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9

Monetary Policy in Europe versus the United States
What Explains the Difference?

Harald Uhlig

9.1 Introduction

Interest rate paths during the last decade or so have been remarkably different in the United States and in Europe (see figure 9.1). What explains the difference?

The analysis of this chapter leads to the conclusion that the difference is due to surprises in productivity as well as surprises in wage demands—moving interest rates in opposite directions in Europe and the United States—but not due to a more sluggish response in Europe to the same shocks or to different monetary policy surprises. To obtain these conclusions, I have specified and estimated a hybrid new-Keynesian dynamic stochastic general equilibrium (DSGE) model and have used it to investigate three potential interpretations for the U.S.-European monetary union (EMU) difference.

The first interpretation is to argue that monetary policy is simply different. A number of observers have argued that the difference in policy shows the difference between an established central bank in the United States (which knows what it is doing and acts decisively, if need be), versus a new central bank in Europe, run by a committee that is too timid and too inertial to anything in time, following the U.S. example with too much caution and delay. A

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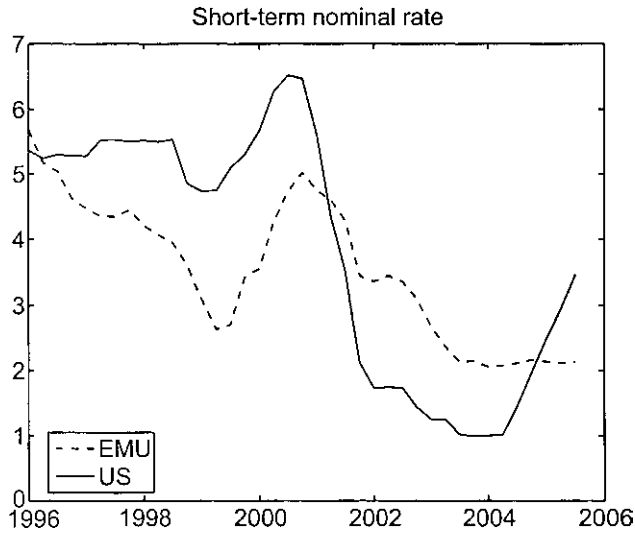


Fig. 9.1 Central bank rates in the EMU and in the United States

more benign interpretation—recently put forth by European Central Bank (ECB) president Trichet in a speech, "Activism and alertness in monetary policy," in Madrid 2006—argues that instead it is the ECB holding the steering wheel steady, while the monetary policy pursued by the Fed is just erratic.

The second interpretation is that the shocks simply have been different. For example, growth in the United States was considerably higher in the second half of the 1990s, giving rise to fear of "overheating" there and thereby possibly necessitating policy interventions, which then needed to be reversed, as the U.S. economy spun into a recession. While the decline in growth rates in EMU may have been similarly large between 2000 and 2002, the growth rate only briefly achieved U.S. levels in 2000 (see figure 9.2).

The third interpretation is that the structure of the economies are simply different. There are three striking differences in particular:

1. Labor markets are more rigid in Europe than in the United States. While one can point to some measures, the evidence here comes more from a variety of sources and qualitative measures, starting with labor market regulations and government interference in the labor market to union memberships and the role of unions in economic policy and the governance of firms.
2. The share of government is larger in Europe than it is in the United States. For the period from 1985 to 2005, mean government consumption to gross domestic product (GDP) was 16 percent in the United States and 20 percent in Europe. For government expenditure, the contrast was even more

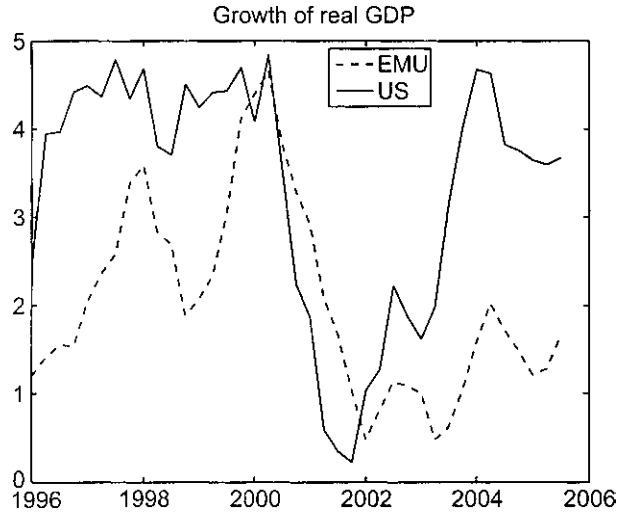


Fig. 9.2 Real GDP growth in the United States and EMU

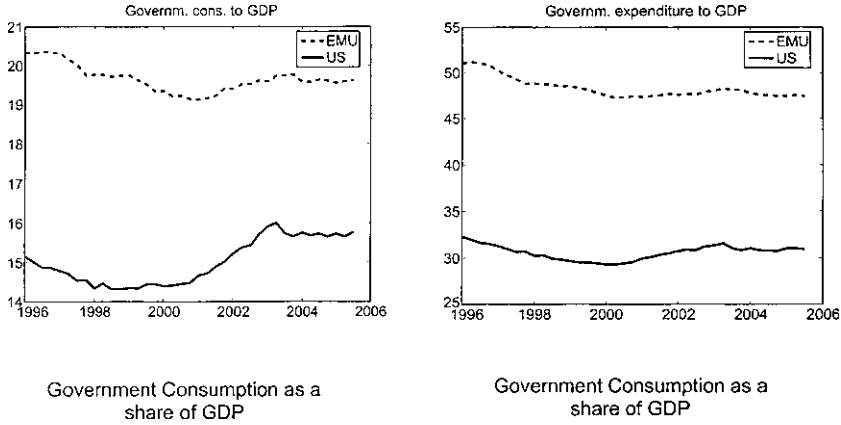


Fig. 9.3 Government consumption and government expenditure

striking, with 32 percent in the United States versus 50 percent in Europe (see also figure 9.3). Furthermore, fiscal policy is arguably more decentralized in Europe, with Brussels playing a minor role vis-à-vis the nation states in Europe compared to the federal government vis-à-vis state and local governments in the United States.

3. A much larger share of business is bank-financed rather than market-financed in EMU, compared to the United States. For example, de Fiore and Uhlig (2006) document that the ratio of debt-to-equity is .41 in the United States and .61 in Europe. Furthermore, the ratio of bank-to-bond

finance is 7.3 in the EMU and thus ten times as high as 0.74, the value for the United States.

It seems a priori plausible that these differences play a significant factor in the explanations for monetary policy. For example, government spending tends to be rather smooth and acyclical: a larger share of government spending might therefore lessen the role of price rigidities for the private economy.

Recent advances in the modeling of dynamic stochastic general equilibrium models—for example, Smets and Wouters (2003), Christiano, Eichenbaum, and Evans (2005) and related work—in particular have made it possible in principle to impose the key structural differences of the economy, estimate monetary policy reaction functions, and quantitatively account for the movements in key variables by a decomposition into the model-specific structural shocks. This avenue is therefore well-suited for answering the question at hand.

These models are built on recent advances in investigating the role of sticky prices for the economy and the new-Keynesian paradigm (see, in particular, Clarida, Galí, and Gertler [1999] and Woodford [2003]). Applying them directly to the task at hand poses three challenges, however.

1. Most of these models emphasize the role of sticky prices and the output gap in driving inflation rates. Frictions from the interaction between financial intermediation and monetary policy typically play no role or a role only insofar as they influence the output gap. This makes it challenging to address the third of the three key differences mentioned previously.
2. The distortionary role of nonmonetary economic policy typically plays a minor role. This makes it hard to address the first and the second of the aforementioned key differences.
3. In quantitative applications and estimations, many observable time series are used. An equivalent number of shocks is then used in order to generate a regular one-step ahead variance-covariance matrix of the prediction errors. This makes it challenging to avoid pushing key dynamic features of the economy into "measurement errors" instead, which then receive a structural interpretation.

There is an earlier literature, emphasizing financial frictions and the reallocational role of monetary injections. For example, Lucas and Stokey (1987) emphasize the role of cash for some of the transactions, while Bernanke, Gertler, and Gilchrist (1999) emphasize credit contracts arising in the presence of asymmetric information. While the new-Keynesian approach in focusing on sticky prices may be appealing for a number of reasons, it is useful for the task at hand to bring lessons of that earlier literature into this framework.

There are important contributions in the literature on which I can draw for

this task. In their seminal paper, Christiano, Eichenbaum, and Evans (2005) (henceforth, CEE) impose a cash-in-advance constraint for firms to pay their wage bill. Firms borrow these funds from financial intermediaries who in turn obtain funds from household deposits as well as central bank cash injection. While this feature of their paper seems there mostly to create some sort of money demand, it opens the possibility of studying financial frictions further. Schmitt-Grohe and Uribe (2004) and Altig et al. (2004) assume an additional cost for purchasing consumption goods, which depends on the velocity of the household's cash balances. Christiano, Motto, and Rostagno (2003) introduce a fairly rich banking sector, allowing for various monetary aggregates such as bank reserves and demand deposits, to study the role of money in the Great Depression.

To keep the model tractable, yet allow for some potentially important avenues, I largely follow the lead of CEE. I additionally allow for a cash-in-advance constraint on consumption good purchases in order to judge the relative importance of private transactions to firm borrowing. I allow for the possibility that not all cash injections are permanent, but instead are taken out of the system again at the end of the period (which one might think of as a one-off reverse transaction). Finally, I explicitly account for the cost of borrowing in the profit maximization problem and price setting problem of the firm, giving rise to an interest rate cost channel (see also Barth and Ramey [2001] and Secchi and Gaiotti [2006]). This is a modest contribution to solving the first of the three challenges listed previously.

I will explicitly allow for distortionary taxation of labor income, used to finance a stock of government as well as a certain level of government expenditure. I view this as a beginning to make progress on the second challenge. Certainly, several—although not all—monetary policy models of recent vintage have allowed for such influence of nonmonetary policy: this model is in the same tradition. In particular, Schmitt-Grohe and Uribe (2006) add distortionary income taxation to CEE.

For the third challenge, I use Dynare and thus off-the-shelves estimation techniques, and discuss some issues arising from mapping the dynamics into the dynamics for few observable series only, employing the "ABCD" framework of Fernandez-Villaverde, Rubio-Ramirez, and Sargent (2007). In particular, I will focus on a small set of observable variables, judiciously chosen, and allow for as many shocks as there are variables. It will turn out that one needs to be careful. It is not just enough to insure an invertible mapping from the shocks to the innovations of the variables, but furthermore, it is important to check invertibility of the Value at Risk (VAR) representation itself. We do this by "visually" inspecting the VAR coefficients in the derived representation (see section 9.4).

In sum, the model can perhaps best be described as a variant of the CEE model, with the following deviations:

1. The costs of adjusting the capital stock arise from the investment-to-capital ratio, not the investment-to-previous-investment ratio.
2. There is a cash-in-advance constraint for household consumption purchases.
3. Only a fraction of the cash injections, which "liquify" the loan market for firms, may permanently increase the money supply.
4. The interest rate costs for borrowing part of the input bill explicitly arises in the objective function of the intermediate good firms.
5. Capital utilization is constant.
6. There is a distortionary tax on wage income and firm profits. There is government debt.
7. There is no indexation.
8. There is real wage sluggishness, following Blanchard and Galí (2005).
9. Monetary policy is assumed to follow a Taylor rule.
10. There are six shocks: a productivity shock, an investment-specific shock, a wage setting shock, a monetary policy shock and two fiscal policy shocks, a tax rate shock, and a spending shock. For estimation, I only "turn on" the tax rate shock.
11. Estimation is in terms of five variables, inverting for the shocks per the recursive law of motion.

The approach of this chapter (as well as the results) share many similarities with the two slightly earlier papers by Sahuc and Smets (2008) and Christiano, Motto, and Rostagno (2007).

Sahuc and Smets (2008, 507) likewise come to the "overall conclusion [. . .] that differences in the size and the persistence of the shocks hitting the two economies is the main driving force behind the different interest rate behaviour." Their model differs from mine in several dimensions. Most notably, perhaps, there is no role for fiscal policy and hardly a role for differences in the financial structure in their paper.

Christiano, Motto, and Rostagno (2007) also share the view with this chapter that "the U.S. economy was aided during the most severe phase of the [2001] recession by favourable productivity shocks, which [. . .] helped keep inflation in check. By contrast, the slowdown in the Euro Area was exacerbated by negative productivity forces which also prevented inflation from ebbing" (5). These authors furthermore emphasize the greater persistence of ECB policy compared to Fed policy. This is in some contrast to our findings: while, for example, monetary policy shocks are more persistent in the EMU than the United States according to our findings, interest rates are not.

The model by Christiano, Motto, and Rostagno (2007) features a much more detailed entrepreneurial sector as well as more details on the banking sector, and therefore makes more progress than this chapter in its ability to address the differences in financial structure between the United States and

Europe. There is no role for fiscal policy in their paper, though. Their model is driven by fifteen shocks, whereas my model features only five. The costs of adjusting capital in their model are determined by the change in investment, whereas it is determined (more classically) by the ratio of investment to capital here.

These two papers therefore complement the investigation here. Despite a number of modeling differences they come to fairly similar conclusions, which ought to provide additional trust in the conclusions drawn.

Section 9.2 explains the model. A technical appendix provides the details for the analysis of the model. Section 9.3 explains the estimation strategy and lists the parameters used for the comparison. Section 9.4 is devoted to the invertibility issue. Section 9.5 provides results. Section 9.6 discusses these results and offers some tentative conclusions.

9.2 The Model

The model is a combination of a cash-in-advance model and a Calvo sticky-price model, amended with a role for a government.

Time is discrete. There are identical households, who supply labor and enjoy final consumption. They own all firms. They use cash for parts of their transactions. There is a competitive sector of final goods producing firms. There is a unit interval of monopolistic intermediate good firms, using labor to produce output and setting sticky prices. They need to borrow a fraction of their input bill from commercial banks. Commercial banks take deposits from households and receive cash injections from the central bank. They lend to intermediate goods firms. The central bank injects cash and thereby sets the nominal interest rate. The government taxes wage income and uses it to finance government purchases as well as debt repayments. Nominal wages are sluggish on the aggregate level.

A period has four parts:

1. Shocks are realized. The new nominal wage for the period is set. The central bank injects cash Ψ_t to banks.
2. A fraction of intermediate good firms is chosen to reset its price. Intermediate good firms "guess" demand and produce accordingly, hiring labor at the market wage. They are assumed to be required to borrow a fixed fraction of the input bill from banks.
3. Households shop, using cash at hand as well. Government shops, using tax receipts as well as a short-term credit line from the central bank.
4. Financial markets open. Firms pay capital rental payments and wages to households. Firms pay interest to banks. They pay profits to households. Households pay taxes to the government. The government issues new bonds and repays old bonds. The household splits the remaining cash into deposits with banks and cash-at-hand for the next period.

