatic seasons. For example, a seasonal change is made in the price of coal to stimulate buying during the summer, and most seasonal goods can be had cheaper in their "off seasons." Thus the effects of climatic seasons are extended by man's contrivance over a wide variety of economic processes.

(2) Conventional seasons have many origins—ancient religious observances, folk customs, fashions, business practices, statute law.

The most pervasive of all the seasonal variations in time series is due to the calendar. February is nearly 10 per cent shorter than January, except in leap years when it is about 6 per cent shorter, and April is nearly 3 per cent shorter than March. Series of monthly aggregates are thus made to show a spurious seasonal variation—spurious in the sense that it bears no relation to the current rate of activity. And this spurious variation is made irregular from year to year by the different ways in which Sundays and holidays are divided among the months.

Many of the conventional seasons have considerable effects on economic behavior. We can count on active retail buying before Christmas, on the Thanksgiving demand for turkeys, on the July demand for fireworks, on the preparations for June weddings, on heavy dividend and interest payments at the beginning of each quarter, on an increase of bankruptcies in January, and so on. One of these conventional seasons is especially troublesome to statisticians, because it is movable. Easter may come as early as March 22d or as late as April 25th. Seasonal variations in series affected by Easter buying are decidedly different in the March and the April years.

From the activities directly affected by climatic or conventional seasons, acting separately or in unison, seasonal influences radiate to all other activities, probably without exception. In part these radiations are due to the conscious efforts already spoken of to counteract seasonal changes in demand or supply; in part they are unplanned consequences of these changes. For example, the fact that American crops are harvested largely in the autumn gives rise to a seasonal demand for currency in the farming districts, to seasonal changes in interest rates (and sometimes stock prices) in the financial centers, to seasonal changes in railway traffic, to seasonal changes in farmers' receipts, to seasonal changes in their payments to creditors, to seasonal changes in the business of country merchants, and to seasonal changes in wage disbursements. So, too, the expectation of heavy buying by consumers in the holiday season leads retailers to increase their stocks at earlier dates. In turn, the prospect of these large orders injects still earlier seasonal variations into manufacturing, into the demand for raw materials, into employment, and into wage payments, thus tending to produce secondary seasonal variations in retail buying itself.

It seems probable that these reflex effects of the primary seasonal disturbances grow smaller in most cases as they radiate to other processes. For example, manufacturers of goods for which the demand is largely concentrated in a few weeks seek to spread the production over a longer period. It is far less costly to provide a moderate equipment which can be used continuously in making a year's supply of goods than to provide a large equipment which must stand idle most of the time in order to produce a year's supply in a rush. Of course the ideal of continuous production is seldom attained in seasonal trades; but the business motives for stabilizing operations are clear enough and strong enough to moderate the effect of seasonal factors in a notable degree.[2]

Not less important, is the fact that both the original and the derived impulses toward seasonal activity are well scattered over the months of the year. For example, coal mining and logging grow brisk while construction work is falling off and farming requires fewer hands. In Great Britain employment reaches its maximum in

[2] Dr. N. I. Stone has pointed out that efforts to stabilize operations usually begin with seasonal variations, and later may or may not extend to cyclical variations. See his chapter on "Methods of Stabilizing Production of Textiles, Clothing, and Novelties," in *Business Cycles and Unemployment,* National Bureau of Economic Research, 1923, pp. 116-133.

References to the rapidly growing literature upon methods of mitigating seasonal and cyclical variations may be found in the bibliography given by Lewisohn, Draper, Commons and Lescohier in *Can Business Prevent Unemployment?* New York, 1925, pp. 217-226, and in the footnotes of H. Feldman's *The Regularization of Employment,* New York, 1925.

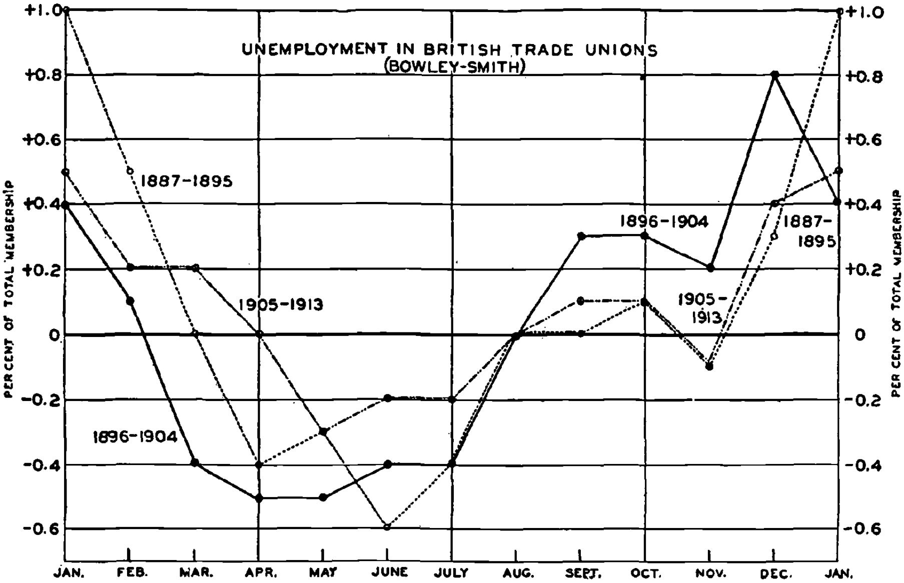
March for shipbuilding; in April and May for the furnishing trades; in June and July for engineering; in August for building; in September for iron mining and iron and steel making; in November for tin-plate and sheet-steel work, the miscellaneous metal trades, and printing; in December for coal mining.[3] This diversity of dates makes the business of a country as a whole far steadier from month to month than are most of the component parts. As the seasonal impulses produced by any factor radiate from the center of disturbance, they encounter the radiating effects of the seasonal variations which have occurred or are expected in other factors. If some time series, like the Bureau of Labor Statistics index number of prices at wholesale or stock prices, seem to be nearly or quite free from seasonal variations, it is presumably because they are affected by many different seasonal influences which cancel one another.

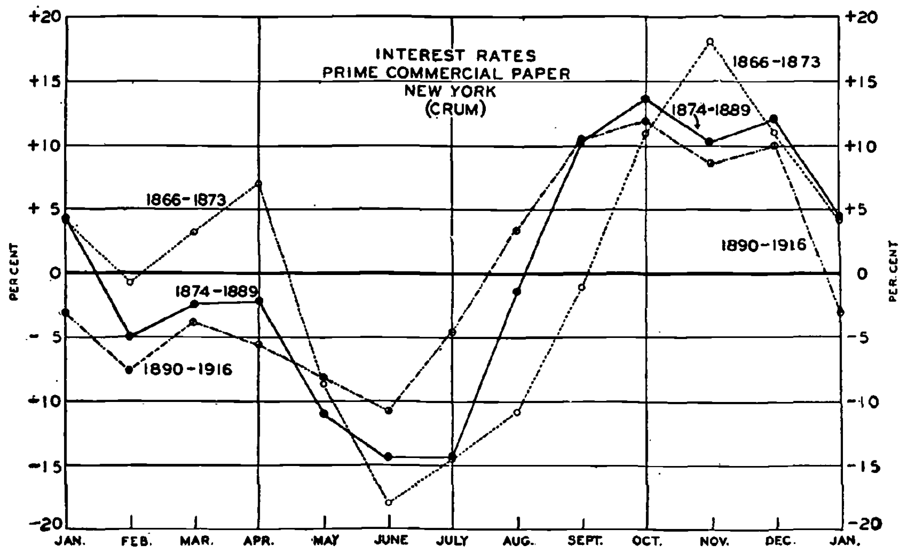
Reflection upon the causes which we assign for seasonal movements suggests that few of them will produce precisely the same effects year after year. An exceptionally cold winter will increase the seasonal swing in coal consumption, in the sale of woolen underwear, and in construction work. The conventional seasons which have not a fixed date in the calendar—particularly Easter—are responsible for other deviations from the seasonal pattern. In careful work, the months with five Sundays must be treated differently from months with four Sundays. And even the conventions tied to fixed dates—like holiday shopping and January and July interest payments—produce different effects when they occur in combination with different phases of business cycles. Over long periods, also, changes in industrial technique, communication, transportation, and business organization alter seasonal oscillations. The autumnal drain upon the money markets for moving the crops, for example, has become decidedly less of a bugbear to operators on the New York Stock Exchange than it was before the organization of the Federal Reserve System. Hence statisticians who break long time series into briefer periods often find that the seasonal variations of the parts differ appreciably. Chart 5 gives two examples of this sort, one taken from

[3] See A. L. Bowley and K. C. Smith, "Seasonal Variations in Finance, Prices and Industry," *London and Cambridge Economic Service,* Special Memorandum, No. 7, July, 1924, pp. 14, 15. The seasonal variations are derived from data for the years 1900-1913.

CHART 5.

EXAMPLES OF SEASONAL VARIATIONS WHICH HAVE CHANGED IN THE COURSE OF TIME.





the British unemployment data, the other from interest rates on prime commercial paper in the New York market.[4]

(2) Methods of Measuring Average Seasonal Variations.

Of ways of measuring seasonal variations, perhaps the most widely used is the ingenious "link-relative" method invented by Professor Warren M. Persons. This method requires six successive operations upon the original data. (1) Compute the link relatives for each successive month (week or quarter), that is, find the percentage of each item to the preceding item. (2) Arrange the link relatives for all the Januaries in a frequency table. Make similar tables for the other months. (3) Find the median in each of these twelve tables. (4) Forge the medians into a chain with January represented by 100. Carry the chain through the twelve months back to January. (5) If, as usually happens, the second January figure differs from the first one, adjust the figures for all the months to make the second January figure 100. (6) Readjust all the figures once more so that the arithmetic mean of the twelve monthly figures shall equal 100.[1]

[4] See the paper of Messrs. Bowley and Smith cited above, and W. L. Crum, "Cycles of Rates on Commercial Paper", *Review of Economic Statistics*, January, 1923, Preliminary vol. v, p. 29.

The data from which Chart 5 is drawn are as follows:

	Seasonal Variations of					
	Percentage of Members Unemployed in British Trade Unions			Interest Rates on Prime Commercial Paper in New York		
	1887–95	1896–1904	1905–13	1866–73	1874–89	1890–1916
January	+1.0	+.4	+.5	+ 4.1	+ 4.4	− 3.0
February	+ .5	+.1	+.2	− .7	− 4.9	− 7.6
March	0	−.4	+.2	+ 3.2	− 2.4	− 3.9
April	− .4	−.5	0	+ 6.9	− 2.3	− 5.6
May	− .3	−.5	−.3	− 8.7	−11.0	− 8.1
June	− .6	−.4	−.2	−18.0	−14.4	−10.8
July	− .4	−.4	−.2	−14.6	−14.3	− 4.6
August	0	0	0	−10.9	− 1.4	+ 3.3
September	0	+.3	+.1	− 1.1	+10.2	+10.5
October	+ .1	+.3	+.1	+10.9	+13.6	+11.8
November	− .1	+.2	−.1	+18.0	+10.3	+ 8.6
December	+ .3	+.8	+.4	+11.1	+12.1	+10.0

[1] For examples of this method see Professor Persons' papers in the *Review of Economic Statistics,* particularly Preliminary volume i. Later expositions are given by Professor Persons in "Correlation of Time Series," *Journal of the American Statistical Association,* June, 1923, vol. xviii, pp. 713-726, and in his contribution to the *Handbook of Mathematical Statistics,* edited by H. L. Rietz, Boston, 1924, chapter x.

A technical improvement upon Persons' method has been suggested by Professor W. L. Crum. Instead of using the medians given by the frequency tables, take "the

It will be noticed that Professor Persons measures the seasonal variation of a series before he has computed its secular trend. It is primarily because of the trend that his second January figure usually differs from the first, and that his fifth step has to be taken. Dr. William L. Hart has argued that it is better first to determine the trend of a series by whatever method is most appropriate; second to eliminate the trend, by subtracting the magnitude it indicates from the original item for each month, and then to compute the arithmetic means of all the Januaries, all the Februaries, etc., as shown by the data in this corrected form.[2]

Several statisticians have pointed out that Dr. Hart's method of "monthly means" does not guard against the influence upon the seasonal index of extreme and irregular deviations.[3] As a remedy for this defect, Dr. Lincoln W. Hall and Miss Helen D. Falkner proposed what the latter calls the "ratio-to-ordinate" method. Like Dr. Hart, they start by determining the secular trend. Next they compute the percentage of each original item to the corresponding value given by the trend line, make frequency tables of these percentages for each month, select from the frequency tables a typical value (usually the mean of a middle group of items), and finally adjust the twelve typical values so that their average for the year is 100.[4]

Still another method, devised by Dr. Fred R. Macaulay, is to make 12-months moving averages of the original items centered at the seventh month, compute the ratio of the original item for each month to its moving-average value, find medians of these ratios for

mean of a middle group of items (two or four if the series contains an even number of items, three if an odd number)." "Use of the Median in Determining Seasonal Variation," *Journal of the American Statistical Association*, March, 1923, vol. xviii, pp. 607-614.

Criticisms of this method may be found in several of the papers presently to be cited, and on p. 26 of the paper by Bowley and Smith cited above. "It is very doubtful . . .," say these writers, "whether the comparison of each month with the preceding is as appropriate for the measurement of seasonal influence as the comparison of each month with the position determined by the general average or by the line of trend." They regard Professor Persons' method as "specially suitable for correcting records for seasonal variation" (since it reveals irregularities in the frequency tables), but hold that the most accurate method of "measuring the seasonal effects" is to use deviations from a 12-months moving average.

[2] William L. Hart, "The Method of Monthly Means for Determination of a Seasonal Variation," *Journal of the American Statistical Association*, September, 1922, vol. xviii, pp. 341-349.

[3] See, for example, W. M. Persons in the *Handbook of Mathematical Statistics*, edited by H. L. Rietz, pp. 155-158.

[4] See Lincoln W. Hall, "Seasonal Variation as a Relative of Secular Trend," *Journal of the American Statistical Association*, June, 1924, vol. xix, pp. 155-166, and Helen D. Falkner, "The Measurement of Seasonal Variation," in the same issue, pp. 167-179.

all the Januaries, Februaries, etc., and convert the medians into percentages which total 1200 for the year.[5]

Choice among these methods is to be guided, of course, by the characteristics of the series to be dealt with and the specific purpose in view. As Messrs. Bowley and Smith point out in their systematic discussion of the problem, the first question to be asked is whether the seasonal variation of any month from the annual average is more appropriately considered as a given amount, or as a given proportion. For example, do the October imports of wheat into England tend to be 50,000 tons more than the average of all months, or do they tend to be 11 per cent above the average? If greater regularity is secured by considering amounts, then seasonal variations should be expressed in percentages borne by the absolute difference between the average amount in each month shown by the data (in their original form or adjusted for trend) and the arithmetic mean for the whole year. If greater regularity is secured by considering proportions, then seasonal variations should be computed from ratios, and geometric means should be used. A second question to be considered in either case is whether the series shows an upward or downward trend. If not, the absolute differences or the relative differences may be computed directly from the original data. If there is a trend to be eliminated, that can be accomplished by using moving averages or by fitting curves, and the absolute or relative differences ascertained from the data corrected for trend. Thus Bowley and Smith discuss and use in different series six methods—three based on amounts and three on proportions, one of each set without correction for trend, one with the trend removed by moving averages, and one with the trend removed by the method of least squares. Of these methods they think the two which use moving averages are the most accurate, though the most laborious. In a few cases they make use also of Persons' link-relative method.[6]

(3) Efforts to Measure Changing Seasonal Variations.

In all the preceding methods the object is to find one set of monthly figures which measures the average seasonal influence. Oc-

[5] See "Index of Production in Selected Basic Industries," *Federal Reserve Bulletin*, December, 1922, vol. iii, pp. 1414, 1415.

[6] A. L. Bowley and K. C. Smith, "Seasonal Variations in Finance, Prices and Industry," *London and Cambridge Economic Service*, Special Memorandum, No. 7, July, 1924.

casionally statisticians who use one of these methods notice that the seasonal fluctuations of their series seem to undergo a change between the beginning and the end of their period. They may then break the whole period into two or more parts, and compute fixed seasonals for each segment in the fashion of Chart 5. A more ambitious plan, that of measuring seasonal variations as they change from one year to the next, has been suggested by Dr. Willford I. King. His successive steps are as follows: (1) Plot the data. (2) Draw a free hand curve through the data representing the cyclical fluctuations. (3) Read from this "preliminary cycle curve" the numerical values which it indicates each month, and (4) divide the actual data by these values. The quotients give the first approximation to the seasonal variations. (5) Smooth out the irregularities of this rough approximation by using 9-period moving averages of all the Januaries, all the Februaries, etc. Plot the results and smooth out the small irregularities which may still remain. (6) Adjust the results so that the sum of the twelve seasonal indexes for each year shall equal 1200.[1]

Dr. King's effort to take account of the changes which seasonal variations undergo from time to time has commended itself to other statisticians; but his method has been questioned because of the free play given to the investigator's personal equation.[2] Dr. W. L. Crum has devised a method of showing progressive changes in seasonal variations which is not open to this objection. The essential feature of his plan is to find the secular trend of all the January items in a series, all the February items, and so on, and to use the ordinates of these 12 monthly trends as the basis for determining the seasonal variations in each successive year.[3]

This method is designed primarily to yield "seasonal *standards* applicable to the study of current phenomena." It applies to series

[1] Willford I. King, "An Improved Method for Measuring the Seasonal Factor," *Journal of the American Statistical Association,* September, 1924, vol. xix, pp. 301-313.

[2] Compare O. Gressens, "On the Measurement of Seasonal Variations," *Journal of the American Statistical Association,* June, 1925, vol. xx, p. 205.

[3] The details of this method differ according as the investigator works with link relatives or with monthly means of the data. See W. L. Crum, "Progressive Variation in Seasonality," *Journal of the American Statistical Association,* March, 1925, vol. xx, pp. 48-64. As Dr. Crum points out, similar suggestions have been made by Dr. E. C. Snow, "Trade Forecasting and Prices," *Journal of the Royal Statistical Society,* May, 1923, vol. lxxxvi, p. 334, and by Mr. Harold Flynn, quoted in *The Problem of Business Forecasting,* Boston and New York, 1924, p. 104.

For an interesting application of the method, see Edwin Frickey, "Bank Clearings Outside New York City, 1875-1914," *Review of Economic Statistics,* October, 1925, vol. vii, pp. 258-262. Mr. Frickey found seasonal variations in the terminal years of a period by fitting straight line trends to link relatives for each month, and then made progressively changing seasonals for the intervening years by straight line interpolations.

whose seasonal variations change slowly and progressively for considerable periods.[4] But it does not take account of yearly changes in seasonal factors, such as are produced, say, by warm and cold winters, or by March and April Easters. Mr. O. Gressens has proposed a method which will show changes of the latter type. He computes the ratio of a variable each month to the average monthly value for its year, corrects these monthly ratios when necessary for the trend within the year, smooths them by the use of moving averages, moderates any widely divergent items which may remain, and adjusts his ratios for each year so that their sum shall be 1200. This method he believes to have the merits of Dr. King's procedure, and to be "mechanically more definite." [5]

(4) Conclusion.

The preceding review shows how much labor statisticians have devoted to the measurement of seasonal variations. That problem has an interest of its own, apart from its bearing upon the isolating of cyclical fluctuations. Efforts to mitigate seasonal reductions in employment, and to reduce costs of production by "budgeting" production and marketing are stimulated and made more effective by knowledge of the magnitude and regularity of the seasonal variations in the numerous activities which have to be considered in laying plans. Happily for us, it is often possible to turn results worked out for practical ends to theoretical uses.

A few illustrations of seasonal variations, as measured by statistical investigators have already been given in Chart 5. Additional illustrations are presented in Chart 6, which shows the same method applied to different series, and Chart 7, which shows various methods applied to the same or different series.

Non-technical readers who feel a bit confused by the variety of methods which have been sketched may take comfort in Chart 7. It illustrates a remark made by Messrs. Bowley and Smith. Having experimented with different ways of measuring seasonal variations perhaps more elaborately than any other investigators, they observe

[4] W. L. Crum, as cited, pp. 60, 61.

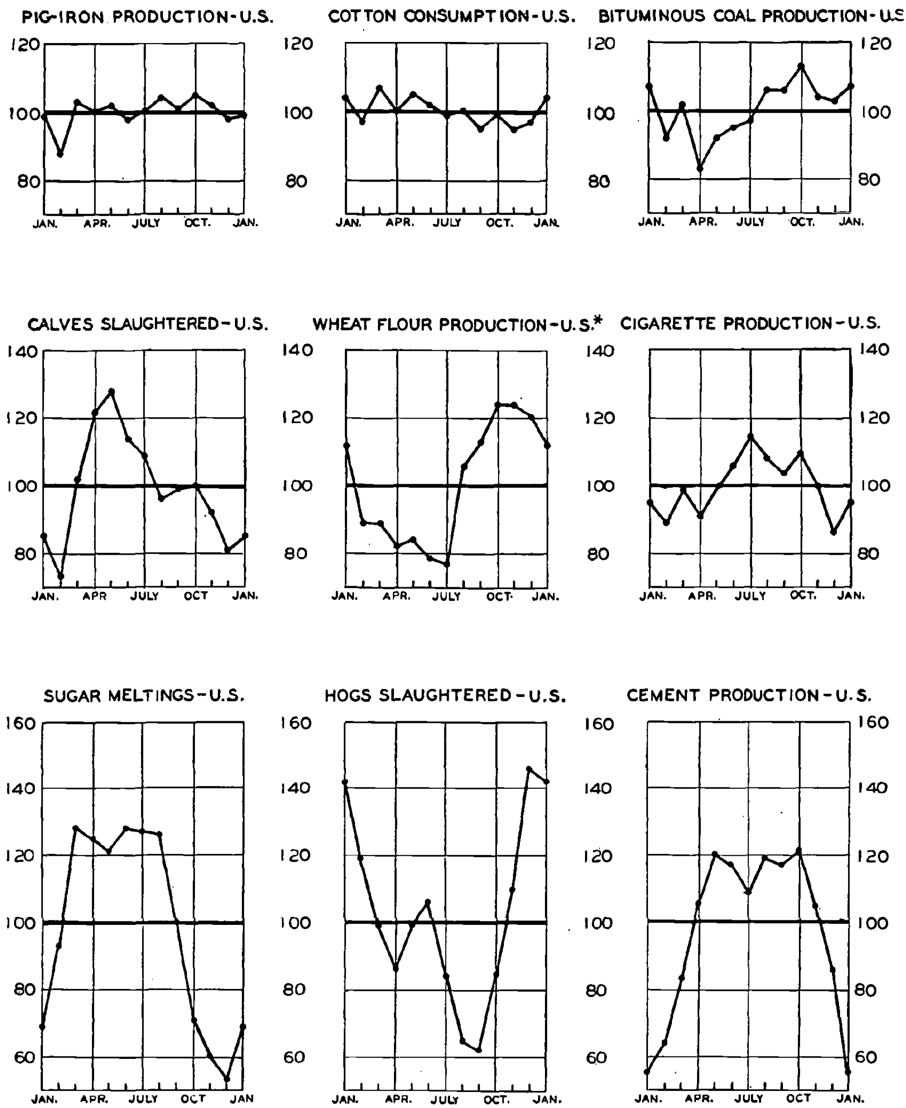
[5] O. Gressens, "On the Measurement of Seasonal Variation," *Journal of the American Statistical Association*, June, 1925, vol. xx, pp. 203-210.

Dr. King, on the other hand, points out that it is difficult to apply either Crum's or Gressens' method to time series in which the seasonal variations are relatively slight and the cyclical fluctuations relatively violent.

CHART 6.

INDEXES OF SEASONAL VARIATIONS.

Made by the moving-average-median method (F. R. Macaulay).

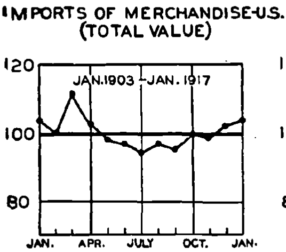


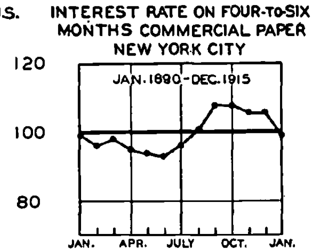
COMPUTATION BASED ON DATA FROM JAN. 1913 TO DEC. 1921
*COMPUTATION BASED ON DATA FROM JAN. 1914 TO DEC. 1921

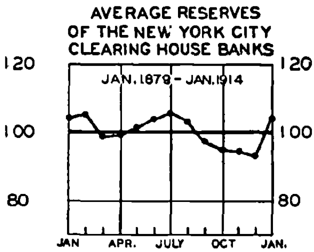
CHART 6 (*Continued*).

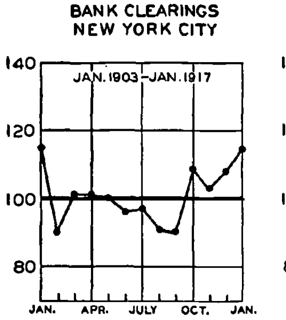
INDEXES OF SEASONAL VARIATIONS.

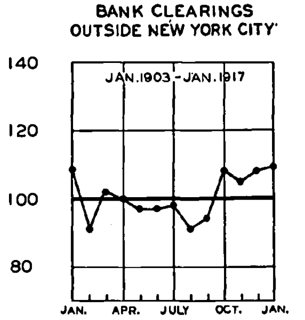
Made by the link-relative method (W. M. Persons).

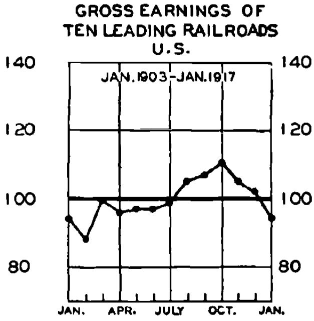


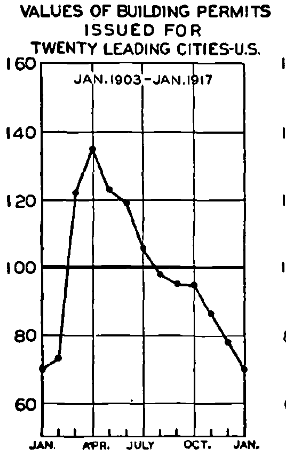


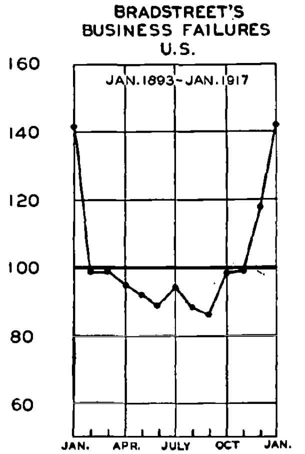












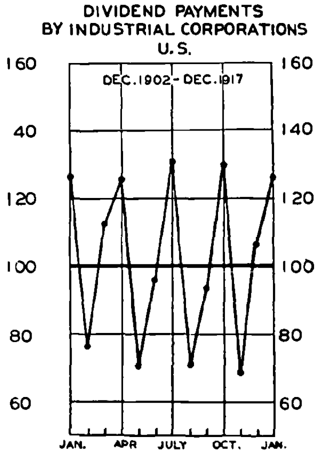
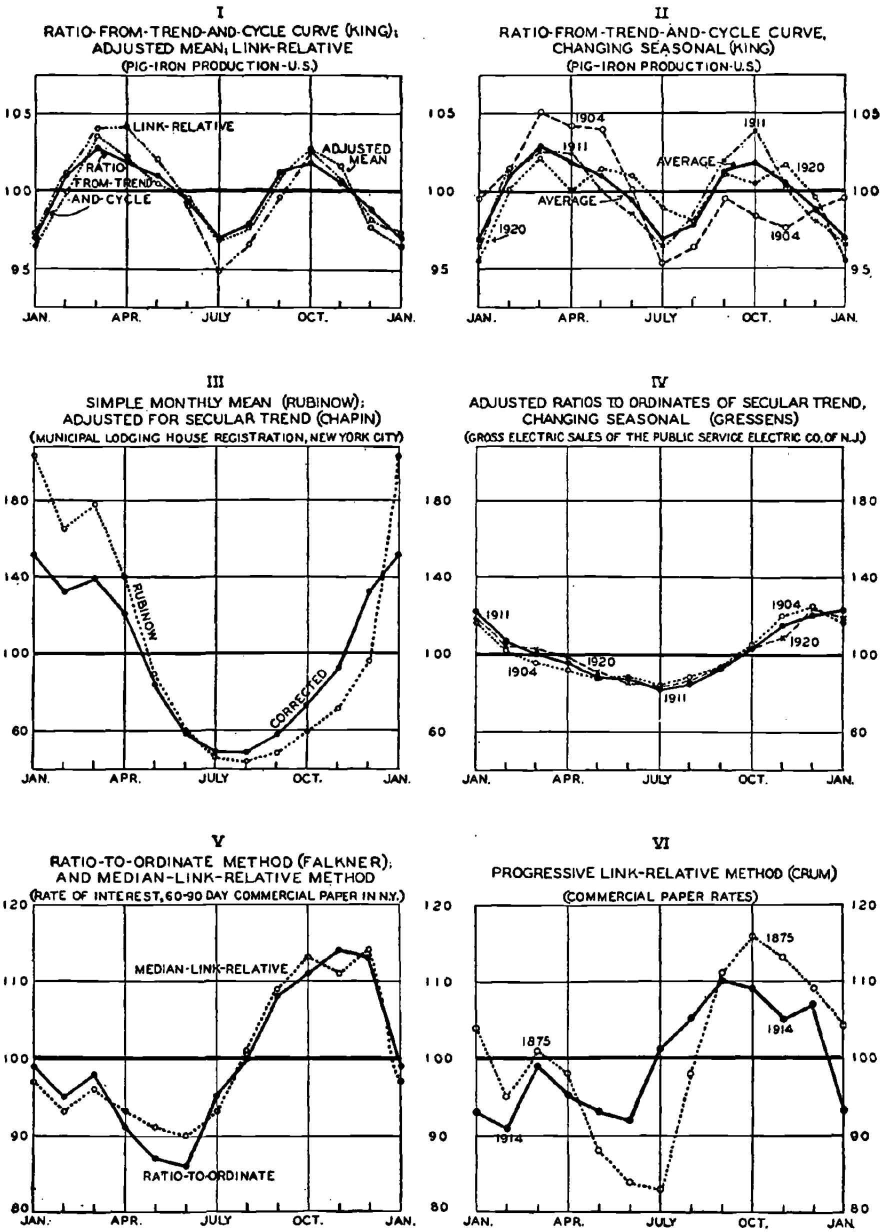


CHART 7.

INDEXES OF SEASONAL VARIATIONS MADE BY VARIOUS METHODS.



that "in fact, these methods give nearly identical results in most cases." [1]

5. The Problem of Irregular Fluctuations.

While less progress has been made in dealing with the irregular fluctuations of time series than in dealing with secular trends and seasonal variations, we are not at liberty to dismiss this topic quite so summarily as do most statisticians. The conceptual difficulties with which the problem bristles are significant for the theory of business cycles. In considering them, we shall find ourselves confronting certain issues fundamental to our further work.

(1) The Concept of Irregular Fluctuations.

So far as known, all social time series without exception present irregularities of contour in their raw state. Statisticians take these irregularities for granted without attempting to define them, as they define secular trends, seasonal variations, and cyclical fluctuations.[1] What little they say upon the subject concerns the causes of irregularities in particular series. For example, they point out that wars or civil insurrections may disturb many economic processes for a considerable period. Less serious disturbances may be caused by such events as earthquakes, conflagrations, floods, droughts, epidemics, insect pests, strikes and lockouts, railway embargoes, inventions, changes in trade routes, discoveries of fresh resources, changes in laws, judicial rulings, and so on through an interminable list. Nor should we forget the effects of changes in the method of compiling statistics, and of inaccurate reporting. The addition of a new town to the list for which bank clearings are published, the disruption of a trade union which had made unemployment returns, changes in the lists of commodities used in a price index, the failure of customs-house clerks to include all July invoices in their July statement of imports, revisions

[1] "Seasonal Variations in Finance, Prices and Industry," *London and Cambridge Economic Service,* July, 1924, Special Memorandum, No. 7, p. 3.

[1] The nearest approach to a definition has been made by Professor Edmund E. Day, who proposes to divide irregular fluctuations into two classes—namely,

"Episodic movements due to specific causes, ordinarily reflected in sharp, pronounced breaks in the record of the variable. . . .

"Fortuitous or accidental movements, of unknown origin, quite irregular in character, but involving only minor disturbances of the general course of the variable."

See Day's *Statistical Analysis,* New York, 1925, pp. 285, 302-306, and 310-312.

in the estimates of monetary stocks to take account of losses, reclassifications of railroad freight, errors in addition, misprints—a thousand such matters may produce purely artificial irregularities in time series.

The idea suggested by this practice of listing causes is that we may classify as an irregular fluctuation any movement of a curve which we do not ascribe to secular, seasonal or cyclical changes. If we had clean-cutting methods of ascertaining what changes in our curves are due to these three sets of factors, the treatment of irregular fluctuations as residuals would be satisfactory. On that basis we might rationalize our procedure as follows: The activities represented by a time series are influenced every day by a host of factors which are not secular, cyclical, or seasonal in character. Most of these random factors are known vaguely, if at all. But the theory of probabilities justifies the assumption that the random factors acting at a given moment cancel one another when they are very numerous, independent in origin, and of the same order of magnitude. Indeed, Professor Edgeworth has shown that these strict conditions may be relaxed:—there will be much canceling if the random factors are not few, if there is a considerable measure of independence among them, and if no two or three preponderate over the rest.[2] It often happens, however, that even the relaxed conditions are not complied with. At any time a group of mutually related factors may dominate the complex, or one or two factors far more powerful than the other random influences may crop up. Under such circumstances, the random factors cease to cancel one another, even roughly. Instead, they produce a large or small deviation from the undulating curve marked out by the secular trend, seasonal variations, and cyclical fluctuations in combination. It is deviations caused by such failure of mutual canceling of the random factors which we call irregular fluctuations.

