

Earnings and consumption dynamics: a nonlinear panel data framework

Arellano, Blundell, Bonhomme
Econometrica (2017)

by Nicolò Russo

This paper

- Develops a nonlinear framework to study:
 - Nature of earnings persistence
 - Impact of earnings shocks on consumption
- Provides:
 - New method of studying earnings persistence and shocks
 - New way of estimating the earnings process
 - New evidence on impact of earnings shocks on consumption

Why we should study the earnings process

- This paper takes earnings as **exogenous**, like BPP (2008)
- Why studying the earnings process:
 1. Size and persistence of income shocks influence consumption decisions
 2. Persistence of earnings affects inequality and drives much of the variation in consumption ▸ Variance of consumption
 3. Designing optimal social insurance and taxation

The literature so far

- **Focus on linear models**

- Permanent/transitory (BPP); AR(1) (HSV)
- Main characteristics of linear models:
 1. All shocks are associated to same persistence
 2. Identification using covariance techniques
 3. Nonlinear transmission of shocks ruled out

- **Approaches to consumption**

1. Take stand on the consumption smoothing mechanisms and take a fully specified model to the data \Rightarrow Gourinchas and Parker (2002), Kaplan and Violante (2014)
2. Estimate the degree of “partial insurance”, without specifying the insurance mechanisms \Rightarrow BPP (2008)

Why getting over linear models

- Empirical evidence in favor of richer earnings dynamics
- Consumption function in BPP (2008) comes from linear approximation of the Euler equation
 - Linear approximations not always accurate
 - Precautionary saving, asset accumulation with borrowing constraints, and nonlinear persistence ruled out

Empirical Results

Working families from the PSID (1999-2009):

1. Impact of earnings shocks varies across households' earnings histories
2. Nonlinear persistence of earnings, where:
 - “Unusual” positive shocks for low earnings households
 - “Unusual” negative shocks for high earnings householdsare associated with lower persistence
3. Asymmetries in consumption responses to earnings shocks that hit households at different points of the income distribution
4. Similar empirical patterns in Norwegian administrative data

Roadmap for today

1. Introduction
2. Quantile Methods
3. Model
4. Identification
5. Estimation
6. Results
7. Life-cycle model
8. Conclusions

A crash course on the quantile function

- Given a probability p , the quantile function for a random variable Y $Q_p(Y)$ gives the values y such that:

$$Q_p(Y) = \inf\{y \in \mathbb{R} : p \leq F(y)\}, \quad \text{for } p \in (0, 1)$$

▶ Example

▶ Intuition

- cdf $F(\cdot)$ continuous and monotonically increasing: $Q = F^{-1}$
- Conditional quantile function at a quantile p for a random variable Y given X is:

$$Q_p(Y|X) = \inf\{y \in \mathbb{R} : p \leq F(y|x)\}$$

Quantile regression

- Models quantiles in the distribution of Y given X
 - e.g. estimate how the 1st and 3rd quartiles in the distribution of $Y|X$ change with X

- Object of interest: conditional quantiles, not mean

- Described by:

$$y_i = x_i' \beta_q + \varepsilon_i$$

where different choices of q yield different estimated β

- Yields one vector of coefficients for every quantile analyzed
- Interpretation depends on which quantile coefficients refer to

Earnings process

“Detrended” log pre-tax labor earnings:

$$y_{i,t} = \eta_{i,t} + \varepsilon_{i,t}, \quad i = 1, \dots, N, \quad t = 1, \dots, T$$

where:

- $\eta_{it} :=$ **persistent component:**

$$\eta_{it} = Q_t(\eta_{i,t-1}, u_{i,t}), \quad (u_{i,t} | \eta_{i,t-1}, \dots) \sim U(0, 1), \quad t = 2, \dots, T$$

where:

- $Q_t(\eta_{i,t-1}, \tau) :=$ τ -th quantile of $\eta_{i,t} | \eta_{i,t-1} \forall \tau \in (0, 1)$
- $\varepsilon_{it} =$ **transitory component:**
 - Zero mean, serially uncorrelated, independent of η
- **Special case:** canonical model of earnings dynamics

$$y_{i,t} = \eta_{i,t} + \varepsilon_{i,t}, \quad \eta_{i,t} = \eta_{i,t-1} + v_{i,t}$$

