Consumption Inequality and Partial Insurance

Richard Blundell, Luigi Pistaferri, and Ian Preston
American Economic Review (2008)

by Nicolò Russo

Question

What is the link between income and consumption inequality?

- The evolution of inequality can be explained by the degree of consumption insurance against income shocks
- Famous for consumption insurance, rather than inequality!

Mainstream approaches to consumption insurance

1. **Complete markets hypothesis**
    - Full insurance against idiosyncratic shocks
    - Rejected in the data → Attanasio and Davis (1996)
2. **Permanent income hypothesis**
    - Consumption reacts one-to-one to permanent shocks and is perfectly insured against transitory shocks
    - In the data:
        - Too little reaction to permanent shocks → Campbell and Deaton (1989)
        - Too much reaction against transitory shocks → Hall and Mishkin (1982)

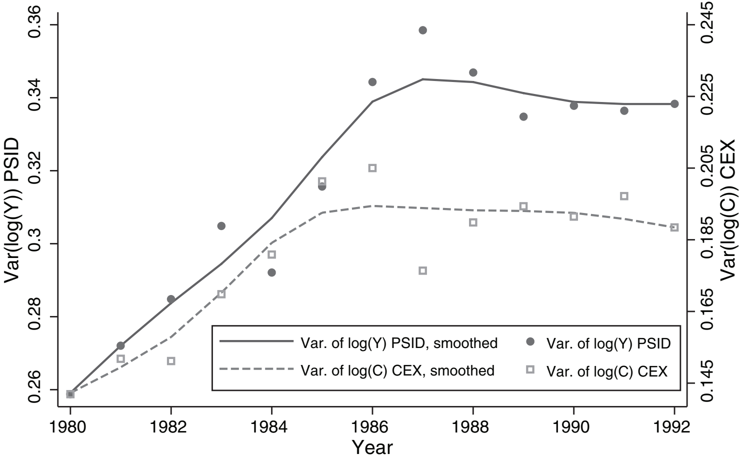
This paper

- Studies **partial insurance** and estimates it
- Takes no a-priori stance on the insurance mechanisms
- Strategy:
  - 1978-1992 PSID and 1980-1992 CEX
  - Specifies income process
  - Uses covariance restrictions to identify insurance parameters
- Findings:
  1. Almost full insurance against transitory shocks
  2. Only partial insurance against permanent shocks

Insurance and inequality

- If there was **full insurance**:
  - Consumption inequality would not react to income inequality
- If there was **no insurance**:
  - Consumption inequality would perfectly track income inequality
- What happens in US data?

Income and consumption inequality

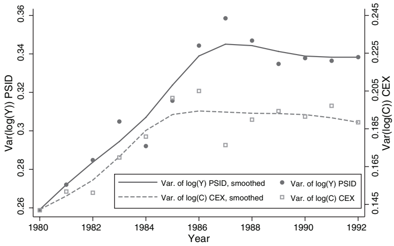


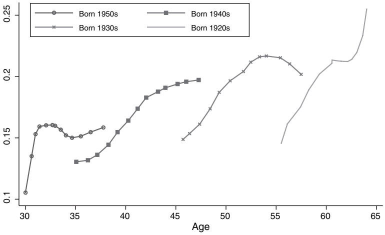
Overall pattern of inequality

Previous literature

- Blundell and Preston (1998)
    - Use growth in consumption inequality to identify growth in permanent income inequality
    - No panel data
- Krueger and Perri (2006)
    - Limited commitment explains the differences between consumption and income inequality
    - No distinction between permanent and transitory shocks
- Heathcote, Storesletten, and Violante (2004) and Heathcote, Storesletten, and Violante (2007)
    - Study partial insurance in "simple" economies

Empirical observations





- Slope of income variance > Slope consumption variance
- Consumption inequality flattens out

- Consumption inequality should be monotonically increasing with age → Deaton and Paxson (1994)
- Broadly true in the sample
- Higher inequality for recent cohorts

What do these empirical observations tell us?

- Identified features of the evolution of inequality
- **But** how did these features come about?
- We do not know:
    1. Nature of changes in the income process
    2. Nature of insurance

A new panel dataset

- Need a panel with both income and consumption
- Not available for sample period!
- They combine PSID (panel) with CEX (cross-section)
- Sample selection:
  - Continuously married couples headed by a male age 30 to 65
  - No households with changes in head or spouse

Imputation procedure

- **Main idea:**
  - Use data from CEX to construct a measure of nondurable consumption for the PSID
- **Steps:**
  1. Start with food consumption → available in both datasets
  2. Estimate demand for food using CEX
  3. Invert demand to obtain nondurable consumption in the PSID

Imputation procedure

- Estimate demand for food in CEX:

$$f_{i,t} = \boldsymbol{W}'_{i,t}\boldsymbol{\mu} + \boldsymbol{p}_t\boldsymbol{\theta} + \beta(D_{i,t})c_{i,t} + e_{i,t} \tag{1}$$

where:

- f:= log of real food expenditure
- W:= vector of demographic variables
- p:= vector of relative prices
- c:= log of nondurable expenditure
- e:= unobserved heterogeneity and measurement error
- $\beta(\cdot)$:= budget elasticity

- c is only in the CEX, all else is in both!
- Estimate and invert to get c in the PSID

▸ Food demand estimates ▸ How good is the imputation?