One doubt concerning this conception can be met by an extension of the argument. As we saw in the last section, some of the causes of seasonal variations vary from year to year. Likewise all secular trends are admitted to be subject to change without notice, and those trends which are ascribed to growth may be inconstant by nature for aught we know. Finally, among the numerous causes of cyclical fluctuations reviewed in the first chapter, there is not one which we should expect to produce perfectly regular cycles. Thus, once we

[2] Compare section ii, 2, above.

adopt the current practice of the statisticians and start discussing irregular fluctuations in terms of their causes, we seem forced to admit that irregularities may occur in the movements which we classify as secular, cyclical and seasonal. In other words, an attempt to treat secular, seasonal and cyclical changes as regular runs counter to much that we know and more that we suspect.

This conclusion may be admitted, and treated as a reason, not for abandoning the classification of fluctuations in time series into the regular and the irregular, but for making it more rigorous. That is, we may conceive the causes of seasonal, secular, and cyclical changes as so many complexes, each made up of one or more causes which act regularly according to some "law," and of random factors which more or less often fail to cancel each other. Then we may throw the irregularities which are connected with the seasons, with the factor of growth, and with business cycles into the same box as the irregularities which we ascribe to wars, earthquakes, epidemics and misprints. In contrast to this heterogeneous collection of irregularities, we have left secular trends which, if not constant, change in some regular way, seasonal variations which, if not uniform, change in some regular way, and what one who took this view would probably call "normal" cycles.

From the theoretical viewpoint this conception seems clear, whatever difficulties might be encountered in applying it to time series. But is it the conception with which business-cycle statisticians work? Perhaps such an idea is implicit in the application of periodogram analysis to time series.[3] Perhaps there are champions of the "40-month cycle" who would accept the notion.[4] The majority of the business-cycle statisticians, however, find certain features of the idea ill adapted to their needs.

The men who analyze time series primarily with an eye to forecasting the future commonly accept the notion of regular trends and seasonal variations. They need standards by which they can test current developments and on which they can base reasoned expectations. Such standards they can make with trends and seasonals which change according to some rule, but not with irregular seasonals and trends. The forecasters believe themselves justified by the past behavior of many time series in setting up the standard trends and

[3] See the remarks on periodogram analysis at the end of the next section.
[4] See below, section vi, 3, (6) "The Duration of Business Cycles."

seasonals they need, and judging what departures from these standards seem probable in the near future. For while they admit that actual trends have altered and actual seasonals have differed in the past, they seldom find these changes very sudden, or very great. But when they come to cyclical fluctuations, they find less warrant in the past behavior of time series for setting up similar standards. The past changes in these fluctuations have been so sudden, so frequent, and so considerable as to make the notion of a "normal cycle" inappropriate. Not feeling justified in imposing a "normal cycle" upon their data, they have no means of distinguishing the regular from the irregular changes in cycles, as they distinguish the regular from the irregular changes in trends and seasonal variations. Nor can they distinguish clearly between the cyclical fluctuations and the irregular fluctuations of a non-cyclical character. All they can do is to note occasional marked departures from the course of events which it seemed reasonable to expect, and to search through descriptive materials for plausible explanations of these gross irregularities.[5]

The statistical analysts who are not attempting to make business forecasts hesitate to accept even the idea of "normal" trends and seasonals. As shown above, they have suggested methods for measuring seasonal variations which change from year to year. Also, secular trends pursue a meandering course when made by the use of free-hand curves or moving averages; they become unsteadier still when obtained by taking ratios to items in other series. Such methods absorb into the seasonals and trends of time series a part of the movements which the methods commonly practiced by forecasters intermingle with the cyclical fluctuations. But even when changing seasonal variations and meandering trends have been eliminated, the residuals show many irregularities.

(2) Irregular Fluctuations in the Theory of Business Cycles.

No method seems to have been devised for segregating and eliminating from the cyclical fluctuations the irregularities not absorbed in seasonals and trends. The nearest approach to such a method is to distribute the irregular fluctuations by the use of moving averages or free-hand curves. Such operations do not show what the cyclical fluctuations would have been in the absence of irregular fluctuations;

[5] Compare Warren M. Persons, "Indices of Business Conditions," *Review of Economic Statistics*, January, 1919, Preliminary vol. i, pp. 33-35.

they merely show the combined cyclical and irregular fluctuations distributed in a new way among the months which are averaged together. And of course we cannot find out what the irregular fluctuations really were by subtracting the successive values of such a smoothed curve from the corresponding original items.

Yet Professor Persons has obtained one result of much theoretical interest by an operation of this character. Taking the value of building permits granted in twenty American cities in the 156 months of 1903-16 (a series in which the irregular fluctuations are marked), he subjected the original data to the following operations: (1) He eliminated the secular trend and the seasonal variations by his favorite methods. The residuals showed the cyclical and irregular fluctuations of the series in combination. (2) He computed the percentages which the twelve months moving averages make of the corresponding ordinates of the secular trend. This process (by averaging) presumably eliminates the irregular variations and seasonal variations, and (by taking ratios) the secular trend; but it does not eliminate the cycles. (3) He subtracted the items found in (2) above from the items described in (1). The irregular variations were present in (1) but not in (2). The resulting differences were, presumably, approximations to the irregular fluctuations. (4) He made the differences thus ascertained into a frequency table. . . . From the data in this table, he drew a rectangular diagram. Finally, to this diagram he fitted a normal curve. From the closeness with which the normal curve fitted the data, Professor Persons concluded that "the distribution of the irregular fluctuations of building permits is normal." [1]

May we not draw a further conclusion? If the irregularities of economic time series over a considerable period are distributed in the same way that errors of observation are distributed, can we not take the combined cyclical and irregular fluctuations of a time series without regard to the temporal order of their occurrence, interpret each fluctuation as one observation upon the behavior of the cyclical factors distorted in some measure by an error, and base our results upon averages drawn with due caution from the array of observations? The confidence we can put in such averages will depend of course upon the number of observations which each time series yields, and upon the way in which the observations in each array are distributed about their central tendency. This procedure is certainly less hazard-

[1] Warren M. Persons, "An Index of General Business Conditions," *Review of Economic Statistics*, April, 1919, Preliminary vol. 1, pp. 137-139.

ous than the attempt to decide what part of a given change in any time series should be ascribed to the failure of random factors to cancel each other. It enables us to utilize all the available statistics, and it gives some basis for judging the probable reliability of the inferences we may draw from them.[2]

In Chapter V, this suggestion will be elaborated, with the aid of the more adequate materials which will then be in hand. Of course, there is no thought of returning to the idea of a "normal cycle." For between the conception of an average empirically determined from the study of statistical arrays, and the conception of "normal" phenomena employed by economic theorists there is a vital difference. The theorist's normal is that which complies with certain conditions which he has laid down. It may approximate average experience, or it may be far removed from the facts of life—all depends upon the manner in which the theorist has chosen the ground for his argument. Even when the two agree closely, they remain conceptually unlike. To speak of average conditions as "normal" is to introduce needless confusion.[3]

In our dealings with irregular fluctuations as theorists, we are confronted again by the problem mentioned at the end of the section

[2] It may be asked: If Professor Persons is justified in computing the irregular fluctuations of building permits in order to find how they are distributed about their central tendency, would he not be justified in eliminating these irregular fluctuations from the residuals left by taking out the trend and the seasonal variations? Would he not thus isolate the cyclical fluctuations? And might not his methods, with this development, be applied freely to other time series?

The reason why statisticians hesitate to follow this obvious line is that in dealing with cycles they cannot accept the rough approximations to irregular fluctuations which will serve in testing the types of distribution to which the latter conform. With 156 irregularities in his frequency table, Professor Persons can suppose that the imperfections of his measurements of irregularities will cancel each other in large degree. He could not make such an assumption regarding the irregular fluctuations which accompanied any specific business cycle. In trying to get a curve representing a succession of cycles, it is small comfort to say that a distortion at one point in one cycle, caused by inaccurate measurement of irregular fluctuations, is probably matched by a distortion of the opposite sort at some unknown point in the same, or another cycle.

The only way in which we can invoke the canceling of random effects in eliminating irregular fluctuations from cyclical fluctuations is the way suggested in the text: namely, by collecting numerous cases showing cyclical and irregular fluctuations in combination, and ascertaining the central tendencies of these arrays.

[3] There is, however, one accredited use of this term in statistics, illustrated in the preceding quotation from Professor Persons: namely, the "normal curve of error" and "normal" distributions, that is, distributions which are described approximately by the "normal" curve. As pointed out by Professor Karl Pearson, who first applied the term "normal" to the curve developed by Laplace and Gauss, the choice was not a happy one. (See Karl Pearson, *Biometrica,* October, 1920, p. 25.) But the usage is so firmly established by this time that more confusion might be caused by departing from than by conforming to it.

upon secular trends. While we desire to discriminate as clearly as we can between the irregular and the cyclical fluctuations of time series, we cannot discard irregular fluctuations offhand as irrelevant to the understanding of business cycles. By doing so we should be tacitly rejecting without investigation some of the working hypotheses presented in Chapter I. Dr. Veblen, for example, holds that in the period from, say, 1816 to 1873 liquidation was "apparently always brought on by some extraneous disturbance," whereas since the 1870's seasons of prosperity "are pretty uniformly traceable to specific causes extraneous to the process of industrial business proper."[4] So also, Professors Arthur B. Adams of Oklahoma and S. A. Pervushin of Moscow argue at length that revival cannot blossom into full prosperity without the aid of some favoring cause which the revival itself does not generate.[5] Several other authorities assign an important though less systematic rôle to "disturbing circumstances" as factors in shaping the course of business cycles. In view of these considered opinions we cannot take it for granted that irregular fluctuations are to be eliminated from our theorizing, much as we should like to eliminate them from our curves. Even statistical elimination is desirable only in the sense that we should like to isolate the irregular as well as the cyclical fluctuations, in order to study intensively both types of changes.

6. The Problem of Isolating Cyclical Fluctuations.

From what has been said regarding the other types of changes found in time series, it is clear that the ambition to isolate cyclical fluctuations has not been attained. Our review of methods of computing secular trends and seasonal variations showed that even for these movements we have approximations rather than measures. Our discussion of irregular fluctuations showed that no statistician ventures to do more than smooth out irregularities in his curves by using moving averages or free-hand constructions. Inability to measure the net effects of secular, seasonal, and random factors separately, or in combination, means that we cannot isolate the cyclical fluctuations of time series by eliminating the three other sets of changes recog-

[4] Thorstein Veblen, *Theory of Business Enterprise,* New York, 1904, pp. 249-255.

[5] Arthur B. Adams, *Economics of Business Cycles,* New York, 1925, pp. 111-158; S. A. Pervushin, *The Business Conjuncture,* Moscow, 1925, pp. 54-61. I am indebted to Dr. Simon S. Kuznets for a synopsis of Professor Pervushin's discussion.

nized by our classification. And no one has yet devised a satisfactory method of measuring the cyclical fluctuations directly.[1]

What we can get from the statisticians, then, are the residual fluctuations of many American and some foreign time series after the secular trends and the seasonal variations (determined by some variant of the methods described above) have been eliminated. The process of eliminating these two types of movements consists in computing, or reading from a chart, the values which the trend, corrected for seasonal variations, would have at successive intervals of time, and then subtracting these values from the corresponding items of the original data, or expressing the deviations of the original data from the corrected trend in percentages.[2]

Chart 8 gives three examples of the results obtained in this way. To apply what has just been said to the chart: the deviations of the

[1] It is true that in his plan for ascertaining seasonal variations which change from year to year, Dr. King draws a "preliminary cycle curve" directly from the raw data—"a free-hand curve representing what was assumed to be the course of the cycle." But neither this curve, nor the "final cycle curve" which Dr. King gets after eliminating his varying seasonal indexes, purports to separate the cyclical from the secular and irregular changes. "An Improved Method for Measuring the Seasonal Factor," *Journal of the American Statistical Association*, September, 1924, vol. xix, pp. 301-313.

[2] To illustrate: Suppose that we have obtained the following results by analyzing a series showing the production of commodity X by months.

Ordinate of secular trend January, 1926, 900 tons.
Monthly increment of secular trend, 5 tons.
Seasonal variations expressed as percentages of mean monthly production:

January	90
February	100
March	110
April	80

Suppose also that the production reported in the opening months of 1926 runs as follows:

January	800 tons
February	1000 tons
March	1100 tons
April	700 tons

Then we can make the following computations:

1926	Ordinates of secular trend		Seasonal variations		Trend adjusted for seasonal variations	Original data
January	900 tons	×	.90	=	810 tons	800 tons
February	905 "	×	1.00	=	905 "	1000 "
March	910 "	×	1.10	=	1001 "	1100 "
April	915 "	×	.80	=	732 "	700 "

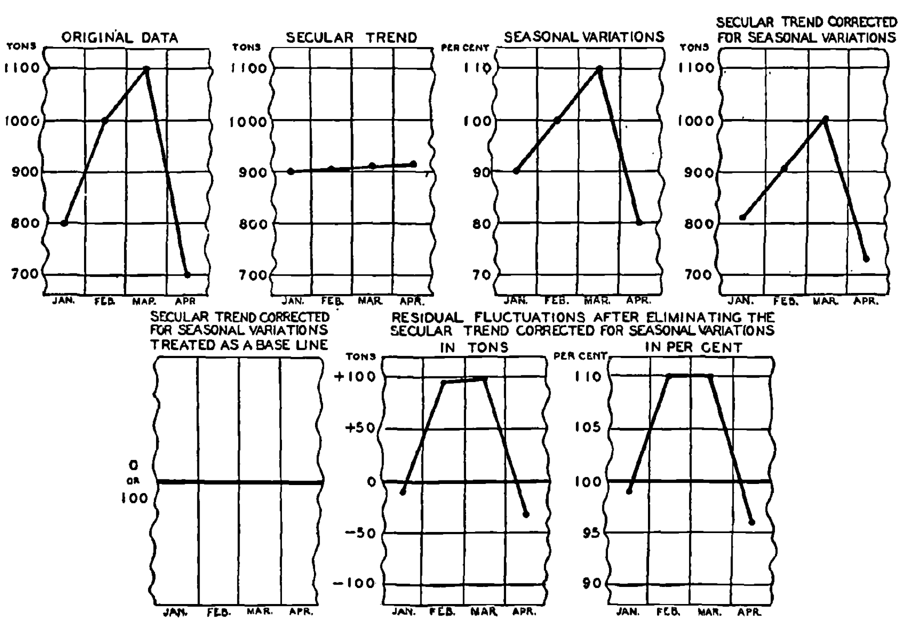
Results after eliminating the secular trend corrected for seasonal variations.

1926	In tons	In percentages of adjusted trend
January	−10 tons	99
February	+95 "	110
March	+99 "	110
April	−32 "	96

curves from their base lines do not show the cyclical fluctuations of the series included. What they do show are the cyclical fluctuations combined with the irregular fluctuations, among which are included the deviations of the actual seasonal and secular changes from the curves chosen to represent them.

Thus statistical technique in its present state enables us to picture cyclical fluctuations only in a distorting combination with irregular fluctuations which we cannot measure. It seems legitimate to believe that the cyclical factor or factors operate with greater regularity than the curves suggest. But we are not entitled to believe that, were the effects of all non-cyclical factors excluded, the deviation and amplitude of all cyclical fluctuations would be uniform. For cyclical factors influencing time series may vary from year to year, as many of the seasonal factors vary. Just as our conviction that many seasonal variations are not uniform from year to year rests upon what we know about their causes, so the opinions we may finally form concerning the uniformity or variability of cyclical fluctuations must be based upon what we can learn about their causes, rather than upon study of such curves as are presented in Chart 8.

Graphically we can represent these successive steps by the following segments of charts:

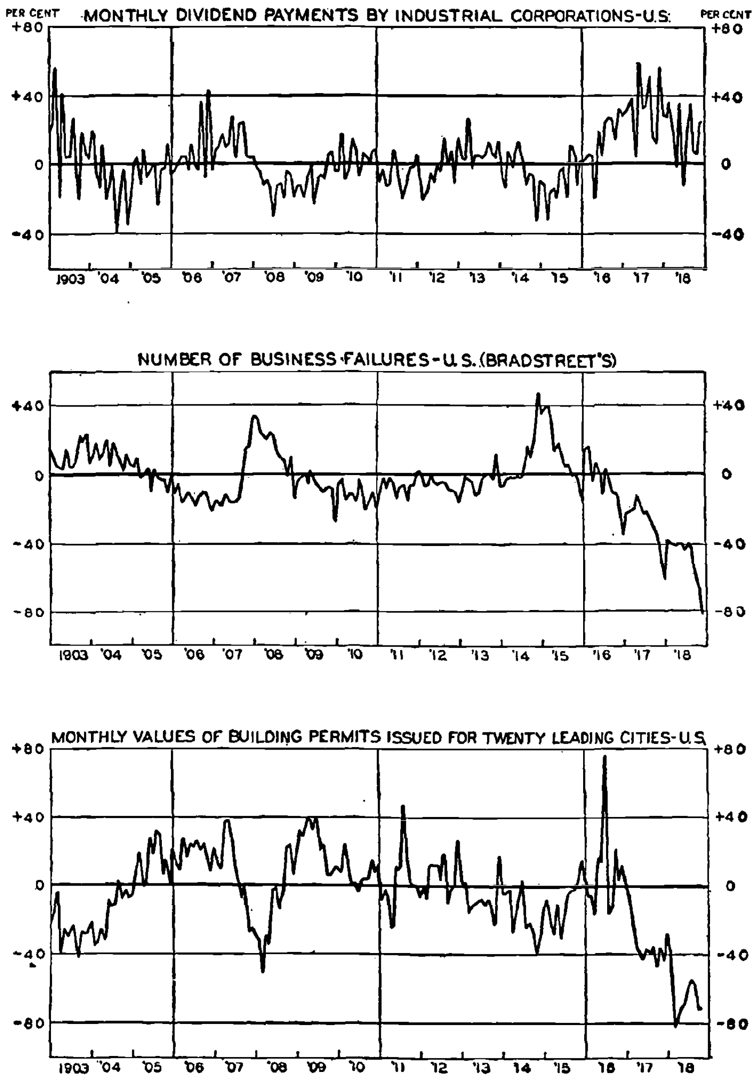


Although the statistical segregation of the factors influencing time series stops short of our desires, a comparison of the refined curves of Chart 8 with the "raw-data" curves of Chart 1 shows that the cyclical

CHART 8.

RESIDUAL FLUCTUATIONS OF TIME SERIES AFTER ELIMINATION OF SECULAR TRENDS AND SEASONAL VARIATIONS.

Percentage deviations of original items from secular trend corrected for seasonal variations.

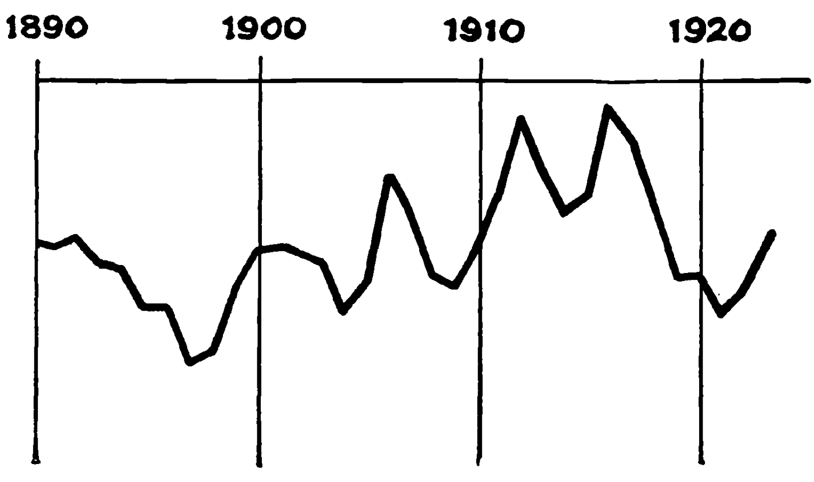


fluctuations stand out more clearly after the statistical trends and seasonal variations have been eliminated, however roughly. We shall therefore make such use as we can of these eliminations in our further work; but instead of ignoring trends and seasonals we shall study them both in their original combinations with cyclical and irregular fluctuations, and in their segregated form.

To readers trained in the natural sciences, it may seem that periodogram analysis should be substituted for the cumbersome and inexact procedure which has been described as the standard method of determining cyclical fluctuations. A few economists, notably Henry L. Moore, Sir William H. Beveridge, and William L. Crum, have made significant experiments with this method.[3] It yields excellent results in many physical processes which show strictly periodic fluctuations of a symmetrical type, and it should reveal any similar periodicities which exist in economic time series. Nor is the method limited to the discovery of simple movements. A periodogram analysis may indicate the existence of several or many periodicities, which when combined with each other give a curve so complicated that the uninitiated reader would not suppose it to be made up of periodic elements.[4] Perhaps it will be found that many of the time series used

[3] For explanations of this rather elaborate method of analysis, and illustrative results, see H. L. Moore, *Economic Cycles: Their Law and Cause,* New York, 1914, and *Generating Economic Cycles,* New York, 1923; Sir W. H. Beveridge, "Wheat Prices and Rainfall in Western Europe," *Journal of the Royal Statistical Society,* May, 1922, vol. lxxxv, pp. 412-459; W. L. Crum, "Cycles of Rates on Commercial Paper," *Review of Economic Statistics,* January, 1923, Preliminary vol. v, pp. 17-29, and "Periodogram Analysis," chapter xi in *Handbook of Mathematical Statistics,* edited by H. L. Rietz, Boston, 1924.

[4] For an example, see the "synthetic curve" made by adding eleven cycles of different lengths, in Beveridge's paper referred to above. (Opposite p. 453.) A section of this curve is reproduced here.



by the business-cycle statisticians can be usefully described by "synthetic curves" formed by adding together several periodic fluctuations which differ in length.

But the systematic application of periodogram analysis to economic series encounters serious obstacles. Comparatively few of the series which we wish to utilize have been maintained over a period long enough to yield satisfactory results when treated in this way. To establish the rather brief cycles in which the business-cycle statistician is most interested, it is necessary to have monthly, or at least quarterly, data, and long series of that character are few. When materials are available in this form, the seasonal variations and the irregular fluctuations so characteristic of economic processes tend to blur the periodograms. To leave out the periods in which irregular fluctuations seem to alter the cyclical movement limits the data available for study still more narrowly, and gives the investigator's personal judgment a considerable influence. To eliminate secular trends and seasonals before applying periodogram analysis is often necessary, but it may bias the results.[5] There remain two doubts more fundamental. Can we assume that the cyclical fluctuations of economic processes are, or tend to be, strictly periodic? If there are tendencies toward periodic fluctuations in given processes at a given point in economic evolution, do such tendencies maintain themselves under changing conditions over a period long enough to be revealed by periodogram analysis?

The most obvious of these obstacles to the systematic use of the periodogram method in business-cycle work—the brevity of the majority of the series which must be analyzed—presumably will shrink with the lapse of time. If the future is less checkered by catastrophes than the past has been, the troubles caused by irregular fluctuations will diminish also. The doubts now harbored about the propriety of adjusting series to get rid of trends and seasonal variations before beginning periodogram analysis may be set at rest by further work. Fuller knowledge may make us readier to accept the working hypothesis that there are true periodicities of various lengths in economic processes, and that these periodicities maintain themselves for long periods of time. Certainly we cannot say that periodogram analysis will not play a large rôle in future economic work. But it seems

[5] Compare Sir W. H. Beveridge's remarks upon this point in the article cited above, pp. 414-415.

equally certain that we cannot yet make it the standard procedure for studying cyclical fluctuations.[6]

IV. On Measuring the Relationships Among Time Series.

While the isolating of cyclical-irregular fluctuations is the end of one set of statistical efforts, it is the beginning of a new set. As we look at Chart 8, we grow eager to attack the problems it presents. The curves resemble each other in that all show at least the major fluctuations in business conditions which occurred during the periods they cover. But this resemblance, though clearer than in Chart 1, before the secular trends and seasonal variations had been eliminated, is still overlaid by striking and persistent differences. If a larger collection is taken than that offered by Chart 8, it is found that the waves in the several curves differ widely in amplitude. They differ also in timing; that is, the crests and troughs come several months later in some curves than in others. They differ finally in shape—some curves have roughly symmetrical waves, others suggest a very choppy sea. What use can we make of these results?

One course is to compile a general index of business cycles from as large and varied a collection of curves as we can assemble, all of them in the form illustrated by Chart 8. If that is our next step, ought we reduce the varying amplitudes of the cyclical-irregular fluctuations characteristic of different series to a common scale? Ought we try to get rid also of the differences in timing, and in shape? Or are the differences in amplitude, timing, and perhaps shape, matters which we do not wish to get rid of, but rather to investigate and use in framing a theory of business cycles? Do we, indeed, wish to make a general index of business cycles? Shall we not gain more by

[6] Colonel M. C. Rorty adds the following comment:

"The harmonic analysis can fairly safely be used to segregate periodicities which are known to be compounded in any series of observations; but I do not believe it can be trusted to establish the existence of periodicities as to the reality of which there is no other definite evidence. The fundamental defect in the harmonic analysis is that it will resolve any ordinary business time series into definite regular periodicities, regardless of whether any real periodicities exist or not. The probability that such resolution has any real meaning would seem to be infinitesimal when the number of cycles subjected to analysis is small and the number of periodicities required for, say, a 90% resolution of the time series exceeds two. Furthermore, when the element of lag is introduced, the value of the harmonic analysis becomes still more tenuous. I doubt whether it is possible to set up a complete and direct mathematical test of the method. Probably the best indirect proof of the lack of value of the harmonic analysis is to create an artificial time series by throwing dice and then analyze this series with and without assumptions as to lag."

concentrating attention upon the fluctuations of particular processes, bringing back into the discussion even the secular trends and the seasonal variations which we have eliminated?

These rhetorical questions suggest their own answers. If we are to make the most of the statistical contribution to business cycles, we must learn all that we can by studying time series separately and studying them with reference to all their characteristics; we must also learn all that we can by studying them in combination, or rather in varying combinations.

For either purpose we need a special technique. Whether we set about comparing the fluctuations of different series, or combining different series into general indexes, we must have some standard method of measuring the relationships among the fluctuations. Such a method has been devised, and we have merely to note how carefully it must be used. But attempts to apply the method lead us on to a subtler problem which statisticians are but beginning to grasp: Precisely what relations among the fluctuations of time series do we wish to measure?

1. The Correlation of Time Series and Its Pitfalls.

Inspection of such curves as are shown in Charts 1 to 8 suggests various conclusions regarding their relationships. But experience has shown that conclusions reached by visual study are strongly biased by the investigator's personal equation. Everyone is likely to see in the curves what he looks for, and not to see relations of which he has no image in his "mind's eye." Moreover, visual comparisons are influenced much more by the conspicuous turning points in the curves—the peaks and the troughs—than by the intermediate segments. Finally, the conclusions yielded by such comparisons are at best vague, and quite incapable of numerical expression. Statisticians have therefore sought some method of measuring the relationships among the fluctuations of time series—particularly among their cyclical-irregular fluctuations—which will be objective, precise, and which will allow due influence to every segment of the curves compared.

Such a method they have found in the correlation calculus invented by Sir Francis Galton for the study of inheritance, developed by Karl Pearson, F. Y. Edgeworth, and G. Udny Yule, and applied to time series by J. Pease Norton and others. The coefficient of correlation expresses the relationship between two series on a scale which runs from + 1.00 (signifying perfect positive agreement), through 0 (meaning no agreement), to — 1.00 (signifying perfect inverse agreement).[1] In adapting this device to measuring the relationship between two time series, the temporal order of the items must of course be kept—a fact which renders the theory of probabilities inapplicable to the data and to the interpretation of the results.[2] Comparisons are made between the deviations of each successive pair of items from the arithmetic means of their respective series. Thus every item in each series has its influence upon the result, and this result is a mathematically precise average, unaffected (so far as the computation is concerned) by the personal equation of the investigator.

Yet the use of coefficients of correlation does not substitute mathematics for personal judgment, or make less necessary the visual study of charts. Just as the representative value of an arithmetic mean must be judged by the distribution of the array from which it is computed, so the significance of a coefficient of correlation must be judged by critical study of the materials combined in getting it.

This critical study should begin with the original data. When secular trends and seasonal variations have been eliminated in order to correlate the cyclical-irregular fluctuations of two series, the "fit" of the two trend lines requires close scrutiny.[3] For in this operation the correlation coefficient averages the relationship between two sets of cyclical-irregular deviations from two trends, and it will show close agreement between the two sets of deviations if the two trend lines misfit their data in similar fashion. One might expect similarity in misfits to be a rare occurrence. On the contrary, it happens often, and the "spurious correlation" it produces vitiates numerous

[1] Compare Warren M. Persons' chapter on "Correlation of Time Series" in the *Handbook of Mathematical Statistics*, edited by H. L. Rietz, Boston, 1924, pp. 160-165. Directions for computing coefficients of correlation can be found in almost all recent textbooks of statistics. For a fuller treatment see A. A. Tschuprow, *Grundbegriffe und Grundprobleme der Korrelationstheorie*, Leipzig & Berlin, 1925.

[2] Hence the significance of the "probable error" of a coefficient of correlation between two time series is not known. It certainly does not represent, as in other applications, an equal chance that a second computation, based upon a different sample, would deviate from the coefficient first found by no more than the limits which the probable error sets. See Persons, as above, pp. 162-163.

[3] There is, indeed, little point in correlating two time series from which the trends have not been eliminated, except when both trends can be representd by horizontal straight lines. For the results will show primarily the relations of the trends themselves—an aim which can be attained less ambiguously by simpler devices. Compare F. C. Mills, *Statistical Methods*, New York, 1924, pp. 410-412.

investigations in the business-cycle field. Similar misfits of trend lines are especially likely to occur in correlating

> economic series covering both the period of declining prices previous to 1897 and the period of rising prices following that year. Nearly all economic series dip below the linear trend in the nineties so that a correlation coefficient between their deviations would indicate that fact rather than the general correspondence of their fluctuations.[4]

Since there is seldom an objective criterion for determining the goodness of a trend line's fit, there is seldom an objective criterion for determining the representative value of a coefficient of correlation between two sets of deviations from two trends. All that correlation coefficients can do for us is to make more precise the comparisons which are warranted by careful study of the original data, the fit of the trends, and the character of the deviations.

There is another source of error in interpreting coefficients of correlation which statisticians are prone to overlook. The time relations between the cyclical-irregular fluctuations of economic processes may shift from phase to phase of business cycles. For example, the production of industrial equipment may lag behind the production of consumers' goods during the phase of recuperation after a depression, and yet decline earlier than the production of consumers' goods when prosperity begins to wane. Again, New York clearings have usually begun to decline after periods of prosperity some months before outside clearings drop; in the opposite phase of recovery the New York clearings show no such lead.[5] When such changes in timing occur, a coefficient of correlation gives an average relationship which not only has little significance, but may actually put the investigator off a promising trail. The only safeguard against being misled in this way is to study charts with close attention to the regularity with which fluctuations in one curve precede or follow fluctuations in the second. This warning is needed, because the chief use of coefficients of correlation in business-cycle work has been to determine the lag of one series in relation to another.[6]

[4] W. M. Persons, in *Mathematical Handbook of Statistics,* p. 164, note.

[5] See section v, 2 below, "The Time Sequence of Cyclical-Irregular Fluctuations.

[6] See Professor Allyn A. Young's discussion of the interpretation of correlation coefficients in his introduction to *Social Consequences of Business Cycles,* by M. B. Hexter Boston and New York, 1925.

2. Transformations of Time Series in the Investigation of Their Relationships.

When economists began to study time series they took the data in their original form, whatever that happened to be. The difficulty of incommensurable units—for example gold production in million ounces and bank discount rates in percentages—could be met by drawing charts on which two arbitrarily adjusted scales were laid off. A neater shift was to turn the original data into the form of "relatives," with 100 to represent the values of both variables in some period chosen as the base. Another plan was to chart the logarithms of the two series. Still another was to drop the original data and compare the percentage differences between the successive items in each series.

Once started upon this career of transforming time series into new shapes for comparison, statisticians have before them a limitless field for the exercise of ingenuity. They are beginning to think of the original data, coming to them in a shape determined largely by administrative convenience, as concealing uniformities which it is theirs to uncover. With more emphasis upon statistical technique than upon rational hypothesis, they are experimenting with all sorts of data, recast in all sorts of ways. Starting with two series having little resemblance in their original shape, they can often transmute one series into "something new and strange," which agrees closely with the other series. In work of this type, they rely upon the coefficient of correlation to test the degree of relationship between the successive transformations.

Two recent examples of such researches may be cited. (1) Mr. Karl G. Karsten has studied the relations between the American data for freight-car surplus and shortage and for interest rates on 60-90 day commercial paper in New York. Between the two series in their original form there appeared to be no correspondence, for the coefficient of correlation was nearly zero (+.02). By making the freight-car data lag behind interest rates eight months, Mr. Karsten got a coefficient of +.402. By cumulating the deviations of the freight-car data from their trend, he obtained a curve which gave a coefficient of +.914 when correlated with the interest curve. By using logarithms of interest rates, he raised the coefficient to +.926, and by omitting the nine months, March to November, 1918, when interest rates were purposefully kept from rising above six per cent, he obtained a coefficient of +.950 between the cumulatives of car shortages and the moving annual averages of interest rates.[1]

(2) Professor Irving Fisher has studied the relation between wholesale-price fluctuations in the United States and the volume of trade. Starting with the Bureau of Labor Statistics index of wholesale prices and Persons' index of the physical volume of trade by months from August, 1915, to March, 1923, he smoothed the latter by a moving average, and found a correlation coefficient between the two series of +.54. Then Fisher turned the price index into figures which showed the rapidity of change in prices, gave this derivative series a seven-months lead over the trade index, and got a coefficient of +.727. Next he made an elaborate set of experiments in distributing this fixed lag over varying numbers of months weighted in different ways "according to the principles of probability." He finally found a scheme of distributing the lag which raised his coefficient to +.941.[2]

These interesting experiments may be the beginning of a long series of efforts to obtain high coefficients of correlation between various pairs of time series by casting them into new forms. Work of this sort is fascinating to the statistical technician. While very expensive, because of the endless experimental computations required, the possibility of finding marketable forecasting series may enable investigators to secure the necessary funds. And such work may yield results of theoretical interest as well as practical value. But it may also lead to grave mistakes, unless soberly controlled.