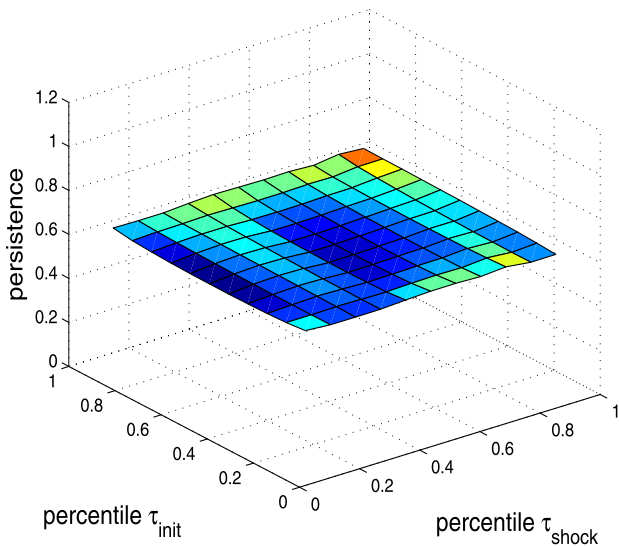
Nonlinear persistence

- Quantile model allows for nonlinear dynamics of earnings
- We are interested in nonlinear persistence
- Measures of persistence of the η component:

$$\rho_t(\eta_{i,t-1}, \tau) = \frac{\partial Q_t(\eta_{i,t-1}, \tau)}{\partial \eta}, \quad \rho_t(\tau) = \mathbb{E} \left[\frac{\partial Q_t(\eta_{i,t-1}, \tau)}{\partial \eta} \right]$$

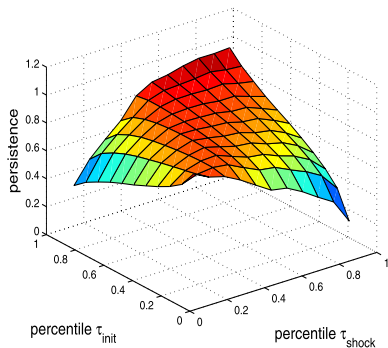
- **Canonical model:** $\rho_t(\eta_{i,t-1}, \tau) = 1$ always
- **Quantile model:** depends on magnitude and direction of $u_{i,t}$

Persistence of earnings in the canonical earnings model

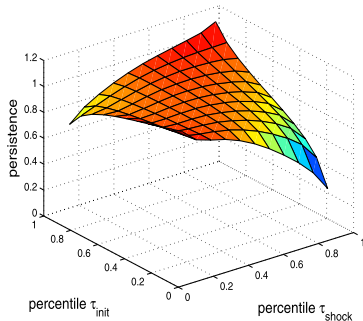


Evidence of nonlinear persistence of earnings

(a) PSID data



(b) Norwegian administrative data



Consumption rule in a simple life-cycle model

- Households choose consumption and savings subject to:

$$A_{i,t} = (1 + r)A_{i,t-1} + Y_{i,t-1} - C_{i,t-1}$$

- Family log-earnings are given by:

$$\log Y_{i,t} = \kappa_t + \eta_{i,t} + \varepsilon_{i,t}$$

- No advance information, no aggregate uncertainty, agents know all distributions.
- Bellman equation in each period:

$$V_t(A_{i,t}, \eta_{i,t}, \varepsilon_{i,t}) = \max u(C_{i,t}) + \beta \mathbb{E}_t[V_{t+1}(A_{i,t+1}, \eta_{i,t+1}, \varepsilon_{i,t+1})]$$

Consumption rule in a simple life-cycle model

- Consumption policy rule:

$$C_{i,t} = G_t(A_{i,t}, \eta_{i,t}, \varepsilon_{i,t})$$

for some age-dependent function G_t

- Possible approaches:
 1. Build a fully structural model and calibrate or estimate it via MSM \Rightarrow Gourinchas and Parker (2002)
 2. Linearize the Euler equation and then use standard covariance based methods \Rightarrow BPP (2008)
 3. Directly estimate the nonlinear consumption rule \Rightarrow This paper!

Empirical consumption rule

- Log-consumption net of age dummies:

$$c_{i,t} = g_t(a_{i,t}, \eta_{i,t}, \varepsilon_{i,t}, \nu_{i,t})$$

$\nu_{i,t} :=$ unobserved arguments of the consumption function.

- Assets:

$$a_{i,t} = h_t(a_{i,t-1}, c_{i,t-1}, y_{i,t-1}, \eta_{i,t-1}, \nu_{i,t})$$

$\nu_{i,t} :=$ i.i.d. and independent of the other arguments of h

Derivative effects

- Average consumption for given assets and earnings components:

$$\mathbb{E}[c_{i,t} | a_{i,t} = a, \eta_{i,t} = \eta, \varepsilon_{i,t} = \varepsilon] = \mathbb{E}[g_t(a, \eta, \varepsilon, \nu_{i,t})]$$

- Average derivative of consumption with respect to η :

$$\phi_t(a, \eta, \varepsilon) = \mathbb{E} \left[\frac{\partial g_t(a, \eta, \varepsilon, \nu_{i,t})}{\partial \eta} \right]$$

- Average derivative effect:

$$\bar{\phi}_t(a) = \mathbb{E}[\phi_t(a, \eta, \varepsilon)]$$

- $1 - \bar{\phi}_t(a)$: := degree of consumption insurance to shocks to persistent earnings component

Identification overview and references

- We have nonlinear models with latent state variables.
- A series of papers has established conditions under which nonlinear models with latent variables are nonparametrically identified under conditional independence restrictions:
 1. Hu and Schennach (2008)
 2. D'Hautfoeuille (2011)
 3. Hu and Shum (2012)
 4. Wilhelm (2015)
 5. Arellano and Bonhomme (2016)
 6. Hu (2019)
- This section covers the gist of the identification strategy, but more details are provided in the paper and in the Supplemental Material available online.

