The Nontradable Goods' Real Exchage Rate Puzzle

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Abstract

This paper studies empirically and theoretically the decomposition of the real exchange rates into tradable and nontradable components, in the spirit of Engel (1999). Empirically, using an extended decomposition, we find that the contribution of the relative price of non-tradable goods to local nontradable output to the overall real exchange rate movements is at best modest. Theoretically, we argue that this finding is a puzzle for the standard models in which the law of one price holds, and fluctuations of the real exchange rate for tradable goods are fully accounted for by the relative price movements of the differentiated home and foreign tradable goods. Specifically, we find that, in the best case scenario, the standard model over-shoots the contribution of non-tradable goods to the overall real exchange rate fluctuations by a factor of two.

1 Introduction

Real exchange rates are amongst the most volatile aggregate prices. Volatility of the real exchange rates in the cross-section of countries is typically a multiple of the volatility of output, with the deviations being highly persistent. As demonstrated by Engel (1999) and Betts et al. (2001, 2006, 2008), the decomposition of real exchange rate movements in the data suggests that the bulk of

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these movements is accounted for by the international deviations in the relative price of tradable goods (or tradable consumption baskets). This finding is grossly at odds with the predictions of the traditional models of real exchange rate determination featuring one homogenous tradable good, and an emphasis on the role of the relative price of non-tradable to tradable goods.

These results — however disturbing from the perspective of the traditional theory — are not readily inconsistent with the predictions of the standard international macro models featuring differentiated tradable output by the country of origin, like the Backus, Kehoe & Kydland (1995) model or the extension of this model featuring non-tradable goods due to Stockman & Tesar (1995). Such models, in principle, can generate volatile tradable real exchange rates through the channel of relative price movements of foreign versus domestic varieties of tradable goods, and perhaps account for all the empirical regularities implied by the decomposition of real exchange rates to tradable and non-tradable components.

Motivated by this state of affairs, our paper asks to what extent the above claim is warranted. More precisely, we ask here the question whether a carefully parameterized standard theory extended to incorporate a non-tradable sector can quantitatively account for the properties of the decomposed real exchange rates, and in particular for the small contribution of the relative price of nontradables to the overall real exchange rate movements.

Our findings suggest that while the parameterized models — with some necessary restrictions on the response of the supply-side of the economy to shocks — can generate volatile and persistent real exchange rates, they fail to account for the modest contribution of the non-tradable goods to the overall real exchange rate movements. All our parameterizations, including the unconventional one rigged to account for the data, grossly overstate the contribution of this relative price (by a factor of at least 2). Key problems, as we explain below, is an overstated absolute volatility of relative price of non-tradable goods, and its strongly negative correlation with the real exchange rate for tradable goods.

To understand the basic intuition behind our results, it is instructive to first review some of the algebra behind the decomposition of the real exchange rate. In what follows, we will use a simplified approach to sketch the main ideas behind the real exchange rate decomposition, and defer the full analysis and discussion of measurement to the data section. In this simplified approach, we assume that the CPI basket in each country has a fixed internal structure, and is comprised of a weighted geometric average of: (i) the composite price index of all the non-tradable components in the basket P_N, (ii) the composite price index of all local tradable components entering the basket P_T, and (iii) the price of imported goods P_I :

$$CPI = P_N^{\zeta}(P_T^{\omega} P_I^{1-\omega})^{1-\zeta}.$$

In such case, the real exchange rate rer can be easily decomposed into three interpretable terms, labelled here rer^T, rer^N and a residual ε:[1]

$$rer = \frac{eCPI^*}{CPI} = \underbrace{\frac{eP_T^*}{P_T}}_{rer^T} \times \underbrace{\frac{\left(\frac{P_N^*}{P_T^*}\right)^{\zeta}}{\left(\frac{P_N}{P_T}\right)^{\zeta}}}_{rer^N} \times \underbrace{\frac{\left(\frac{P_I^*}{P_T^*}\right)^{(1-\omega)(1-\zeta)}}{\left(\frac{P_I}{P_T}\right)^{(1-\omega)(1-\zeta)}}}_{residual\ \varepsilon},$$

where the asterisk refers to foreign analogs of the variables. In this decomposition, rer^T measures international deviations of the relative price of country's tradable output. This is the price, which in the data carries most of the volatility. rer^N, in turn, measures the international deviation of the relative price of non-tradable output in terms of local tradable output, and turns out to play only a modest role in the data. Finally, the residual term — which plays a scant role in the data — measures the international deviation of the relative price of tradable consumption basket in terms of local tradable output.

In light of the above decomposition, the puzzle of accounting for the modest contribution of rer^N can be understood as follows. In the standard models — under actually the most favorable assumption of perfect labor mobility between tradable and nontradable sectors of the economy — the log of rer^N is trivially proportional to the log of international ratio of relative productivity in tradable to non-tradable sectors,

$$\frac{z_T^*}{z_N^*} \Big/ \frac{z_T}{z_N}\ , \tag{1}$$

with the coefficient of proportionality equal to the share of non-tradable output in total output.

[1] By *real exchange rate,* we mean the CPI-based bilateral real exchange rate that is defined as the ratio of foreign to domestic CPIs converted to a common currency unit using the official bilateral nominal exchange rate.

As a consequence, the volatility of rer^N is equal to the product of the share of nontradables in output and the volatility of the international ratio of relative productivities stated above. Hence, when the model is quantitatively disciplined using the data, the volatility of rer^N implied by the model is bounded from below by $78\% \times 6.06\% = 4.7\%$, where the first number is the median share of non-tradables in output and the second number is the volatility of the productivity ratio (1) in our sample. By comparison, the median volatility of rer^N in the data is only 2.5% — which is half as much. To top it off, the standard model also fails to replicate the data correlation of -0.26 between rer^N and rer^T, and implies a correlation of -0.8 . Consequently, given that the model already overstates the absolute volatility of rer^N to begin with, even if it matches the volatility of rer^T from the data (which it does in some of our parameterizations), it is bound to overpredict the contribution of rer^N at least by a factor of 2 relative to the volatility of the product $rer^T \times rer^N$.

The above findings lead us to the conclusion that the modest contribution of non-tradable goods is a puzzle with respect to the standard model, and more theory is needed to understand how rer^T and rer^N move in the data. The biggest problem seems to be a pervasive prediction of the standard theory of a strong negative correlation between rer^N and rer^T, and the tight link between rer^N and and relative productivity ratio (1). Theories meant to account for this fact would have to break this link.

In terms of the literature, our work is related to the papers on the decomposition of the real exchange rates following Engel (1999). In this line of research, Betts and Kehoe (2001, 2006, 2008) extend Engel's original results by looking at a broad cross-section of country pairs, consider a broader set of decompositions, and propose a model with endogenous tradability of goods to account for real exchange rate dynamics between US and Mexico. Burstein, Eighenbaum & Rebelo (2006) study Engel's decomposition using import and export prices, and conclude that with such indices used as trade prices the verdict is more favorable for the traditional trade theory of real exchange rate determination.

Relative to these papers, our contribution is to document how the standard international macro models fare in light of these findings. Specifically, we ask whether these properties can be accounted for by the simple theories. We view our contribution as important in clearing the path for further research, which, to our best knowledge, lacked such motivation so far. In terms of measurement, to be consistent between productivity and output prices, we focus here solely on the value added deflators as measures of tradable and nontradable output, and also consider a different, more detailed decomposition. Relative to the results presented both in Engel (1999) and Betts and Kehoe (2006, 2008), our approach of using value added deflators makes the results on the relative contribution of non-tradable goods in the data higher[2] — which nevertheless remains still modest in light of the predictions of the model.

2 Empirical Regularities

This section documents the key empirical regularities of the time-series of the bilateral real exchange rates. Specifically, here we decompose the overall bilateral real exchange rates into a tradable part and a nontradable part, and asses the relative contribution of these two separate parts. Our finding is that the nontradable part we extract has at best a moderate contribution to the dynamics of the overall bilateral real exchange rates. This finding is broadly consistent with the related literature — even though our measurement methodology is different.

2.1 Conceptual Framework

Our approach to decompose the movements of the real exchange rates extends the sketch provided in the introduction to avoid some of the measurement issues its implementation actually involves. Specifically, our goal here is to decompose real exchange rates into a tradable part and nontradable components, while at the same time avoid assuming a very restrictive internal structure of the CPIs that we made in the introduction. The key idea is to use auxiliary prices for tradables and nontradables and load the measurement issues connected to the internal structure of the CPI onto the residual term. What we gain — in light of the particular properties of the data that actually makes the residual term quantitatively unimportant — is a sharp decomposition that is free some measurement issues present in alternative approaches.

More precisely, given the definition of the real exchange rate as the ratio of CPIs translated to

[2]See Betts & Kehoe (2006) for a discussion of the differences in estimates when value added deflators are involved.

a common currency unit using the nominal exchange rate e,

$$rer = \frac{eCPI^*}{CPI}, \tag{2}$$

we use an auxiliary price of each country's tradable P_T and non-tradable output P_N and an identify relation to decompose the CPI real exchange into tradable goods' real exchange rate rer^T, nontradable goods' real exchange rate rer^N, and a residual term ε:

$$rer = \frac{eCPI^*}{CPI} = \underbrace{\frac{eP_T^*}{P_T}}_{rer^T} \times \underbrace{\frac{\left(\frac{P_N^*}{P_T^*}\right)^{\zeta}}{\left(\frac{P_N}{P_T}\right)^{\zeta}}}_{rer^N} \times \underbrace{\frac{\frac{CPI^*}{P_N^{*\zeta}P_T^{*1-\zeta}}}{\frac{CPI}{P_N^{*\zeta}P_T^{*1-\zeta}}}}_{residual \;\; \varepsilon}. \tag{3}$$

In this decomposition, the first term captures international deviations in the relative price of local tradable output of each country — which we label *the tradable goods real exchange rate* — the second term captures international deviations in the relative price of non-tradable output to local tradable output — which we label the *non-tradable goods real exchange rate* — and the third term is intended to convey information how the composite price index obtained by weighting two auxiliary prices by ζ relates to the overall CPI level of the respective country. Even thought we refer to the last term as a residual, we should stress at this point that it may be potentially large. It is the implementation of our method, and the choice of P_T, P_N, that will render this term small.