Framework

- **Main object of interest:**
  - % response of consumption to a 1% change in income
- **Assumptions:**
  1. Income: net of taxes
  2. Preferences: separable between consumption and leisure

Income process

Real log-income:

$$\log Y_{it} = \boldsymbol{Z}'_{it}\varphi_t + P_{it} + \nu_{it} \tag{2}$$

where:

- Z:= observable known characteristics
- P:= permanent component of income
- ν:= transitory component of income

Income components

- **Permanent component:** random walk

$$P_{it} = P_{i,t-1} + \zeta_{it} \tag{3}$$

where ζ_{it} is serially uncorrelated

- **Transitory component:** MA(q)

$$\nu_{it} = \sum_{j=0}^{q} \theta_j \varepsilon_{i,t-j} \tag{4}$$

where:

- $\theta_0 = 1$
- q will be determined empirically

How flexible is this income process?

Unexplained income growth

- "Detrended" log-income:

$$y_{it} = \log Y_{it} - \boldsymbol{Z}'_{it}\varphi_t$$

- Unexplained income growth:

$$\Delta y_{i,t} = \zeta_{i,t} + \Delta\nu_{i,t} \tag{5}$$

Transmission of income shocks to consumption

- Unexplained change in log-consumption:

$$\Delta c_{it} = \phi_{it}\zeta_{it} + \psi_{it}\varepsilon_{it} + \xi_{it} \qquad (6)$$

- Partial insurance parameters:
  - ϕ:= insurance against permanent shocks
  - ψ:= insurance against transitory shocks

Insurance benchmarks

- **Full insurance**

$$\phi_{it} = \psi_{it} = 0$$

- **No insurance**

$$\phi_{it} = \psi_{it} = 1$$

- **Partial insurance**

$$0 < \phi_{it} < 1, \quad 0 < \psi_{it} < 1$$

Models of partial insurance

1. PIH with self insurance through precautionary savings ▸ Details
2. Excess smoothness and “excess” insurance ▸ Details
3. Advance information ▸ Details

Identification of income process

- **WANT:** identification of ϕ and ψ
- Start from the income process
- **Assumptions**
  1. ζ, ν, ε mutually uncorrelated
  2. ν is an MA(0) $\rightarrow \Delta y_{it} = \zeta_{it} + \Delta\varepsilon_{it}$
- Can show that:

$$var(\zeta_t) = cov(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})$$
$$var(\varepsilon_t) = -cov(\Delta y_t, \Delta y_{t+1})$$

Derivation

Identification of income process

- Can show that:

$$cov(\Delta y_t, \Delta y_{t+s}) = \begin{cases} var(\zeta_t) + var(\Delta v_t) & \text{for } s = 0 \\ cov(\Delta \nu_t, \Delta \nu_{t+s}) & \text{for } s \neq 0 \end{cases}$$

- Use this to identify order of MA process for ν:
  - If ν is an MA(q):

$$cov(\Delta y_t, \Delta y_{t+s}) = 0 \quad \forall \quad |s| > q + 1$$

  - If ν is serially uncorrelated ($\nu_{it} = \varepsilon_{it}$):

$$cov(\Delta \nu_t, \Delta \nu_{t+s}) = -\sigma_\varepsilon^2, \qquad \text{for } s = 1$$
$$cov(\Delta \nu_t, \Delta \nu_{t+s}) = 0, \qquad \text{for } s \geq 2$$

Derivation ACF for MA(1)

Identification of insurance coefficients

- Can show that:

$$cov(\Delta c_t, \Delta y_{t+s}) = \begin{cases} \phi_t var(\zeta_t) + \psi_t var(\varepsilon_t) & \text{for } s = 0 \\ \psi_t cov(\varepsilon_t, \Delta \nu_{t+s}) & \text{for } s \neq 0 \end{cases}$$

- Can identify ϕ and ψ with:

$$var(\zeta_t) = cov(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})$$
$$var(\varepsilon_t) = -cov(\Delta y_t, \Delta y_{t+1})$$

▸ Derivation ▸ Solution

Consumption growth inequality

- Recall that:

$$\Delta c_{it} = \phi_{it}\zeta_{it} + \psi_{it}\varepsilon_{it} + \xi_{it}$$

- Can show that:

$$cov(\Delta c_t, \Delta c_{t+s}) = \begin{cases} \phi_t^2 var(\zeta_t) + \psi_t^2 var(\varepsilon_t) + var(\xi_t) & \text{for } s = 0 \\ 0 \quad \text{for } s \neq 0 \end{cases}$$

- Consumption growth inequality ($s = 0$) can increase because:

1. Decline in insurance (increase in ϕ and ψ)
2. Increase in the variance of income shocks

Derivation

Autocovariances of income growth

Table 3—The Autocovariance Matrix of Income Growth

Year	$var(\Delta y_t)$	$cov(\Delta y_{t+1}, \Delta y_t)$	$cov(\Delta y_{t+2}, \Delta y_t)$
1980	0.0832	−0.0196	−0.0018
	(0.0089)	(0.0035)	(0.0032)
1981	0.0717	−0.0220	−0.0074
	(0.0075)	(0.0034)	(0.0037)
1982	0.0718	−0.0226	−0.0081
	(0.0051)	(0.0035)	(0.0026)
1983	0.0783	−0.0209	−0.0094
	(0.0066)	(0.0034)	(0.0042)
1984	0.0805	−0.0288	−0.0034
	(0.0055)	(0.0036)	(0.0032)
1985	0.1090	−0.0379	−0.0019
	(0.0180)	(0.0074)	(0.0038)
1986	0.1023	−0.0354	−0.0115
	(0.0077)	(0.0054)	(0.0038)
1987	0.1116	−0.0375	0.0016
	(0.0097)	(0.0051)	(0.0046)
1988	0.0925	−0.0313	−0.0021
	(0.0080)	(0.0042)	(0.0032)
1989	0.0883	−0.0280	−0.0035
	(0.0067)	(0.0059)	(0.0034)
1990	0.0924	−0.0296	−0.0067
	(0.0095)	(0.0049)	(0.0050)
1991	0.0818	−0.0299	NA
	(0.0059)	(0.0040)	
1992	0.1177	NA	NA
	(0.0079)		

- $var(\Delta y_t) \uparrow$
- $cov(\Delta y_{t+1}, \Delta y_t) \uparrow$ until mid-80s
- $cov(\Delta y_{t+2}, \Delta y_t)$ small, so MA(1)

Autocovariances of consumption growth

Table 4—The Autocovariance Matrix of Consumption Growth

Year	$var(\Delta c_t)$	$cov(\Delta c_{t+1}, \Delta c_t)$	$cov(\Delta c_{t+2}, \Delta c_t)$
1980	0.1275	−0.0526	0.0022
	(0.0097)	(0.0076)	(0.0056)
1981	0.1197	−0.0573	0.0025
	(0.0116)	(0.0084)	(0.0043)
1982	0.1322	−0.0641	0.0006
	(0.0110)	(0.0087)	(0.0060)
1983	0.1532	−0.0691	−0.0056
	(0.0159)	(0.0100)	(0.0067)
1984	0.1869	−0.1003	−0.0131
	(0.0173)	(0.0163)	(0.0089)
1985	0.2019	−0.0872	NA
	(0.0244)	(0.0194)	
1986	0.1628	NA	NA
	(0.0184)		
1987	NA	NA	NA
1988	NA	NA	NA
1989	NA	NA	NA
1990	0.1751	−0.0602	−0.0057
	(0.0221)	(0.0062)	(0.0067)
1991	0.1646	−0.0696	NA
	(0.0142)	(0.0100)	
1992	0.1467	NA	NA
	(0.0130)		

Imputation error

- $var(\Delta c_t) \uparrow$ until 1985, then flattens
- $var(\Delta c_t)$ large
- $cov(\Delta c_{t+1}, \Delta c_t)$ large, so large imputation error
- $cov(\Delta c_{t+2}, \Delta c_t)$ very small

Income-Consumption growth covariance

Table 5—The Consumption-Income Growth Covariance Matrix

Year	$\text{cov}(\Delta y_t, \Delta c_t)$	$\text{cov}(\Delta y_{t+1}, \Delta c_t)$	$\text{cov}(\Delta y_t, \Delta c_{t+1})$
1980	0.0040	0.0013	0.0053
	(0.0041)	(0.0039)	(0.0037)
1981	0.0116	−0.0056	−0.0043
	(0.0036)	(0.0032)	(0.0036)
1982	0.0165	−0.0064	−0.0006
	(0.0036)	(0.0031)	(0.0039)
1983	0.0215	−0.0085	−0.0075
	(0.0045)	(0.0049)	(0.0043)
1984	0.0230	−0.0030	−0.0119
	(0.0052)	(0.0043)	(0.0050)
1985	0.0197	−0.0035	−0.0035
	(0.0068)	(0.0047)	(0.0065)
1986	0.0179	−0.0015	NA
	(0.0048)	(0.0052)	
1987	NA	NA	NA
1988	NA	NA	NA
1989	NA	NA	0.0030
			(0.0040)
1990	0.0077	0.0045	−0.0016
	(0.0045)	(0.0065)	(0.0042)
1991	0.0112	0.0011	−0.0071
	(0.0044)	(0.0049)	(0.0042)
1992	0.0082	NA	NA
	(0.0048)		
Test $\text{cov}(\Delta y_{t+1}, \Delta c_t) = 0$ for all t			p-value 25%
Test $\text{cov}(\Delta y_{t+2}, \Delta c_t) = 0$ for all t			p-value 27%
Test $\text{cov}(\Delta y_{t+3}, \Delta c_t) = 0$ for all t			p-value 74%
Test $\text{cov}(\Delta y_{t+4}, \Delta c_t) = 0$ for all t			p-value 68%

- $cov(\Delta y_t, \Delta c_t) \uparrow$ until 1985
- $cov(\Delta y_{t+1}, \Delta c_t)$ close to 0, so almost full insurance against transitory shocks
- Tests reject advance information

Estimation

- Objects of interest:
  - Variance of income shocks: σ_ζ^2, σ_ε^2
  - Insurance parameters: ϕ, ψ

- Allow for:
  - Measurement error
  - Time varying variance in measurement error and shocks
  - MA(1) transitory component of income
  - Unobserved heterogeneity

- Three samples:
  1. Baseline
  2. Separated by education
  3. Separated by cohort

- Use diagonally weighted minimum distance (DWMD)