The proposition may be ventured that a competent statistician, with sufficient clerical assistance and time at his command, can take almost any pair of time series for a given period and work them into forms which will yield coefficients of correlation exceeding ±.9. It has long been known that a mathematician can fit a curve to any time series which will pass through every point of the data. Performances of the latter sort have no significance, however, unless the mathematically computed curve continues to agree with the data when projected beyond the period for which it is fitted. So work of the sort which Mr. Karsten and Professor Fisher have shown how to do must be judged, not by the coefficients of correlation obtained within the periods for which they have manipulated the data, but by

[1] See Karl G. Karsten, "The Theory of Quadrature in Economics," *Journal of the American Statistical Association*, March, 1924, vol. xix, pp. 14-29.

[2] Irving Fisher, "Our Unstable Dollar and the So-called Business Cycle," *Journal of the American Statistical Association*, June, 1925, vol. xx, pp. 179-202. The method devised for distributing the lag will be described below in section v, 2.

the coefficients which they get in earlier or later periods to which their formulas may be applied. Mr. Karsten points out that his coefficient of correlation between adjusted cumulatives of freight-car shortage and the logarithms of the moving average of interest rates sinks from +.95 in 1915-23 to +.856 in 1907-14. Similarly Professor Fisher shows that his coefficient between the rate of change in prices with a distributed lag and the physical volume of trade sinks from +.941 in 1915-23, to +.58 in 1877-99, to +.67 in 1903-15, and to +.78 in 1923-24.[3] Controlled by such tests, the methods of Karsten and Fisher, or rather an endless variety of methods not less intricate, may be applied to the study of the relations among time series *ad libitum* without scruple—provided investigators are chary of interpreting their coefficients as demonstrating causal connections.

This caution is important. Statisticians know that such an average of relationships between paired items of two time series as the coefficient of correlation yields does not prove that the changes in one of the series produce the changes in the second series, even when the latter series has been made to lag in the pairing of dates. Careful workers bear this fact in mind. Mr. Karsten, for example, though he obtained a high coefficient of correlation between his two series for years preceding his trial period, did not suggest that the number of idle freight cars is controlled by interest rates in New York. Nor would a coefficient of correlation, however high, warrant such an inference, unless independent evidence of causal connection could be adduced. Professor Fisher has reason to believe that the rate of change in prices influences the physical volume of trade, and gives a causal interpretation to his results.[4] But even in Professor Fisher's position an investigator should be cautious. It is not unlikely that by taking equal pains another worker studying the relations between the physical volume of trade and (say) reserve ratios, interest rates, profits, or payroll disbursements within Professor Fisher's period might get very high coefficients of correlation, and argue that he too had found "an almost complete explanation of fluctuations in the volume of trade,"—or several "almost complete" explanations.[5]

[3] See the two papers cited above, pp. 23 (Karsten) and 201 (Fisher).

[4] Something will be said about the causal relationship later in the present chapter (see footnote on p. 286); but the problem will be treated more fully in the second volume.

[5] Compare Professor Fisher's conclusion, "These correlations are so high as to leave little or no doubt that changes in the price level afford an almost complete explanation of fluctuations in the volume of trade for the above period beginning in 1915 and ending in 1923. . . . With a correlation of nearly 100 per cent between trade and projected price-change, there is little left to explain." As cited above, p. 191.

While we cannot account for more than 100 per cent of the variations in one time series by any combination of causal relationships, it is not absurd to have several coefficients of correlation adding up to more than 1.00 between a given series and a number of other series which are regarded as exercising a causal influence over the first. Were such a set of results before us, we should have two quite different lines of explanation. The more obvious explanation is that the several variables correlated with the series whose fluctuations we are trying to account for are not independent of each other. In other words, the same causal influences are represented in two or more of the variables. There is much overlapping of this sort among economic time series. The second explanation is that in a theoretically perfect case of causal explanation by the joint action of two or more strictly independent factors, the two or more coefficients of correlation between the various series which represent causes and the one series which represents effects will add up to more than 1.00. Coefficients of correlation are not percentages, though the fact that they run on a scale from — 1.00 to + 1.00 has seemed to Professor Fisher sufficient warrant for calling them percentages. In the perfect case of exhaustive explanation referred to, it is not the several coefficients which equal 1.00 but the sum of their squares.[6] In con-

[6] To illustrate by Fisher's case: Waiving all question about the significance of a relation made to fit one period which does not maintain itself in other periods, let us accept his coefficient of "94 per cent" between price-change with a distributed lag and the volume of trade in 1915-23, and ask what part of the fluctuations in trade is left unaccounted for. The answer is not given by the formula 100 per cent — 94 per cent = 6 per cent; but by the formula $k^2 + r^2 = 1$, in which k stands for the "coefficient of alienation" and r for the coefficient of correlation. The coefficient of alienation measures the lack of agreement between two variables, as the coefficient of correlation measures the degrees of relationship. Substituting .94 for r in the equation, we get $k^2 = 1 - .8836 = .1164$, and $k = .34$. Of course, the coefficient of alienation is no more a percentage than the coefficient of correlation. We must not add 94 per cent and 34 per cent, concluding that we have accounted for 128 per cent of the variability of trade! But we may add $.94^2$ and $.34^2$, and say that their sum, 1.00, represents a theoretically complete explanation. If we insist upon using a percentage scale it should be that of the squared coefficients: $.8836 + .1156 = 1.00$.

Similarly with the results which Professor Fisher gets on applying his method of connecting price-change with volume of trade in other periods than 1915-23. The significance of his correlation of "58 per cent" in 1879-99 is to be judged from the equation $.58^2 + .82^2 = .3364 + .6724 = 1$; the significance of his correlation of "67 per cent" in 1903-15 is to be judged from the equation $.67^2 + .74^2 = .4489 + .5476 = 1$; and the significance of his coefficient of "78 per cent" in 1923-24 is to be judged from the equation $.78^2 + .63^2 = .6084 + .3969 = 1$.

Though calling his coefficients of correlation percentages, Professor Fisher does not commit the error of saying that a coefficient of "94 per cent" explains all but 6 per cent of the variations of the series which lags; he says merely that this coefficient affords "an almost complete explanation." (See preceding note.)

On the coefficient of alienation and its uses, see Truman L. Kelley, *Statistical Method,* New York, 1923, pp. 173, 174.

sidering how much significance attaches to a given coefficient of correlation, therefore, one should take the square of that coefficient, rather than the coefficient itself, as indicating the degree of relationship between the two variables, and compare with it the square of the "coefficient of alienation" as indicating the lack of relationship. And one should always remember that coefficients of correlation, however high, do not suffice to establish relationships of cause and effect.

3. Conclusion

All these cautions about the interpretation of results do not mean that one should hesitate to turn any series into a form which will agree better than the original figures with some variable one wishes to explain. On the contrary, search for relationships which are hidden by the form in which series happen to be compiled is one of the most promising, though one of the most arduous, lines of statistical research. Simple transformations into relatives, logarithms, and first differences have long been practiced with general approval; more elaborate transformations need no justification beyond fruitfulness.

Some hidden relationships between time series may be discovered by accident or by strictly empirical work. But the search is most likely to prosper if guided by rational hypotheses. These hypotheses usually occur to our minds in terms of cause and effect. What we know from non-statistical sources about business processes may suggest that the activities represented by one time series lead to consequences shown by one or more other series. Before plunging into the computations which such a notion suggests, it is wise to think out the hypothesis with care. Precisely what feature of the first series is causally important—the actual magnitudes as reported, the changes in these magnitudes from date to date, the percentage rates of change, the accumulated changes, the excess beyond some critical range, the ratio of the causal factor to some other variable, or what? Similarly: upon what feature of the series regarded as showing effects is the causal effect exercised? The suggestions just listed are possible answers to this question also. Is the relationship direct, or inverse? Is the effect immediate or postponed? Is the effect cumulative? Does the effect change with the phases of business cycles? All these matters, and in many cases others, should be considered. Often it is only by trial computations that one can decide the issues raised; but they are best raised before computations are begun, and then thought out again in the light of what the computations suggest.

In judging the relationship between any two series, how low a coefficient of correlation should one accept as "significant"? That is a question which statisticians often raise, but to which they do not give categorical answers, because much depends upon the character of the data and the purpose in view. When the aim is merely to find whether two phenomena are unrelated to each other, or related in some degree, interest centers less in the absolute size of the coefficient obtained, than in its size compared with that of its standard (or probable) error. Provided a coefficient is several times its standard error, a figure in the forties, or even in the twenties, suffices to show the existence of some relationship. But when the coefficient of correlation is used in estimating the value of one variable from given values of other variables—the problem usually met in correlating time series—a much higher standard must be set. Coefficients ranging from .40 to .50, which often pleased earlier students of cyclical fluctuations, and even coefficients of .60 to .70, are not very imposing when squared, as they should be in thinking about their significance for making such estimates. In many cases a result of this order is best taken as a sign that the investigator has found a promising trail, but is not close to his goal. A reconsideration of the causal relationships involved, and further experimental computations, may lead to much higher coefficients. An expert in research of this type becomes exacting; Mr. Karsten, for example, remarks: "in my own forecasting work I do not consider of much value a coefficient below .90." [1]

V. The Amplitude and the Timing of Cyclical-Irregular Fluctuations in Different Processes.

From the preceding discussion of methods of analyzing time series and their relations we score two gains. One is understanding of and ability to use the results reached by other investigators. The second is guidance in analytic work of our own.

Anyone who takes the statistical approach to business cycles develops a longing to assemble all the pertinent series and analyze them afresh upon some consistent plan, which shall incorporate the best ideas of his predecessors with improvements of his own. But, as must be clear by this time, the analysis of time series is a laborious

[1] Karl G. Karsten, "The Harvard Business Indexes—A New Interpretation," *Journal of the American Statistical Association*, December, 1926, vol. xxi, p. 409.

and expensive process, only less expensive and laborious than compiling the original data. Moreover, the changes made in results by alterations in method are often slight. Hence every investigator does well to go as far as he can in utilizing the results obtained by others, even when these results are not precisely adapted to his needs. Limits are set upon such borrowings by radical differences in methods, in periods covered, and in data treated. When he approaches these limits, the investigator must give up the quantitative approach for the qualitative, or he must undertake the heavy burden of making statistical analyses for himself.

On three topics of great interest we can learn much by the simple process of assembling and comparing the results reached by others: the relative amplitude of the cyclical-irregular fluctuations characteristic of different economic activities, the temporal order in which different activities increase or diminish, and the way in which series differing in amplitude and timing can be combined to throw light upon the cyclical movement as a whole.

1. The Amplitude of Cyclical-Irregular Fluctuations.

While Chart 8 makes it plain that economic processes differ widely in the amplitude of their cyclical-irregular fluctuations, it leaves us with rather vague impressions, and we want measurements. A considerable variety of such measurements, however, lies ready to hand. Analytic statisticians often compute the standard deviations of the cyclical-irregular fluctuations of their time series expressed as percentages of the ordinates of secular trend corrected for seasonal changes.[1] These standard deviations may be used as measures of the average amplitude of cyclical variations in the economic processes concerned, if certain precautions are observed. Technical defects in the method of fitting trends and ascertaining seasonals affect the percentage magnitudes of the cyclical-irregular deviations, and therefore of their standard deviations. Differences in the periods covered also may influence the results; for the cyclical-irregular fluctuations of a given time series are likely to vary somewhat from decade to decade, even in percentage form. Still further doubts are raised by

[1] The standard deviation of a statistical series, conventionally represented by sigma (σ), is computed by taking the arithmetic mean, finding the deviation of each item from this mean, squaring the deviations, adding the squares, dividing the sum by the number of items, and extracting the square root of the quotient. All modern text-books of statistics discuss this device.

differences in the form of the original data analyzed, of which more presently. But if we make a considerable collection of standard deviations computed in similar ways for various periods and countries, confine our observations to broad differences, and avoid some obvious pitfalls, we can reach conclusions of importance.

In computing the standard deviations which we shall borrow, the aim of statisticians has usually been to get similar units in terms of which they can express the cyclical-irregular fluctuations of their time series, in order to make these fluctuations more comparable. Logically, the procedure is analogous to expressing the prices of the different commodities used in constructing an index number as relatives of the actual quotations for each commodity at some base period. Our aim is different. Several of the theories of business cycles reviewed in Chapter I, notably those of Mr. Lawrence K. Frank and Dr. T. W. Mitchell, rely largely upon differences in the amplitude of the cyclical fluctuations characteristic of different processes to explain the origin or the propagation of cyclical impulses. Like certain other features of time series which give trouble to statisticians, these differences of amplitude may give help to the theorist. At least we must learn what we can about them.

In Table 11 there are assembled several collections of standard deviations of cyclical-irregular fluctuations measured by percentage deviations from ordinates of secular trend corrected, when necessary, for seasonal variations. The rather miscellaneous array is classified first by countries, secondly by periods, and thirdly by the magnitude of the standard deviations themselves.

A glance over the various sections of the table shows that economic activities are characterized by marked differences in the amplitude of their cyclical-irregular fluctuations. In part these differences are due to the form in which the original data are gathered. The most notable case is the extreme variability of employment, which is represented in sections H, I and J of the table by percentages of reporting trade-union membership unemployed at successive periods. If the same data were converted into percentages employed, the standard deviations would be greatly reduced. Again, the standard deviations of the British series showing the market values of certain types of securities outstanding are not strictly comparable with the standard deviations of American series showing the average market prices of similar types of securities. Once more, standard deviations computed from data in monthly, quarterly, and annual form are not strictly comparable. Finally, there are differences of business usage between the United States, Great Britain and Germany which interfere with the comparison of, say, bank clearings, bank loans, and discount rates in the three countries. Most of our comparisons must be limited to items within a given section of the table.

Observing these restrictions does not much reduce the spread of the standard deviations; for the differences between economic processes in the same country and period are much wider than the differences between analogous processes in different countries and periods. In several sections of the table, the standard deviations run from 2 or 3 per cent of the ordinates of secular trend to 30, 40, 50 or 60 per cent. Even in closely related processes, like various banking operations, wholesale and retail trade, the prices of different types of securities, the more variable series of cyclical irregular fluctuations have standard deviations which are two or three times the standard deviations of the stabler series. This table is the statistical justification for the remark made in Chapter I, that we must conceive of business cycles as congeries of cyclical fluctuations in different processes which have widely different amplitudes.

TABLE 11

Relative Amplitude of the Cyclical-Irregular Fluctuations of Various Economic Processes

A. American Series, 1860, 1862 or 1866 to 1880

Standard deviations, arranged in order of magnitude, of the relative deviations of the original data from lines of secular trend, corrected when necessary for seasonal variations.
Compiled from the Appendix to Persons, Tuttle and Frickey, "Business and Financial Conditions following the Civil War in the United States," *Review of Economic Statistics*, Preliminary vol. ii, Supplement, July, 1920.

	Standard Deviations
Call-loan rate on the New York Stock Exchange, monthly, 1866–80....	38.7
Clearings of the New York City banks, monthly, 1862–80............	24.8
Interest rates on prime commercial paper, New York City, monthly, 1866–80..............	23.12
Interest rates on prime commercial paper, Boston, monthly, 1860–80..	23.0
Yield of U. S. Government 6's of 1881, monthly, 1862–80............	21.4
Price of ten common railroad stocks, monthly, 1866–80..............	18.1
Reserves of all National Banks, 5 calls yearly, 1866–80..............	11.7
Ratio of reserves to deposits, New York clearing-house banks, monthly 1866–80..............	9.6
Loans and deposits of all National Banks, 5 calls yearly, 1866–80......	8.9
Loans of New York clearing-house banks, monthly, 1866–80..........	6.85
Wholesale price index (W. C. Mitchell), quarterly, 1860–80..........	5.63

TABLE 11—*Continued*

RELATIVE AMPLITUDE OF THE CYCLICAL-IRREGULAR FLUCTUATIONS OF VARIOUS ECONOMIC PROCESSES

B. American Series, 1903–14 or 1903–18, by Months

Standard deviations, arranged in order of magnitude, of the relative deviations of the original data from lines of secular trend corrected for seasonal variation.

Compiled from articles by Warren M. Persons, *Review of Economic Statistics*, Preliminary vol. i, pp. 36 and 191.

	Standard Deviations
Shares traded on the New York Stock Exchange	49.6
Unfilled orders of the U. S. Steel Corporation (quarterly before 1910)	32.3
Value of building permits issued for 20 American cities	20.4
Bank clearings in New York City	20.3
Interest rates on 60– to 90–day paper in New York City	19.66
Production of pig iron	19.15
Interest rates on 4– to 6–months paper in New York City	16.46
Average price of 12 industrial stocks	15.03
Dividend payments by industrial corporations	14.96
Number of business failures (Bradstreet's)	13.55
Imports of merchandise	11.91
Reserves of the New York clearing-house banks	10.83
Average price of 20 railroad stocks	10.18
Bank clearings outside of New York City	8.62
Deposits of New York clearing-house banks	8.20
Gross earnings of 10 leading railroads	6.07
Loans of New York clearing-house banks	5.37
Bradstreet's wholesale-price index	3.68
Interest yield on 10 railroad bonds	2.82
Bureau of Labor Statistics wholesale-price index	2.60

C. American Series, 1879–96 and 1897–1913

Standard deviations, arranged in order of magnitude, of the relative deviations of the original data from lines of secular trend, corrected when necessary for seasonal variations.

From Warren M. Persons, "An Index of General Business Conditions, 1875–1913," *Review of Economic Statistics*, January, 1927, vol. ix, p. 28.

	Standard Deviations 1879–1896	Standard Deviations 1897–1913
New York bank clearings	24.55	18.21
Interest rates on prime commercial paper, New York	21.71 *	17.53 *
Pig-iron production	19.30	15.65
Industrial stock prices	14.11	15.07
Bank clearings outside New York City	12.03	7.98
Average of industrial and railroad stock prices	11.91	11.94
Railroad stock prices	10.89	10.12
Wholesale commodity prices (J. L. Snider's series)	7.78	3.77
Loan-deposit ratios of New York clearing-house banks	5.93	3.19
Loan-liability ratios of National Banks outside New York City	2.22	1.84

* Computed from percentage deviations from 5 per cent, adjusted for seasonal variations.

TABLE 11—*Continued*

RELATIVE AMPLITUDE OF THE CYCLICAL-IRREGULAR FLUCTUATIONS OF VARIOUS ECONOMIC PROCESSES

D. American National Banking Series, 1901–14, by 5 "calls" yearly

Standard deviations of percentage deviations from trend, corrected for seasonal variation.

From Allyn A. Young, "An Analysis of Bank Statistics for the United States," *Review of Economic Statistics*, January and April, 1925, vol. vii, pp. 36 and 101–104.

	Standard Deviations				
	New York City	Outside New York City	Boston	Chicago	San Francisco
Lawful money held.....	11.62	4.25			
Net deposits..........	9.64	3.41			
Investments...........	9.59	4.59			
Individual deposits.....	9.12	3.14	5.51	5.40	17.2
Loans and discounts...	6.80	2.96	3.84	4.02	17.0

	Individual deposits less clearing-house exchanges	Loans and discounts	Investments, except securities against notes and U. S. deposits
Pacific states..........................	7.09	7.96	7.29
Southern states........................	5.49	5.06	7.16
Western states.........................	5.36	4.92	3.62
New England states....................	2.98	1.98	7.55
Eastern states, excluding New York City.	2.34	2.77	4.19
Middle Western states..................	2.30	2.62	6.97
Money in National Banks...............		5.9	
Money in circulation not in banks.......		3.74	

E. Velocity of Bank Deposits in American Cities, by Months, 1919–February, 1923

Standard deviations of the monthly velocity after adjustment for seasonal variation.

From W. Randolph Burgess, "Velocity of Bank Deposits," *Journal of the American Statistical Association*, June, 1923, vol. xviii, p. 738.

	Standard Deviations		Standard Deviations
New York City....................	5.98	Chicago.................	2.79
Syracuse..........................	4.74	San Francisco............	2.23
Albany...........................	4.36	Buffalo..................	1.79
Boston...........................	3.64	Rochester...............	1.22

TABLE 11—*Continued*

RELATIVE AMPLITUDE OF THE CYCLICAL-IRREGULAR FLUCTUATIONS OF VARIOUS ECONOMIC PROCESSES

F. American Series Showing Volume of Wholesale and Retail Trade by Months 1919–1925

Standard deviations of percentage deviations from secular trends corrected for seasonal variations.

From Simon S. Kuznets, *Cyclical Fluctuations: Retail and Wholesale Trade*, pp. 37, 41, 102, 114.

	Standard Deviations Retail Sales	Wholesale Sales
Mail-order houses	16.4	....
Music-store chains	11.8	....
Dry goods	11.5	16.5
Grocery-store chains	10.6	14.4
Shoe-store chains	9.6	18.1
Tobacco- and cigar-store chains	7.9	....
Candy-store chains	7.9	14.7
Department stores	6.3	15.8
Five- and ten-cent store chains	5.1	....
Drug-store chains	4.4	6.2
Hardware	....	13.4
General index	....	14.8

Series "Deflated" by Dividing Dollar Volume of Sales by Appropriate Index Number of Prices

	Standard Deviations		
	Retail Trade	Wholesale Trade	Production
Shoe-store chains	9.9	14.6	19.3
Department store chains	6.3	....	....
Grocery-store chains	4.5	7.6	10.8
Dry Goods	...	13.3	....
Hardware	....	10.0	
Drugs	....	5.1	
General index	....	6.7	

G. British Series, by Quarters, before 1850

Standard deviations, arranged in order of magnitude, of the percentage deviations from the line of secular trend, adjusted when necessary for seasonal variations.

From Norman J. Silberling, "British Prices and British Cycles, 1779–1850," *Review of Economic Statistics*, October, 1923, Preliminary vol. v, Supplement 2, pp. 254–257.

	Standard Deviations
Quarterly average market rate of discount in London, best bills, 1824–1850	30.3
General commodity prices at wholesale, by quarters, 1779–1850	5.3

TABLE 11—*Continued*

RELATIVE AMPLITUDE OF THE CYCLICAL-IRREGULAR FLUCTUATIONS OF VARIOUS ECONOMIC PROCESSES

H. British Series, by Quarters, 1903–June 30, 1914

Standard deviations of the percentage deviations from secular trends.

Compiled from Persons, Silberling, and Berridge, "An Index of British Economic Conditions," *Review of Economic Statistics*, Preliminary vol. iv, Supplement 2, June, 1922, p. 189.

	Standard Deviations
Stores of Cleveland pig-iron in public warehouses	64.8
Percentage unemployed in all trades	43.8
Discount rate on 3-months paper in London	27.2
London bank clearings on stock-settling days	13.3
Exports of iron and steel (quantities)	13.2
Number of blast furnaces in blast	8.1
Exports of British produce (values)	7.50
Imports of raw materials, excluding cotton (values)	7.41
Bank clearings in 5 provincial cities	5.99
Sauerbeck Statist index number of wholesale prices of "all materials"	5.68
Market value of securities yielding variable incomes, Bankers' Magazine	4.3
Market value of selected British railway ordinary stocks	3.62
County bank clearings through London	3.34
Market value of local-government bonds	2.14

I. British Series, by Years, various dates—1913

Standard deviations of the percentage deviations from secular trends.

From Dorothy S. Thomas, *Social Aspects of the Business Cycle*, London, 1925, pp. 187, 200, 203 and 211.

	Standard Deviations
Percentage unemployed, "all trades," 1860–1913	54.6
Emigrants of British origin leaving U. K. for U. S. A., 1870–1913	21.3
Total emigrants of British origin from U. K., 1862–1913	19.1
Casual pauperism, 1883–1913	10.5
Production of pig iron, U. K., 1865–1913	8.77
Exports of British produce, 1854–1913	8.08
Sauerbeck's index number, wholesale prices of "all materials," 1854–1913	7.40
Indoor pauperism, 1857–1913	6.45
Provincial bank clearings, 1887–1913	5.25
Outdoor pauperism, 1857–1913	4.33
Per-capita consumption of spirits, 1856–1913	4.08
Per-capita consumption of beer, 1856–1913	3.83
Production of coal, 1865–1913	3.59
Railway freight traffic receipts, 1881–1913	2.69

TABLE 11—*Continued*

RELATIVE AMPLITUDE OF THE CYCLICAL-IRREGULAR FLUCTUATIONS OF VARIOUS ECONOMIC PROCESSES

J. British Series, by Quarters, various dates to 1914

Standard deviations of the percentage deviations from secular trend corrected for seasonal variations.

From Dorothy S. Thomas, unpublished data.

	Standard Deviations
Unemployed iron founders, 1855–1914	60.52
Unemployment, "all trades," 1887–1914	45.20
Value of total exports of British produce, 1855–1914	9.25
Sauerbeck's wholesale-price index number, "all materials," 1885–1914	6.44
Blast furnaces in blast, 1897–1914	6.30
Provincial bank clearings (Manchester and Birmingham), 1887–1914	6.03
Railway freight traffic receipts, 1881–1914	3.93

K. German series by Quarters or Months, various dates to 1913–14

Standard deviations, arranged in order of magnitude, of the percentage deviations from lines of secular trend adjusted when necessary for seasonal variations.

Compiled from Emerson W. Axe and Harold M Flinn, "An Index of General Business Conditions for Germany, 1898–1914," *Review of Economic Statistics*, October, 1925, vol. vii, p. 287.

	Standard Deviations
Market discount rates in Berlin, 1868–1914	27.90
Discounts and advances of the Reichsbank, 1872–1914	10.65
Ten-commodity price index of business cycles, 1898–1913	9.43
Index of stock prices on the Berlin Bourse, 1900–1914	8.45
Receipts from the *Wechselstempelsteuer*, 1900–1913	7.66
Value of commodities imported into Germany, 1892–1914	7.06
German bank clearings, 1898–1914	6.83
German pig-iron production, 1882–1914	6.64
Value of commodities exported from Germany, 1892–1914	5.53
Males enrolled in employees' insurance plan, 1904–1913	3.32
Monthly quotation of German *Reichsanleihung*, 1899–1914	2.36

Proceeding to particulars, we may set out certain conclusions which the figures suggest in quasi-tabular form. Regarding the volume of trade and production, the table indicates that

Retail trade shows fluctuations of smaller amplitude than wholesale trade in the same commodities.

Wholesale trade shows fluctuations of smaller amplitude than production of the same commodities, so far as our very limited evidence goes.

Judged by bank clearings, the volume of payments made in the great financial centers is far more variable than the volume of payments made in smaller towns.

The volume of foreign trade seems to be subject to wider fluctuations than domestic business outside the financial centers.

The volume of construction work in the United States, as shown by building permits, varies about as much as New York clearings.

Regarding prices we have no standard deviations for retail index numbers, which would presumably be small; but it does appear that

The cyclical-irregular fluctuations of wholesale-price index numbers have low standard deviations when many commodities are included, and moderate standard deviations (approaching 10.0) only when the indexes are made on purpose to exhibit cyclical fluctuations in a clear light. The "general level of wholesale prices" is one of the relatively stable factors in business, when monetary systems are not subject to grave disturbances, such as those caused by paper standards and great wars.

Stock prices are highly variable as compared with wholesale commodity prices, at least in the United States. (The British and German materials in the table do not admit of satisfactory comparisons with other series.)

Bond prices are even more stable than wholesale commodity prices, if we exclude the bonds of debtors whose credit is doubtful.

Regarding interest rates on short loans in the financial centers, for which alone the table gives data, we find that

The standard deviations are always rather high.[1] They range from 16.5 to 38.7. The London and Berlin rates in 1903-14

[1] It does not follow that the interest rates paid by most commercial borrowers are subject to wide variations. See Carl Snyder, "The Influence of the Interest Rate on the Business Cycle," *American Economic Review*, December, 1925, vol. xv, pp. 684-699.

fluctuate even more than the New York rates, a statistical result which may or may not be significant.

Regarding banking operations we have relatively full results for the National Banks as a whole, and by sections of the United States.

In all the operations for which the standard deviations of cyclical-irregular fluctuations have been computed, the New York figures are more than double the "outside" figures.

As among different operations, lending seems to be the least variable. The most variable item outside New York is the volume of investments; in New York it is the amount of lawful money held.

Another conclusion of theoretical importance is that the volume of coin and paper money held by the banks has larger standard deviations than the volume of coin and paper money in the hands of the public.

2. The Time Sequence of Cyclical-Irregular Fluctuations.

When several time series from which the secular trends and the seasonal variations have been eliminated are plotted by months one above another on the same time scale for a considerable period, the business cycles of that period can usually be traced in most if not all of the curves. But it is highly improbable that all the curves will reach the crests and troughs of their successive cycles in the same months. As a rule the crests and the troughs of the various curves are distributed over periods of several months—often over periods of more than a year.

Closer inspection shows that the order in which the curves reach their crests and decline, or reach their troughs and rise, presents that mixture of uniformity and differences with which economic statistics commonly confront us. The crests of a given curve may precede those of a second curve in some cycles and follow those of the latter curve in other cycles. But other comparisons show tolerably regular time relations over long periods. That is, the cyclical changes in certain economic processes appear to lead or lag behind the corresponding changes in certain other economic processes by intervals of time which are fairly constant.

This feature of the behavior of time series has been turned to account by statisticians interested in business forecasting. Indeed, it forms the corner-stone of several forecasting systems. If certain changes in banking operations regularly preceded certain other changes in discount rates by a regular interval, the latter changes could be foretold as soon as the former changes had been reported. Further, if an invariable series of such time relations between the cyclical fluctuations of different economic processes could be discovered, and if this series returned upon itself in the sense that the last set of changes in one cycle preceded the first set of changes in the next cycle by a regular interval, then business forecasting could be raised to a quasi-mechanical level. Needless to say, no such chain of events with links of unchanging length has been discovered. Perhaps no statistician has expected to find such a chain. But it has been a leading aim of statistical research to determine the time sequence in which important series pass through the successive phases of business cycles, to find cases in which this sequence is fairly regular, and in such cases to measure the average intervals by which certain series or groups of series lead or lag behind others.

The standard procedure in studying the time relationships among cyclical-irregular fluctuations is to start by plotting each series to be studied on a strip of translucent paper laid off with a uniform time scale. Any one of these strips can then be placed above any other and shifted to right or left until that position is found which seems to make the two series of cyclical-irregular fluctuations match best with each other. This matching may be closest when the same dates on the two strips are put together, or it may be closest when one series is made to lag behind the other by several months, by a year, or even more. Sometimes the best matching can be determined with confidence; sometimes it is so uncertain that two trained observers will differ in their opinions.

To test the conclusions suggested by this simple procedure, and to decide the doubtful cases, which are numerous, the statisticians resort to a more objective method. They compute several coefficients of correlation between two series, pairing the items in different ways. For example, if visual comparison of the curves suggests that the cyclical fluctuations of series A agree best with those of series B when A lags two months behind B, the investigator may compute (say) seven coefficients between the two series, one coefficient when

TABLE 12

TIME SEQUENCE IN THE CYCLICAL-IRREGULAR FLUCTUATIONS OF VARIOUS ECONOMIC SERIES

A. American Series, by Months, January, 1903–July, 1914

Compiled from Warren M. Persons, "An Index of General Business Conditions," *Review of Economic Statistics*, April, 1919, Preliminary vol. i, pp. 129, 182.

All the series included in each group have maximum coefficients of inter-correlation when concurrent months are paired, and these coefficients are of significant size. The coefficients are computed from cyclical-irregular fluctuations after elimination of secular trends, and, when necessary, of seasonal variations.

	Groups based upon Time Sequence of Cyclical-Irregular Fluctuations	Comparison with Bradstreet's price index: Lead or lag in months	Comparison with Bradstreet's price index: Coefficient of correlation
I	Series which precede other groups in time sequence		
	Yield of 10 railroad bonds	leads 10	− .72
	Price of 20 railroad stocks	leads 10	+ .76
	Price of industrial stocks	leads 10	+ .63
II	Series which lag behind Group I by 2–4 months		
	Shares traded in the N. Y. Stock Exchange	leads 12	+ .44
	Value of building permits in 20 American cities	leads 6	+ .61
	New York bank clearings	leads 6	+ .60
III	Series which lag behind Group II by 2–4 months		
	Production of pig iron	leads 2	+ .75
	Bank clearings outside of New York City	leads 2	+ .70
	Imports of merchandise	leads 2	+ .77
	Business failures	leads 2	− .67
IV	Series which lag behind Group III by 2–4 months		
	Bradstreet's index of wholesale prices	concurrent	+1.00
	Bureau of Labor Statistics index of wholesale prices	concurrent	....
	Gross earnings of railroads	concurrent	+ .77
	Reserves of New York City banks	lags 2	− .78
V	Series which lag behind Group IV by 4–6 months		
	Dividend payments	concurrent	+ .65
	Loans of New York City banks	lags 4	− .67
	Rate on 4- to 6-months paper	lags 4	+ .80

the Augusts in series A are paired with the Junes in series B, the Septembers in A with the Julys in B, the Octobers with the Augusts, and so on; a second coefficient when the Julys in A are paired with the Junes in B, etc.; a third when the Junes in A are paired with the Junes in B, etc.; a fourth pairing Mays in A with Junes in B, etc.; a fifth, sixth, and seventh when the Septembers, Octobers and Novembers in A are paired successively with the Junes in B. When all these coefficients have been computed, the investigator concludes that the closest time relationship between the cyclical-irregular fluctuations in the two series is that indicated by the highest of the co-

TABLE 12—*Continued*

TIME SEQUENCE IN THE CYCLICAL-IRREGULAR FLUCTUATIONS OF VARIOUS ECONOMIC SERIES

B. American Series, by Months, January, 1902, to December, 1908

Compiled from Alvin H. Hansen, *Cycles of Prosperity and Depression in the United States, Great Britain and Germany*, Madison, Wisconsin, 1921, pp. 26, 30, 33, 38, 39

Groups based upon Time-Sequence of Fluctuations	Comparison with Series Chosen as Standard for the Group: Lead or lag in months	Coefficient of correlation
I Banking Group which precedes other groups in timing		
Cash reserves of N. Y. Clearing-house banks—*Standard*	concurrent	+1.000
Call-loan rates, New York	concurrent	− .477
Deposits N. Y. Clearing-house banks	concurrent	+ .956
Loans N. Y. Clearing-house banks	concurrent	+ .889
* Commercial paper rates New York	lags 3	− .686
II Investment Group, lags 12 months behind Banking Group		
* Prices of 10 railroad bonds	leads 2	+ .808
Liabilities of business failures	leads 1	− .542
Prices of 10 investment stocks. *Standard*	concurrent	+1.000
Prices of 40 common stocks	concurrent	+ .904
Shares traded on New York Stock Exchange	concurrent	+ .580
* Total bank clearings in the U. S.	lags 3	+ .557
* Building permits in 20 American cities	lags 3	+ .482
* Railroad net earnings	lags 6	+ .473
III Industrial Group, lags 8 months behind Investment Group		
* Railroad net earnings	leads 4	+ .756
* Unemployment (Hornell Hart)	leads 3	− .719
Pig iron production	leads 1	+ .797
Imports of merchandise	leads 1	+ .905
Bureau of Labor Statistics index of wholesale prices. *Standard*	concurrent	+1.000
Railroad gross earnings	concurrent	+ .857
Immigration	concurrent	+ .696
* Exports of merchandise	lags 4	+ .758
Commercial paper rates (see Group I) compared with Industrial Group	lags 5	+ .688

* Series omitted from Group indexes because of lead or lag.

efficients. If the coefficient is highest when the Septembers in A are paired with the Junes in B, the investigator will say that series A lags behind series B by three months; if the coefficient is highest when the Mays in A are compared with the Junes in B, he will say that series A leads series B by one month; if the coefficient is highest when the Junes in both series are paired, he will say that the two series fluctuate synchronously.