In the next paragraph, we discuss how we implement the above decomposition method using the time-series that are available for a wide cross-section of countries. Our goal here is to come as close as possible to the interpretation given to each term above, while still be able to obtain enough data to cover a wide cross-section of countries.[3]

[3]It is important to note that our definition of nontradable real exchange rate is slightly different than the one used by Engel (1999) or Betts & Kehoe (2008). Engel (1999) uses the relative nontradable CPI to tradable CPI in each country, while Betts & Kehoe (2008) use overall CPI relative to PPI in each country. Both the tradable CPI and the overall CPI include imported goods, while our method includes only locally produced and sold goods.

2.2 Implementation

To implement our decomposition, we use value added deflators as an auxiliary measure of each country's local price of tradable P^T and non-tradable output P^N. The weight ζ is assumed common across countries, and equal to the median share of each sector in the total output in these countries of .78.[4] The advantage of using value added deflators is their wide availability in the OECD STAN database. The major concern — which is not going to be important for our results — is that they may not necessarily coincide with the price of tradable and non-tradable goods at any level of aggregation, and worse, may potentially overstate the 'true' volatility of these prices. The source of this problem comes from outsourcing of services by the tradable sectors of the economy — which given their high volatility may increase volatility of value added deflators for non-tradable goods relative to other measures of non-tradable good prices.[5] In fact, Betts & Kehoe (2006) discuss this issue in detail, and by comparing value added deflators to gross output deflators, find that the bias of value added decomposition is generating a significantly higher variance of the nontradable goods real exchange rate. Since our intention here is to find the upper bound of the contribution of non-tradable prices to real exchange rate fluctuations, we are comfortable with this bias. The central advantage of our approach is the consistency of these definitions with our later measure of productivity used to estimate the forcing process.

To construct the key objects present in the decomposition (3), we use annual data from 1970 to 2005 for 21 countries[6], giving us the total of 210 country pairs. Our raw data includes: all-items Consumer Price Index (CPI_i) from the Word Bank Development Indicators database, the official nominal exchange rates (e_{ij}) from the same source, Manufacturing Value Added deflators (P_i^T) from the OECD Stan database, and Total Services Value Added deflator (P_i^N) from the same source. All statistics that we report below have been first logged and then HP filtered with an annual smoothing parameter 100. Time-series illustrated in the figures are always linearly detrended.

[4]We found modest variation of this weight, and also experimented with weights varying by country. It did not change any of the results.

[5]For example, when a firm producing a tradable good outsources some of the activities to a firm from a non-tradable sector (e.g. business service sector), this activity may artificially inflate the volatility of the value added deflator due to potential demand-side links between them. Yet, the value of the outsourced non-tradable service will be included in the final good prices of tradables, but not in the final good prices of non-tradables (these non-tradables are intermediate goods).

[6]This is the widest date range. For some pairs, data is limited to fewer years.

For each of the 210 country pairs in our sample, we construct the following objects:

- The *CPI-based bilateral real exchange rate between country i and j*

$$rer = \frac{e_{ij} CPI_j}{CPI_i}, \tag{4}$$

- value added composite output deflator in country i constructed by us from the manufacturing and services value added deflators (STAN series: 1537, 5099) according to the formula,

$$P_i = \left(P_i^N\right)^{\zeta} \left(P_i^T\right)^{1-\zeta}, \tag{5}$$

  where ζ is assumed common and set equal to the median value of .78 in the entire sample of countries,

- the *deflator-based bilateral real exchange rate between country i and j*

$$rer^D = \frac{e_{ij} P_j}{P_i}, \tag{6}$$

- the *tradable goods' bilateral real exchange*

$$rer^T = \frac{e_{ij} P_j^T}{P_i^T}, \tag{7}$$

- the *nontradable goods' bilateral real exchange rate*

$$rer^N = \left(\frac{P_j^N}{P_j^T}\right)^{\zeta} \Big/ \left(\frac{P_i^N}{P_i^T}\right)^{\zeta}, \tag{8}$$

- and the residual of the decomposition

$$\varepsilon = \left(\frac{CPI_j}{P_j}\right) \Big/ \left(\frac{CPI_i}{P_i}\right).$$

Together, these objects together decompose the movements in the overall CPI-based bilateral real exchange rate (4) into movements coming from the tradable goods real exchange rate (7), the nontradable goods real exchange rate (8), and the residual:[7]

$$
\begin{aligned}
rer &= rer_{ij}^{D} \times \varepsilon_{ij}, \\
rer^{D} &= rer_{ij}^{T} \times rer_{ij}^{N}.
\end{aligned} \tag{9}
$$

We now proceed to studying the properties of the above objects in the data. In what follows, we report results for prices for the whole sample as well as for the sample excluding European countries. We also report results when the whole sample is split into low and high bilateral intensity pairs — to relate it to the work of Betts & Kehoe (2006) and justify our modeling choices made later in the paper.

2.3 Findings

Equations (9) are the main relations we focus on. Below, we first study the first equation of the decomposition (9) to document the relation between CPI- and deflator-based bilateral real exchange rates, and then proceed to study the contribution of the tradable and nontradable real exchange rate to the movements of the deflator-based real exchange rates, as implied by the second equation in (9).

2.3.1 Selected US Pairs

To give a better feel of our data, we will first briefly look at the case of four major US trade relationships. The four trading partners that we consider here constituted over 40% of US imports in 1987 (middle of the sample period), and in our sample were the four largest US trading partners in that year. Figure 1 illustrates linearly detrended series for these selected pairs. The left-hand side panels in the figure show the relation between CPI-based real exchange rate rer and the deflator-based real exchange rate rer^D. The right-hand side panels look at the decomposition to rer^N and

[7] Note that, in essence, our decomposition is actually a decomposition of rer^D, which is then related back to the CPI-based real exchange rate by introducing an explicit residual term.

Table 1: Prices in Major US Trade Partnerships

	US-Canada	US-Germany	US-Japan	US-UK
Comparison of rer and rer^D				
- $\sigma(rer)$	6.1%	10.6%	11.1%	10.5%
- $\sigma(rer^D)/\sigma(rer)$	99%	101%	100%	112%
- $\rho(rer, rer^D)$	0.97	1.00	0.99	0.99
Comparison of rer^D and rer^N				
- $\sigma(rer^D)$	6.1%	10.7%	11.1%	11.9%
- $\sigma(rer^N)/\sigma(rer^D)$	48%	15.6%	15.2%	25%
- $\rho(rer^N, rer^D)$	0.52	0.48	0.15	0.40
Comparison of rer^T and rer^N				
- $\sigma(rer^T)$	5.20	10.0%	11.0%	11.0%
- $\sigma(rer^N)/\sigma(rer^T)$	56%	16.7%	15.4%	26%
- $\sigma(rer^T, rer^N)$	0.04	0.34	0.00	0.17

σ - refers to standard deviation of a logged and HP filtered variable (with annual smoothing parameter 100), and ρ refers to correlation of similarly treated series. Data sources as described in text.

rer^T.

As we can see in the figure, the residual of the decomposition is relatively unimportant. The right-hand panels of the figure also reveal that the volatility of rer^N is very small relative to overall deflator-based real exchange rate rer^D, perhaps with the exception of the intensive in trade relation with Canada. In other cases, rer^N barely moves, but when it does, it tends to be rather positively correlated with rer^D. As we will later show, these features are also true in our full sample, and the standard model fails to replicate the positive correlation between the two series as well as the modest relative volatility of rer^N. Table 1 summarizes the exact statistics calculated for these four country pairs.

Our findings for US pairs on the relative volatility of the nontradable real exchange rate are slightly higher than the numbers reported by Betts & Kehoe (2006) based on gross output deflators – confirming that using value added deflators produces slightly higher values of the volatility of the nontradable real exchange rate. Nevertheless, the overall contribution of rer^N remains small.

