Community Structure and Market Outcomes: Towards a Theory of Repeated Games in Networks*

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Abstract

This paper investigates the role of community structure and institutions in determining outcomes of markets with asymmetric information and moral hazard. We introduce a framework for studying repeated games in buyer-seller networks, and show that networks that facilitate cooperation are sparse, balanced with respect to the degrees of buyers and sellers, and divide the society into segregated communities. Moreover, there is an inherent trade-offs between sustaining cooperation and: (1) maximizing the volume of trade, (2) providing equal opportunities to market participants. When demand and supply fluctuations exist, the first best cannot be achieved and the second best is a balance between 'old world' tight and sparse networks and a 'global village'. Institutions such as reputation systems, litigation, and third-party evaluation services, can restore efficiency and equality of opportunities. Our results are consistent with observations from labor and credit markets, supply networks, and development economics. (JEL: A14, C73, D82, D85, L14)

Keywords: Buyer-seller networks, globalization, repeated games, moral hazard, asymmetric information, trust, institutions.

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1 Introduction

In markets with asymmetries of information and moral hazard, transactions often rely on cooperation and trust for providing accurate information, paying a debt, or supplying a quality product. Many times, cooperation relies on repeated interaction between individuals or small groups.

Using a model of repeated games in buyer-seller networks we show how market structure and patterns of interactions affects the ability of market participants to cooperate with each other. As a result, certain patterns of interactions are more likely to develop. We find that the need to enforce cooperation limits efficiency and equality of opportunities in almost every market by restricting the volume of trade and excluding some individuals from the market. In particular, the need to enforce cooperation prevents markets from reaping the potential gains of globalization: an increase in trade opportunities and accessibility to markets.

Recovering efficiency and equality of opportunities can be achieved by introducing institutions that support the ability to enforce cooperation. We formally model three such institutions: *Reputation Systems*, *Litigation*, and *Third-Party Evaluation Services*. We show that these institutions are complementary to the network in enforcing cooperation and allow for more efficiency and equality enhancing network structures to exist.

To the best of our knowledge, this paper is the first to offer a theory that incorporates: (1) the role of the network structure in facilitating repeated interactions and cooperation; (2) the role of institutions; and (3) the interaction between institutions and networks. In section 8, we discuss related literature that study separately each of these aspects.

Methodologically, we propose a tractable framework for the analysis of a general class of repeated bilateral games in buyer-seller networks, which allows us to characterize networks that allow for long term cooperation by every buyer and seller who are connected. Such a model is missing from previous literature despite the fact that long term incentives and cooperation in markets are at the heart of many economic interactions. Karlan et. al. (forthcoming) suggest that this is because *"networks are complicated structures, and combining them with repeated interaction can make the analysis intractable."*

At the core of our model is the understanding that an individual has different values for future interactions with different individuals. A seller s who produces one unit of good every period and is connected to buyer b will ask herself: "What is the probability that I will be able to sell the good to buyer b and not be able to sell it to any other buyer?". The answer reflects the probability that seller s *needs* buyer b in a given period. In evaluating the connection that the seller has with buyer b, seller s will count only these periods in which she is expected to *need* buyer b. If the seller has high value for the connection, she will not risk losing the connection by cheating buyer b.

As the network structure describes the possible interactions between buyers and sellers, the ability to cooperate depends on the network structure in two ways. First, the network structure determines the frequency of interactions between seller s and buyer b; when the frequency of interaction rises, so does the value of the connection. Second, the network structure determines the option value if seller s was not connected to buyer b; when this option value increases the value of the connection between seller s and buyer b decreases. As a result, a deviation by a seller in interaction with a buyer, that leads the buyer to disconnect the relationship, affects in a systematic way other links of both the buyer and the seller, as well as other links in the network, and can lead to further deviations by other sellers. Figure 1 provides an example.

Figure 1a **Figure 1b**

Figure 1: In every period, let meetings between buyers and sellers occur in a random order conditional on having a link between them. Successful interactions between seller s and buyer b in figure 1a are more frequent than in figure 1b. However, in figure 1a the seller has a guaranteed outside option, as buyers b' and b'' cannot transact with any other seller, while in figure 1b, there is a positive probability that buyer b will be the only buyer ready to buy from seller s and the connection has a higher value that might be enough to sustain cooperation. Focusing on figure 1b, if seller s' defects in an interaction with buyer b, the connection between seller s' and buyer b is eliminated. As a result, the connection between seller s and buyer b becomes more valuable due to higher expected frequency of interaction, but the connections between seller s and buyers b' and b'' become less valuable due to an improved outside option for seller s, possibly inducing a defection by seller s.

Networks that are especially good in sustaining cooperation are: (1) Balanced: no close agents (buyers or sellers) have degrees that are too different; (2) Sparse: the number of connections in the network is low; and (3) Segregated: sellers that have one buyer in common, have connections to similar sets of buyers overall. This is consistent with observations of supplier-consumer loyalty in markets (Kirman and Vriend 2000), limited economic reach of firms in the absence of reputation mechanisms (Fafchamps 1996), and evidence that a firm trusts its customer enough to offer credit when the customer finds it hard to locate an alternative supplier (McMillan and Woodruff 1999).

The advantage of segregation in facilitating cooperation suggests an explanation to several observations in markets in the age of globalization, when physical boundaries are less of a constraint, but market participants still choose a segmented market structure. In labor markets, groups of firms coordinate on interviewing from the same pool of workers rather than sample workers from various pools (Lee and Schwarz 2008), and in the microfinance industry in developing countries, lenders focus on borrowers from the same village (lending institutions usually open a separate branch to deal with each group of small villages).

In environments with demand and supply fluctuations, we find a trade-off between enforcing cooperation and facilitating high volumes of trade. Efficiency in these environments requires a dense and global network. This leads us to expect that networks in fluctuating environments be denser and more global, especially as globalization enables connections between remote individuals. However, globalization is limited to the extent that allows the repeated nature of the interactions to sustain cooperation.

The trade-off between sustaining cooperation and facilitating maximal trade volumes can be mitigated by the introduction of trust enhancing institutions. We study three institutions: *Reputation Systems*, *Litigation*, and *Third-Party Evaluation Services*. The first affects players strategy spaces and consequently increases the implied punishment for a seller who deviates, while the second and third affect the punishment (litigation) and the profit from deviation (third-party evaluation services) directly. In the presence of institutions, networks are denser and more global and still facilitate the necessary cooperation, leading to optimal trade volumes. This is consistent with the empirical literature on institutions and markets.[1]

We extend prior literature on games in networks in several ways. Most notably, while most of the literature focuses on static games,[2] we analyze repeated games, and allow for complex strategic interactions incorporating asymmetric information and moral hazard. In addition, to date, the literature focused either on complete information of the network structure (e.g. Ballester 2006, Galeotti 2005, Goyal and Moraga-Gonzalez 2001) or on extreme levels of incomplete information where a player knows only her own degree and the degree distribution of others in the network (see Galeotti et. al. 2008 and references therein). We suggest a realistic form of knowledge of the network structure, in which an individual knows her own degree as well as the degree of her direct neighbors, the degree distribution in the population, and some aggregate information regarding the network structure such as the average level of overlap.

The rest of the paper is organized as follows. The following section motivates our analysis by laying out an example of job recommendations in labor markets. In section 3, we present the model and derive the value of a connection in a network. Section 4 describes community structures that best support cooperation, and in section 5 we

[1] Hall and Jones (1999) show that differences in institutions explain much of the variation in product per worker across countries. Johnson et. al. (2002) show that the main effect of belief in the court system is to encourage the formation of new relationships.

[2] Exceptions include Lippert and Spagnolo (2006), work in progress by Miller and Nageeb, and Kinateder (2008). All three are network based generalizations of the community enforcement literature referred to section 8, and are different both in methodology and in motivation from the current paper.

analyze the effect of globalization on network structure and cooperation. Section 6 evaluates the welfare characteristics of different networks and investigates the trade-off between cooperation and trade volume. In section 7 we focus on institutions that enhance efficiency and equality in markets. Section 8 offers a discussion of related literature and empirical evidence supporting our findings, and section 9 offers concluding remarks.

2 Example: job recommendations

Our framework is useful for the analysis of many applications with both *Moral Hazard* and *Asymmetric Information*. To motivate our analysis, we briefly describe one important application that demonstrates many of the features of our model. More applications and a discussion of the testable implications of our results in the different markets are provided in section 8 and in a technical appendix available from the author.

The importance of social networks for getting jobs has been long recognized. Granovetter (1974) documents that more than half of (white-collar) workers use personal connections to obtain a job. 24 other relevant U.S. studies that point to similar results can be found in Bewley (1999). Fainmesser (2009) shows that transmission of information over social networks affects the timing of hiring in entry-level labor markets.

Consider a group of teachers that have students of different qualities graduating periodically (one student per teacher each period) and a group of firms that are seeking to hire high quality graduating students.

A teacher receives a utility of ν from getting a job for her student. Without loss of generality, assume that a student's quality is either high (with probability μ) or low. With some positive probability I_b, the student's quality is observable to the firm after she is hired.

A firm wants to hire only a high quality student, and does not hire based on recommendations of teachers that have recommended a low quality student in previous periods.

Let $FV_{s,b}$ be the future value for teacher s from interactions with firm b if the firm continues to trust the recommendation of teacher s. A teacher who has a low quality student can decide whether to recommend him to the firm as a high quality student or not. To make her decision, the teacher weighs the immediate profit of ν from the student being hired versus a positive probability I_b of losing a connection worth $FV_{s,b}$.

3 Model

Consider a large market with a set S of sellers (teachers) and a set B of buyers (firms) ($|B|, |S| \to \infty$). Time is discrete. Sellers live forever and seller s has a discount factor δ_s. In every period, seller s has unit capacity with probability μ_s and zero capacity with probability $(1 - \mu_s)$.[3,4]

When buyer b and seller s interact the seller can either defect or cooperate.[5] In real scenarios the seller's gains from deviating depends on the nature of the deviation. The seller can say that a good is of high quality when it is low and make a sale that would not have taken place otherwise, or she can save costs by failing to put the necessary effort in producing or supplying a good or a service. For our needs, it is sufficient to say that seller s that cooperates receives a payoff of $\pi_{s,b}$ if she has a unit supply and zero if she does not have unit supply, and that the maximal profit from deviation that seller s can make in one period is $\Pi^D_{s,b}$.

To enforce cooperation, a buyer that is cheated can punish a seller that cheated her by not purchasing goods from the seller in subsequent periods. The maximal level of cooperation can be enforced when a buyer uses the maximal punishment and essentially disconnects from a seller that cheated her. Without loss of generality, we focus on this

[3] The assumption that a seller has at most unit capacity is without loss of generality. A seller is considered to have a unit capacity if she is able to produce a high quality product. A seller has zero capacity if she is able to produce low quality only, or unable to produce.

[4] We remain agnostic to whether buyers live for one period or more. It will become clear that it does not matter for the analysis.

