Dynamic Monetary Economics: Paradoxes and a Non-Ricardian Resolution

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Introduction

The object of this article is to construct rigorous models that bridge the current gap between a number of monetary intuitions and facts, and recent macroeconomic modeling in the "dynamic general equilibrium" line. Currently the most popular models in this area are "Ricardian" (in a sense that will be made clear just below). These models have been successful on several points, but on the other hand produce disturbing puzzles and paradoxes on a number of important monetary issues. A central theme of this article is that moving to "non-Ricardian" models allows to solve many of these problems in one shot.

0.0.1 Ricardian versus non-Ricardian dynamic models

One of the most important developments in macroeconomics in recent years has been the replacement of traditional "ad-hoc" macroeconomic models by "dynamic stochastic general equilibrium" (DSGE) macromodels, where all decisions are taken by fully maximizing agents (consumers and firms).

Of course there are many possible types of DSGE models, as there are many types of general equilibrium models. The most popular DSGE model is a stochastic version of the famous Ramsey (1928) model: households in the economy are represented as a homogeneous family of infinitely lived individuals. We shall call such economies and models "Ricardian", because they have the famous "Ricardian equivalence" property (Barro, 1974), according to which, as long as the government fulfills its intertemporal budget constraint, the repartition of (lump sum) taxes across time is irrelevant. Another striking property is that in such models bonds do not represent real wealth for households.

On the other hand in this article by non-Ricardian we mean models where, due for example to the birth of new agents as in the overlapping generations (OLG) model of Samuelson (1958), Ricardian equivalence does not hold. In such models the precise timing of fiscal policy does matter, and government bonds, or at least a fraction of them, represent real wealth for the agents.

0.0.2 Ricardian monetary models: puzzles and paradoxes

When people started studying monetary phenomena within the DSGE framework, they quite naturally continued to use the Ricardian model, adding traditional devices (cash in advance, or money in the utility function) that allowed money to coexist with other financial assets.

Although the Ricardian model has been successful on a number of points, it turned out that the introduction of money delivered surprising and paradoxical results on a number of important monetary issues. We shall mention three examples (which are all treated in this article):

1. The standard Ricardian model predicts that, under realistic monetary processes, the nominal interest rate will go up if there is a positive shock on money. On the other hand, in traditional models and, apparently, in reality, the nominal interest rate goes down (the liquidity effect).

2. Following Sargent and Wallace (1975) it has been shown that in these models interest rate pegging leads to "nominal indeterminacy" (which means that if a sequence of prices is an equilibrium one, then any proportional price sequence is also an equilibrium one). This is quite bothering since, from a normative point of view, many optimal policy packages include the "Friedman rule" according to which the nominal interest rate should be equal to zero. This means that such policies could lead to price indeterminacies.

3. Another condition for determinacy of the price level has been developed in recent years, called the fiscal theory of the price level (FTPL). This theory says that if interest rates do not react strongly enough to inflation, price determinacy can nevertheless be achieved if the government follows a rather adventurous fiscal policy, consisting in expanding government liabilities at such a high rate that intertemporal government solvency is achieved for a unique value of the price (hence the determinacy result). This is clearly not a policy one would want to advise.

0.0.3 Non-Ricardian models: solving the paradoxes

These puzzles and paradoxes might cast some doubts on the relevance of the "DSGE" methodology for monetary economics. We want to have a more positive attitude, and we shall now argue that moving to "non-Ricardian" models will actually allow us to solve all these problems (and others) with one single modification. The modification we shall implement here consists

in considering non-Ricardian economies by assuming (realistically) that new agents are born over time, whereas there are no births in the Ricardian model.

As it turns out, moving from a Ricardian to a non-Ricardian framework changes many properties:

1. In non-Ricardian models a liquidity effect naturally appears, through which an increase in the quantity of money leads to a decrease in the interest rate.

2. In non-Ricardian models price determinacy is consistent with interest rate pegging, under the condition that the pegged interest rate leads to a high enough return on financial assets.