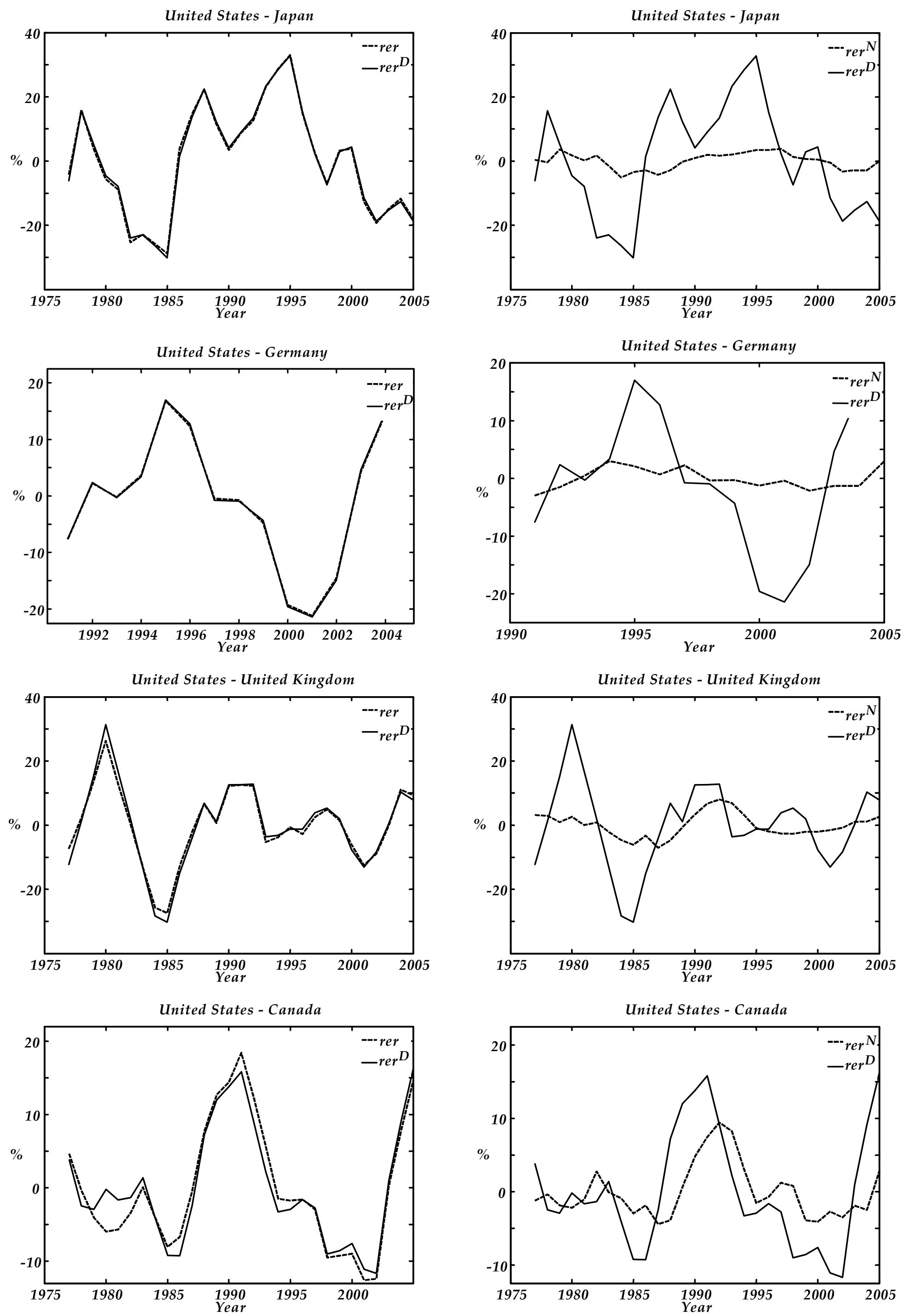

Figure 1: Bilateral real exchange rates in selected US pairs (deviation from linear trend).

2.3.2 All Country Pairs

We now proceed to the results of our decomposition in light of the the full sample of 210 country pairs. To summarize the properties of the data, we report the median values and one or two percentiles to give a feel how spread out are the observations. Are main conclusions are:

1. The residual of the decomposition plays very minor role, and rer^D and rer are almost the same object in the data.

2. rer^N and rer^T are only moderately negatively correlated in the data, and rer^N carries about 1/3 volatility of rer^T.

3. Majority of the volatiltiy present in rer^T carries through to rer^D, and by 1 to rer, so that rer^N carries moderate volatility of about 1/3 also relative to these overall indices.

The first panel of Table 2 describes the relation of the CPI and deflator-based real exchange rates. As we can see from the summary statistics included in this table, both the CPI- and deflator-based bilateral real exchange rates are highly volatile — consistent with observation of highly volatile real effective exchange rates. Importantly, the two series exhibit almost identical behavior — just like in the US case with countries other than Canada. The median correlation between rer and rer^D in the full sample is 0.98, with 90% of the observations falling in the interval above 0.86. In terms of volatility, the deflator-based real exchange rate is slightly more volatile than the CPI-based index, with the median ratio of the volatility of deflator-based index to the CPI-based index of 106%, with 90% of observations lying below 126%. On this basis, we conclude that the residual of the decomposition does not play any significant role in the data.

Let us now turn to the analysis of the second equation from the decomposition given by (9). The second panel of Table 2 presents summary descriptive statistics of our constructed series. As we can see from this panel, the high volatility of the bilateral real exchange rate is driven predominantly by the tradable real exchange rate, with a the nontradable real exchange rate contributing only a small part. The real exchange rate for nontradable goods rer^N carries only 38.2% percent of the volatility of the deflator-based real exchange rate as measured by the ratio $\sigma(rer^N)/\sigma(rer^D)$, and exhibits only slight positive median correlation with the overall index rer^D (0.09). The third panel of the same table reveals the source of this small contribution of rer^N to the overall index rer^D. First, rer^T is much more volatile than rer^N – in fact it is more volatile than the overall index rer^D. Second, rer^T and rer^N are weakly correlated, with the median correlation of -0.26, and a very wide range of numbers across pairs (this property of the data will be imported in the subsequent analysis). This means that these two components rarely offset each other in the data, and hence both of their volatilities contribute to the volatility of the overall index.

Table 2: Prices in All Country Pairs

	Median	10^{th} percentile	90^{th} percentile
Comparison of rer and rer^D			
- $\sigma(rer)$	5.95%	2.23%	11.2%
- $\sigma(rer^D)/\sigma(rer)$	106%	96%	126%
- $\rho(rer, rer^D)$	0.98	0.86	0.99
Comparison of rer^D and rer^N			
- $\sigma(rer^D)$	6.55%	2.9%	11.4%
- $\sigma(rer^N)/\sigma(rer^D)$	38.2%	18.7%	80.1%
- $\rho(rer^N, rer^D)$	0.09	-0.38	0.47
Comparison of rer^T and rer^N			
- $\sigma(rer^T)$	7.0%	3.5%	11.3%
- $\sigma(rer^N)/\sigma(rer^T)$	38.1%	18.5%	65%
- $\rho(rer^T, rer^N)$	-0.26	-0.72	0.16

Notes under Table 1 apply.

Since European countries look very differently in terms of trade statistics and policies, in Tables 3 and 4 we analyze the decomposition by splitting the sample to non-European country pairs and European pairs. We find that this split does matter for the key statistics. In the non-European case the relative volatility of the nontradable real exchange rate is lower, bringing the relative volatility $\sigma(rer^N)/\sigma(rer_D)$ down to 25%. This is not the case in the European case, which behaves similarly to the trade relation between US and Canada illustrated in Figure 1. We conjecture that these differences are attributed to the fact that European countries pegged their nominal currencies and trade very intensively with each other.

Superficially, one could be even tempted to think that standard model could be more successful in accounting for this part of the sample. However, since these countries trade very intensively —

and rer^T is not that volatile in this case — the model does only slightly better when required to match these statistics. We thus do not discuss this split any further.

Table 3: Prices in Non-European Country Pairs Only

	Median	10^{th} percentile	90^{th} percentile
Comparison of rer and rer^D			
- $\sigma(rer)$	10.1%	6.53%	11.35%
- $\sigma(rer^D)/\sigma(rer)$	102%	98%	110%
- $\rho(rer, rer^D)$	0.98	0.97	0.99
Comparison of rer^D and rer^N			
- $\sigma(rer^D)$	10.4%	6.46%	11.7%
- $\sigma(rer^N)/\sigma(rer^D)$	25%	17.9%	46.6%
- $\rho(rer^N, rer^D)$	0.27	-0.27	0.53
Comparison of rer^T and rer^N			
- $\sigma(rer^T)$	10.0%	6.6%	11.5%
- $\sigma(rer^N)/\sigma(rer^T)$	26%	17.4%	47%
- $\rho(rer^T, rer^N)$	0.02	-0.52	0.26

Notes under Table 1 apply.

2.3.3 Dependence on Trade

To complete our analysis, here we report to what extent there are systematic differences in our statistics depending on trade intensity between country pairs. For this purpose, we define trade intensity of any country pair reported by country i as the ratio of bilateral imports I_{ji} from country j to country i to total imports I_i of country i,

$$BTI_{ij} = \frac{I_{ji}}{I_i},$$

and split the sample into two groups: (i) high trade intensity pairs (top 25% of country pairs), and (ii) low trade intensity pairs (bottom 25%). If such dependence is clearly present in the data, it should be controlled for in the theoretical exercise with the model.

As documented in Table 5, our key patterns turns out quite robust to splitting the sample into a low and a high bilateral trade intensity countries — which led us to use only the median

Table 4: Prices in European Pairs Only

	Median	10^{th} percentile	90^{th} percentile
Comparison of rer and rer^D			
- $\sigma(rer)$	4.43%	2.1%	7.2%
- $\sigma(rer^D)/\sigma(rer)$	109%	97%	143%
- $\rho(rer, rer^D)$	0.96	0.75	0.99
Comparison of rer^D and rer^N			
- $\sigma(rer^D)$	4.87%	2.2%	7.7%
- $\sigma(rer^N)/\sigma(rer^D)$	50.4%	28.8%	109%
- $\rho(rer^N, rer^D)$	0.08	-0.42	0.47
Comparison of rer^T and rer^N			
- $\sigma(rer^T)$	5.3%	2.8%	8.1%
- $\sigma(rer^N)/\sigma(rer^T)$	46.7%	29%	70%
- $\rho(rer^T, rer^N)$	-0.42	-0.86	0.00

Notes under Table 1 apply.

values for trade in our quantitative exercise with the model. As we can see from these tables, although the numbers differ slightly across these subsamples, the overall pattern of low volatility and correlation of rer^N relative to rer^D remains. The most significant difference is the correlation of the nontradable real exchange rate with the deflator-based real exchange rate, which goes from 0.03 for low bilateral trade pairs to 0.18 for high bilateral trade pairs[8].

3 The Model

In this section, we formally set up the standard model of international business cycle under complete markets. To highlight the links between prices and quantities, we set up the model as a decentralized equilibrium. The model is closely related to the setup in Stockman & Tesar (1995), with a few modifications. Our model nests an additional distribution sector that will later allow us to flexibly accommodate two alternative calibrations of the model. In addition, we should mention here that

[8]These results do not contradict the ones in Betts & Kehoe (2008), These authors use different measures of tradable prices, the Producer Price Index. Our analysis of CPI and PPI based indices confirms their results.