Table 12 shows two sets of results reached in this way by Professors Warren M. Persons and Alvin H. Hansen. Both these men publish the coefficients for several monthly pairings of each set of series, but only the maximum coefficients which they take to indicate leads or lags are entered in the table. Often the maximum coefficients are but little greater than those for a somewhat longer or shorter lead or lag. This fact raises a problem which requires illustration and discussion.

A rather elaborate illustration may be borrowed from Dr. Frederick C. Mills, who has worked out not less than twelve coefficients of correlation for different monthly pairings of an index of prices of industrial stocks and the index of "general business activity" compiled by the statistical division of the American Telephone and Telegraph Company. Dr. Mills gives these twelve coefficients for two separate periods and shows that the results differ considerably—a point to which we shall presently recur. The point of immediate interest is that in both periods the coefficients are much alike for no less than five different pairings. In the first period, with stock prices leading "general business" by 3, 4, 5, 6, and 7 months the coefficients

TABLE 13

COEFFICIENTS OF CORRELATION BETWEEN THE CYCLICAL-IRREGULAR FLUCTUATIONS OF INDUSTRIAL STOCK PRICES AND AN INDEX OF "GENERAL BUSINESS"

Based upon American data, by Months, 1903–14, and 1919–23
From Frederick C. Mills, *Statistical Methods*, New York, 1924, pp. 424 and 426

	Coefficient of correlation 1903–14	1919–23
Stock prices concurrent with business index	+.55	+.75
Stock prices preceding general business by 1 mo.	+.65	+.83
" " " " " " 2 mos.	+.70	+.87
" " " " " " 3 "	+.73	+.88
" " " " " " 4 "	+.76	+.85
" " " " " " 5 "	+.76	+.82
" " " " " " 6 "	+.76	+.77
" " " " " " 7 "	+.74	+.72
" " " " " " 8 "	+.71	+.66
" " " " " " 9 "	+.67	+.57
" " " " " " 10 "	+.61	+.46
" " " " " " 11 "	+.54	+.33

range between +.73 and +.76. In the second period, the coefficients range between +.82 and +.88, when stock prices lead by 1, 2, 3, 4, and 5 months.

Whenever a close relation like this is found between the coefficients computed for several different pairings, the statement that one series lags behind another by some definite interval, such as five months, hardly suggests the facts, and may suggest quite false ideas. The definite lag should be thought of as the central tendency of an elaborate array of time relations between the fluctuations of two variables. When we speak of a lag of say five months we should accustom ourselves to bear in mind the other intervals which may show agreements nearly as close. And when, as sometimes happens, the array of coefficients for lags of different length shows no central tendency, but varies in an irregular fashion, we should be extremely cautious about saying that there is a definite lag.

The importance of the full array of time relations becomes clearer when we consider the causal interpretation of leads and lags. A theorist in thinking about the relations between changes in the price and changes in the production of a commodity does not suppose that an increase in prices made in January will affect production in just one future month, say June. On the contrary, he supposes that the January increase will begin to influence production policies, and perhaps actual output, as soon as it is announced, which may be in advance of the actual change in prices; or, rather, as soon as it is anticipated, which may be in advance of the announcement. The increase in prices actually made in January may have influenced production in the preceding December. Nor is the influence of the price change upon output likely to exhaust itself quickly; it may grow stronger for several months, reach a maximum, and then gradually decline. Conversely, the price influences upon the production of a given month, say, June, are not the result of changes made in some one preceding month, say, January; but the net resultant of price changes made in many preceding months, combined perhaps with price changes which are anticipated in the months to follow—changes which may not take place. Finally, the theorist does not trace the causal relationship in one direction only. He realizes that production reacts upon prices. The January change in prices, of which we have spoken, was probably influenced by the production of many months which preceded, and by anticipation of the production in months still to come.

So complicated a set of causal interrelationships can hardly be followed by statistical methods. But Professor Irving Fisher has shown how to take one step in advance by replacing a fixed lag with a lag which is distributed over many months. On comparing the rate of change from month to month in the wholesale-price index of the Bureau of Labor Statistics with Professor Persons' index of the physical volume of trade from August, 1915, to March, 1923, Fisher found that the usual methods gave the highest coefficient of correlation (+.727) for a lag of seven months in volume of trade. Six and eight months, however, gave coefficients nearly as high (+.719 and +.715). Giving a causal interpretation to the relation between price change and volume of trade, Fisher conceived the hypothesis that the effects of a given price change in one month upon volume of trade are distributed over succeeding months in accordance with the probability curve. He set himself to find out by experiment just what probability distribution of the price changes over time gives the index that correlates best with the volume of trade. As the result of many trials, he concluded that the best fit is obtained when the price changes are distributed along a "normal" curve having a logarithmic time axis, a mode lagging 9½ months behind the change in prices, and "probable error points" lagging about 5 and 18 months. When the price changes of 1915-23 were redistributed in this way, the coefficient of correlation between the new index and the volume of trade index proved to be +.941.[1]

[1] Irving Fisher, "Our Unstable Dollar and the So-called Business Cycle," *Journal of the American Statistical Association,* June, 1925, vol. xx, pp. 179-203.

Our present concern is with the method of treating lags which Professor Fisher has devised; not with the time relations between fluctuations in prices and "trade." But we should note that statisticians interested in the latter problem have questioned the comparability of the two series which Fisher uses, and the interpretation which he gives of the results.

Mr. Carl Snyder points out that the index of physical volume of trade which Fisher borrows from Persons represents essentially fluctuations in basic industrial activity, while his price index is very heavily weighted by agricultural prices. That is, Fisher compares the price changes of one list of commodities with the trade in a decidedly different list.

Dr. Willford I. King suggests that the lag may be explained as follows: wholesale prices are made when orders are placed or contracts drawn; goods are manufactured at a later date; shipments and transfers come later still. Price indexes are based upon current quotations. Volume of trade indexes are based mainly upon manufacturing output and deliveries. Thus volume-of-trade indexes are "post-dated" in comparison with the price indexes. If we could take both price and volume-of-trade records at the same stage of given transactions—that is, if we could take volume of transactions entered into at the time prices are made, or if we could take the prices at which current output and deliveries are being made—the relations between changes in prices and changes in volume of trade might appear in a new light. As matters stand, what is more natural than that the changes in a price index should precede in date the changes in a trade index? But does this lead due to post-dating justify the conclusion that price change causes changes in volume of trade?

As Professor Fisher suggests, the principle involved in this study of time relationship "would seem to be a general and useful one." In practical application to any case, however, the method is exceedingly laborious and costly, because there is no way of determining, except by actual computation, precisely what form of the probability distribution applied to the curve which leads will turn it into the form which correlates best with the curve which lags. Until some far quicker way of distributing lags is devised, the method will have but limited use.[2]

The problem still remains of treating statistically the influence of the variable which lags upon the variable which leads. To apply this remark to Professor Fisher's problem: economic theory suggests that changes in the physical volume of trade react upon prices. Can this reaction be demonstrated and measured by the use of time series? So far as the writer knows, no statistician has attacked this problem. It would be interesting to see what results could be obtained, for example, by treating the volume-of-trade index, or some derivative of it, as the variable which leads, and the price-change index, or some derivative of it, as the variable which lags, presumably choosing a period in which monetary disturbances were less extraordinary than in the years which Professor Fisher took as the base for his experiments. If a significant relation were shown to exist not only between price-change and volume of trade, but also between changes in volume of trade and subsequent changes in prices, the statistician would have come nearer presenting the complicated relations which the theorist contemplates as real.

Time sequence among the cyclical-irregular fluctuations of different series is no more constant than are secular trends or seasonal variations. One illustration of change has already been given in

[2] Professor Fisher himself suggests a "short cut."

"By this method the influence of any given price-change is assumed to begin at a maximum immediately (i.e., at the very next month or quarter following the given price-change), and then to taper off by equal reductions for each successive interval of time. . . . By having only *one* parameter to vary . . . the labor is reduced by at least four-fifths. The only variable becomes the number of months in which the influence tapers off to zero."

This short method yielded somewhat higher coefficients of correlation than the long method in all cases which Fisher worked out in both ways. For example, in 1915-23, the short method, with a tapering off period of 25 months, when compared with the physical volume of trade, gave a coefficient of +.95, against +.94 for the long method. On Professor Fisher's own logic, the short method seems to be better as well as shorter. But, though taking only one-fifth as much time as the long method, it is still laborious.

See the paper referred to above, p. 198, note.

Table 13. While the period covered by the second column of that table (1919-23) is too brief to yield final conclusions,

> The results indicate that since the war the movements of general business have followed more closely behind stock-price movements than during the pre-war days. A maximum correlation is secured when stock prices precede the business index by 3 months (instead of 4-6 months as in 1903-14).[3]

So, too, Professor Persons found that the time sequences shown in section A of Table 12 were not maintained in 1914-18.

> The systematic relation of the fluctuations which held during the pre-war period was shattered by the outbreak of war.[4]

Apart from such catastrophic events, the improvement of communication and transportation speeds up many effects which economic processes exert upon each other. Statistically this quickening means a gradual reduction of the intervals by which certain time series lag behind others, as well as a shifting of seasonal variations.

A formal discrepancy between the two parts of Table 12 may have troubled the reader's mind. According to Professor Persons, in 1903-14 New York City bank loans and deposits belonged in the group of series which fluctuated last in the sequence, while the reserves of these banks belonged in the group which was next to the last. On the contrary, according to Professor Hansen, in 1902-08 these three banking series belong in the group which preceded all other groups in the time sequence. To reconcile these opposing results is easy on the theory that business fluctuations have the character of recurrent cycles. One can break into a round of events which keeps repeating itself in time at any point and follow the sequence back to its starting point. A second investigator can follow the sequence equally well by starting where the first man stopped. Our two authorities have merely chosen different starting points for their analyses of time sequence. By linking Professor Persons' last group to his first, one gets Professor Hansen's start; by linking Hansen's last group to his first, one gets Persons' ending.

Another point which requires notice here has already been mentioned in discussing the correlation of time series; [5] but it is so im-

[3] Frederick C. Mills, *Statistical Methods*, New York, 1924, p. 426.
[4] Warren M. Persons, "An Index of General Business Conditions," *Review of Economic Statistics*, April, 1919, Preliminary vol. i, p. 116.
[5] See section iv, i, above.

portant and so seldom attended to that repetition is in order. The time sequence of two series sometimes changes from phase to phase of business cycles. For example, one series may lead a second in reviving after depression, but lag behind the second in declining after prosperity. In such cases a coefficient of correlation computed in the usual fashion will conceal in an average two opposite sequences which may be highly significant. To guard against such misfortunes, the computation of coefficients of correlation to determine time relations should always be preceded by close study of the plotted curves to see whether the sequence of fluctuations changes during the cycles. If such changes do occur it may be feasible to compute separate coefficients for the periods of revival and recession, of prosperity and depression.

Finally, we should recall in connection with the study of time sequences what was said in the preceding section about hunting for the particular phase of one economic process which affects a second process, and for the particular phase of the second process which experiences the causal effects of the first. Economic theorists, like economic statisticians, have been prone to argue from chronological priority to causal relationship, without intensive analysis of the way in which the causal influence is exerted. To illustrate the danger of drawing hasty conclusions: A decline in one activity generally precedes, and so seems to cause, a decline in a second activity; but changes in the volume of the first activity may be controlled by changes in the rate of growth in the second. In that case, throwing the second time series into the form of first derivatives will show that changes in its rate of growth regularly precede the changes observed in the first series, and reverse the inference concerning cause and effect. No feature of business cycles presents more misleading cues than does the apparent chronological order among the cyclical fluctuations of different processes, and no other feature requires from the investigator a finer blend of theoretical insight with statistical skill.[6]

[6] A concrete example may make clearer the type of issues involved in such work. Mr. Karl G. Karsten's paper, "The Harvard Business Indexes—A New Interpretation," (*Journal of the American Statistical Association,* December, 1926) will serve.

The Harvard indexes in question, more fully described in the next section, are represented by three curves: Curve A showing changes in speculation; Curve B showing changes in general business, and Curve C showing changes in discount rates. Index A moves first. Index B, lagging six months behind A, shows a coefficient of correlation with A of +.77 in the pre-war period and of +.73 in the post-war period. Index C, lagging four months behind B, shows a coefficient of correlation with B of +.80 in the pre-war period and (according to Karsten) of +.36 in the post-war period.

Mr. Karsten develops a new set of working hypotheses concerning the causal rela-

VI. Indexes of Business Conditions.

So far we have been dealing with the statistical analysis of time series taken one at a time, or with statistical comparisons of time series taken in pairs. In ascertaining secular trends and seasonal variations, in isolating cyclical-irregular fluctuations, in turning series into new forms, and in computing the standard deviations of cyclical-irregular fluctuations, the investigator treats each series by itself. In computing coefficients of correlation, and in studying leads and lags, he works with two series at a time.

The chief value of statistical methods and results for the theory of business cycles lies in these intensive studies of the fluctuations of particular processes and of their relations to each other. But the theorist can profit also by a further stage in the statistician's program —the effort to combine time series representing different economic processes into general indexes of business conditions. If such indexes can really be made, they should help to solve many theoretical problems.

To find what the indexes of general business mean, what they do and do not tell about business cycles, it is necessary to examine the

tionships among the activities represented by these indexes. (1) He supposes changes in the security prices of Curve A to be determined primarily by the amount of "money" which in dull times flows out of business into the stock market for investment, and which in brisk times is withdrawn from the stock market for business use. (2) What counts in this causal relationship is not the volume of money flowing into or out of the stock market in a given month, but the fund accumulated in the stock market by the net influx and efflux of past months—just as the amount of a bank deposit account is determined by cumulated withdrawals and deposits over a period. (3) On this hypothesis, the movements of Index B should lead the movements of Index A, not lag behind the latter, as the Harvard interpretation holds. (4) Also, the causal relation between general business and speculation is inverse. Dull business tends to produce stock-market activity; active business to produce stock-market dullness. (5) Discount rates are controlled by the cumulated demand for loan funds from both general business and the securities market. The volume of the former demand exceeds that of the latter, perhaps by three to one.

Computations which Mr. Karsten made to test these hypotheses show that the cumulation of Index B, inverted, and given a lead of two and one-half months over Index A, yields coefficients of correlation with A of +.94 in the pre-war period and of +.96 in the post-war period. Similarly, by cumulating Indexes A and B, weighted one and three respectively, Karsten gets an index which yields coefficients of +.85 and +.94 when correlated with Index C in the pre-war and post-war periods.

Since these coefficients are on a decidedly higher level than those obtained by the Harvard Committee of Economic Research, Mr. Karsten argues that his interpretation of the causal and chronological relationships among the three Harvard indexes has more theoretical justification and affords a better basis for forecasting than the Harvard interpretation.

That Mr. Karsten's results cannot be regarded as definitely established until they have been subjected to critical examination, makes his paper all the better as an illustration of the argument in the text.

leading specimens, beginning with simple devices and advancing toward more elaborate constructions.

1. A Collection of Indexes of Business Conditions.

(1) Beveridge's Chart of "The Pulse of the Nation," Great Britain, 1856-1907.

Sir William H. Beveridge's chart of "The Pulse of the Nation," made in 1908 and reproduced on the next page, was one of the early efforts to present graphically that "remarkable phenomenon . . . the . . . cyclical fluctuation of industrial activity." Seven important variables are represented:—foreign trade, the bank rate, employment, marriage rate, indoor pauperism, consumption of beer, and capital of new companies registered. Sir William puts the trade-union employment returns in the central position, but he reverses the figures,

> so as to represent an unemployed percentage of 2.5 as an employed percentage of 97.5 and so on. Lines drawn through the successive lowest points of this curve—1868, 1879, 1885, 1894, 1904—cut it up into waves of unequal length, representing successive industrial cycles. The crest of each wave is at about 98; the depressions are anywhere between 89 and 94. The point of the chart is this, that the same lines cut up every one of the other curves into corresponding waves.[1]

No adjustments are practiced upon the data beyond reducing all the series, except the percentage figures, to a per-capita basis. Of course there are differences in the movements of the seven curves; but these differences are matters of detail, and Sir William concludes his picture of business cycles by remarking,

> It would be possible to extend almost indefinitely the . . . review of economic statistics and almost everywhere to meet the same familiar phenomenon. . . . It is hardly too much to say that, apart from the death rate, the only prominent social and economic records in which the pulsation of the nation's aggregate activities cannot be traced as a significant factor, whether cause or symptom, are the price of Consols and the price of wheat.[2]

[1] Sir William H. Beveridge, *Unemployment, a Problem of Industry,* 2d ed., London, 1910, p. 41.

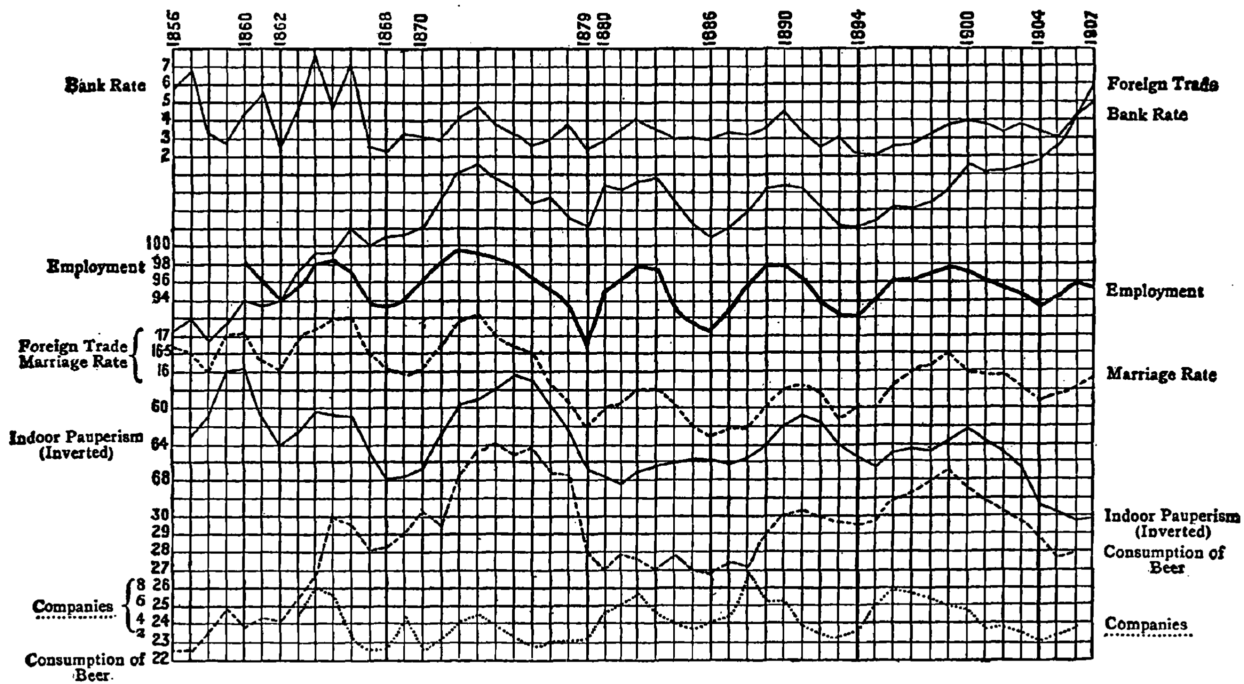
[2] The same, pp. 50, 51.

CHART 9.

SIR WILLIAM H. BEVERIDGE'S CHART OF "THE PULSE OF THE NATION."

GREAT BRITAIN, BY YEARS, 1856-1907.

(Reproduced from the chart in *Unemployment, a Problem of Industry,* 2d ed., London, p. 44.)



The scales at the left-hand side indicate respectively the actual Bank Rate of discount per cent., the percentage of trade union members not returned as unemployed, the number of marriages in England and Wales per 1000 of the population, the number of indoor paupers in England and Wales per 10,000 of the population, the gallons of beer consumed, and the nominal capital of new companies registered, in pounds, per head of the population. No scale is given for the Foreign Trade Curve. Unless the contrary is stated, all figures apply to United Kingdom generally.

With such a picture before them and with such materials to handle, statisticians wished to get results better adapted to analytic work than a collection of separate curves. Index numbers which reduce the price fluctuations of many commodities to a single series, raised the question whether it is possible to get a single curve to represent business cycles. And since business cycles cover but a few years, the year seemed too large a unit of measurement.

(2) Persons' "Index of General Business Conditions," United States, 1903-14.

By developing a more elaborate technique, Professor Warren M. Persons was able to draw a simpler chart of business cycles in 1919. He first isolated the cyclical-irregular fluctuations in a considerable number of time series in monthly form by the methods described above, and expressed these fluctuations as percentage deviations from the ordinates of secular trend corrected for seasonal variations. Since these deviations presented wide differences of amplitude from series to series, differences which seemed irrelevant for his purposes, Persons next reduced the cyclical-irregular fluctuations of each series to terms of their standard deviation. Then he made elaborate studies of the timing of the fluctuations of his various series expressed in this form, using coefficients of correlation to determine what series varied concurrently, what series lagged behind others, and how long were the lags. Finally, he averaged together the series which varied concurrently. In this way Persons reduced thirteen series to three indexes which he presented as "The Index of General Business Conditions." One index he called an "index of speculation," the second "an index of physical productivity and commodity prices combined," the third "an index of the financial situation in New York." [1]

[1] Warren M. Persons, "An Index of General Business Conditions," *Review of Economic Statistics*, April, 1919, Preliminary vol. i, pp. 111-114.

The series combined to make the three curves are as follows:

Index of speculation: yields of ten railroad bonds, prices of industrial stocks, prices of twenty railroad bonds, bank clearings in New York City.

Index of physical productivity and commodity prices: pig-iron production, outside clearings, Bradstreet's price index, Bureau of Labor Statistics price index, reserves of New York City banks.

Index of the financial situation in New York: rate on 4 to 6 months paper, rate on 60 to 90 day paper, loans of New York City banks, deposits of New York City banks.

For the revised form of this chart used for the post-war period, 1919 to date, see W. L. Crum, "The Interpretation of the Index of General Business Conditions," *Review of Economic Statistics*, Supplement, 1925, vol. vii, p. 226.

Recently Professor Persons has made a similar three-curve index which covers a much longer period than his first one. See his article, "An Index of General Business Conditions, 1875-1913," *Review of Economic Statistics*, January, 1927, vol. ix, pp. 20-29.

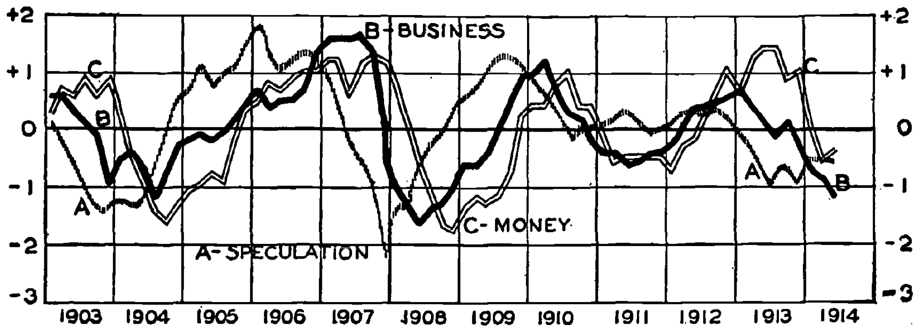
Although Persons called his chart "The Index of General Business Conditions," his aim was less to picture business cycles graphically, than to provide a basis for forecasting business changes. The merit of his index for this purpose was that, during the pre-war period, the cyclical fluctuations of the index of speculation systematically preceded in time those of the index of physical productivity and commodity prices, and that the cyclical fluctuations of the latter index systematically preceded those of the index of financial conditions.[2]

CHART 10.

PERSONS' "INDEX OF GENERAL BUSINESS CONDITIONS."

BI-MONTHLY AVERAGES, 1903 TO JUNE, 1914.

From *Review of Economic Statistics*, April, 1919, Preliminary vol. i, pp. 112-113.



(3) The American Telephone and Telegraph Company's "Index of General Business Compared with Normal," United States, 1877 to date.

Later investigators have used Professor Persons' technique to combine numerous time series into a single index of business conditions. Of such constructions the one which covers the longest period of American business is the chart of "General Business Compared with Normal," 1877 to date, made by the statistical division of the American Telephone and Telegraph Company. In carrying their chart back to 1877 by months, the statisticians of the telephone company had to use such data as they could find. From 1877 to 1884 pig-iron output was the only series available. In 1885, bank clearings outside New York City and blast-furnace capacity were added. In 1892 Brad-

[2] See Professor Persons' paper, cited above, p. 114. Persons' latest statements concerning these lags are quoted below in this section, 2, (4) "Forecasting Sequences."

street's wholesale price index came in. Further changes were made in 1903, 1909, 1913, 1919 and 1921; but the compilers believed that

> Variation in the number of series used and in weights is of little importance, since all the series move together in the business cycle.[1]

Finally in April, 1922, it was decided to drop all data which contained a price factor, and to add several series of recent origin. As Colonel M. C. Rorty explained in publishing the chart,

> . . . it includes no measure of agricultural activity or retail trade, except as such items are indirectly reflected in freight movements and bank clearings, and it includes only a very limited list of non-agricultural raw materials. It is, therefore, primarily a measure of manufacturing activity and the physical movements of commodities. Nevertheless, with all these limitations, it represents, perhaps, as serviceable an approach as can be made to a single "all purpose" business index.[2]

In making this index, secular trends and seasonal variations were eliminated, the cyclical-irregular fluctuations were expressed as multiples of their standard deviations, in this form the several series were weighted according to their values as representatives of business conditions, and weighted totals were cast.

(4) Snyder's "Index of the Volume of Trade," United States, 1919-25.

While the telephone company's statisticians desired to cover as long a period as is feasible in their index, in the first of his indexes Mr. Carl Snyder of the Federal Reserve Bank of New York sought to cover a brief period as thoroughly as possible. For the years 1919 to 1923 Snyder was able to find no less than 56 series in monthly form

[1] *Method of Construction of "General Business" Curve.* American Telephone and Telegraph Company, Office of the Chief Statistician. New York, July 8, 1921, p. 3. (Manuscript.)

[2] M. C. Rorty, "The Statistical Control of Business Activities," *Harvard Business Review*, January, 1923, vol. i, pp. 159, 160. After the revisions of 1922 the items included in the index, and their weights, were as follows:

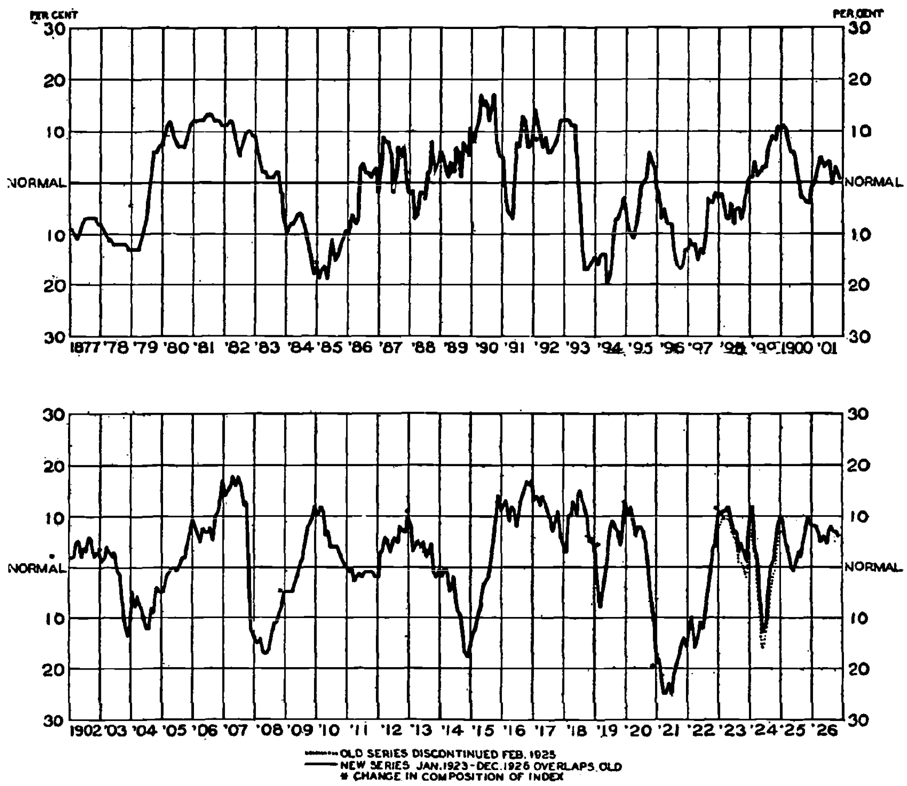
	Weight		Weight
Pig-iron production	20	Cotton consumption	10
Unfilled orders, U. S. Steel Corporation	10	Activity of wool machinery	10
Freight-car demand	10	Paper production	10
Car loadings	5	Lumber production	5
Net freight ton miles	5	Leather production	5
Coal production	5	Power production	5

which reflected some aspect of the fluctuations in the volume of trade. These series he arranged in 28 groups, which he classified in turn under five general heads, namely: productive activity, primary distribution, distribution to consumers, general business, and financial

CHART 11.

THE AMERICAN TELEPHONE AND TELEGRAPH COMPANY'S INDEX OF "GENERAL BUSINESS COMPARED WITH NORMAL."

By Months, 1877 to Date.



business. In working up his materials, Mr. Snyder divided the series expressed in dollars by appropriate index numbers of prices to eliminate the wild price perturbations of his period, computed secular trends and seasonal variations where necessary, expressed the cyclical-irregular fluctuations as percentages of the trends, and weighted each series according to the importance of the element which it represents in the country's trade. He did not reduce the percentage deviations from trends to multiples of their standard deviations; but averaged them as they stood. For each of his five general heads and for each of his 28 groups Mr. Snyder computed an index, which can be compared with his index for all of the series. Chart 12 gives his leading results as revised and extended to 1925.[1]

(5) Persons' "Index of Trade," United States, 1903-25.

Valuable for comparison both with the American Telephone and Telegraph Company's index, which covers so many years, and with Snyder's index, which covers so many processes, is the "Index of Trade" which Warren Persons made in 1923. This series is

> designed to give a view of the combined fluctuations of trade, transportation, manufacturing activity and industrial employment in the United States, month by month, since 1903.

[1] Carl Snyder, "A New Index of the Volume of Trade," *Journal of the American Statistical Association*, December, 1923, vol. xviii, pp. 949–963, and "The Revised Index of the Volume of Trade," the same, September, 1925, vol. xx, pp. 397–404.

