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Consumption over the life cycle

Gourinchas and Parker, Econometrica 2002

By Mariacristina De Nardi

An excellent example of how to write an abstract

- **Question**: This paper estimates a structural model of optimal life-cycle consumption expenditure in the presence of realistic labor income uncertainty

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Also an excellent example of how to write an introduction

- **What does this paper do?**

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Dynamic stochastic model of life-cycle saving behavior

- Quantitative life cycle model that focuses on the working period
- Households face an exogenous, stochastic labor income process
- Estimates structural preference parameters using Method of Simulated Moments
- Characterizes optimal behavior

Key contributions

- First structural estimation of consumption functions that incorporates precautionary savings
- Characterization of the implied precautionary and life cycle savings over the life cycle and consumption function

How is this done?

1. Construct profiles across working lives for different groups of
   households (by education and occupation), CEX data
   • Average total consumption
   • Income

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1. Construct profiles across working lives for different groups of households (by education and occupation), CEX data
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2. Estimate labor income uncertainty, PSID data
3. Use 1. and 2. to estimate stochastic life-cycle model of consumer behavior
   • Solve numerically for policy functions, and aggregate to generate a simulated life-cycle profile for many values of key parameters
   • Match simulated profile to its empirical counterparts to estimate parameters (MSM)

Forces at play

- When
    - Expected income growth
    - Household's discount rate
    - Are HIGH compared to the interest rate ⇒ Consumers behave as buffer-stock agents. They wish to borrow from the future but save a bit to self-insure against shocks

Forces at play

- When
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- When
    - Expected income growth
    - Household's discount rate
    - Are LOW compared to the interest rate $\Rightarrow$ Consumers behave almost as in the certainty equivalent, life-cycle hypothesis (CEQ-LCH) model

Forces at play

- Heterogeneity in savings by age because income growth changes by age

Forces at play

- Heterogeneity in savings by age because income growth changes by age
- Households behave as buffer stock when young

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Forces at play

- Heterogeneity in savings by age because income growth changes by age
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Forces at play

- Heterogeneity in savings by age because income growth changes by age
- Households behave as buffer stock when young
- Households behave ad CEQ-LCH as retirement nears
- The relative shapes of consumption and income profiles reveal the role for precautionary and retirement savings in liquid assets.

Findings

- Fitted model matches
  - Correlation between consumption and income at young ages
  - General concavity of the consumption profile observed in data

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Findings

- Fitted model matches
  - Correlation between consumption and income at young ages
  - General concavity of the consumption profile observed in data
- The average household has
  - A discount rate of 4.0-4.5%
  - A coefficient of risk aversion between 0.5 and 1.4
  - Marginal propensity to consume out of liquid assets at retirement is 6-7% (reasonable)
  - Marginal propensity to consume out of illiquid wealth is zero (unreasonable)

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More findings

- Use model to assess determinants of wealth accumulation
    - Wealth is accumulated early in life for precautionary reasons
    - Households over 40 instead save mostly for retirement and bequests

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- Use model to assess determinants of wealth accumulation
    - Wealth is accumulated early in life for precautionary reasons
    - Households over 40 instead save mostly for retirement and bequests
- The importance of precautionary motive early in life implies
    - Between 60 and 70% of non-pension wealth is due to precautionary wealth

Model: main structure

$$E\Big[\sum_{t=1}^{N}\beta^t u(C_t, Z_t) + \beta^{N+1} V_{N+1}(W_{N+1})\Big]$$

- C_t total consumption at age t
- Z_t vector of deterministic household characteristics
- W_t total financial wealth
- V_{N+1} value to the consumer of assets left at time of death

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- T, N exogenous and fixed. No mortality until final period
- N periods of life, work for T periods. Retired for $N - T$