3. In non-Ricardian models the risky policies implicitly advocated by the fiscal theory of the price level can be replaced by much more traditional policies.

0.0.4 The Pigou effect

Of course one may wonder why the introduction of an (apparently unrelated) "demographic" assumption changes so many things in the properties of monetary models. We shall argue that one fundamental key to the differences is the so called "Pigou effect" (Pigou, 1943), which is absent from the Ricardian model, and present in the non-Ricardian one. The Pigou effect has been notably studied and developed by Patinkin (1956) under the name of "real balance effect", and was central to many macroeconomic debates in the 1950's and 1960's. Unfortunately it has been by and large forgotten since then by many theorists.

In a nutshell, there is a Pigou effect when aggregate financial wealth matters for the behavior of agents and for the dynamics of the economy. This will be the case, for example, when the aggregate consumption depends positively on this aggregate wealth. It will be seen below that the presence of this financial wealth in the dynamic equations changes many properties.

We should finally give here a brief intuition, due to Weil (1991), as to why a Pigou effect appears in non-Ricardian economies, whereas it is absent from the Ricardian ones. If one writes the intertemporal budget constraint of the government, one sees that every dollar of financial wealth is matched by an equal amount of discounted taxes. If there is a single infinitely lived family, these taxes fall hundred percent on the agents alive, so they match exactly their financial assets. As a result financial assets and taxes cancel each other, and the assets disappear from the intertemporal budget constraints and the agents' behavioral equations. Now if there are births in this economy,

the newborn agents in all future periods will pay part of these taxes, and consequently only part of the financial assets of agents currently alive will be matched by taxes. The rest will represent real wealth to them, leading to a Pigou effect.

Chapter 1

The Ricardian Issue and the Pigou Effect

1.1 Introduction

Since a central theme of this article is the importance of the Ricardian versus non-Ricardian distinction, we shall in this chapter highlight the differences between the two types of models by comparing two polar models: first we shall describe a standard Ricardian monetary model, and then a non-Ricardian monetary model based on an overlapping generations structure. We shall see that major differences appear between the two.

We shall further see that these differences are intimately connected to the presence or absence of the "Pigou effect" (Pigou, 1943, Patinkin, 1956). We shall see why this Pigou effect, which is present in the overlapping generations model, disappears completely from the Ricardian one.

1.2 The traditional Ricardian model

We shall first describe a standard monetary Ricardian model, notably associated with the names of Ramsey (1928), Sidrauski (1967) and Brock (1974, 1975)[1], where the consumer side is represented by a single dynasty of identical infinitely lived households. There is no birth, and nobody ever dies.

[1]The Sidrauski and Brock models had money in the utility function, whereas we use cash in advance. It turns out that the results for the two specifications are highly similar.

1.2.1 Households

In each period t the representative household receives an exogenous real income Y_t and consumes an amount C_t. He chooses the sequence of his consumptions so as to maximize an intertemporal utility function. Consider a particular period t. The utility of the representative household from period t onwards is:

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} Log\, C_s \tag{1.1}$$

This household is submitted in each period to a "cash in advance" constraint "à la Clower (1967)":

$$P_t C_t \leq M_t \qquad \forall t \tag{1.2}$$

The household enters period t with a financial wealth Ω_t. Things happen in two successive steps. The household first visits the bonds market where he splits this wealth between bonds B_t (which he lends at the nominal interest rate i_t) and money M_t:

$$M_t + B_t = \Omega_t \tag{1.3}$$

Then the goods market opens, and the household sells his endowment Y_t, pays taxes T_t in real terms and consumes C_t, subject to the cash constraint (2). Consequently his financial wealth at the beginning of the next period Ω_{t+1} is given by the budget constraint:

$$\Omega_{t+1} = (1 + i_t) B_t + M_t + P_t Y_t - P_t T_t - P_t C_t \tag{1.4}$$

which, using (3), can be rewritten as:

$$\Omega_{t+1} = (1 + i_t) \Omega_t - i_t M_t + P_t Y_t - P_t T_t - P_t C_t \tag{1.5}$$