9.2.1 Households

Households enjoy final consumption c_t and dislike labor n_t according to

$$U = E\left[\sum_{t=0}^{\infty} \beta^t(\log(c_t - \chi c_{t-1}) - An_t^{1+\sigma})\right], \tag{1}$$

where $0 \leq \chi < 1$ is a habit parameter and $1/\sigma > 0$ is the Frisch elasticity of labor supply. Households enter period t, holding deposits D_{t-1} at financial intermediaries and cash-at-hand H_{t-1}. In the second part of the period, they supply labor n_t according to demand at the market wage W_t. In the third part, they use cash-at-hand to shop for a fraction η of consumption,

$$H_{t,res} + \eta P_t c_t = H_{t-1}, \tag{2}$$

holding residual cash $H_{t,res} \geq 0$. I essentially assume that there are cash goods and credit goods as in Lucas and Stokey (1987), but that these cash and credit goods are purchased in fixed proportion for consumption, and that investment goods are always credit goods.[1] The latter would be implied by a Leontieff specification for the preferences in cash and credit goods. In principle, the household may spend less cash than available. However, I shall assume that shocks and parameters are such that the constraint on residual cash is binding, $H_{t,res} = 0$.

In the fourth part of the period, households receive after-tax nominal wages and trade all contingent claims, as well as firm shares and government bonds, and pay for the remaining $(1 - \eta)$ share of their purchases ("credit goods"). Netting out all household-to-household trades, the financial market budget constraint is

$$\begin{aligned} H_t + D_t + q_t B_t + (1 - \eta)P_t c_t + P_t x_t = \\ (1 - \tau_t)W_t n_t + (1 + i_t)D_{t-1} + P_t r_t K_{t-1} + (1 - \tau_V)V_t + B_{t-1} + H_{t,res}, \end{aligned} \tag{3}$$

where H_t is cash-at-hand for the next period, D_t is deposited with banks, q_t is the discount price for government bonds B_t, $1 + i_t$ is the return paid by banks on deposits D_{t-1}, $P_t r_t$ is the nominal rental rate for capital, V_t is the value added of intermediate good firms, and B_{t-1} are the debt repayments by the government.

One can extend this budget constraint with between-household trades. In particular, let $\Lambda_{t,t+k}$ be the discount price on the financial market at t for an extra unit of cash on the financial market at date $t + k$.

Also, households produce new capital subject according to

$$k_t = \left(1 - \delta + \varphi\left((1 + u_{x,t})\frac{x_t}{k_{t-1}}\right)\right)k_{t-1}, \tag{4}$$

1. A key reason for introducing the cash-in-advance constraint on only a fraction of the goods is that otherwise the money stock becomes quantitatively large in this model, implying that seignorage is a substantial fraction of the government budget constraint.

where the adjustment cost function $\varphi(\cdot)$ satisfies

$$\varphi(\delta) = \delta,\ \varphi'(\delta) = 1,\ \delta\varphi''(\delta) = -\frac{1}{\varpi}$$

for some $\varpi > 0$ (see Jermann [1998]), and where $u_{x,t}$ is a possibly persistent investment-specific disturbance,

$$u_{x,t} = \rho_x u_{x,t-1} + \varepsilon_{x,t}, \tag{5}$$

following Fisher (2006).

9.2.2 Final Good Firms

Final good firms take inputs $y_{t,j}$ to produce a final good y_t according to the production function

$$y_t = \left(\int_0^1 y_{t,j}^{1/(1+\mu)} dj \right)^{1+\mu}. \tag{6}$$

They purchase intermediate goods at price $P_{t,i}$ per unit and sell the final good at price P_t.

9.2.3 Intermediate Good Firms

Given a current intermediate goods price $P_{t,j}$, intermediate good firms "guess" their demand $y_{t,j}$ resulting from the demand of final good firms, see equation (62). They thus hire labor $n_{t,j}$ at nominal wages W_t and rent capital $k_{t,j}$ at nominal rental rates $P_t r_t$ to produce output according to

$$y_{t,j} = \begin{cases} \gamma_t k_{t,j}^{\theta} n_{t,j}^{1-\theta} - \Phi & \text{if } \gamma_t n_{t,j} > \Phi \\ 0 & \text{otherwise,} \end{cases} \tag{7}$$

where γ_t is an exogenous process for the change in technology and Φ is a parameter of the production function and might be thought of as a fixed cost of production. Let $\hat{\gamma}_t = \log(\gamma_t) - \log(\overline{\gamma})$ for some appropriate $\overline{\gamma}$, and assume

$$\begin{aligned} \hat{\gamma}_t &= \rho_{\gamma,L} \hat{\gamma}_{t-1} + u_{\gamma,t} \\ u_{\gamma,t} &= \rho_{\gamma,u} u_{\gamma,t-1} + \varepsilon_{\gamma,t}. \end{aligned} \tag{8}$$

I assume that the firm needs to obtain a loan $L_{t,j}$ for a fraction ξ_t of the input bill, on which a nominal market interest rate i_t needs to be paid. The rest of the input bill is paid for per trade credit (or more efficient market instruments) to be settled at the end of the period, on which no interest needs to be paid. That is, let MC_t be the nominal marginal costs of producing an extra unit of output, excluding the additional costs of borrowing (see equation [54]). Then,

$$L_{t,j} = \xi \mathrm{MC}_t y_{t,j}$$

and the value added of this firm (or, equivalently, end-of-period profits) are

$$V_{t,j} = (P_{t,j} - (1 + \xi i_t)\mathrm{MC}_t)y_{t,j}. \tag{9}$$

Firms get to reoptimize prices with probability $1 - \alpha$, independently of their past. If they cannot reoptimize prices, they will be adjusted at the average inflation rate; that is,

$$P_{t,j} = \bar{\pi} P_{t-1,j}. \tag{10}$$

When given a chance to reoptimize prices, they will choose it so as to maximize discounted value added along the no-optimization-of-prices path[2]

$$\mathrm{NPV}_{t,j} = E\left[\sum_{k=0}^{\infty} \alpha^k \Lambda_{t,t+k} V_{t,j}\right], \tag{11}$$

where $\Lambda_{t,t+k}$ is the market price at date t for an extra unit of cash at date $t + k$ on the financial markets in part four of the period.

9.2.4 Commercial Banks

Banks compete for deposits from households and can borrow from the central bank. They then compete for giving loans to firms. Banks collect the returns on their loans in the fourth part of the period, and then repay households as well as the central bank. In equilibrium, banks make zero profits. Thus, there will be a market nominal rate of return i_t on loans, deposits, and central bank money.

9.2.5 The Central Bank

The central bank provides cash Ψ_t into the economy via providing loans to the commercial banks at the nominal interest rate i_t. It may be best to think of this as open market operations. The interest earnings on this open market operation constitute seignorage. Additionally, the central bank declares a fraction ν of the cash injection to be seignorage, not to be taken out of the system after repayment by the commercial banks. Thus, the government receives a central bank profit transfer of $(\nu + i_t)\Psi_t$ in part four of the period.

Note that only $\nu\Psi_t$, but not the interest earnings on the cash injection (or even the entire cash injection) constitute an increase in the money supply,

$$M_t = M_{t-1} + \nu\Psi_t. \tag{12}$$

The parameter ν allows the distinction between a short-run liquidity injection and a long-run increase in money supply. If $\nu = 0$, liquidity is provided only temporarily, and taken out of the economy after the injection. Seignorage is then given only by the interest earned on the short-term injection. By

2. Note that I assume that value added or profits are taxed at rate τ_V. Because I hold this rate constant, maximizing the net present value of before-tax value added is equivalent to maximizing the net present value of after-tax value added, which would be the more appropriate objective.

contrast, $\nu = 1$ means that any short-term injection also increases money supply in the long run.

Recall that the output gap is defined as the difference between actual output and the output that would emerge in the absence of sticky prices and absence of stickiness in wages; that is, for $\alpha = 0$ and $\omega = 1$, but keeping the friction of borrowing from banks. In an economy without sticky prices and sticky wages and without the need to borrow from banks, real marginal costs will be constant. The percent deviation of actual real marginal costs

$$\text{mc}_t = \frac{\text{MC}_t}{P_t} \tag{13}$$

from its steady-state level can therefore serve as a proxy for the output gap.

I therefore assume that the central bank follows a Taylor rule in setting interest rates, using this ratio that

$$i_t = \bar{i} + \rho_{i,L} i_{t-1} + (1 - \rho_{i,L})\left(\zeta_\pi\left(\frac{\pi_t}{\bar{\pi}} - 1\right) + \zeta_x\left(\frac{\text{mc}_t}{\overline{\text{mc}}} - 1\right) + u_{i,t}\right), \tag{14}$$

where

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{15}$$

is inflation, where $\bar{\pi}$ is the inflation target, $\bar{i}$ is the steady-state nominal rate, ζ_π and ζ_x are coefficients of the policy rule, and where

$$u_{i,t} = \rho_{i,u} u_{i,t-1} + \varepsilon_{i,t} \tag{16}$$

is a possibly persistent distortion to the Taylor rule, driven by the monetary policy shock $\varepsilon_{i,t}$.

9.2.6 The Government

The budget constraint of the government at the end of the period is given by

$$q_t B_t = B_{t-1} + P_t g_t - \tau_t W_t n_t - \tau_V V_t - (\nu + i_t)\Psi_t. \tag{17}$$

The government does not carry cash from one period to the next. However, the government is assumed to finance its purchases within the period via a short-term credit from the central bank. Thus, government spending $P_t g_t$ is akin to a short-term cash injection on the demand side. This is consistent with the view that the central bank acts as the "checking account" bank to the government. Note that I do not allow the government to borrow from the central bank in the long term.

Define real debt

$$b_t = \frac{B_t}{P_t}, \tag{18}$$

real seignorage

$$\psi_t = \frac{\Psi_t}{P_t}, \tag{19}$$

real value added

$$v_t = \frac{V_t}{P_t}, \tag{20}$$

as well as real wages

$$w_t = \frac{W_t}{P_t}. \tag{21}$$

I assume that the government aims at some steady-state debt-to-GDP ratio $\overline{b}/\overline{y}$, as well as some steady-state level government-spending-to-GDP ratio $\overline{g}/\overline{y}$. Given all other parameters, let $\overline{\tau}$ be the steady-state tax rate on wage income consistent with these targets.

I assume that the government follows the policy rule of adjusting future tax and spending plans, if the current debt level b_t deviates from its target level $\overline{b}$,

$$\tau_t - \overline{\tau} = \zeta_\tau \left(\frac{b_{t-1} - \overline{b}}{\overline{y}} - 1 \right) + u_{\tau,t} \tag{22}$$

$$\frac{g_t}{\overline{y}} = \zeta_g \left(\frac{b_{t-1} - \overline{b}}{\overline{y}} - 1 \right) + u_{g,t}, \tag{23}$$

where $\zeta_\tau \geq 0$ and $\zeta_g \leq 0$ such that the dynamics of government debt remains stable, and where both equations are driven by possibly persistent distortions

$$u_{\tau,t} = \rho_\tau u_{\tau,t-1} + \varepsilon_{\tau,t} \tag{24}$$

$$u_{g,t} = \rho_g u_{g,t-1} + \varepsilon_{g,t}, \tag{25}$$

driven by the fiscal tax shock $\varepsilon_{\tau,t}$ and the fiscal spending shock $\varepsilon_{g,t}$.

9.2.7 Labor Markets and Wage Setting

I assume that wages move sluggishly on the aggregate level. A common form to generate nominal wage sluggishness is to assume Calvo wage stickiness for wage setters (see Erceg, Henderson, and Levin [2000]). A different literature has emphasized frictions or sluggishness stemming from bargaining as the route cause (see Shimer [2005] or Hall [2005]), giving direct rise to real wage sluggishness. The form I use here has been adapted from Blanchard and Galí (2005) and has been used, for example, in Uhlig (2007).

More specifically, let $W_{t,f}$ be the wage emerging from the first-order condition of the households' maximization problem. I assume that

$$W_t = ((1 - \omega)\pi_t W_{t-1} + \omega \Upsilon W_{t,f})(1 + u_{w,t}) \tag{26}$$

for some $\Upsilon > 1$ and a possibly persistent stochastic distortion

$$u_{w,t} = \rho_w u_{w,t-1} + \varepsilon_{w,t}. \tag{27}$$

An alternative interpretation of the distortion $u_{w,t}$ is to view it as being driven by fluctuations in the preference parameter A, manifested in stochastic fluctuations of the market-clearing wage $W_{t,f}$. This perspective may be a reasonable shortcut in order to account for the fluctuations in female labor supply, for example.

Assuming moderate-size fluctuations, actual wages will exceed the wage stemming from the first-order condition, $W_t > W_{t,f}$, and thus, labor markets will be demand constrained. That is, I assume that households always supply labor at the going wage. Note that (26) can be rewritten in terms of real wages as

$$w_t = ((1 - \omega)w_{t-1} + \omega\Upsilon w_{t,f})(1 + u_{w,t}), \tag{28}$$

where $w_{t,f} = W_{t,f}/P_t$.

9.2.8 Aggregation and Market Clearing

1. Money market: Post-injection money supply equals end-of-period money demand. This is given by

$$M_t = D_t + H_t. \tag{29}$$

2. Final goods market:

$$g_t + c_t + x_t = y_t. \tag{30}$$

3. Labor market:

$$n_t = \int_0^1 n_{t,j} dj. \tag{31}$$

4. Capital market:

$$k_{t-1} = \int_0^1 k_{t-1,j} dj. \tag{32}$$

5. Loan market:

$$D_{t-1} + \Psi_t = L_t = \int_0^1 L_{t,j} dj. \tag{33}$$

9.2.9 Equilibrium and Solution

An equilibrium is an allocation, policy parameters and prices (including returns and profits) such that

1. The allocation solves the problem of the representative household, given prices and policy parameters.

2. The allocation solves the firms maximization problems, given prices and policy parameters.
3. The constraints for the government and the central bank hold.
4. Markets clear.

To solve for the equilibrium, I characterize the first-order conditions, explicitly solve for the steady state, and characterize the dynamics per log-linearization around the steady state. I then compute the recursive law of motion solving these log-linearized equations. Details are available in a technical appendix.

9.3 Data and Estimation

I assume that $u_{g,t} \equiv 0$; that is, I assume that there are no fiscal spending shocks. This is reasonable in light of the smoothness[3] in figure 9.3.

There are five shocks in the model: I therefore need observations on five time series to solve for these shocks:

1. Inflation, π_t. I calculate it using the GDP deflator, since I am using real GDP in some other measures. A popular alternative is to use the consumer price index (CPI).
2. The central bank interest rate or short rate, i_t.
3. Labor productivity, y_t/n_t. For y_t, I use real GDP. For n_t, I use employment rather than hours worked. In a boom, more part-time labor will be hired, but also, more "uncounted" hours are worked by employees: thus, it may be that employment rather than hours is a more reasonable variable to measure fluctuations in labor input. It was also the series that was more easily available.
4. The consumption-to-GDP ratio, c_t/y_t. Cochrane (1994) in particular has shown that this ratio has predictive power for GDP growth and a number of other variables. Theory implies that this statistic indeed provides key information, so it is included here.
5. The debt-to-GDP ratio, b_t/y_t.