[5] An analogous analysis can be performed for a market in which buyers have the ability to deviate.

maximal punishment in our analysis.[6]

3.1 Community structure and knowledge of the network

We define the pattern of which sellers cooperate with which buyers as a network: seller s is connected to buyer b if and only if they are able to cooperate and seller s does not deviate in any interaction with buyer b (in the next section we endogenize the decision to cooperate).

Formally, Let d_s be the number of buyers that seller s is connected to, and d_b be the number of sellers that buyer b is connected to. We call d_s the degree of seller s and d_b the degree of buyer b. Let the links in the graph be chosen randomly conditional on a fixed degree distribution $G = \left\langle G^S, G^B \right\rangle$, where G^S and G^B are the degree distributions of sellers and buyers respectively.[7]

Buyers and sellers have incomplete knowledge of the network. In particular they know their close local environment, captured by their own degree and the degrees of every buyer or seller that is connected directly to them, as well as the global network's aggregate characteristics including the degree distribution G, and the random nature of the network.[8]

Within a period, meetings between connected buyers and sellers occur in a random order, i.i.d. across periods. Unconnected sellers and buyers do not meet. Formally, let E be the set of connections (links) in the network. Links from E are drawn randomly without replacement (all links are chosen in each period). When a link is chosen, the seller and buyer at the ends of the link meet and get an opportunity to trade. Buyers and sellers that do not manage to trade in a given period have utility 0.

[6] This punishment scheme can be supported as a part of a Nash equilibrium. The analysis of shorter or random punishments schemes is qualitatively identical.

[7] See Newman, Strogatz, and Watts (2001) for a review of the literature on random graphs with arbitrary degree distributions. See also Galeotti et. al. (2008) for an application of random graphs with arbitrary degree distributions in static games.

[8] The latter provides knowledge of the structure of the network beyond degree distribution, such as knowledge of aggregate measures of overlap in the sets of common connected buyers and sellers, and the expected length of cycles in the graph. We find it realistic to assume that agents have some insights on the structure of the society as a whole.

Models of incomplete information have been used in the economic literature for the analysis of static games in networks (see Galeotti et. al. 2008 and references therein). In the remainder of this subsection we discuss the adaptation of the incomplete information environment to markets with repeated games, which is a methodological contribution of the current paper. This is useful as it sheds light on the type of markets to which our analysis applies most closely. As the discussion is not used throughout most of the paper (with the exception of subsection 7.4), the less technical reader can go directly to the next subsection about the value of a relationship.

Community structure and incomplete information of the network structure. Our model can be interpreted in several ways. In one scenario the network is drawn once and stays fixed for all periods. The assumption of incomplete information of the network fits environments in which agents can learn their local neighborhood structure (including their own and close neighbors' degrees) and some characteristics of the global community (including degree distribution and overlap). However, the network is too big, and the market structure is sufficiently complex that even agents who live for a long time do not find it profitable (or feasible) to learn the complete network structure.

Alternatively, the network might undergo little changes across periods. While affecting only slightly the value of a connection, the changes decrease the ability or even the benefits from gathering complete information of the network structure. Allowing for slight changes in the network structure creates a realistic market environment for many applications. For example, consider markets in which buyers are divided into communities that share (at least partially) information on past transactions. Formally, let buyers live in different locations, in every location $l \in L$ there is a set B^l of buyers of which a subset of size $b^{l,active}$ is active in every period. A buyer from location l is connected to d_l sellers and an active buyer has unit demand. Every period, active buyers are chosen randomly and i.i.d. across locations and periods with the following restriction: a seller s has access to one active buyer from each location from a (fixed) set of d_s locations in every period.

The degree distribution of the network between sellers and active buyers, $G = \langle G^S, G^B \rangle$, can be inferred from L, $\{d_l\}_{l \in L}$, S, $\{d_s\}_{s \in S}$, and $\{b^l\}_{l \in L}$, and is constant across periods. As we are interested in large markets, we focus on the case where $|B|, |S|, |L|, |B^l| \to \infty$ for every l. We let $b^{l,active} < \infty$. Therefore, each location is only a small part of the active market in every period.

Focusing on large markets with random selection of active sellers and buyers creates an environment in which the network structure changes over time without changes to agents' local environments or to the basic aggregate descriptors of the global network (degree distribution, density, overlap, etc.). Consequently, complete knowledge of the network is obsolete, as the actual structure of the network undergo moderate changes across periods, allowing our analysis to hold without any changes for anything between agents who know the full network structure and agents who know only basic information that includes their own degree, the degree of their direct neighbors, and the degree distribution G.

The explicit model of community structure with locations is useful in several ways. First, allowing for some collective memory within location, it fits well into the job recommendations example. Consider a business area as a location, there are many firms and each firm has various departments, each department can be considered a separate hiring entity. While only a subset of departments are recruiting in every given period, departments in the same firm, and to some extent, departments in other firms in the same business area, observe the outcomes of the hiring. For example, if a department in firm A observes that some department in firm B hired a worker who was trained in a certain institution and turned out to be a low quality worker, firm A is less likely to hire graduates of the same institution.[9] Second, the model allows for an explicit study of reputation systems (see section 7.4).

[9] While the evidence on sharing information between firms in the same location about hiring of workers is only suggestive, there is empirical evidence of sharing other types of information within locations, i.e. Hong et. al. 2004.

For most of the paper, with the exception of the analysis of reputation systems in section 7.4, we rely directly on the direct analysis of the network between buyers and sellers.

3.2 The value of a relationship

Our framework allows us to pin down the marginal future value for seller s from a connection with buyer b. To see how, we start with one seller who has unit capacity every period with probability one and one buyer; a scenario that was heavily studied in the literature. In this simple case, with probability one, seller s *needs* buyer b in order to trade with a payoff $\pi_{s,b}$. For the seller, the future value of the link is $\frac{\delta_s}{1-\delta_s} \cdot \pi_{s,b}$.

Within a network, different links have different marginal future values even if sellers are homogenous with respect to their discount factors and the production process (μ_s). In fact, this is true even if the payoffs from different interactions over the network are independent of the buyer and seller transacting ($\pi_{s,b} = \pi$).

As we are interested in the effect of the network structure on the value of links, let $\delta_s = \delta$, $\mu_s = \mu$ and $\pi_{s,b} = \pi$. These can be relaxed with some technical burden.[10] Departing from the analysis of bargaining in networks and making the payoffs exogenous is useful for our purposes and fits well in various applications; in some, as in the job recommendations example in the previous section, there are no money transfers and the payoffs represent intrinsic utilities from trade. In others, the network represents which buyers trust which sellers, but as far as accessibility goes, all buyers can view (posted) prices and trade with all sellers.

Let seller s be connected to buyers $\{b_k\}_{k=1}^{d_s}$. We can write down the future value for seller s from a connection with buyer b as

[10] The assumptions that $\delta_s = \delta$ and $\mu_s = \mu$ are without loss of generality and save notation (the latter is relaxed in a technical appendix). Assuming that $\pi_{s,b} = \pi$ simplifies the analysis as it guarantees the existence of a Nash equilibrium in which a seller with unit capacity and a buyer with unit demand that meet, transact with probability one and do not prefer to wait for subsequent meetings in the same period. Extending the analysis to heterogenous payoffs introduces (manageable) complexity without much added insight. Endogenizing payoffs is beyond the scope of the paper and is the focus of the literature on bargaining in networks (see also Manea 2008, and Abreu and Manea 2008).

$$FV_{s,b} = FV\left(b, \{b_k\}_{k=1,..,d_s}, G, \mu, \delta, \pi\right) = \frac{\delta^e_{s,b}}{1-\delta^e_{s,b}} \cdot \pi \tag{1}$$

for some *effective discount factor* $\delta^e_{s,b}$. Intuitively, in a repeated interaction, seller s will behave towards buyer b *as if* she has a discount factor of $\delta^e_{s,b}$.

To derive the effective discount factor one should ask the following key question: How likely is it, that in a given period, seller s can sell a good to buyer b, and cannot sell the good to any other buyer? This is the probability that *the seller needs the buyer in a given period*, or $\mu \cdot P(b) \cdot \left\{\Pi_{k=1,..,d^S_s, b_k \neq b}\left[1 - P(b_k)\right]\right\}$, where $P(b) = P(b|G, \mu)$ is the within period ex-ante probability that buyer b has not purchased a good before the link with seller s, who is connected to her, was chosen. Therefore,

$$\frac{\delta^e_{s,b}}{1-\delta^e_{s,b}} = \mu \cdot P(b) \cdot \{\Pi_{k=1,..,d_s, b_k \neq b}\left[1 - P(b_k)\right]\} \cdot \frac{\delta}{1-\delta} \tag{2}$$

4 Market structure and cooperation

In this section we examine the effects of changes in the network structure on the value that a certain seller s have for a connection with buyer b and the effective discount factor used by s in interactions with b. We characterize the the future value of a connection as a function of the degrees of the buyer and the seller that are connected, the degrees of immediate neighbors, and the degree distribution of buyers and sellers in the population.

To this end, we assume that all *other* links in the network have a high enough effective discount factor to prevent the breaking of those links and analyze the implied effective discount factor for the link between seller s and a buyer b. Applying our results to each link separately, allows us to find the 'weakest link' in a network and to define and characterize networks that sustain high links' values and facilitate cooperation in repeated games. In the following sections we analyze aggregate characteristics of such networks.

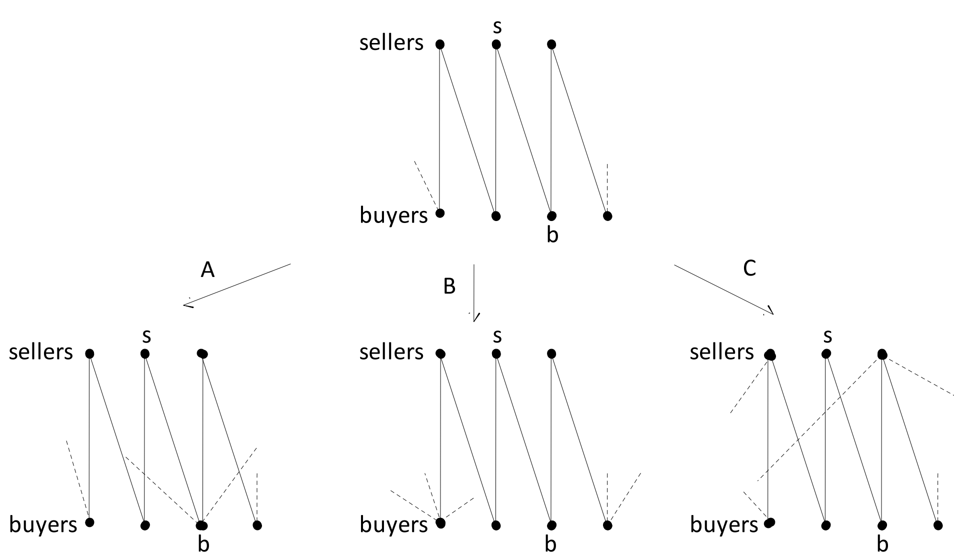
The following lemma is key in evaluating (2) and summarizes useful characteristics of $P(b)$ (the within period ex ante probability that buyer b has not purchased a good before the link with seller s, who is connected to her, was chosen). The proof is deferred to the appendix.

