Heterogeneous life-cycle models

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Overview of the lecture

- Main purpose: basic ideas on how to solve heterogeneous-agents, life-cycle models, simulate them, and possibly estimate their parameters
- I will assume that people have never written any program to solve these types of model
- Use a simple model (Cagetti 2003) to illustrate key ideas and procedures
    - Review the model (already discussed in the first lecture of the class)
    - Numerical solution
    - Simulation
    - Parameter estimation

Overview of the model

- Marco Cagetti: "Wealth accumulation over the life-cycle and precautionary savings", *Journal of Business and Economic Statistics* (2003).
- Based on Carroll, "Buffer stock saving and the life-cycle permanent income hypothesis", QJE (1997), and Gourinchas and Parker, "Consumption over the life-cycle", *Econometrica* (2002)
- Basic model of intertemporal choice of consumption and saving over the life cycle with stochastic income:
  - Two basic motives for saving: retirement and precautionary purposes
  - Key drivers: income shocks and preferences (patience and risk aversion)

The problem

- Household works for 40 years, then retires. Uncertain life-span and possible bequests.
- While working: stochastic income process with permanent and transitory shocks
- While retired: receive pensions and consume accumulated wealth.
- Each period: choice of consuming and saving in a risk-free asset.
- Borrowing constraint.

Formally, the decision problem is:

$$\max_{W_{t+1}, C_t} E\left(\sum_{t=0}^{\tilde{T}} \beta^t e^{Z_t \theta} \frac{C_t^{1-\gamma} - 1}{1-\gamma} + \beta^{\tilde{T}+1} e^{Z_{\tilde{T}} \theta} S(W_{\tilde{T}+1})\right),$$

subject to

$$W_{t+1} = RW_t + Y_t - C_t + B_t, \quad \forall t$$

and

$$W_{t+1} \geq 0, \quad \forall t.$$

Earnings process

- Stochastic earnings:

$$\log Y_t^i = G_t^i + u_t^i$$

$$G_t^i = gt + F(\text{age}^i,\ \text{education}^i),$$

$$u_t^i = u_{t-1}^i + \eta_t^i - \psi \eta_{t-1}^i.$$

Note:

- The assumptions on utility, earnings, and additional assumptions on the bequest function and the replacement rate of pensions imply that the model is homogeneous in earnings (ie it scales up with earnings).
- So for this particular model we have two state variable, cash-in-hand (assets + current income) / income, and earnings shocks
- The model is easy to solve and the solution is easy to understand, but this has very serious shortcomings….

Quick discussion:

- Looking back at all that you have learnt in the class, can you describe some of the shortcomings of the model?
- What would you change/add to the model…
  - If you wanted to focus on the behavior of retired people
  - If you wanted to focus on poorer households
  - If you wanted to focus on the top 1% of the income and wealth distributions?

Solving the model

- Key steps:
  - Discretize the state space, ie define a grid for the state variables (income and earnings shocks)
  - Work backwards from the last period, finding the optimal consumption and saving choice for all gridpoints of the state space
  - For some models, it is not possible to work backwards, since they behave like an infinitely lived agents. In this case, one has to iterate on the solution (much more complicated-we won’t go into this here)
- (For those interested, Fortran codes available online at http://www.cagetti.com/Research/)

Step 1 - Creating relevant variables

Example of entries from parameter module

!sizes of vectors and other relevant numbers numbers

INTEGER, PARAMETER :: nworkyears=40, nretyears=25

INTEGER, PARAMETER :: ngrid1=60,ngrid2=40,ngrid=ngrid1+ngrid2-1

INTEGER, PARAMETER :: nshocky=5, nagents=5000

…

! Grid for cash-in-hand state variable

REAL, DIMENSION(ngrid) :: Xhand

REAL :: xhmax,xhmed,xhmin

Step 1 - Creating grid for state variables…

- In general, one needs to have more gridpoints where the solution changes more, there are kinks, etc. Where the function is nearly linear, less need to have gridpoints, since one can interpolate easily
- For this problem, more gridpoints for small cash in hand values, and fewer for higher levels, eg.

```
CALL linspace(xhmin,xhmed,ngrid1,Xhand(1:ngrid1))
CALL linspace(xhmed,xhmax,ngrid2,Xhand(ngrid1:ngrid))
```

Step 1 - Creating grid for state variables…

- Discretized income shock process can be described by
  - N (typically a small number) gridpoints for income
  - A transition matrix NxN that gives the probability of moving from one income state to the other
- Various discretization techniques (eg Tauchen-Hussey for normal shocks) to choose gridpoints efficiently
- Problem is related to the vast field of numerical integration (or quadrature) . Ie computing integrals under various density functions.