Insurance parameters

TABLE 6—MINIMUM-DISTANCE PARTIAL INSURANCE AND VARIANCE ESTIMATES

	Whole sample	No college	College	Born 1940s	Born 1930s
θ	0.1132	0.1268	0.1086	0.1324	0.1706
(Serial correl. trans. shock)	(0.0247)	(0.0318)	(0.0341)	(0.0442)	(0.0470)
σ_ξ^2	0.0105	0.0074	0.0141	0.0122	0.0001
(Variance unobs. slope heterog.)	(0.0041)	(0.0079)	(0.0040)	(0.0064)	(0.0090)

- MA parameter θ small
- Variance of unobserved heterogeneity small but significant

Insurance parameters

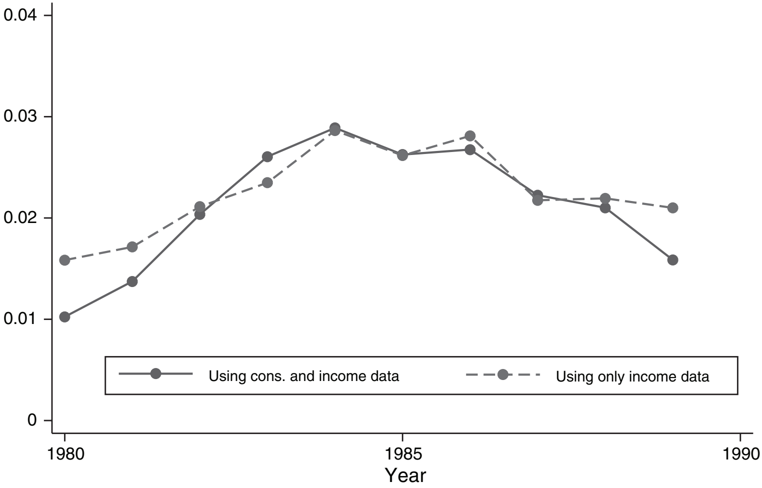
TABLE 6—MINIMUM-DISTANCE PARTIAL INSURANCE AND VARIANCE ESTIMATES

	Whole sample	No college	College	Born 1940s	Born 1930s
ϕ	0.6423	0.9439	0.4194	0.7928	0.6889
(Partial insurance perm. shock)	(0.0945)	(0.1783)	(0.0924)	(0.1848)	(0.2393)
ψ	0.0533	0.0768	0.0273	0.0675	−0.0381
(Partial insurance trans. shock)	(0.0435)	(0.0602)	(0.0550)	(0.0705)	(0.0737)
p-value test of equal ϕ	23%	99%	8%	81%	18%
p-value test of equal ψ	75%	33%	29%	76%	4%

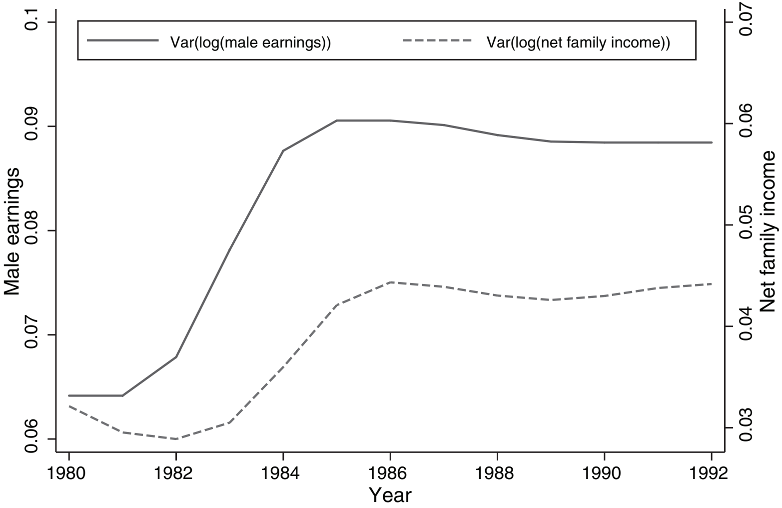
- $\phi = 0.6423 \rightarrow$ partial insurance against permanent shocks
- $\psi = 0.0533 \rightarrow$ almost full insurance against transitory shocks
- ϕ changes by education

Insurance or pass-through?

Variance of permanent shocks



Variance of transitory shocks



Variance of consumption growth

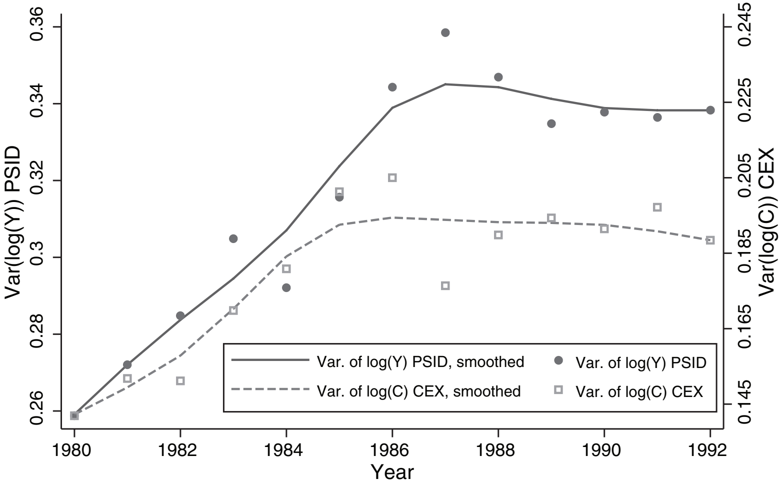
$$\Delta var(\Delta c_t) \approx var(\zeta_t)\Delta\phi_t^2 + \phi_{t-1}^2 \Delta var(\zeta_t) + var(\varepsilon_t)\Delta\psi_t^2 + \psi_{t-1}^2 \Delta var(\varepsilon_t)$$

