The Collective Model of the Household

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Plan for today

1. Static models of the household
2. Static models of the household
3. Mazzocco (2007)
4. Some interesting applications of the collective model
5. Voena (2015)
6. Summary

Unitary model

- Most used model of the household.
- Household behaves as a single rational agent.
- Pros:
    1. Simplicity
- Cons:
    1. Assumes an aggregation result.
    2. Often rejected by the data (more on this later).

Static Unitary model

$$\max_{(X,x,l^1,l^2,d^1,D^1,d^2,D^2)} U^H(Q,q,l^1,l^2)$$

$$\text{s.t (BC) } pX + p(x^1+x^2) + \sum_{i=1}^{2} \omega^i(l^i + D^i + d^i) = Y$$

$$\text{(HP) } Q = F(X,D) \text{ , } q^1 + q^2 = f(x,d)$$

- q , (Q): Consumption of the private or public good.
- l: leisure.
- x, (X): expenditures in private or public good.
- d , (D): time devoted to production of private or public good.

Testable predictions

- Model predicts that only total household non-labor income, rather individual non-labor incomes, matters for allocations: **Income Pooling**.
- Standard consumer theory applies. The Slutsky matrix has to be symmetric and negative semi-definite.
    - The i, j term of the Slutsky matrix is given by:

$$s_{ij} = \frac{\partial \xi_i}{\partial p_j} + \frac{\partial \xi_i}{\partial Y} \xi_i$$

ξ: Marshallian demand (solution to the Utility Maximization Problem)

Collective Model

- Acknowledges that a household is formed by many individuals.
- Does not assume a particular protocol for how decisions within the household are made.
- Assumes that allocations are Pareto efficient.
- Allows each spouse's position on the Pareto Frontier to depend on **distribution factors**.
- These are variables that do not affect preferences or the budget set, but rather "bargaining power".

Collective Model

$$\max_{(X,x,l^1,l^2,d^1,D^1,d^2,D^2)} \mu_1(Z)U^1(Q,q^1,l^1) + \mu_2(Z)U^2(Q,q^2,l^2)$$

$$\text{s.t} \quad (\text{BC})\ pX + p(x^1 + x^2) + \sum_{i=1}^{2} \omega^i(l^i + D^i + d^i) = Y$$

$$(\text{HP})\ Q = F(X,D)\ ,\ q^1 + q^2 = f(x,d)$$

Browning and Chiappori (1998)

- Income pooling and the properties of the Slutsky matrix predicted by the unitary model are rejected in a sample of married households.
- The properties of the Slutsky matrix are neither rejected in a sample of single women, nor in a sample of single men.
- A collective model generates a set of testable restrictions on Marshallian demands and the Slutsky matrix that are not rejected by the data.

Why should we care?

- The collective model is more empirically supported than the unitary model.
- Why does this matter?
- Some policy recommendations are different for the unitary and the collective model.
- Example: Cash-transfer program to poor households, like Bolsa Familia in Brazil or Progresa in Mexico.
- Unitary model: It doesn't matter whether the wife or the husband receives the transfer.
- Collective model: This may matter. Moreover, since the objective of this program is to improve outcomes of children, it is very likely that giving the money to mothers will be more effective.

Mazzocco (2007)

- Extends the collective model to a dynamic setting.
- At least two ways of doing that:
  1. Upon household formation, individuals can commit to any budget-feasible contingent plan: Dynamic Collective Model with Full Commitment.
  2. Individuals cannot commit to allocations that violate a Participation Constraint: Dynamic Collective Model with Limited Commitment.
- Mazzocco (2007) shows how to empirically distinguish one model from the other using household-level consumption data (as opposed to individual-level consumption data).

Dynamic Unitary Model

$$\max_{\{C_t, Q_t, s_t\}_{t\in T, \omega\in\Omega}} \mathbb{E}_0 \left[\sum_{t=0}^{T} \beta^t U\left(C_t, Q_t\right) \right]$$

$$\text{s.t } C_t + P_t Q_t + s_t \leq \sum_{i=1}^{2} y_t^i + R_t s_{t-1}, \ \forall \ t, \omega$$

$$s_T \geq 0, \forall \ \omega$$

where:

- C_t Total HH consumption of the private good.
- Q_t Total HH consumption of the public good.
- s_t Savings in riskless asset.
- R_t Gross return of the riskless asset at t.
- y_t^i Income of spouse i (exogenous labor supply).

