# The Distribution of Wealth and Fiscal Policy in Economies With Finitely Lived Agents

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presented by

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# Introduction

# Research Question

## Some Stylized Facts

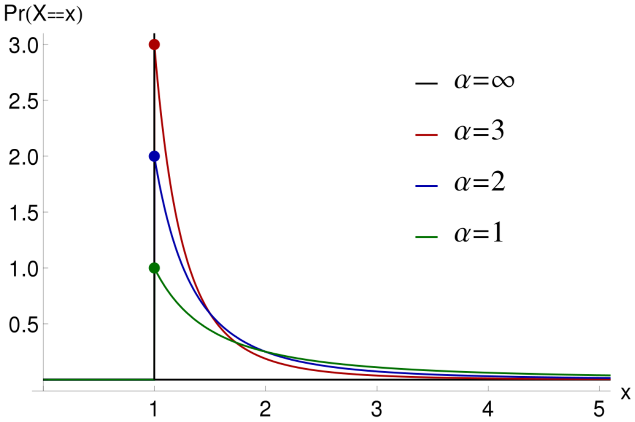
- Income/Wealth distributions are **skewed to the right**.
  - Top 1% of the richest households in the US hold 33% of the wealth.
- Income/Wealth distributions have **heavy upper tails.**
  - Top wealth shares decline slowly.
  - Top end of wealth distributuion follows a **Pareto law**.
- Models with uninsurable idiosyncratic labor income risk can generate some skewness, but not heavy tails.

### Question

Which features of the wealth accumulation process explain these stylized facts, focusing on the heavy upper tail?

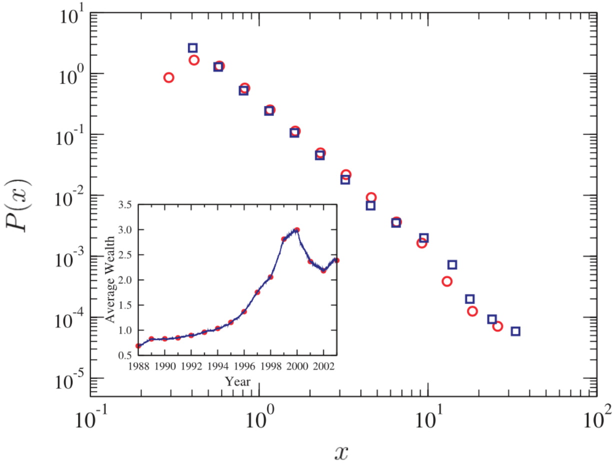
# Pareto (power-law) distribution

$$Pr(X > x) \sim kx^{-\alpha},\ \alpha > 1$$



# The Wealth of the Forbes 400

## Klass, Biham, Levy, Malcai, and Solomon (2007)



# This paper
## OLG model

- "Standard" features:
  - Continuous time OLG
  - Finitely lived agents
  - "Joy of giving" bequest motive
- Non-standard features
  - Labor income has **uninsurable idiosyncratic component** and **trend-stationary component across generations**
  - Capital income is subject to **stationary idiosyncratic shocks**, possibly persistent across generations (in the data, due to housing and private business equity)

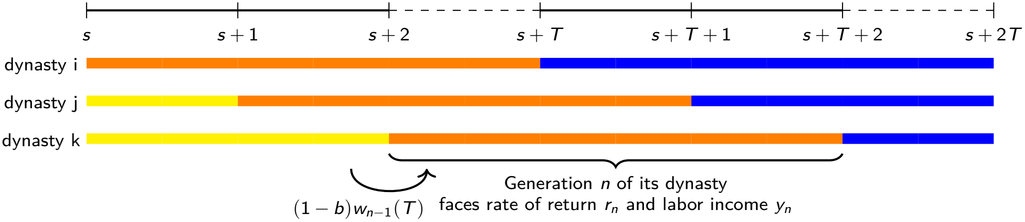
**Result**

Capital income risk, and not stochastic labor income, drives the properties of the right tail of the wealth distribution.