Lemma 1 *$P(b|G,\mu)$ is decreasing in d_b and μ. Moreover, let $\widehat{G} = \left\langle \widehat{G^B}, \widehat{G^S} \right\rangle$ such that $\widehat{G^B}$ first order stochastic dominates (FOSD) G^B, and G^S FOSD $\widehat{G^S}$; thus, $P(b|G,\mu) > P\left(b|\widehat{G},\mu\right)$.*

The network changes mentioned in lemma 1 are shown in figure 2. Note that changes in the degree distribution of only one side of the market require a change in the ratio $|S|/|B|$.[11]

Remark 1 *When investigating the effects of changes in the degree distribution on the value of a link between seller s and buyer b, we keep the degrees of s and all of her connected buyers (including b) fixed unless mentioned otherwise.*

Figure 2



[11] In such cases, assume that there is a common pool of buyers and sellers that are not connected. These buyers and sellers can be added to our network, or absorb agents that are disconnected from the network.

Figure 2: Changes in network structure (the broken lines represent links to agents that are not in the diagram). With respect to the benchmark network at the upper part of the figure, the arrows lead to the following changes with respect to the link between seller s and buyer b: (A) An increase in d_b; (B) A FOSD increase in the degree distribution of buyers $\left(G^B\right)$; and (C) A FOSD increase in the degree distribution of sellers $\left(G^S\right)$.

As d_b and μ increase, buyer b has access to more sellers that she can buy from and is more likely to have bought from one of them before meeting seller s. The relation to the degree distribution of other buyers (not directly connected to seller s) and other sellers is more subtle. When buyers that are connected to other sellers that are in turn connected to buyer b are more connected, the sellers that are connected to buyer b are less likely to sell to the other buyers and more likely to sell to her, this is true even with more distant sellers. Roughly speaking, an increase in the degree distribution of buyers (sellers) corresponds to an increase in supply (demand) within the network, which affect also buyers and sellers for which there was no change in degree through the effect of competition over the goods produced by other sellers in the network.

A connection between a seller and a buyer that are connected between them but not to any other buyer or seller, provides the highest future value of a connection. In this case (1) becomes $FV_{s,b} = \mu \cdot 1 \cdot \frac{\delta}{1-\delta} \cdot \pi$, which is the maximum value of FV possible. Hence, beyond one to one connections, more connectivity reduces the ability to sustain bilateral cooperation.[12] More formally, we can state the following result.

Proposition 1 *An increase in the number of connections of either seller s or buyer b that are connected, decreases the future value $(FV_{s,b})$ and the effective discount factor $\left(\delta^e_{s,b}\right)$ applied by s to interactions with b.*

Proof. The effect of an increase in the seller's degree is immediate from (2). To see the effect of an increase in the buyer's degree note that $\frac{\partial FV_{s,b}(\cdot)}{\partial P(b)} \geq 0$ and that by lemma 1, $P(b)$ is decreasing in d_b. ■

[12]This observation should not be interpreted as determining that more connectivity is bad, but rather that there are trade-offs when adding more connections. We investigate the welfare implications of more connectivity in section 6.

As a seller becomes more connected, she has more options in the case of elimination of the link with each of her buyers. As a buyer becomes more connected, her frequency of interaction with the corresponding seller decreases. Both effects lead to a devaluation of the connection.

On the other hand, additional links are also partially public goods; as other buyers connected to seller s have higher degrees, s is more likely to need buyer b and is less likely to mislead her.

Proposition 2 *An increase in the number of connections of a buyer $b' \neq b$ who is connected to seller s, increases the future value $(FV_{s,b})$ and the effective discount factor $\left(\delta^e_{s,b}\right)$ applied by seller s to interactions with b.*

Proof. From (1) and (2) we have that $\frac{\partial FV_{s,b}(\cdot)}{\partial P(b')} \leq 0$ $\left(\frac{\partial \delta^e_{s,b}(\cdot)}{\partial P(b')} \leq 0\right)$ for every $b' \neq b$. Lemma 1 states that $P(b)$ is decreasing in d_b which completes the proof. ■

In an extreme case, if a buyer is connected to only one seller, this seller has a future value of 0 from each additional link, and will apply a discount factor of $\delta^e = 0$ to interactions over additional links. Similarly, in order to maintain the value of a link, an increase in the degree of a buyer should be accompanied by an increase in the degrees of the other buyers that are connected to the same seller, and the network needs to be balanced with respect to buyers' degrees.

Propositions 1 and 2 can be used to demonstrate the effect of defection by one seller on the possibility that other sellers defect. For example, in figure 1b, if seller s' defects in an interaction with buyer b, the connection between seller s' and buyer b is eliminated. As a result, the connection between seller s and buyer b becomes more valuable, but the connections between seller s and buyers b' and b'' become less valuable, and in some interactions that might induce defection by seller s.

Even beyond the immediate neighborhood, an increase in sellers' (buyers') degree that is not accompanied by a corresponding increase in buyers' (sellers') degree leads to a decrease in the future values of links and the effective discount factors that sellers apply in interactions over their links.

Proposition 3 *A FOSD increase in the degree distribution of buyers and a FOSD decrease in the degree distribution of sellers lead to an increase in the future values of links leading to sellers with high degrees, and a decrease in the future values of links leading to sellers with low degrees.*

Formally, let $\widehat{G} = \left\langle \widehat{G^B}, \widehat{G^S} \right\rangle$ *be such that either* $\widehat{G^B}$ *FOSD* G^B, *or* G^S *FOSD* $\widehat{G^S}$. *Then there exist* $\overline{d_s} < \infty$ *and* $\underline{d_s} \geq 1$ *such that* $FV_{s,b}\left(\cdot|\widehat{G}\right) \leq FV_{s,b}\left(\cdot|G\right)$ *and* $\delta^e_{s,b}\left(\cdot|\widehat{G}\right) \leq \delta^e_{s,b}\left(\cdot|G\right)$ *for every link of every seller* s *with* $d_s < \underline{d_s}$, *and* $FV_{s,b}\left(\cdot|\widehat{G}\right) \geq FV_{s,b}\left(\cdot|G\right)$ *and* $\delta^e_{s,b}\left(\cdot|\widehat{G}\right) \geq \delta^e_{s,b}\left(\cdot|G\right)$ *for every link of every seller* s *with* $d_s > \overline{d_s}$.

Proof. From lemma 1, the change from G to $\widehat{G}$ leads to a decrease in $P\left(b|G,\mu\right)$. We are just left to plug this in (2). ■

From proposition 1, links in the network have low values if either the buyer has many links or the seller has many links. Proposition 3 implies that as sellers as a group become connected to more buyers, some links in which the sellers are more connected lose their value even if the degrees of the buyers at the end of these links did not vary. Others links, in which sellers were less connected become more valuable.

To better understand proposition 3, recall that a FOSD increase in the degree distribution of sellers, is accompanied by an increase in the number of buyers that are connected through the network, keeping the degree distribution of buyers constant. Consequently, when sellers become more connected, demand grows, and a seller s who is connected to many buyers can allow herself to have less buyers in the future without significant expected future loss. On the other extreme, when a seller s has only one connection, an increase in the degree distribution of sellers make her connected buyer less likely to buy before seller s meets her buyer, and the seller will need her more. This affects even sellers for which the number of connection has not changed, as they are affected by a reduction in competition as other sellers are more likely to sell to other buyers. The effect of changes in buyers degrees has a similar intuition, but goes in the opposite direction.

Proposition 3 introduces another layer of balance that the network is required to exhibit in order to maintain high values of the network's links. There cannot be too many buyers relative to sellers or vice versa. With respect to connectivity, this implies that buyers' (sellers') connectivity cannot grow without a corresponding growth in the degrees of sellers (buyers) by just adding more sellers (buyers) to the networked market. Propositions 4 and 5 in the next section strengthen this result.

The result can also be viewed as a claim about reputation and market power. Competition that is too weak (too few sellers so that buyers have only few connections) or too fierce (too many sellers and highly connected buyers) reduces the ability to sustain reputation, whereas a medium level of competition enhances reputation creation. We predict high level of cooperation in environments with positive, but moderate competition.[13] This effect is demonstrated in figure 3.

So far we characterized changes in the network that are 'value enhancing'. A network has high values for links if no seller or buyer have an abnormally high number of connections, and if there is a balance between the connectivity of buyers and the connectivity of sellers. The latter requires that the market will not have too many or too few sellers. Consequently, we expect a balance in the structure of networks that facilitate cooperation in markets.

In the next section, motivated by observations of changes in the structure of society following processes of globalization, we investigate the effects of an increase in overall connectivity and of the breaching of geographical and social boundaries that occur naturally when communication and transportation costs decline, and when cities as well as other communities grow.

5 Globalization

Globalization is said to make the world more connected. But what does it mean 'more connected'? Does each person have more connections? Or is there something in the

[13] In a related work on competition and seller's reputation in an environment with price competition, Bar-Isaac (2005) finds that competition can both aid and hinder reputation for quality.

pattern of connections that changes? [14]

Globalization can lead to people and businesses having more connections. With mobile phones, E-mail, and online chatting, communicating with other people became cheap. Transportation also became more affordable. This lowers the cost of creating and maintaining a connection.

Another impact of the reduction in communication and transportation costs is the virtual collapse of geographic borders, leading to growth of individual communities and to a decrease in segregation. As distance matters less, individuals and businesses choose their connections from a bigger set, and 'close' individuals have only little overlap in their acquaintances. While in the past people from the same village community knew (and traded with) the same set of people, nowadays as the world becomes a 'global village', the overlap in the sets of connections of different people declines.[15]

In this section we evaluate the effects of increasing the number of links, and the breaking of geographic borders on the values of links and on the ability to maintain cooperation over links in the network. In particular, we aim to shed light on the following questions: Is it possible that a seller with limited supply has valuable links to a large number of consumers? Does a global network facilitate higher or lower link values, and ability to cooperate, than a segregated local one? In section 6, we explore the welfare consequences of globalization.

5.1 Congestion

Both the market design literature and the social networks literature recognize circumstances in which lack of coordination in markets creates congestion that reduces the volume of transactions in markets and harms assortative and Pareto efficiency.[16] In the market design literature, congestion is often a consequence of lack of time to complete search and transactions in the market. In both literatures congestion is a result of

[14] See Watts (2003) for a non technical survey.
[15] See also Mobius and Rosenblat (2004).
[16] See Roth nand Xing (1997), Calvo-Armengol and Zenou (2005), and Fainmesser (2009).

a lack of ability to coordinate on who transacts with whom. This leads to increased randomness that makes it hard for buyers and sellers to find each other.