Table 5: Prices for Low- and High Trade Bilateral Intensity Pairs

	Top 25% of bilateral trade	Bottom 25% of bilateral trade
Comparison of rer and rer^D		
- $\sigma(rer)$	6.98%	6.5%
- $\sigma(rer^D)/\sigma(rer)$	104%	104%
- $\rho(rer, rer^D)$	0.98	0.97
Comparison of rer^T and rer^N		
- $\sigma(rer^T)$	7.0%	7.8%
- $\sigma(rer^N)/\sigma(rer^T)$	32.5%	38.5%
- $\rho(rer^T, rer^N)$	-0.18	-0.29
Comparison of rer^D and rer^N		
- $\sigma(rer^D)$	6.98%	7.15%
- $\sigma(rer^N)/\sigma(rer^D)$	35%	41%
- $\rho(rer^N, rer^D)$	0.18	0.03

Notes from Table 1 apply.

below we use a three country setup as opposed to a more standard two country setup.[9]

3.1 Physical Environment

The world economy comprises of three countries: two small countries called home (H) and foreign (F) and one large country, called the rest of the world (G). Time is discrete and horizon infinite, $t = 0, 1, 2....$

Each country produces a local non-tradable good and a country specific tradable good. The tradable good produced in the home country is labeled H (home good), the tradable good produced by the foreign country is labeled F (foreign good), and the tradable good produced by the rest of the world world is labeled G (global good). The nontradable good in each country is labeled N.

There are three sectors in the economy: a production sector for tradable goods, a production sector for non-tradable goods and a distribution sector. Producers in the tradable and nontradable

[9]This choice is dictated by two considerations, important given the nature of our exercise. First, a three country setup allows to better control shock spillovers between countries and the rest of the world, as it allows for a natural structure of the productivity process. Second, it also disconnects bilateral trade intensity from trade openness that are the same thing in two country models, and thus presumably disciplines better the endogenous demand spillovers across countries depending on how open they are versus how much they trade with each other.

sectors employ labor supplied by households and produce a country specific tradable good or the non-tradable good, depending on the sector. Agents in the distribution sector, the distributors, pay a nontradable distribution cost to deliver tradable goods from producers in each country to households in their local country. Households consume tradables and nontradables, supply labor and trade a complete set of state contingent assets. All markets are perfectly competitive.

Notation

Variables in the model are subscripted and superscripted. Our convention is that the subscript denotes the country in which a given activity takes place, whereas, depending on the context, the superscript denotes the country/sector of origin or the type of good involved. For example, the price of the home good in the foreign country is denoted as P_f^H.

All variables in the model are history dependent, where the history of shocks up to and including period t is denoted by $s^t = (s_0, s_1, s_2, ..., s_t)$. The seed value s_0, as well as the time invariant product probability measure over the space of all possible histories are assumed given, and denoted by $\mu(\cdot)$ and S^t.

Output and productivity

In each country i, local producers have access to a linear production function that uses labor as the only input and is subject to country- and sector-specific productivity shock z_i^j

$$y_i^j(l) = z_i^j l,\ \ j = T, N.$$

Productivity z_i^j is stochastic and is assumed to follow a joint $AR(1)$ process,

$$\mathbf{z}(s^t) = A\mathbf{z}(s^{t-1}) + \boldsymbol{\varepsilon}, \tag{10}$$

where

$$\mathbf{z} = \left[\begin{array}{cccccc} z_H^T & z_H^N & z_F^T & z_F^N & z_G^T & z_G^N \end{array}\right],$$

and $\boldsymbol{\varepsilon}$ – identified with the primitive event s_t – is assumed to be an i.i.d. random variable with zero mean and a finite variance-covariance matrix Σ. Stochastic productivity is the only source of uncertainty in this economy.

In what follows next, we exploit the assumption of constant returns to scale and summarize all production constraints by the marginal production cost, which are given by

$$v_i^T(s^t) = \frac{w_i(s^t)}{z_i^T(s^t)} \tag{11}$$

for the tradable sector, and by

$$v_i^N(s^t) = \frac{w_i(s^t)}{z_i^N(s^t)} \tag{12}$$

for the non-tradable and distribution sectors.

Household's problem

In each country i, there is a measure n_i of households, each endowed with one unit of labor. The population size is assumed equal between home and foreign country, and larger in the rest of the world ($n_h = n_f < n_g$).

Households supply labor inelastically, purchase tradable and non-tradable goods in the local markets, and trade a complete set of financial assets in an integrated world asset market. Their objective is to maximize the expected discounted stream of flow utility from the composite consumption c_i,

$$\sum_{t=0}^{\infty} \beta^t \int_{S^t} u(c_i(s^t)) d\mu(s^t),$$

where c_i is determined by consumption of tradable and non-tradable goods through CES aggregators $c(\cdot)$ and $c^T(\cdot)$:

$$c_i = c(c^T(c_i^H, c_i^F, c_i^G), c_i^N). \tag{13}$$

In their choice, the households are constrained by a sequence of budget constraints given by

$$\sum_{j=H,F,G,N} P_i^j c_i^j + \int Q(s_{t+1}, s^t) b_i(s_{t+1}, s^t) d\mu(s_{t+1}) = b_i(s^t) + w_i(s^t) n_i + \Pi_i(s^t).$$

The expenditures are comprised of consumption expenditures on all four commodities,

$$P_i^H c_i^H + P_i^F c_i^F + P_i^G c_i^G + P_i^N c_i^N$$

and purchases of a set of one-period forward state contingent bonds $b_i(s_{t+1}, s^t)$,

$$\int Q_i(s_{t+1}, s^t) b_i(s_{t+1}, s^t) d\mu(s_{t+1}),$$

priced by kernel $Q_i(s_{t+1}, s^t)$, and promising a payoff of $b(s_{t+1}, s^t)$ units of the consumption in case contingency s_{t+1} arises next period.

Household's income is derived from the payoff of previously purchased bonds $b_i(s^t)$, labor income $w_i(s^t)n_i$, and dividends paid out by home producers. To avoid Ponzi schemes, bond holdings of the household are assumed to be bounded from below.

The numeraire in each country is assumed to be the s^t-composite consumption c_i. By uncovered interest rate parity condition, we can recover the evolution of the relative price of the composite consumption in country j in the units of country i from the pricing kernels:

$$\frac{x_i^j(s^{t+1})}{x_i^j(s^t)} = \frac{Q_j(s_{t+1}, s^t)}{Q_i(s_{t+1}, s^t)}. \tag{14}$$

By definition, the above price is the ideal consumption-based real exchange rate, and the condition states that at any history node s^t, the difference between country i and country j s_{t+1}-contingent consumption-based rate of interest must be equal to the rate of depreciation/appreciation of the real exchange rate between these two states.

This formula can actually be restated in a much simpler form by exploiting the first order condition

$$Q_i(s^t) = \beta \frac{u'(c_i(s_{t+1}))}{u'(c_i(s_t))},$$

and iterating backwards on equation (14). It then converges to the familiar efficient risk sharing condition

$$x_i^j(s^t) = x_i^j(0) \frac{u'(c_j(s^t))}{u'(c_i(s^t))}, \tag{15}$$

which says that the households perfectly share consumption risk in the sense of equalizing cross-country marginal rate of substitution from consumption with the relative price of this consumption. The constant $x_i^j(0)$ in the above expression is there to guarantee that in expected present discounted value terms there will be no net flows of wealth between countries – as implied by the budget constraints.

Producers and Distributors

Both producers an distributors operate in a perfectly competitive market. Producers of tradable goods sell their goods to distributors, who incur the distribution cost and resell the goods to households. Nontradable goods have no distribution cost.

Producers sell their respective goods in a perfectly competitive market and face a marginal cost of production $v_i^j\left(s^t\right)$. Profit in state s^t of a producer of good k in country i is

$$\pi\left(s^t\right) = y_i^k\left(p_i^k - v_i^k\right),$$

subject to

$$y_i^k \leq z_i^k L_i^k.$$

The zero profit conditions imply that producer prices are going to be equal to the marginal cost of production, i.e.

$$\begin{aligned} p_i^N &= v_i^N \\ p_i^k &= v_i^k, \;\; k = H, F, G. \end{aligned}$$

Distributors purchase tradable goods from producers and resell them in the local, perfectly competitive market. The distribution cost, denoted ξ, is paid in the local nontradable good. Zero profit condition for distributors ensures that the consumer prices are

$$P_i^j(s^t) = p_i^j(s^t) + \xi v_i^N(s^t), \;\; j = H, F, G$$

Market Clearing and Feasibility

Equilibrium requires several market clearing and feasibility conditions to be satisfied. Consumption of tradables by all countries has to be equal to production

$$y_i^T = z_i^T L_i^T \tag{16}$$

$$\sum_{i=h,f,g} c_i^j = y_j^T. \tag{17}$$

Consumption of nontradables in each country and expenditures on distribution must be equal to the production of nontradables

$$c_i^N + \xi \left(c_i^H + c_i^F + c_i^G\right) = z_i^N L_i^N.$$

Labor markets must clear

$$L_i^N + L_i^T = n_i.$$

The definition of equilibrium is straightforward and will be omitted.

4 Dynamics of Prices and Quantities in the Model

This section studies the response of key prices to a sectoral productivity shock. To facilitate the exposition, we consider below here a simplified auxiliary partial equilibrium setup that captures all essential elements of the theory, and explains how these components together qualitatively generate the observed responses in the key variables of the model. We will alternate between this setup to highlight key forces, and comment on the full setup.

Below, we first lay out our auxiliary setup. Next, using this setup, we explain the dynamics of rer^T in response to shocks. We then return to the full model to explain movements in the nontradable real exchange rate rer^N. Since our theory provides a very clear connection of this price to primitives, we will not need to resort to the auxiliary setup in this case at all. Finally, we link our analysis of tradable and nontradable real exchange rate back to the decomposition of the real exchange rate in the full model and to the failure of the full model to account for the modest contribution of the relative price of non-tradable goods to the overall real exchange rate fluctuations.

Auxiliary framework for model analysis

This restricted setup of the model focuses solely on the problem of the domestic producer of tradable goods. The key simplifying assumption is that the producer faces an exogenous demand function for its good at home an abroad. We also restrict our attention to a two country framework, and use an asterisk to distinguish home and foreign variables throughout.

The producer of the home tradable good is assumed to face a simple isoelastic demand function at home

$$p_H = \omega Q^{-\gamma},$$

and abroad

$$p_H^* = (1-\omega) Q^{*-\gamma},$$

where $\omega > 1/2$, and Q is the total quantity supplied to the market. These demand functions weigh quantities differently — depending on the value of $\omega > 1/2$ — reflecting the fact that trade intensity between the two countries is below 50%. In the full model, these demand functions are generated by the CES aggregator $c^T(\cdot)$, and in principle can shift due to the behavior of foreign producers. We will ignore this dependence, as it does not play any essential role in the full model. (Production cost is assumed exogenous.)