Earnings process

- Assume that the data contain T consecutive periods.
- The goal is to identify the joint distributions of $(\eta_{i,1}, \dots, \eta_{i,T})$ and $(\varepsilon_{i,1}, \dots, \varepsilon_{i,T})$ given data from $(y_{i,1}, \dots, y_{i,T})$
- These are identified under conditions which follow directly from Hu and Schennach (2008) and Wilhelm (2015).
- These conditions rely on the distributions of $(y_{i,t}|y_{i,t-1})$ and $(\eta_{i,t}|y_{i,t-1})$ being **complete**.
 - The distribution of $(y_{i,t}|y_{i,t-1})$ is complete if the only function h satisfying $\mathbb{E}[h(y_{i,t}|y_{i,t-1})] = 0$ is $h = 0$.
- Completeness is a common assumption in nonparametric instrumental variables problems.

Consumption rule without unobserved heterogeneity

- For a generic variable z , let $z_i^t = (z_{i1}, \dots, z_{it})$. Then make the following assumption:

ASSUMPTION 1: For all $t \geq 1$:

- $u_{i,t+s}$ and $\varepsilon_{i,t+s}$, for all $s \geq 0$, are independent of a_i^t , η_i^{t-1} , and y_i^{t-1} . ε_{i1} is independent of a_{i1} and η_{i1} .
 - $a_{i,t+1}$ is independent of $(a_i^{t-1}, c_i^{t-1}, y_i^{t-1}, \eta_t^{t-1})$ conditional on $(a_{it}, c_{it}, y_{it}, \eta_{it})$.
 - The taste shifter ν_{it} is independent of η_{i1} , $(u_{is}, \varepsilon_{is})$ for all s , ν_{is} for all $s \neq t$, and a_i^t .
- The identification argument proceeds in a sequential way:
 - Start with the first period.
 - Proceed to the second period using the assets.
 - Using the second period consumption move to subsequent periods and use induction.

Consumption rule - First period

- Let $y_i = (y_{i1}, \dots, y_{iT})$ denote the whole history of earnings for agent i and f denote a density function.
- Using Assumption 1.1 we have that

$$f(a_1|y) = \mathbb{E}[f(a_1|\eta_{i1})|y_i = y] \quad (1)$$

- Given that distribution of $(\eta_{i1}|y_i)$ is complete, $f(a_1|\eta_1)$ is identified from (1).
- Using the consumption rule and Assumption 1.3 we have that:

$$f(c_1|a_1, y) = \mathbb{E}[f(c_1|a_{i1}, \eta_{i1}, y_{i1})|a_{i1} = a_1, y_i = y] \quad (2)$$

- Given that the distribution of $(\eta_{i1}|a_{i1}, y_i)$ is complete, $f(c_1|a_1, \eta_1, y_1)$ and $f(c_1, \eta_1|a_1, y)$ are identified from (2).
- Identification of the consumption function for $t = 1$ follows.

Consumption rule - Second period

- Using Assumption 1.1 and 1.3, we have that:

$$f(a_2|a_1, c_1, y) = \int f(a_2|a_1, c_1, \eta_1, y_1)f(\eta_1|a_1, c_1, y)d\eta_1 \quad (3)$$

- Given that the distribution of $(\eta_{i1}|c_{i1}, a_{i1}, y_i)$ is complete, $f(a_2|a_1, c_1, \eta_1, y_1)$ is identified from (3).
- Using Bayes' rule and Assumption 1.1 and 1.3 we have that:

$$f(\eta_2|a_1, a_2, c_1, y) = \int \frac{f(y|\eta_1, \eta_2, y_1)f(\eta_1, \eta_2|a_1, a_2, c_1, y_1)}{f(y|a_1, a_2, c_1, y)}d\eta_1$$

- As the density $f(\eta_1|a_1, a_2, c_1, y)$ is identified from above, and, by Assumption 1, we have that:

$$f(\eta_1, \eta_2|a_1, a_2, c_1, y_1) = f(\eta_1|a_1, a_2, c_1, y_1)f(\eta_2|\eta_1)$$

it follows that $f(\eta_2|a_1, a_2, c_1, y)$ is identified.