# Model

# Savings and Bequests

## OLG structure



# Savings and Bequests

## Some notation

- Consumption and wealth of a household at $t$ depends on
  - the generation $n$ through $r_n$ and $y_n$
  - its age $\tau = t - s$
  - $r_n$ and $y_n$ are stochastic across generations and idiosyncratic across individuals
- Consumption for household of generation $n = \frac{s}{T}$ at time $t$: $c(s\ t) = c_n(t - s)$
- Wealth for household of generation $n = \frac{s}{T}$ at time $t$: $w(s\ t) = w_n(t - s)$
- Estate tax: $b < 1$
  - Household of generation $n$ inherits $w_n(0) = (1 - b)w_{n-1}(T)$.

# Savings and Bequests

## Households

### Household Problem

$$\max_{c_n(\tau)} \int_0^T e^{-\rho\tau} u(c_n(\tau)) d\tau + e^{-\rho T}\phi(w_{n+1}(0))$$

subject to

$$\dot{w}_n(\tau) = r_n w_n(\tau) + y_n - c_n(\tau)$$

$$w_{n+1}(0) = (1-b) w_n(T)$$

Preferences satisfy

$$u(c_n(\tau)) = \frac{c_n(\tau)^{1-\sigma}}{1-\sigma}, \quad \phi(w_{n+1}(0)) = \chi \frac{w_{n+1}(0)^{1-\sigma}}{1-\sigma}$$

# Savings and Bequests

## Analytical Solution I

### The Dynamics of Wealth Across Generations

Let $w_n = w_n(0)$ denote the initial wealth of the $n$'th generation. Then,

$$w_{n+1} = \alpha_n w_n + \beta_n,$$

where $(\alpha_n, \beta_n)_n = (\alpha(r_n), \beta(r_n, y_n)_n)$ is a stochastic process.

- $\alpha_n$: lifetime rate of return on initial wealth from one generation to the next minus fraction of lifetime wealth consumed
- $\beta_n$: lifetime labor income minus lifetime wealth consumed.

# Savings and Bequests

## Analytical Solution II

### The Dynamics of Individual Wealth as a Function of Age

$$w_n(\tau) = \sigma_w(r_n, \tau)w_n + \sigma_y(r_n, \tau)y_n$$

This is a deterministic map since $r_n$ and $y_n$ are fixed for any household.

# The Stationary Distribution of Wealth

## Initial Wealth I

Recall the dynamics of initial wealth:

$$w_{n+1} = \alpha(r_n)w_n + \beta(r_n, y_n)$$

Suppose $r_n$ and $y_n$ (and therefore $\alpha_n$ and $\beta_n$) are $i.i.d$. Then, wealth converges to stationary distribution with a Pareto law:

$$Pr(w_n > w) \sim kw^{-\mu}$$

But the $i.i.d$ assumption is very restrictive:

- Autocorrelation in $r_n$ and $y_n$: Captures variations in social mobility
- Correlation between $r_n$ and $y_n$: Higher labor income correlated with higher return on wealth in financial markets

# The Stationary Distribution of Wealth

## Initial Wealth II

### Theorem 1

Consider,

$$w_{n+1} = \alpha(r_n)w_n + \beta(r_n, y_n), \quad w_0 > 0.$$

Under certain assumptions on $(r_n, y_n)_n$ the tail of the stationary distribution of $w_n$, $Pr(w_n > w)$, is asymptotic to a Pareto law

$$Pr(w_n > w) \sim kw^{-\mu}$$

where $\mu > 1$ satisfies $\lim_{N\to\infty} \left(\mathbb{E} \prod_{n=0}^{N-1} (\alpha_{-n})^{\mu}\right)^{1/N}$.

# The Stationary Distribution of Wealth

## Wealth in the Population I

We want to find distribution of wealth $w$ in the population.

- We need to aggregate over wealth of households of different ages, $\tau = 0, \ldots, T$.
- Recall the dynamics of wealth of generation $n$ at age $\tau$:

$$w_n(\tau) = \sigma_w(r_n, \tau) w_n + \sigma_y(r_n, \tau) y_n$$

- Define cdf of $w_n(\tau)$: $F(w; \tau) = 1 - Pr(w_n(\tau) > w)$
- Then, cdf of $w$ is $F(w) = \int_0^T F(w; \tau) \frac{1}{T} d\tau$

# The Stationary Distribution of Wealth

## Wealth in the Population II

### Theorem 2

Suppose the tail of the stationary distribution of initial wealth $w_n = w_n(0)$ is asymptotic to a Pareto law, $Pr(w_n > w) \sim kw^{-\mu}$. Then the stationary distribution of wealth in the population has a power tail with the same exponent $\mu$.