Full Commitment Collective Model

$$\max_{\{c_t^1, c_t^2, Q_t, s_t\}_{t \in \mathbb{T}, \omega \in \Omega}} \mu_1(Z)\mathbb{E}_0 \left[\sum_{t=0}^{T} \beta_1^t u^1(c_t^1, Q_t) \right] + \mu_2(Z)\mathbb{E}_0 \left[\sum_{t=0}^{T} \beta_2^t u^2(c_t^2, Q_t) \right]$$

$$\text{s.t } \sum_{i=1}^{2} c_t^i + P_t Q_t + s_t \leq \sum_{i=1}^{2} y_t^i + R_t s_{t-1}, \ \forall \ t, \omega$$

$$s_T \geq 0, \forall \ \omega$$

where:

- μ_i is the Pareto weight of individual i.
- Z contains variables that affect the "bargaining power" of individuals within the HH, these are called *distribution factors*.
- Examples of distribution factors can be the relative income of each household member or the local sex ratio.

Limited Commitment Collective Model

$$\max_{\{c_t^1, c_t^2, Q_t, s_t\}_{t\in\mathbb{T},\omega\in\Omega}} \mu_1(Z)\mathbb{E}_0\left[\sum_{t=0}^{T} \beta_1^t u^1(c_t^1, Q_t)\right] + \mu_2(Z)\mathbb{E}_0\left[\sum_{t=0}^{T} \beta_2^t u^2(c_t^2, Q_t)\right]$$

$$\text{s.t } \mathbb{E}_\tau\left[\sum_{t=0}^{T-\tau} \beta_i^t u^i(c_{t+\tau}^i, Q_{t+\tau}^i)\right] \geq \underline{u}_{i,\tau}(Z), \; \forall\omega, \tau > 0, i = 1, 2$$

$$\sum_{i=1}^{2} y_t^i + P_t Q_t + s_t \leq \sum_{i=1}^{2} y_t^i + R_t s_{t-1}, \; \forall \; t, \omega$$

$$s_T \geq 0, \forall \; \omega$$

- New: Participation Constraint.
- Each household member can decide to leave the Household at any point in time.

Testing

- Mazzocco (2007) tests:
  1. Dynamic collective model with and without commitment.
  2. Dynamic unitary vs dynamic collective.
- To perform this test:
  1. Derives the EE for aggregate private and public consumption for each model.
  2. Takes a second order log-linear approximation and shows that Z enters differently in EE.
  3. More concretely:
     - The approximate EE for the collective model nests the one for the unitary model
     - The approximate EE for LC nests the one for FC model
  4. This implies:
     - Testing for commitment can be done with a simple parametric test.
     - Same can be said for unitary vs collective.

Comment on the test

- Note that the test:
  - Uses consumption at the household level.
  - Doesn't track individuals over time.
- An easier to interpret test could be achieved if we could observe how individual shares of consumption respond to wage shocks.
- This kind of test requires:
  1. Household-level panel data.
  2. Detailed information on individual consumption.
- This test is performed in Lise and Yamada (2018).
- They conclude that:
  - Pareto weights respond to large wage shocks...
  - ... but not to small wage shocks.
- This can be interpreted as a rejection of Full Commitment.

Some interesting applications of the Collective Model

- Lundberg, Startz and Stillman 2003: Marital Bargaining can rationalize the Consumption-Retirement puzzle if intra-household bargaining power depends on relative incomes and wives expect to live longer.
- Lise and Seitz 2011: Adult-equivalent measures of consumption get the level and the trend of consumption inequality wrong.
  1. The **level** of consumption inequality is higher than measured by using equivalence scales because of intra-household inequality.
  2. The **trend** is flatter because decreasing intra-household inequality offsets increasing within-household inequality.
- Voena 2015: Examines how divorce laws affect intertemporal choices of couples and welfare.