Since our producer sells in two different markets, it is essential how the price abroad compares with price at home. To replicate the key aspects of the full environment here, we will assume that frictionless arbitrage guarantees the law of one price:

$$p_H = x p_H^*.$$

In addition, to link the supply-side response of the producers to real exchange rates, we add a simplified auxiliary structure in the spirit of the perfect risk sharing condition (14). More precisely, we assume that the initial value of the real exchange rate x is 1, and its subsequent change (appreciation/depreciation) is proportional to the ratio of quantities of home good sold in each market:

$$\Delta x \ \sim \ \Delta(\frac{Q}{Q^*}). \tag{18}$$

Recall from the discussion in the setup, that the condition (18) says in this context that whenever the quantity supplied by the home producers Q at home is larger than the quantity supplied by them to the foreign market Q^*, the real exchange rate depreciates. In the full model, this link is through consumption.

Using the above framework, in what follows, we study the forces behind rer^T and rer^N movements in the model, and in particular, their response to a positive productivity shock in each sector.

Dynamic response of rer^T to sectoral shocks

Tradable sector productivity shock The response of the key objects of the auxiliary model to a positive productivity shock in the home tradable sector is illustrated in Figure 2. In this figure, the left-hand side panel illustrates the market for the home good in the home country, and the right-hand side panel illustrates the market for the home good in the foreign country. The foreign demand is plotted steeper to reflect the fact that there is home bias ($\omega > 1/2$). This property plays actually a critical role in generating CPI-real exchange rate x movements in the model. Point A in the figure is the initial point, and point B is the final point after all adjustments take place.

The key driving force of the responses illustrated in Figure 2 is the fall in production cost faced by the home producers in the home tradable sector as a result of a positive productivity shock, and their subsequent attempt to expand supply both at home and abroad. We note from the figure that this behavior — due to differences in the slope of the demand lines — results in an asymmetric fall in of the price of home good in the home market, and a simultaneous real exchange rate depreciation.

The mechanics behind these responses is as follows. When the home producers attempt to expand quantity sold in both markets, they face a steeper demand abroad than at home – due to home bias. Thus, due to arbitrage considerations, more quantity is directed to the home market than to the foreign market. This, however, leads to an increase in the overall consumption at home

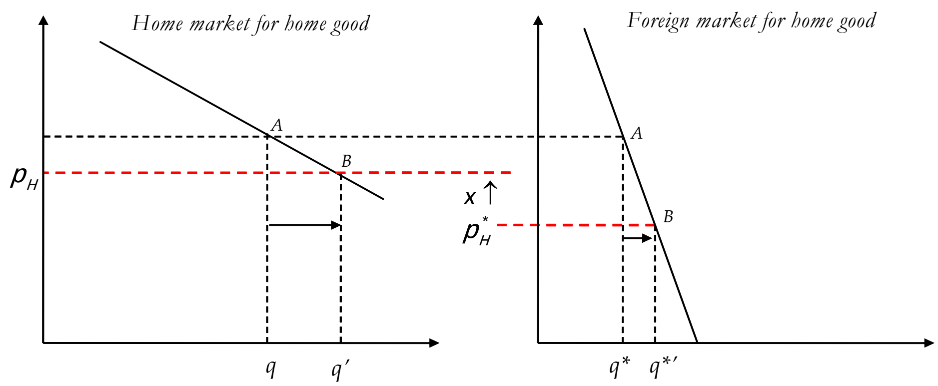

Figure 2: Response of prices to positive shock to home country tradable sector.

relative to the consumption abroad, and by risk sharing condition (18 or 15), to a simultaneous depreciation of the real exchange rate. At the same time — assuming an exogenous and fixed price of the foreign price of the foreign tradable good p_F^* in the auxiliary setup — the price of the foreign good at home xp_F^* goes up. Consequently, given the approximate[10] formula for rer^T,

$$rer^T = \frac{xp_F^*}{p_H},$$

an even more forceful depreciation of this object (relative to x) is observed in the model.

In Figure 3, we plot impulse response functions for our parameterized model (standard parameterization) to a one time positive productivity shock in the tradable sector. As we can see, abstracting all quantitative considerations that are the subject of the next section, the response of the full model is consistent with the description given above. Following the shock, real exchange rate x depreciates, and home price of home good falls. Abroad, not much is happening. As a result, rer^T goes up, but rer^N falls — as home price of home goods p_H enters the denominator.

[10]In the model, we take great care to measure data analogously to way it is measured in the model. Therefore, this is only an approximate formula, and in our quantitative exercise rer^T is defined using deflator prices of sectoral output. These prices, however, turn out almost identical to the actual prices.

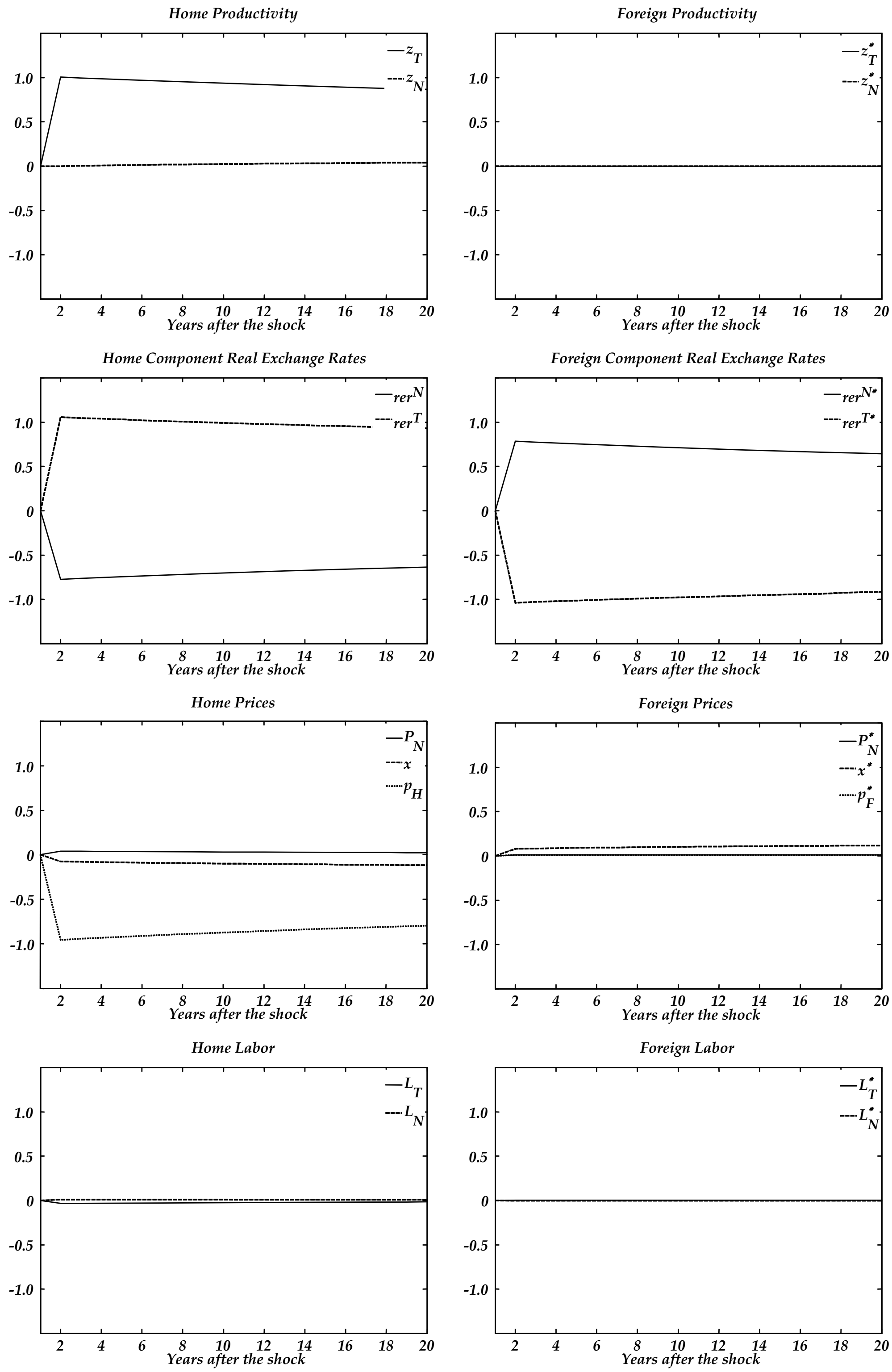

Figure 3: Response to 1% productivity shock in the tradable sector.