Consumption rule - Subsequent periods

- Consider second period's consumption. Using Assumption 1.3 we have that:

$$f(c_2|a_1, a_2, c_1, y) = \int f(c_2|a_2, \eta_2, y_2)f(\eta_2|a_1, a_2, c_1, y)d\eta_2 \quad (4)$$

- Given that the distribution of $(\eta_{i2}|a_{i1}, a_{i2}, c_{i1}, y_i)$ is complete, $f(c_2|a_2, \eta_2, y_2)$ is identified from (4).
- By induction, using in addition Assumption 1 from the third period onward, the joint density of η 's, consumption, assets, and earnings is identified, provided, for all $t \geq 1$, the distributions of $(\eta_{it}|c_i^t, a_i^t, y_i)$ and $(\eta_{it}|c_i^{t-1}, a_i^t, y_i)$ are complete in $(c_i^{t-1}, a_i^{t-1}, y_i^{t-1}, y_{i,t+1}, \dots, y_{iT})$.

Overview of estimation

- Estimate jointly quantile regressions of:
 1. Markovian transitions Q
 2. Transitory component ε
 3. Initial persistent component η_1
- Use these to estimate:
 1. Consumption c
 2. First period assets a_1
 3. Evolution of assets

Empirical specification - Earnings

- Quantile function of ε_{it} (for $t = 1, \dots, T$) given age_{it} :

$$Q_{\varepsilon}(age_{it}, \tau) = \sum_{k=0}^K a_k^{\varepsilon}(\tau) \varphi_k(age_{it})$$

where:

- φ_k for $k = 0, 1, \dots :=$ polynomial
- In practice, they will use Hermite polynomials
- $a_k(\varepsilon) :=$ scalars that will be estimated

▶ Hermite polynomials

- In practice:

$$Q_{\varepsilon}(age_{it}, \tau) = a_1(\tau) + a_2(\tau)age_{it} + a_3(\tau)(age_{it}^2 - 1) + a_4(\tau)(age_{it}^3 - 3age_{it}) + \dots$$

Empirical specification - Earnings

- Quantile function of η_{i1} given age_{i1} :

$$Q_{\eta_1}(age_{i1}, \tau) = \sum_{k=0}^K a_k^{\eta_1}(\tau) \varphi_k(age_{i1})$$

- Markovian transitions of persistent component:

$$Q_t(\eta_{i,t-1}, \tau) = Q(\eta_{i,t-1}, age_{it}, \tau) = \sum_{k=0}^K a_k^Q(\tau) \varphi_k(\eta_{i,t-1}, age_{it})$$

- In practice:

$$Q_t(\eta_{i,t-1}, \tau) = a_1(\tau) + a_2(\tau)\eta_{i,t-1} + a_3(\tau)age_{it} + a_4(\tau)\eta_{i,t-1}age_{it} \\ + a_5(\tau)(\eta_{i,t-1}^2 - 1)age_{it} + a_6(\tau)\eta_{i,t-1}(age_{it}^2 - 1) + \dots$$

Empirical specification - Consumption rule

- Empirical specification for consumption:

$$c_{i,t} = g_t(a_{i,t}, \eta_{i,t}, \varepsilon_{i,t}, \nu_{i,t})$$

- Conditional distribution of consumption given assets and earnings components:

$$\begin{aligned} g_t(a_{it}, \eta_{it}, \varepsilon_{it}, \tau) &= g(a_{it}, \eta_{it}, \varepsilon_{it}, \text{age}_{it}, \tau) \\ &= \sum_{k=1}^K b_k^g \tilde{\varphi}_k(a_{it}, \eta_{it}, \varepsilon_{it}, \text{age}_{it}) + b_0^g(\tau) \end{aligned}$$

where:

- b_k^g and $b_0^g(\tau)$ are scalars to be estimated
- $\tilde{\varphi}_k$ is a product of Hermite polynomials

Empirical specification - Assets evolution

- Distribution of initial assets a_{i1} conditional on η_{i1} and age_{i1} :

$$Q_a(\eta_{i1}, age_{i1}, \tau) = \sum_{k=0}^K b_k^a(\tau) \tilde{\varphi}_k(\eta_{i1}, age_{i1})$$

- Evolution of assets:

$$a_{i,t} = h_t(a_{i,t-1}, c_{i,t-1}, y_{i,t-1}, \eta_{i,t-1}, v_{i,t})$$

- Assets evolution is specified as:

$$\begin{aligned} h_t(a_{i,t-1}, c_{i,t-1}, y_{i,t-1}, \eta_{i,t-1}, \tau) &= h(a_{i,t-1}, c_{i,t-1}, y_{i,t-1}, \eta_{i,t-1}, age_{it}, \tau) \\ &= \sum_{k=1}^K b_k^h \tilde{\varphi}_k(a_{i,t-1}, c_{i,t-1}, y_{i,t-1}, \eta_{i,t-1}, age_{it}) + \\ &\quad b_0^k(\tau) \end{aligned}$$