Voena 2015

- Divorce laws likely to affect the risk-sharing ability of households and bargaining power within the household. This matters for the level and the intra-household distribution of welfare.
  1. Unilateral divorce introduces the possibility of walking away from marriage. This reduces the ability of couples to commit and therefore to share risks.
  2. How property is divided upon divorce is likely to affect the bargaining power of household members. This effect on bargaining power is likely to depend on whether or not unilateral divorce is possible.
- Given this, they also matter for household choices

Institutional Framework

- **Divorce grounds:**
  1. Mutual consent: Divorce takes place if both partners agree to it.
  2. Unilateral Divorce: Any partner can end the marriage at any point in time.
- **Property Division:**
  1. Title-based regimes: Assets are allocated after divorce according to ownership.
  2. Community property: Assets are divided equally upon divorce.
  3. Equitable distribution: Courts decide the division of assets upon divorce.

Model

Environment

- Husband and Wife live for T periods and retire exogenously at age R.
- Each period, choose how much to save, how to allocate consumption and whether or not to stay together. Before retirement they also choose labor force participation for the wife.

Model

Preferences

- Husband and wife derive utility from consumption c_t^j, participation P_t^j and a subjective taste for marriage shock ξ_t^j
- Period utilities:

$$u^j_{married} = u(c^j_t, P^j_t) + \xi^j_t \qquad u^j_{divorced} = u(c^j_t, P^j_t)$$

- ξ_t^j evolves according to:

$$\xi^j_t = \xi^j_{t-1} + \epsilon^j_t, , \xi^j_1 = \epsilon_1, \quad \epsilon^j_t \sim N(0, \sigma^2)$$

- $u(c, P) = \frac{c^{1-\gamma}}{1-\gamma} - \psi P$, with $\gamma \geq 0$ $\psi \geq 0$

Model

Economies of scale and children

- Childbirth is exogenous and happens at predetermined ages.
- Let x be the level of consumption expenditures. The inverse production function of consumption is given by:

$$x = F(c_t^H, c_t^W)e(k) = \left[(c_t^H)^\rho + (c_t^W)^\rho\right]^{\frac{1}{\rho}} e(k)$$

  where $e(k)$ is an equivalence scale that depends on the number of children.
- This function tries to capture economies of scale in consumption.
- Key to include this if economies of scale are a big advantage of marriage.

Model

Income over the Life-Cycle

- Labor income of spouse j (y_t^j) depends on her human capital h_t^j and her permanent component of income (z_t^j):

$$\ln y_t^j = \ln h_t^j + z_t^j$$

where:

$$z_t^j = z_{t-1}^j + \zeta_t^j \text{ and } z_1^j = \zeta_1^j, \quad \zeta_t^j \sim N(0, \sigma_{\zeta^j})$$

with ζ_t^j iid across time and correlated across spouses.

- Human capital evolves according to:

$$\ln h_t^j = \ln h_{t-1}^j - \delta(1 - P_{t-1}^j) + (\lambda_0^j + \lambda_1^j t) P_{t-1}^j$$

Model

Budget Constraints

- Married couple:

$$A_{t+1} - (1+r)A_t + x_t = y_t^H + (y_t^W - d_t^W)P_t^W$$

where $A_t = A_t^H + A_t^W$

  - Keeping track of individual asset holdings matters only in title-based regimes.
  - d_t^k captures child-care costs that the household has to incur if both parents work.

- Divorcee:

$$A_{t+1}^j - (1+r)A_t^j + c_t^j e(k_t) = \left(y_t^j - \frac{d_t^k}{2}\right) P_t^j \quad j = H, W$$

- Division of assets upon divorce:
  - In a title-based regime, each spouse keeps A_t^j after divorce.
  - In a community property regime, assets are divided equally.
  - In an equitable distribution regime, assets are divided randomly.