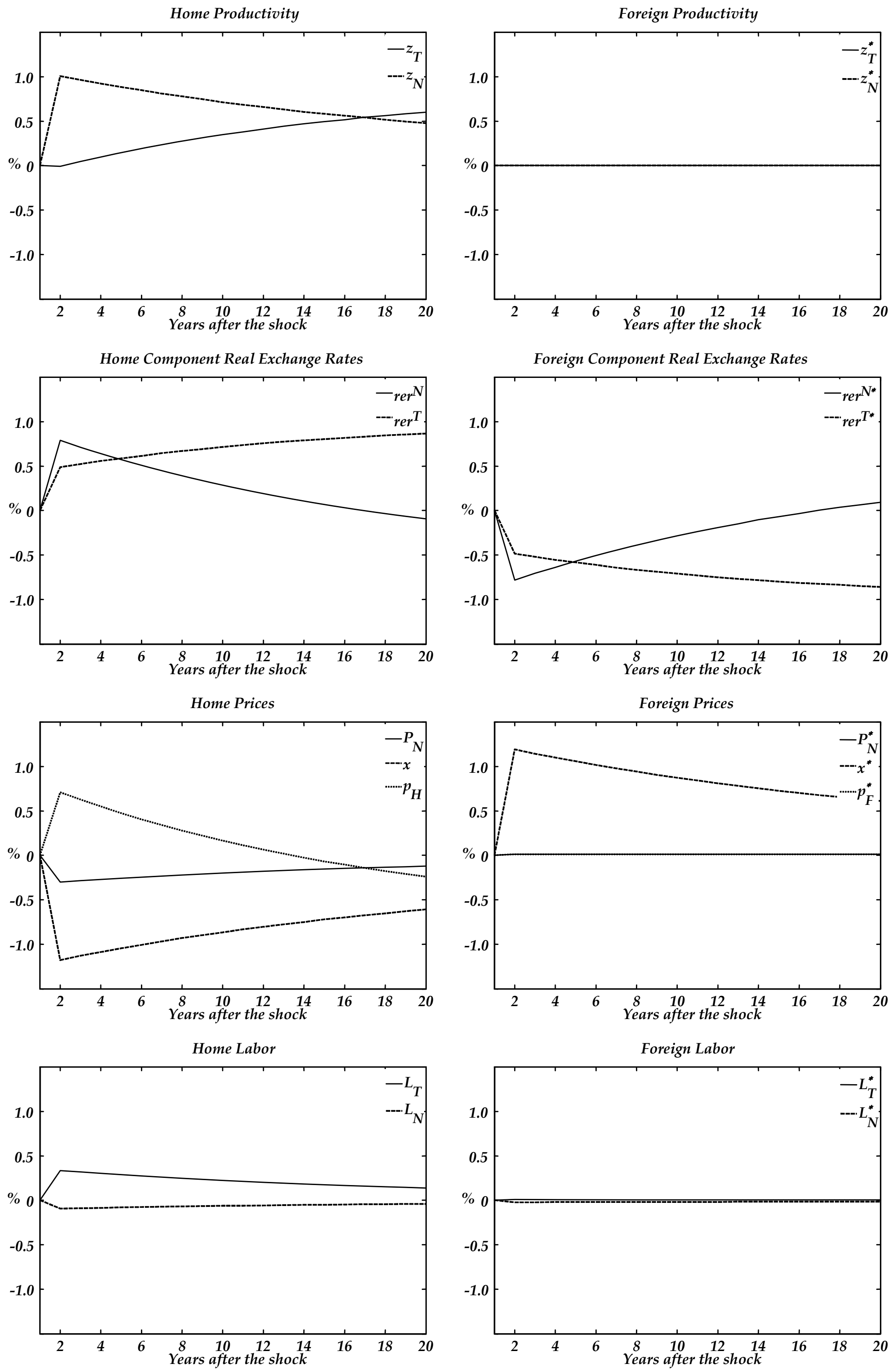

Figure 4: Response to 1% positive productivity shock in the nontradable sector.

Nontradable sector productivity shock The increase in the productivity of the non-tradable sector at home results in a fall of the price of non-tradable output relative to the local tradable output. As a result, rer^N depreciates (goes up) — for obvious reasons. However, due to risk sharing and increase in home consumption, by (14), the CPI-based real exchange rate x depreciates. The real exchange rate depreciation, in turn, results in a simultaneous reallocation of labor from non-tradable sector to the now more profitable home tradable sector (foreign price level increased). The subsequent behavior is then similar to the situation illustrated in Figure 2, leading to a simultaneous depreciation of rer^T. Note that here rer^N and rer^T actually positively comove.

The dynamic response of the full model is illustrated in Figure 4. By comparing this figure to Figure 3, we should note that the non-tradable shock is the main driver of rer^D and rer due to the offsetting effects rer^N and rer^T have in case of tradable shocks — in the case of non-tradable sector productivity shock they are actually positively correlated. However, because in the quantitative model non-tradable shock will play only a minor role due to its low volatility, the response to tradable shock will play bigger role in generating the fluctuations of rer and rer^D — thus resulting in their counterfactually low volatility due to offsetting effects rer^T and rer^N have on each other.

Dynamic response of rer^N to sectoral shocks

Let us now return to the full model from Section 3 to focus on the real exchange rate for nontradable goods rer^N. Due to perfect labor mobility between the two sectors, in the full model relative wages across sectors of the economy are equalized, and the response of rer^N is trivially hardwired to the international relative productivity ratio. To see this implication of the model, note that the prices of tradable and non-tradable output at home and abroad are given by:

$$\begin{aligned} p_H &= w z_H^T, \\ p_F^* &= w^* z_F^{T*}, \\ P_N &= w z_H^N, \\ P_N^* &= w^* z_H^{N*}, \end{aligned}$$

and therefore

$$rer^N = \left(\frac{P_N^*}{p_F^{*T}}\right)^{\zeta} \Big/ \left(\frac{P_N}{p_H^T}\right)^{\zeta} = \left(\frac{z_N^*}{z_T^*}\right)^{\zeta} \Big/ \left(\frac{z_N}{z_T}\right)^{\zeta}. \tag{19}$$

As a result, the dynamics of rer^N is directly pinned down by the evolution of these exogenous shocks, and the share of non-tradable goods in the overall output ζ.

We should stress, however, that perfect labor mobility is actually the most favorable assumption for the standard model to account for the data. Any frictions that obstruct the flow of labor would actually make rer^N more volatile, as it would then be additionally driven by the international differences in the relative wage between the two sectors:

$$rer^N = \left(\frac{w_N^* z_F^{N*}}{w_T^* z_F^{T*}}\right)^{\zeta} \Big/ \left(\frac{w_N z_N}{w_T z_H^T}\right)^{\zeta} = \left[\left(\frac{w_N^*}{w_T^*}\right)^{\zeta} \Big/ \left(\frac{w_N}{w_T}\right)^{\zeta}\right] \times \left[\left(\frac{z_N^*}{z_T^*}\right)^{\zeta} \Big/ \left(\frac{z_N}{z_T}\right)^{\zeta}\right].$$

Clearly, with such frictions in place, higher productivity of in one of the sectors would result in also higher wages in this sector, making the two terms in the above formula positively correlated, and consequently raising the relative volatility of rer^N even further.

Implications for component real exchange rates

In the quantitative section, we argue that the standard model fails to account for the modest relative volatility of rer^N to rer^D, and the low correlation between rer^N and rer^T, which, to a first approximation[11] can be identified here with the following objects in the model:

$$\begin{aligned} rer^T &= \frac{xp_F^*}{p_H}, \\ rer^N &= \left(\frac{P_N^*}{p_F^*}\right)^{\zeta} \Big/ \left(\frac{P_N}{p_H}\right)^{\zeta}. \end{aligned}$$

The key reason why this happens is the described above theoretical link between rer^N and the international deviations in the relative productivity between the two sectors. As a result, even if the model was required to match the volatility of $rer^T = xp_F^*/p_H$ from the data (about 7%), given that

[11]In the quantitative section, we construct exactly analogous objects to the ones in the data. The objects used here are very good approximations of the actual deflator based objects we construct.

it needs to replicate the volatility of z^T/z^N , as well as its comovement with the foreign ratio, by (19), it has little endogenous flexibility left in matching the relative volatility of rer^N and rer^T. So, even in the best case scenario when the model replicates volatility of rer^T, by the sheer properties of the productivity process in the data, this statistic already off by 30% in our parameterization.

To top it off, rer^N and rer^T are very highly negatively correlated. This is because the tradable sector turns out to be the main driver of volatility in the economy, and following a shock in the tradable sector rer^T and rer^N move in the opposite direction — as illustrated in impulse response figure 3. The exact value of the correlation between these objects in our parameterized model is -0.8. Hence, given the volatility of rer^T that matches the value from the data, once the already too volatile rer^N works through through the decomposition $rer^D = rer^T \times rer^N$, it renders rer^D less volatile than rer^T, overshooting quite grossly its contribution measured by $\sigma(rer^N)/\sigma(rer^D)$.

5 Quantitative Analysis

5.1 Parameterization

This section describes the choice of functional forms and the associated with them parameters. We propose three conceptually different parameterizations, labeled *Standard* calibration, *Preferred* calibration and *High Risk Aversion* calibration. In the standard calibration, there is no distribution sector and $\xi = 0$. In the preferred calibration, distribution sector is present and accounts for about 50% of the price of each final tradable goods — as suggested by the estimates from Burstein, Neves & Rebelo (2000).

Functional Forms

The utility function is assumed to be CRRA with the intertemporal elasticity parameter denoted by σ :

$$u(c_i) = \frac{c^{1-\sigma}}{1-\sigma}$$

The aggregator between tradable c_i^T and non-tradable consumption c_i^N is assumed to be Cobb-

Douglas, with the share on non-tradables parameterized by ζ:

$$c(c_i^T, c_i^N) = (c_i^N)^{\zeta}(c_i^T)^{1-\zeta}.$$

The build in assumption of a unit elasticity between the two consumption components in the above aggregator are on the high side of the elasticity numbers used in the literature. For example, Stockman & Tesar (1995) use 0.44, and Corsetti, Dedola & Leduc (2008) use 0.75. A lower value of the elasticity parameter acts similarly as an increase in σ, but does not have a big effect quantitatively. We will consider it in the sensitivity analysis.

Tradable consumption is assumed to be a composite of home good consumption c_i^H, foreign good consumption c_i^F, and global good consumption c_i^G. It is modeled by a CES aggregator:

$$c_i^T(c_i^H, c_i^F, c_i^G) = (\sum_{j=H,F,G} \omega_i^j (c_i^j)^{\frac{\gamma-1}{\gamma}})^{\frac{\gamma}{\gamma-1}},$$

where $\sum_j \omega_i^j = 1$, and $\gamma > 0$ is the elasticity of substitution. The remaining functional forms are stated in the setup of the model.

Parameter Values

The model is calibrated to match a set of selected moments from the data for all bilateral pairs of countries discussed in the data section. In this section, we discuss different parameterizations of the model, labeled: (i) standard, (ii) preferred, and (iii) high σ. The non-standard values of the parameters across these parameterization are the parameters governing the process A and Σ — estimated to account for productivity fluctuations in a three country-two sector system — and in the last calibration (high sigma) the value of σ — chosen to match the volatility of rer^T. The values of all the parameters are listed in Table 7. Below, we provide a detailed description how we have chosen these values, and which moments from the data we used as our calibration targets.