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Model: Preferences

Utility function

$$u(C, Z) = v(Z)\frac{C^{1-\rho}}{1-\rho}$$

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Model: Preferences

Certain income: consumer chooses consumption path such that

$$\frac{C_t}{C_{t+1}} = \left(\beta R \frac{v(Z_{t+1})}{v(Z_t)}\right)^{\left(\frac{1}{\rho}\right)}$$

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  - $\Rightarrow$ Consumption growth rate is constant and independent on the income profile
- Varying $v(Z_t)$ the growth rate of consumption may change, for instance with family size
- Individual earnings uncertainty
  - $\Rightarrow$ Households hold liquid wealth for self-insurance
  - $\Rightarrow$ Increases the slope of the consumption profile

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Income

- Income at **age** t, Y_t: permanent and transitory components:

$$Y_t = \left\{ \begin{array}{ll} P_t U_t, & t = 1, 2, \ldots, T \\ 0, & t = T + 1, \ldots, N \end{array} \right\}$$

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- Permanent component P_t: age-adjusted random walk:

$$P_t = \left\{ \begin{array}{ll} G_t P_{t-1} N_t, & t = 1, 2, \ldots, T, \\ P_T, & t = T + 1, \ldots, N \end{array} \right\}$$

$$G_t = \text{deterministic growth}$$

$$N_t = \text{permanent shock, i.i.d., } \log(N_t) \sim N(0, \sigma_n^2)$$

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$$G_t = \text{deterministic growth}$$

$$N_t = \text{permanent shock, i.i.d., } \log(N_t) \sim N(0, \sigma_n^2)$$

- Transitory component u_t is an i.i.d. process with catastrophes:

$$U_t = 0, \text{ with probability } p > 0$$

$$\sim \log(U_t) \sim N(0, \sigma_u^2), \text{ with probability } (1-p)$$

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Wealth, illiquid assets

- Model one liquid asset, W_t
- There is an illiquid asset that accumulates exogenously and is available after retirement
- Illiquid (pension) wealth, H_{T+1}, accumulates as

$$H_{T+1} = hP_{T+1} = hP_T$$

and cannot be borrowed against

Wealth, liquid assets

- Liquid wealth, W_t, follows standard asset dynamics:

$$W_{t+1} = R(W_t + Y_t - C_t)$$

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- Terminal condition for assets

$$W_{N+1} \geq 0$$

- But for any $t \leq T$:
  - $\exists$ a non-zero probability that $Y_t = Y_{t+1} = \ldots = Y_T = 0$.
  - Illiquid wealth is not available
  - Because preferences obey the Inada conditions, consumer will never borrow against future income. By backward induction $\Rightarrow$

$$W_t > 0$$

  - Self-imposed borrowing constraint because of possible zero earnings

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Model notes

- Innovations to "permanent component": as permanent as remaining length of working life
  ⇒ Propensity to consume out of "permanent" shocks varies with age

- Consumers never borrow against future labor income. Precautionary motive acts as a self-imposed borrowing constraint

- Not so, if earnings uncertainty is eliminated

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Model, more assumptions

Age variations in $v(Z_t)$:

- Homogeneous across households age t
- Deterministic
- Comes from family size, which affects utility from consumption

Model: Retirement

- Use Bellman's optimality principle to avoid modeling retirement
- Impose exogenous retirement value function summarizing consumer's problem at retirement

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Model: COH

Define cash on hand
$$X_{t+1} = R(X_t - C_t) + Y_{t+1} = W_{t+1} + Y_{t+1}$$

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Model: Recursive problem

$$
V_t(X_t, P_t) = \begin{cases}
\max_{0 \leq C_t \leq X_t} \left\{ \upsilon(Z_t)\dfrac{1}{1-\rho} C_t^{1-\rho} + \right. \\
\left. \beta E_t \left[V_{t+1}(R(X_t - C_t) + Y_{t+1}, P_{t+1})\right] \right\} & t \leq T \\
\kappa \upsilon(Z_t)\dfrac{1}{1-\rho} \left[X_t + H_t \right]^{1-\rho} & t = T+1
\end{cases}
$$

subject to

$$
X_{t+1} = R(X_t - C_t) + Y_{t+1}, \quad X_{T+1} \geq 0
$$

$$
Y_t = P_t \, U_t
$$

$$
P_t = G_t \, P_{t-1} \, N_t
$$

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Note on V_{T+1} (retirement time)