- Evidence that $\Delta\phi_t^2 = \Delta\psi_t^2 = 0$, so:

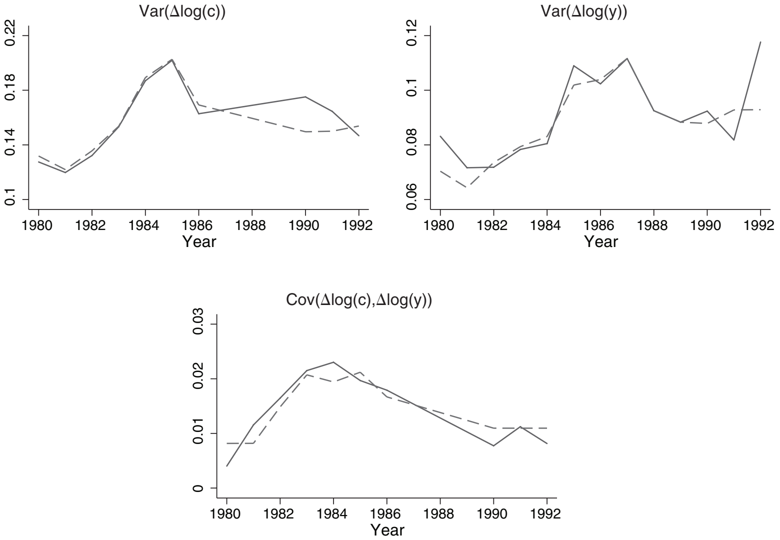
$$\Delta var(\Delta c_t) \approx \phi_{t-1}^2 \Delta var(\zeta_t) + \psi_{t-1}^2 \Delta var(\varepsilon_t)$$

- Early part of the sample:
  - Variance of permanent shock and of consumption $\uparrow$
  - But attenuation due to insurance
- Later part of the sample:
  - Variance of transitory shocks $\uparrow$
  - But ψ close to 0, so little effect on consumption inequality

Variance of income and consumption



Goodness of fit



Taxes, transfers, and family labor supply

TABLE 7—MINIMUM-DISTANCE PARTIAL INSURANCE AND VARIANCE ESTIMATES

Consumption: Income: Sample:	Nondurable net income baseline	Nondurable earnings only baseline	Nondurable male earnings baseline
ϕ	0.6423	0.3100	0.2245
(Partial insurance perm. shock)	(0.0945)	(0.0574)	(0.0493)
ψ	0.0533	0.0633	0.0502
(Partial insurance trans. shock)	(0.0435)	(0.0309)	(0.0294)

- Replace net family income with family or male earnings
- $\phi \downarrow$, so insurance $\uparrow$
- Important role for taxes, transfers, and family labor supply

Private transfers and low wealth

TABLE 8—MINIMUM-DISTANCE PARTIAL INSURANCE AND VARIANCE ESTIMATES, VARIOUS SENSITIVITY ANALYSES

Consumption: Income: Sample:	Nondurable net income baseline	Nondurable excluding help baseline	Nondurable net income low wealth	Nondurable net income high wealth	Total net income low wealth	Nondurable net income baseline+SEO
ϕ	0.6423	0.6215	0.8489	0.6248	1.0342	0.7652
(Partial insurance perm. shock)	(0.0945)	(0.0895)	(0.2848)	(0.0999)	(0.3517)	(0.1031)
ψ	0.0533	0.0500	0.2877	0.0106	0.3683	0.1211
(Partial insurance trans. shock)	(0.0435)	(0.0434)	(0.1143)	(0.0414)	(0.1465)	(0.0354)

- Negligible impact of help from friends and relatives
- Low wealth individuals are less insured
- Durable purchases and timing of durable replacement might act as insurance for low wealth individuals

Conclusions

- The evolution of permanent and transitory income shocks can explain the disjuncture between income and consumption inequality
- Partial insurance against permanent shocks, almost full insurance against transitory shocks
- Less insurance of low-wealth, more insurance for more educated
- Tax and welfare system play important role for insurance

Comments

- Role of income process?
- Advance information and expectations?
- What are the insurance mechanisms?
- Role of borrowing constraints?