# Wealth Inequality: Comparative Statics

# Comparative Statics

## The Tail Index

- Recall from Theorem 1 that initial wealth $w_n$ asymptotically follows a Pareto law:

$$Pr(w_n > w) \sim kw^{-\mu}$$

- The tail index $\mu$ is inversely related with wealth inequality
- Gini coefficient of the tail: $G = \frac{1}{2\mu - 1}$
- Four exercises: What is the relationshop of $\mu$ with:
  - Capital and labor income risk
  - Preferences, particularly the bequest motive
  - Capital income and estate taxes
  - Social mobility

# Comparative Statics

## Capital and Labor Income Risk

Recall,

$$w_{n+1} = \alpha_n w_n + \beta_n$$

Theorem 1 implies,

- $(\beta_n)_n$ has **no effect on tail** of stationary wealth distribution
- High capital income risk $(Pr(\alpha_n > 1) > 0)$ is **necessary for heavy tails** in the distribution
  - If $Pr(\alpha_n < 1) = 1$ the stationary wealth distributution is bounded at $\frac{\bar{\beta}}{1-\bar{\alpha}}$.

**Proposition 1**

The tail index $\mu$ decreases with the idiosyncratic risk on return on capital.

# Comparative Statics

## Bequest Motive

Recall,

$$\phi(w_{n+1}(0)) = \chi \frac{w_{n+1}(0)^{1-\sigma}}{1-\sigma}$$

- If $\chi$ is high, **households save more** and accumulate wealth faster
- Effective rate of return $\alpha_n$ increases
- This increases wealth inequality

**Proposition 2**

The tail index $\mu$ decreases with bequest motive $\chi$.

# Comparative Statics

## Fiscal Policy

- Let $\xi$ be a tax on capital
  - Post tax return on capital: $(1-\xi)r_n$
- When $\xi$ increases, **capital income risk decreases**
  - By proposition 1, $\mu$ increases
- Bequests can partly offset this effect, but cannot change the direction of the response

**Proposition 3**

The tail index $\mu$ increases with the estate tax $b$ and capital income tax $\xi$.

# Comparative Statics

## Social Mobility

- Social mobility is higher, when $(r_n)_n$ and $(\beta_n)_n$ are **less autocorrelated**
- They consider the AR(1) and the MA(1) case:

$$\log \alpha_n = \eta_n + \theta \eta_{n-1}$$
$$\log \alpha_n = \theta \log \alpha_{n-1} + \eta_n$$

**Proposition 4**

The tail index $\mu$ decreases with $\theta$ in both, the AR(1) and the MA(1), cases.

# Calibration

# Calibration

## Matching US Lorenz Curve

- They calibrate these parameters following standard US data:
  - $\sigma = 2$ ; $\rho = 0.04$ ; $\chi = 0.25$ ; $T = 45$
  - $y_n$ has mean of 42000\$ and standard deviation of 95000\$. It grows at a rate $g$ of 1% per year.
  - Data is from Diaz-Gimenez, Quadrini, Ríos-Rull, and Rodríguez (2002), who used the 1998 survey of consumer finances.
- For the cross-sectional distribution of the rate of return on wealth - $r_n$, they:
  - distinguish two components of $r_n$: a common economy-wide rate of return $r^E$ and an idiosyncratic component $r_n^I$
  - According to the Survey of Consumer Finances: $r_n = \frac{r^E}{2} + \frac{r_n^I}{2}$
  - They are set between 7% and 9%, and their processes follow from Angeletos (2007)

# Calibration

## Social mobility

- They model the variations in $r_n$ from generation to generation as a Markov Chain
  - $r_n = (0.08, 0.12, 0.15, 0.32)$ and

$$Pr(r_{n+1}|r_n) = \begin{pmatrix} 0.8 + \epsilon_{low} & 0.12 - \frac{\epsilon_{low}}{3} & 0.07 - \frac{\epsilon_{low}}{3} & 0.01 - \frac{\epsilon_{low}}{3} \\ 0.8 & 0.12 & 0.07 & 0.01 \\ 0.8 & 0.12 & 0.07 & 0.01 \\ 0.8 - \frac{\epsilon_{high}}{3} & 0.12 - \frac{\epsilon_{high}}{3} & 0.07 - \frac{\epsilon_{high}}{3} & 0.01 + \epsilon_{high} \end{pmatrix}$$