- Assume retiree's expected utility

$$V_{T+1}(W_{T+1}) = \max_{\{C_t\}_{T+1}^N} \sum_{j=1}^{N-T} \beta^{j-1} S_{T+1}(j) \frac{1}{1-\rho} C_{T+j}^{1-\rho}$$

$$s.t. \quad W_{t+1} = R(W_t - C_t)$$

$$W_{T+1} \text{ given}$$

$S_{T+1}(j) = (\Pi_{k=1}^{T+j} s_k)/(\Pi_{k=1}^{T+1} s_k)$: cond. surv. prob.

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- FOC

$$C_{T+j}^{-\rho} = \beta R \frac{S_{T+1}(j+1)}{S_{T+1}(j)} C_{T+j+1}^{-\rho} \Rightarrow \frac{C_{T+j+1}}{C_{T+j}} = [\beta R s_{T+j+1}]^{1/\rho}$$

$$\Rightarrow C_{T+j} = \left[(\beta R)^{j-1} S_{T+1}(j) \right]^{1/\rho} C_{T+1}$$

Note on V_{T+1} (cont.)

- Retiree's present value budget constraint

$$
\begin{aligned}
W_{T+1} &= \sum_{j=1}^{N-T} \frac{1}{R^{j-1}} C_{T+j} \\
&= \sum_{j=1}^{N-T} \frac{1}{R^{j-1}} \left[(\beta R)^{j-1} S_{T+1}(j) \right]^{1/\rho} C_{T+1} \\
&\equiv \Lambda^{-1} C_{T+1}
\end{aligned}
$$

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&V_{T+1}(W_{T+1}) \\
&= \sum_{j=1}^{N-T} \beta^{j-1} S_{T+1}(j) \frac{1}{1-\rho} \left(\left[(\beta R)^{j-1} S_{T+1}(j) \right]^{1/\rho} \Lambda W_{T+1} \right)^{1-\rho} \\
&\equiv \Gamma \frac{1}{1-\rho} W_{T+1}^{1-\rho}
\end{aligned}
$$

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Normalization

- Problem is homogeneous of degree $1 - \rho$
- Normalize by P_t, to simplify:

$$x_t \equiv \frac{X_t}{P_t}; \quad c_t \equiv \frac{C_t}{P_t}$$

- Rewrite the asset accumulation equation as

$$
\begin{aligned}
X_{t+1} &= R(X_t - C_t) + Y_{t+1} \\
&= P_{t+1}\left[R\frac{P_t}{P_{t+1}}\left(\frac{X_t - C_t}{P_t}\right) + u_{t+1}\right] \\
\Rightarrow x_{t+1} &= \frac{R}{G_{t+1}N_{t+1}}(x_t - c_t) + u_{t+1} \quad (1)
\end{aligned}
$$

$$x_{T+1} = R(x_T - c_T) \quad (2)$$

- Eq. (2) used $P_{T+1} = P_T$, $u_{T+1} \equiv 0$

Euler Equations

- For periods $1, 2, \ldots T - 1$, the first-order conditions are

$$C_t^{-\rho} = \beta R \frac{\upsilon(Z_{t+1})}{\upsilon(Z_t)} E_t \left(C_{t+1}^{-\rho} \right)$$

$$\Rightarrow c_t^{-\rho} = \beta R G_{t+1}^{-\rho} \frac{\upsilon(Z_{t+1})}{\upsilon(Z_t)} E_t \left(c_{t+1}^{-\rho} N_{t+1}^{-\rho} \right) \qquad (3)$$