For the EMU, the data has been obtained from the ECB, and is in use for the area-wide model. For the United States, the data has been obtained from the Federal Reserve Bank of St. Louis. For debt, I have used the series GFDEBTN; that is, debt on the federal level.

I have used quarterly data from 1985 to 2005, striking a compromise between getting a reasonably long time span for data and relying on a reasonably stable monetary policy environment. While EMU only exists since

3. It also appeared to be initially sensible for the invertibility issue discussed in the next section, when doing an exploration of the model properties with freely chosen parameters. That issue seemed to disappear with the estimated parameters, though.

1999, one might argue that the Bundesbank has effectively played the role of a European central bank in the time before.

I am comparing the model in its log-linearized version—that is, in terms of log-deviations from the steady state—to the data. I therefore take logs of all variables, and removed the means. The resulting five time series used in estimation can therefore be seen in figure 9.4. In particular, I have linearly

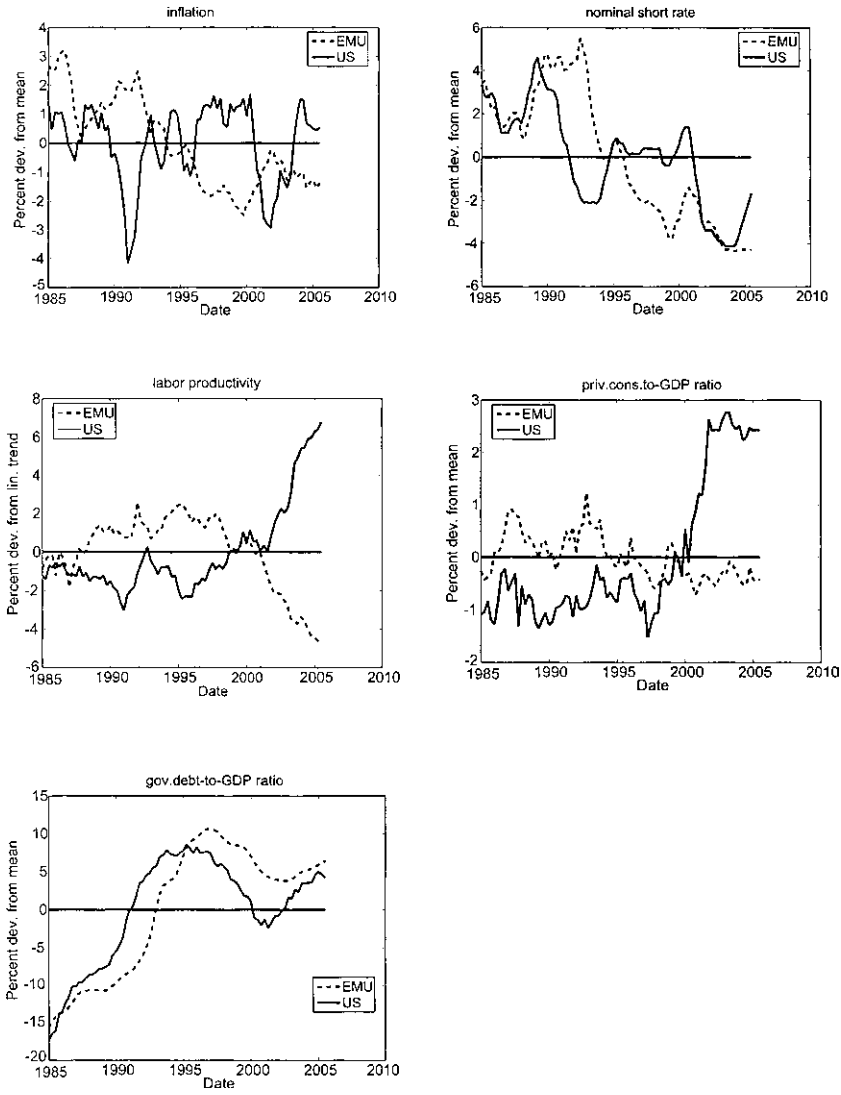


Fig. 9.4 Data used for estimation

Table 9.1 **List of parameters fixed a priori**

Parameter	U.S.	EMU, if different	Interpretation
γ	1		Productivity
β	0.99		Discount factor
θ	0.36		Capital share
δ	0.02		Depreciation rate
σ	1		Inverse Frisch elasticity
Φ	$0.8 \cdot$ markup		Fixed cost
μ	0.5		Markup
ϖ	2		Cost of adjustment of capital
Υ	1.1		Wage markup
ξ	0.1	0.5	Bank financing share
g/y	0.15	0.2	Gov. spending to GDP
b/y	0.62		Debt-to-GDP ratio
π	1.033		Inflation
τ_V	0.2		Profit or value added tax

detrended labor productivity. If there is a constant time trend in γ_t, it is fairly straightforward to correct all equations for it: essentially, this amounts to a slight correction in the discount rate. If the time trend is stochastic, the correction would imply a different set of equations, comparing everything to the current level of productivity. Since I log-linearized the model around a steady state with constant productivity, the linear detrending method is therefore more compatible with the theory.

The linearized model has been estimated using Dynare. In so doing, I have fixed a number of parameters, and estimated others. A list is given in tables 9.1 and 9.2.

For the parameters fixed a priori, I have set $\overline{n} = 1$, backing out the preference parameter A, rather than vice versa. In order to capture the different importance of banking in Europe versus the United States, I have fixed $\xi = 0.5$ for Europe, and $\xi = 0.1$ for the United States. A good calibration for these numbers would be sensible: the results here instead should be taken as indicative for what would happen for reasonable, although perhaps not sufficiently carefully calibrated, values for these parameters. The factor five was chosen to roughly reflect the approximately fivefold financing of firms through banks (rather than capital market instruments and stocks) in Europe compared to the United States. I have used 1 for the inverse Frisch elasticity σ of labor supply. All the other parameters are fairly standard.

For the estimated parameters, I have chosen rather uninformative priors. For parameters that should sensibly be in the unit interval, I used a uniform distribution, or, equivalently, a beta distribution with mean 0.5 and a standard deviation of $1/\sqrt{12} = 0.29$. For parameters that ought to be positive, I have used an inverted gamma distribution with infinite variance. I have used

Table 9.2 List of estimated parameters

Parameter	Distribution	Mean	Standard deviation	Interpretation
η	beta	0.5	0.29	Cash-in-advance share
ν	beta	0.5	0.29	Permanent liquidity
α	beta	0.5	0.29	Calvo prob. of stickiness
χ	beta	0.5	0.29	Habit share
ω	beta	0.5	0.29	Wage sluggishness
ρ_w	beta	0.5	0.29	Autocorr. wage disturb.
$\rho_{\gamma,L}$	beta	0.5	0.29	Autoregr. techn.
$\rho_{\gamma,u}$	beta	0.5	0.29	Autocorr. techn. disturb.
ρ_x	beta	0.5	0.29	Autocorr. inv disturb.
$\rho_{i,L}$	beta	0.5	0.29	Autoregr. int. rate
$\rho_{i,u}$	beta	0.5	0.29	Autocorr. int. rate disturb.
ρ_τ	beta	0.5	0.29	Autocorr. tax disturb.
ζ_τ	inv.gamma	0.5	∞	Tax rule
ζ_g	normal	–0.2	1	Spending rule
ζ_π	inv.gamma	1.5	∞	Taylor rule: on inflation
ζ_x	inv.gamma	0.5	∞	Taylor rule: on markup
stderr(ε_γ)	inv.gamma	0.2	∞	Std. err techn.
stderr(ε_i)	inv.gamma	0.2	∞	Std. err int. rate
stderr(ε_τ)	inv.gamma	0.2	∞	Std. err tax rate
stderr(ε_x)	inv.gamma	0.2	∞	Std. err inv. shock
stderr(ε_w)	inv.gamma	0.2	∞	Std. err wage shock

Note: std. err. = standard error.

a normal distribution centered at zero and a standard deviation of 1 for ζ_g, which is certainly wide.

9.4 A, B, C, and D's of VARs

When estimating a model with just a subset of variables, the issue of invertibility may be of concern. Invertibility may matter even more for recovering the sequence of shocks explaining the observations. I use the ABCD framework of Fernandez-Villaverde, Rubio-Ramirez, and Sargent (2007) to investigate the issue: the name of their paper has inspired the choice of the title for this subsection.

Let x_t be the list of log-deviations from steady state for all variables in the economy, including the exogenous disturbances $u_{i,t}$, and so forth. Let y_t be a list of observable variables, and let ε_t be the vector of i.i.d. shocks driving the system. Solving the linearized model with, for example, the methods exposited in Uhlig (1999), provides a recursive law of motion

$$x_t = Ax_{t-1} + B\varepsilon_t \tag{34}$$

$$y_t = Cx_{t-1} + D\varepsilon_t. \tag{35}$$

Assume that D is square and invertible, and that the eigenvalues of $(A - BD^{-1}C)$ are strictly less than one in modulus. Fernandez-Villaverde, Rubio-Ramirez, and Sargent (2007) show that

$$y_t = C\sum_{j=0}^{\infty}(A - BD^{-1}C)^{-j}y_{t-j-1} + D\varepsilon_t \tag{36}$$

is an (infinite-order) vector autoregression for y_t, and that $D\varepsilon_t$ are the one-step ahead forecasts for y_t.

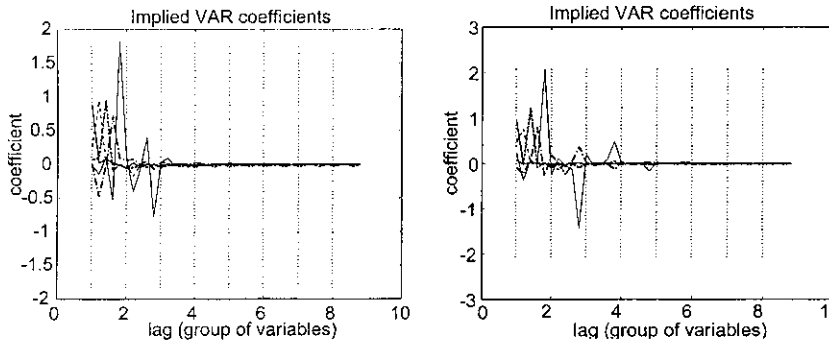
Let

$$y_t = C\sum_{j=0}^{k}(A - BD^{-1}C)^{-1}y_{t-j-1} + D\varepsilon_t + \vartheta_{k,t}, \tag{37}$$

be a finite-order approximation to the infinite-order VAR in (36), defining the approximation error $\vartheta_{k,t}$. Given a recursive law of motion as in (34) and (35), and assuming D to be square and invertible, it is always possible to calculate the finite-order approximation (37). In practice, one would drop $\vartheta_{k,t}$ from this equation, hoping that it is small. Equation (37) then provides for a convenient procedure to recover the residuals ε_t driving the data.

But $\vartheta_{k,t}$ may not be small, either, because the eigenvalues of $(A - BD^{-1}C)$ are not strictly less than one in modulus, or because they are only just below one, with the coefficients in (36) only gradually dying out with increasing lag length. The latter is the problem emphasized by Chari, Kehoe, and McGrattan (2005).

It may thus be useful to examine how fast the coefficients in (37) die out at a specific parameterization of the model. Grouping the coefficients together according to lag length, I do this in figure 9.5 for the coefficient specifications



VAR coefficients for y_t
US parameterization

VAR coefficients for y_t
EMU parameterization

Fig. 9.5 Coefficients in the derived VAR representation for y_t

following for the United States and EMU. Note that the VAR coefficients die out quite fast.

9.5 Results

9.5.1 Estimates

The results of the estimation are provided in table 9.3. The results are taken directly from Dynare, using standard settings. While some of the confidence intervals are perhaps too tight—most likely pointing to yet insufficient sampling—the estimates all appear to be reasonable.

Taking these estimates at face value, there are some interesting differences as well as similarities in the comparison of the United States to EMU. Surprisingly, according to these estimates, wages actually appear to be more flexible and less sluggish in the EMU rather than the United States, with $\omega = 0.18$ and $\rho_w = 0.88$ there, as opposed to $\omega = 0.06$ and $\rho_w = 0.686$ in the EMU. Less surprisingly, prices appear to be more sticky in EMU with $\alpha = 0.778$ than the United States, with $\alpha = 0.668$. Productivity (or, for the United

Table 9.3 **Estimation results**

	U.S.		EMU	
Parameter	Mean	Conf. interval	Mean	Conf. interval
η	0.57	[0.23, 0.90]	0.38	[0.16, 0.60]
ν	0.27	[0.00, 0.68]	0.981	[0.974, 0.995]
α	0.668	[0.667, 0.676]	0.778	[0.769, 0.793]
χ	0.64	[0.64, 0.65]	0.35	[0.30, 0.42]
ω	0.18	[0.17, 0.18]	0.061	[0.055, 0.078]
ρ_w	0.88	[0.88, 0.89]	0.686	[0.682, 0.685]
$\rho_{\gamma,L}$	0.00	[0.00, 0.00]	0.979	[0.978, 0.980]
$\rho_{\gamma,u}$	0.93	[0.93, 0.96]	0.266	[0.248, 0.262]
ρ_x	1	[1, 1]	0.962	[0.958, 0.964]
$\rho_{i,L}$	0.73	[0.73, 0.74]	0.289	[0.285, 0.290]
$\rho_{i,u}$	0.24	[0.22, 0.24]	0.496	[0.495, 0.498]
ρ_τ	0.99	[0.99, 0.99]	0.985	[0.982, 0.984]
ζ_τ	0.18	[0.18, 0.18]	0.356	[0.352, 0.355]
ζ_g	–0.058	[–0.058, –0.058]	0.079	[0.078, 0.079]
ζ_π	1.07	[1.07, 1.07]	1.192	[1.192, 1.193]
ζ_x	0.35	[0.35, 0.35]	0.211	[0.210, 0.211]
std. err.(ε_γ)	0.26	[0.24, 0.26]	0.30	[0.29, 0.33]
std. err.(ε_i)	2.38	[2.38, 2.55]	1.08	[1.05, 1.24]
std. err.(ε_τ)	1.56	[1.58, 1.64]	0.52	[0.49, 0.58]
std. err.(ε_x)	2.15	[2.07, 2.25]	1.95	[1.81, 2.16]
std. err.(ε_w)	0.84	[0.80, 0.87]	0.49	[0.45, 0.56]

Note: std. err. = standard error.

States, the productivity disturbance), tax disturbances, and investment-specific disturbances are all essentially random walks.

The fraction η of cash required for consumption transactions is about one-third in EMU and about one-half in the United States. Monetary injections seem to be temporary in the United States, $\nu = 0.27$, but permanent in Europe, $\nu = 0.981$. Interest rates show a persistence of $\rho_{i,L} = 0.289$: one-third in EMU and about three-quarters in the United States: if anything, interest rate choices appear to be more sluggish in the United States. The Taylor rule coefficients are about 1.2 on inflation and 0.2 on markup in the EMU, which is reasonable. They are slightly lower for inflation and slightly higher on markup for the United States.