In this paper, we focus on a different aspect of the ability to transact. Instead of looking at time constraints we examine constraints on the ability to cooperate. We show that congestion can occur also at the more fundamental level of deciding who to cooperate with after finding trading partners.

To abstract from changes to the degrees of specific subgroups in a network, and without loss of generality, we focus in this section on networks in which all buyers are linked to the same number of sellers, and all sellers are connected to the same number of buyers. Let d^B and d^S be the degrees of buyers and sellers respectively. We show that while a balance in the network, with respect to the ratio $d^B : d^S$, can facilitate high links' values; when d^B and d^S grow too large, a network cannot facilitate significant cooperation. Proofs are deferred to the appendix.

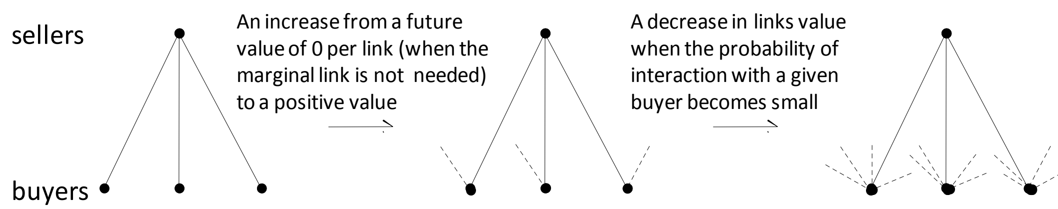
Propositions 4 and 5 corroborate the intuition summarized in section 4 for uniform changes in degrees of buyers and sellers and reinforce our intuition that a network is required to be balanced in order to maintain high value of links as it grows. More directly, they show that too many connections on either side of the market make cooperation hard to enforce.

Proposition 4 *An increase in buyers' degree d^B leads to a decrease (increase) in the future value of a link and of the effective discount factor used in the network when d^B and μ are large (small) and when d^S is small (large).*

Proposition 5 *An increase in sellers' degree d^S leads to a decrease in the future value of a link and of the effective discount factor used in the network when d^S is large and when d^B and μ are small.*

It is interesting to see how proposition 4 aggregates our results from propositions 1 - 3: an increase in d^B corresponds to a combination of an increase in the degree of a specific buyer (proposition 1), an increase in the degrees of other buyers connected to the same seller (proposition 2) and an increase in the degree distribution of buyers altogether (proposition 3). The first decreases the value of the link, the second increases it via the public good nature of a links, and the third has a non-monotonic effect. The result is non-monotonic in nature. When d^B is low, a seller has many good options and is guaranteed to sell even if she has fewer connections, raising d^B a little decreases the probability of a sale and the seller needs more connections. However, raising d^B too much reduces the frequency with which a seller interacts with each buyer and the value of each link. This non-monotonic effect is demonstrated in figure 3.

Figure 3



To understand the role of μ, recall that in our job recommendations example, low μ implies that only a small fraction of the teachers have high ability students in every period. Hence, it is easy to add more teachers, by increasing the number of connections that firms have, and sustain high value for every link. On the other hand, high μ implies that a large fraction of the teachers have high ability students in every period and in order to maintain the value of links the network is required to have sufficient demand through a low degree for firms or a high degree for teachers.

However, no matter how balanced the network is or what is the fraction of sellers that are active in every period, if we increase the number of links too much, the values of links decline.

Proposition 6 *Let $D = d^B$ and $\alpha \cdot D = d^S$, then for every α, δ, μ, and $V > 0$ there exist $\overline{D}(\alpha, \delta, \mu, V)$ such that for every $D > \overline{D}$ the future value of a link is smaller than V.*

Proof. Substituting $D = d^B$ and $\alpha \cdot D = d^S$ in (1) yields $FV(\cdot) = \mu \cdot P(\cdot) \cdot [1 - P(\cdot)]^{\alpha \cdot D - 1} \cdot \frac{\delta}{1-\delta} \cdot \pi$. As $0 \leq P(\cdot) \leq 1$; $P(\cdot) \cdot [1 - P(\cdot)]^{\alpha \cdot D - 1} \to 0$ when $D \to \infty$. ■

Intuitively, the pivotal probability that a seller s manages to sell to a specific buyer b, but would not have managed to sell to any other buyer, becomes negligible when sellers and buyers have many connections.

Proposition 6 ties back to the discussion about coordination and congestion. When anyone can potentially cooperate with everyone else, the value of a cooperating partner goes down as each partner has only a small influence on outcomes. This leads to a congested market, in which being potentially able to cooperate with everyone means that there is no ability to really cooperate with anyone. The real value creating role of the network is to provide coordination and specify who cooperates with whom. This necessary coordination is lost when the network becomes too dense.

5.2 Community size and segregation

In the age of the world wide web, it is hard to define the boundaries for human interactions. The growth of multi-national corporations and the constant increase in the ratio of international trade to GDP since World War II is yet additional evidence that economic interactions are not constrained by borders. At the individual level, as communication and travel become more accessible, people develop sets of acquaintances that are not limited by physical locations.

In this section, we investigate how the weakening of physical borders affects the ability to sustain high values of connections and cooperation. We show that in general, the ability to define small communities by creating artificial borders increases the value of links and facilitates cooperation. In our job recommendations example it suggests why groups of firms often interview and hire from the same sets of schools.[17] Proofs are deferred to the appendix.

We extend our analysis to allow networks to be divided into islands (connected

[17] See also Lee and Schwarz (2008).

components), each with $\Psi \cdot d^B$ sellers and $\Psi \cdot d^S$ active buyers in every period ($\Psi \in \mathbb{N}$). Ψ represents the size of each community within the society. The definition of 'a community' for applied work depends on the application and can vary from a community defined by geographical region, interests, race, social status, culture, etc.[18]

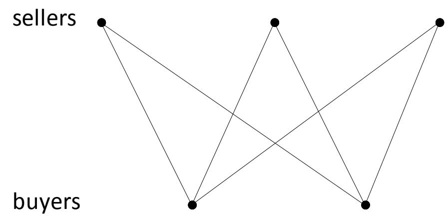
In the absence of network data, there are two exogenous and observable dimensions of comparisons that can be mapped into this framework: (1) Time - as communication and transportation costs decline, communities are becoming inter-connected, and people know people from outside their community of origin. (2) Community size - in small towns a person is likely to know the same group of individuals as her peers, whereas in large cities, even a short subway commute connects neighbors from the same building to different sets of acquaintances. Note that as Ψ varies, the degree of an individual buyer or seller are held constant. This allows us to discriminate between the effect of the size of the community, and of the degree of each individual.

Varying Ψ continuously raises technical difficulties and is beyond the scope of this paper. Instead, we focus on limit cases that shed light on the role of community size. When $\Psi \to \infty$ we call the network '*the global network*' and the analysis is identical to the previous sections; there is one big island and all locations and sellers are interconnected. On the other extreme, when $\Psi = 1$, we encounter '*the segregated network*', which is divided into islands in which each of a group of d^S buyers is connected to each of a group of d^B sellers. Figure 4 provides an example of an island with $d^S = 2$ and $d^B = 3$ $(\Psi = 1)$.[19]

[18] An additional form of segregation can come through some form of product differentiation. If each small set of firms invests in producing according to the specific requirements of a small set of buyers, we get a segregated network in which crossing segregation lines require investment in modifications of the product. For more on differentiation and trust, see also Fainmesser (2009b).

[19] It is impossible to draw a figure of a global network ($\Psi \to \infty$) with a small number of buyers and sellers. For our results, the important characteristic of the global network is that there is very low expected overlap between the sets of buyers that different sellers are connected to, which is the opposite of the perfect overlap in the segregated network.

Figure 4



The setup naturally raises two 'dual' questions: (1) Are connections more valued in a locally segregated society or in a global one? and (2) Where can a network grow thicker while maintaining the value of a connection, in the segregated or in the global world? On a deeper level, one also wonders how will society be structured; proposition 7 suggests that in some environment segregation will persist even when geography is no longer a constraint.

As the analysis of the two types of networks employs different methods and does not produce closed form solutions, answering these questions is not straightforward. Nevertheless, focusing on the cases in which each seller is connected to very few buyers (low d^S) or to many buyers (high d^S) makes the problem more tractable and is insightful.[20]

Proposition 7 *Let FV_g and δ_g^e be the future value of a link and the effective discount factor that a seller uses for interactions over a link when the network is global, and FV_s and δ_s^e be the corresponding values when the network is segregated and divided to small communities. Then,*

1. *There exists $\overline{d^S} > 1$ such that for every $d^S \leq \overline{d^S}$:*

    (a) *(Balanced network) There exist $\underline{d^B} > 1$ such that $d^B \leq \underline{d^B}$ implies that $FV_s \geq FV_g$ and $\delta_s^e \geq \delta_g^e$.*

[20] It is not necessary to go over the complete analysis of the segregated network in order to establish our analytic results. For completeness, it is presented in a technical appendix, available from the author.

(b) *(Unbalanced network) For every* $\mu < \frac{1}{2}$ *there exists* $\overline{d^B}(\mu)$ *such that* $d^B \geq \overline{d^B}$ *implies that* $FV_s \leq FV_g$ *and* $\delta_s^e \leq \delta_g^e$.

2. *If* $d^S > d^B$ *then* $FV_s \leq FV_g$ *and* $\delta_s^e \leq \delta_g^e$.

There are two countervailing forces in action. Those can be demonstrated using the networks in figure 5.

Figure 5a

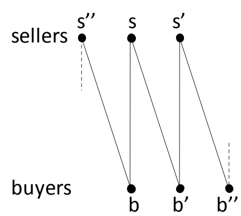
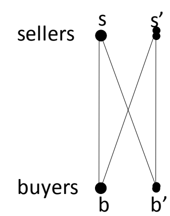


Figure 5b



On the one hand, *without* the link connecting seller s and buyer b in both networks, the segregated network in figure 5b provides seller s with a higher probability of trading, conditional on producing, than the global network in figure 5a. This is because in figure 5b seller s' does not face any competition for selling to b, while in figure 5a s' faces competition for selling to buyer b''. Therefore, seller s' is more likely not to sell to buyer b' in figure 5b. This reduces the value of the link between seller s and buyer b in figure 5b relative to that in figure 5a.

On the other hand, adding the link connecting s and b in figure 5a adds another independent opportunity for seller s to sell the good (to buyer b), while in figure 5b it adds an opportunity that is negatively correlated with the opportunity to trade with b'. In fact, in the segregated network in figure 5b, seller s is guaranteed to be able to trade if she has the link to buyer b. In this example, the second force dominates and the value of the link between seller s and buyer b is higher in the segregated network.

When d^B grows (holding d^S constant), the negative correlation is weakened as not being able to trade with one buyer indicates only that one has one less competitor for trading with the second buyer. However, it is still the case that a seller with one link has higher probability of trading in the segregated network.

The intuition for the second part of proposition 7 is trivial as when there are more buyers than sellers, the value of each link in a segregated network is zero because a seller is guaranteed to trade with or without her marginal link. This is not true for a global network. In this case, the coordination between firms in the segregated networks allows sellers with limited capacity to be local monopolists, while in the global network, there is no strong notion of locality and competition increases.