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- With $P_{T+1} = P_T$, the Euler equation for period T is

$$c_T^{-\rho} = \max \left\{ x_T^{-\rho}, \beta R \frac{\upsilon(Z_{T+1})}{\upsilon(Z_T)} c_{T+1}^{-\rho} \right\} \qquad (4)$$

Because the worker might be liquidity constrained

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Because the worker might be liquidity constrained

- The solution to the retiree's problem is

$$C_{T+1} = \Lambda W_{T+1} = \Lambda(hP_{T+1} + X_{T+1})$$

$$\Rightarrow c_{T+1} = \gamma_0 + \gamma_1 x_{T+1}$$

$$\gamma_0 = \Lambda h; \quad \gamma_1 = \Lambda$$

Euler Equations

Euler equation for $t < T$

$$u'(c_t(x_t)) = \beta R E\Big[\frac{v(Z_{t+1})}{v(Z_t)} u'(c_{t+1}(x_{t+1}) G_{t+1} N_{t+1})\Big]$$

Intro   Findings preview   **Model**   Estimation   Results   Cagetti   Conclusions
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Numerical Solution

Goal: find the set of decision rules $\{c_t(x_t)\}_{t=1}^{T+1}$

1. Begin at age $T + 1$: $c_{T+1}(x_{T+1}) = \gamma_0 + \gamma_1 x_{T+1}$
2. Move to age T: Insert $c_{T+1}(x_{T+1})$ and equation (2) into RHS of equation (4) to get $c_T(x_T)$
3. Move to age $T - 1$: Insert $c_T(x_T)$ and equation (1) into RHS of equation (5) to get $c_{T-1}(x_{T-1})$
4. Repeat step 3 until $t = 1$

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Estimation: MSM

- Given all model parameters, solve numerically for age-dependent optimal consumption rules

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Estimation: MSM

- Given all model parameters, solve numerically for age-dependent optimal consumption rules
- For given consumption rules, numerically simulate associated expected consumption as a function of age only

Estimation: MSM

- Given all model parameters, solve numerically for age-dependent optimal consumption rules

- For given consumption rules, numerically simulate associated expected consumption as a function of age only

- Minimize distance between simulated and empirical consumption profiles

Estimation

- $s_{i,t} \equiv (x_{i,t}, P_{i,t})$

Estimation

- $s_{i,t} \equiv (x_{i,t}, P_{i,t})$
- Postulate DGP for each age t:

$$log\ C_{i,t} = log\ C_t(s_{i,t}; \psi) + \epsilon_{i,t}$$

  $\epsilon_{i,t} =$ measurement error in consumption levels

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- Goal: estimate ψ and make inference on consumption behavior

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- Goal: estimate ψ and make inference on consumption behavior
- Individual-level consumption data contaminated by measurement error

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Estimation

- Observe average log consumption at each age:

$$log \, \bar{C}_t \equiv \frac{1}{I_t} \sum_{i=1}^{I_t} log \, C_{i,t}$$

- I_t number of observations at age t

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Estimation

- Study unconditional expectation of consumption at each age:

$$log\ C_t(\psi) \equiv E\{log\ C_t(s_t; \psi)|\psi\} = \int log\ C_t(s; \psi)dF_t(s; \psi) \tag{5}$$

$F_t(s; \psi)$: unconditional cumulative distribution at age t of

- Normalized cash on hand
- Permanent component of income
- Note: it depends on ψ

Estimation

Define

- $\psi_0 =$ true parameter vector
- $log\ C_i = \{log\ C_{i,t}\}_{t=1}^T$
- $\zeta_t(log\ C_i; \psi) = log\ C_{i,t} - logC_t(\psi)$
- $\zeta(log\ C_i, \psi) \in \mathbb{R}^T$, with $t-$th as above