The feedback coefficients for fiscal policy differ in an interesting way. In response to a higher debt burden, the United States moderately raises taxes, $\zeta_\tau = 0.18$, and cuts spending, $\zeta_g = -0.058$, while the Europeans actually increase spending, $\zeta_g = 0.079$, and finance it by raising taxes even more, $\zeta_\tau = 0.356$.

Monetary policy shocks, tax shocks, and wage shocks show considerably larger standard deviations in the United States than in EMU.

As a postscriptum, the estimation results and therefore the conclusions based on them should be viewed with a considerable degree of caution. Note that the parameters are estimated rather indirectly: identification is achieved through their impact on the dynamics of the whole system, rather than some more direct consequence. It is likely that misspecification of the model can easily thwart the attempt to draw reasonable inference here: investigating that issue is beyond the scope of this chapter.

Even with the route taken here, it turns out that the model and its estimation appear to be quite sensitive, in particular with respect to the parameters ξ, as well as the fiscal policy parameters $\overline{g}/\overline{y}$, ζ_τ, ζ_g. For example, it is fairly easy to find parameter combinations where Dynare delivers nonsensical results or complains about violations of the Blanchard-Kahn condition for the prior, while it is still possible to calculate solutions with my "toolkit": as an example, take $\xi = 1$, $\overline{g}/\overline{y} = 0.35$, $\zeta_\tau = 1$, $\zeta_g = 0$, and otherwise taking prior means for all other variables. For some other parameter settings, one obtains warnings about badly scaled matrices and difficulties in starting the Markov chain. It is also not unusual that the posterior maximization procedure encounters a cliff shortly before it declare the maximum to be reached. This is true in particular for the estimation of the U.S. model, possibly explaining the unplausibly tight confidence bands for several parameters. The estimation results can also depend quite substantially on $\overline{g}/\overline{y}$ and ξ, which have been fixed a priori. In sum, either the model or the estimation procedure is ill-behaved in certain aspects. Exploring these sensitivities and the reasons further would be interesting, but beyond the scope of this chapter.

9.5.2 Impulse Responses

To understand the properties of the model, I have calculated the impulse responses to shocks 1 percent in size, with the estimated parameters set at the posterior means rounded to two digits. Figure 9.6 shows the impulse response of the nominal interest rate. Figure 9.7 shows the response of out-

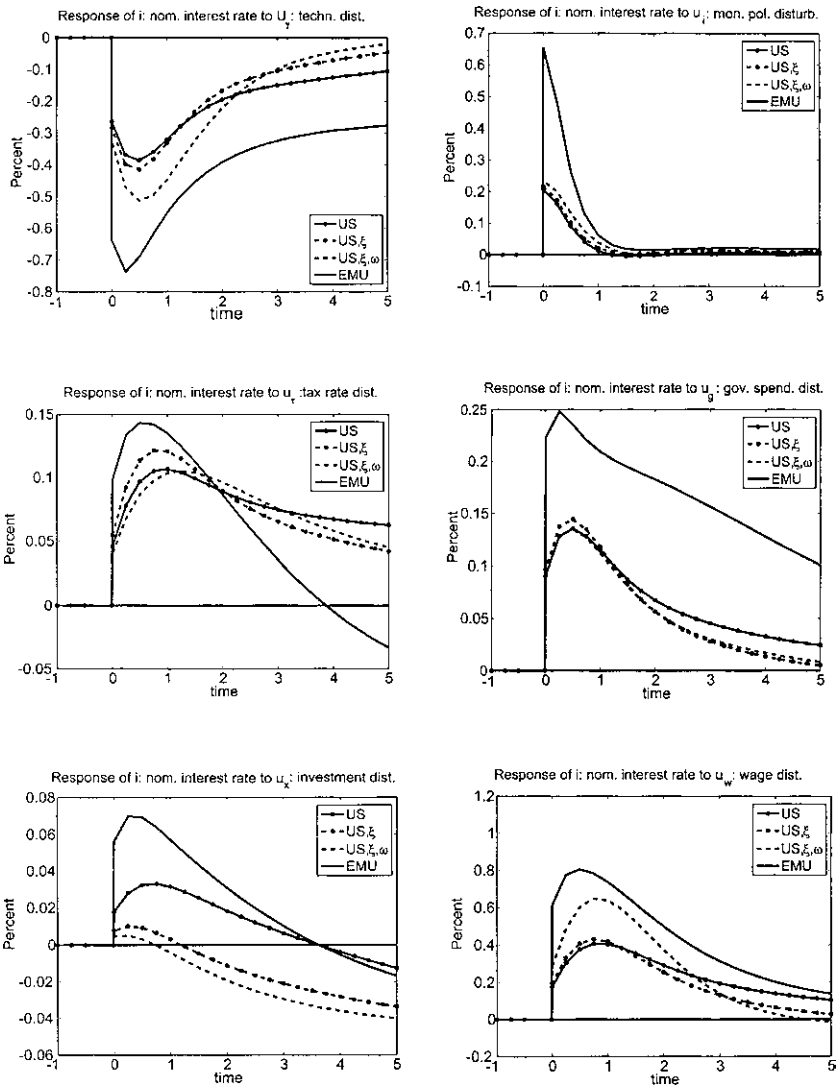


Fig. 9.6 Impulse responses of interest rates

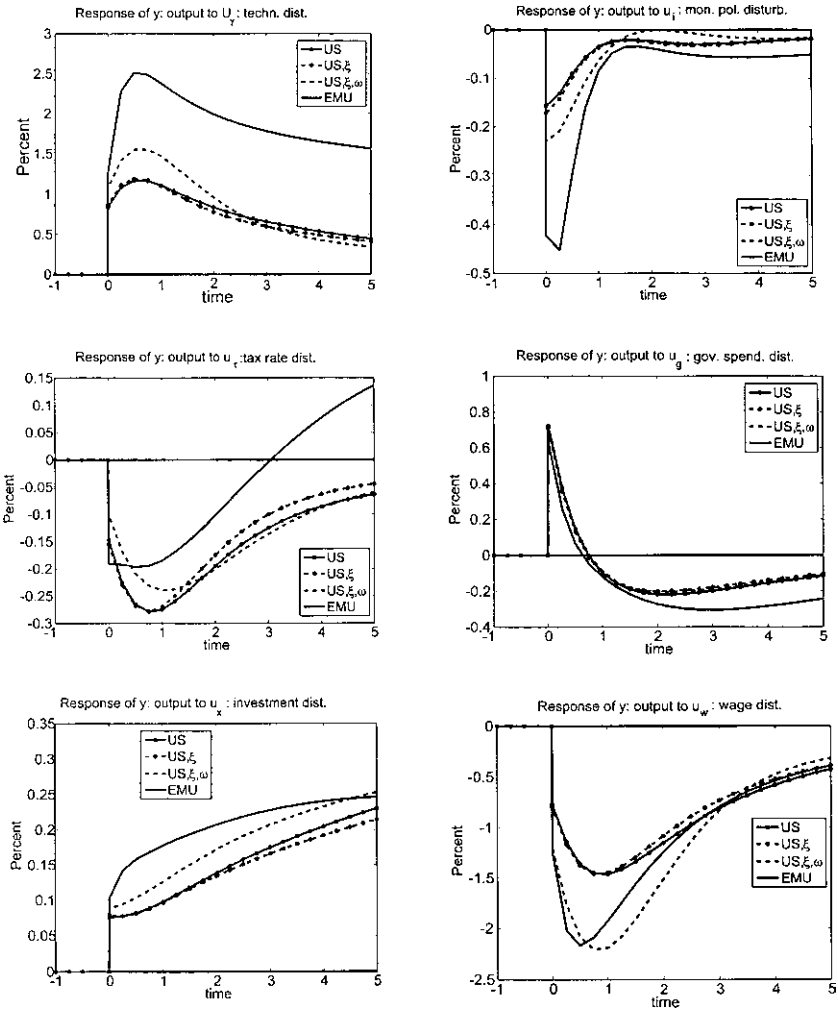


Fig. 9.7 Impulse responses of interest rates

put and figure 9.8 shows the response of inflation. A technical appendix also shows the impulse responses of the remaining variables used for estimating the model; that is, labor productivity, the consumption-to-output ratio, and the debt-to-output ratio.

In these figures, I have also considered two "intermediate" parameterizations to judge the contribution of two features in particular: the higher (assumed) requirement for bank lending in the EMU parameterization, and the parameterization of the labor market with ω and ρ_w. Starting from the U.S. parameterization, I have first only changed the parameter ξ from 0.2

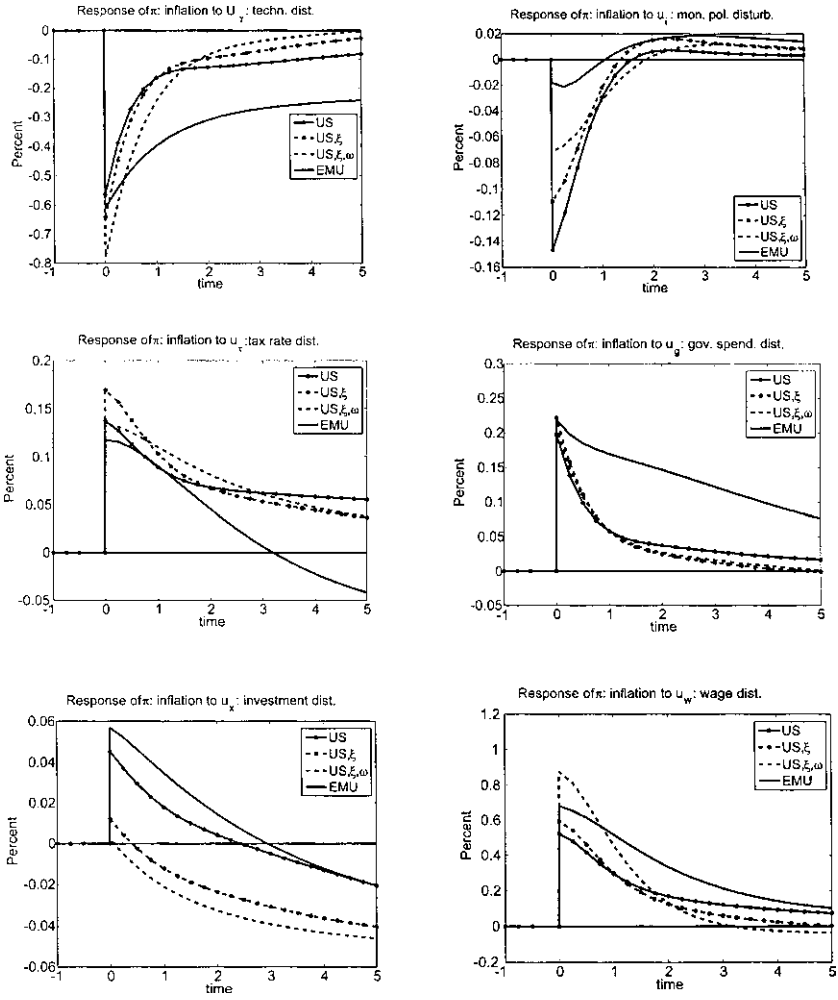


Fig. 9.8 Impulse responses of interest rates

to 1. Next I also have changed the parameters for the labor market to the EMU estimates.

It turns out that the banking requirement ξ matters only for a few key responses and variables. For example, the response of nominal interest rates as well as inflation to investment disturbances moves sizably, when changing ξ. The change in labor market parameters matters in particular in the response of inflation to monetary policy shocks—which becomes less pronounced in the United States, if using EMU labor market parameters—as well as the reaction to wage disturbances of all three variables.

Note also that the difference in the monetary policy reaction function in

EMU compared to the United States shows, if anything, a more pronounced reaction to shocks one standard deviation in size, which then is counterbalanced by the fact that these shocks appear to be smaller. The shape and thus the speed of the reaction looks similar across both regions. That is, by and large, the EMU monetary policy reaction function looks like the U.S. monetary policy reaction function, scaled up a bit, perhaps by a factor of two. This is inconsistent with the view that monetary policy in EMU is sklerotic or that it is indecisive decision-making by a committee of monetary policy makers in Europe.

9.5.3 Answering the Question

Equipped with these tools, I can finally provide an answer to the question with which this chapter started out. The answer is provided[4] graphically in figure 9.9. Note that all figures there have been drawn on the same scale for comparison. This figure decomposes the surprise movements in the United States and the EMU into the five shocks, and adds up their contributions to the cumulative forecast error, compared to the no-shock prediction in 1998. That is, the sequence of shocks, shown in figure 9.10, give rise to impulse responses of the short-term interest rate or central bank interest rate: these impulse responses are cumulated at each point in time, for all present and past shocks (back to 1998) shown.

It turns out that three main sources of the movements come from technology shocks, from monetary policy shocks, and finally, from wage shocks. Interestingly, the monetary policy shocks provide a fairly similar pattern for both Europe and the United States. The top right-hand plot in figure 9.9 shows that monetary policy was tighter in both the United States and EMU in 2000, but considerably looser in 2004, than can be explained by all other variables and historical experience. If one views these shocks as policy mistakes, one would conclude that pretty much the same mistakes have been made in both regions, and that, if anything, the Fed seemed to follow the ECB rather than the other way around.

Surprise movements in productivity provide for a key difference between the United States and EMU. Note that movements in labor productivity in the new millenium were sharply different in the United States and in EMU, as evidenced by the left figure in the second row of figure 9.4. Figure 9.6 shows that monetary policy reacts to surprise rises in productivity and thus the surprise fall in marginal costs by lowering interest rates, see the top left panel. The central bank can afford to do so, since inflation is falling anyhow, as a result, see the top left panel in figure 9.8. Together, it then may no longer surprise that the productivity movements in this millenium led to a consid-

4. It would be even better to provide standard errors in these graphs, based on the posterior distribution for the previously given parameters.