The 28 series or groups of series included in the revised form of the index and their weights are as follows:

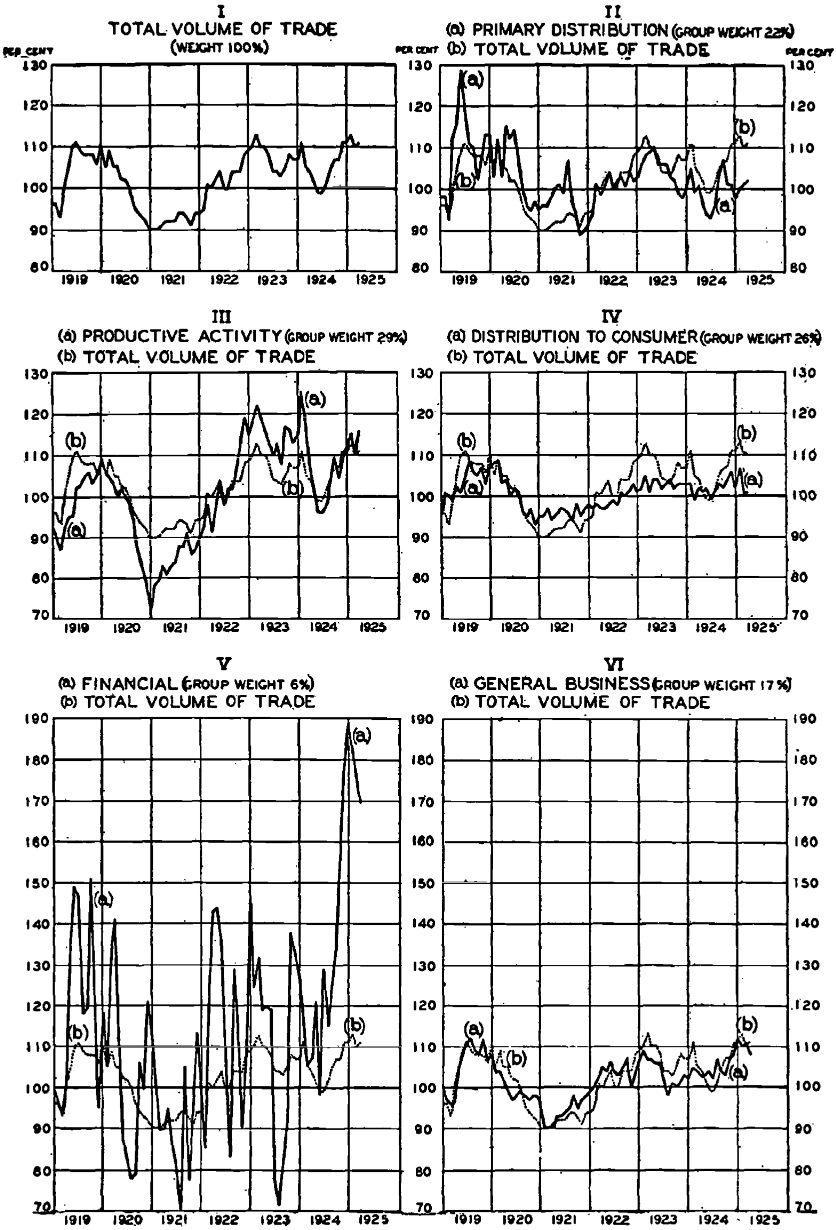
	Weights		Weights
Productive Activity		Distribution to Consumers	
Producers' goods	9%	Department store sales	8%
Consumers' goods	8	Chain store sales	3
Employment	6	Chain grocery sales	6
Motor vehicles	2	Mail order sales	3
Building permits	4	Life insurance written	2
		Real estate transfers	2
	29	Advertising	2
Primary Distribution			
Merchandise car loadings	5		26
Other car loadings	2		
Wholesale trade	8	Financial Business	
Exports	3	New securities issued	2
Imports	2	Stock sales	2
Cereal exports	1	Grain sales	1
Panama Canal	1	Cotton sales	1
	22		6
General Business			
Debits outside N. Y	8		100
Debits in N. Y	5		
Postal receipts	1		
Electrical power	2		
Series not published	1		
	17		

Since this chapter was written, Mr. Snyder has assembled in a book his various studies in this field: *Business Cycles and Business Measurements*, New York, 1927.

CHART 12.

Carl Snyder's Index of the Volume of Trade

United States, by Months, 1919–April, 1925.



The technical methods employed are modifications of those used in constructing the three-curve "Index of General Business Conditions." Changes in the materials available for different parts of his period, and changes in economic conditions led Persons to make his index in three overlapping segments, 1903-15, 1915-19, and 1919-23. Before the war Persons thought it safe to use materials containing a price factor (outside clearings and values of imports); but from 1915 onward he excluded all series expressed in dollars.[1] Chart 13 shows his results.

(6) Miss Thomas' Quarterly Index of "British Business Cycles," 1855-1914.

Indexes of business cycles similar in character to the American series reviewed above are available for Great Britain and Germany.

The British index made by Dr. Dorothy Swaine Thomas, is a mean of percentage deviations from secular trends corrected for seasonal variations and expressed in units of standard deviations. The number of series included rises from two in 1855-80 to six in 1887-96, and seven in 1897 to 1914. Chart 14 shows the quarterly averages for the full 60 years covered.[1]

[1] Warren M. Persons, "An Index of Trade for the United States," *Review of Economic Statistics*, April, 1923, Preliminary vol. v, pp. 71–78.

The materials and methods used were as follows:

	Series used	Weights	Character of average
1903–15:	Bank clearings outside New York Imports of merchandise Gross earnings of leading railroads Production of pig iron Industrial employment		Simple arithmetic means of the percentage deviations from ordinates of secular trend corrected for seasonal variations, expressed in terms of standard deviations. The averages were multiplied by 8.62, the standard deviation of outside clearings.
1915–19:	Net ton-miles of freight carried by railroads	2	Weighted arithmetic means of percentage deviations from linear trends corrected for seasonal variations.
	Production of pig iron	1	
	Raw cotton consumed	1	
	Industrial employment	2	
1919–23:	Total railroad car loadings	6	Weighted arithmetic means of percentage deviations from linear trends corrected for seasonal variations.
	Production of pig iron	1	
	Production of steel ingots	1	
	Raw cotton consumed	1	
	Industrial employment	3	

[1] The series used are as follows:

Value of total exports of British produce, average of monthly items, 1855-1914.
Percentages of iron founders unemployed, inverted, 1855-1914.
Railway freight traffic receipts, 1881-1914.

(Continued on p. 302.)

CHART 13.

PERSONS' INDEX OF TRADE.

United States, by Months, 1903-26.

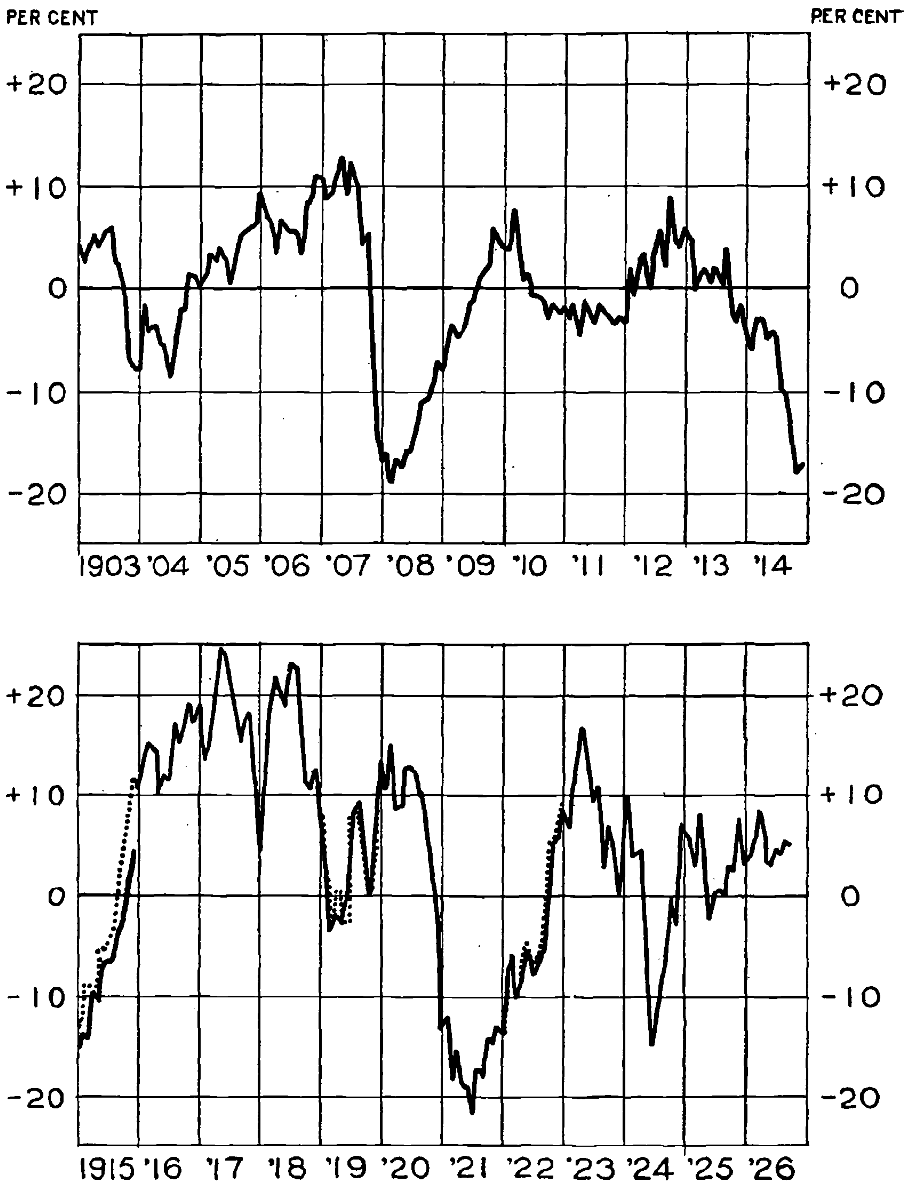
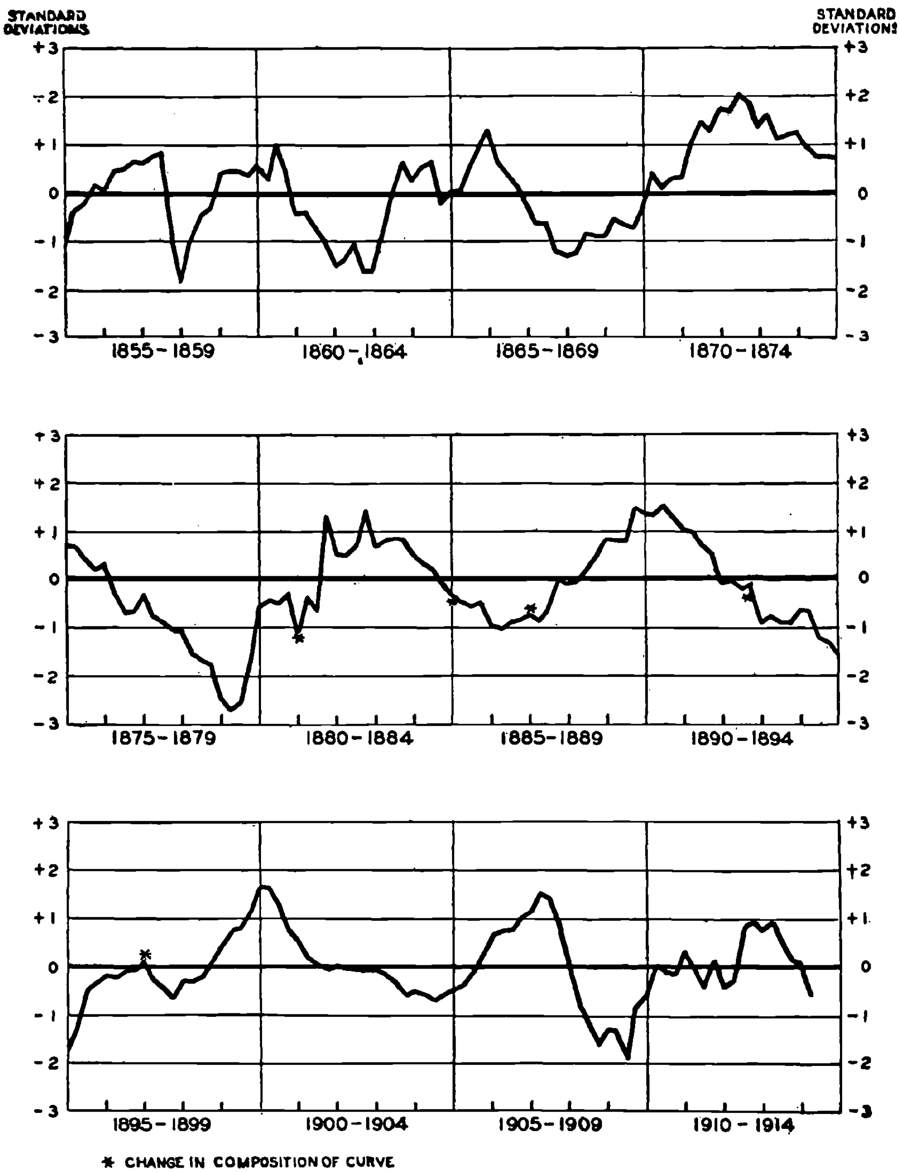


CHART 14.

MISS THOMAS' INDEX OF "BRITISH BUSINESS CYCLES."

By Quarters, 1855 to June, 1914.



(7) Axe and Flinn's "Index of General Business Conditions for Germany, 1898-1914."

This is a three-curve index, covering the years 1898 to June, 1914, bi-monthly, corresponding in character to Persons' "Index of General Business Conditions in the United States" (number 2 above). It was made by Emerson Wirt Axe and Harold M. Flinn from nine series chosen to represent the fluctuations of "speculation," "business," and "money." The results are averages of percentage deviations from secular trends corrected for seasonal variations and expressed in units of standard deviations. In time sequence the speculation index precedes the business index, and the latter precedes the money-market index—relations corresponding to those found by Persons in America.[1] The three curves are shown in Chart 15.

(8) Annual Indexes of Business Cycles.

In addition to the monthly, bi-monthly, and quarterly indexes of business cycles made by averaging the cyclical-irregular fluctuations of more or less numerous series, several indexes have been made by similar methods from annual data. The most valuable of these series are:

Sauerbeck's wholesale price index, "all materials," 1885-1914.
Provincial bank clearings, Manchester and Birmingham, 1887-1914.
Percentage unemployed,—"all trades," 1887-1914.
Blast furnaces in blast, averages of monthly items, 1897-1914.

Dr. Thomas kindly put her quarterly index at the disposal of the National Bureau in advance of its publication. ("An Index of British Business Cycles," *Journal of the American Statistical Association,* March, 1926, vol. xxi, pp. 60-63). A chart of all the seven series which compose the index forms the frontispiece of her *Social Aspects of the Business Cycle,* London, 1925.

Another "Index of British Economic Conditions," covering the years 1902-14 by quarters, has been made by W. M. Persons, Norman J. Silberling, and W. A. Berridge. It is a three-curve construction, corresponding to number 2 above and to number 7 below. See *Review of Economic Statistics,* Preliminary vol. iv, Supplement, June, 1922, pp. 158-189.

[1] Emerson Wirt Axe and Harold M. Flinn, "An Index of General Business Conditions for Germany, 1898-1914," *Review of Economic Statistics,* October, 1925, vol. vii, pp. 263-287.

The series used in making the three curves of the index are:

Speculation:	Industrial stock prices.
Business:	10-commodity price index, averaged with a volume of business index, made from the following series:
	Pig-iron production.
	Domestic-bill tax receipts.
	Value of merchandise exports.
	Value of merchandise imports.
Money:	Berlin open-market discount rates.
	Discounts and advances of the Reichsbank.
	Bond prices.

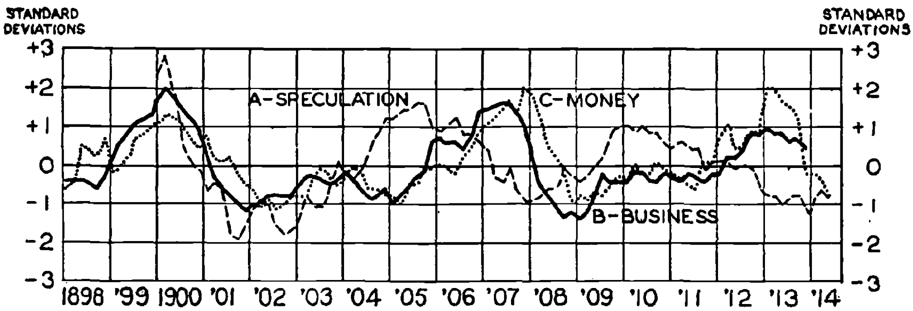
CHART 15.

AXE AND FLINN'S "INDEX OF GENERAL BUSINESS CONDITIONS FOR GERMANY."

I.

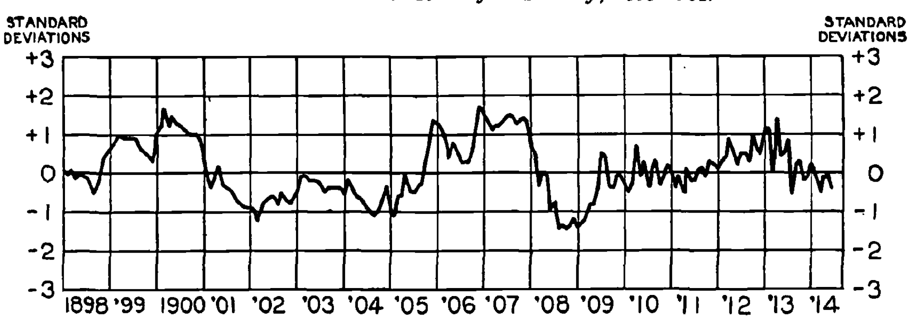
The Adjusted Index of German Economic Conditions, A—Speculation, B—Business, and C—Money: Bi-Monthly, 1898-1914.

(Cycles).



II.

Volume of Business Activity: Monthly, 1898-1914.



An index of business cycles in the United States, 1870-1920, compiled by William F. Ogburn and Dorothy S. Thomas. Based upon 9 series.

An index of business cycles in Great Britain, 1854-1913, compiled by Dorothy S. Thomas. Based upon 7 series.

"Industrial Composites" for Great Britain, Germany and Italy, 1870-1913, compiled by Harry Jerome. Based upon 5, 4 and 2 series respectively.[1]

[1] For detailed methods of construction and results see Ogburn and Thomas, "The Influence of the Business Cycle on Certain Social Conditions," *Journal of the American Statistical Association,* September, 1922, vol. xviii, pp. 324-340. Reprinted, without the tables, in the book next cited, pp. 53-77. Thomas, *Social Aspects of the Business Cycle,* London, 1925, pp. 12-19, and 166-188. Jerome, *Migration and Business Cycles,* National Bureau of Economic Research, New York, 1926, p. 175.

While annual indexes are well adapted to certain uses, and while they may be employed in business-cycle work when data for briefer intervals cannot be had, they frequently give misleading indications of the trend of developments. When business activity declines rapidly in one year from a high to a low point, and recuperates less rapidly in the next year, annual averages may betray an investigator into thinking of the first year as fairly prosperous and the second year as very dull. Thus cyclical fluctuations get not merely obscured but actually distorted in the annual indexes. And of course no satisfactory measurement of periods so short as business cycles can be made with a unit so large as 12 months. Hence the annual business indexes may be passed by without more ado.

(9) Snyder's "Clearings Index of Business," United States, 1875-1924.

There remain the "single-factor" indexes, that is, the indexes made by treating some time series showing the fluctuations of an economic process which is affected by so many types of business activity as to become itself more or less representative of the general trend. The statistical case for accepting such an index has been presented by Mr. Carl Snyder.

After making his comprehensive "Index of the Physical Volume of Trade" in 1919-23 (number 4 above), Snyder sought some method of getting comparable results for earlier years. He thought that the record of bank clearings outside of New York, which can be followed by months back to 1875, might serve as a basis, provided he could eliminate the influence of price fluctuations. Of course the volume of clearings is affected by changes in the prices of all the goods men pay for in checks—commodities at wholesale and retail, securities, real estate, labor, and so on. For the chief categories of prices, Mr. Snyder could find, or make, fairly good index numbers. The problem was how to weight the several indexes so that they would yield an index of "the general price level," which could be used to transform clearings into a record of changes in the physical volume of goods exchanged. His criterion was that the best scheme of weights would be the one which, when applied to "outside debits," would yield the curve agreeing most closely with his comprehensive index of trade in 1919-23. After much experimenting, he concluded that the best results were given by weighting his index of commodity prices at wholesale by 2, his "composite of wage payments" by 3.5, his series showing "elements in the cost of living" by 3.5, his index of rents by 1, and by omitting security prices.[1]

Finding that "outside debits" divided by an "index of the general price level," made on this plan, gave a curve which agreed closely with the "index of the volume of trade" in 1919-23, Mr. Snyder felt justified in switching from "outside debits" to "outside clearings," and carrying his computations backward for 45 years. The construction of the index of the general price level by months for so long a period involved a bold use of scanty data. Even wholesale price indexes were not then available on a monthly basis before 1900, and the records of living costs, wages, salaries, and rents are meager indeed. But Mr. Snyder drove through his computations with the best materials he could gather, interpolated freely, and published his index for every month from 1875 to 1924.[2]

With his price index in hand, Snyder compiled the record of outside clearings from *The Public* and the *Commercial and Financial Chronicle,* and divided the total for each month by the corresponding item in the price index. To the resulting series, which he interpreted as showing fluctuations in the physical volume of trade, he fitted a parabolic trend. The percentage deviations from the trend, corrected for seasonal variations, and smoothed by a three-months moving average, constitute the "Clearings Index of Business," which is presented in Chart 16.[3]

(10) Frickey's Index of Outside Clearings, United States, 1875-1914.

On the same chart is presented a second index of outside clearings made upon a different plan by Mr. Edwin Frickey. Believing that, in the period before 1903, the introduction of new clearing houses at irregular dates appreciably distorted the series, Frickey decided to base his index upon the clearings in seven cities for which substantially complete figures can be had monthly since 1875, and which in the test period 1903-14 give totals agreeing closely in their cyclical-irregular fluctuations with the aggregates for all clearing houses outside of New York. The cities selected are Baltimore, Chicago, Cincinnati, Cleveland, Philadelphia, Pittsburgh, and San Francisco.

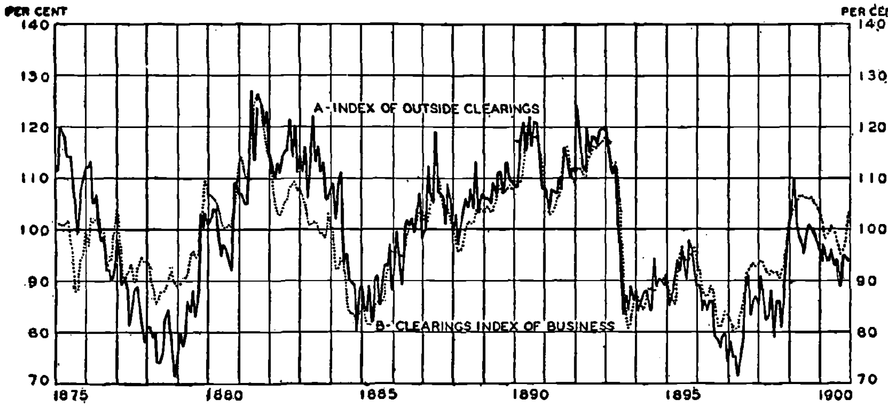
[1] It may be noted that these weights differ widely from those which our data concerning the volume of wholesale and retail trade, and wage payments would suggest. See Chapter II, section v.

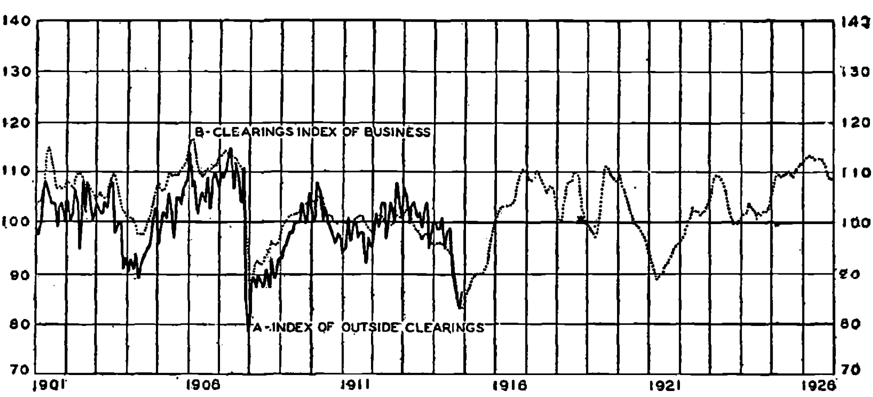
[2] Carl Snyder, "A New Index of the General Price Level from 1875," *Journal of the American Statistical Association,* June, 1924, vol. xix, pp. 189-195.

[3] Carl Snyder, "A New Clearings Index of Business for Fifty Years," *Journal of the American Statistical Association,* September, 1924, vol. xix, pp. 329-335.

CHART 16.

SNYDER'S "CLEARINGS INDEX OF BUSINESS" AND FRICKEY'S "INDEX OF OUTSIDE CLEARINGS." United States, by Months, 1875-1926 and 1875-1914.





A—Frickey's "Index of Outside Clearings" (Aggregate Bank Clearings for 7 Selected Cities): Relatives to ordinates of trend, adjusted for seasonal variations.

B—Snyder's "Clearings Index of Business" (Bank Clearings Outside New York City, 1875-1918; Debits Outside New York City, 1919-1926): Three-months' moving averages of relatives to ordinates of trend, adjusted for seasonal variations and price changes.

* Change in composition of index.

Casting up their total clearings each month, Mr. Frickey fitted a line of secular trend, made corrections for seasonal variations, and found the percentage deviations from the moving base thus obtained. Unlike Mr. Snyder, he attempted no correction for changes in prices; but broke off his index in 1914, just before the great price revolution began.[1]

(11) Snyder's "Index of Deposits Activity," United States, 1875-1924.

From data collected by the Federal Reserve Bank of New York, Dr. W. Randolph Burgess showed that the rate of turnover of bank deposits in eight reporting cities is subject to a wide cyclical swing.[1] A comparison of the individual deposits in about 760 banks belonging to the Federal Reserve System with clearings in 141 leading centers confirmed this conclusion. Further, the fluctuations in the turnover rate of the deposits in these banks, duly adjusted, agreed closely with the fluctuations of Snyder's comprehensive index of the physical volume of trade in 1919-23.

These facts suggested to Mr. Snyder that the velocity of bank deposits constitutes another index of business conditions, an index which he determined to carry back by months to 1875 as a check upon his Clearings Index. As data Snyder used the total bank clearings of the country, and the individual deposits of all National Banks. The latter figures are given at slightly irregular intervals five (now four) times a year. Resorting to interpolation, Mr. Snyder made up a table of individual deposits by months, divided these figures into monthly clearings, used seven-year moving averages as a trend, found the percentage deviations of his ratios from this trend, corrected the percentages for seasonal variations, and finally smoothed his curve by three-months moving averages. Thus he got an "Index of Deposits Activity," which he thought showed a gratifying correspondence with his Clearings Index. The two curves are compared in Chart 17.[2]

2. A Critique of the Indexes of Business Conditions.

Before making use of the indexes assembled in the preceding section, we must examine them critically.

[1] Edwin Frickey, "Bank Clearings Outside New York City, 1875-1914," *Review of Economic Statistics*, October, 1925, vol. vii, pp. 252-262.

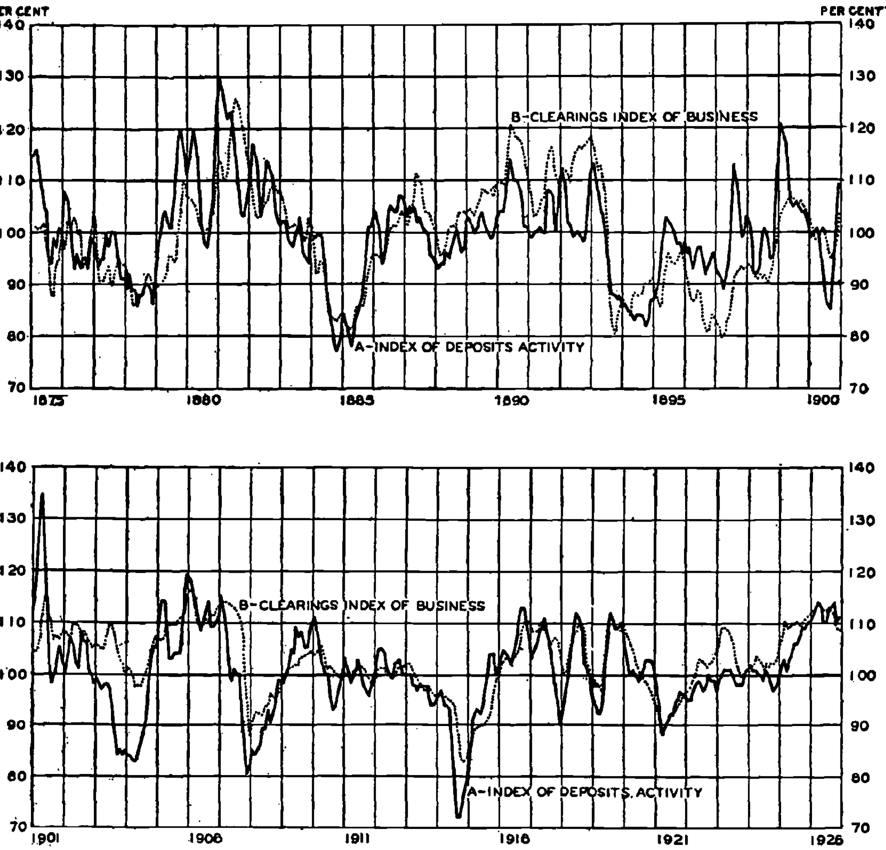
[1] See above, Chapter II, section iv, 4.

[2] Carl Snyder, "A New Index of Business Activity," *Journal of the American Statistical Association*, March, 1924, vol. xix, pp. 36-41, and "Deposits Activity as a Measure of Business Activity," *Review of Economic Statistics*, October, 1924, vol. vi, pp. 253-259.

CHART 17.

SNYDER'S "INDEX OF DEPOSITS ACTIVITY" COMPARED WITH HIS "CLEARINGS INDEX OF BUSINESS."

United States, by Months, 1875-1926.



A—Index of Deposits Activity: three-months' moving averages adjusted for seasonal variations and moving average trend.

B—Clearings Index of Business (bank clearings outside New York City, 1875-1918; debits outside New York City, 1919-1926): three-months' moving averages of deviations from trend, adjusted for seasonal variations and price changes.

At the outset, it should be noted that none of the indexes give, or are meant to give, an adequate picture of business cycles. For, as has been said several times, business cycles are congeries of cyclical fluctuations in a large number of economic activities, fluctuations which differ widely in amplitude and considerably in timing. Such intricate phenomena cannot be presented adequately by any simple device. A real chart of one business cycle would be a hopelessly complex tangle of hundreds of curves. Doubtless the shading produced in such a chart by the concentration of lines in certain areas and their dispersion in others would give an interesting total effect. A faint impression of this effect may be gathered by plotting on a large scale for one cycle 27 of the series, or groups of series, which Carl Snyder uses in the construction of his volume-of-trade index.

Anyone who dwells upon the intricacies of Chart 18, made in this way, will grant not only that the business indexes fail to picture business cycles, but also that faithful pictures would be of doubtful value. In dealing with price fluctuations we have learned to use index numbers which represent in a single time series the net resultants of very many dissimilar changes. These price indexes are far from adequate to show all we need to know about price fluctuations; but they are an indispensable tool even to those investigators who are beginning to go back of them in order to study the successive arrays of price changes which the index numbers condense into a single set of averages. Business fluctuations are far more complicated than price fluctuations; for they include the latter as just one strand interwoven with fluctuations in employment, incomes, consumption, production, transportation, commerce, and finance. Precisely because the full facts are so complicated, we need a device, or devices, for showing simply the general drift of all the changes. Such "indexes of general business conditions" may mislead us, of course, just as price indexes may mislead the unwary; but they may also prove a most useful instrument for gaining clear insights, if we remember their limitations. They will not enable us to dispense with more elaborate studies of the interrelationships among particular series, but they should supplement and summarize what we can learn by more intensive analysis.

Do the existing indexes meet our need? Do they show the general drift of business cycles accurately? How can they be improved?

It may seem that the neatest way to treat indexes of business conditions is to define the purpose they serve, to show what materials and methods are appropriate to that purpose, and to evaluate

CHART 18.

ONE BUSINESS CYCLE, UNITED STATES, 1919-21, SHOWN BY A PLOT OF 27 OF ITS COMPONENTS.

Twelve of the Components Used in Snyder's Volume-of-Trade Index to Represent *Productive Activity* and *Primary Distribution.*

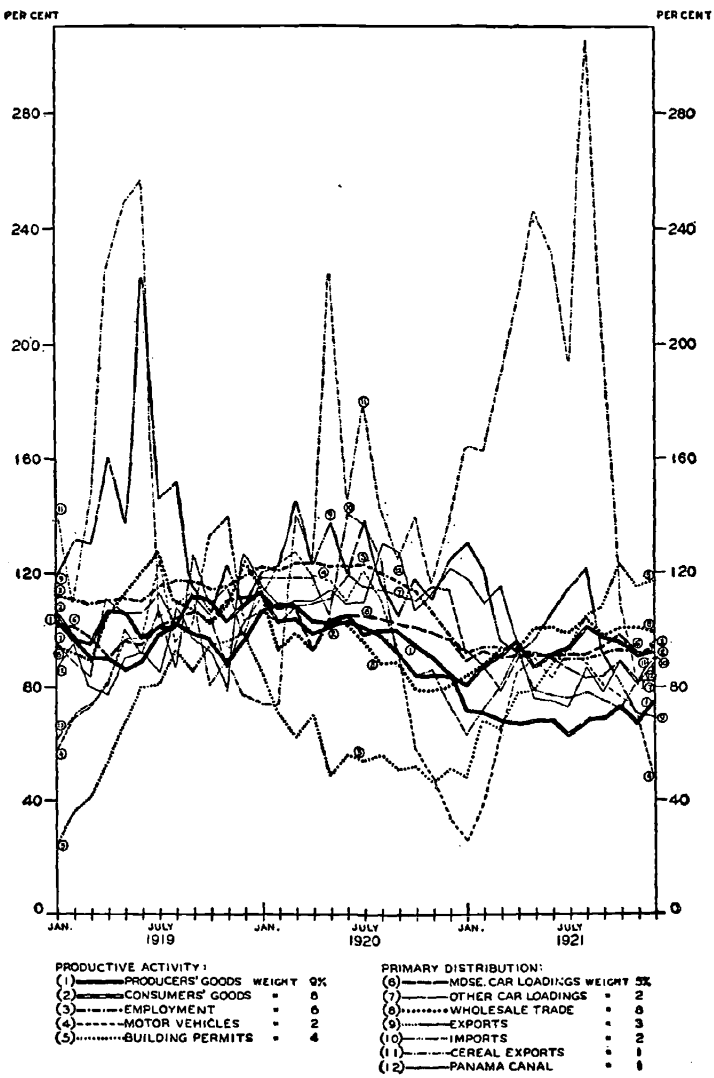
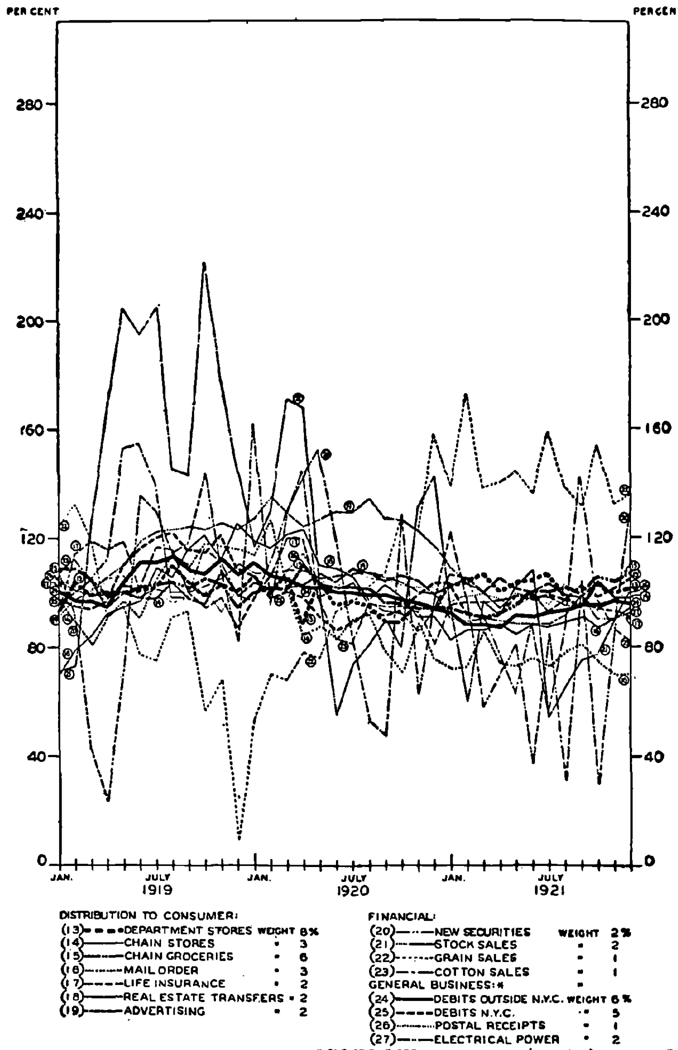


CHART 18 (*Continued*).