Estimate model using following moment conditions:

$$E\big[\zeta(log\ C_i; \psi_0)\big] = 0$$

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Method of Simulated Moments

- Get initial distributions of X_{s0} and P_{s0} from unbiased sources.
- Simulate life histories and calculate

$$\ln C_t^{IS} = \frac{1}{IS} \sum_{s=1}^{IS} \ln C_t(X_{st}, P_{st}; \psi)$$

where I is the number of observations in the data and S is the number of simulations per observation

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- The moment vector $\zeta(\ln C_i; \psi)$ is

$$\zeta(\ln C_i; \psi) = \begin{pmatrix} \ln(C_{i1}) - \ln C_1^{IS} \\ \ln(C_{i2}) - \ln C_2^{IS} \\ \vdots \\ \ln(C_{iT}) - \ln C_T^{IS} \end{pmatrix}$$

- Estimate ψ by trying to set $\frac{1}{I} \sum_i \zeta\left(\ln C_i; \widehat{\psi}\right) = \mathbf{0}$

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Estimation

Difficult to estimate all of ψ in one step
$\Rightarrow$ Two-stage estimation procedure

- Partition ψ in two subsets, θ and χ
- First stage: estimate χ using additional data
- Second stage: $E[\mu(\chi)] = 0$, get $\Rightarrow \hat{\chi}$

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MSM

- Generate (U_{t+1}, N_{t+1}): L households, T periods
- Start from $F_1(s_1)$, θ, and $\hat{\chi}$
- Solve the consumer problem and simulate unconditional expectations

$$log \, \hat{C}_t(\theta, \hat{\chi}) = \frac{1}{L} \sum_{l=1}^{L} log \, C_t(\hat{s}_{l,t}, \theta, \hat{\chi})$$

- Compute

$$\hat{\zeta}_t(log \, C_i; \theta, \hat{\chi}) = log \, C_{i,t} - log \, \hat{C}_t(\theta; \hat{\chi})$$

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MSM

Estimation: make simulated empirical moments as close as possible
to theoretical values using sample averages

$$g_t(\theta; \hat{\chi}) = \frac{1}{I_t} \sum_{i=1}^{I_t} \hat{\zeta}_t(log\ C_{i,t}; \theta, \hat{\chi})$$

$$= \frac{1}{I_t} \sum_{i=1}^{I_t} log\ C_{i,t} - log\ \hat{C}_t(\theta, \hat{\chi})$$

$$= log\ \bar{C}_t - log\ \hat{C}_t(\theta, \hat{\chi})$$

MSM

Second-stage estimation: MSM that minimizes over θ:

$$g(\theta; \hat{\chi})' \, W \, g(\theta; \hat{\chi})$$

$g(\theta; \hat{\chi}) = (g_1, ..., g_T)'$,
W weighting matrix
If $W = I_t$, est. proc. is equivalent to minimizing sum of squared residuals:

$$S(\theta; \hat{\chi}) = \sum_{t=1}^{T} (\log \bar{c}_t - \log \hat{C}_t(\theta; \hat{\chi}))^2$$

First-stage parameters

- Estimate
  - R = real interest rate on tax-free bonds = 1.0334
  - Variance of income shocks (from PSID): $\sigma^2_{\ln N} = 2.1\%$; $\sigma^2_{\ln u} = 4.4\%$
  - p = probability of catastrophe = 0.3% per year
  - Initial distribution of assets and permanent income (from CEX)
- Account for variance of these parameters in the standard errors of the second-stage parameters

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Profiles estimation, what do we need?