To understand our results better, we focus on the tractable case where $d^S = 2$ and compute the value of a link in both the segregated and global networks for different values of d^B and μ, holding d^S, δ and π fixed.[21,22] For each (μ, d^B) pair, we subtract the value of a link in the segregated network $FV_s(d^S = 2, d^B, \mu, \cdot)$, from the corresponding link value in the global network $FV_g(d^S = 2, d^B, \mu, \cdot)$. The result, that is presented graphically in figure 6, is the added link value due to segregation, or the cost of globalization in terms of the value of links and the ability to sustain cooperation in a network with $d^S = 2, d^B$, and μ.

Figure 6

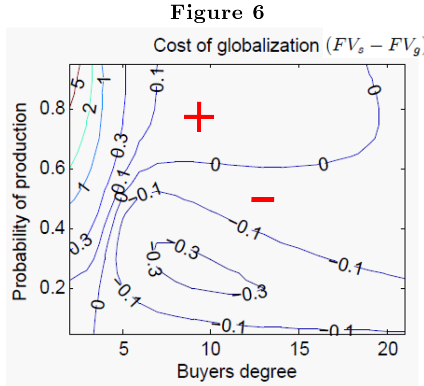


Figure 6: the differences between the values of links in the segregated network and the values of links in a global network $(FV_s - FV_g)$ for different (μ, d^B) pairs. Each contour lines describe plots the different (μ, d^B) pairs for which the difference between the value of a link in the segregated network with $d^S = 2, d^B$, and μ, and the value of a link in the corresponding global network, equal the value written on the line itself. The 0 value contour line divides the plain into (μ, d^B) pairs for which the values of links in the segregated network are higher (to the left and above the 0 value contour line, where

[21] We set $\delta = 0.99$ and $\pi = 1$; this has no affect on the relation between the values of links in the two networks, and has no qualitative impact on the graphs presented here.

[22] The analysis was conducted using Matlab. The code is available upon request from the author.

contour lines have positive values) and (μ, d^B) pairs for which the values of links in the global network are higher (to the right and below the 0 value contour line, where contour lines have negative values).

In light of proposition 7, the numerical results are not surprising. A segregated network facilitates higher link values in balanced networks, as can be seen by the positive values on the contour lines to the left and above the 0 contour line, where d^B is small (recall that $d^S = 2$ for the entire graph). A global network facilitates higher link values in unbalanced networks, as can be seen by the negative values on the contour lines to the right and below the 0 contour line.

Overall, it seems that part 1a of proposition 7 is more suggestive of the general relation between the two networks; when a segregated network facilitates higher links' values, the difference is sizeable (up to a difference of 5 in figure 6). Conversely, when a global network facilitates higher links' values, it is only of a small magnitudes (not more that 0.3 as indicated by the negative values in figure 6). So in general, a segregated network facilitates higher or almost identical links' values.

Beyond the direct comparison between the different network structures, proposition 7 and figure 6 shed light on the role of the noise in the production process, μ, in the de-segregation involved in the globalization process. Although the numerical analysis suggests that the requirement that $\mu < \frac{1}{2}$ it is not necessary, it shows that when μ is large, the segregated network facilitate higher links' values than the global one for a larger set of networks. This is suggested by the upward slope of the zero difference contour line in figure 6, and is driven by differences in the aggregate volume of trade across networks, discussed in the following section. Intuitively, when μ is large sellers produce with higher certainty. In a deterministic environment, the fixed coordination that the segregated network provides is very efficient, while in a changing environment, with small μ, the global network provides more opportunities for trade.

6 Welfare

Welfare enhancing networks facilitate high levels of cooperation and high volumes of trade in each period. In previous sections, we focused on comparing the maximal levels of cooperation that different networks facilitate through a comparison of effective discount factors; higher discount factors are efficiency and welfare enhancing. However, not every transaction requires the highest effective discount factor. In such cases, it is of interest to understand the effect of the network structure on the volume of trade for fixed sets of buyers (B) and sellers (S). In this section, we focus on the effect of the network structure on the volume of trade and discuss the trade-offs between high links' values and high trade volumes in networks.

6.1 The connectivity trade-off

In this section, we show that in environments with non-negligible local fluctuations of the supply vs. demand ratio, the networks that maintain high effective discount factors, and the networks that facilitate high volume of trade are inherently different. As a result, there is a trade-off between the ability to sustain cooperation and achieving the maximal volume of trade in every period. On the other hand, in environments that do not have such fluctuations sparse and segregated networks facilitate maximal welfare by allowing for high effective discount factors and high trade volumes. This suggests that one of the reasons that networks grow is to handle local demand and supply fluctuations.

Before proceeding, we need to introduce some new notation and definitions.

Definition 1 *For fixed S and B, let N be the set of all networks. A network is* Complete (C) *if all sellers in S are connected to all buyers in B. A network is* Minimally Connected *if it has the smallest number of links among all networks in which all buyers and sellers have at least one link.*[23] *Let MC be the set of minimally connected net-*

[23] In a minimally connected network the degree of any agent on the long side of the market is 1.

works. Finally, a Pairs Network *has the maximal number of links such that no buyer or seller has more than one link; let PN be the set of pairs networks. Denote by V^n the expected volume of trade in one period of the infinitely repeated game in network n, conditional on cooperation over all of the network n's links.*[24]

In light of proposition 6, the following result shows that there is a possible trade-off between links' values and the volume of trade.

Proposition 8 $max_{n \in N} \{V^n\} = V^C$*, so the complete network facilitates the maximal volume of trade possible for any fixed set of buyers and sellers.*

Proof. Let AS_t be the set of sellers with unit supply (active sellers) in period t. In every period either $|AS_t| \geq |B|$ or $|AS_t| < |B|$. If $|AS_t| \geq |B|$ then $V^C = |B|$, otherwise $V^C = |AS_t|$. In both cases the volume of trade is maximal. ■

So in the absence of cooperation restrictions, the complete network maximizes the aggregate welfare. To show that the maximization of the volume of trade is not a trivial requirement, and to make precise the claim that in many environments there is a real trade-off between sustaining cooperation and maximizing trade, we need the following definition.

Definition 2 *An environment is constantly over- (under-) demanded if there are more (less) active buyers than sellers with unit supply in every period t.*

The definition is quite restrictive and many environments are not constantly over or under demanded. More specifically,

Corollary 1 *An environment is constantly over-demanded if and only if $|S| \leq |B|$; and constantly under-demanded if and only if $|S| \geq |B|$ and $\mu = 1$.*

Using corollary 1, we can show the following result.

[24]Note that if $|B| \neq |S|$ then $PN \subsetneqq MC$ and $PN \supsetneqq MC$ as some buyers or sellers will be left unconnected in every maximal pairs network. However, if $|B| = |S|$ then $PN = MC$.

Proposition 9 *For every minimally connected network $\widehat{n} \in MC$, $\widehat{n}$ facilitates the maximal volume of trade $V^{\widehat{n}} = V^C = max_{n \in N} \{V^n\}$ if and only if the environment is either constantly over-demanded or constantly under-demanded.*

Proof. It is easy to see that when the environment is either constantly over-demanded or constantly under-demanded a minimally connected network facilitates the maximal volume of trade. For the converse, assume that the environment is neither constantly over-demanded nor constantly under-demanded, so $|S| > |B|$ and $\mu < 1$. As $|S| > |B|$, in a minimally connected network, each seller has exactly one connection, and there exists at least one buyer b who is connected to more than one seller. As $\mu < 1$, there is a positive probability that more than one seller who is connected to b has unit supply, whereas all sellers that are connected to buyer $b' \neq b$ do not have unit supply. In this realization, at least one of the sellers connected to b will have unit supply and will not trade, and at the same time, buyer b' will have an unfulfilled unit demand, and the volume of trade will not be maximal. ■

A similar argument leads to the following result.

Corollary 2 *For every pairs network $n' \in PN$, n' facilitates the maximal volume of trade $V^{n'} = V^C = max_{n \in N} \{V^n\}$ if and only if the environment is either constantly over-demanded or constantly under-demanded.*

In environments that are never fluctuating between over demand and over supply, minimally connected networks, and even pairs networks, generate as much trade as the complete network, and there is no necessary trade-off between cooperation and volume of trade. Nevertheless, the trade-off is not completely eliminated.

Corollary 3 *Adding links to a network can increase or decrease the expected volume of trade. In particular,*

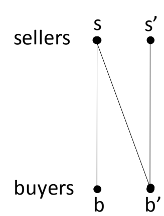
1. *Adding a link that completes the network increases the expected volume of trade; and*

2. *In an environment that is either constantly over-demanded or constantly under-demanded, adding a link to a minimally connected network or to a pairs network decreases the expected volume of trade.*

Proof. Part 1: From proposition 8, adding a link that completes the network never decreases the expected volume of trade. To see that it might strictly increase the volume of trade, consider an environment in which $|S| = |B|$ and $\mu = 1$. In this environment, the complete network guarantees a maximal volume of trade $V^C = |S|$ in every period. However, in the network that is complete apart from the link between seller s and buyer b, there is a positive probability that sellers $S\backslash \{s\}$ trade with buyers $B\backslash \{b\}$ and seller s and buyer b are left without trading partners.

Part 2: From proposition 9 and corollary 2, in an environment that is either constantly over-demanded or constantly under-demanded, adding a link to a minimally connected network or to a pairs network cannot increases the expected volume of trade. To see that it might strictly decrease the volume of trade, consider again the environment in which $|S| = |B|$ and $\mu = 1$. A network that consists of separated pairs guarantees a maximal volume of trade $V^{MC} = |S|$ in every period. Adding a single link creates a connected component of the shape in figure 7.

Figure 7



There is a positive probability that seller s trades with buyer b', so that seller s' and buyer b are left without trading partners. ■

6.2 The segregation trade-off

For a large family of balanced networks, we have shown that segregation enhances cooperation (proposition 7). We now show, that in environments with local fluctuations of supply versus demand, segregated networks are not efficient, motivating trends of globalization observed in the world. In the absence of fluctuations, segregated networks generate the maximal volume of trade and are expected not to exhibit trends of globalization. For consistency we consider segregated networks in which the degrees of all sellers are identical $\left(d^S\right)$ and so are the degrees of all buyers $\left(d^B\right)$.