1.2.2 Government

Another important part of the model is the government. The households' financial wealth Ω_t has as a counterpart an identical amount Ω_t of financial liabilities of the government. The evolution of these liabilities is described by the government's budget constraint:

$$\Omega_{t+1} = (1 + i_t) \Omega_t - i_t M_t - P_t T_t \tag{1.6}$$

The government has two types of policy instruments. Fiscal policy consists in setting taxes T_t, expressed in real terms. The second tool is monetary policy. This policy can take several forms. Two classic monetary policies consist for example in:

(a) Setting the interest rate i_t, letting the market determine M_t.

(b) Setting the quantity of money M_t, letting the market equilibrium determine i_t.

We shall actually consider both policies in what follows. All policies, fiscal and monetary, are announced at the beginning of each period.

1.2.3 The first order conditions

We shall now derive the first order conditions. We assume that i_t is strictly positive. Then the household will always want to satisfy the "cash in advance" constraint (2) exactly, so that $M_t = P_t C_t$ and the budget constraint (5) becomes:

$$\Omega_{t+1} = (1 + i_t)\,\Omega_t + P_t Y_t - P_t T_t - (1 + i_t)\,P_t C_t \tag{1.7}$$

Maximizing the utility function (1) subject to the sequence of budget constraints (7) yields the following first-order conditions:

$$\frac{1}{P_t C_t} = \beta\,(1 + i_t)\,E_t\left(\frac{1}{P_{t+1} C_{t+1}}\right) \tag{1.8}$$

1.3 Monetary puzzles and paradoxes

We shall now consider three central questions in monetary economics, and see that the Ricardian model delivers surprising answers. We shall see next in section 1.4 below that an overlapping generations model with similar features delivers the answers one would expect.

1.3.1 The liquidity puzzle

We shall begin with an issue, the liquidity effect, that dates back at least to Keynes (1936). We want to know what is the response of the nominal interest rate to a monetary expansion. The traditional answer to this question is that there is a "liquidity effect", i.e. a negative response of the nominal interest rate to monetary injections. That liquidity effect was already present in the famous IS-LM model (Hicks, 1937), and it appears to be found in the data (see, for example, Christiano, Eichenbaum and Evans, 1997).

As it turns out, this liquidity effect has been found difficult to obtain in standard monetary DSGE models. The reason is an "inflationary expectations effect" which actually tends to raise the nominal interest rate in response to a monetary injection.

Let us briefly outline the mechanism behind this inflationary expectations effect. It is found in the data that money increases are positively correlated in time. So when an unexpected money increase occurs, this creates the expectation of further money increases in the future, which will itself create the expectation of future inflation. Now from Fisher's equation the nominal interest rate is the sum of expected inflation and the real interest rate, so that, ceteris paribus, this will tend to raise the nominal interest rate.

Let us now make things more formal, and consider the first order condition (8), assuming that interest rates are positive. In that case the cash in advance constraint (2) is satisfied with equality in all periods, and the first order condition is rewritten:

$$\frac{1}{M_t} = \beta \left(1 + i_t\right) E_t \left(\frac{1}{M_{t+1}}\right) \tag{1.9}$$

which is immediately solved in the interest rate as:

$$\frac{1}{1 + i_t} = \beta E_t \left(\frac{M_t}{M_{t+1}}\right) \tag{1.10}$$

What we need to know in order to find the effect of a monetary shock is the response of $E_t \left(M_t / M_{t+1}\right)$ to a shock on M_t. Many authors describe the monetary process under the form of an autoregressive process:

$$\log \left(\frac{M_t}{M_{t-1}}\right) = \frac{\varepsilon_t}{1 - \rho \mathcal{L}} \qquad 0 \leq \rho < 1 \tag{1.11}$$

In this formula ε_t is an i.i.d. stochastic variable and $\mathcal{L}$ is the lag operator which, for any time series x_t, is defined by:

$$\mathcal{L}^j x_t = x_{t-j} \tag{1.12}$$

Most empirical evaluations find a value of ρ around .5. In such a case $E_t \left(M_t / M_{t+1}\right)$ is decreasing in M_t and therefore the nominal interest rate will *increase* in response to a positive monetary shock. This is the "inflationary expectations effect", which causes the nominal interest rate to go in the direction opposite to that predicted by the traditional liquidity effect.