Estimate profiles using household level data

- Average consumption age-profile $\{\bar{C}_t\}_{t=26}^{65}$
- Average income profile $\{\bar{Y}_t\}_{t=26}^{65}$
- $\Rightarrow$ Expected income growth $\{G_t\}_{t=26}^{65}$
- Profile for typical shifts in marginal utility $\{v(Z_t)\}_{t=26}^{65}$

Cleaning the data

- Data reflect several features not incorporated in the model
  - Idiosyncratic demographic variation
  - Time (e.g., business cycle) effects

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- Data reflect several features not incorporated in the model
  - Idiosyncratic demographic variation
  - Time (e.g., business cycle) effects
  - Cohort effects
- Cohort bias (Shorrocks, 1975)
  - Older people were born in earlier years
  - Because of economic growth, people born in earlier years have less lifetime income
  - Therefore, older people will be observed to consume less and have lower assets in part because they were born earlier
  - This biases our estimates of life-cycle effects

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Cleaning the data

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  - Because of economic growth, people born in earlier years have less lifetime income
  - Therefore, older people will be observed to consume less and have lower assets in part because they were born earlier
  - This biases our estimates of life-cycle effects
- Remove all these effects by "cleaning" the data
  - Illustrate for consumption
  - Also construct cleaned estimates of $\{G_t\}_t$

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Cleaning the data, rewrite Euler equation

$$
\begin{aligned}
\eta_{i,t+1} &\equiv \frac{E_t\left(C_{t+1}^{-\rho}\right)}{C_{i,t+1}^{\rho}} \\
C_{it}^{-\rho} &= \beta R \frac{\upsilon\left(Z_{i,t+1}\right)}{\upsilon\left(Z_{i,t}\right)} C_{i,t+1}^{-\rho} \eta_{i,t+1} \\
C_{it} &= (\beta R)^{t/\rho} \left(\frac{\upsilon\left(Z_{i,t}\right)}{\upsilon\left(Z_{i,0}\right)}\right)^{1/\rho} \left(\prod_{j=1}^{t} \eta_{i,j}^{1/\rho}\right) C_{i0} \quad\quad (6)
\end{aligned}
$$

Cleaning the data, note

- First term: effect of family size variation btw age 26 and t
- Second term: drift in marginal utility (function of interest rate, discount factor, and intertemporal elasticity of subs).
- Third term: uncertainty and precautionary savings, and age
- Fourth term: initial variation in COH at age 26

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Cleaning the Data, what else is in the observed data?

- Measurement error (assumed classical and to enter multiplicatively)
- Business cycle (assumed to enter multiplicatively)
- Cohort effects (assume time shifts profiles in a parallel fashion+initial conditions)
- Take logs of previous equation + add measurement error, heterogeneity in initial conditions, and time effects get linear regression to estimate

Cleaning the Data (cont.)

- Let τ denote calendar time. Estimate equation (6) from the CEX:

$$\ln(C_{i\tau}) = \widehat{\pi}_1 f_{i\tau} + \widehat{\pi}_2 b_i + \widehat{\pi}_3 a_{i\tau} + \widehat{\pi}_4 U_{i\tau} + \widehat{\pi}_5 ret_{i\tau} + \widehat{\xi}_{i\tau}$$

$$f_{i\tau} = \text{vector of family size dummies}$$

$$b_i = \text{vector of cohort (birth-year) dummies}$$

$$a_{i\tau} = \text{vector of age dummies}$$

$$U_{i\tau} = \text{regional unemployment rate (time effects)}$$

$$ret_{i\tau} = \text{retirement dummy}$$

$$\xi_{i\tau} = \text{residual}$$

- Cleaned data for workers:

$$\ln(\widehat{C}_{it}) = \widehat{\pi}_1 \overline{f}_t + \widehat{\pi}_2 \overline{b} + \widehat{\pi}_3 a_t + \widehat{\pi}_4 \overline{U} + \widehat{\xi}_{i\tau}$$

Profiles estimation, how to?

- Cleaned data for workers:

$$\ln(\widehat{C}_{it}) = \widehat{\pi}_1 \bar{f}_t + \widehat{\pi}_2 \bar{b} + \widehat{\pi}_3 a_t + \widehat{\pi}_4 \overline{U} + \widehat{\xi}_{i\tau}.$$

- Average these data across households to get average age-profiles to be matched

- $\widehat{\pi}_1 \bar{f}_t$ also provides $\upsilon(Z_t)$

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Profiles estimation, how to?