Model

Problem of the divorcee

- $\omega_t^H = \{A_t^j, y_t^j, \Omega_t\}$: State for the divorced husband.
  - Ω_t: Divorce laws.
- $q_t^j = (c_t^j, A_{t+1}^j)$ be the control.
- For the wife h_t^W is added as a state and P_t^W is added as a control.
- In each period, divorcees remarry with an exogenous probability $\pi_t^{j\Omega}$.
- Problem of the divorcee:

$$V_t^{jD}(\omega_t) = \max_{q_t^j} u(c_t^{jD}, P_t^{jD}) + \beta \left\{ \pi_{t+1}^{jD} E\left[V_{t+1}^{jR}(\omega_{t+1}|\omega_t) \right] + (1 - \pi_{t+1}^{j\Omega}) E\left[V_{t+1}^{jD}(\omega_{t+1}|\omega_t) \right] \right\}$$

subject to Divorcee B.C for $j = H, W$

- Let $V^{jDR} = \pi_t^{j\Omega} V_t^{jR}(\omega) + (1 - \pi_t^{j\Omega}) V_t^{jD}(\omega)$ be the expected value of entering t as a divorcee.

Model

Problem of the married couple under Mutual Consent

- State: $\omega_t = (A_t^H, A_t^W, y_t^H, y_t^W, \xi_t^H, \xi_t^W, h_t^W, \Omega_t)$
- Choice vector: $q_t = \{c_t^H, c_t^W, P_t^W, A_{t+1}^H, A_{t+1}^W, D_t\}$
- Value function:

$$V_t(\omega_t) = \max_{q_t}(1 - D_t)\left\{\theta u(c_t^H, P_t^H; \xi_t^H) + (1-\theta)u(c_t^W, P_t^W; \xi_t^W) + \beta E[V_{t+1}(\omega_{t+1}|\omega_t)]\right\}$$
$$+ D_t\left\{\theta\left[u(c_t^H, P_t^H) + \beta E[V_{t+1}^{HDR}(\omega_{T+1}|\omega_T)]\right] + (1-\theta)\left[u(c_t^W, P_t^W) + \beta E[V_{t+1}^{WDR}(\omega_{t+1}|\omega_t)]\right]\right\}$$

s.t B.C in marriage holds if $D_t = 0$

s.t B.C in divorce holds if $D_t = 1$

$$u(c_t^j, P_t^j) + \beta E[V_{t+1}^{jDR}(\omega_{T+1}|\omega_T)] > V_t^{jM} \text{ for } j = H, W \text{ if } D_t = 1$$

Model

Problem of the married couple under Unilateral Divorce

- ω_t now includes the within-period Pareto weights, $\tilde{\theta}_t^j$, which change over time to ensure that the participation constraint is satisfied.
- These evolve according to:

$$\tilde{\theta}_{t+1}^j = \tilde{\theta}_t^j + \mu_t^j \text{ for } j = H, W$$

- Other difference with respect to the Mutual Consent Regime is that now the Participation Constraints say that the value of marriage now has to be higher than the value of divorce for **both** partners.
- Upon divorce, assets are divided according to the division rule in place.

Divorce Laws and Household Outcomes

- Mutual Consent:

$$\frac{u_c(c_t^H, P_t^H)}{u_c(c_t^W, P_t^W)} = \frac{1-\theta}{\theta}$$

- Unilateral Divorce:

$$\frac{u_c(c_t^H, P_t^H)}{u_c(c_t^W, P_t^W)} = \frac{\tilde{\theta}_t^W + \mu_t^W}{\tilde{\theta}_t^H + \mu_t^H}$$

- Mutual consent: allocations in marriage are not affected by the division rule upon divorce.
- Unilateral divorce: This is no longer the case. Allocations in marriage depend on division rule through the Lagrange multipliers.
- Finally, note that this is partial equilibrium (θ is fixed across institutional arrangements).

Data and Estimation

- Data:
  1. Panel Study of Income Dynamics (PSID)
  2. National Longitudinal Survey of Young Women and National Longitudinal Survey of Mature Women (NLS-YW and NLS-MW)
- Three groups of model parameters:
  1. Some parameters are taken from external sources.
  2. Earnings processes: Estimated without solving the model.These are estimated using Non-linear least squares and using a correction procedure to account for selection of women into the labor force.
  3. (θ, ψ, σ) estimated via indirect inference.