Extensions

- Kaplan and Violante (2010)
  - Advance information, borrowing constraints, performance of BPP estimator in incomplete markets model
- Blundell, Pistaferri, and Saporta-Eksten (2016) and Blundell, Pistaferri, and Saporta-Eksten (2018)
  - Family labor supply and children
- Blundell, Borella, Commault, and De Nardi (2020) and Russo (2020)
  - Role of health

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Food demand estimates

TABLE 2—THE DEMAND FOR FOOD IN THE CEX

Variable	Estimate	Variable	Estimate	Variable	Estimate
$\ln c$	0.8503 (0.1511) [0.012]	$\ln c \times 1992$	0.0037 (0.0056) [0.083]	Family size	0.0272 (0.0090)
$\ln c \times$ high school dropout	0.0730 (0.0718) [0.050]	$\ln c \times$ one child	0.0202 (0.0336) [0.150]	$\ln p_{food}$	−0.9784 (0.2160)
$\ln c \times$ high school graduate	0.0827 (0.0890) [0.027]	$\ln c \times$ two children	−0.0250 (0.0383) [0.120]	$\ln p_{transports}$	5.5376 (8.0500)
$\ln c \times 1981$	0.1151 (0.1123) [0.053]	$\ln c \times$ three children+	0.0087 (0.0340) [0.197]	$\ln p_{fuel+utils}$	−0.6670 (4.7351)
$\ln c \times 1982$	0.0630 (0.0837) [0.052]	One child	−0.1568 (0.3215)	$\ln p_{alcohol+tobacco}$	−1.8684 (4.1425)
$\ln c \times 1983$	0.0508 (0.0704) [0.048]	Two children	0.3214 (0.3650)	Born 1955−59	−0.0385 (0.0554)
$\ln c \times 1984$	0.0478 (0.0662) [0.051]	Three children+	0.0132 (0.3259)	Born 1950−54	−0.0085 (0.0477)
$\ln c \times 1985$	0.0304 (0.0638) [0.064]	High school dropout	−0.7030 (0.6741)	Born 1945−49	−0.0060 (0.0406)
$\ln c \times 1986$	0.0223 (0.0587) [0.068]	High school graduate	−0.8458 (0.8298)	Born 1940−44	−0.0051 (0.0348)
$\ln c \times 1987$	0.0528 (0.0599) [0.065]	Age	0.0122 (0.0085)	Born 1935−39	−0.0044 (0.0273)
$\ln c \times 1988$	0.0416 (0.0458) [0.049]	Age^2	−0.0001 (0.0001)	Born 1930−34	0.0032 (0.0193)
$\ln c \times 1989$	0.0370 (0.0373) [0.046]	Northeast	0.0087 (0.0065)	Born 1925−29	−0.0051 (0.0140)
$\ln c \times 1990$	0.0187 (0.0295) [0.060]	Midwest	−0.0213 (0.0105)	White	0.0769 (0.0129)
$\ln c \times 1991$	−0.0004 (0.0318) [0.111]	South	−0.0269 (0.0096)	Constant	−0.6404 (0.9266)
Test of overidentifying restrictions			20.92 (d.f. 18; χ^2 p-value 28%)		
Test that income elasticity does not vary over time			27.69 (d.f. 12; χ^2 p-value 0.6%)		

Notes: This table reports IV estimates of the demand equation for (the logarithm of) food spending in the CEX. We instrument the log of total nondurable expenditure (and its interaction with time, education, and kids dummies) with the cohort-education-year specific average of the log of the husband's hourly wage and the cohort-education-year specific average of the log of the wife's hourly wage (and their interactions with time, education, and kids dummies). Standard errors are in parentheses, the Shea's partial R^2 for the relevance of instruments in brackets. In all cases, the p-value of the F-test on the excluded instrument is < 0.01 percent.

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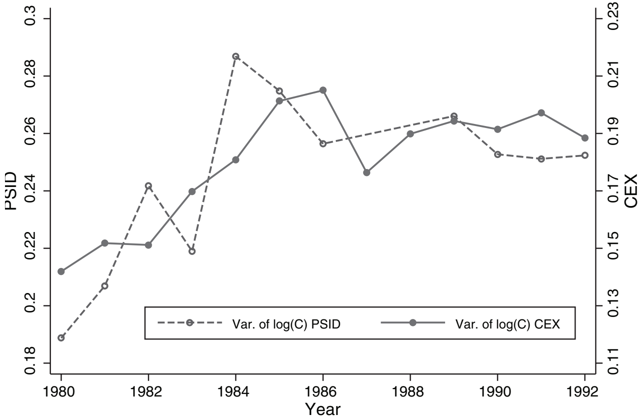
How good is the imputation?



CEX and new PSID compared

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How flexible is this income process?

- This is a **linear** income process
- Identification is relatively easy
- All shocks are associated to the same persistence
- Non-linear transmission of shocks is ruled out

Back

PIH with self insurance

- π_{it}:= share of future labor income in current human and financial wealth
- γ_{tL}:= age-increasing annuitization factor
- One can show that:

$$\phi_{it} \approx \pi_{it}, \quad \psi_{it} \approx \gamma_{tL}\pi_{it}$$

- Precautionary saving can only provide effective self-insurance if π_{it} is small.

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Excess smoothness

- Two alternative insurance configurations:
  1. Public information but limited enforcement of contracts
  2. Private information but full enforcement
- Self-insurance is Pareto inefficient
- More insurance than with a single noncontingent bond, but less than with complete markets.
- Relationship between income shocks and consumption depends on the degree of persistence of income shocks
- Another reason for partial insurance is moral hazard $\rightarrow$ Attanasio and Pavoni (2011) $\rightarrow$ when individuals have hidden access to a simple credit market, some partial insurance is possible.

Back

Advance information

- If the agents knew in advance some parts of the shocks these would already be incorporated into current plans and would not directly affect consumption growth
- Estimated $\phi_{i,t}$ has to be interpreted as reflecting a combination of insurance and information.
- We would be overestimating insurance and thus underestimating parameters
- With no extra data, this combination cannot be untangled $\rightarrow$ BPP provide evidence that advance information is not a serious problem in their sample.