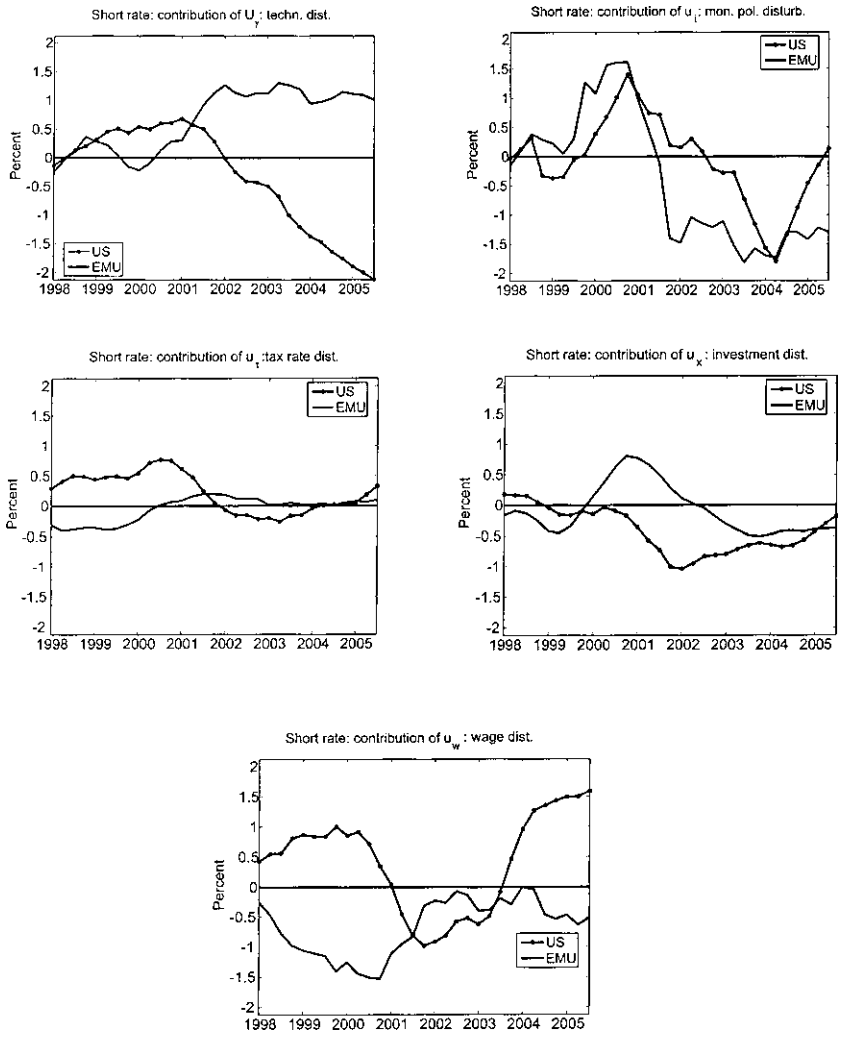


Fig. 9.9 Contribution of each shock to the cumulative forecast error, compared to the no-shock prediction starting in 1996

erable downward drift of interest rates in the United States, but upward pressure in the EMU (see the top left panel in figure 9.9).

The main additional difference then arises due to surprise wage movements. In the United States, they have contributed to raising interest rates before 2000 and after 2004, with the opposite movements in the EMU (see the bottom panel in figure 9.9).

While the reaction function of U.S. and EMU monetary policy to both

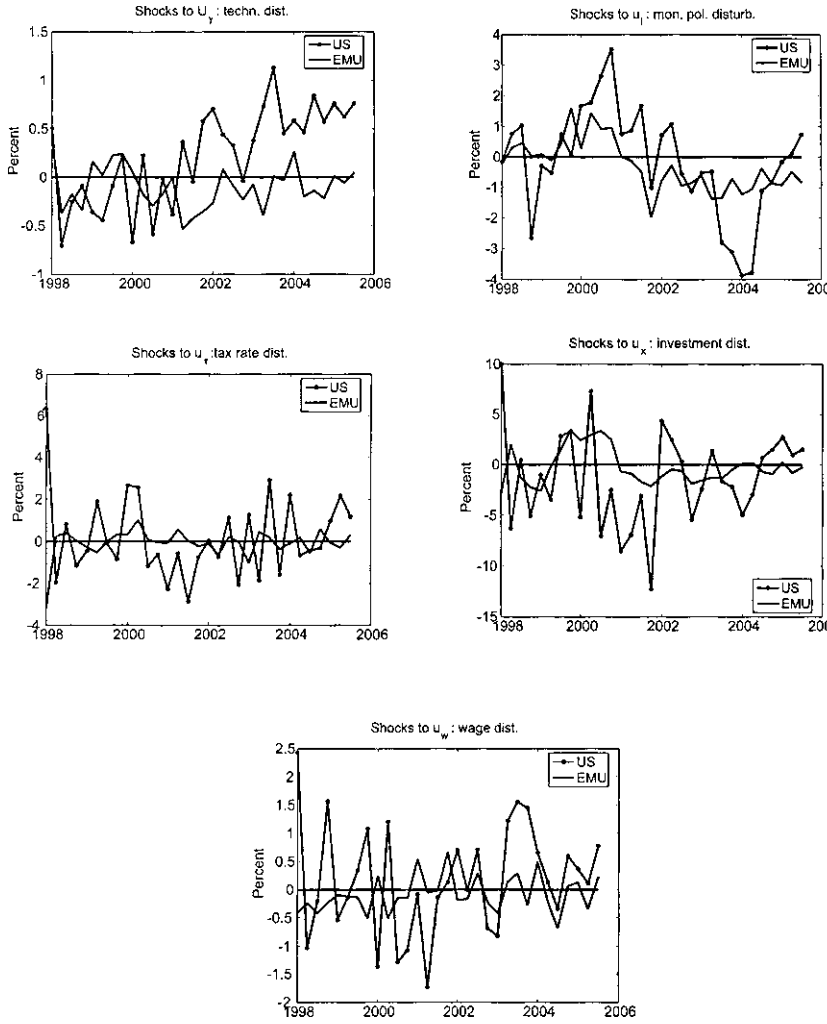


Fig. 9.10 Sequence of shocks

wage shocks and productivity shocks differ quantitatively (see figure 9.6), they do not differ qualitatively. The differences in the interest rate movements in figure 9.9 arises due to different shocks, actually almost moving in oppositive direction for both variables.

In sum, it appears that the difference between the two monetary policies seen in figure 9.1 is due to both surprises in productivity as well as surprises in wage demands, moving interest rates in opposite directions in Europe and the United States, but not due to a more sluggish response in Europe to the same shocks or to different monetary policy surprises.

9.6 Discussion and Conclusion

The conclusion from this quantitative exercise appears to be that the difference between the two monetary policies seen in figure 9.1 is due to both surprises in productivity as well as surprises in wage demands, moving interest rates in opposite directions in Europe and the United States, but not due to a more sluggish response in Europe to the same shocks or to different monetary policy surprises. If anything, it appears that monetary policy in EMU reacts more strongly to shocks, when they appear.

But a number of words of caution are in order. First, these conclusions hinge on a particular choice of shocks propagating in the economy. There is a trade-off between missing an important disturbance as explanation versus adding spurious shocks and thus risking to misinterpret important economic dynamics as movements in these spurious disturbances instead.

Second, the conclusions hinge on the particular model chosen. Is there any sense that they are correct across a wide range of models or approaches? The model may be faulty in a number of crucial features, or improve on these features compared to other models. How are we to judge this? Acknowledging misspecification of the theory and seeking robust approaches to answer the key question may be a way to proceed further (see, e.g., Hansen and Sargent 2001).

Third, while the chapter has provided an accounting method for explaining the different interest paths in the United States and the EMU, it has not asked whether this difference is, in fact, optimal or what the optimal reaction function should have been. That is, it may be the case that U.S. monetary policy has behaved badly and EMU monetary policy has done the right thing, or the other way around. The previous analysis has not addressed this issue all. The tools for pursuing this question are provided in, for example, Schmitt-Grohe and Uribe (2004, 2005) or Levin et al. (2006). One could even combine the perspective of optimality with the acknowledgment of misspecification and a desire for robustness (see, e.g., Levin and Williams 2003).

At the end of the day, there appears to be little else than delivering quantitative answers, based on thoughtfully chosen assumptions. This chapter hopes to make a contribution to that end. Along its novel features it has provided a possibility for considering traditional lending channels of monetary policy alongside the sticky-price perspective pursued by the more recent new-Keynesian literature. To that end, a hybrid new-Keynesian cash in advance model has been provided, estimated, and used to quantitatively answer the question at hand.

Some progress has been made. But much more needs to be done.

Technical Appendix

Analysis

First-Order Conditions

Households

Households solve

$$\max_{c_t,b_t,d_t,h_t} E\left[\sum_{t=0}^{\infty} \beta^t(\log(c_t - \chi c_{t-1}) - An_t^{1+\sigma})\right] \tag{38}$$

$$s.t.\ \eta c_t = \frac{h_{t-1}}{\pi_t}$$

$$h_t + d_t + q_t b_t + (1-\eta)c_t + x_t = (1-\tau_t)w_t n_t + \frac{1+i_t}{\pi_t} d_{t-1} + r_t k_{t-1} + (1-\tau_V)v_t + \frac{b_{t-1}}{\pi_t} \tag{39}$$

$$k_t = \left(1 - \delta + \varphi\left((1+u_{x,t})\frac{x_t}{k_{t-1}}\right)\right)k_{t-1}. \tag{40}$$

Let ϱ_t be the Lagrange multiplier on the first constraint (38), λ_t on the second constraint (39), and ς_t the Lagrange multiplier on the third constraint (40). Note that

$$\Lambda_t = \frac{\lambda_t}{P_t} \tag{41}$$

would therefore be the Lagrange multiplier on the second constraint written in nominal terms. Therefore,

$$\Lambda_{t,t+k} = \beta^k \frac{\Lambda_{t+k}}{\Lambda_t} = \frac{\lambda_{t+k}P_t}{\lambda_t P_{t+k}}. \tag{42}$$

The first-order conditions are

$$\frac{\partial L}{\partial c_t}: \eta\varrho_t + (1-\eta)\lambda_t = \frac{1}{c_t - \chi c_{t-1}} - \beta\chi E_t\left[\frac{1}{c_{t+1} - \chi c_t}\right] \tag{43}$$

$$\frac{\partial L}{\partial h_t}: \lambda_t = \beta E_t\left[\frac{\varrho_{t+1}}{\pi_{t+1}}\right] \tag{44}$$

$$\frac{\partial L}{\partial d_t}: \lambda_t = \beta E_t\left[\lambda_{t+1}\frac{1+i_{t+1}}{\pi_{t+1}}\right] \tag{45}$$

$$\frac{\partial L}{\partial b_t}: \lambda_t q_t = \beta E_t\left[\frac{\lambda_{t+1}}{\pi_{t+1}}\right] \tag{46}$$

$$\frac{\partial L}{\partial x_t}: \lambda_t = (1+u_{x,t})\varphi'\left((1+u_{x,t})\frac{x_t}{k_{t-1}}\right)\varsigma_t \tag{47}$$

$$\frac{\partial L}{\partial k_t}: \varsigma_t = \beta E_t[\lambda_{t+1} r_{t+1}] + \beta E_t\left[\varsigma_{t+1}\left(1 - \delta + \varphi\left((1 + u_{x,t+1})\frac{x_{t+1}}{k_t}\right)\right)\right] - \beta E_t\left[\varsigma_{t+1}\left((1 + u_{x,t+1})\varphi'\left((1 + u_{x,t+1})\frac{x_{t+1}}{k_t}\right)\frac{x_{t+1}}{k_t}\right)\right]. \tag{48}$$

Also note that the first-order condition with respect to labor determines the target real wage $w_{t,f}$,

$$\frac{\partial L}{\partial n_t}: \lambda_t(1 - \tau_t) w_{t,f} = (1 + \sigma) A n_t^{\sigma}. \tag{49}$$

Final Good Firms

Maximizing profits

$$P_t y_t - \int_0^1 P_{t,j} y_{t,j} dj,$$

subject to the production function (6) results in the demand function

$$y_{t,j} = \left(\frac{P_t}{P_{t,j}}\right)^{(1+\mu)/\mu} y_t \tag{50}$$

and the price aggregation

$$P_t = \left(\int_0^1 P_{t,j}^{-1/\mu} dj\right)^{-\mu} \tag{51}$$

$$= ((1 - \alpha)(P_t^*)^{-1/\mu} + \alpha(\overline{\pi} P_{t-1})^{-1/\mu})^{-\mu}. \tag{52}$$

Intermediate Good Firms

Cost minimization leads to the nominal marginal costs of producing an extra unit of output,

$$\mathrm{MC}_t = P_t \theta^{-\theta}(1 - \theta)^{\theta-1} \gamma_t^{-1} r_t^{\theta} w_t^{1-\theta}, \tag{53}$$

and therefore to the real marginal costs

$$\mathrm{mc}_t = \theta^{-\theta}(1 - \theta)^{\theta-1} \gamma_t^{-1} r_t^{\theta} w_t^{1-\theta}, \tag{54}$$

excluding the costs of borrowing from bank.

Cost minimization also implies that

$$r_t k_{t,j} = \theta \mathrm{mc}_t \gamma_t k_{t,j}^{\theta} n_{t,j}^{1-\theta} \tag{55}$$

$$w_t n_{t,j} = (1 - \theta)\mathrm{mc}_t \gamma_t k_{t,j}^{\theta} n_{t,j}^{1-\theta}. \tag{56}$$

Therefore, the capital-labor ratio $k_{t,j}/n_{t,j}$ is the same across all firms, and equal to the aggregate ratio k_{t-1}/n_t. Aggregating (55) and (56) across all firms yields

$$r_t k_{t-1} = \theta \mathrm{mc}_t \gamma_t k_{t-1}^{\theta} n_t^{1-\theta} \tag{57}$$
$$w_t n_t = (1-\theta)\mathrm{mc}_t \gamma_t k_{t-1}^{\theta} n_t^{1-\theta}.$$

Note that (57) follows from (57) with (54) or vice-versa. I will therefore drop (57) when collecting all equations following. Alternatively, observe that (57) and (54) imply the more intuitive equation

$$\frac{1}{\theta} r_t k_{t-1} = \frac{1}{1-\theta} w_t n_t. \tag{58}$$

To calculate the aggregate production function, observe that

$$\gamma_t k_{t-1}^{\theta} n_t^{1-\theta} = \int_0^1 (y_{t,j} + \Phi) dj = \left(\int_0^1 \left(\frac{P_t}{P_{t,j}} \right)^{(1+\mu)/\mu} dj \right) y_t + \Phi,$$

so that

$$y_t = \left(\frac{S_t}{P_t} \right)^{1+\mu/\mu} (\gamma_t k_{t-1}^{\theta} n_t^{1-\theta} - \Phi), \tag{59}$$

where

$$S_t = \left(\int_0^1 P_{t,j}^{-(1+\mu)/\mu} dj \right)^{-\mu/(1+\mu)} \tag{60}$$
$$= ((1-\alpha)(P_t^*)^{-(1+\mu)/\mu} + \alpha(\overline{\pi} S_{t-1})^{-(1+\mu)/\mu})^{-\mu/(1+\mu)},$$

and where $(S_t/P_t)^{(1+\mu)/\mu}$ can be thought of as a correction of the Solow residual due to sticky prices. This correction is known to disappear in a first-order log-linear approximation (see also [123]), but it may be relevant in higher-order approximations.