1.3.2 Interest rate pegging and nominal price indeterminacy

We shall now consider a different monetary experiment, where the government pegs the nominal interest rate, letting the quantity of money adapt endogenously. As we shall see, in the traditional model this may lead prices to be totally indeterminate.

There is nominal indeterminacy if, whenever a price sequence is an equilibrium, then any price sequence multiple of the first one is also an equilibrium. It was first pointed out by Sargent and Wallace (1975) that pegging nominal interest rates could lead to such nominal indeterminacy. At the time they did not use a model with explicit intertemporal maximization, so it is useful to restate the problem in the framework of the maximizing model we just described. To make things simplest, assume there is no uncertainty and that the nominal interest rate is pegged at the value i_t in period t. The first order condition (8) is then rewritten:

$$\frac{1}{P_t C_t} = \beta \left(1 + i_t\right) \frac{1}{P_{t+1} C_{t+1}} \tag{1.13}$$

It is shown below that these first order conditions together with the intertemporal budget constraint of the consumer yield the following consumption function (equation 55):

$$D_t P_t C_t = (1 - \beta) \sum_{s=t}^{\infty} D_s P_s Y_s \tag{1.14}$$

where the D_t's are discount rates equal to:

$$D_t = \prod_{s=0}^{t-1} \frac{1}{1 + i_s} \qquad D_0 = 1 \tag{1.15}$$

Now since markets clear, $C_t = Y_t$ for all t, and inserting this into (14) we obtain the equilibrium equations:

$$D_t P_t Y_t = (1 - \beta) \sum_{s=t}^{\infty} D_s P_s Y_s \qquad \forall t \tag{1.16}$$

We see first that financial wealth Ω_t does not appear in these equations, so that there is no Pigou effect. Also equations (16) are homogeneous of degree 1 in prices, so that if a sequence P_t is a solution of all these equations, then any sequence multiple of that one will also be a solution. There is thus

nominal indeterminacy, and the Sargent and Wallace (1975) result is valid in this maximizing framework.

We can now compute relative intertemporal prices. Replacing C_t and C_{t+1} by Y_t and Y_{t+1} in equation (13) we find:

$$\frac{P_{t+1}}{P_t} = \beta\left(1+i_t\right)\frac{Y_t}{Y_{t+1}} \tag{1.17}$$

We thus see that, although absolute prices are indeterminate, setting nominal interest rates determines the ratios between intertemporal prices P_{t+1}/P_t. We can also compute the net and gross real interest rate r_t and R_t:

$$1+r_t = R_t = \frac{\left(1+i_t\right)P_t}{P_{t+1}} = \frac{1}{\beta}\frac{Y_{t+1}}{Y_t} \tag{1.18}$$

If we assume, as we shall do in subsequent chapters, that output per head grows at the rate ζ, then $Y_{t+1}/Y_t = \zeta$ and:

$$1+r_t = R_t = \frac{\zeta}{\beta} \tag{1.19}$$

1.3.3 The fiscal theory of the price level

In the recent years a challenging theory of price determinacy in monetary economies has developed, the fiscal theory of the price level (FTPL). What the FTPL says is that, even in circumstances where monetary policy is not sufficient to bring determinacy, for example in the case of a pure interest rate peg, adequate fiscal policies can restore determinacy. The fiscal policies that achieve determinacy are such that the government's intertemporal budget constraint is not balanced in all circumstances. In fact the intuition behind the theory is that, unless one starts from a particular price level, the government's real liabilities will explode in time. The problem with the FTPL is that the corresponding fiscal policies are rather adventurous since the government does not plan to balance its budget in all situations, and this can lead in many circumstances to explosive real liabilities.