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Identification of variances of shocks I

$$
\begin{aligned}
&cov(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}) = \\
&= cov(\zeta_t + \Delta\varepsilon_t, \zeta_{t-1} + \Delta\varepsilon_{t-1} + \zeta_t + \Delta\varepsilon_t + \zeta_{t+1} + \Delta\varepsilon_{t+1}) \\
&= cov(\zeta_t + \Delta\varepsilon_t, \zeta_{t-1} + \varepsilon_{t-1} - \varepsilon_{t-2} + \zeta_t + \varepsilon_t - \varepsilon_{t-1} + \zeta_{t+1} + \varepsilon_{t+1} - \varepsilon_t) \\
&= cov(\zeta_t + \Delta\varepsilon_t, \zeta_{t-1} + \zeta_t + \zeta_{t+1} + \varepsilon_{t+1} - \varepsilon_{t-2})
\end{aligned}
$$

Since $\zeta_{i,t}$, $\nu_{i,t}$ and $\varepsilon_{i,t}$ are assumed to be mutually uncorrelated and both ζ and ε are serially uncorrelated, this yields:

$$
\begin{aligned}
cov(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}) &= cov(\zeta_t, \zeta_t) \\
&= var(\zeta_t)
\end{aligned}
$$

Identification of variances of shocks II

Moreover, we have that:

$$\begin{aligned} -cov(\Delta y_t, \Delta y_{t+1}) &= -cov(\zeta_t + \Delta\varepsilon_t, \zeta_{t+1} + \Delta\varepsilon_{t+1}) \\ &= -cov(\zeta_t + \varepsilon_t - \varepsilon_{t-1}, \zeta_{t+1} + \varepsilon_{t+1} - \varepsilon_t) \end{aligned}$$

Since $\zeta_{i,t}$, $\nu_{i,t}$ and $\varepsilon_{i,t}$ are assumed to be mutually uncorrelated and both ζ and ε are serially uncorrelated, this yields:

$$\begin{aligned} -cov(\Delta y_t, \Delta y_{t+1}) &= -cov(\varepsilon_t, -\varepsilon_t) \\ &= var(\varepsilon_t) \end{aligned}$$

Back

Identification of income process

Using $\Delta y_{it} = \zeta_{it} + \Delta \nu_{it}$:

$$cov(\Delta y_t, \Delta y_{t+s}) = cov(\zeta_t + \Delta \nu_t, \zeta_{t+s} + \Delta \nu_{t+s})$$

This implies:

$$\begin{aligned} cov(\Delta y_t, \Delta y_{t+s}) &= cov[(\zeta_t + \Delta \nu_t)(\zeta_{t+s} + \Delta \nu_{t+s})] \\ &= cov[\zeta_t, \zeta_{t+s}] + cov[\zeta_t, \Delta \nu_{t+s}] + \\ &+ cov[\Delta \nu_t, \zeta_{t+s}] + cov[\Delta \nu_t, \Delta \nu_{t+s}] \end{aligned}$$

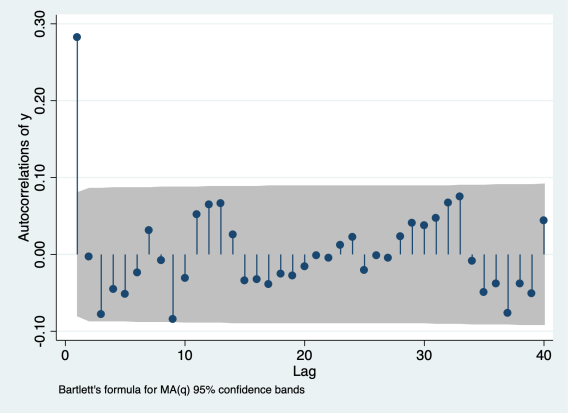
Now, recall that $\zeta_{i,t}$, $\nu_{i,t}$ and $\varepsilon_{i,t}$ are mutually uncorrelated and that $\zeta_{i,t}$ is serially uncorrelated. Then, we have that

$$cov(\Delta y_t, \Delta y_{t+s}) = \begin{cases} var(\zeta_t) + var(\Delta v_t) & \text{for } s = 0 \\ cov(\Delta \nu_t, \Delta \nu_{t+s}) & \text{for } s \neq 0 \end{cases} \quad (7)$$

Back

ACF for MA(1) process

$$y = \varepsilon_t + 0.25\varepsilon_{t-1}$$



Back

Identification of insurance coefficients

$$\begin{aligned} cov(\Delta c_t, \Delta y_{t+s}) &= cov[(\phi_t \zeta_t + \psi_t \varepsilon_t + \xi_t)(\zeta_{t+s} + \Delta \nu_{t+s})] \\ &= \phi_t cov[\zeta_t \zeta_{t+s}] + \phi_t cov[\zeta_t \Delta \nu_{t+s}] + \psi_t cov[\varepsilon_t \zeta_{t+s}] + \\ &+ \psi_t cov[\varepsilon_t \Delta \nu_{t+s}] + cov[\xi_t \zeta_{t+s}] + cov[\xi_t \Delta \nu_{t+s}] \end{aligned}$$

which gives that:

$$cov(\Delta c_t, \Delta y_{t+s}) = \begin{cases} \phi_t var(\zeta_t) + \psi_t var(\varepsilon_t) & \text{for } s = 0 \\ \psi_t cov(\varepsilon_t, \Delta \nu_{t+s}) & \text{for } s \neq 0 \end{cases}$$