Overview of the estimation algorithm

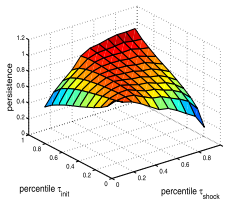
- Adaptation of the techniques developed in Arellano and Bonhomme (2016) to a setting with time-varying latent variables
- Sequential algorithm:
 1. Recover estimates of the earnings parameters $a_k^Q, a_k^\varepsilon, a_k^{\eta_1}$.
 2. Given the estimates of $a_k^Q, a_k^\varepsilon, a_k^{\eta_1}$, recover the consumption and asset parameters b_0^g, b_0^h, b_k^a and b_1^g, \dots, b_K^g and b_1^h, \dots, b_K^h .
- Parameters not estimated jointly, because $a_k^Q, a_k^\varepsilon, a_k^{\eta_1}$ are identified from the earnings process alone
- Closely related to the “Stochastic EM” algorithm (see Celeux and Diebolt (1993)), but based on quantile regression rather than on maximum likelihood

Data

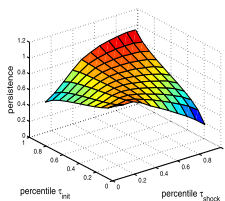
- PSID for 1999-2009
- Y_{it} total pre-tax household labor earnings. y_{it} residual of a regression of $\log Y_{it}$ on demographics
- C_{it} consumption of nondurables and services. c_{it} residual of a regression of $\log C_{it}$ on same demographics
- A_{it} sum of financial and non-financial assets, net of mortgages and debt. a_{it} as residual of regression of $\log A_{it}$ on same demographics
- Sample selection from Blundell, Pistaferri, and Saporta-Eksten (2016)

Persistence of earnings

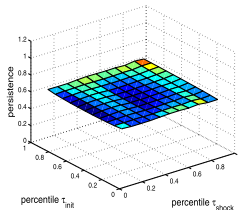
(a) Earnings, PSID data



(b) Earnings, nonlinear model

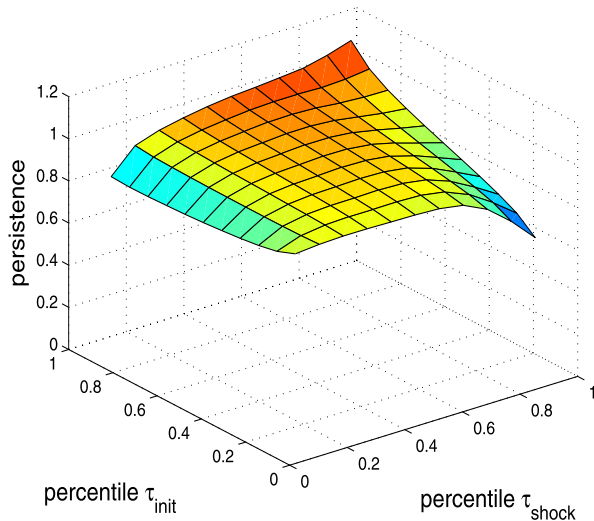


(c) Earnings, canonical model



Persistence of η - Simulated Data

► Other figures

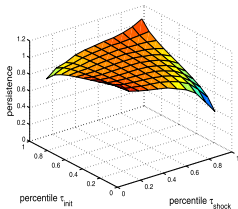


Norwegian population register data

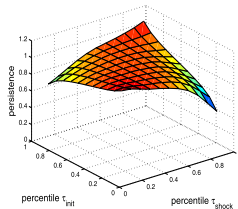
- In order to corroborate their findings using a different and larger data set, they use Norwegian administrative data.
- They consider a balanced sample of 2873 households in the 2000-2005 period.
 - Male, non-immigrant, residents between the age 30 and 60 and their spouses.
 - Continuously married males, with household disposable income above the threshold of substantial gainful activity (\$14,000 in 2014).
- Part of Blundell, Graber, Mogstad (2015)