Preset parameters

TABLE 3—PRESET PARAMETERS OF THE MODEL

Parameter	Value	Reference
Initial age	23	
Years in each period	3	
Age at death	82	
Retirement age	65	
Economies of scale in couple (ρ)	1.4023	McClements scale
Economies of scale for children ($e(k)$)		McClements scale
RRA (γ)	1.5	Attanasio et al. (2008)
Market returns on assets (r)	0.03	
Discount factor (β)	0.98	Attanasio et al. (2008)
Retirement income	1992 Soc. Sec. rules	Casanova (2010)
W's age at childbearing	26 and 29	PSID
Child care costs (g^k)		Attanasio et al. (2008)
Remarriage probabilities $\pi_t^{j\Omega}$		PSID
Cost of divorce (CD)		Rosen law firm fee calculator

Parameters of Income Processes

TABLE 4—PARAMETERS OF THE INCOME PROCESS

Parameter	Symbol	Estimate	Standard error
W's returns to experience (constant)	λ_0^W	0.083	(0.014)
W's returns to experience (age)	λ_1^W	−0.004	(0.001)
W's human capital depreciation	δ	0.080	(0.043)
H's returns to experience (constant)	λ_0^H	0.055	(0.0027)
H's returns to experience (age)	λ_1^H	−0.0066	(0.0004)
Offer wage gender gap	$\frac{y_1^W}{y_1^H}$	0.59	(0.0365)
Variance of *W*'s income shock	$\sigma^2_{\zeta^W}$	0.074	(0.006)
Variance of *H*'s income shock	$\sigma^2_{\zeta^H}$	0.042	(0.001)
Covariance of *H*'s and *W*'s income shocks	$\sigma_{\zeta^H \zeta^W}$	0.007	(0.002)

Notes: Income process parameters estimated by nonlinear least squares using PSID data of couples married before divorce law reforms and of divorcees. Standard errors in parentheses computed by bootstrap to account for first-stage estimation errors.

Parameters estimated by indirect inference

1. σ: The standard deviation of the marriage quality shock.
2. ψ: The utility cost of participation.
3. θ: The Pareto weight of the husband.

- Auxiliary models (estimated on the sub-sample of couples living in community property states):

$$assets_{i,s,t} = \beta Unilateral_{s,t} + \gamma' Z_{i,t} + \delta_t + f_i + v_{1,i,s,t}$$

$$\phi_1 = \frac{\beta}{average\ assets}$$

$$employment_{i,s,t} = \phi_2 Unilateral_{s,t} + \gamma' Z_{i,t} + \delta_t + f_i + v_{2,i,s,t}$$

$$employment_{i,s,t} = \phi_3 + v_{3,i,s,t}$$

$$ever\ divorced = \phi_4 + v_{4,i,s}$$

Identification

- ϕ_2 is informative of θ.
  - When θ is high, wife is more likely to be better off when divorced. When unilateral divorce is introduced, her Pareto weight shifts. Since she values leisure, her labor supply goes down.
  - When θ is low, wife is better off in marriage. When unilateral divorce is introduced, divorce and the associated drop in consumption are more likely. This increases wife's labor supply due to a consumption-smoothing motive.
- ϕ_3 is informative of ψ and θ.
  - ψ The higher the disutility of working, the lower the participation rate.
  - θ The higher the Pareto weight of the wife, the larger her consumption of leisure and the lower her participation rate.
- ϕ_4, (share of women ever divorced) is informative of σ. The higher σ, the larger the probability of a very negative marriage quality shock, and the more likely divorce is.