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Solution to identification problem

Start from:

$$cov(\Delta c_t, \Delta y_{t+s}) = \begin{cases} \phi_t var(\zeta_t) + \psi_t var(\varepsilon_t) & \text{for } s = 0 \\ \psi_t cov(\varepsilon_t, \Delta \nu_{t+s}) & \text{for } s \neq 0 \end{cases}$$

For $s = 1$ and using the fact that ν is an MA(0):

$$cov(\Delta c_t, \Delta y_{t+s}) = \begin{cases} \phi_t var(\zeta_t) + \psi_t var(\varepsilon_t) & \text{for } s = 0 \\ \psi_t cov(\varepsilon_t, \Delta \varepsilon_{t+1}) & \text{for } s = 1 \end{cases}$$

which yields:

$$cov(\Delta c_t, \Delta y_{t+s}) = \begin{cases} \phi_t var(\zeta_t) + \psi_t var(\varepsilon_t) & \text{for } s = 0 \\ -\psi_t var(\varepsilon_t) & \text{for } s = 1 \end{cases}$$

Since you observe $cov(\Delta c_t, \Delta y_{t+s})$ from the data and you have identified the variances of the shocks before, this is a system of two equations in two unknowns, which you can solve to find ψ and ϕ. Back

Consumption growth inequality

$$
\begin{aligned}
cov(\Delta c_t, \Delta c_{t+s}) &= cov[(\phi_t \zeta_t + \psi_t \varepsilon_t + \xi_t)(\phi_{t+s}\zeta_{t+s} + \psi_{t+s}\varepsilon_{t+s} + \xi_{t+s})] \\
&= \phi_t \phi_{t+s} cov[\zeta_t \zeta_{t+s}] + \phi_t \psi_{t+s} cov[\zeta_t \varepsilon_{t+s}] + \\
&+ \phi_t cov[\zeta_t \xi_{t+s}] + \psi_t \phi_{t+s} cov[\varepsilon_t \zeta_{t+s}] + \\
&+ \psi_t \psi_{t+s} cov[\varepsilon_t \varepsilon_{t+s}] + \psi_t cov[\varepsilon_t \xi_{t+s}] + \\
&+ \phi_{t+s} cov[\xi_t \zeta_{t+s}] + \psi_{t+s} cov[\xi_t \varepsilon_{t+s}] + cov[\xi_t \xi_{t+s}]
\end{aligned}
$$

This gives that:

$$
cov(\Delta c_t, \Delta c_{t+s}) = \begin{cases} \phi_t^2 var(\zeta_t) + \psi_t^2 var(\varepsilon_t) + var(\xi_t) & \text{for } s = 0 \\ 0 & \text{for } s \neq 0 \end{cases}
$$

Back

Imputation error

Suppose that consumption is measured with error. Then, we have:

$$c^*_{i,t} = c_{i,t} + u^c_{i,t}$$

where c^* denotes measured consumption, c is true consumption and u^c is measurement error. Measurement error induces serial correlation in consumption growth. Now, suppose that $c_{i,t}$ is a random walk, that is: $c_{i,t} = c_{i,t-1} + \eta_{i,t}$ where $\eta_{i,t}$ is i.i.d. Then $\Delta c_{i,t} = \eta_{i,t}$. Then, we have that $\Delta c^*_{i,t} = \Delta c_{i,t} + \Delta u^c_{i,t} = \eta_{i,t} + \Delta u^c_{i,t}$. This implies that:

$$\begin{aligned} E[\Delta c^*_{i,t}\Delta c^*_{i,t-1}] &= E[(\eta_{i,t} + u^c_{i,t} - u^c_{i,t-1})(\eta_{i,t-1} + u^c_{i,t-1} - u^c_{i,t-2})] \\ &= -E[u^c_{i,t-1}u^c_{i,t-1}] \\ &= -\sigma^2_u && \text{(Since } u \sim iid\text{)} \end{aligned}$$

Moreover we have that:

$$\begin{aligned} E[\Delta c^*_{i,t}\Delta c^*_{i,t+1}] &= E[(\eta_{i,t} + u^c_{i,t} - u^c_{i,t-1})(\eta_{i,t+1} + u^c_{i,t+1} - u^c_{i,t})] \\ &= -E[u^c_{i,t}u^c_{i,t}] \\ &= -\sigma^2_u && \text{(Since } u \sim iid\text{)} \end{aligned}$$

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Insurance or pass-through coefficients?

Consider an idiosyncratic shock x_{it}. The pass-through coefficient measures the share of the variance of the shock that is passed to log-consumption (Kaplan and Violante (2010)):

$$\phi^x = \frac{cov(\Delta c_{it}, x_{it})}{var(x_{it})}$$

The insurance coefficient is the share of the variance of the shocks which is **not** passed to consumption, so $1 - \phi^x$.
BPP use the pass-through and call it insurance coefficient, be careful with reference values:

- Pass-through: 0 (full insurance), 1 (no insurance)
- Insurance: 1 (full insurance), 0 (no insurance)

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