- Similarly, construct profiles for income and typical family size
- Smooth profiles using polynomials in age and year of birth instead of dummies
- Smoothed profiles for income and family size are used as inputs of the model

Estimates

TABLE III

STRUCTURAL ESTIMATION RESULTS

MSM Estimation	Robust Weighting	Optimal Weighting
Discount Factor (β)	0.9598	0.9569
S.E.(A)	(0.0101)	
S.E.(B)	(0.0179)	(0.0150)
Discount Rate ($\beta^{-1}-1$)(%)	4.188	4.507
S.E.(A)	(1.098)	
S.E.(B)	(1.949)	(1.641)
Risk Aversion (ρ)	0.5140	1.3969
S.E.(A)	(0.1690)	
S.E.(B)	(0.1707)	(0.1137)
Retirement Rule:		
γ_0	0.0015	$5.68\,10^{-6}$
S.E.(A)	(3.84)	
S.E.(B)	(3.85)	(16.49)
γ_1	0.0710	0.0613
S.E.(A)	(0.1215)	
S.E.(B)	(0.1244)	(0.0511)
χ^2(A)	175.25	
χ^2(B)	174.10	185.67

Note: MSM estimation for entire group. Standard errors calculated without (A) and with (B) correction for first stage estimation. Cell size is 36,691 households. The last row reports a test of the overidentifying restrictions distributed as a Chi-squared with 36 degrees of freedom. The critical value at 5% is 50.71. Efficient estimates are calculated with a weighting matrix $\hat{\Omega}$ computed from the robust estimates.

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Results

- Parameter estimates and model fit
  - $\widehat{\beta} = 0.96$, tightly identified
  - $\rho \in \{0.5, 1.4\}$
  - Model fails overidentification test
  - $\gamma_1 = $ MPC out of retirement wealth $= 7\%$; imprecisely estimated
  - $\gamma_0 \approx 0 \Rightarrow$ pension wealth small relative to PI and/or low MPC out of pension wealth; not reasonable
  - Model overstates wealth at ages 55-64: $9\times$income in model, $7.4 \times$ in data. Consistent with low γ_0

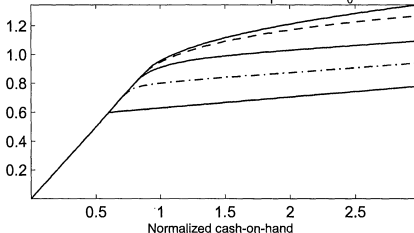# Consumption policy functions
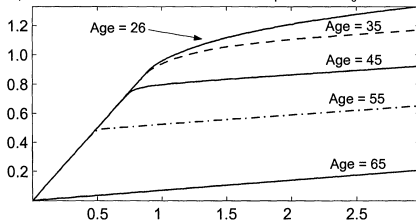
FIGURE 1.—A tale of two households.

Consumption policy functions

- At various ages, for a typical household working from age 25 to 65, for given consumption rule after retirement
- Consumption is increasing and concave in COH
- Expected permanent component of income $= 1$
- Young: "buffer stock".
  - At low levels of COH ($x < 1$): consume most, but not all, financial wealth and save little
  - At high levels of COH: precautionary motive is small, $\Rightarrow$ consume more they expect to receive (1) and run down liquid assets
- Old: Save for retirement

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Decision Rules

- As people age, switch from buffer stock to life-cycle saving.
  - Young have high values of G_t, lifetime wealth mostly in future earnings.
  - Old have low values of G_t, lifetime wealth mostly in assets.