When a firm can reoptimize its price $P_t^* = P_{t,j}$, it seeks to maximize the objective (11), taking into account the dependence of demand on its chosen price in future dates, if prices cannot be reoptimized, and taking into account the costs of borrowing from banks,

$$y_t(P_{t-k}^*) = \left(\frac{P_t}{\overline{\pi}^k P_{t-k}^*} \right)^{(1+\mu)/\mu} y_t. \tag{61}$$

This problem can be rewritten as

$$\max_{P_t^*} E\left[\sum_{k=0}^{\infty} \alpha^k \Lambda_{t,t+k} y_{t+k}(P_t^*)(P_t^* - P_{t+k}(1+\xi i_{t+k})\mathrm{mc}_{t+k}) \right]. \tag{62}$$

The first-order condition becomes—as usual (or with some calculation)—

$$P_t^* E_t\left[\sum_{k=0}^{\infty} \alpha^k \Lambda_{t,t+k} \overline{\pi}^k y_{t+k}(P_t^*) \right] = \tag{63}$$
$$(1+\mu) E_t\left[\sum_{k=0}^{\infty} \alpha^k \Lambda_{t,t+k} y_{t+k}(P_t^*) P_{t+k}(1+\xi i_{t+k})\mathrm{mc}_{t+k} \right],$$

which essentially says that P^* is a markup of μ over nominal marginal costs inclusive of the costs of borrowing,

$$P_{t+k}(1 + \xi i_{t+k})\mathrm{mc}_{t+k},$$

appropriately discounted.

Aggregating (9) across all firms delivers

$$v_t = y_t - (1 + \xi i_t)\mathrm{mc}_t \gamma_t k_{t-1}^{\theta} n_t^{1-\theta}. \tag{64}$$

Banks

Note that the required loan quantity per intermediate good firms is

$$L_{t,j} = \xi \mathrm{MC}_t(y_{t,j} + \Phi). \tag{65}$$

Aggregating, and equalizing to available funds yields in real terms

$$\frac{d_{t-1}}{\pi_t} + \Psi_t = \xi \mathrm{mc}_t \gamma_t k_{t-1}^{\theta} n_t^{1-\theta}, \tag{66}$$

which I shall use instead of (33).

Parameters

The fundamental parameters are

$$A, \beta, \chi, \sigma, \theta, \delta, \Phi, \mu, \overline{\gamma}, \rho_{\gamma,L}, \rho_{\gamma,u}, \rho_x, \varpi,$$

and the variance of the technology shock and investment-specific shock. The parameters for prices, wages, and credit markets are

$$\Upsilon, \alpha, \omega, \xi, \rho_w, \eta,$$

and the variance of the wage shock. The policy parameters are

$$\overline{g}, \overline{b}, \overline{\pi}, \nu, \tau_V,$$

as well as the feedback coefficients

$$\zeta_\tau, \zeta_g, \zeta_\pi, \zeta_x, \rho_{i,L}, \rho_{i,u}, \rho_\tau, \rho_g,$$

and variances of the policy shocks.

Collecting the Equations

The equations characterizing the equilibrium are (HH: "household"; "FG": final good firms; "IG": intermediate good firms; "CB": central bank; "GOV": government; "MC": labor market and market clearing):

$$\text{HH: } 0 = -\eta c_t + \frac{h_{t-1}}{\pi_t} \tag{67}$$

$$\text{HH: } 0 = -h_t - d_t - q_t b_t - (1 - \eta)c_t - x_t + (1 - \tau_t)w_t n_t + r_t k_{t-1} + \frac{1 + i_t}{\pi_t} d_{t-1} + (1 - \tau_V)v_t + \frac{b_{t-1}}{\pi_t} \tag{68}$$

$$\text{HH: } 0 = -k_t + \left(1 - \delta + \varphi\left(\frac{x_t}{k_{t-1}}\right)\right)k_{t-1} \tag{69}$$

$$\text{HH: } 0 = -\eta\varrho_t - (1-\eta)\lambda_t + \frac{1}{c_t - \chi c_{t-1}} - \beta\chi E_t\left[\frac{1}{c_{t+1} - \chi c_t}\right] \tag{70}$$

$$\text{HH: } 0 = -\lambda_t + \beta E_t\left[\frac{\varrho_{t+1}}{\pi_{t+1}}\right] \tag{71}$$

$$\text{HH: } 0 = -\lambda_t + \beta E_t\left[\lambda_{t+1}\frac{1+i_{t+1}}{\pi_{t+1}}\right] \tag{72}$$

$$\text{HH: } 0 = -\lambda_t q_t + \beta E_t\left[\frac{\lambda_{t+1}}{\pi_{t+1}}\right] \tag{73}$$

$$\text{HH: } 0 = -\lambda_t + (1+u_{x,t})\varphi'\left((1+u_{x,t})\frac{x_t}{k_{t-1}}\right)\varsigma_t \tag{74}$$

$$\begin{aligned}\text{HH: } 0 = &-\varsigma_t + \beta E_t[\lambda_{t+1}r_{t+1}] \\ &+\beta E_t\left[\varsigma_{t+1}\left(1 - \delta + \varphi\left((1+u_{x,t+1})\frac{x_{t+1}}{k_t}\right)\right)\right] \\ &-\beta E_t\left[\varsigma_{t+1}\left((1+u_{x,t+1})\varphi'\left((1+u_{x,t+1})\frac{x_{t+1}}{k_t}\right)\frac{x_{t+1}}{k_t}\right)\right]\end{aligned} \tag{75}$$

$$\text{HH: } 0 = -\lambda_t(1-\tau_t)w_{t,f} + (1+\sigma)An_t^{\sigma} \tag{76}$$

$$\text{FG: } 0 = -y_t(P_{t-k}^*) + \left(\frac{P_t}{\overline{\pi}^k P_{t-k}^*}\right)^{(1+\mu)/\mu} y_t \tag{77}$$

$$\text{FG: } 0 = -P_t + ((1-\alpha)(P_t^*)^{-1/\mu} + \alpha(\overline{\pi}P_{t-1})^{-1/\mu})^{-\mu} \tag{78}$$

$$\begin{aligned}\text{IG: } 0 = &-S_t + ((1-\alpha)(P_t^*)^{-(1+\mu)/\mu} \\ &+ \alpha(\overline{\pi}S_{t-1})^{-(1+\mu)/\mu})^{-\mu/(1+\mu)}\end{aligned} \tag{79}$$

$$\text{IG: } 0 = -y_t + \left(\frac{S_t}{P_t}\right)^{(1+\mu)/\mu}(\gamma_t k_{t-1}^{\theta} n_t^{1-\theta} - \Phi) \tag{80}$$

$$\text{IG: } 0 = -\text{mc}_t + \theta^{-\theta}(1-\theta)^{\theta-1}\gamma_t^{-1} r_t^{\theta} w_t^{1-\theta} \tag{81}$$

$$\text{IG: } 0 = -r_t k_{t-1} + \theta \text{mc}_t \gamma_t k_{t-1}^{\theta} n_t^{1-\theta} \tag{82}$$

$$\begin{aligned}\text{IG: } 0 = &-P_t^* E_t\left[\sum_{k=0}^{\infty}(\alpha\beta)^k \frac{\lambda_{t+k}P_t}{\lambda_t P_{t+k}}\overline{\pi}^k y_{t+k}(P_t^*)\right] \\ &+(1+\mu)E_t\left[\sum_{k=0}^{\infty}(\alpha\beta)^k \frac{\lambda_{t+k}P_t}{\lambda_t P_{t+k}} y_{t+k}(P_t^*)P_{t+k}(1+\xi i_{t+k})\text{mc}_{t+k}\right]\end{aligned} \tag{83}$$

$$\text{IG: } 0 = v_t - y_t + (1+\xi i_t)\text{mc}_t \gamma_t k_{t-1}^{\theta} n_t^{1-\theta} \tag{84}$$

$$(85)\quad \text{CB: } 0 = -i_t + \bar{i} + \rho_{i,L} i_{t-1} + (1-\rho_{i,L})\left(\zeta_\pi\left(\frac{\pi_t}{\overline{\pi}} - 1\right) + \zeta_x\left(\frac{\text{mc}_t}{\overline{\text{mc}}} - 1\right) + u_{i,t}\right)$$

$$(86)\quad \text{CB: } 0 = -m_t + \frac{m_{t-1}}{\pi_t} + \nu\Psi_t$$

$$(87)\quad \text{GOV: } 0 = -q_t b_t + \frac{b_{t-1}}{\pi_t} + g_t - \tau_t w_t n_t - \tau_V v_t - (\nu + i_t)\Psi_t$$

$$(88)\quad \text{GOV: } 0 = -\tau_t + \overline{\tau} + \zeta_\tau\left(\frac{b_{t-1} - \overline{b}}{\overline{y}} - 1\right) + u_{\tau,t}$$

$$(89)\quad \text{GOV: } 0 = -\frac{g_t}{\overline{y}} + \zeta_g\left(\frac{b_{t-1} - \overline{b}}{\overline{b}} - 1\right) + u_{g,t}$$

$$(90)\quad \text{MC: } 0 = -w_t + ((1-\omega)w_{t-1} + \omega\Upsilon w_{t,f})(1 + u_{w,t})$$

$$(91)\quad \text{MC: } 0 = -m_t + d_t + h_t$$

$$(92)\quad \text{MC: } 0 = -y_t + g_t + c_t + x_t$$

$$(93)\quad \text{MC: } 0 = -\frac{d_{t-1}}{\pi_t} - \Psi_t + \xi \text{mc}_t \gamma_t k_{t-1}^{\theta} n_t^{1-\theta}$$

$$(94)\quad \text{MC: } 0 = -\pi_t + \frac{P_t}{P_{t-1}}$$

together with the specification for the exogenous processes

$$(95)\quad \text{techn.: } \hat{\gamma}_t = \rho_{\gamma,L}\hat{\gamma}_{t-1} + \varepsilon_{\gamma,t}$$

$$(96)\quad \text{mon.pol: } u_{i,t} = \rho_{i,u} u_{i,t-1} + \varepsilon_{i,t}$$

$$(97)\quad \text{taxes: } u_{\tau,t} = \rho_\tau u_{\tau,t-1} + \varepsilon_{\tau,t}$$

$$(98)\quad \text{gov.spend.: } u_{g,t} = \rho_g u_{g,t-1} + \varepsilon_{g,t}$$

$$(99)\quad \text{investment: } u_{x,t} = \rho_x u_{x,t-1} + \varepsilon_{x,t}$$

$$(100)\quad \text{wages: } u_{w,t} = \rho_w u_{w,t-1} + \varepsilon_{w,t}.$$

The previous equations determine the quantities

$$b_t, c_t, g_t, n_t, v_t, y_t, \text{mc}_t, k_t, x_t, r_t,$$

the demand function

$$y_t(P^*_{t-k}),$$

real money balances

$$d_t, h_t, m_t, \Psi_t,$$

multipliers

$$\varrho_t, \lambda_t, \varsigma_t,$$

prices and tax rate

$$i_t, P_t, P_t^*, q_t, S_t, w_t, w_{t,f}, \pi_t, \tau_t,$$

as well as the exogenous processes

$$\gamma_t, u_{i,t}, u_{r,t}, u_{g,t}, u_{x,t}, u_{w,t}.$$

Note that these are 34 equations for 33 variables. One may drop either the household budget constraint, the government budget constraint, or one of the market-clearing conditions, due to Walras' law.

Steady State

Household

To calculate the steady state, and since my focus is not on a steady-state comparison across various parameters, I assume a value for $\overline{n}$ and instead back out the compatible preference parameter A. The capital accumulation equation (69) implies

$$\overline{x} = \delta \overline{k}. \tag{101}$$

The first-order conditions (71) and (74) of the households imply

$$\overline{\varrho} = \frac{\overline{\pi}}{\beta} \overline{\lambda} \tag{102}$$

$$\overline{\varsigma} = \overline{\lambda}. \tag{103}$$

For the rental rate of capital, the first-order condition (75) implies

$$\overline{r} = \frac{1}{\beta} - 1 + \delta. \tag{104}$$

The first-order conditions (72) and (73) imply

$$1 + \overline{\imath} = \frac{1}{\overline{q}} = \frac{\overline{\pi}}{\beta}. \tag{105}$$

Firms

We shall assume that the parameters imply $\overline{v} > 0$. Equations (78), (79), and (94) deliver

$$\overline{P}_t^* = \overline{S}_t = \overline{P}_t = \overline{\pi} \overline{P}_{t-1}.$$

With equation (77),

$$y(\overline{P}^*_{t-k}) = \overline{y}.$$

The markup equation (83) for the intermediate good implies

$$\overline{\mathrm{mc}} = \frac{1}{(1+\mu)(1+\xi\overline{\imath})}. \tag{106}$$

This and equation (82) imply

$$r\overline{k} = \theta\overline{\mathrm{mc}}\,\overline{\gamma}\overline{k}^{\theta}\overline{n}^{1-\theta},$$

or

$$\overline{k} = \left(\frac{\theta\overline{\mathrm{mc}}\,\overline{\gamma}}{\overline{r}}\right)^{1/(1-\theta)}\overline{n}. \tag{107}$$

From this and (81) or, equivalently, (57), obtain

$$\overline{w} = (1-\theta)\overline{\mathrm{mc}}\,\gamma\left(\frac{\overline{k}}{\overline{n}}\right)^{\theta} = (1-\theta)\theta^{\theta/(1-\theta)}(\overline{\mathrm{mc}}\,\overline{\gamma})^{1/(1-\theta)}\overline{r}^{-\theta/(1-\theta)}. \tag{108}$$

With this as well as equations (80, 92, 70, 90, 84)

$$\overline{y} = \overline{\gamma}\overline{k}^{\theta}\overline{n}^{1-\theta} - \Phi \tag{109}$$

$$\overline{c} = \overline{y} - \overline{g} - \delta\overline{k} \tag{110}$$

$$\overline{\lambda} = \left(\eta\frac{\overline{\pi}}{\beta} + 1 - \eta\right)^{-1}\frac{1-\beta\chi}{1-\chi}\,\frac{1}{\overline{c}} \tag{111}$$

$$\overline{w}_f = \frac{\overline{w}}{\Upsilon} \tag{112}$$

$$\overline{v} = \overline{y} - (1+\xi\overline{\imath})\overline{\mathrm{mc}}\,\overline{\gamma}\overline{k}^{\theta}\overline{n}^{1-\theta} \tag{113}$$

$$= \frac{\mu}{1+\mu}\overline{\gamma}\overline{k}^{\theta}\overline{n}^{1-\theta} - \Phi \tag{114}$$

$$= \overline{y} - \frac{1}{1+\mu}(\overline{y} + \Phi), \tag{115}$$

which now allows to solve for the steady-state values of the Lagrange multipliers in (102).