Population n_i of the relative rest of the world (country G) has been set so that country G is 20 times bigger than H or F. The value of the intertemporal discount β is 0.96, and in the stationary equilibrium it implies a real risk free interest rate of 4%. Factoring in an expected world growth of about $2-3\%$, it implies a real interest rate of about $6-7\%$.

The value of risk aversion parameter σ has been set to the standard value in the business cycle literature in the first two baseline calibrations (standard and preferred), and is equal to 2. In the *High Risk Aversion* calibration, we increase this value to 9 so that the model matches exactly the volatility of the tradable value added deflator-based real exchange rate rer^T of 7%. (This is the most favorable parameterization of the model that we have found.)

In the *Preferred* calibration of the model, the share of consumption of non-tradable goods in the final consumption ζ and the distribution cost ξ have been selected to account for the median 78% share of non-tradable sectors in total output of countries from our sample in year 2000,[12] and the estimate of a 50% share of non-tradable inputs in the price of final goods on the consumer level as estimated by Burstein, Neves & Rebelo (2000). To account for these two numbers, we have chosen the value of ξ so that 50% non-tradable content is added to every tradable good reaching the consumer, and then set the value of ζ to account for the overall 78% share of services in the total output. In the *Standard* calibration $\xi = 0$, and ζ has been selected to account for 78% share of services in total output.

The elasticity of substitution γ has been selected to match the so called short-run elasticity of trade flows — a measure how trade flows between countries respond to a relative price changes seen in the time-series. However, here, instead of relying on micro-level estimates of such elasticity typically used in the literature, we use our own methodology based on the aggregate data. The advantage of our methodology is a more natural mapping between the aggregate model and the data, and the avoidance of the use of correlation — which in simple regressions of this sort may create a bias due to lagged adjustment of quantities to prices (J-curve). The details of our approach closely follow Drozd & Nosal (2008), with the key idea relying on the fact that in a large class of models, the demand for domestic and foreign good is modeled by a CES aggregator of the form

$$G(d_t, f_t) = \left(\omega_t d_t^{\frac{\gamma-1}{\gamma}} + (1-\omega_t) f_t^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}.$$

When such aggregator is present the theory, the import ratio $\frac{f_t}{d_t}$ is intimately related to the relative

[12] The data cames from STAN database. To obtain this number we evaluated the ratio of value added in total services to total value added in all sectors.

price of domestic and imported goods $\frac{p_{d,t}}{p_{f,t}}$:

$$\log\frac{f_t}{d_t} = \gamma \log\frac{p_{d,t}}{p_{f,t}} + \log\frac{\omega_t}{1-\omega_t}. \tag{20}$$

Thus, if the weights ω are not time-varying, the following object called *volatility ratio,*

$$VR \equiv \sigma(\log\frac{f_t}{d_t})/\sigma(\log\frac{p_{d,t}}{p_{f,t}}), \tag{21}$$

is intimately related to the value of elasticity γ. In the case of time-varying weights ω, the above approach give the upper bound value for the value of this parameter. To see how, note that under normal conditions, i.e. an increasing supply curve wrt price, and lack of correlation between ω_t-demand shocks and supply shocks, the correlation between $\log\frac{\omega_t}{1-\omega_t}$ and $\log\frac{p_{d,t}}{p_{f,t}}$ should be positive, and thus

$$\gamma = \sigma(\log\frac{f_t}{d_t})/\sigma(\log\frac{p_{d,t}}{p_{f,t}} + \frac{1}{\gamma}\log\frac{\omega_t}{1-\omega_t}) \le \sigma(\log\frac{f_t}{d_t})/\sigma(\log\frac{p_{d,t}}{p_{f,t}}) = VR. \tag{22}$$

To implement our method, we use here annual data for manufacturing value added volume index to measure d, annual data on imports in constant prices to measure f, and their respective deflators to measure their corresponding relative price.[13] We then set γ so that the model matches the median volatility ratio obtained from the data.

The results are listed in Table[14] 6. Based on the median value from this table, in the *Standard* calibration we choose $\gamma = .93$, which is close to the implied by model value of volatility ratio, but due to the existence of 3 countries with independent shocks not exactly equal. In the *Preferred* calibration, we have obtained the value of parameter γ of about 2 – a value twice as high as the volatility ratio implied by the model. The reason behind such discrepancy between γ and the implied by the model volatility ratio in this case is the non-tradable contents of tradable goods

[13]We have corrected the nominal price of imports so that it excludes highly volatile fuels — a feature of the data that is not modeled in our theory. Using data pulled out from the World Bank Development Indicators on local currency value of total imports, total imports of merchandise products, and the share of imports of merchandise products excluding fuels, we have constructed the time series of local currency value of imports less fuels. Data range varies by country, but most series extend from 1980-2006. See online appendix for the details.

[14]Low values of the elasticity are consistent with other microlevel studies on import prices and quantities (e.g. Blonigen & Wilson (1999).). We have also verified the numbers for US using the BLS series of import prices excluding fuels. We have obtained the value 1.04, which very close to the number reported in the table.

Table 6: Volatility Ratio in Selected OECD Countries

	Detrending method	
Country	HP-100	Linear[a]
Australia	0.82	0.72
Austria	1.58	1.41
Belgium	0.71	0.66
Canada	1.02	1.02
Denmark	1.15	1.56
Finland	0.88	0.64
France	0.84	0.71
Germany	0.82	1.04
Italy	0.92	1.69
Japan	0.71	1.00
Korea	0.99	1.32
Netherlands	0.68	1.00
Norway	1.33	1.33
Spain	1.08	1.11
Sweden	1.62	1.31
Switzerland	1.48	1.59
United States	1.11	0.92
United Kingdom	0.59	1.15
MEDIAN	0.96	1.07

Results based on the annual data pulled out from the STAN database, 1970-2006. The *volatility ratio* is measured as the ratio of standard deviations of the manufacturing output to imports measured in constant prices to the standard deviation of the corresponding ratio of their price deflators. The deflator price of imports has been corrected from the influence of highly volatile fuels (described in text).
[a]Linear trend has been subtracted from logged raw series (in both cases).

which smooths retail prices of these goods on the level of the aggregator, and volatility of quantities (quantity ratio).

The weights ω_i^j on each tradable good have been chosen to account for the median bilateral trade intensity (imports from selected partner country to total imports) and the median trade openness (ratio of imports to GDP) in the selected set of bilateral pairs of countries from our sample. The set of countries excludes all European country pairs (also European with non-European pairs), to take into account the fact that our model does not make any distinction between gross and net trade flows, and is ill-suited to match trade openness of countries in which import figures are inflated by cross-border production sharing[15]. The targeted trade openness is 17.5% (versus 28% in full sample) — and the targeted bilateral trade intensity is 3.45% (versus 1.25% in the full sample). To obtain the target for trade openness of the relative rest of the world, we have calculated the median imports of the world from pair of countries from our sample (excluding European pairs) evaluated relative to the world GDP (less the GDP of the selected pair of countries), which gave 0.8%.[16] Our conservative approach to matching the trade numbers only reinforces our results, as more trade in this model deteriorates its performance.

Since our results very much depend on it, we took great care to estimate the stochastic process governing the two sectors. Parameters governing the forcing process, A and Σ, have been obtained by fitting an $AR(1)$ process to the constructed by us panel of annual sectoral productivity. The productivity series have been calculated from output and employment series available from the STAN database. As sectoral measure of output, we have used value added volumes (STAN code: VALK 1537 and 5099). These two groupings account for about 85-90% of total output in a median economy from our sample — we left out agriculture, mining, and construction. (Note that this measurement of sectoral productivity exactly aligns with the way me measure prices of the corresponding sectoral output.)

To construct labor productivity series from output and employment series, we have divided sectoral output by total hours worked (HRSN), and when not available, we have used instead total employment series (EMPN).[17] We have obtained the relative rest of the world productivity time-series by aggregating all the countries together, except for the two from a randomly selected pair. To build this aggregate, we have first normalized individual productivity series so that the number in year 2000 corresponds to the share of each sector in the total output, and then we have multiplied each country series by the corresponding PPP-based GDP in year 2000[18] to weight them properly. Finally, to render the resulting productivity series stationary, we have subtracted exponential trend growth for each country pair equal to the growth rate of the respective relative

[15]With the share of non-tradable goods unchanged, our model is not-capable to match any numbers in excess of 22% for trade openess. Trade openess in the full sample is 28%, and bilateral trade intensity is 1.25%.

[16]Data source: Directions of Trade Statistics database, IMF 2005.

[17]We have not included capital in the analysis. However, capital stock rarely affects the results in this kind of analysis.

[18]Obtained from Penn World Tables.

rest of the world (by sector).[19] Using OLS, we have obtained the following spillover matrix for the order of variables $z_H^T, z_H^N, z_F^T, z_F^N, z_G^T, z_G^N$:

$$A = \begin{bmatrix} 0.99 & 0.059 & 0 & 0 & -0.094 & 0.140 \\ 0.004 & 0.956 & 0 & 0 & 0.011 & -0.031 \\ 0 & 0 & 0.99 & 0.059 & 0.14 & -0.094 \\ 0 & 0 & 0.004 & 0.956 & -0.031 & 0.011 \\ 0 & 0 & 0 & 0 & 0.994 & 0.006 \\ 0 & 0 & 0 & 0 & 0.050 & 0.902 \end{bmatrix},$$

and derived the following variance-covariance matrix for the shocks:

$$\Sigma = \begin{bmatrix} 0.959 & 0.092 & 0.106 & 0.036 & 0.105 & 0.030 \\ 0.092 & 0.230 & 0.035 & 0.012 & 0.019 & 0.007 \\ 0.106 & 0.035 & 0.975 & 0.092 & 0.098 & 0.023 \\ 0.036 & 0.012 & 0.092 & 0.229 & 0.019 & 0.006 \\ 0.105 & 0.019 & 0.098 & 0.019 & 0.195 & 0.023 \\ 0.030 & 0.007 & 0.023 & 0.006 & 0.023 & 0.027 \end{bmatrix} \times 10^{-3}.$$

5.2 Quantitative Results

This section presents main quantitative results. We will present the price statistics generated by different parameterizations of the standard model from the perspective of the variance decomposition[20] given by

$$\sigma_{rer^D} = \sqrt{\sigma_{rer^T} + \sigma_{rer^N} + 2(\rho(rer^T, rer^N) - 1)\sigma_{rer^T}\sigma_{rer^N}}. \tag{23}$$

It is worth emphasizing at this point that the median values obtained from the data are consistent with the implications of the variance-covariance decomposition — which may not be true in general.