Norwegian population register data results

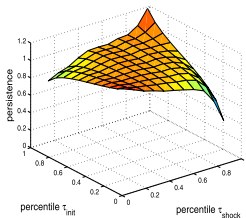
(a) Earnings, Norwegian data



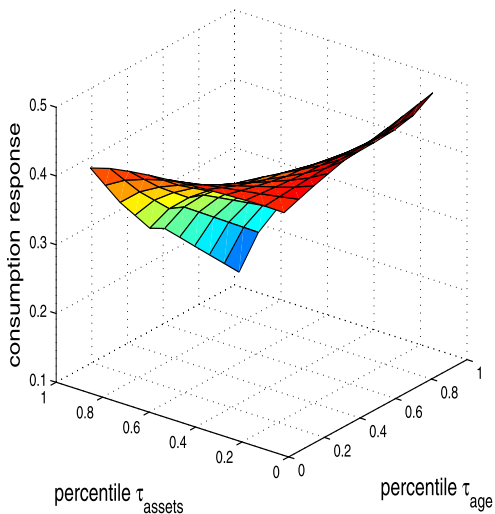
(b) Earnings, nonlinear model



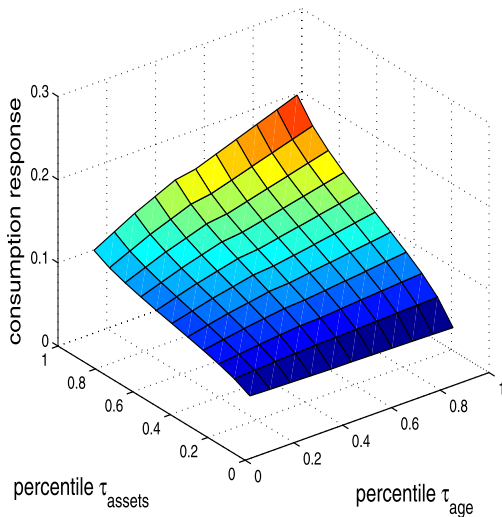
(c) Persistent component η_{it} , nonlinear model



Consumption response to η - Simulated Data

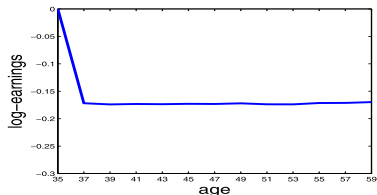


Consumption response to assets - Simulated Data

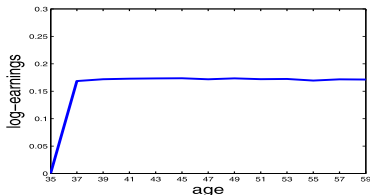


Impact of persistent earnings shocks on earnings - Canonical model

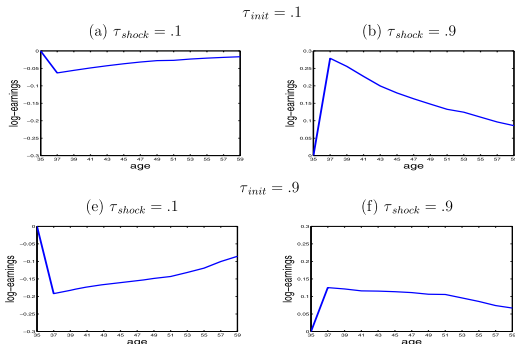
(g) $\tau_{shock} = .1$



(h) $\tau_{shock} = .9$



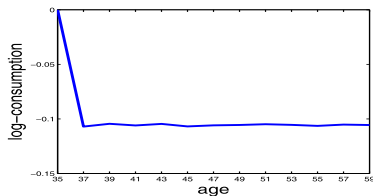
Impact of persistent earnings shocks on earnings - Nonlinear model



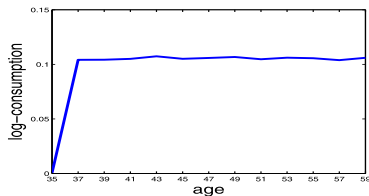
- Earnings responses change, based on the rank of the household in the income distribution and the magnitude of the earnings shock.

Impact of persistent earnings shocks on consumption - Canonical model

(g) $\tau_{shock} = .1$

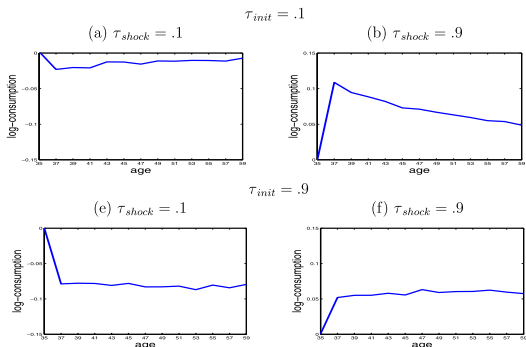


(h) $\tau_{shock} = .9$



► Different timing of shocks

Impact of persistent earnings shocks on consumption - Nonlinear model



- Consumption responses change, based on the rank of the household in the income distribution and the magnitude of the earnings shock.

Simulating a life-cycle model

- Want to study the possible implications of the nonlinearity in the earnings process.
- Simulate consumption and assets using the life-cycle model of Kaplan and Violante (2010).
- Compare the canonical linear model with a simple nonlinear earnings model, with “unusual” earnings shocks.

Some details of the simulation

- Each household enters the labor market at age 25, works until 60, and dies with certainty at 95.
- After retirement, households receive Social Security transfers Y_i^{SS} , which are functions of the entire realizations of labor income.
- Utility is CRRA.
- Single risk-free, one period bond, with constant return is $1 + r$.
- Period-by-period budget constraint.
- Natural borrowing limit (households cannot die in debt)

The earnings process

- During working years, after-tax earnings are described by:

$$\log Y_{it} = \kappa_t + y_{it}$$

$$y_{it} = \eta_{it} + \varepsilon_{it}$$

where:

- κ is a deterministic experience profile.
- η is the persistent component of earnings, ε is the transitory one.
- The process for the persistent component of earnings is:

$$\eta_{it} = \rho_t(\eta_{i,t-1}, v_{it})\eta_{i,t-1} + v_{it}$$

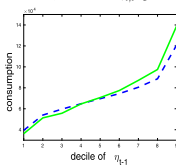
where two specifications are compared:

- $\rho_t = 1$ and v_{it} is normally distributed in the canonical earnings process.
- ρ_t is nonlinear and follows the rich process estimated from the PSID.