Identification

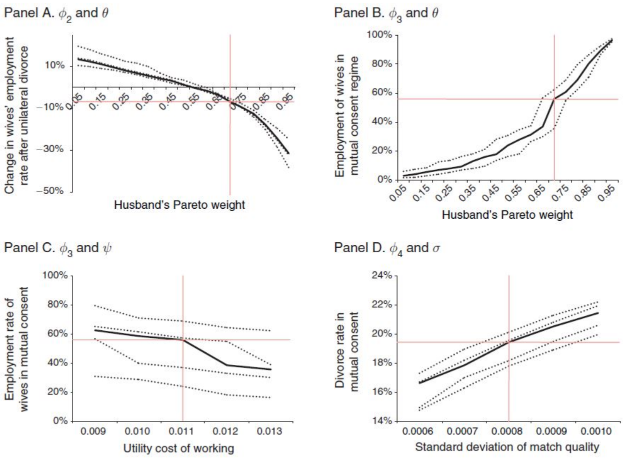

FIGURE 2. IDENTIFICATION OF THE PARAMETERS

Notes: Relationship between a parameter of the structural model and a parameter of the auxiliary model obtained by simulation. The solid line is computed using the other estimated parameters. The dotted lines are computed using random values of the other structural parameters.

Estimated Parameters

TABLE 5—ESTIMATED STRUCTURAL PARAMETERS AND MATCH OF THE AUXILIARY MODEL

Parameter	Symbol	Estimate	Standard error
Standard deviation of preference shocks	σ	0.0008	0.0004
Disutility from labor market participation	ψ	0.0107	0.0025
Husbands' Pareto weight	θ	0.7	0.0155
Auxiliary model parameter	**Symbol**	**Target**	**Simulated**
Effect of uni. divorce on savings in CP	ϕ_1	13.54 percent	13.43 percent
Effect of uni. divorce on participation in CP	ϕ_2	−6.93 pcpt	−6.86 pcpt
Baseline participation rate in CP	ϕ_3	55.97 percent	56.03 percent
Baseline divorce probability in CP	ϕ_4	19.44 percent	19.44 percent

Notes: Parameters of the dynamic model $\{\sigma, \psi, \theta\}$ estimated by indirect inference. The parameters of the auxiliary model are $\{\phi 1\ ,\ \phi 2\ ,\ \phi_3, \phi_4\}$.

Welfare analysis of divorce laws reform

- Use model to evaluate how introduction of unilateral divorce affected distribution of resources within marriage and upon divorce.
- At the estimated value of $\theta = 0.7$, the average share of resources that goes to the wife increases from 39% to 41%.
- This is driven by the 19% of couples for whose Pareto weights are modified after the introduction of unilateral divorce.
- The distribution of assets upon divorce also changes. In a title-based regime, the average share of assets upon divorce is 40%, while this share is 42% under equitable distribution and 50% under community property by construction.

Divorce laws and consumption insurance

- From the risk-sharing with limited commitment literature we know that the possibility of walking out of the marriage reduces the ability to share risk.
- In this models the introduction of unilateral divorce worsens the commitment technology.
- If we had individual consumption data we could just run BPP regressions on individual consumption under mutual consent and unilateral divorce and see how those coefficients change.
- We do not have data on individual consumption, but we can use simulations from the model.
- The paper also examines how individual consumption responds to divorce in both regimes.

$$\Delta \log(c_{it}^j) = \kappa^j + \mu^j \Delta \log(y_{it}^j) + \nu^{j\prime} X_{it}^j + \epsilon_{it}^j$$

$$\log(c_{it}^j) = \chi^j + \eta^j Divorced_{it} + \psi^{j\prime} X_{it}^j + \rho_i^j + v_{it}^j$$

Divorce laws and consumption insurance

- Consumption is more responsive to the husband's income under unilateral divorce for all property division regimes.
- This is not true for the wife's income, because under unilateral divorce wives work less.
- Moreover, divorce is associated with a drop in consumption in all institutional arrangements for both wife and husband.
- Husbands suffer the least from divorce under unilateral divorce and a title-based regime.
- Wives suffer the least under mutual consent and a community property regime.

Summary

- Introduced the static unitary and collective models of the household, and revised some literature that suggests that the collective model has more empirical support.
- Use an influential paper to see how these static models can be extended to a dynamic context.
- When dynamics are introduced, commitment becomes an issue. Revised how to empirically distinguish a model with commitment from a model without commitment.
- Revised an interesting application of the dynamic collective model.
- From this application, we learned that divorce laws affect intertemporal choices of married couples and their welfare by changing their ability to commit and the distribution of resources upon divorce.