The above formula points to the importance of three moments in generating the volatility of the

[19]This implies that on a corresponding balance growth path of our model, all agents effectively expect to see the same growth rate, equal to the growth rate in the rest of the world. We have experimented with several other detrending methods, and the number do change quantitatively, but qualitatively all results stand.

[20]We transformed the variance decomposition equation from variances into standard deviations.

Table 7: Parameter Values

		Calibration		
Description	Symbol	Preferred	Standard[a]	High Risk Aversion[a]
Common parameters:				
Discount facor	β	0.96		
Risk aversion	σ	2		9.0
Share of N consumption	ζ	0.44	0.78	0.78
Distribution cost	ξ	1.0	0.0	0.0
Elasticity between T goods	γ	2.06	0.93	0.6
Stochastic process	A,Σ	in text		$\varepsilon_i^T \equiv 0$
Country specific parameters:				
**Home country*				
- weight on home good	ω_H^H	0.24	0.173	0.0375
- weight on foreign good	ω_H^F	0.09	0.02	0.0013
- weight on global good	ω_H^W	0.67	0.807	0.9612
- population size	n_H	5		
**Relative rest of the world*				
- weight on home good	ω_G^G	0.835	0.682	0.9192
- weight on foreign good	ω_G^H	0.083	0.159	0.0404
- weight on global good	ω_G^F	0.083	0.159	0.0404
- population size	n_G	100		

[a]Values reported only when different from the preferred calibration.

deflator-based real exchange rate rer^D : the volatilities σ (standard deviation) of rer^N and rer^T, together with their correlation ρ. As mentioned in the introduction, the parameterizations *Standard* and *Preferred* will come up short on the volatility of the tradable goods' real exchange rate rer^T, and will predict a strongly negative relation between tradable goods' and nontradable goods' indices. By (23), these two statistical properties of these object in the theory diminishes the implied by them volatility of rer^D, and thus keeps the relative volatility of rer^N high. The parameterization of the model called *High Risk Aversion* parameterization matches exactly the volatility of the tradable goods' real exchange rate. Nevertheless, it still suffers from the high negative correlation mentioned above. As a result, despite the fact that it roughly replicated the total volatility $\sigma_{rer^T} + \sigma_{rer^N}$, it suffers from the same problem as other, more conservative parameterizations.

We now proceed to discuss the details of our results. We start with presentation of the results for quantities (Table 8) — to argue that our model is reasonable on this side — and then proceed to the main results for price indices (Table 9).

5.3 Quantities

Before we proceed to the discussion of the results for prices, we need to take a brief look at what the models predict for quantities. Given its simplified supply-side structure, we would like to make sure that the models do not imply excessive reallocations of labor over the business cycle, excess volatility of output and productivity. We would like to especially make sure that the volatility of relative sectoral productivity z_H^T/z_H^N is consistent with the data, since it is critical for the pricing predictions that follow.

In the measurement of employment in the model, it is important to note that labor is in fixed supply and therefore for consistency we will compare them with the share of each sector in total employment in the case of the data. The corresponding volatility numbers for employment are lower than in the data for the *Standard* and *High Risk Aversion* parameterizations, and come quite close for the *Preferred* parameterization. This suggests to us that the elastcity between tradable and non-tradable goods is reasonable, and perhaps even on the low side.

Table 8 presents the complete results for quantities. As we can see, all the parameterizations of the model pass the preliminary test – the productivity process statistics are matched for all models. Most of the statistics are within the range of the data, and the volatility of z_H^T/z_H^N is matched very closely. We conclude that the volatilities of the key drivers — critical for the results — behind relative price movements across sectors match the primitive objects they correspond to in the data.

We now proceed to the discussion of our main results for prices.

Table 8: Quantities: Data versus Models

		Parameterization		
Statistic	Data	Standard	Preferred	High Risk Aversion
Standard deviations (in %)				
- GDP	1.76	1.37	1.21	1.36
- GDP^T	3.39	2.87	2.20	2.66
- GDP^N	1.49	1.28	1.20	1.19
- L^T/L	1.90	0.46	1.41	1.06
- L^N/L	0.55	0.13	0.23	0.30
- z_h^T	2.41	2.88	2.88	2.88
- z_h^N	1.13	1.40	1.40	1.40
- z_h^T/z_h^N	2.48	2.93	2.93	2.93
- z_{rw}^T	1.29	1.26	1.26	1.26
- z_{rw}^N	0.64	0.50	0.50	0.50
Correlations				
- GDP^T, GDP^N	0.62	0.37	0.48	0.61
- $L^T/L, L^N/L$	-0.97	-1.00	-0.99	-1.00
- z_h^T, z_h^N	0.19	0.20	0.20	0.20

Data has been logged and HP filtered with a smoothing parameter 100.

5.4 Prices

The results for prices are reported in Table 9, which follows the structure of the tables in the data section. The first panel of Table 9 shows the connection of the CPI- versus deflator-based real exchange rates. All parameterizations of the model replicate the high correlation between the two series and come close to replicating their relative volatility. Only the *High Risk Aversion* parameterization, however, comes close to replicating the level of standard deviation of these prices. The *Standard* and *Preferred* parameterizations fall short in this respect for both indexes, as well

Table 9: Prices: Data versus Models

Statistic	Data	Parameterization Standard	Preferred	High Risk Aversion
Comparison of rer_{ij} and rer_{ij}^D				
- $\sigma(rer)$	5.9%	2.3%	2.0%	4.5%
- $\sigma(rer^D)/\sigma(rer)$	106%	117%	126%	118.4%
- $\rho(rer, rer^D)$	0.98	0.95	0.96	0.98
Comparison of rer^D and rer^N				
$\sigma(rer^D)$	6.6%	2.7%	2.6%	5.3%
$\sigma(rer^N)/\sigma(rer^D)$	38.2%	133.5%	132.4%	89%
$\rho(rer^N, rer^D)$	0.09	-0.01	0.14	0.03
Comparison of rer^T and rer^N				
- $\sigma(rer^T)$	7.0%	4.5%	4.0%	7.0%
- $\sigma(rer^N)/\sigma(rer^T)$	38.1%	79.6%	86%	67.5%
- $\rho(rer^T, rer^N)$	-0.26	-0.80	-0.77	-0.65

All series refer to bilateral statistics between home and foreign country. Notes from table 1 apply.

as for rer^T. Inspection of the second panel of the table reveals that in terms of relation of rer^N to the overall index rer^D, matters look even worse.

The second panel of the table tells us about the source of this failure of the model. As we can see, the *Standard* and *Preferred* parameterizations somewhat underpredict the volatility of the tradable goods' real exchange rate rer^T (4.46% and 4% in the models versus 7% in the data), and thus since the volatility of rer^N is fully determined by the parameterized productivity process, they quite grossly overpredict the relative volatility of rer^N to rer^T. Moreover, because the models imply a strongly negative relation of rer^T and rer^N (correlation of around -0.8 versus -0.26 in the data), the volatility of the overall index rer^D is severely dampened with respect to rer^T (around 2.7% in models versus 6.5% in data). Consequently, rer^N turns out to be even more volatile relative to the overall index rer^D (around 130% in models versus 38.2% in data).

Given tha the first two parameterizations fail to account for the absolute volatility of rer^T to begin with, our next attempt is to take a less conservative approach and increase risk aversion σ to actually 'force' the model to replicate the overall volatility of rer^T (*High Risk Aversion).* It turns out that it does not help much, as the relative volatility of rer^N to rer^D is still more than twice the number in the data (89.1% in the model versus 38.2% in the data) due to their high negative correlation. Moreover, given that in this exercise we have matched the level of the volatility of rer^T, the overstanted relative volatility of rer^N to rer^T reveals that the estimated productivity process implies a way too high volatility of rer^N relative to the data. This suggests to us that in order to understand the underpinnings of the behavior of rer^N, we need a theory that disconnects the rer^N from the relative productivities.

Finally, as a side comment, we should stress here that the particular channel of generating real exchange rate movements in our complete markets economy is not really essential for the results. In fact, we obtain exactly the same results under the assumption of financial autarky. Under financial autarky, the tight link between real exchange rate and consumption is severed, and all our results still stand.

6 Conclusions

In this paper, we have studied whether standard models, when extended to include non-tradable sectors in a disciplined manner, can account for the decomposition of the real exchange rate. We found that even though these models, under some restrictive conditions on the supply side of the economy, can generate volatile and persistent real exchange rates for tradable goods, they have difficulty in accounting for the modest share of the relative price of non-tradable goods to tradable output, and thus the high relative volatility of the CPI real exchange rates. Because this is a pervasive feature of the theory across all our parameterizations, we conclude that this property of the data should be thought of as a puzzle with respect to the standard models.

What can possibly account for the this puzzle? Our conjecture, based on the fact that mechanically more cushion is needed to isolate domestic prices from the volatile international prices, theories featuring some form of the deviations from the law of one price may be more successful in accounting for the facts. Such resolution of the puzzle would be also consistent with the anecdotal evidence suggesting stability of the relative prices of home to foreign goods in the data. The list of promising theories would then include the models of pricing-to-market or sticky price models featuring local currency pricing. One example would be pricing-to-market theory by Drozd & Nosal (2008) — in which goods are endogenously differentiated by the target market rather than country of origin, in contrast to standard theory. Future research will show, to what extent plausibly parameterized extended theory can match the actual data.

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