Decision Rules

- As people age, switch from buffer stock to life-cycle saving.
    - Young have high values of G_t, lifetime wealth mostly in future earnings.
    - Old have low values of G_t, lifetime wealth mostly in assets.
- Decision rules depend on retirement parameter γ_0.
    - Reflects pension wealth and Marginal Propensity to Consume (MPC) from it.
    - Higher value of $\gamma_0 \Rightarrow$ more consumption upon retirement $\Rightarrow$ more consumption while working.
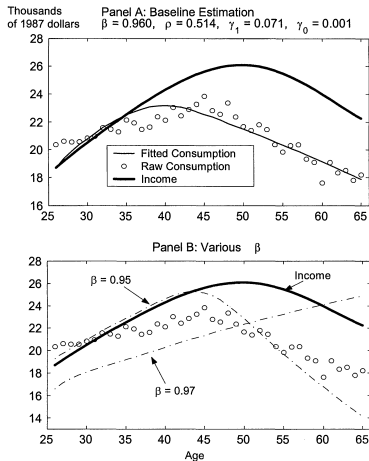
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Model and data

FIGURE 5.—The fitted consumption profile.

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Saving motives

- Use model to predict saving in no-risk environment $\equiv$ life-cycle saving

- Split total saving between life-cycle and buffer stock saving (the remainder)

- Results show that consumers switch from buffer stock to CEQ-LCH saving around age 40

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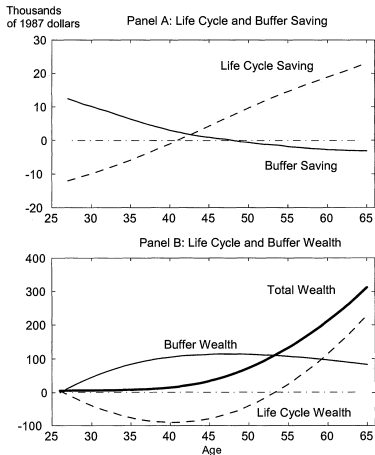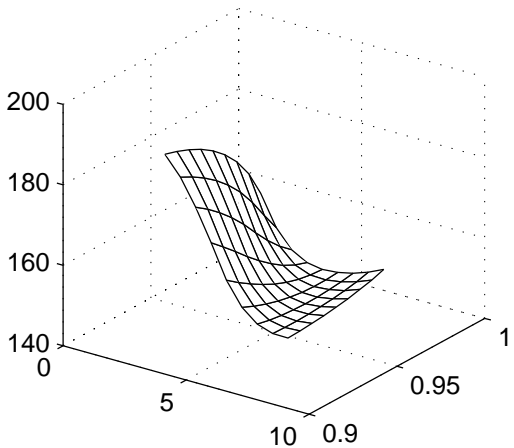# Life cycle and precautionary saving

FIGURE 7.—The role of risk in saving and wealth accumulation.

Related work: Cagetti, 2001 and 2003

Differences in approach compared to G-P 2002

- Model both working period and retirement period explicitly (no terminal ad-hoc condition)

- Allow for uncertain lifetimes

- People receive inheritances (as in data and as function of their state variables)

- Explicit modeling of a bequest motive

- Instead of average consumption, match mean or median wealth by age

- PSID and SCF data

Cagetti, 2003. Identification of β and γ

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Cagetti, 2003

Differences in results

- More sensible results for patience across education groups
  (match medians, PSID data)

    - No high school: $\beta = 0.948$, $\gamma = 2.74$
    - High school: $\beta = 0.952$, $\gamma = 3.27$
    - College graduates: $\beta = 0.989$, $\gamma = 4.26$

- Higher risk aversion and less patience than G-P $\Rightarrow$ Low
  elasticity of savings to interest rate

Cagetti, 2001: Elasticity of saving to interest rate fixing σ (choosing β to match median net worth at retirement)

| | Risk aversion | | |
Education	1	2	3
No high school	1.96	0.31	0.13
High school	1.18	0.19	0.07
College	1.31	0.26	0.10

- For similar assets at retirement, elasticity of savings to the interest rate is decreasing in γ

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Final comments on GP 2002 and Cagetti 2003

- Match both consumption and wealth. What are the trade offs and how do they contribute to identification?
- Model labor supply but more general wage processes
- Role of medical expenses and health shocks?
- Home production