Step 2 - Solving the model backwards:

- T-1: two period decision problem. In this case, one can use Euler equations (ie first order conditions) to compute optimal consumption
- More generally, one can use “brute force” value function backward iteration, that is, compute objective function over a grid of c and pick the maximizing c*

V(…, T-1) = max u(…) + Bequest utility (T)

- With the solution for T-1, go backwards in time

V(…, T-2) =max u() + beta V(…,T-1)

- Example on the program (show relevant lines of file cagettiJB)

Simulating the model

- With the optimal rules c* in hand, one can compute the relevant statistics implied by the model.
- One way to compute them is to simulate the model
- Key steps
    - Start from a large number of households at age zero
    - Initialize households over an assumed initial distribution of state variables (cash-in-hand and income)
    - Draw a series of shocks (income)
    - Compute the implied optimal consumption and saving for all the households
    - Compute sample statistics

Simulating the model

- Most software packets include routines to draw uniform or normal shocks. For other processes, one can draw from a uniform distribution and invert the cumulative distribution function
- For a discretized income shock process like ours, map uniform on transition matrix
- Looking at some code…
- Note: you need to use the exact same shocks across different iterations, or the parameters may not converge to the desired precision

Estimating key parameters

- Use (simulated method of moment)
- Key steps
  - Decide if you want to estimate certain parameters
  - Choose a set of meaningful moments M
  - Use (simulated) method of moments
  - Use an appropriate minimization routine to find the min

Estimating key parameters

- Looking at some code …
- Optimization routines depend on problem
- One parameter: quite straightforward
- Two parameters: basic techniques based on gradient usually work (if problem is well behaved)
- More parameters: gradient methods may or may not work. More advanced algorithms that “test” the function in various directions and update accordingly.

Estimating key parameters

- Eg: GP estimate time preference and patience (beta and gamma), the key parameters of the model. So does Cagetti.
- As target moments, I use wealth at different times of the working age for 5-year age groups. This results in 8 target moments.

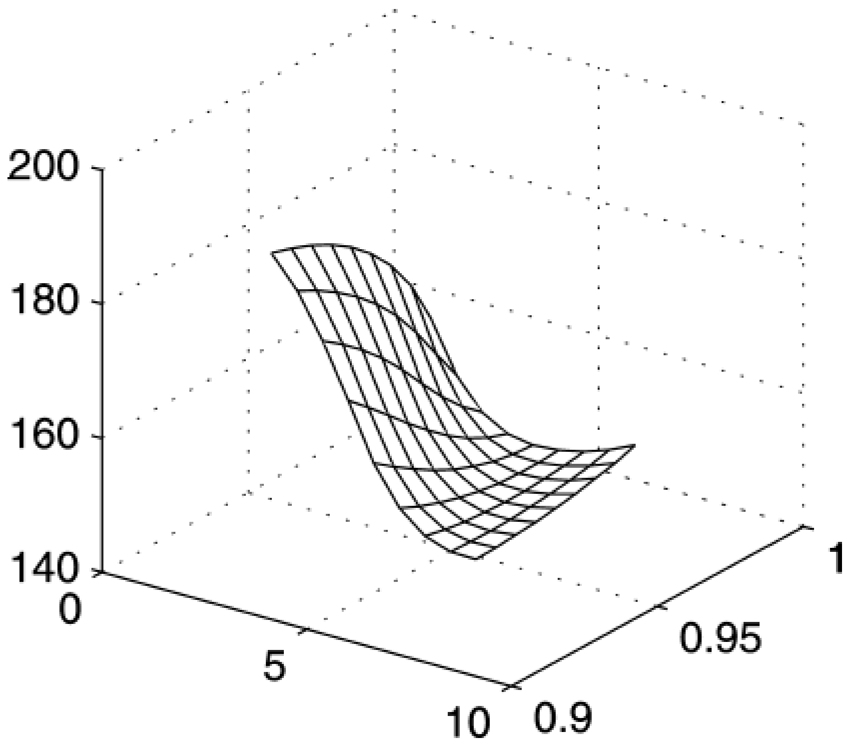
$$\widehat{M}(\beta, \gamma) = 1/N \sum (w_i^t - W^t(\beta, \gamma))$$

$$\min_{\beta,\gamma} \widehat{M}(\beta, \gamma)' T \widehat{M}(\beta, \gamma),$$

Estimating key parameters

- Calibration vs estimation vs exogenously imposed parameters
- Choose moments that are key for the model and that depend crucially on the estimated parameter
- Limit the number of estimated parameters
- Sensitivity analysis
  - Try a few parameters and check how they affect the target moments
  - If few parameters, try checking a rough grid

Estimating key parameters



Estimating key parameters

- Cagetti uses simulated medians (ie quantiles) rather than means.
- Simulation allows one to compute quantiles just as easily as means.
- However, the distributional properties of the estimator are not as well known as those for medians
- Solutions
  - Ignore issue and work with medians as you would with means
  - Check with econometricians if somebody has provided the distribution of the estimator

Estimating key parameters: means vs medians

- Why could one use quantiles instead of means/variances?
  - Means and medians are very different
  - Means can be influenced by outliers or by the tails of the distribution
  - The model is not designed to capture the tails of the distribution
  - Eg: wealth is concentrated among the top 1%. If the model doesn't explain their behavior, then using means is inappropriate.