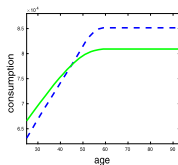
Consumption and assets

Solid lines are for the canonical earnings model, dashed lines for the nonlinear one.

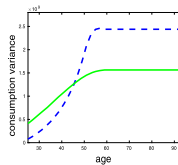
(a) Consumption, age 37
by decile of $\eta_{i,t-1}$



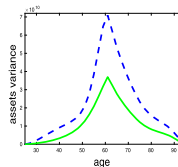
(b) Average consumption
over the life-cycle



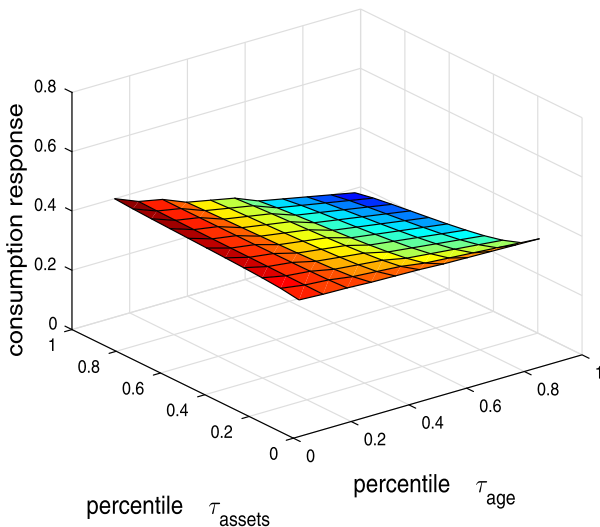
(c) Consumption variance
over the life-cycle



(d) Assets variance
over the life-cycle



Consumption response to earnings



Conclusions

- Develops a nonlinear framework for modeling persistence
- Reveals asymmetric persistence patterns, with “unusual” shocks associated with a drop in persistence
- Provides conditions for the nonparametric identification and develops a simulation-based quantile regression method for estimation
- Nonlinear persistence is an important feature of earnings processes
- Consumption responses vary with the position of the household in the income distribution, age, and assets

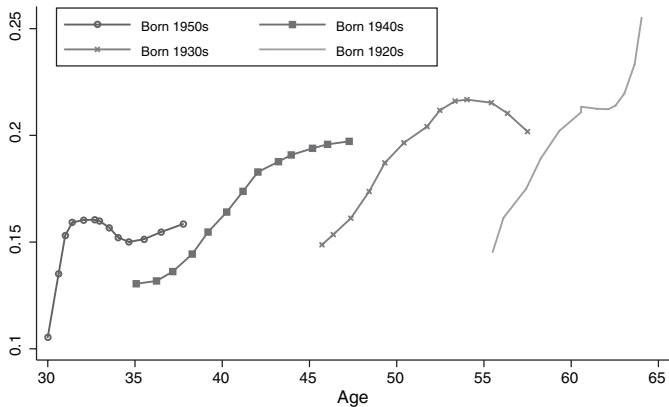
Extensions

- Combine this framework with more structural approaches.
- Extend the analysis to consider the effects of business cycles.
 - Gonzalo Paz-Pardo's JMP (2019)
- Extend the analysis to incorporate family labor supply à la Blundell, Pistaferri, and Saporta-Eksten (2016)
- Extend the analysis to older households

Appendix

BPP (2008) - Variance of log-consumption

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Derivation of a quantile function

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The cdf of *Exponential*(λ) is:

$$F(x; \lambda) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

To find the quantile function we need to find the value of x such that $\Pr(X \leq x) = p$. That is:

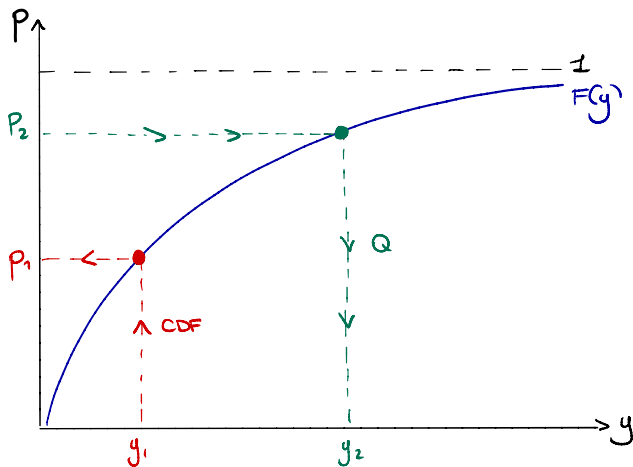
$$1 - e^{-\lambda x} = p \Rightarrow -\lambda x = \log(1 - p) \Rightarrow x = -\frac{\log(1 - p)}{\lambda}$$

which means that the quantile function is:

$$Q(p; \lambda) = -\frac{\log(1 - p)}{\lambda}$$

This means that in order to find the value of X for which, say, $\Pr(X \leq x) = 0.5$, you feed $p = 0.5$ to the quantile function.