Proposition 10 *For every segregated network $n' \neq C$ that is not the complete network; n' facilitates the maximal volume of trade $V^{n'} = V^C = max_{n \in N}\{V^n\}$ if and only if the environment is constantly over-demanded or constantly under-demanded.*

Proof. As each connected components is a complete network, it is immediate that if the environment is constantly over-demanded or constantly under-demanded, n' generates the maximal volume of trade $V^{n'} = V^C$ in every period. For the converse, assume that the environment is neither constantly over-demanded nor constantly under-demanded, so $|S| > |B|$ and $\mu < 1$. Therefore, this is true for each of the connected components. Pick two of the components n_1 and n_2. There is a positive probability that in a certain period the number of active sellers in n_1 $(|AS_t^{n_1}|)$ is bigger than the number of buyers in n_1, and that the opposite is true for n_2, so $|AS_t^{n_1}| > |B^{n_1}|$ and $|AS_t^{n_2}| < |B^{n_2}|$. In this case the volume of trade in components n_1 and n_2 is $V^{n_1 \cup n_2} = |B^{n_1}| + |AS_t^{n_2}|$ which is strictly smaller than $min\left\{|AS_t^{n_1}| + |AS_t^{n_2}|, |B^{n_1}| + |B^{n_2}|\right\}$ which is the maximal trade in the complete network on the same sets of buyers and sellers. ■

So when there are local demand and supply fluctuations the segregated network loses its ability to facilitate maximal trade.[25,26]

[25] In a related work, Lee and Schwarz (2008) analyze interviewing decisions in labor markets. As the decision to interview is made after knowing that workers are of at least some minimal quality, there are no demand and supply fluctuation. Lee and Schwarz find that complete overlap in the interviewing decisions among groups of firms maximizes that number of position filled.

[26] Note that our results are different in nature from Hall's theorem that characterizes the networks in which there exists a maximal matching. We ask that a network will *guarantee* a maximal matching

7 Institutions and markets

In order to achieve cooperation in a market and facilitate large trade volumes, ones needs to find alternative ways to raise the value of links, or to make it less profitable for sellers to deviate. In the following sections, we suggest three institutions. Even though the institutions fulfill a similar role to the network by sustaining cooperation, their true strength is via the strong complementarity with the network in facilitating efficiency in markets; institutions allow networks to grow and become dense and global, without losing the ability to sustain cooperation.

Our analysis suggests that the difference between market with and without cooperation enhancing institutions cannot be measured by the scope of cooperation between two parties, but rather by the volume of trade in the market, and by the ability of a market to grow. This new theoretical prediction is supported by empirical work on institutions and markets. Hall and Jones (1999) show that differences in institutions explain much of the variation in product per worker across countries, and Johnson et. al. (2002) show that the main effect of belief in the court system is to encourage the formation of new relationships. The proofs for the claims in this sections are immediate from construction and are omitted.

In addition, we suggest that in markets where there are imbalances between supply and demand, institutions can allow for networks that are not balanced to sustain cooperation, allowing more buyers or sellers to participate in the market and restoring equality of opportunity among market participants, as well as increasing the volume of trade (see also Maggi 1999).

Remark 2 *We present in this section a high level view of institutions and focus on a reduced form model that helps to understand their effect on the cooperation over*

in a decentralized setting. As a result, Hall's theorem provide a larger set of networks that allow for the maximal matching.

By showing that not many networks allow for the maximal matching in a decentralized setting we motivate the design of many markets for indivisible goods, such as the market for kidney exchanges, the market for medical internships, and other markets studied in the market design literature. For a survey see Roth (2008).

our buyer-seller network. We defer a thorough micro level analysis of each of the institutions to future work. Incidentally, looking at different social networks, as the social network among buyers, sheds light on the way these institutions work.

7.1 Communal institutions - reputation systems

In section 3.1. we suggested a model of community structure that incorporates a positive probability that a buyer shares her trading history with other buyers in her location. This probability is affected by reputation systems used within a location.

We can interpret I_b as the probability that a buyer b shares information regarding a given transaction with other buyers in the same location. Let the realizations of I_b be i.i.d. with respect to both buyers and periods. Also, let $\Pi_{s,b}^D$ be the maximal additional present profit for seller s from 'deviating' in an interaction with buyer b by taking an action that does not satisfy the buyer, and $FV_{s,b}$ be the future value of the connection between s and the location of buyer b.

Proposition 11 *Seller s will deviate in interactions with buyer b if and only if $\Pi_{s,b}^D > I_b \cdot FV_{s,b}$.*

As a result, locations with high quality reputation systems (buyers with high I_b) can have more trustworthy links, including links with sellers that buyers from locations with low quality reputation systems cannot trust. More important, an increase in I_b for some b while not decreasing $I_{b'}$ for any $b' \neq b$ allows for denser and more global networks to sustain cooperation.

Corollary 4 *Let $I = \{I_b\}_{b \in B}$ describe the qualities of the reputation system of all buyers in a market, and let $N^I \subseteq N$ be the set of networks that sustain cooperation over all the links in the network with reputation system I. If $\widehat{I} = \left\{\widehat{I}_b\right\}_{b \in B}$ is such that $I_b \leq \widehat{I}_b$ for every $b \in B$ then $N^I \subseteq N^{\widehat{I}}$.*

Improving the reputation system increases the set of networks that can sustain cooperation, and allows for more efficient trade patterns to arise. A different type of reputation systems are those that enable information sharing across locations. It is easy to see that allowing buyers to punish sellers that deviated in other locations increases further the set of networks that can sustain cooperation. In the extreme case that all buyers are informed of all transactions in the market, the complete network sustains as much cooperation as any other network for a fixed set of buyers and sellers.

Our model also suggests that markets that are denser and have a network with less overlap (e.g. centers of metropolitan areas) benefit more from developing alternative mechanisms to maintain reputation. This is due to the lack of ability to sustain cooperation when the network is simply too dense and global. Examples include the online communities such as Yelp and Chowhound, both create a large accessible database of recommendations.

7.2 Transaction oriented institutions - litigation and third-party evaluation services

Litigation allows buyers that were 'cheated' to prosecute the deviating seller in order to get compensation and punish the seller directly with some positive probability. Third-party evaluation services inspect the goods before trade occurs in order to expose cheating before trade has taken place with positive probability, decreasing the potential gains from deviating.

Formally, let β^L be the probability that a buyer who was cheated succeeds in prosecuting the deviating seller and receives a compensation λ from the seller (without loss of generality, λ is also the fine that the seller pays). Let β^E be the probability that a third-party evaluation institution find a low quality good to be low quality, in which case, trade does not occur even if the seller recommended the good.[27] As the deviation of the seller is exposed, buyers can punish the deviating seller even if trade was prevented. Building on the notation from the previous section we can state the

[27] For simplicity, we do not allow for mistakes in which an evaluation institution mistakes a good product to be of low quality.

following result.

Proposition 12 *Seller s will deviate in interactions with buyer b if and only if* $\left(1-\beta^{E}\right)\cdot\left(\pi^{D}_{s,b}-\pi^{C}_{s,b}\right) > I_b \cdot FV_{s,b} + \beta^{L}\cdot\lambda$.

Improving the institutions, by increasing β^{L}, λ, or β^{E}, increases the set of feasible network and allow for efficient, trade enhancing networks to sustain cooperation.

As discussed above, technological progress through the reduction in communication and transportation costs is the driver of globalization, as it reduces the costs of creating links. Propositions 11 and 12 and corollary 4, suggest that a necessary condition for the rise of dense and global networks is having the appropriate institutions in place. Table 1 describes the optimal network structure as determined by the quality of institutions, the cost of creating links, and the noise in the market environment (the local demand and supply fluctuations). In the presence of non-negligible local demand and supply fluctuations, first best can be achieved only with high levels of technological progress AND good cooperation enhancing institutions.

Table 1

No local fluctuations of demand versus supply

Reduction in the cost of creating a link →

Improvement in the quality of institutions ↓

Local / sparse	Local / sparse
Local / sparse	Local / sparse

Non negligible local demand and supply fluctuations

Reduction in the cost of creating a link →

Improvement in the quality of institutions ↓

Local / sparse	Intermediate
Intermediate	Global / dense

Table 1: the predicted (optimal) network structure as affected by the cost of linking (as affected by transportation and communication costs), the institutional quality (I_b, β^{L}, λ, and β^{E}), and the local demand and supply fluctuations.

In the absence of local demand versus supply fluctuations local and sparse networks are efficient. However, The requirement on no fluctuation is strong, and we expect most markets to be affected by the quality of the institutions and the cost of linking.

In the absence of any non-negligible local demand and supply fluctuation, optimal networks are easy to achieve: pairs networks are optimal both for sustaining cooperation, and for allowing the maximal volume of trade. Even when pairs networks are not realistic, other segregated networks provide the maximal volume of trade coupled with high links' values. As a result, in the absence of local demand and supply fluctuation, and especially when interactions require a high level of trust and cooperation, we expect to find little or no effect of globalization on the network structure and the resulting trade patterns; networks will maintain low degrees and high levels of overlap (and segregation). This suggests that 'difficult' moral hazard problems are solved by interactions between family members and a small subset of close friends and is consistent with the evidence on strong ties (see also, Granovetter 1974 and Karlan et. al. forthcoming).[28]

On the other hand, when local demand and supply fluctuations are large, there are no optimal solutions; pairs networks still facilitate high levels of cooperation, but no longer allow for maximal trade volume to take place. Moreover, other segregated networks do not offer a solution to this problem as they no longer facilitate favorable conditions for cooperation, nor do they allow for maximal trade. Our results suggest, that in this 'noisy' environment there is an inherent trade-off between cooperation and volume of trade. A dense network allows for more trade, conditional on facilitating cooperation, but is limited with respect to the maximal level of cooperation that it allows.

Therefore, in the presence of local demand and supply fluctuation, the network structure is expected to rely heavily on the level of trust required from the interaction. Interactions that require little trust will be conducted over dense networks, while interactions that require higher levels of trust are expected to have more sparsely connected networks and smaller trade volumes as a result. Moreover, when the trade-off is faced, we also expect to find buyers' and sellers' organizations that allow for collective

[28] Moral hazard problems are less likely to exhibit supply and demand fluctuations, as opposed to adverse selection problems, for which the supply is exogenous and often probabilistic.

reputation to substitute the personal reputation in our model.

For example, our findings are consistent with the high volume of trade in generic goods that is conducted over the internet between individuals, as well as with the concentration of sales of used and experience goods around brands and website that provide reliable ways of transacting and reputation systems (eBay, Amazon, etc.).

8 Discussion

The generality of our framework allows us to derive practical implications and approach questions from the literatures on Labor Economics, Industrial Organization, Development Economics, and Market Design. In this section, we discuss the predictions of our model, review evidence, and discuss the relation to the different strands of the literature. For some applications, the adjustments of our model involve simple renaming of variables and is deferred to a technical appendix available from the author.

8.1 Community structure and cooperation

The model predicts that each buyer can trust only a limited number of sellers, who in turn interact with a small number of other buyers. Networks that are especially good in sustaining cooperation are: (1) *balanced*: no close buyers or sellers have degrees that are too different; (2) *sparse*: the number of connections of sellers and buyers are limited; and (3) *segregated*: sellers who have a buyer in common have connections to similar sets of buyers overall.