  - $\epsilon_{low}$ controls persistence of lowest rate of return
  - $\epsilon_{high}$ controls persistence of highest rate of return

# Calibration

## Results

- Good match of top percentiles

TABLE II

PERCENTILES OF THE TOP TAIL; $\varepsilon_{\text{low}} = .01$

| | Percentiles | | |
|---|---|---|---|
| Economy | 90th–95th | 95th–99th | 99th–100th |
| United States | .113 | .231 | .347 |
| $\varepsilon_{\text{high}} = 0$ | .118 | .204 | .261 |
| $\varepsilon_{\text{high}} = .01$ | .116 | .202 | .275 |
| $\varepsilon_{\text{high}} = .02$ | .105 | .182 | .341 |
| $\varepsilon_{\text{high}} = .05$ | .087 | .151 | .457 |

# Calibration

## Results

- They they claim that $\epsilon_{high} = 0.02$ has the best fit here, but it's not that clear

TABLE III

TAIL INDEX, GINI, AND QUINTILES; $\varepsilon_{\text{low}} = .01$

| | | | Quintiles | | | | |
|---|---|---|---|---|---|---|---|
| Economy | Tail Index $\mu$ | Gini | First | Second | Third | Fourth | Fifth |
| United States | 1.49 | .803 | −.003 | .013 | .05 | .122 | .817 |
| $\varepsilon_{\text{high}} = 0$ | 1.796 | .646 | .033 | .058 | .08 | .123 | .707 |
| $\varepsilon_{\text{high}} = .01$ | 1.256 | .655 | .032 | .056 | .078 | .12 | .714 |
| $\varepsilon_{\text{high}} = .02$ | 1.038 | .685 | .029 | .051 | .071 | .11 | .739 |
| $\varepsilon_{\text{high}} = .05$ | .716 | .742 | .024 | .042 | .058 | .09 | .786 |

# Tax Experiments

# Tax Experiments

## Results

- Keeping $\epsilon_{high} = 0.02$ and $\epsilon_{low} = 0.01$, they run experiments with $b$ - estate tax and $\zeta$ - capital income tax.
- Taxes have a significant effect on the inequality of the wealth distribution as measured by the tail index. This is especially the case for the capital income tax.

TABLE IX

TAX EXPERIMENTS—TAIL INDEX $\mu$

| $b\backslash\zeta$ | 0 | .05 | .15 | .2 |
|---|---|---|---|---|
| 0 | .68 | .76 | .994 | 1.177 |
| .1 | .689 | .772 | 1.014 | 1.205 |
| .2 | .7 | .785 | 1.038 | 1.238 |
| .25 | .706 | .793 | 1.051 | 1.257 |

TABLE X

TAX EXPERIMENTS—GINI

| $b\backslash\zeta$ | 0 | .05 | .15 | .2 |
|---|---|---|---|---|
| 0 | .779 | .769 | .695 | .674 |
| .1 | .768 | .730 | .693 | .677 |
| .2 | .778 | .724 | .679 | .674 |
| .3 | .754 | .726 | .680 | .677 |

# Tax Experiments

## Results

- Castaneda, Diaz Gimenez, and Rios-Rull (2003) and Cagetti and De Nardi (2007) found very small (or even opposite) effects of eliminating bequest taxes in their calibrations in models with a skewed distribution of earnings but no capital income risk.
- This paper has a different result.

# Conclusion

# Conclusion

## Some comments

- The model results in a **good fit to the data** while still using **classical model structures**
- However:
  - The implication that **labour income** has little to no impact on wealth inequality at the tail is at odds with other modern papers, and seems unrealistic.
  - There is no role for **entrepreneurship** in this model, which has lately been shown to be a main factor in determining wealth distribution.
  - The study of **social mobility** is still limited.
  - **Age and dynasty size** are a determining factor, while data suggests that the super-rich are often self-made and even young.