Intuition for quantile function



Orthogonal polynomials

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- Hermite polynomials are an orthogonal polynomial sequence.
 - An orthogonal polynomial sequence is such that any two different polynomials in the sequence are orthogonal to each other under some inner product.
- The polynomials $p_0(x) = 1$, $p_1(x) = x$, $p_2(x) = 3x^2 - 1$ constitute a sequence of orthogonal polynomials under the inner product:

$$\langle g, h \rangle = \int_{-1}^1 g(x)h(x)dx$$

This is because

$$\langle p_0, p_1 \rangle = \int_{-1}^1 1x = \frac{x^2}{2} \Big|_{-1}^1 = 0, \quad \langle p_0, p_2 \rangle = \int_{-1}^1 1 \cdot (3x^2 - 1)dx = x^3 - x \Big|_{-1}^1 = 0,$$

$$\langle p_1, p_2 \rangle = \int_{-1}^1 x \cdot (3x^2 - 1)dx = \frac{3}{4}x^4 - \frac{1}{2}x^2 \Big|_{-1}^1 = 0$$

Hermite polynomials

◀ Back

- The first four probabilists' Hermite polynomials are:

$$He_0(x) = 1$$

$$He_1(x) = x$$

$$He_2(x) = x^2 - 1$$

$$He_3(x) = x^3 - 3x$$

- Hermite polynomials are orthogonal with respect to a weight function $w(x)$.
- In particular, the probabilist Hermite polynomials are orthogonal with respect to the standard normal probability density function, that is:

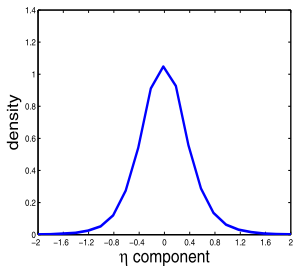
$$\int_{-\infty}^{\infty} He_m(x) He_n(x) e^{-\frac{x^2}{2}} = \sqrt{2\pi n!} \delta_{nm}$$

where δ is the Kronecker delta, $\delta_{nm} = 0$ if $n \neq m$ and 1 otherwise.

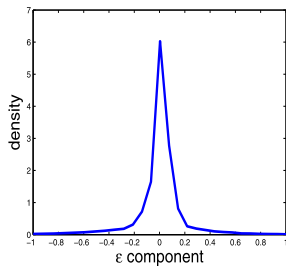
Densities of earnings components

← Back

(a) Persistent component η_{it}



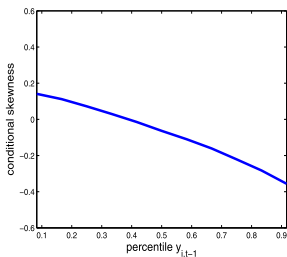
(b) Transitory component ε_{it}



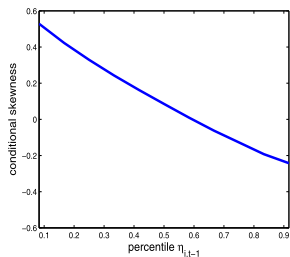
Conditional skewness of earnings components

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(a) Log-earnings residuals y_{it}



(b) Persistent component η_{it}



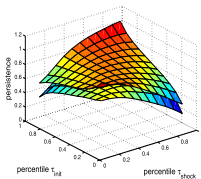
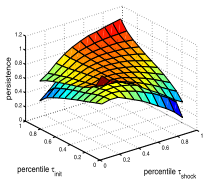
Confidence bands

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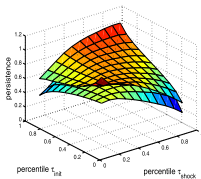
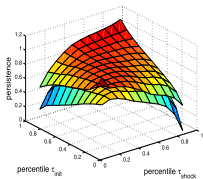
(a) PSID data

(b) Nonlinear model

Parametric bootstrap



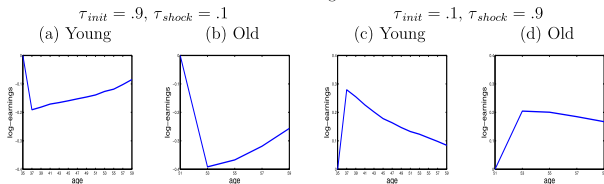
Nonparametric bootstrap



IRF's with different timing of shocks

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Earnings



Consumption