ONE BUSINESS CYCLE, UNITED STATES, 1919-1921, SHOWN BY A PLOT OF 27 OF ITS COMPONENTS.

Fifteen of the Components Used in Snyder's Volume-of-Trade Index to Represent *Distribution to Consumers, Finance,* and *General Business.*



the existing indexes by the canons thus provided. But that procedure assumes some single purpose to be served by business-cycle indexes, a purpose so definite that it dictates an ideal set of materials and an ideal set of methods. This assumption is no more valid in our field than in the better-explored field of wholesale-price indexes. Certainly the purposes for which the statisticians whose indexes we have reviewed designed their series were not all the same, and certainly there is a use to which every one of their series can be put.

We can accomplish more by reversing the procedure. Instead of starting with some single purpose to be served and criticizing the existing indexes for their ill adaptation to this use, we shall start with the materials and methods employed in making the existing indexes, and consider what the results mean and to what uses they may be put. Constructive suggestions of practical value in future work are more likely to come out of a discussion which regards divergent purposes and limited data than out of a speculation spun from some definition.

Round the technique of making index numbers of prices an extensive literature has grown up. This literature warns us that we are entering a field where none but statistical experts feel comfortable. On the making of business indexes there has been but little critical discussion so far, unless we put under that head the preliminary processes of eliminating secular trends and seasonal variations. Our problem stands now much as the price-index problem stood in Jevons' youth. Having little to guide me, I cannot go far, and I may readily go wrong.

The chief conclusion to which analysis leads is that the various methods used in making indexes of business conditions produce, not rival measures of the same variable, but indexes of different variables. If we can become clear just what the variable is which each index relates to, we can tell for what uses the several indexes are adapted.

(1) Indexes of the Pecuniary Volume of Business.

Of the variables to which the indexes relate, the most comprehensive, and yet the simplest, is the pecuniary volume of business transactions. We conceive of it as the aggregate made up of all the values exchanged in a country month by month—not merely commodities in the usual sense, but also real estate, securities, funds loaned and repaid, interest, rents, services of all sorts from manual labor to professional advice, transportation, storage, publicity—to repeat, every good exchanged for a price, counted at every exchange.

The only *measure* of this variable is its own aggregate value. In the United States we now have data which enable us to fix the order of magnitude attained by the dollar volume of payments, and even to venture rough estimates of how many billions it comes to. Such estimates, indeed, have already been given on an annual basis in Chapter II.[1]

To these rough estimates we might add a monthly *index* of changes in the dollar volume of payments, built up from series showing bank debits, retail sales, payroll disbursements, amusement receipts, and the like. We could argue that such series, when properly selected and combined with an eye to their relative importance and to overlaps, give partial totals whose monthly variations probably represent approximately the variations in the aggregate volume.

If we felt sufficient confidence in the figures to elaborate still further, we might tie the monthly index to the annual estimates of the aggregate volume of payments, and say that we were approximating the monthly aggregates. Finally, we might eliminate the secular trend and seasonal variations of our series, if we had data for a sufficient period, and present the results in billions of dollars or in percentages, as showing the cyclical-irregular fluctuations of the pecuniary volume of transactions.

Only one of the business indexes reviewed above relates directly to this variable—Mr. Frickey's series for bank clearings in seven American cities from 1875 to 1914. In technical parlance, however, Frickey's results are not index numbers, but a series of relatives computed on a moving base (each month the ordinate of secular trend, adjusted for seasonal variation, equals 100). Mr. Frickey calls them "a continuous monthly index of business activity as indicated by the fluctuations of bank clearings." [2] The only form of business activity which bank clearings show directly, is the amount of payments made.

Doubtless, this partial record of payments is the best single indicator of the pecuniary volume of trade for the years before bank debits were compiled. But we are not sure whether bank clearings,

[1] See Table 9 above, Chapter II, section v, 4, "The Flow of Payments Among Business Enterprises."

[2] Edwin Frickey, "Bank Clearings Outside New York City, 1875-1914," *Review of Economic Statistics,* October, 1925, vol. vii, p. 252.

even with New York City omitted, give a just impression of the amplitude of the cyclical-irregular fluctuations in the aggregate of payments. For the fluctuations of clearings probably have a larger amplitude than the fluctuations of payments in coin and paper money. And it must be recalled that the volume of payments made does not parallel closely the dollar volume of goods exchanged day by day. Many of the checks cleared are "cash" payments for goods just bought; a few are advance payments for goods to be received in the future, but the majority are payments for goods received some time in the past.[3] Much business is done three times over so to speak, advance orders are placed, or contracts for future performances are made; somewhat later the goods ordered are shipped or the contract work is performed; later still the payments are made. When borrowed funds are used, the scheme is still more complicated, and the pecuniary volume of exchanging growing out of a given operation is enhanced by the making and repaying of loans. We have little information about the volume of the several types of operations, or about the average lags of deliveries behind orders, and of payments behind deliveries. An ideally complete index of pecuniary transactions would have at least three curves, one showing commitments entered into each month, one showing current transfers of goods in dollar values, and one showing payments. The third curve is the only one we can draw with any confidence. But it is important to remember that the two curves we cannot draw represent actual processes of great moment, which certainly differ in timing, and probably differ in amplitude of fluctuation from the process typified by bank clearings.

The reason why few investigators have dealt with the pecuniary volume of trade must be that they have believed other aspects of business activity to be more significant. Almost all of the business indexes have been compiled since the price revolution of the war produced violent oscillations in the pecuniary volume of trade. Even Mr. Frickey, it will be noticed, did not think it worth while to continue his series beyond 1914. While economists have been deeply interested in measuring price changes, they have sought to measure the reaction of prices, not on the amount of exchanging done in dollars, but on the physical volume of goods exchanged.

[3] Compare Chapter II, section iv, 5, above; "The Quantity Theory and Business Cycles."

(2) Indexes of the Physical Volume of Trade or of Production.

Like the pecuniary volume of trade, the physical volume of trade is a definite quantity. It consists of all the goods exchanged in a country, during specified intervals, reckoned in physical units—hours of labor, cubic feet of buildings occupied under leases, numbers of securities transferred, ton miles of freight carried one mile, lines of advertising printed, and so on. Conceivably one might make an inventory of the goods exchanged each month, expressed in such units. But to make from the inventories totals which could be compared month by month, it is necessary to assign each good a money price. If the prices are kept fixed month after month, the aggregates, though expressed in billions of dollars, will show changes in the physical volume of trade weighted by dollar values.

While we lack data to approximate even roughly the physical inventories of goods exchanged annually in the United States, we might build up from series now published an aggregative index of the physical volume of trade which would possess much interest. Such an index would correspond in character to the U. S. Bureau of Labor Statistics index number of wholesale prices. That price index shows changing prices weighted by fixed physical quantities; the volume-of-trade index would show changing physical quantities weighted by fixed prices. With this series in hand, we might eliminate the secular trend and seasonal variations (if any), and thus get an index of the cyclical-irregular fluctuations in the physical volume of trade, expressed in dollars, or converted into ratios of the adjusted trend values.

The current indexes of the cyclical-irregular fluctuations in the physical volume of trade are not made on this simple plan. Instead of starting with an index of the volume of trade and ascertaining its trend, the compilers ascertain the trend of each time series separately, turn the original data into relatives of their trends, weight these relatives, and compute their arithmetic means. Snyder's index of the volume of trade in 1919-25, and the two later segments of Persons' index of the volume of trade (the segments for 1915-19, and 1919-23) are made in this way. Thus they are not quite what their names suggest, indexes of the volume of trade, but weighted arithmetic means of the percentage deviations of certain time series from their adjusted trends. Their analogues among price indexes would be a new type:—weighted arithmetic means of relative prices, computed on the base: ordinates of secular trends, corrected for seasonal variations, equal 100.

It is now generally recognized that weighted arithmetic means of relatives are less desirable price indexes for most uses than relatives made from aggregates of actual prices weighted by physical quantities.[1] If we wish to make an index of changes in the physical volume of trade, we can apply this conclusion with confidence, and say that the aggregative form of index is more serviceable, on the whole, than the weighted mean of relative quantities. But does it follow that an index of the cyclical-irregular fluctuations in the volume of trade is better made by the aggregative method than by weighting and averaging the relatives of the constituent series?

It is clear from their formulas that the two methods do not yield identical results.[2] In seeking to choose between them we get

[1] Of course, the averages of relatives have their advantages; but I think the conclusion stated in the text is valid.

On the relative merits of different forms of price indexes, see Irving Fisher, *The Making of Index Numbers*, Boston and New York, 1922, and Truman L. Kelley, *Statistical Method*, New York, 1924, chapter xiii. A summary of the chief findings is given by Frederick C. Mills, *Statistical Methods*, New York, 1924, pp. 207-221.

[2] The formula for an index of deviations from a secular trend made by the aggregative method, may be written:

$$\frac{\Sigma q_1\, p_w}{T_1} \qquad (1)$$

Here q_1 represents the quantities of the goods exchanged in time "1"; p_w represents the fixed prices used as weights, and T_1 represents the adjusted ordinate of the secular trend of $\Sigma q\, p_w$ in time 1. We may regard this trend as itself the aggregate of the trends of the constituent series, each weighted by its appropriate price, provided that the trends fitted to all the constituent series and to the aggregate are straight lines, or (perhaps) if some single type of curve other than a straight line is fitted throughout. Then the formula becomes:

$$\frac{\Sigma q_1\, p_w}{\Sigma t_1\, p_w} \qquad (2)$$

The formula for a similar index made by averaging relatives weighted by fixed values (v_w) is:

$$\frac{\Sigma\left(\frac{q_1}{t_1}\right) v_w}{\Sigma v_w} \qquad (3)$$

But the values used as weights are products of quantities and prices. Therefore we may replace v_w by $q_w p_w$ and write:

$$\frac{\Sigma\left(\frac{q_1}{t_1}\right) q_w p_w}{\Sigma q_w p_w} \qquad (4)$$

If $q_w = t_1$, formula (4) reduces to formula (1). That is, the two methods yield identical results only when the quantities used in the value-weights applied to the relalittle help from the discussions of price indexes, for they deal with the problem of comparing the relative values of the same variables in different periods, and our problem is to compare two values of the same variable at the same periods—the values shown by averaging deviations from the adjusted trends of the individual series and the values shown by computing deviations from the adjusted trend of the group aggregate.[3] But by falling back upon broader considerations we can justify a choice. Ease of computation, and, what is more important, ease of comprehension, speak for the aggregative method. That method requires the computation of but one trend and one set of seasonal variations, while the method of averaging relatives requires the computation of as many trends and sets of seasonals as there are component series. An aggregative index of physical volume of trade is relatively easy to conceive; so are its trend, its seasonal variations, and the deviations from its trend adjusted for seasonals. When one tries to think what reality is represented by a weighted average of many sets of deviations made in this way, the conception grows more complicated. Finally, we have need of an index of the physical volume of trade, as well as need of an index of its cyclical-irregular fluctuations. The method of aggregates gives us the first index as well as the second. The average of relatives gives us the second index, but not the first, though in compensation it offers the cyclical-irregular fluctuations of all the component series, for which we can find use. An ideal procedure would be to deal with our quantity series as the federal Bureau of Labor Statistics deals with price series—make the general index from weighted aggregates, but also publish relatives for each series separately, and, of course, add indexes

tives equal the adjusted ordinate of secular trend at the time for which the computation is made. This coincidence may well happen once in a period for which two indexes are made from identical materials by the two methods; but it is most unlikely to happen twice.

When the trend-lines used do not make the ordinates of the trend of the aggregate equal to the sum of the ordinates of the trends of the constituent series, there is no assurance that the two methods will give identical results. This is the commoner case in statistical practice. But the differences between the two sets of results may be small.

[3] For example, in our problem the time bias of arithmetic means of relative prices and of various systems of weighting does not enter. Nor is the "circular test" applicable in the usual way, if at all. The "factor reversal" test can be used, but it is inconclusive, because neither method can pass it. Kelley's test of reliability is pertinent: one might divide a body of data into two samples, compute indexes for both samples by both methods, and see which pair of indexes showed the higher coefficient of correlation. But the test would be laborious, and trial with one set of data might not prove conclusive.

Mathematical statisticians have in this problem a promising field for work.

of the cyclical-irregular deviations from adjusted trends both for the aggregates and for all the components.[4]

The practice of expressing cyclical-irregular fluctuations in terms of their standard deviations is not proper in making volume-of-trade indexes. For differences in the amplitude of these variations are present in the inventories to which such indexes relate. To cover up the differences by reducing the fluctuations of all the series used to a common scale, is to make measurement of changes in the aggregate volume impossible. A series constructed in this way may have some interest as indicating the time at which cyclical-irregular fluctuations have occurred, and as indicating the relative amplitude of successive cycles as pictured by the series in question, so long as it is made of uniform materials; but it is of little value for comparing the amplitude of fluctuations in the physical volume of trade with that of fluctuations in production, employment, prices, or any other variable.

To get reliable indexes of the physical volume of trade and of its cyclical-irregular fluctuations we need representative samples drawn from all parts of the field. Even for recent years in the United States, the materials are not ample. The trading done by farmers is most inadequately covered. There is reason to suspect that the best

The only practical test of the agreement between the two methods of which I know is the following:

In making his "index of physical production for all manufacture" in the United States by years since 1899, Dean Edmund E. Day started with 33 time series showing the output of various types of goods. These series he arranged in 10 industrial groups.

(1) For each group he made an "unadjusted index" by reducing the annual items of the component series to relatives on the base, production in 1909 equals 100, weighting these relatives, computing geometric means, and making certain corrections on the basis of Census data which do not now concern us. Next he weighted the 10 group indexes, and used their geometric means as an "unadjusted index of physical production for manufacture."

(2) To measure the cyclical fluctuations of manufacturing, Day returned to his 33 original series, fitted a trend line to each, expressed the actual figures as percentages of the corresponding trend values, weighted these relatives, computed averages (this time arithmetic) for each of the 10 industries, then weighted these group averages, and finally took their arithmetic means as his "adjusted index."

Day found two objections to the latter procedure. The determination of the lines of secular trend for the 33 original series involved difficult choices of period and line. The computations were laborious. For both reasons he sought a simpler method.

(3) This method was to fit a trend line directly to the "unadjusted index," and turn its annual values into percentages of the corresponding ordinates of this trend. The differences between the results of methods (2) and (3) in the 21 years 1899-1919 did not exceed three points in the percentage scale in any year. Dean Day concluded that the "case for the simpler method of getting the adjusted index is conclusive." I should be inclined to argue that the simpler method is preferable on logical grounds, as well as on practical grounds.

See Edmund E. Day, "An Index of the Physical Volume of Production," *Review of Economic Statistics*, November and December, 1920, Preliminary vol. ii, pp. 310, 311, 332-337, 362-365.

totals we can now make show variations somewhat in excess of the truth; for the types of trading not represented at all, particularly the retail sales of the great mass of small, independent shops, are probably rather steady. By long odds the most comprehensive index we have is Snyder's series for 1919-25.

When compilers go back to pre-war years, they are forced to resort to one of two undesirable shifts. Either they must change the composition of their indexes from time to time, accepting less satisfactory and smaller samples as they work backward, or they must use materials which do not show the physical volume of trade. Mr. Snyder chose the latter alternative in making his "clearings index of business." Instead of treating bank clearings outside of New York City as a sample of fluctuations in the pecuniary volume of transactions, Snyder sought to transmute these dollars figures into a physical-volume index. Of course, it is always questionable how accurately division of such data by a price index really shows the corresponding changes in physical quantities. Mr. Snyder was able to test his procedure more adequately than is often possible in such cases, by making sure that his price index when applied to outside bank debits in 1919-23 gave results which fitted closely his physical-volume index. Yet a doubt remains whether a "deflating series" adjusted to bank debits in the period of violent price fluctuations just after the war is well adapted to deflating bank clearings for distant years, when prices were relatively stable. And at best, clearings is only one sample of volume of trade—though the best single sample.[5]

What has been said about methods of making indexes of the physical volume of trade and its cyclical-irregular fluctuations applies also to indexes of physical production. Production is not limited to the output of tangible commodities; it includes such services as fabrication, transportation, storage, and distribution, which suggests that production indexes should be made on a "value-added" basis. For example, if we have data showing retail sales of shoes in pairs and dollars, wholesale sales, and manufacture in the same double form, leather used by shoemakers in physical units and value, and hides and skins tanned into shoe leather, we can include the output of all these agencies in an index of physical production. Starting with the

[5] These doubts about the reliability of long-period indexes made from changing materials or by deflating clearings are shared by the compilers. It is necessary to note them, but it is also just to add that the criticisms apply to the data, and not to the men who have sought to learn all they can from what records the past provides.

pairs of shoes sold at retail, we can weight the service of the shopkeepers by the average value which they add to the wholesale price, and work back in this way to the fresh hides. Of course, to make the result show changes in physical production, we must keep constant the "values added" which we use as weights. Constructed in this way from exhaustive data, production indexes would cover most of the transactions included in the volume-of-trade indexes; but the weights of the two indexes would differ widely. For example, in a trade index the retail sales of shoes in pairs would be multiplied by the full retail price, instead of by the retailers' margin. Both indexes would run in billions of dollars, but the trade index would show many more billions. There would also be differences in timing. In a trade index the aim is to record exchanges when they are made; in a production index the aim is to record productive services when they are rendered. Manufacturers make goods before they exchange them, and merchants render their productive service to the community from the time they buy goods to the time they deliver packages to the ultimate consumer. These time relations are of great moment in the theory of business cycles.

As we should like three indexes of the pecuniary volume of trade showing commitments entered into, goods transferred, and payments made, so we should like three indexes of production in physical terms, —one showing goods ordered, one showing productive services rendered, and one showing goods delivered. To complete our modest requirements, add that these three indexes should be made to show separately for each industry the operations of retail dealers, wholesale merchants, manufacturers of consumers' and producers' goods, producers of raw materials, and builders of industrial equipment. In all this work adequate samples should be used, the aggregative method followed, and the cyclical-irregular fluctuations taken in their original amplitudes, not in units of their standard deviations.

(3) Indexes of "General Business Conditions."

Most of the indexes under review relate neither to the pecuniary nor to the physical volume of trade, but to the "general condition of business." This concept is much less definite than the others. It corresponds to no sum in dollars, to no inventory of goods, to nothing we can count. By nature it is not an aggregate amount, but a synthesis of relatives. We can say that retail sales in dollars are larger now than they were last month or last year, that the percentage of unemployment is less, that more tons of steel are being produced, railway traffic is heavier, prices are higher, "money is tighter," and so on. Or we can compare each of these variables with the standard we have made from its secular trends and seasonal variations. But when we combine these series we are not measuring any quantity, or the variations in any quantity. We are merely summarizing our observations upon the values of certain variables from time to time in terms of other values of the same variables. And these variables in their original form are largely incommensurable, or rather are divided among incommensurable groups, prices, physical quantities of many sorts, values, ratios.

Of course, there is no technical difficulty in combining or averaging time series of any sort when they are expressed as relatives, percentage deviations from trends, or multiples of standard deviations. Combinations of incommensurable series are frequently made in the business indexes. The American Telephone and Telegraph Company's index for a time included Bradstreet's price series, the production of pig iron in tons, and bank clearings outside of New York, though it is now made of more homogeneous materials. Persons also used these three series, among others, in making one of the curves in his "Index of General Business Conditions." In her British index, Miss Thomas went even further; she combined unemployment percentages with series expressed in price relatives, physical units, and pounds sterling.

In defense of this practice, it can be urged that the phenomena of business cycles include changes in prices, physical quantities, pecuniary volumes, and ratios—for example, the ratio of bank reserves to demand liabilities. Not only do these four types of variables coexist; they also act and react upon one another. When the statistician combines measurements of the changes in these variables, he is merely trying to represent in one series of figures the net resultants of changes which are genuine and which do interact. Anyone who can conceive of business cycles as congeries of fluctuations in many processes should be able to grasp what is meant by an index made from sample fluctuations. It is true that indexes confined to one type of changes, such as prices, pecuniary volume of trade, or physical production, have a more definite meaning. But what they gain in definiteness they lose in breadth. No business index deserves the term "general" unless it includes samples of the various types of business activities.

By considering the method of weighting the series used in a "general business" index, we can put this elusive problem in the clearest light. Conceivably, a logical basis for weighting even the series which belong to incommensurable groups might be found in the relation which every business series bears to the pecuniary volume of trade.[1] But do we wish such a scheme of weights? Certainly we do not make an index of general business conditions as an indirect way of showing fluctuations in the pecuniary volume of business transactions. As said above, the condition of business is not itself an aggregate of goods or of values, and the components we use in making an index do not derive their importance from the influence they exercise upon anything which can be counted. But to what other criterion can we appeal to make sure that every series we use shall have due chance to influence the results?

In practice, statisticians have not lingered long upon this question. They usually proceed promptly to turn the original data into percentage deviations from their adjusted secular trends, to express each set of deviations in units of its standard deviation, and to take arithmetic means of the series in this form. Sometimes they apply rough weights, but more often not.[2] Reduction of the several sets of deviations to multiples of their own standard deviations is itself a scheme of weighting that gives each series a chance to influence the results in inverse proportion to its variability in terms of its standard deviation; applying different weights to the multiples gives each series an

[1] Many series are fractions of that aggregate—bank debits, railway receipts, payroll disbursements, import and export values, and the like. Price data become fractions of the aggregate when multiplied by physical volume of trade; physical-volume and production data become similar fractions when multiplied by prices. A place can be made even for the ratios:—unemployment percentages, for instance, can be related to payroll disbursements, and bank-reserve ratios to the volume of financial transactions. The elaboration of a scheme of weights upon this basis would require much ingenuity, and many conjectural estimates. Not only are there gaps in the data, but there is also much overlapping to be allowed for, particularly between bank debits and other factors. Yet the task is not an impossible one.

[2] For example, for some time the American Telephone and Telegraph Company's "General Business Curve" was made from the following series and weights:

	Weights
Outside clearings	25
Pig-iron production	20
Railroad traffic	15
Failures (number)	10
Copper production	5
Cotton consumption	10
Coal production	5
Commodity prices	10
	100

influence upon the results proportioned to the investigator's rating of its relative importance. That is, the original differences among the series as components of averages are wiped out, and new differences are written in. No single criterion of the importance of the several series is set up; but statisticians easily agree upon the more important and less important business indicators in any list of series, and agree also that the rankings within these groups must be rather arbitrary. In some lists they judge that even weighting is quite as good as differential weighting.

Of course such general agreement upon schemes of weights for practical use is not a satisfactory solution of the problem how to make an index of general business conditions; but it is the nearest approach to a solution which has been worked out. The need of further methodological research at this point is pressing.

Indexes made in the way described are clearly more representative of general business conditions when they are based upon large and varied lists of series than when the lists are small or one-sided. The rule is sometimes laid down that only series in which the cyclical-irregular fluctuations are synchronous should be included. This rule is proper if the compiler wishes merely to exhibit the cyclical element in business changes; it is not proper if he wishes an index of general business conditions. For we have seen how considerably and how systematically the cyclical-irregular fluctuations of certain processes lag behind those of other processes. To exclude any series from the index merely because it differs in timing from others, and therefore blurs the cycles, is to distort the general business index in the interests of symmetry. The only way to observe the rule and remain faithful to fact is to give up the plan of making a single index, and follow Persons in making a three-, possibly a four-, or a five-curve index, including as many trustworthy series as possible, but throwing them into groups on the basis of synchronous fluctuations. It is not consonant with the aim of representing general business conditions to exclude even series which show no cyclical characteristics.

Granted that general-business indexes combining the changes in many economic activities are not irrational, the question remains, What are they good for? One use at least can be claimed for them. So long as such a series is made from uniform materials by uniform methods, it enables one to compare successive cycles with respect to duration, amplitude and the character of their several phases. The wider the range of materials included and the longer the period covered, the more significant these comparisons become. For intensive theoretical work they are of slight value. As said before, they do not measure any magnitude. And their value for the extensive work to which they are adapted is compromised by the small number and changing character of the data available for the long periods which they should cover.[3]

(4) Forecasting Sequences.

While the three-curve charts which Professor Persons and his coworkers have made for the United States, Great Britain and Germany are called "Indexes of General Business Conditions," they are made primarily as forecasting sequences. That is, the series used in each curve were chosen for the regularity with which their cyclical-irregular fluctuations are synchronized, and for the regularity with which their cyclical-irregular fluctuations precede or follow those of the series used in the other curves. Such constructions are to be judged on principles somewhat different from those laid down concerning general business indexes proper.

[3] Dr. Frederick C. Mills points out that when standard deviations are used as units in which to measure cyclical deviations from trend lines, they cannot be interpreted in the usual fashion. The standard deviation of a "normal distribution" has a precise meaning; we know what percentage of the total number of cases in such a distribution will deviate from the mean by more (or less) than any given multiple of the standard deviation; we know also what the odds are that a given deviation from the mean will be exceeded by a random observation. These precise rules of the normal distribution apply approximately to a wider variety of actual distributions. But they are frequently violated by the distribution of deviations of time series from their secular trends.

(1) A deviation equal to 6 standard deviations below the mean would occur in a normal frequency distribution once in 1,000,000,000 times. Deviations of that order occur not infrequently among deviations from a secular trend, in consequence of such disturbances as strikes, railway embargoes, wars, panics.

(2) Such extraordinary deviations are particularly common among deviations measured from projected trends. For example, in the publications of the Harvard Committee on Economic Research we find the Bureau of Labor Statistics index number of prices at wholesale represented by a positive deviation of 15.5 times the standard deviation in February, 1920, and Bradstreet's price index represented by a negative deviation of 10.6 times the standard deviation in July, 1921. Deviations reaching or exceeding 4.5 times the standard deviation are somewhat common in this valuable source. In a normal distribution, a deviation of this size occurs three times in a million.

(3) Since these extreme deviations are commoner in some series than in others, we do not quite get away from the danger of distorting our averages by using the standard deviations of the series as units.

In short, Professor Warren M. Persons' argument concerning the non-applicability of the concept of the probable error to time series seems to apply to the use of the standard deviation for measuring departures from a projected trend, if not to its use in measuring departures from a fitted trend. (For the argument see above section iv, 1, "The Correlation of Time Series and Its Pitfalls," note 2.)

A forecasting sequence cannot be expected to utilize all the materials which are available for making a general business index, and which should be included in the latter to render it "general." Comparatively few of the series lead or lag behind others with sufficient regularity to give reliable forecasts. But it may be argued that slenderness of materials is no defect in a forecasting sequence; it is better to limit the series rigidly to those showing the closest approach to perfect regularity of sequence than to gain comprehensiveness at the cost of uncertainty. If it does not matter what is to be forecasted, this view is valid. Strictly speaking, all that can be inferred from a three-curve chart is the movements of the particular series represented by the curves which lag. Both for practical and for theoretical purposes the whole operation is highly important or a curiosity, according as the curves whose movements are forecasted represent activities of large or of slight significance.

As for technical methods, the only criterion applicable to the making of forecasting sequences is supplied by the results. The original data can be made into aggregates, treated as relatives to some base period, computed as deviations from adjusted trends, expressed in units of their standard deviations, or thrown into any other form which brings out most clearly the regularity of the time sequences. The methods developed by Professor Persons serve well, and have been accepted as models by many other investigators.

The chief difficulty in applying these methods lies in securing indexes which maintain fairly regular relationships in the timing of their movements. For the period 1903 to 1914, Professor Persons finds:

> first, that the cyclical fluctuations of curve B, business, lagged eight months, on the average, after those of curve A, speculation; second, that the cyclical fluctuations of curve C, money rates, lagged four months, on the average, after those of curve B, business, and third, that the cyclical fluctuations of curve C, money rates, lagged twelve months, on the average, after those of curve A, speculation.

To complete the full round of events, it is necessary to ascertain the average period by which the movements of curve A, speculation, in one cycle, lag behind the movements of curve C, money rates, in the preceding cycle. Supplemental computations made for the purpose show

> that the interval of lag of speculation after money rates was extremely variable and averaged 6-12 months. . . .[1]

All these averages are ascertained by finding the period of lag which yields the highest coefficient of correlation between the indexes paired. The maximum coefficients are not exceedingly high. They run for curves B and A +.81; for curves C and B +.83; for curves C and A +.74; for curves A and C —.67. Moreover, every pairing shows one or two other lags with coefficients nearly equal to the maximum.[2] Of course, this means that the time relationships among the three indexes of the Harvard sequence are not sufficiently regular to afford an assured mechanical forecast of the successive movements which occur within a cycle and which form the transition from one cycle to the next. What is claimed for the sequence is that,

> Although the *lag*—the time by which the movements of one curve lag behind those of another—is not invariable in length, it is much more nearly uniform than is the length of the cycle itself. Furthermore, variations in the duration of lag can in a measure be foreseen by a careful examination of the relations subsisting between the curves at the time of forecast.[3]

3. What the Indexes of Business Conditions Show About Business Cycles.

A classification of certain business indexes according to the economic processes to which they relate will summarize the preceding

[1] See Warren M. Persons and Edwin Frickey, "Money Rates and Security Prices," *Review of Economic Statistics*, January, 1926, vol. viii, pp. 30 and 32.

[2] The full array of coefficients given by Professor Persons in the article quoted (pp 30 and 32) is as follows:

Coefficients of Correlation between Curves A, B, and C, of the Index of General Business Conditions. Bi-monthly, 1903–June, 1914

	Lag in Months								
Curves Correlated	0	2	4	6	8	10	12	14	16
B follows A........	..	..	.72	.80	.81	.76	..	..	..
C " B........	..	.75	.83	.81	.70	..	..	..	..
C " A........	..	..	..	..	..	.69	.74	.71	.62
A " C........	−.29	−.44	−.58	−.63	−.67	−.65	−.62	..	..

Somewhat different results concerning the lags between curves A and B, A and C, and B and C of the "revised index," are given by Professor W. L. Crum, in "The Pre-War Indexes of General Business Conditions," *Review of Economic Statistics*, January, 1924, vol. vi, p. 19. But since Professor Persons uses the results obtained from the old index in the latest issue I have seen, I follow his example. See W. M. Persons, "An Index of General Business Conditions, 1875–1913," *Review of Economic Statistics*, January, 1927, vol. ix, p. 26.

[3] W. L. Crum, "The Interpretation of the Index of General Business Conditions," *Review of Economic Statistics*, Supplement, September, 1925, vol. vii, p. 223.

critique in part, and prepare for the following constructive comparisons. The indexes selected all refer to the same country, cover relatively long periods by months, and present their results in readily comparable form.

INDEX RELATING TO THE PECUNIARY VOLUME OF TRANSACTIONS.

Edwin Frickey's "Index of Outside Clearings," United States, 1875-1914.

Relatives (not strictly an index) of bank clearings in seven cities (used as a sample of all clearings outside of New York), computed on the base, monthly ordinate of secular trend, adjusted for seasonal variation, equals 100.

The transactions which give rise to the drawing of checks cover many types of economic relations, but small transactions in general and rural transactions in particular are under-represented. The exclusion of New York City reduces the representation of financial and of speculative transactions. In time, the clearings of a particular day cover transactions ranging from the re-payment of debts incurred years before to the advance of funds to be returned years later. On the average, clearings probably lag some weeks behind the exchange of goods to which they relate.

INDEXES RELATING TO THE PHYSICAL VOLUME OF TRADE.

Carl Snyder's "Clearings Index of Business," United States, 1875-1924.

Relatives (not strictly an index) of all outside clearings, "deflated" by an index of the "general price level," computed on the base, monthly ordinate of secular trend equals 100, corrected for seasonal variations, and smoothed by a three-months moving average.

What is said above concerning the activities represented by outside clearings applies here. It is, of course, questionable how far an aggregate in dollars can be made to show fluctuations in physical volume through division by a price index.

Warren M. Persons' "Index of Trade," United States, 1903-23.

First segment, 1903-15.

A mixed index of physical and pecuniary volume of trade.

Simple arithmetic means of relatives of seven series computed on the base, monthly ordinate of secular trend, corrected for seasonal variations, equals 100, expressed in multiples of their several standard deviations. The arithmetic means are multiplied by the standard deviation of outside clearings (8.62).