The literature on repeated games in networks to date provided various network based extensions of the literature on community enforcement (i.e. Kandori (1992), Greif (1993) and Ellison (1994)). This literature is focused on the ability of the community to coordinate on punishing a deviator.[29] Our focus is different; we study cooperation

[29] In Lippert and Spagnolo (2006), and in work in progress by Miller and Nageeb, the network is used both for information transmission and for punishment, in Kinateder (2008) a deviator is punished by all members of society and the network is used only for coordination on the punishing behavior.

between individuals or groups that are not coordinated on a joint punishment and can cooperate and punish only bilaterally. As a result, our framework allows links to be substitutes or complements and additional links can improve or harm cooperation, while in the community enforcement literature additional links always improve the ability to sustain cooperation.

More related to our research goals is work by Karlan et. al. (2009) and by Ambrus, Mobius, and Szeidl (2008). In their work links carry exogenous values, and the risk of breaking links facilitates cooperation and trust between members of the network. Their model allows for trust between remote agents in the network through the value of all links along the path(s) connecting the agents. They remain agnostic as to the origin and magnitude of the values of links. To that extent, their model is especially suitable for transactions that are small, or sufficiently infrequent and do not affect the values of links via which the transactions take place. Moreover, as the value of links in their model is held fixed, adding links can only improve cooperation in the network.

The empirical literature provides ample evidence on the role of networks in markets and generally find that the network is not as dense as one would expect and that additional links can either improve or harm cooperation. The literature on the microfinance industry in developing countries finds exceptionally low default rate even without a centralized credit bureau. Consistent with our model, microfinance institutions are very local. In every developing country there are significant parts of the population that have no access to loans, independent of their economic status, while others can take multiple loans simultaneously, suggesting that some individuals are part of the market's network while others are not. Moreover, our model predicts that strategic default occurs when many lending institutions offer loans to the same borrowers, and do not condition the loan on repayment of debt to some of the other lenders. While the evidence is far from being conclusive, Chaudhury and Matin (2002) and McIntosh and Wydick (2005) find suggestive evidence that is consistent with this observation.

Research of other markets in developing and transition economies provide further evidence. Fafchamps (1996) surveys manufacturing and trading firms in Ghana and finds that firms rely on repeated bilateral interactions to enforce contracts and that the absence of reputation mechanisms limits the economic reach of firms. McMillan and Woodruff (1999) who study trading networks in Vietnam find that a firm trusts its customer enough to offer credit when the customer finds it hard to locate an alternative supplier.

In developed countries, market structure is also found to be influenced by connections and repeated interactions. Hardle and Kirman (1995), Kirman and Vriend (2000), and Weisbuch et. al. (1996) find consistent loyalty of buyers to sellers within the fish market in Marseille. Kirman and Vriend assert that the standard asymmetric information model "seems a too loose application of the textbook argument". They explain that this is because there is a fixed population of buyers and sellers in this market and "every buyer (loyal or not) is a potential repeat buyer" so "a seller would have an incentive to deliver good quality to every single buyer". Incidentally, the selective supply of high quality by sellers to a subset of the fixed population of buyers is consistent with our model as we show that sellers do not have the incentives to maintain reputation with all of the buyers, even if all are potential repeated customers. The networks that facilitate trust in our model describe a pattern of loyalty of consumers to different sellers.

In labor markets, as illustrated by our example in section 2, many hiring decisions are affected by patterns of connections in the market. Fainmesser (2009) allow for truthful revelation of workers' qualities along connections in a network and demonstrates that the patterns of connections affect not only the number of workers hired, but also the timing of hiring in entry level labor markets.[30] It is therefore important to understand what networks will facilitate truthful revelation of private information. In a technical appendix we show, using our repeated games model, that the timing of hiring does not affect the feasible network structure. This allows us to separate the analysis to two stages: (1) characterizing the network structures that sustain truthful

[30] Entry level markets are studied extensively in the market design literature (for a survey, see Roth 2008).

revelation of information, which is the focus of this paper; and (2) analyzing the effect of the network structure on the timing of hiring, which is the focus in Fainmesser (2009).

8.2 Trade, institutions, and growth

Countries with better institutions and countries that trade more grow faster, and countries with better institutions also tend to trade more (for evidence see Dollar and Kraay 2003 and reference there). Hall and Jones (1999) show that differences in institutions also explain much of the variation in product per worker across countries. In particular, the lack of institutions can explain why some developing countries are 'left behind' while other countries enjoy high levels of productivity. In developing countries, Fafchamps (1996) finds that the absence of reputation mechanisms limits the economic reach of manufacturing and trading firms in Ghana and Johnson et. al. (2002) show that the main effect of belief in the court system is to encourage the formation of new relationships. The complementarity of network and institutions in the context of the transition to market economies in Eastern Europe is also documented by Woodruff (2002).

Our model provides a connection between the micro evidence on informal contract enforcement and the macro level observations on institutions and growth. Our findings suggest that in the absence of trust facilitating institutions, developing countries cannot catch up with the developed world (even if they achieve similar technological progress) as they are forced to compromise on the volume of trade in order to sustain trust. Our results also suggest that the effect of the missing institutions on trust might not be as visible as previously suspected. Even when there is evidence of high level of trust in individual transactions, developing countries are paying the cost of lack of trust facilitating institutions through a reduction in overall trade and growing inequalities.

9 Conclusion

This paper presents a framework for analyzing repeated bilateral games over buyer-seller networks. The model can be applied to a variety of markets with asymmetric information and moral hazard. Methodologically, the framework simplifies greatly the analysis of repeated games in networks and yet provides a rich setup that can be applied to real markets.[31]

Our results show that the network structure matters. On one hand, increasing the number of links in a network and making the network more global, has the potential to increase trade. On the other hand, the same changes threatens the ability to sustain cooperation in environments with asymmetric information and moral hazard. Without cooperation in these environments, there is a risk that no trade will take place (see Akerlof 1970).

Consistent with evidence, we show that the benefits of globalization are regained when proper institutions are present. Consequently, improving transportation and communication is not enough to promote markets in developing countries.

In different than previous literature on networks and markets (see Kranton 1996 and references therein), we do not analyze markets and networks as two mutual exclusive ways to conduct the same activity. We rather focus on markets that are networked; even though every agent in the market can potentially approach any other agent, the need to trust ones partners puts a constraint on the actual trade in the market based on a network of trust and cooperation. We rely on evidence that networks are present in many market interactions and suggest that understanding their role will improve our understanding of markets.

[31] A natural extension is to symmetric games in one-sided networks. An example of a simple one-sided network structure for which some of our results follow naturally is presented in a technical appendix available from the author.

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11 Appendices

11.1 Appendix A: Analysis accompanying section 4

To prove lemma 1, we need some definitions and additional lemmas.

Let $p(d_b, t) = p(d_b, t|G, \mu)$ be the ex-ante probability that a buyer that has degree d_b has not bought a good when exactly t links have already been drawn. Clearly, $p(d, 0) = 1$. Similarly, let $\varphi(d_s, t)$ be the conditional ex-ante probability that a (random) seller who has unit capacity and has degree d_s has not sold the good when t links have already been drawn, so $\varphi(d, 0) = 1$. Let g be the pdf of G. We can write p and φ in the following way.

$$p\left(d_b,t\right)=\left[p\left(1,t\right)\right]^{d_b} \tag{3}$$

where

$$\begin{aligned} p\left(1,t\right) &= \frac{|E|-t}{|E|}+\frac{t\cdot\left(1-\mu\right)}{|E|}+\frac{t\cdot\mu}{|E|}\cdot\sum_{d=1}^{\infty}g^{S}\left(d\right)\cdot\frac{1}{t}\sum_{\tau=0}^{t-1}\left[1-\varphi\left(d-1,\tau\right)\right]= \\ &= 1-\frac{\mu}{|E|}\cdot\sum_{d=1}^{\infty}g^{S}\left(d\right)\cdot\sum_{\tau=0}^{t-1}\left[\varphi\left(1,\tau\right)\right]^{d-1} \end{aligned} \tag{4}$$

$$\varphi\left(d_s,t\right)=\left[\varphi\left(1,t\right)\right]^{d_s} \tag{5}$$

and

$$\begin{aligned} \varphi\left(1,t\right) &= \frac{|E|-t}{|E|}+\frac{t}{|E|}\cdot\sum_{d=1}^{\infty}g^{B}\left(d\right)\cdot\frac{1}{t}\sum_{\tau=0}^{t-1}\left[1-p\left(d-1,\tau\right)\right]= \\ &= 1-\frac{1}{|E|}\sum_{d=1}^{\infty}g^{B}\left(d\right)\cdot\sum_{\tau=0}^{t-1}\left[p\left(1,\tau\right)\right]^{d-1} \end{aligned} \tag{6}$$

Lemma 2 *$p\left(d_b,t\right)$ is decreasing in d_b and in t.*

Proof. Note that $0\leq\varphi\left(1,\tau\right),p\left(1,t\right)\leq 1$. Therefore, the result is immediate from (3) and (4). ■

Lemma 3 *$\varphi\left(d_s,t\right)$ is decreasing in d_s and in t.*

Proof. Note that $0\leq\varphi\left(1,\tau\right),p\left(1,t\right)\leq 1$. Therefore, the result is immediate from (5) and (6). ■

Lemma 4 *$p\left(d_b,t|G,\mu\right)$ is decreasing in μ and $\varphi\left(d_s,t|G,\mu\right)$ is increasing in μ.*

Proof. Note that $\frac{\partial p(1,t)}{\partial\varphi(1,\tau)}\leq 0$ for all t and $\tau<t$. Also, $\frac{\partial\varphi(1,t)}{\partial p(1,\tau)}\leq 0$ for all t and $\tau<t$. Finally, $p\left(1,t\right)$ is directly decreasing with μ. ■

Lemma 5 *Let $\widehat{G} = \left\langle \widehat{G^B}, G^S \right\rangle$ and $\widehat{G^B}$ FOSD G^B. Hence $p(d,t|G,\mu) \geq p\left(d,t|\widehat{G},\mu\right)$ for every d, and t.*

Proof. A change in the degree distribution involves a corresponding change in the ratio of connected sellers to connected buyers. As the market is large, we can restrict our attention to a fixed number of links $|E|$ without loss of generality. We prove the lemma by induction on t.

For $t = 0$, $1 = p(d,0|G,\mu) \geq p\left(d,0|\widehat{G},\mu\right) = 1$. Assume that this is true for every $t < t'$; we prove the claim for t'.

$[p(1,\tau-1)]^{d_b-1}$ is decreasing in d_b, so it is guaranteed that $\varphi\left(1,t|G^B,\mu\right) \leq \varphi\left(1,t|\widehat{G^B},\mu\right)$ as long as $p\left(1,\tau-1|G^B,\mu\right) \geq p\left(1,\tau-1|\widehat{G^B},\mu\right)$. This is true by the induction assumption since according to (4) we are interested in $\tau < t'$.