Monetary Quantities

Cash demand is given by (67) or

$$\overline{h} = \eta\overline{\pi}\,\overline{c}.$$

To calculate the other monetary quantities, combine the three steady-state relationships of (86, 91, 93),

$$\nu\overline{\psi} = \overline{m}\left(1 - \frac{1}{\overline{\pi}}\right)$$

$$\overline{m} = \overline{d} + \overline{h}$$

$$\xi\overline{\mathrm{mc}}(\overline{y} - \Phi) = \frac{\overline{d}}{\overline{\pi}} + \overline{\psi},$$

to obtain

$$\overline{m} = \frac{\overline{\pi}\nu}{\overline{\pi} + \nu - 1}\left(\xi\overline{\mathrm{mc}}(\overline{y} - \Phi) + \frac{\overline{h}}{\overline{\pi}}\right) \tag{116}$$

$$\overline{\psi} = \frac{\overline{\pi} - 1}{\overline{\pi}\nu}\overline{m} \tag{117}$$

$$\overline{d} = \overline{m} - \overline{h}. \tag{118}$$

Note that the fraction appearing in the equation for $\overline{m}$ equals 1, if either $\overline{\pi} = 1$ or $\nu = 1$. For technical reasons, one must set $\nu \neq 0$. Note that $\overline{\pi} = 1$ implies $\overline{\psi} = 0$.

Remaining Equations

The steady-state government budget constraint (88)

$$\overline{\tau}\,\overline{w}\,\overline{n} = \frac{1 - \beta}{\overline{\pi}}\overline{b} + \overline{g} - \tau_V\overline{v} - (\nu + \bar{\imath})\overline{\psi}, \tag{119}$$

can be solved for the steady-state level of taxes $\overline{\tau}$. With this and (76), calculate the preference parameter A per

$$A = \frac{\overline{\lambda}(1 - \overline{\tau})}{1 + \sigma}\overline{n}^{-\sigma}\overline{w}_f. \tag{120}$$

Note finally that (95) to (98) deliver

$$\overline{z} = \overline{a} = \overline{f}_\tau = \overline{f}_g = 0.$$

Log-Linearization

Let hat on variables denote the logarithmic deviation from steady-state values; for example, $\hat{c}_t = \log(c_t)$–$\log(\overline{c})$. For nominal quantities, in particular prices, I use, likewise, $\hat{P}_t = \log(P_t)$–$\log(\overline{P}_t)$, where I note that $\overline{P}_t = \overline{\pi}^k\overline{P}_0$, starting from some initial level $\overline{P}_0$. I make the following exceptions for the notation, so as to allow zero values in steady state or to obtain meaningful quantities:

$$r_t = \overline{r} + \hat{r}_t$$

$$i_t = \overline{\imath} + \hat{\imath}_t$$

$$\tau_t = \overline{\tau} + \hat{\tau}_t$$
$$g_t = \overline{g} + \hat{g}_t \overline{y}$$
$$b_t = \overline{b} + \hat{b}_t \overline{y}$$
$$\psi_t = \overline{\psi} + \overline{m}\hat{\psi}_t.$$

Hence, $\hat{r}_t$, $\hat{\tau}_t$, and $\hat{i}_t$ are in percent, $\hat{g}_t$ and $\hat{b}_t$ are in percent of steady-state output, and $\hat{\psi}_t$ is in percent of the steady-state money supply. Most equations can be log-linearized in a straightforward manner, but some equations require a bit more thought. They are explained now.

Pricing Decisions

The following derivation is standard in the literature on new-Keynesian models and is replicated here for completeness.

Equations (78) and (79) log-linearize to

$$\hat{P}_t = (1-\alpha)\hat{P}_t^* + \alpha \hat{P}_{t-1} \tag{121}$$
$$\hat{S}_t = (1-\alpha)\hat{P}_t^* + \alpha \hat{S}_{t-1}$$

and thus

$$\hat{S}_t = \hat{P}_t. \tag{122}$$

This substantiates the claim that the correction to the Solow residual in (59) vanishes in a first-order approximation.

The first-order condition (83) of the intermediate good firms log-linearizes to

$$\hat{P}_t^* = (1-\alpha\beta)E_t\left[\sum_{k=0}^{\infty}(\alpha\beta)^k\left(\frac{\xi}{1+\xi\bar{\imath}}\hat{\imath}_{t+k} + \widehat{\mathrm{mc}}_{t+k} + \hat{P}_{t+k}\right)\right]. \tag{123}$$

A rather "pedestrian" but fail-safe way to see this is to indeed replace all variables, say x_{t+k}, with their log-linearized counterpart $x(1+\hat{x}_{t+k})$, drop all products of hat-variables as "higher order" (or better, do not write them down—there are many). Simplify the constants, employing equation (106). A slightly more sophisticated approach is to immediately log-linearize products, say $x_t y_t z_t$ to $xyz(1+\hat{x}_t+\hat{y}_t+\hat{z}_t)$.

The previous equation can be rewritten as

$$\hat{P}_t^* = (1-\alpha\beta)\left(\frac{\xi}{1+\xi\bar{\imath}}\hat{\imath}_t + \widehat{\mathrm{mc}}_t + \hat{P}_t\right) + \alpha\beta E_t[\hat{P}_{t+1}^*]. \tag{124}$$

From equation (122), substitute $\hat{P}_t^*$ and $\hat{P}_{t+1}^*$ per

$$\hat{P}_t^* = \frac{1}{1-\alpha}(\hat{P}_t - \alpha\hat{P}_{t-1}).$$

Combine terms to obtain the new-Keynesian Phillips curve

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \kappa\left(\frac{\xi}{1+\xi\bar{\imath}}\hat{\imath}_t + \widehat{\mathrm{mc}}_t\right), \tag{125}$$

where

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}. \tag{126}$$

One may view the driving term

$$\frac{\xi}{1+\xi\bar{\imath}}\hat{\imath}_t + \widehat{\mathrm{mc}}_t$$

either as reflecting marginal costs inclusive of the costs of borrowing or as a correction to net marginal cost by an interest rate cost channel, as emphasized by Christiano et al. (2003).

Collecting Log-Linearized Equations Without Expectations

We shall drop the budget constraint of the household—appealing to Walras' law—as well as equations from pricing decisions and demand, which are no longer needed. All remaining equations without expectations are, in log-linearized form:

$$\text{HH: } 0 = -\hat{c}_t + \hat{h}_{t-1} - \hat{\pi}_t \tag{127}$$

$$\text{HH: } 0 = -\hat{k}_t + \delta\hat{x}_t + (1-\delta)\hat{k}_{t-1} + \delta u_{x,t} \tag{128}$$

$$\text{HH: } 0 = -\hat{\varsigma}_t + \hat{\lambda}_t + \frac{1}{\varpi}(\hat{x}_t - \hat{k}_{t-1}) - \left(1 - \frac{1}{\varpi}\right)u_{x,t} \tag{129}$$

$$\text{HH: } 0 = -\hat{\lambda}_t + \frac{\hat{\tau}_t}{1-\bar{\tau}} - \hat{w}_{t,f} + \sigma\hat{n}_t \tag{130}$$

$$\text{IG: } 0 = -\frac{\bar{y}}{\bar{y}+\Phi}\hat{y}_t + \hat{\gamma}_t + \theta\hat{k}_{t-1} + (1-\theta)\hat{n}_t \tag{131}$$

$$\text{IG: } 0 = -\widehat{\mathrm{mc}}_t + \theta\frac{\hat{r}_t}{\bar{r}} + (1-\theta)\hat{w}_t - \hat{\gamma}_t \tag{132}$$

$$\text{IG: } 0 = -\frac{\hat{r}_t}{\bar{r}} + \widehat{\mathrm{mc}}_t + \gamma_t + (1-\theta)(\hat{n}_t - \hat{k}_{t-1}) \tag{133}$$

$$\text{IG: } 0 = \frac{\bar{v}}{\bar{y}-\bar{v}}\hat{v}_t - \frac{\bar{y}}{\bar{y}-\bar{v}}\hat{y}_t + \frac{\xi}{1+\xi\bar{\imath}}\hat{\imath}_t + \widehat{\mathrm{mc}}_t + \hat{\gamma}_t + \theta\hat{k}_{t-1} + (1-\theta)n_t \tag{134}$$

$$\text{CB: } 0 = -\hat{\imath}_t + \rho_{i,L}\hat{\imath}_{t-1} + (1-\rho_{i,L})(\zeta_\pi\hat{\pi}_t + \zeta_x\widehat{\mathrm{mc}}_t + u_{i,t}). \tag{135}$$

$$\text{(136)}\quad \text{CB: } 0 = -\hat{m}_t + \frac{1}{\overline{\pi}}(\hat{m}_{t-1} - \hat{\pi}_t) + \nu\hat{\psi}_t$$

$$\text{(137)}\quad \text{GOV: } 0 = -\overline{q}(\overline{b}\hat{q}_t + \overline{y}\hat{b}_t) + \frac{1}{\overline{\pi}}(\overline{y}\hat{b}_{t-1} - \overline{b}\hat{\pi}_t) + \overline{y}\hat{g}_t$$
$$-\overline{w}\,\overline{n}\hat{\tau}_t - \overline{\tau}\,\overline{w}\,\overline{n}(\hat{w}_t + \hat{n}_t) - \tau_V\overline{v}\hat{v}_t - (\nu + \bar{\imath})\overline{m}\hat{\psi}_t - \overline{\psi}\hat{\imath}_t$$

$$\text{(138)}\quad \text{GOV: } 0 = -\hat{\tau}_t + \zeta_\tau\hat{b}_{t-1} + u_{\tau,t}$$

$$\text{(139)}\quad \text{GOV: } 0 = -\hat{g}_t + \zeta_g\hat{b}_{t-1} + u_{g,t}$$

$$\text{(140)}\quad \text{MC: } 0 = -\hat{w}_t + (1-\omega)\hat{w}_{t-1} + \omega\hat{w}_{t,f} + u_{w,t}$$

$$\text{(141)}\quad \text{MC: } 0 = -\overline{m}\hat{m}_t + \overline{d}\hat{d}_t + \overline{h}\hat{h}_t$$

$$\text{(142)}\quad \text{MC: } 0 = -\overline{y}\hat{y}_t + \overline{y}\hat{g}_t + \overline{c}\hat{c}_t + \overline{x}\hat{x}_t$$

$$\text{(143)}\quad \text{MC: } 0 = \frac{\overline{d}}{\overline{d} + \overline{\pi}\,\overline{\psi}}(\hat{d}_{t-1} - \hat{\pi}_t) + \frac{\overline{\pi}\,\overline{m}}{\overline{d} + \overline{\pi}\,\overline{\psi}}\hat{\psi}_t$$
$$+ \widehat{\text{mc}}_t + \hat{\gamma}_t + \theta\hat{k}_{t-1} + (1-\theta)\hat{n}_t$$

together with the specification for the exogenous processes

$$\text{(144)}\quad \text{techn.: } \hat{\gamma}_t = \rho_{\gamma,L}\hat{\gamma}_{t-1} + u_{\gamma,t}$$

$$\text{(145)}\quad u_{\gamma,t} = \rho_{\gamma,u}u_{\gamma,t-1} + \varepsilon_{\gamma,t}$$

$$\text{(146)}\quad \text{mon.pol: } u_{i,t} = \rho_{i,u}u_{i,t-1} + \varepsilon_{i,t}$$

$$\text{(147)}\quad \text{taxes: } u_{\tau,t} = \rho_\tau u_{\tau,t-1} + \varepsilon_{\tau,t}$$

$$\text{(148)}\quad \text{gov.spend.: } u_{g,t} = \rho_g u_{g,t-1} + \varepsilon_{g,t}$$

$$\text{(149)}\quad \text{investment: } u_{x,t} = \rho_x u_{x,t-1} + \varepsilon_{x,t}$$

$$\text{(150)}\quad \text{wages: } u_{w,t} = \rho_w u_{w,t-1} + \varepsilon_{w,t}.$$

Collecting Log-Linearized Equations with Expectations

All equations with expectations in log-linearized form are:

$$\text{(151)}\quad \text{HH: } 0 = -\frac{(1-\beta\chi)(1-\chi)}{\eta(\overline{\pi}/\beta) + 1 - \eta}\left(\eta\frac{\overline{\pi}}{\beta}\hat{\varrho}_t + (1-\eta)\hat{\lambda}_t\right)$$
$$+\chi\hat{c}_{t-1} - (1+\beta\chi^2)\hat{c}_t + \beta\chi E_t[\hat{c}_{t+1}]$$

$$\text{(152)}\quad \text{HH: } 0 = -\hat{\lambda}_t + E_t[\hat{\varrho}_{t+1} - \hat{\pi}_{t+1}]$$

$$\text{(153)}\quad \text{HH: } 0 = -\hat{\lambda}_t + E_t[\hat{\lambda}_{t+1} + \frac{\hat{\imath}_{t+1}}{1+\bar{\imath}} - \hat{\pi}_{t+1}]$$

$$\text{(154)}\quad \text{HH: } 0 = -\hat{\lambda}_t - \hat{q}_t + E_t[\hat{\lambda}_{t+1} - \hat{\pi}_{t+1}]$$

$$\text{(155)}\qquad \text{HH: } 0 = -\hat{\varsigma}_t + \beta E_t[\bar{r}\hat{\lambda}_{t+1} + \hat{r}_{t+1}] + \beta E_t[(1-\delta)\hat{\varsigma}_{t+1} + \frac{\delta}{\varpi}(\hat{x}_{t+1} - \hat{k}_t + u_{x,t+1})]$$

$$\text{(156)}\qquad \text{FG, IG: } 0 = -\hat{\pi}_t + \beta E_t[\hat{\pi}_{t+1}] + \kappa\left(\frac{\xi}{1+\xi\bar{i}}\hat{i}_t + \widehat{\text{mc}}_t\right).$$

These equations and the equations without expectations determine the evolution of the log-deviations for the quantities

$$\hat{b}_t, \hat{c}_t, \hat{g}_t, \hat{n}_t, \hat{v}_t, \hat{y}_t, \widehat{\text{mc}}_t, \hat{k}_t, \hat{x}_t, \hat{r}_t,$$

real money balances

$$\hat{d}_t, \hat{h}_t, \hat{m}_t, \hat{\psi}_t,$$

multipliers

$$\hat{\varrho}_t, \hat{\lambda}_t, \hat{\varsigma}_t,$$

prices and tax rate

$$\hat{i}_t, \hat{q}_t, \hat{w}_t, \hat{w}_{t,f}, \hat{\pi}_t, \hat{\tau}_t.$$

Note that there are twenty-three equations for twenty-three variables, plus the equations for the exogenous processes.

Note that q_t is the inverse of the one-period risk free return $R_{t,f}$ from period t to $t+1$. Hence

$$\hat{R}_{t,f} = -\hat{q}_t.$$

Note that generally $\hat{i}_t \neq \hat{R}_{t-1,f}$, since $\hat{i}_t$ can react to shocks within period t.

Define

$$\text{(157)}\qquad \hat{r}_t^* = \hat{r}_t + \frac{1}{\varpi}(\hat{x}_t - \hat{k}_{t-1}) - \left(1 - \delta - \frac{1}{\varpi}\right)u_{x,t} - \frac{1}{\beta\varpi}(\hat{x}_{t-1} - \hat{k}_{t-2}) + \frac{1}{\beta}\left(1 - \frac{1}{\varpi}\right)u_{x,t-1}.$$

One may interpret this as the log-deviation of the return to capital, taking into account the cost of adjustment and the additional discounting due to the extra period of being able to spend the rental rate on consumption. With this definition and the help of equation (130), one can rewrite (155) as

$$\text{(158)}\qquad 0 = -\hat{\lambda}_t + \eta(\hat{\lambda}_t - \hat{\varrho}_t) + E_t[\lambda_{t+1} + \beta\eta(1-\delta)(\hat{\varrho}_{t+1} - \hat{\lambda}_{t+1}) + \beta\hat{r}_{t+1}^*],$$

which may be a more intuitive or familiar expression.