Second and third segments, 1915-19 and 1919-23.

Indexes of physical volume only.

Weighted arithemtic means of relatives of four (1915-19) and of five (1919-23) series, computed on the base, monthly ordinate of linear trend, corrected for seasonal variation, equals 100.

INDEXES RELATING TO GENERAL BUSINESS CONDITIONS.

American Telephone and Telegraph Company's index of "General Business Compared with Normal," United States, 1877-1925.

From 1877 to 1884, when pig-iron output is the only series used, this index relates to physical volume of production. From 1922, when all "dollar series" are dropped, to date, it relates to physical volume of production and of trade. In the intervening years, 1885-1921, it relates to "general business conditions."

Weighted arithmetic means of relatives computed on the base, monthly ordinate of secular trend, corrected for seasonal variations, equals 100, expressed in multiples of standard deviations. The arithmetic means are finally put into percentage form through multiplying by 10 (the approximate weighted average of the standard deviations of the constituent series in percentages).

Carl Snyder's "Index of Deposits Activity," United States, 1875 to 1924.

Relatives (not strictly an index) of the ratios of individual deposits in all National Banks to total clearings, computed on the base, monthly ordinate of secular trend equals 100, corrected for seasonal variations, and smoothed by three-months moving averages.

This interesting series is best classed as relating to general business conditions. The inclusion of New York City clearings (as well as New York City deposits) gives dealings in securities far more weight in this series than they have in Frickey's and Snyder's relatives of outside clearings.

None of the series here described is comprehensive enough to pass as an index of business cycles. Each series relates to but one or a few economic activities, and these activities differ widely. In every case the representative character of the data used is open to question, and the methods of isolating cyclical-irregular fluctuations lack precision. Two series change character from period to period. Three series are relatives measuring the fluctuations of a variable about its adjusted trend; two series show arithmetic means of such relatives reduced to units of their standard deviations. By no means uniform materials, one would say.

If these series made by different hands, with different methods, from different data, to show different things, agree with each other in large measure, it must be that business cycles manifest themselves in much the same way over a wide variety of economic activities, that these fluctuations are recorded with reasonable accuracy in numerous time series, and that the diversities of method make no great difference in the results. So far as the series differ, we may infer that variety of method does make some difference in the results; that the data are unreliable in different ways; or that the activities to which the series relate have characteristically different fluctuations. Perhaps all these explanations, and others too, are applicable to every difference.

Thus in comparing the five indexes of business conditions in the United States which cover by months the longest periods of time, we shall be testing the underlying statistics, testing the methods of isolating cyclical-irregular fluctuations, and testing the hypothesis that similar cycles occur in different economic activities. Of course, we should prefer to test these matters separately, but must content ourselves with testing them in combination. Further, we shall be testing in one country for about 50 years the regularity of business cycles in respect to duration and amplitude.

(1) The "Saw-Tooth" Contour of the Business Indexes.

All five of our indexes present business cycles, not as sweeping smoothly upward from depressions to a single peak of prosperity and then declining steadily to a new trough, but as moving in a jerky fashion. Even the two curves which Snyder smoothed by moving averages are made up of serrated segments.

Counting shows that the indexes change direction on the average every three months, every two months, or even oftener.[1] We cannot

[1] The following table gives details.

FREQUENCY OF CHANGES IN DIRECTION OF THE CURVES TRACED BY FIVE MONTHLY INDEXES OF BUSINESS CONDITIONS IN THE UNITED STATES, 1877–1922

1877–1922	Number of Times Curves Change Direction: From rise to fall or from fall to rise	From rise or fall to horizontal, or *vice versa*	Total changes of direction	No. of months covered	Proportion of months in which curves change direction
A. T. and T. Index.......	178	134	312	550	57%
Frickey's Clearings Index*	222	59	281	456	62
Snyder's Clearings Index..	103	87	190	552	34
Snyder's Deposits Index..	127	141	268	552	49
1903–1922					
Persons' Trade Index.....	95	61	156	238	66
A. T. and T. Index......	91	39	130	240	54
Frickey's Clearings Index*	69	18	87	144	60
Snyder's Clearings Index..	36	86	121	240	50
Snyder's Deposits Index..	47	69	116	240	48

* To 1914 only.

be sure that these frequent minor irregularities are due wholly to the failure of random influences to cancel each other. They may be due in part to the averaging together of series which differ in timing. And perhaps the cyclical movements themselves keep producing and overcoming small checks.

Reversals of direction are more frequent near the climax of prosperity and in the trough of depression than during the transitions between these extreme states. Charts 11 to 17 suggest that business has a ceiling and a floor, both somewhat elastic or irregular. Between these limiting planes it can glide up or down on a slant rather smoothly. But when business nears the ceiling or the floor it bumps up and down in a jerky fashion for a while before it goes off on the next glide.

(2) Month-to-Month Changes.

A second resemblance among the five indexes concerns the amplitude of their month-to-month changes. The "points" in which the monthly changes are expressed are relatives to ordinates of secular trend, averages of such relatives reduced to percentage form, or averages of such relatives multiplied by the standard deviations of "outside" clearings. In practice, the scales cover similar ranges.

Chart 19 shows the distribution of these monthly changes in percentages of the total number of cases covered by each series.[1]

[1] The data from which the chart is drawn are as follows:

FREQUENCY DISTRIBUTION OF THE MONTH-TO-MONTH CHANGES IN FIVE INDEXES OF BUSINESS ACTIVITY

Unit in each case is one point in the scale of deviations from adjusted trends. These scales differ somewhat from each other. See text.

Percentage Basis

Direction and magnitude of month-to-month changes	A. T. and T. Index 1877–1925	Frickey's Clearings Index 1875–1914	Snyder's Clearings Index 1875–1923	Snyder's Deposits Index 1875–1923	Persons' Index of Trade 1903–1924
+17		.2			
+16		.2			
+15		.2			
+14		.2			
+13		.6			
+12		.2		.2	
+11				.5	
+10	.2	.6			
+ 9		.8		.3	.4
+ 8	.3	1.9	.2	1.2	.7
+ 7	.8	1.7		1.7	1.5
+ 6	1.2	4.0	.5	1.9	1.5

As economic data go, these distributions are remarkably symmetrical and show a high degree of concentration around their central tendencies, most marked in Snyder's Clearings Index and least marked in Frickey's series. The difference between the two clearings indexes in this respect is probably due to the facts (1) that Snyder smoothed his index by a three-months moving average, thus reducing the amplitude of the extreme movements and increasing the number of minor movements, and (2) that Frickey used data from only seven cities, while Snyder took all outside clearings. In each case, the distribution is slightly elongated toward the left; that is, the most violent declines exceed the most considerable advances. The abrupt declines

FREQUENCY DISTRIBUTION OF THE MONTH-TO-MONTH CHANGES IN FIVE INDEXES OF BUSINESS ACTIVITY

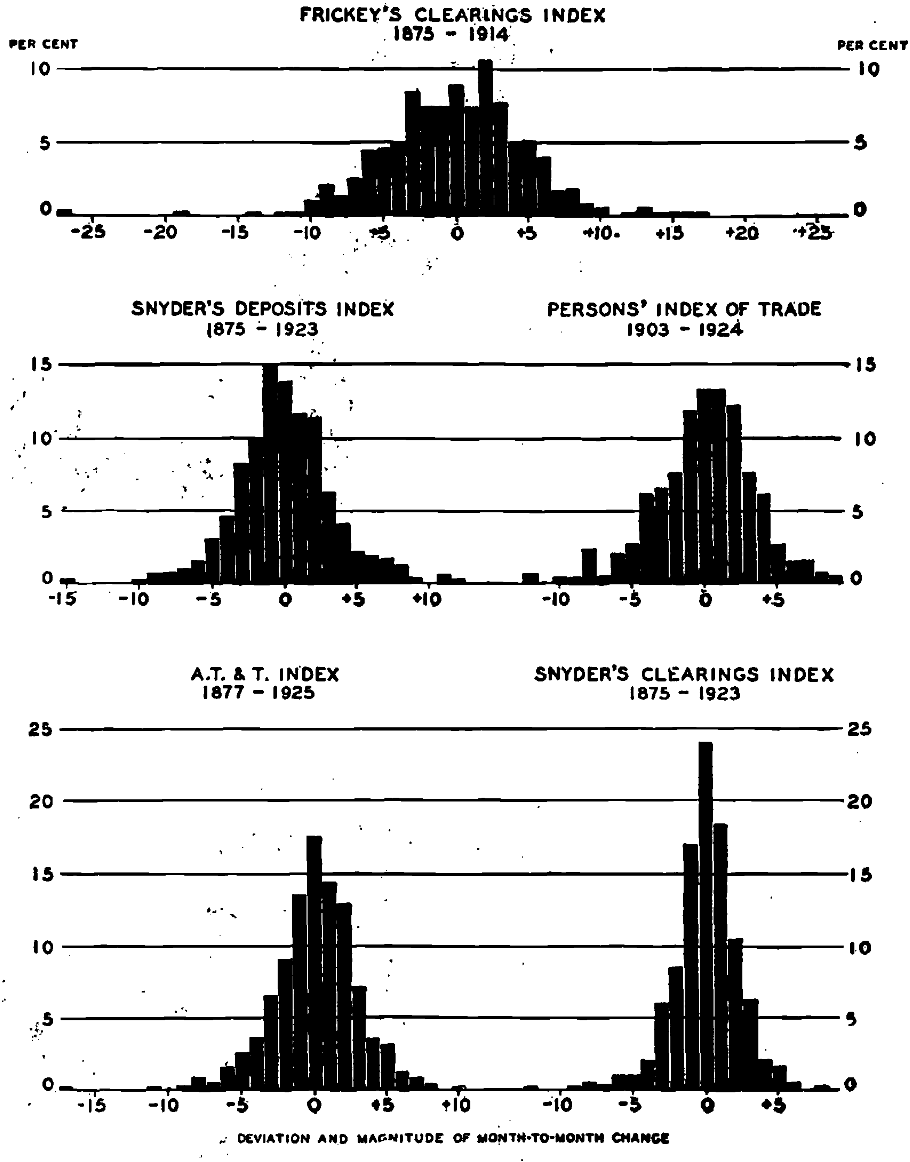
Direction and magnitude of month-to-month changes	A. T. and T. Index 1877–1925	Frickey's Clearings Index 1875–1914	Snyder's Clearings Index 1875–1923	Snyder's Deposits Index 1875–1923	Persons' Index of Trade 1903–1924
+ 5	3.2	5.2	1.7	2.2	2.7
+ 4	3.6	5.0	2.2	4.1	6.1
+ 3	7.2	7.7	6.2	6.1	7.6
+ 2	12.8	10.5	10.4	11.3	12.2
+ 1	14.3	7.3	18.3	11.6	13.3
0	17.5	8.8	24.0	13.8	13.3
− 1	13.5	7.3	16.9	15.0	11.8
− 2	9.0	7.3	8.5	9.9	7.6
− 3	6.5	8.4	6.0	8.2	6.5
− 4	3.7	5.0	2.1	4.6	6.1
− 5	2.6	4.6	.9	3.1	2.7
− 6	1.7	4.4	.9	1.5	1.9
− 7	.5	2.5	.3	1.0	.4
− 8	.8	1.3	.5	.7	2.3
− 9	.2	2.1	.2	.7	.4
−10		1.0		.2	.4
−11	.2	.2			
−12		.2	.2		.7
−13					
−14		.2			
−15				.2	
−16					
−17	.2				
−18					
−19		.2			
....					
−27		.2			
	100.0	100.0	100.0	100.0	100.0

+ Indicates a rise.

− Indicates a fall

CHART 19.

FREQUENCY DISTRIBUTION OF MONTH-TO-MONTH CHANGES IN FIVE INDEXES OF BUSINESS ACTIVITY.



usually occur in crises; the greatest gains occur in periods of revival, or come in other phases of the cycle as reactions after sudden drops.[2] Except in Snyder's deposits index, the number of declines is smaller than the number of advances, but the average magnitude of the declines is greater.[3] Business contraction seems to be a briefer and more violent process than business expansion. Why Snyder's deposits index give opposite indications will appear presently. But most of the monthly changes in both directions have small amplitudes. Less

[2] The dates of the largest month-to-month changes in each of the series have some interest. As above, + indicates an advance, − a decline. Readers whose recollection of business chronology is hazy may care to use the "Conspectus of Business Conditions," given in the next chapter to interpret this table.

A. T. and T. Index	Frickey's Clearings Index	Snyder's Clearings Index
−17 Nov., 1907	−27 Nov., 1907	−12 Aug., 1893
−11 Aug., 1893	−19 June, 1884	− 9 March 1884
− 9 Nov., 1890	−14 July, 1881	− 8 Sept., 1876
		− 8 Nov., 1907
		− 8 Dec., 1907
+10 June, 1886	+17 Nov., 1880	+ 8 June 1881
+ 8 July, 1891	+16 June, 1881	+ 6 Nov., 1875
+ 8 Aug., 1894	+15 Oct., 1875	+ 6 Nov., 1879
		+ 6 May, 1890

Snyder's Deposits Index	Persons' Index of Trade
−15 July, 1901	−12 Jan., 1921
−10 June, 1901	−12 June, 1924
− 9 Aug., 1875	−10 Nov., 1907
− 9 June, 1880	− 9 Dec., 1907
− 9 May, 1907	
− 9 Sept., 1914	
+12 April, 1901	+ 9 March, 1918
+11 Dec., 1880	+ 8 July, 1919
+11 Dec., 1898	+ 8 Nov., 1922
+11 Dec., 1900	+ 7 Feb., 1904
+ 9 Feb., 1876	+ 7 Oct., 1912
+ 9 July, 1897	+ 7 Dec., 1919
	+ 7 Dec., 1924

[3] The details are as follows:

Number and Average Magnitude of Advances and Declines in the Month-to-Month Changes in Five Indexes of Business Conditions

	Number of cases of			Average magnitude of	
	Advance	No Change	Decline	Advance Points	Decline Points
A. T. and T. business index	256	103	228	2.5	2.7
Frickey, clearings	222	42	215	4.0	4.2
Snyder, clearings	231	141	213	2.0	2.2
Snyder, deposits	241	81	264	3.0	2.8
Persons, trade	121	35	107	2.8	3.3

than half exceed two points in the scales used, except in Frickey's index, and, with the same exception, only a tenth exceed five points.[4]

(3) On Identifying Business Cycles by Use of the Business Indexes.

The irregularities of contour in the business indexes cause considerable difficulty when one tries to count the number of business cycles in a given period. The memorable cycles which culminated in 1882, in 1893, in 1907, in 1917, and in 1920 stand out clearly in all our curves. But in all the curves there are stretches when the cyclical fluctuations are less easy to identify; for example, the later 1880's, the middle 1890's, the early 1900's, the years 1910-13, and 1923-24. While the five monthly indexes agree in presenting this contrast between major and minor fluctuations, it would be hard from study of the curves to lay down rules for determining precisely what movements of an index shall be counted a business cycle. In the less pronounced cycles, the period of greatest activity occasionally (2 times out of 56) remains below the base line in some one of the indexes. In other cycles, the period of least activity occasionally (2 times out of 55) remains above the base line in some index. Often there are double or triple peaks, and double or triple troughs. At times one suspects that irregular fluctuations are dominating the cyclical factors. Nor is the duration of business cycles uniform enough to be used as a criterion in doubtful cases. But one who studies all five curves with care can draw up a list of business cycles which anyone else can identify with confidence in every curve.

Such a list is best made by noting the successive turning points in the business indexes. One may count either the successive crests, or the successive troughs of the waves. These two ways of reckoning usually give different measurements of duration for particular cycles; but the average duration over a period of considerable length must

[4] Once more the details are of interest:

PERCENTAGES OF THE MONTH-TO-MONTH CHANGES IN FIVE INDEXES OF BUSINESS CONDITIONS WHICH FALL WITHIN CERTAIN LIMITS

	± 1 point	± 2 points	± 5 points	± 10 points
A. T. and T. business index.....	45.3%	67.1%	93.9%	99.6%
Frickey's clearings index........	23.4	41.2	77.1	97.4
Snyder's clearings index........	59.2	78.1	97.2	99.8
Snyder's deposits index.........	40.4	61.6	89.9	99.1
Persons' trade index............	38.4	85.2	89.9	99.4

come out nearly the same whether one counts from trough to trough or from crest to crest.

Table 14 shows that 13 business cycles occurred in the United

TABLE 14

DATES OF THE TROUGHS AND THE CRESTS OF AMERICAN BUSINESS CYCLES IN 1878–1924 ACCORDING TO FIVE MONTHLY INDEXES OF BUSINESS ACTIVITY

	A. T. and T. business index	Frickey's clearings index	Snyder's clearings index	Snyder's deposits index	Persons' trade index
Trough....	Dec. '78–Apr. '79	Dec. '78	May '78	June and Dec. '78	
Crest......	June-Aug. '81	June '81	Aug. '81	Feb. '81	
Trough....	Feb., May '85	Nov. '84	Apr. '85	Nov. '84	
Crest......	March '87	June '87	June '87	Nov., Dec. '86	
Trough....	March '88	March '88	March '88	Feb. '88	
Crest......	May, Oct. '90	July '90	July '90	June '90	
Trough....	May '91	March '91	March '91	Jan., Feb. '91	
Crest......	Feb. '92	June '92 Jan. '93	Jan. '93	Feb. '93	
Trough....	June '94	Aug. '93	Oct. '93	Oct. '94	
Crest......	Oct. '95	Oct. '95	Dec. '95	June '95	
Trough....	Oct. '96	May '97	Mar. '97	Apr. '97	
Crest......	Dec. '99 Feb. '00	March '99	June '99	Feb. '99	
Trough....	Nov., Dec. '00	Sept. '00	Sept. '00	Sept. '00	
Crest......	Sept. '02	May '01 Sept. '02 July '03	June '01	Apr. '01 May '01	
Trough....	Dec. '03	May '04	July '04	Apr., May '04	July '04
Crest......	May, July '07	May '07	Feb. '06	Jan. '06	May '07
Trough....	May, June '08	Dec. '07	Jan. '08	Dec. '07	March '08
Crest......	Jan., March '10	March '10	Apr. '10	Feb. '10	March '10
Trough....	April '11	Oct. '11	Dec. '11	Sept. '10	April '11
Crest......	Jan. '13	Oct. '12	Feb. '13	March., Apr. '12	Oct. '12
Trough....	Dec. '14	Nov. '14	Dec. '14	Sept., Oct. '14	Nov. '14
Crest......	Nov. '16 Jan. '17		Dec. '16 Jan. '17	Oct., Nov. '16	May '17
Trough....	March '19		March '19	March '19	June '19
Crest......	Jan. '20		Aug., Sept. '19	July '19	March '20
Trough....	Apr., May, July '21		March '21	Mar., July '21	July '21
Crest......	May '23		May '23	March '23	May '23
Trough....	June '24				June '24

States between 1878 and 1923. Each of these cycles is traced by each of our indexes. The table gives the dates both of the crests and the troughs, as they appear in the several curves. Closer agreement among the dates could be secured by smoothing out their serrations with free-hand curves, and making a single crest and trough for each successive cycle. That process is legitimate; but it is well to show how frequent are the multiple peaks.

(4) Time Relationships Among the Business Indexes.

Of the several comparisons which Table 14 suggests, the simplest concerns the time sequence in which the five indexes reach their turning points. For this purpose, we need to replace the multiple peaks and troughs by single dates. The method followed is arbitrary and does nothing to lessen the differences among the curves: we place the crest midway between the months showing the highest points revealed by a given wave of activity, and date the troughs in corresponding fashion.[1] With this simplification, we can manage the data easily.

On no occasion do all of our indexes reach the crest or the trough of a given wave in the same month. Four times three out of four series then available agree; [2] but there is always one series which leads or lags behind the others by a month or more. Seemingly, we should think, not of turning points in business cycles, but of turning periods. As a rule, these turning periods are relatively long in the violent cycles and relatively short in the mild cycles. If we count from the date when the first of our indexes turns a given corner to the date when the last one turns the same corner, we get periods which run from one month in the trough of 1888, or two months in the trough of 1914 and in the crests of 1890, 1910, and 1923, to 14 months in the trough of 1893-94, 15 months in the trough of 1910-11, 16 months in the crests of 1901-02, and 17 months in the crest of 1906-07. On the average these turning periods are longer at the crest (8.0 months) than in the troughs (6.1 months).

Snyder's deposits index is the first to reach the crest 12 times out of 13 (the exception occurred in 1892-93). It also leads 8 times out of 13 in reaching the troughs. It will be recalled that this index is made by dividing individual deposits in all National Banks into total clearings. Of total clearings, the New York City figures make

[1] If there is a double crest in two adjacent months (for example, October and November, 1916), we choose the later month to avoid fractions. Similarly, in dealing with a double crest in May and October, 1890, we put the single crest in August, instead of July. There are 23 double or multiple crests or troughs among 111 turning points in Table 14.

[2] These dates are March, 1888, September, 1900, March, 1919, and May. 1923.

roughly half, and the New York City figures are influenced largely by the current volume of transactions on the Stock Exchange. In the other indexes this type of business activity counts for little. We may infer, then, that the pecuniary volume of trading in stocks almost always reaches its peak and begins to decline in a business cycle, before other types of business have culminated. With decidedly less regularity, trading in stocks also precedes other types of business in recovering from a depression.

As for the other indexes, their average order in reaching peaks and troughs is (2) Snyder's clearings index, (3) Frickey's clearings index, (4) the American Telephone and Telegraph Company's general business index, and (5) Person's index of trade. In each of these cases the lags average longer at the crests than in the troughs.[3]

(5) Duration of Periods of Expansion and Contraction.

Since all five indexes never reach the crests or the troughs of cyclical waves at the same time, and since neither the time sequences among the indexes nor the lags are constant, we find considerable differences among our measurements of the duration of periods of rising and declining activity. Table 15 presents the details.

Here we have a double complexity—five different measures of phenomena which themselves vary widely from case to case. But when we strike averages we approach uniformity. According to four of the indexes, periods of business expansion have lasted about two years on the average (23-25 months); while periods of business contraction have averaged little more than a year and a half (18, 18, 19 and 21 months according to the several indexes). That the longest periods of decline are greater than the longest periods of advance makes this average the more striking. It links with what has already been said about the greater number and smaller average value of the upward month-to-month changes. The average (and the modal) American cycle seems to be made up of two unequal segments, a two-year period of gradually increasing activity, and a period, four to six months shorter, of less gradually shrinking activity.

[3] The average lags of the several indexes behind the leaders are as follows:

	A. T. and T.	Frickey	Snyder clearings	Snyder deposits	Persons
In troughs	3.7 mo.	3.2 mo.	3.0 mo.	2.4 mo.	4.2 mo.
At crests	5.8 "	6.1 "	4.2 "	0.9 "	6.5 "
Number of cases	26	21	26	26	12

TABLE 15

Duration of Alternate Periods of Business Expansion and Business Contraction in the United States, 1878–1923, According to Five Indexes of Business Activity.

Based upon Table 14

Business Cycles of		A. T. and T. business index Months Rise	A. T. and T. business index Months Fall	Frickey's clearings index Months Rise	Frickey's clearings index Months Fall	Snyder's clearings index Months Rise	Snyder's clearings index Months Fall	Snyder's deposits index Months Rise	Snyder's deposits index Months Fall	Persons' trade index Months Rise	Persons' trade index Months Fall
1878–85	Rise.........	29		30		39		29			
	Fall..........		45		41		44		45		
1885–88	Rise..........	23		31		26		25			
	Fall..........		12		9		9		14		
1888–91	Rise..........	29		28		28		28			
	Fall..........		9		8		8		8		
1891–94	Rise..........	9		19		22		24			
	Fall..........		28		10		9		20		
1894–97	Rise..........	16		26		26		8			
	Fall..........		12		19		15		22		
1897–00	Rise..........	39		22		27		22			
	Fall..........		11		18		15		19		
1900–04	Rise..........	21		22		9		8			
	Fall..........		15		22		37		36		
1904–08	Rise..........	42		36		19		20		34	
	Fall..........		12		7		23		23		10
1908–11	Rise..........	20		27		27		26		24	
	Fall..........		14		19		20		7		13
1911–14	Rise..........	21		12		14		19		18	
	Fall..........		23		25		22		30		25
1914–19	Rise..........	24				25		25		30	
	Fall..........		27				26		28		25
1919–21	Rise..........	10				6		4		9	
	Fall..........		16				18		22		16
1921–23	Rise..........	24				26		22		22	
Maximum	Rise.......	42		36		39		29		34	
	Fall........		45		41		44		45		25
Minimum	Rise.......	9		12		6		4		9	
	Fall........		9		7		8		7		10
Average	Rise.......	24		25		23		20		23	
	Fall........		19		18		21		23		18

Snyder's deposits index gives a different result. As it almost always leads the other indexes in attaining the crest of a business wave, and less uniformly leads them in reaching the trough, so it makes the periods of increasing activity relatively brief, and the periods of declining activity relatively long. Combined with what we know of its composition and its month-to-month changes, this suggests (though it does not prove) that the cycles in financial activity differ from those characteristic of general business in being made up of a shorter section of relatively rapid advance, and a longer section of decline.

All that has just been said applies strictly to the five indexes under consideration, for one country, in the years 1878-1923. Whether the generalizations are applicable to other indexes, other countries, and other periods remains an open question. In view of the diversity of the items which enter into the averages, we cannot judge the representative value of the averages themselves until a wider array of data is available. In the meanwhile, we may note that in respect to variability, our measures of the durations of business cycles compare not unfavorably with the measures made of many other social phenomena. The period of decline in business cycles is decidedly more variable in duration than the period of advance; the latter period in turn is appreciably more variable in duration than are whole cycles. For the latter measurements our materials show a coefficient of variation of 29.4 per cent.[1]

(6) The Duration of Business Cycles.

In Table 16 the lengths of the American cycles of 1878-1925 are measured by adding together first each period of advance and its subsequent period of decline, secondly each period of decline and its subsequent period of advance. From what we have already seen about the variability of these periods, it is clear that the two ways of measuring seldom give identical results. No reason appears for regarding one set of measurements as more significant than the other.

[1] Using all of the observations given in Table 15, we get the following coefficients of variation (that is, standard deviations as percentages of the corresponding arithmetic means):

	Number of observations	Mean duration	Standard deviation	Coefficient of variation
Period of advance..........	55	22.75 mo.	8.34 mo.	36.7%
Period of decline...........	51	19.82 "	10.10 "	51.0%
Whole cycles..............	101	42.02 "	12.37 "	29.4%

Further, our five indexes often given five different measurements for a given cycle counted in the same way. The differences range from

TABLE 16

Duration of Business Cycles in the United States, 1878–1925, According to Five Indexes of Business Activity

Based upon Table 15

		A. T. and T. business index Months		Frickey's clearings index Months		Snyder's clearings index Months		Snyder's deposits index Months		Persons' trade index Months	
		T–T	C–C	T–T	C–C	T–T	C–C	T–T	C–C	T–T	C–C
Trough-trough	1878–85	74		71		83		74			
Crest-crest	1881–87		68		72		70		70		
Trough-trough	1885–88	35		40		35		39			
Crest-crest	1887–90		41		37		37		42		
Trough-trough	1888–91	38		36		36		36			
Crest-crest	1890–92		18		27		30		32		
Trough-trough	1891–94	37		29		31		44			
Crest-crest	1892–95		44		36		35		28		
Trough-trough	1894–97	28		45		41		30			
Crest-crest	1895–99		51		41		42		44		
Trough-trough	1897–00	50		40		42		41			
Crest-crest	1899–02		32		40		24		27		
Trough-trough	1900–04	36		44		46		44			
Crest-crest	1902–07		57		58		56		56		
Trough-trough	1904–08	54		43		42		43		44	
Crest-crest	1907–10		32		34		50		49		34
Trough-trough	1908–11	34		46		47		33		37	
Crest-crest	1910–12		35		31		34		26		31
Trough-trough	1911–14	44		37		36		49		43	
Crest-crest	1912–17		47				47		55		55
Trough-trough	1914–19	51				51		53		55	
Crest-crest	1917–20		37				32		32		34
Trough-trough	1919–21	26				24		26		25	
Crest-crest	1920–23		40				44		44		38
Trough-trough	Maximum	74		71		83		74		55	
Crest-crest	"		68		72		70		70		55
Trough-trough	Minimum	26		29		24		26		25	
Crest-crest	"		18		27		24		26		31
Trough-trough	Average	42		43		43		43		41	
Crest-crest	"		42		42		42		42		38

2 to 18 months, and average nearly 10 months. Yet even a period of about 45 years is long enough to make the average length of busi-

ness cycles come out nearly the same whatever index is used, and whether the measurements are taken from crest to crest or from trough to trough. The eight averages covering this period all come out 42 or 43 months. Even Persons' index, which overs only 20 years, gives averages of 38 months from crest to crest and 41 months from trough to trough.

Once more we must note the limited scope of the data under analysis and question the representative value of the averages. But these are the most precise measurements of the duration of business cycles we can get for the present; they refer to the country and the period which interests us most, and they come from five different sources. We may therefore consider the distribution of the measurements in some detail. We shall treat each measurement given by each series for each cycle, whether taken from crest to crest or from trough to trough, as one observation upon the duration of business cycles.

The 101 observations which this procedure lets us count are scattered over a range which runs from 18 to 83 months.[1] Half of the

[1] The full arrays, tabulated from Table 16, may be given.

Observations upon the Duration of Business Cycles in the United States, 1878–1923, Made from Five Indexes of Business Activity

Duration in Months	Number of Cases: Trough to Trough	Number of Cases: Crest to Crest	Total	Duration in Months	Number of Cases: Trough to Trough	Number of Cases: Crest to Crest	Total
18		1	1	45	1		1
24	1	1	2	46	2		2
25	1		1	47	1	2	3
26	2	1	3	49	1	1	2
27		2	2	50	1	1	2
28	1	1	2	51	2	1	3
29	1		1	53	1		1
30	1	1	2	54	1		1
31	1	2	3	55	1	2	3
32		5	5	56		2	2
33	1		1	57		1	1
34	1	4	5	58		1	1
35	2	2	4	68		1	1
36	5	1	6	70		2	2
37	3	3	6	71	1		1
38	1	1	2	72		1	1
39	1		1	74	2		2
40	2	2	4	83	1		1
41	2	2	4				
42	2	2	4		51	50	101
43	3		3				
44	5	4	9				

(Note continued on p. 342.)

observations, however, are concentrated between 34 and 47 months, an interval of little more than a year.

As often happens in dealing with a frequency table, we get more significant results by grouping the intervals. Here we may combine months into quarters. This we can do in three ways, treating each month as the end, the middle, or the beginning of a quarter. Thus our time scales might be based upon any of these groupings:

1st grouping	2d grouping	3d grouping
22–24 mo.	23–25 mo.	24–26 mo.
25–27 "	26–28 "	27–29 "
28–30 "	29–31 "	30–32 "
etc.	etc.	etc.

These three groupings give appreciably different distributions, as the following figures show:

		Crude Primary Mode	Crude Secondary Mode
1st grouping	(22–24 mo., etc.)	15 observations at 34–36 mo.	13 observations at 43–45 mo.
2d "	(23–25 " ")	16 observations at 35–37 mo.	12 observations at 44–46 mo.
3d "	(24–26 " ")	16 observations at 42–44 mo.	14 observations at 36–38 mo.

Such shifting in the positions of the crude mode as the grouping of the months is altered from one arbitrary scheme to another, makes us wish for an average of the three groupings. In averaging, we can once more arrange the items in three ways, putting (say) the 26-28 month interval first in a combination with 27-29, and 28-30 months; second in a combination with 25-27 and 27-29 months; or last in a combination with 24-26 months and 25-27 months. As before, we have no reason for preferring any of these arrangements to the others. We may therefore use an average of all three. That plan will give as our final distribution of the 101 observations a distribution in which the class-frequencies are weighted averages of the frequencies secured in nine different groupings of the observations. The central

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Two measures of the central tendencies of the arrays may be added.

	Observations taken From trough to trough	From crest to crest	In both ways
Arithmetic means	42.5 months	41.5 months	42.0 months
First Quartile	36	32	34
Median	41	39	40
First Quartile	46	49	47

points of the intervals used are 16 months, 19 months, 22 months, etc.[2]

Chart 20, in which the results appear, shows that all our averaging of different arrangements does not completely smooth out the irregularities. The two modes persist in the intervals centering on 37 and 43 months. They are separated by a lower point at 40 months, which happens to be the median of the series.

The conclusion is clear that within the period and country represented by our indexes, business cycles, while varying in length from a year and a half to nearly seven years, have a modal length in the neighborhood of three to three and one half years. They are far from uniform in duration, but their durations are distributed about a well marked central tendency in a tolerably regular fashion. This distribution differs from the type described by the "normal curve" in being prolonged toward the upper end of the time scale somewhat farther than toward the lower end.

There we may leave the topic for the present, planning to return to it in the next chapter, when we shall have for analysis observations upon a larger number of business cycles, over a longer period, and from seventeen countries instead of one.

(7) The Amplitude of Business Cycles.

All our indexes measure the amplitude of business fluctuations in percentage deviations from base lines, which represent the loci of the ordinates of secular trend corrected for seasonal variation.[1] We have noted certain technical differences in methods of construction, but have found that the average magnitudes of the published figures are of the same order in all the indexes. The greatest differences, indeed, are between two indexes which are alike in method of construction—Snyder's series for clearings and for deposits.

Table 17 assembles all the measurements of extreme deviations from the base line at the troughs and crests of successive business

[2] Under this plan, the average for the interval centering on 19 months, for example, is made from the observations for 16-18 months weighted 1, the observations for 17-19 months weighted 2, the observations for 18-20 months weighted 3, the observations for 19-21 months weighted 2, and the observations for 20-22 months weighted 1. By months, this arrangement weights the observation at 16 months 1, at 17 months 3, at 18 months 6, at 19 months 7, at 20 months 6, at 21 months 3, and at 22 months 1.