As $\frac{\partial p(1,t)}{\partial \varphi(1,\tau)} \leq 0$ for all t, $p(d,t'|G,\mu) \geq p\left(d,t'|\widehat{G},\mu\right)$ as requested. ■

Lemma 6 *Let $\widehat{G} = \left\langle G^B, \widehat{G^S} \right\rangle$ and G^S FOSD $\widehat{G^S}$. Hence $p(d,t|G,\mu) > p\left(d,t|\widehat{G},\mu\right)$ for every t.*

Proof. The proof is identical to the one of lemma 5 (substituting p with φ, and d_b with d_s) and is omitted. ■

Lemma 1 - **Proof.** As $P(l|G,\mu) = \frac{1}{|E|}\sum_{t=1,\ldots,|E|} p(d_b-1,t-1) = \frac{1}{|E|}\sum_{t=1,\ldots,|E|} p(1,t-1)^{d_b-1}$, lemma 1 is a direct implication of lemmas 2, and 4 - 6. ■

11.2 Appendix B: Analysis accompanying section 5

Even though there are no buyers with $d \neq d^B$ or sellers with $d \neq d^S$ we use an analysis that builds strongly on our analysis of the previous sections and let $p(d,t)$ and $\varphi(d,t)$ be defined over every d as auxiliary functions. Expressions (4) and (6) become $p(1,t) = 1 - \frac{1}{|E|}\cdot\mu\cdot\sum_{\tau=0}^{t-1}[\varphi(1,\tau)]^{d^S-1}$, and $\varphi(1,t) = 1 - \frac{1}{|E|}\cdot\sum_{\tau=0}^{t-1}[p(1,\tau)]^{d^B-1}$.

We use the following results in the proofs of propositions 4 and 5.

Lemma 7 *$p(1,t)$ is decreasing in d^B and μ and increasing in d^S.*

Proof. We start with showing that $p(1,t)$ is decreasing in d^B by induction on t. Note that $p(1,0)=1$ is independent of d^B and assume that $p(1,t)$ is decreasing in d^B for every $t<t'$. It is immediate that $\varphi(1,t)=1-\frac{1}{|E|}\cdot\sum_{\tau=0}^{t-1}[p(1,\tau)]^{d^B-1}$ is directly increasing in d^B and decreasing in $p(1,\tau)$ for $\tau<t$. Using the induction assumption, we get that $\varphi(1,t)$ is increasing in d^B for every $\tau\leq t'$. To conclude, note that $p(1,t')$ is decreasing in $\varphi(1,t)$ for every $t\leq t'$ so $p(1,t')$ is decreasing in d^B.

To see that $p(1,t)$ is increasing in d^S note that $p(1,0)=1$ is independent of d^S and assume $p(1,t)$ is increasing in d^S for every $t<t'$. As $\varphi(1,t)$ is not affected directly by d^S (but only through p), by the induction assumption $\varphi(1,t)$ is decreasing in d^S for every $t<t'$. As $p(1,t)=1-\frac{1}{|E|}\cdot\mu\cdot\sum_{\tau=0}^{t-1}[\varphi(1,\tau)]^{d^S-1}$ is directly increasing in d^S and decreasing in $\varphi(1,\tau)$ for $\tau<t$, $p(1,t)$ is increasing in d^S for every d^S.

The proof for μ is immediate from lemma 4. ■

Corollary 5 *$p\left(d^B-1,t\right)$ is decreasing in d^B and μ, and increasing in d^S.*

Corollary 6 *$P(\cdot)$ is decreasing in d^B and μ, and increasing in d^S.*

Proposition 4 - Proof. For the proof, let d^B be a continuous variable. We than show that $\partial FV(\cdot)/\partial d^B<0$ for large μ and d^B, and small d^S and vice versa. Later, we show that there exists 'legal' parameters for which $FV(\cdot)$ is both increasing and decreasing in d^B as required.

As $\partial FV(\cdot)/\partial d^B=\mu\cdot\frac{\delta}{1-\delta}\cdot[1-P(\cdot)]^{d^S-2}\cdot\left[\partial P(\cdot)/\partial d^B\right]\left\{1-P(\cdot)\cdot d^S\right\}\cdot\pi$ its sign is determined as the opposite of the sign of $\left\{1-P(\cdot)\cdot d^S\right\}$ (recall that $P(\cdot)$ is decreasing in d^B by corollary 8). If $P(\cdot|\mu)$ and d^S are small, $1-P(\cdot)\cdot d^S>0$ and $\partial FV(\cdot)/\partial d^B<0$, and vice versa. It is only left to note that by corollary 8, $P(l|G,\mu)$ is decreasing in d^B and μ, and increasing in d^S. ■

Proposition 5 - Proof. As $FV(\cdot)=\mu\cdot P(\cdot)\cdot[1-P(\cdot)]^{d^S-1}\cdot\frac{\delta}{1-\delta}\cdot\pi$, corollary 8 implies that when d^S is large and when d^B and μ are small, $P(\cdot)$ is large and an increase in d^S decreases $P(\cdot)\cdot[1-P(\cdot)]^{d^S-1}$ by both increasing $P(\cdot)$ and the power argument. ■

Proposition 7 - Proof. Part 1a:

consider the case where $d^S = 2$ and $d^B = 2$.

In the segregated network the probability that a link is necessary (conditional on having a unit supply) is $1 - \left(1 - \frac{1}{3}\mu\right)$ as when a seller has both links she knows that a sale is guaranteed, and when she has only one link, a sale is guaranteed unless the other seller has unit supply and his link to the same buyer was the first one to be chosen in the connected component.

In the global network, the equivalent probability is $x\,(1-x)$ where $1-x$ is the probability that a link cannot be used when it is chosen due to lack of demand on the side of the buyer. $1-x$ can be decomposed into the sum of $\frac{1}{2}\mu\,(1-x)$ for the case that the other link of the competitor is not useful, plus $\mu\left(\frac{1}{3}-\varepsilon\right)x$ for the case that the other link of the competitor is useful but his relevant link was chosen first; $\varepsilon > 0$ as the fact that the link was useful indicates that it is more likely to have been chosen early before the relevant link.

From $1 - x = \frac{1}{2}\mu\,(1-x) + \mu\left(\frac{1}{3}-\varepsilon\right)x$ it is immediate that $x = \frac{6-3\mu}{6-\mu-6\varepsilon\mu}$ and the segregated network provides higher link value and effective discount factor than the global network as long as $\frac{6-3\mu}{6-\mu-6\varepsilon\mu}\left(1-\frac{6-3\mu}{6-\mu-6\varepsilon\mu}\right) \leq 1 - \left(1-\frac{1}{3}\mu\right)$. This can be simplified to $0 \leq 108\varepsilon - 126\mu\varepsilon + 6\mu + \mu^2 + 12\varepsilon\mu^2 + 36\varepsilon^2\mu^2$ and is true for every $0 \leq \mu$ and $\varepsilon \leq 1$. As in the case that $d^B = 1$ the two networks are identical, this completes part 1 of the proof.

Part 1b:

In this part we are focused on the case where $d^B \to \infty$.

In the segregated network, a seller with one link has $\mu \cdot \left(d^B - 2\right)$ competitors (as long as $\mu \nrightarrow 0$ one of the other sellers that have unit supply will trade with the other buyer with probability $\to 1$).

In the global network a seller with one link has $\mu \cdot \left(d^B - 1\right) \cdot \left[(1-x) + x \cdot \left(\frac{1}{2} - \varepsilon\right)\right]$ competitors so $x = \frac{1}{\mu \cdot (d^B - 1) \cdot \left[1 - \frac{1}{2}x - \varepsilon x\right]}$. $\varepsilon > 0$ as the fact that the competitor's link still had unit demand, implies that it was chosen early, so the probability that the relevant link was chosen before is less than $\frac{1}{2}$.

The probability that a seller with unit supply needs her marginal link is $(1-x)\,x$ in the global network and $\left(1-\frac{1}{\mu\cdot(d^B-1)}\right)\cdot\frac{1}{\mu\cdot(d^B-2)}$ in the segregated network.

As $\mu \nrightarrow 0$ and $d^B \to \infty$, x is small (and in particular $x<\frac{1}{2}$), so a lower bound on x provides a lower bound on $x\,(1-x)$ and we can focus on demonstrating that $x\,(1-x) \geq \left(1-\frac{1}{\mu\cdot(d^B-1)}\right)\cdot\frac{1}{\mu\cdot(d^B-2)}$ for $\mu<\frac{1}{2}$.

From $x=\frac{1}{\mu\cdot(d^B-1)\cdot\left[1-\frac{1}{2}x-\varepsilon x\right]}$ we get that $1+\varepsilon x^2\cdot\mu\cdot\left(d^B-1\right) = x\cdot\mu\cdot\left(d^B-1\right) - \frac{1}{2}x^2\cdot\mu\cdot\left(d^B-1\right)$ and $\varepsilon=0$ provides a lower bound on x. Denote this lower bound as $\underline{x}$ such that $\frac{1}{\mu\cdot(d^B-1)} = \underline{x}-\frac{1}{2}\underline{x}^2$ and $\underline{x} = \frac{1}{\mu\cdot(d^B-1)}+\frac{1}{2}\underline{x}^2 \geq \frac{1}{\mu\cdot(d^B-1)}$. Consequently, $\underline{x} \geq \frac{1}{\mu\cdot(d^B-1)}+\frac{1}{2}\left(\frac{1}{\mu\cdot(d^B-1)}\right)^2$. Plugging $\underline{\underline{x}} = \frac{1}{\mu\cdot(d^B-1)}+\frac{1}{2}\left(\frac{1}{\mu\cdot(d^B-1)}\right)^2$ into $x\,(1-x)$ yields that $(1-x)\,x \geq \left[1-\left(\frac{1}{\mu\cdot(d^B-1)}+\frac{1}{2}\left(\frac{1}{\mu\cdot(d^B-1)}\right)^2\right)\right]\cdot\left(\frac{1}{\mu\cdot(d^B-1)}+\frac{1}{2}\left(\frac{1}{\mu\cdot(d^B-1)}\right)^2\right)$ and it is sufficient to show that $\left[1-\left(\frac{1}{\mu\cdot(d^B-1)}+\frac{1}{2}\left(\frac{1}{\mu\cdot(d^B-1)}\right)^2\right)\right]\cdot\left(\frac{1}{\mu\cdot(d^B-1)}+\frac{1}{2}\left(\frac{1}{\mu\cdot(d^B-1)}\right)^2\right) \geq \left(1-\frac{1}{\mu\cdot(d^B-1)}\right)\cdot\frac{1}{\mu\cdot(d^B-2)}$ for every $\mu<\frac{1}{2}$.

With some algebra this becomes $\mu+\frac{1}{2}\cdot\frac{\left(d^B-2\right)}{\left(d^B-1\right)}+\frac{1}{\mu}\cdot\frac{\left(d^B-2\right)}{\left(d^B-1\right)^2}+\frac{1}{4\mu^2}\cdot\frac{\left(d^B-2\right)}{\left(d^B-1\right)^3} \leq 1$. Recalling that $\mu>0$ and $d\to\infty$ this is simplified to $\mu+\frac{1}{2}+0+0\leq 1$ which hold for every $\mu<\frac{1}{2}$. As in the case that $\mu\to 0$ the two networks provide identical link's value, this completes the proof. ■