Figures

Shown here are the impulse responses of the three variables used for estimating the model, not shown in the body of the chapter.

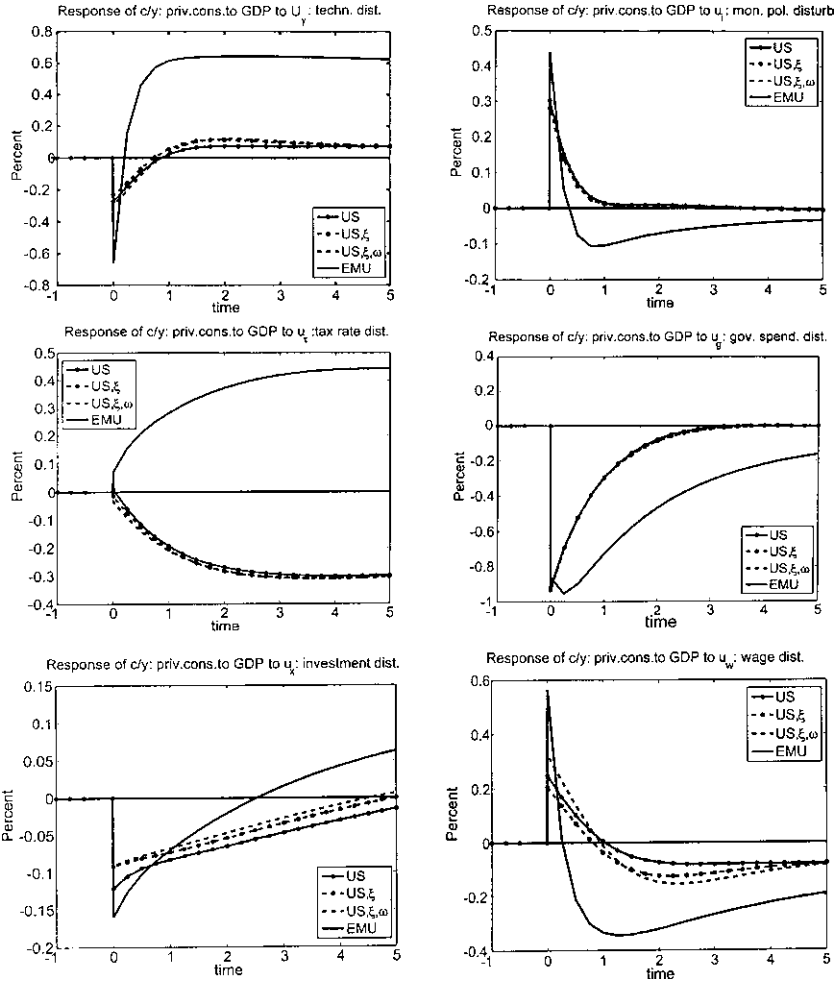


Fig. 9A.1 Impulse response of the log consumption-output ratio to six shocks, each providing a comparison of three model specifications for the United States and one specification for the EMU

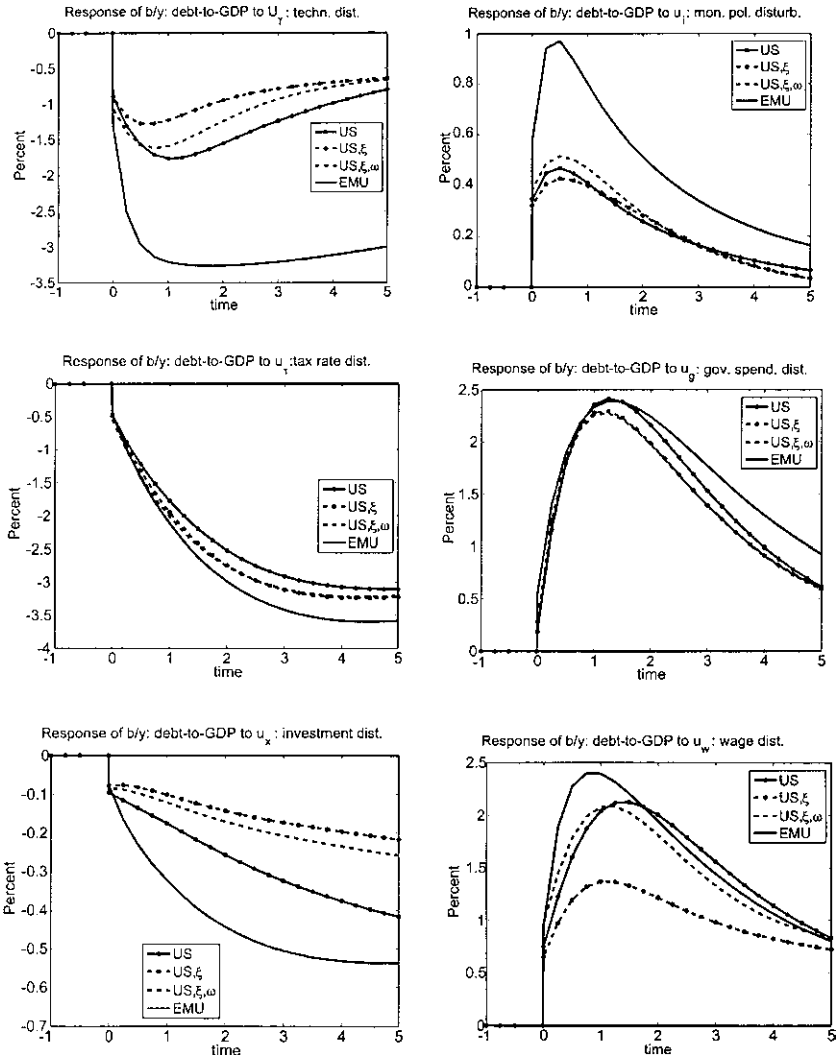


Fig. 9A.2 Impulse response of the debt-output ratio to six shocks, each providing a comparison of three model specifications for the United States and one specification for the EMU

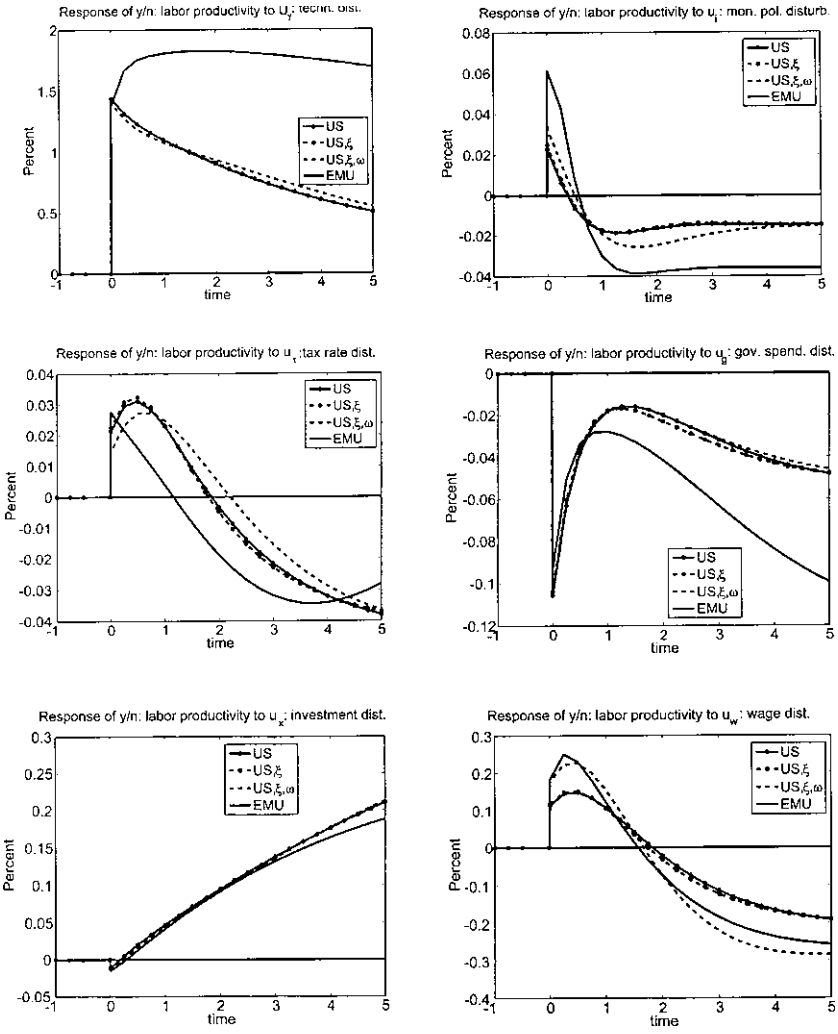


Fig. 9A.3 Impulse response of the log output-labor ratio (i.e., the log labor productivity) to six shocks, each providing a comparison of three model specifications for the United States and one specification for the EMU

References

Altig, D., L. J. Christiano, M. Eichenbaum, and J. Linde. 2004. Firm-specific capital, nominal rigidities, and the business cycle. Sveriges Riksbank (Central Bank of Sweden), Working Paper Series no. 176.

Barth, M., and V. Ramey. 2001. The cost channel of monetary transmission. *NBER Macroeconomics Annual 2001,* ed. B. S. Bernanke and K. S. Rogoff, 1991–240. Cambridge, MA: MIT Press.

Bernanke, B. S., M. Gertler, and S. Gilchrist. 1999. The financial accelerator in a quantitative business cycle framework. In *Handbook of macroeconomics, Volume 1C. Handbooks in economics, vol. 15,* ed. J. B. Taylor, and M. Woodford, 1341–93. Amsterdam: Elsevier Science, North-Holland.

Blanchard, O., and J. Galí. 2005. Real wage rigidities and the new Keynesian model. Massachusetts Institute of Technology, Department of Economics Working Paper no. 05-27.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan. 2005. A critique of structural VARs using real business cycle theory. Federal Reserve Bank of Minneapolis, Working Paper no. 631.

Christiano, L., M. Eichenbaum, and C. Evans. 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113 (1): 1–45.

Christiano, L., R. Motto, and M. Rostagno. 2003. The Great Depression and the Friedman-Schwartz hypothesis. Federal Reserve Bank of Cleveland Working Paper.

———. 2007. Shocks, structures or monetary policies? European Central Bank (ECB) Working Paper no. 774, July.

Clarida, R., J. Galí, and M. Gertler. 1999. The science of monetary policy: A new Keynesian perspective. *Journal of Economic Literature* 37 (4): 1661–1707.

Cochrane, J. 1994. Permanent and transitory components of GNP and stock prices. *Quarterly Journal of Economics* 109 (February): 241–66.

de Fiore, F., and H. Uhlig. 2006. Bank finance versus bond finance: What explains the differences between U.S. and Europe? ECB Working Paper no. 547.

Erceg, C. J., D. W. Henderson, and A. T. Levin. 2000. Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics* 46 (2): 281–313.

Fernandez-Villaverde, J., J. F. Rubio-Ramirez, and T. J. Sargent. 2007. A,B,C's (and D's) for understanding VARS. *American Economic Review* 97 (3): 1021–26.

Fisher, J. D. M. 2006. The dynamic effects of neutral and investment-specific technology shocks. *Journal of Political Economy* 114 (3): 413–51.

Hall, R. E. 2005. Employment fluctuations with equilibrium wage stickiness. *American Economic Review* 95 (1): 50–65.

Hansen, L. P., and T. J. Sargent. 2001. Acknowledging misspecification in macroeconomic theory. *Review of Economic Dynamics* 4 (3): 519–35.

Jermann, U. J. 1998. Asset pricing in production economies. *Journal of Monetary Economics* 41 (2): 257–75.

Levin, A., A. Onatski, J. Williams, and N. Williams. 2006. Monetary policy under uncertainty in micro-founded macroeconometric models. In *NBER macroeconomics annual 2005,* ed. M. Gertler and K. Rogoff, 229–87. Cambridge, MA: MIT Press.

Levin, A., and J. Williams. 2003. Robust monetary policy rules with competing reference models. *Journal of Monetary Economics* 50:945–75.

Lucas, R. E., Jr., and N. L. Stokey. 1987. Money and interest in a cash-in-advance economy. *Econometrica* 55 (3): 491–513.
Sahuc, J.-G., and F. Smets. 2008. Differences in interest rate policy at the ECB and the Fed: An investigation with a medium-scale DSGE model. *Journal of Money, Credit and Banking* 40 (2–3): 505–21.
Schmitt-Grohe, S., and M. Uribe. 2004. Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory* 114 (2): 198–230.
———. 2005. Optimal fiscal and monetary policy in a medium-scale macroeconomic model. In *NBER macroeconomic annual 2005,* ed. M. Gertler and K. Rogoff, 383–425. Cambridge, MA: MIT Press.
Secchi, A., and E. Gaiotti. 2006. Is there a cost channel of monetary policy transmission? An investigation into the pricing behavior of 2,000 firms. *Journal of Money, Credit, and Banking* 38 (8): 2013–37.
Shimer, R. 2005. The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review* 95 (1): 25–49.
Smets, F., and R. Wouters. 2003. An estimated dynamic stochastic general equilibrium model of the Euro area. *Journal of the European Economic Association* 1 (5): 1123–75.
Uhlig, H. 1999. A toolkit for analysing nonlinear dynamic stochastic models easily. In *Computational methods for the study of dynamic economies,* ed. R. Marimon and A. Scott, 30–61. New York: Oxford University Press.
———. 2007. Explaining asset prices with external habits and wage rigidities in a DSGE model. *American Economic Review Papers and Proceedings* 97 (2): 239–43.
Woodford, M. 2003. *Interest and prices: Foundations of a theory of monetary policy.* Princeton, NJ: Princeton University Press.

Comment

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Over the past decade or so, researchers at academic institutions and central banks have been active in specifying and estimating dynamic stochastic general equilibrium (DSGE) models that can be used for the analysis of monetary policy.[1] While the first generation models were relatively small and stylized, more recent models typically embed a much more elaborate dynamic structure aimed at capturing key aspects of the aggregate data.[2] Indeed, a number of central banks are now employing DSGE models in the forecasting process and in formulating and communicating policy strategies.

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The views expressed in this comment are solely those of the author, and should not be interpreted as representing the views of the Board of Governors of the Federal Reserve System nor of anyone else associated with the Federal Reserve System.

1. Pioneering early studies include King and Wolman (1996, 1999); Goodfriend and King (1997); Rotemberg and Woodford (1997, 1999); Clarida, Galí, and Gertler (1999); and McCallum and Nelson (1999).

2. See Christiano, Eichenbaum, and Evans (2005); Smets and Wouters (2003); Levin et al. (2006); and Schmitt-Gröhe and Uribe (2006).