[1] For the present purpose, Snyder's practice of eliminating the seasonal variations *after* the deviations from secular trends have been computed is not an important departure from the methods followed by Persons, Frickey, and the statisticians of the American Telephone and Telegraph Company.

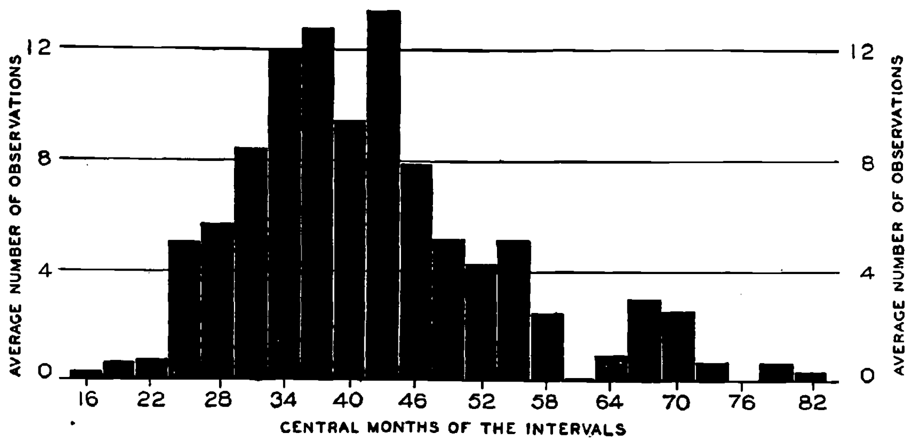
cycles, and Table 18 shows the magnitude of the successive swings from trough to crest and from crest to trough.

CHART 20.

FREQUENCY DISTRIBUTION OF 101 OBSERVATIONS UPON THE DURATION OF BUSINESS CYCLES: UNITED STATES, 1878-1923.

Based upon Table 16.

Averages obtained by combining several different groupings of the observations. See text



NUMERICAL VALUES

Central Months of the intervals	Average Number of observations	Central months of the intervals	Average Number of observations
16	.3	52	4.2
19	.7	55	5.1
22	.8	58	2.4
25	5.0	61	.1
28	5.7	64	.9
31	8.4	67	2.9
34	11.9	70	2.5
37	12.7	73	.7
49	9.4	76	. .
43	13.3	79	.7
46	7.8	82	.3
49	5.1		
			100.9

The highest pitch of prosperity in the whole period covered was attained in 1881 according to Frickey's and Snyder's clearings indexes, in 1901 according to Snyder's deposits index, in 1907 according to the American Telephone and Telegraph Company's index, and in 1917 according to Persons' index (which does not go back of 1903). These differences are not to be regarded as discrepancies, but rather as probably reliable indications that the processes to which the indexes specifically relate really attained their highest levels above their base lines in different cycles.

The most interesting case is the maximum shown by Snyder's deposits index in 1901,—a moderate crest according to the two clearings indexes, and the lowest crest in the list according to the American Telephone and Telegraph Company's series. That year saw the great "Northern Pacific corner" on the New York Stock Exchange. Financial activity in New York did perhaps rise higher above its trend then than at any other time between 1878 and 1923. The series which is much influenced by New York City clearings reflects this feverish activity. Outside clearings were stimulated in moderate degree. But "general business," as shown by the telephone company's series, expanded only a little.

The deepest depressions occurred in 1878 and 1896 according to Frickey's index, in 1894 and 1896 according to Snyder's clearings index, in 1914 according to Snyder's deposits index, and in 1921 according to the telephone company's and Persons' indexes. All these were unquestionably periods of extreme hardship, and the differences among the several indexes concerning their relative severity need give us no concern.

The periods of maximum and minimum advance and decline in business activity according to the five indexes are best presented in tabular form. The greatest "boom" in the whole period was that which followed the prolonged period of depression in the 1870's if we consider outside clearings, that which culminated in the Northern Pacific corner of 1901 if we give metropolitan clearings their share in the national total, and that which accompanied or followed the World War if we take the trade indexes. Similarly the most catastrophic declines were those which followed on the greatest booms; for even Snyder's deposits index makes the drop of 1881-1884 slightly larger than that following the Northern Pacific corner. The mildest periods of expansion, on the other hand, were those which culminated in 1895, 1902, and 1912-13, while the mildest depressions came in 1886-88, 1900, and 1910-11. The decline in 1895-97 was also slight; but that was because the preceding period of activity was mild, not because the depression lacked severity. All this is quite consistent with what is known from other sources concerning the major and minor business cycles of the period.

TABLE 17

PERCENTAGE DEVIATIONS FROM THEIR BASE LINES OF FIVE INDEXES OF BUSINESS ACTIVITY AT THE CRESTS AND TROUGHS OF SUCCESSIVE BUSINESS CYCLES, UNITED STATES: 1878–1923.

		A. T. and T. business index		Frickey's clearings index		Snyder's clearings index		Snyder's deposits index		Persons' trade index	
		Trough	Crest	Trough	Crest	Trough	Crest	Trough	Crest	Trough	Crest
'78	Trough......	−13		−29		−14		−14			
'81	Crest.......		+13		+27		+26		+30		
'84–'85	Trough......	−19		−20		−19		−23			
'86–'87	Crest.......		+ 9		+19		+11		+ 7		
'88	Trough......	− 7		− 3		− 4		− 7			
'90	Crest.......		+17		+22		+21		+14		
'91	Trough......	− 7		+ 4		+ 3		− 1			
'92–'93	Crest.......		+14		+20		+18		+13		
'93–'94	Trough......	−20		−17		−20		−18			
'95	Crest.......		+ 6		− 2		− 3		+ 3		
'96–'97	Trough......	−17		−29		−20		−11			
'99	Crest.......		+11		+10		+ 7		+21		
'00	Trough......	− 4		−11		− 5		−15			
'01–'02	Crest.......		+ 6		+ 8		+15		+34		
'03–'04	Trough......	−14		−11		− 2		−17		− 9	
'06–'07	Crest.......		+18		+15		+16		+19		+13
'07–'08	Trough......	−17		−22		−12		−20		−19	
'10	Crest.......		+12		+ 8		+ 6		+11		+ 8
'10–'11	Trough......	− 3		− 8		− 2		− 7		− 4	
'12–'13	Crest.......		+10		+ 8		+ 3		+ 5		+ 9
'14	Trough......	−18		−17		−17		−28		−18	
'16–'17	Crest.......		+17				+10		+13		+25
'19	Trough......	− 8				− 2		− 6		− 3	
'19–'20	Crest.......		+13				+ 9		+11		+15
'21	Trough......	−25				−10		− 7		−22	
'23	Crest.......		+10				+11		+ 6		+17
Minimum...........		− 3	+ 6	+ 4	− 2	+ 3	− 3	− 1	+ 3	− 3	+ 8
Maximum...........		−25	+18	−29	+27	−20	+26	−28	+34	−22	+25
Average............		−13.2	+12.0	−14.8	+13.5	− 9.5	+11.5	−13.4	+14.4	−12.5	+14.5

By far the most important difference among the indexes is that Snyder's clearings series, and less clearly Frickey's companion piece, suggest that business cycles have been growing progressively milder decade by decade, whereas the other indexes show no such cheering

TABLE 18

AMPLITUDE OF THE RISE FROM TROUGH TO CREST AND OF THE DECLINE FROM CREST TO TROUGH IN THE AMERICAN BUSINESS CYCLES OF 1878–1923 AS SHOWN BY FIVE INDEXES OF BUSINESS ACTIVITY.

	A. T. and T. business index		Frickey's clearings index		Snyder's clearings index		Snyder's deposits index		Persons trade index	
	Rise	Fall	Rise	Fall	Rise	Fall	Rise	Fall	Rise	Fall
Rise 1878–'81	26		56		40		44			
Fall 1881–'84, '85		32		47		45		53		
Rise 1884, '85–'86, '87	28		39		30		30			
Fall 1886, '87–'88		16		22		15		14		
Rise 1888–'90	24		25		25		21			
Fall 1890–'91		24		18		18		15		
Rise 1891–'92, '93	21		16		15		14			
Fall 1892, '93–'93, '94		34		37		38		31		
Rise 1893, '94–'95	26		15		17		21			
Fall 1895–'96, '97		23		27		17		14		
Rise 1896, '97–'99	28		39		27		32			
Fall 1899–'00		15		21		12		36		
Rise 1900–'01, '02	10		19		20		49			
Fall 1901, '02–'03, '04		20		19		17		51		
Rise 1903, '04–'06, '07	32		26		18		36		22	
Fall 1906, '07–'07, '08		35		37		28		39		32
Rise 1907, '08–'10	29		30		18		31		27	
Fall 1910–'11		15		16		8		18		12
Rise 1911–'12, '13	13		16		5		12		13	
Fall 1912, '13–'14		28		25		20		33		27
Rise 1914–'16, '17	35				27		41		43	
Fall 1916, '17–'19		25				12		19		28
Rise 1919–'19, '20	21				11		17		18	
Fall 1919, '20–'21		38				19		18		37
Rise 1921–'23	35				21		13		39	
Minima	10	15	15	16	5	8	12	14	13	12
Maxima	35	38	56	47	40	45	49	53	43	37
Average rise	25		28		21		28		27	
Average fall		25		27		21		28		27

drift. If we confine attention to the most violent cycles of 1878-1923 (including the cycle of 1900-04 in Snyder's deposits index), and com-

TABLE 19

DATES OF THE MOST AND THE LEAST VIOLENT CYCLICAL FLUCTUATIONS IN AMERICAN BUSINESS, 1878–1923, ACCORDING TO FIVE INDEXES OF BUSINESS ACTIVITY

	A. T. and T. business index	Frickey's clearings index	Snyder's clearings index	Snyder's deposits index	Persons' trade index
Greatest rise...	1914–17 1921–23	1878–81	1878–81	1900–01	1914–17
Greatest fall...	1920–21	1881–84	1881–85	1881–84	1920–21
Smallest rise...	1900–02	1893–95	1911–13	1910–12	1911–12
Smallest fall....	1899–00 1910–11	1910–11	1910–11	1886–88 1895–97	1910–11

bine the points of rise and fall shown in Table 18, we get the following results:

TABLE 20

AMPLITUDE OF COMBINED RISE AND FALL IN VIOLENT BUSINESS CYCLES, ACCORDING TO FIVE INDEXES OF BUSINESS ACTIVITY

Based upon Table 18

Business Cycles of	A. T. and T. business index	Frickey's clearings index	Snyder's clearings index	Snyder's deposits index	Persons' trade index
1878–85..........	58	103	85	97	
1891–94..........	55	53	53	45	
1900–04..........	..	..	..	100	
1904–08..........	67	63	46	75	54
1914–19..........	60		39	60	71
1919–21..........	59		30	35	55

On the face of the returns we must conclude that, since the early 1880's, the cyclical fluctuations of outside clearings have been greatly reduced; but that the lessened fluctuations of outside clearings have not led to greater stability in other types of business.[2] It does not necessarily follow, however, that business processes of any one of the many types which affect outside clearings have become more stable. In the periods covered by Snyder's and Frickey's indexes, checks have come into wider use in retail trade, and in paying rents, salaries, and even wages. Transactions of this type have cyclical-irregular fluctuations of notably smaller amplitude than the wholesale transactions which have long been settled with checks. Further, Mr. Snyder's series, which includes all outside clearings, tends to become stabler

[2] It may be remarked that the evidence of the A. T. and T. index on this point cannot be thrown out because of the many changes in the series used in making it. For these changes have been mainly of a sort which tend to render the averages less variable.

from the addition of new clearing houses to the list. In view of these considerations and of the contrary evidence borne by other business indexes, it is rash to say that business cycles are growing milder because the cyclical fluctuations of outside clearings are less now than they were some 40 years ago. The severity of the crisis of 1920 and the depression of 1921 is attested by abundant evidence.

By way of summary we may assemble our 111 observations upon the extreme amplitudes of the crests and troughs of business cycles in another frequency table.[3] Again the question arises how the observations made at successive points shall be grouped to bring out their significance. Taking a span of 5 points in the scale of deviations, I have made groups centered around deviations of 0, 5, 10 . . . points; 1, 6, 11 . . . points; 2, 7, 12 . . . points; 3, 8, 13 . . . points; and 4, 9, 14 . . . points. In all of these groupings, the distributions of the maximum deviations at the troughs of cycles show two crude modes. One or both of these modes shift their locations and their relative prominence on every change in the groupings of the observations. On the other hand, all 5 of the distributions of the maximum deviations at the crests of cycles rise step by step to a

[3] The full array is as follows:

Scale of deviations from base lines in Points		Number of Observations		Scale of deviations from base lines in Points		Number of Observations	
Troughs	Crests	Troughs	Crests	Troughs	Crests	Troughs	Crests
+ 4	− 4	1		−16	+16		1
+ 3	− 3	1	1	−17	+17	6	3
+ 2	− 2		1	−18	+18	3	2
+ 1	− 1			−19	+19	3	2
0	0			−20	+20	5	1
− 1	+ 1	1		−21	+21		2
− 2	+ 2	3		−22	+22	2	1
− 3	+ 3	3	2	−23	+23	1	
− 4	+ 4	3		−24	+24		
− 5	+ 5	1	1	−25	+25	1	1
− 6	+ 6	1	4	−26	+26		1
− 7	+ 7	5	2	−27	+27		1
− 8	+ 8	2	4	−28	+28	1	
− 9	+ 9	1	3	−29	+29	2	
−10	+10	1	4	−30	+30		1
−11	+11	3	5	−31	+31		
−12	+12	1	1	−32	+32		
−13	+13	1	5	−33	+33		
−14	+14	3	2	−34	+34		1
−15	+15	1	3				
					Totals	56	55

single mode and decline again step by step. Only once is this mode shifted to a new position by altering the groupings. If the observations analyzed are representative, the crests of business cycles are more regularly distributed about their central tendency than are the troughs.

On adding together the groupings for troughs and crests, we get five total distributions which are not quite so regular as the distributions of the crests, but decidedly more regular than the distributions of the troughs. In two cases there are two crude modes separated by a lower interval; in one case two adjacent intervals show the same maximum figure; in the other two cases there is a single mode coinciding in position with the mode of the distribution of crests.

Perhaps the best way to establish the broad characteristics of these distributions is to strike averages of all five groupings. Table 21 shows the results of that operation. The number of observations recorded at each point in the scale of deviations from the trend lines of the series appear five times in as many different combinations. In dividing these interlocking combinations for averaging I have used a scale which centers the successive groups around deviations from the trends of 2 points, 7 points, 12 points, etc. Since two troughs are found slightly above and two crests slightly below the trend lines, the first group in the scale centers around + 8 for the troughs and — 3 for the crests.

It will be seen that the double mode persists in the averaged distribution of the troughs. The primary mode is in the group centering about 17 per cent below the trend lines, the secondary mode in the group centering about minus 7 per cent. The intermediate interval, 12 per cent, is the point of greatest concentration of the crests. The distribution which combines crests and troughs has almost equal values at 7, 12 and 17 per cent deviations from trend values.

On the whole, there is no clear evidence of the existence of two distinct types of business cycles—major and minor, or violent and mild—in these observations concerning extreme deviations from the trends. Of course there are major business cycles and minor ones, just as there are tall men and short men in every race; but when all the deviations at the crests and troughs of business cycles are put together they suggest a homogeneous group of phenomena rather than a mixture of two species. Yet the volume of data is not sufficient to close the question.

TABLE 21

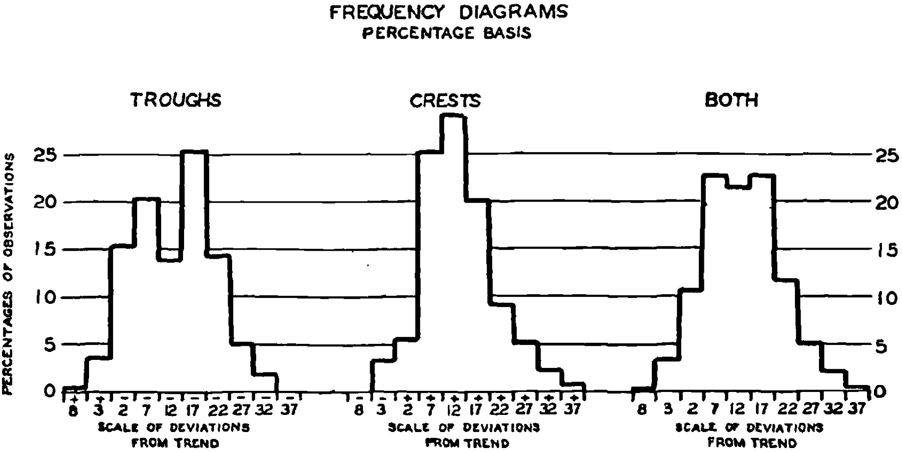
FREQUENCY DISTRIBUTIONS OF 111 OBSERVATIONS UPON THE AMPLITUDE OF THE PERCENTAGE DEVIATIONS OF FIVE BUSINESS INDEXES FROM THEIR RESPECTIVE TRENDS AT THE TROUGHS AND CRESTS OF AMERICAN BUSINESS CYCLES: 1878–1923.

Data from Table 17

Averages of five groupings of the observations. See text

Scales of Percentage Deviations: Central points of the groups averaged		Number of Observations					
		Actual Numbers			Percentages of Totals		
Troughs	Crests	Troughs	Crests	Both	Troughs	Crests	Both
+ 8	− 8	0.2	...	0.2	0.4	...	0.2
+ 3	− 3	2.0	1.8	3.8	3.6	3.3	3.4
− 2	+ 2	8.6	3.0	11.6	15.4	5.5	10.5
− 7	+ 7	11.4	13.8	25.2	20.4	25.1	22.7
−12	+12	7.8	16.0	23.8	13.9	29.1	21.4
−17	+17	14.2	11.0	25.2	25.4	20.0	22.7
−22	+22	8.0	5.0	13.0	14.3	9.1	11.7
−27	+27	2.8	2.8	5.6	5.0	5.1	5.0
−32	+32	1.0	1.2	2.2	1.8	2.2	2.0
−37	+37	...	0.4	0.4	...	0.7	0.4
	Totals	56.0	55.0	111.0	100.2	100.1	100.0

	Arithmetic Means	Medians	Standard deviations	Coefficients of variation
Troughs........	12.6	13.5	7.88	63%
Crests..........	13.0	12.0	7.54	58%
Both...........	12.8	12.7	7.92	52%



Additional evidence upon this important point can be obtained by analyzing in similar fashion the figures given in Table 18 concerning the percentage amplitude of the swings from crest to trough, and from trough to crest of successive cycles. Of course, the same materials underlie this table and the one we have just been using; but the deviations, instead of being taken always with reference to the base lines, are combined in two new ways: first a deviation in a trough is added (without regard to signs) to the next deviation at a crest; then the latter deviation is added to that at the following trough, and so on. Such re-groupings of data are most useful in trying to determine whether multiple modes reveal features which are accidental or characteristic of the series in which they occur.

Again I have used a span of 5 points; centered my groups successively around movements of 2, 7, 12 . . ., 3, 8, 13 . . ., 4, 9, 14 . . ., 5, 10, 15 . . ., and 6, 11, 16 . . . per cent of trend values, and finally averaged all the groupings together. Double modes appear in most of the groupings; but they shift location as the groupings change, and they dwindle when the five groupings are combined in a general average.

Thus Table 22, which presents the final outcome of these operations, confirms the impression made by Table 21. The diagram for the declines from crest to trough shows a slight secondary mode at 34 per cent separated from the primary mode by a wider interval than appeared in Table 21, while the diagram for the advances from trough to crest shows almost equal frequencies at 19 and 29 per cent, and a slight increase from 34 to 39 per cent. But the diagram which includes both declines and advances rises rapidly to a single mode and then falls gradually step by step with one slight arrest in the descent. On the whole our frequency distributions of the amplitudes of cyclical fluctuations are somewhat more regular than our distributions of their durations, and afford even less basis for supposing that there are two or more distinct species of business cycles.

That business cycles bring enormous economic losses upon a country is clear from this study of their amplitudes. Reckoned in percentages of ordinates of secular trend, the declines from the crests to the troughs of business cycles in the United States from 1878 to 1923 averaged more than a fifth in Snyder's clearings index, and more than a quarter in all the other series. The grand average of all the observations is 25.5 per cent. In extreme cases, these declines exceeded

TABLE 22

FREQUENCY DISTRIBUTIONS OF 106 OBSERVATIONS, MADE FROM FIVE INDEXES OF BUSINESS CONDITIONS, UPON THE AMPLITUDE OF THE RISE FROM TROUGH TO CREST AND THE DECLINE FROM CREST TO TROUGH IN AMERICAN BUSINESS CYCLES, 1878–1923, RECKONED IN PERCENTAGES OF TREND VALUES.

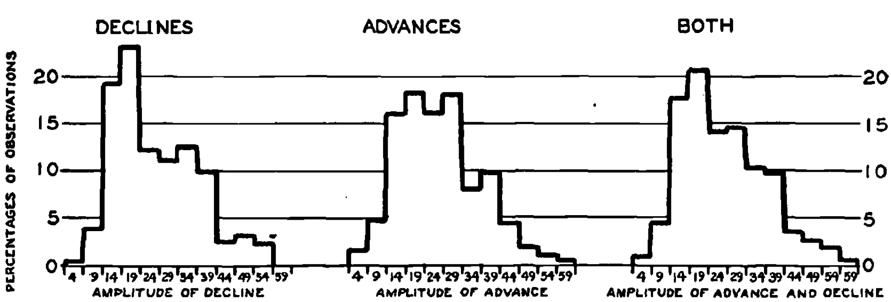
Data from Table 18

Averages of five groupings of the observations. See text

Scale of percentage rise or decline	Number of Observations					
	Actual Numbers			Percentages of Totals		
Central points of the groups averaged	Decline	Rise	Both	Decline	Rise	Both
4	0.2	0.8	1.0	0.4	1.5	0.9
9	2.0	2.6	4.6	3.9	4.7	4.3
14	9.8	8.8	18.6	19.2	16.0	17.5
19	11.8	10.0	21.8	23.1	18.2	20.6
24	6.2	8.8	15.0	12.2	16.0	14.1
29	5.6	9.8	15.4	11.0	17.8	14.5
34	6.4	4.4	10.8	12.5	8.0	10.2
39	5.0	5.4	10.4	9.8	9.8	9.8
44	1.2	2.4	3.6	2.4	4.4	3.4
49	1.6	1.0	2.6	3.1	1.8	2.5
54	1.2	0.6	1.8	2.4	1.1	1.7
59	...	0.4	0.4	...	0.7	0.4
Totals	51.0	55.0	106.0	100.0	100.0	99.9

	Arithmetic means	Medians	Standard deviations	Coefficients of variation
Decline	25.5	23	10.99	43%
Rise	25.6	25	10.97	43%
Both	25.5	24.2	10.89	43%

FREQUENCY DIAGRAMS
PERCENTAGE BASIS



35, 45, and even 50 per cent of the levels indicated by the trends. Perhaps the data from which these indexes are made overweight the activities which are particularly susceptible to the business-cycle hazard; that is another question which requires the study of elaborate evidence which would be confusing here. But one might make large reductions on this score and still leave a huge charge against depressions, without counting the lasting impairment of efficiency produced by business demoralization and unemployment.

(8) The Distinctive Character of Each Business Cycle.

As one analyzes successive business cycles in various ways, one finds evidence, even in the bleak statistical records here used, that each cycle has special characteristics of its own, or rather a special combination of characteristics. More intensive study carried over a wider range in time and space would strengthen this impression. Strictly speaking, every business cycle is a unique historical episode, differing in significant ways from all its predecessors, and never to be repeated in the future. Of course, the theory of business cycles aims primarily to find generalizations which can be applied to all cases. But it is wise for those on theory bent to realize clearly the multi-faceted variability of their cases. Such knowledge may even help them in the process of establishing generalizations. For one who is familiar with the idiosyncrasies of particular cycles will sometimes see that a given rule does apply to cases which at first sight seem to form exceptions. Both as a contribution to our general understanding of business cycles and as an aid to our later inquiries, we may note briefly some of the salient characteristics of the American cycles of 1878 to 1923.

(1) The cycle of 1878-85 followed an exceptionally long and exceptionally severe period of depression. When it finally started, revival was rapid; prosperity was sustained at a high level for an unusually long time; the recession was gentle, and the decline long drawn out. The only season of acute financial strain in this cycle came, not at the downward turning point, but late in the decline—the so-called "crisis of 1884."

(2) While the period of expansion was decidedly shorter than the period of contraction in the cycle of 1878-85, the reverse was true in the cycle of 1885-88. Again the recession was mild, there was no period of acute financial stress, and the depression was not severe.

(3) In the cycle of 1888-91 prosperity attained a higher pitch than in its predecessor, though not so high as in 1881. The recession of 1890 was accompanied by more financial strain than occurred in 1882 or 1887; but the difficulties seem to have been due largely to foreign influences connected with the collapse of Baring Brothers in London. The period of decline was even briefer and milder than in the preceding cycle, being cut short by an unusual harvest situation. The world crop of wheat was poor in 1891, the American crop abundant. Hence our farmers sold a large amount of grain at prices high for that period. Their prosperity, shared by the "granger" railroads and distributors in the agricultural districts, contributed powerfully to an early resumption of activity.

(4) The next period of expansion, 1891-93, was briefer than any of its predecessors shown by our indexes. It was terminated by the great panic of 1893, one of the longest and severest crises in American business history. While this business wave had not risen to a high crest, its trough was very low.

(5) We may call the fluctuations of 1894-97 a submerged cycle. Although the amplitudes of the rise and the decline were not far from the average amplitudes shown by Table 18, the preceding and the following troughs were so low that the crest of the wave did not quite reach the base line in two of our indexes, and barely rose above the base line in two others.

(6) Rising slowly from the low point of 1897, business had not attained a very high level when it was interrupted by the mild reaction of 1900. As in 1890, foreign difficulties seem to have been largely responsible for the recession. The period of contraction was both brief and mild.

(7) The cycle of 1900-04 contained the Northern Pacific corner of 1901, and the peculiar "rich-man's panic" of 1903. In financial circles the fluctuations were of great amplitude, as Snyder's deposits index shows. But business of other sorts was affected relatively little. The American Telephone and Telegraph Company's index makes this cycle the mildest in our list, while Snyder's deposits index makes it the most intense.

(8) Perhaps better than any other case in our period, the fluctuations of 1904-08 answer to the generalized conception of a business cycle presented in the theoretical treatises. From the depression of 1904, business made a fine recovery in 1905, maintained a high pitch of prosperity for some two years, passed through a severe crisis in the

autumn of 1907, and plunged into a new depression in 1908. All the familiar phenomena appeared in standard succession and sharply defined.

(9) and (10) The next two cycles (1908-11 and 1911-14), on the other hand, were mild affairs. While the revival from the depression of 1908 was vigorous, it did not lead to a boom; the recession in 1910 was not sharp, the depression of 1911 was not severe, and the succeeding period of expansion of 1912-13 was terminated early by another mild recession. But the depression which closed the second of these cycles gained dramatic intensity when it was accentuated by the outbreak of war at the end of July, 1914.

(11) Of course the war-time cycle of 1914-18 was distinguished by unusual features—extraordinary price fluctuations, a not less extraordinary shift in the character of production, extreme scarcity of labor, abundance of loan funds, and, toward the end, by government intervention in business on an unprecedented scale.

(12) Hardly less exceptional was the first post-war cycle of 1918-21. After the brief and mild depression ushered in by the Armistice of November, 1918, business started on a boom so sudden that the period which can be labeled "revival" was very brief. Again the price fluctuations were extremely violent. The crisis was of exceptional severity so far as industry was concerned, and, while the Federal Reserve System bore the financial strain with marked success, the subsequent depression was one of the worst in American experience. Yet one who realizes how profoundly economic activities in the United States were affected by the great war, from the time when its sudden onset shattered confidence to the time when industry won back to a peace basis, must wonder that it altered the usual round of business cycles so little. A person who did not know when the great war occurred, could not date it from inspection of our five business indexes, though it would stand out clearly in indexes made largely from price series.

(13) During the cycle of 1921-24, American business gradually returned to more settled conditions. While price fluctuations continued greater than they had been from 1878 to 1914, the price system attained a new equilibrium. After a rather slow recovery from the depression of 1921, business had a short period of almost feverish activity early in 1923, suffered a check, recovered in the opening months of 1924, and then entered upon a sharper decline.

(14) From this trough in the middle of 1924 we may date the

beginning of the cycle in the later stages of which this account is written.

VII. The Need of Combining Theory, Statistics, and History.

From the outset of this inquiry into business cycles, the need of statistical work has been clear. The first demonstration came from a quarter which few might expect—a review of theories made on non-statistical lines. By showing how many processes are involved in business cycles, these theories raised a series of essentially quantitative problems. Which of the causes of cyclical fluctuations stressed by different theorists are the most important? How considerable are the effects produced by these causes, directly and indirectly? What changes occur simultaneously? In what sequence, and after what intervals do other changes follow? How regular are cyclical fluctuations? All these are obviously questions which call for measured observations—very many measured observations upon diverse processes, systematically made in numerous markets over long series of years—in short, the type of observations which constitute statistics. Indeed, the idea suggested itself at the close of Chapter I, that the whole inquiry might shift from a search for causes conducted in the light of common reason to a quasi-mathematical study of the interrelations among a number of complex variables.

Again in Chapter II, when discussing the economic organization within which business cycles run their course, we found ourselves facing quantitative issues at every turn. From the section which dealt with the proportions of "real" income which families produce for themselves and the proportions which they buy with money, to the section on international differences of organization, we had to answer as best we could questions of how much and how often.

But now that in the present chapter we have surveyed the statistical materials and methods of particular concern to students of business cycles, we see that there are grave limitations upon the help we can expect from this source. Rapid progress has been made by the last generation in gathering and in utilizing statistical data; yet we are far from the goal of establishing the study of business cycles upon a strictly statistical basis.

On the technical side, our methods of determining both the secular trends and the seasonal variations of time series are rough. So far, no one has segregated irregular from cyclical fluctuations. While a method of approximating that result was suggested above and will be elaborated in Chapter V, it yields only averages, and is applicable only to series which cover a considerable number of cycles. Neither the visual study of charts nor the coefficient of correlation are wholly satisfactory methods of determining the relationships among fluctuations. In precisely what forms series should be expressed to bring out their most significant relationships is a problem which statisticians have posed rather than solved. Little systematic work has been done toward measuring the amplitude of the cyclical-irregular fluctuations characteristic of different processes. More attention has been given to the problems of time sequence, but the results are neither comprehensive nor secure. Finally, we have no index numbers of business cycles, nor any definite program for making them. As substitutes we must use a somewhat haphazard collection of indexes relating to such processes as happen to have been recorded in statistical form for considerable periods.

Yet graver limitations are imposed by the paucity of statistical materials. Of the various processes which the theories reviewed in Chapter I represent as of crucial importance, we have satisfactory data concerning not one. Wholesale prices, foreign trade, banking, railway transportation, the metropolitan money and securities markets are the fields best covered in the United States; but the investigator who works on any of these subjects develops many problems for which he cannot get solutions from his data. About profits, savings, advance orders and other future commitments, the production of consumers' goods and industrial equipment, the amount of income disbursed to consumers and their spending, our information is fragmentary. In certain respects other countries offer better data than the United States—the British unemployment returns, and the German receipts from the tax on domestic bills of exchange are examples; but broadly speaking the foreign records are more deficient than our own.

To overcome this handicap so far as possible, the National Bureau of Economic Research has made a systematic collection of economic and social statistics for the United States, Great Britain, France and Germany, which it hopes to publish in the near future for the benefit of all workers in the social sciences. Each series is described, annotated, and presented by months or quarters, if possible, for the full period since its start. The collection covers in considerable detail all types of economic activity, and the leading indicia of social changes.

By enlisting the coöperation of foreign experts, the compilers, Dr. Willard L. Thorp and Mr. Harold Villard, hope to keep errors of omission and commission to a minimum. In addition, the National Bureau is compiling various new series of data which throw light upon salient aspects of business cycles. Dr. Frederick R. Macaulay's studies of bond yields and interest rates since 1857, Dr. Harry Jerome's monograph upon *Migratión and Business Cycles,* Dr. Leo Wolman's critical investigations of the labor market, Dr. Frederick C. Mills' intensive work upon the interrelations among price fluctuations, and Dr. Simon S. Kuznets' study of secular trends all promise contributions of importance. Incidental use can be made also of our earlier studies of unemployment, and of Dr. Willford I. King's continuing estimates of income. By utilizing the general collection of statistics, together with the special studies made by the National Bureau and other agencies, we shall be doing what we can to give our inquiry a secure foundation of measurements.

One other source we have, intermediate between statistics and casual observation. This is the collection of *Business Annals,* recently published by the National Bureau. Diligent ransacking and critical comparison of many reports, periodicals, and pamphlets enabled Dr. Thorp to trace the course of business cycles over a longer period and a wider area than is covered by any but the most meager statistics. From his systematic records we can learn certain broad facts about the characteristics of recent cycles in countries of varying culture, and of early cycles in the countries which concern us most. These annals give the best opportunity for studying the international relationships of business fluctuations. They enable us even to make crude measurements of the duration of business cycles, which by covering many more cases supplement and broaden the conclusions drawn above from the business indexes. Exploiting this fresh source is our next task.

Just as a review of theories of business cycles made us see the need of statistics, so our review of statistics makes us see the need of economic history. Of course our historical survey must be condensed, like our summary of theories. When that survey has been completed, we shall not dwell upon the limitations of business annals —they will be obvious.

It is wise to face the shortcomings of each of these approaches to one problem—the theoretical, the statistical, and the historical. But critical though we must be of all our materials and methods, we can put the criticisms to constructive uses. What has just been said concerning the limitations of statistics should not check, but guide our dealings with tables and charts. The diversity of theories sketched in Chapter I, all plausible and each claiming to reveal the cause of most importance, seemed rather disconcerting. But from each explanation we may get some suggestion of value, and certainly neither our statistics nor our condensed business histories will enable us to do without much reasoning of the sort relied upon by economic theorists. To win as much knowledge as we can of business cycles, we must combine all that we can learn from theory, statistics, and history.