To show the mechanics of the FTPL we shall consider a simple policy of interest rate pegging, which is the typical situation where the FTPL holds. To simplify the exposition we shall assume that the pegged interest rate is constant in time, so that:

$$i_t = i_0 \qquad \forall t \tag{1.20}$$

As for fiscal policy we shall assume that the government has policies of the form:

$$P_t T_t = i_t B_t + (1-\gamma)\,\Omega_t + \delta P_t Y_t \qquad \gamma \geq 0 \qquad \delta \geq 0 \tag{1.21}$$

This formula has three terms:

(a) The term $i_t B_t$ is interest paid on bonds. If there was only this term government budget would be balanced at all times.

(b) The term $\delta P_t Y_t$, says that the governement taxes a fraction δ of national income.

(c) The term $(1-\gamma)\,\Omega_t$, says that the government may want to withdraw a fraction $1-\gamma$ of its outstanding financial liabilities. If γ is greater than 1, this actually corresponds to an expansion of government liabilities. The FTPL, as we shall see, corresponds notably to a "large" value of γ.

Let us recall the government budget equation:

$$\Omega_{t+1} = (1+i_t)\,\Omega_t - i_t M_t - P_t T_t \tag{1.22}$$

Combining (21) and (22) with $\Omega_t = M_t + B_t$, we find:

$$\Omega_{t+1} = \gamma \Omega_t - \delta P_t Y_t \tag{1.23}$$

Turning now to nominal income $P_t Y_t$, we saw above (formula 17) that its dynamics is given by, under the interest peg $i_t = i_0$:

$$P_{t+1} Y_{t+1} = \beta\,(1+i_0)\,P_t Y_t \tag{1.24}$$

Dividing (23) by (24) we obtain:

$$\frac{\Omega_{t+1}}{P_{t+1} Y_{t+1}} = \frac{\gamma}{\beta\,(1+i_0)} \frac{\Omega_t}{P_t Y_t} - \frac{\delta}{\beta\,(1+i_0)} \tag{1.25}$$

The condition for determinacy is:

$$\gamma > \beta\,(1+i_0) \tag{1.26}$$

Combining (25) and (26) we see that, in order to achieve determinacy, the parameter γ must be chosen high enough so that the ratio of government liabilities to income is explosive. This is clearly a highly adventurous policy.

From (25) we can also compute the steady state value of $\Omega_t / P_t Y_t$:

$$\frac{\Omega_t}{P_t Y_t} = \frac{\delta}{\gamma - \beta\,(1+i_0)} > 0 \tag{1.27}$$

1.4 An overlapping generations model

We shall now consider an alternative model, a monetary overlapping generations model in the tradition of Samuelson (1958). We shall see that it delivers answers strikingly different from those obtained in the Ricardian model.

1.4.1 The model

The household side is represented by overlapping generations of consumers. Households born in period t live for two periods, t and $t+1$, and receive real income Y_t when young. They consume C_{1t} in period t, C_{2t+1} in period $t+1$, and their utility is:

$$U_t = \alpha \log C_{1t} + \log C_{2t+1} \tag{1.28}$$

In each period of his life a household born in period t is submitted to a cash in advance constraint:

$$P_t C_{1t} \leq M_{1t} \qquad P_{t+1} C_{2t+1} \leq M_{2t+1} \tag{1.29}$$

Total consumption and money are:

$$C_t = C_{1t} + C_{2t} \qquad M_t = M_{1t} + M_{2t} \tag{1.30}$$

As in the previous Ricardian model, let us call Ω_t the total amount of financial assets that the agents have at the beginning of period t. Since young households are born without any assets, Ω_t is entirely in the hands of old households. To simplify the exposition, we assume that taxes T_t are levied only on young households.

1.4.2 Equilibrium

Let us start with the old households who arrive in period t with financial assets Ω_t. In view of the hundred percent cash in advance constraint (formula 29), their consumption is equal to:

$$C_{2t} = \frac{\Omega_t}{P_t} \tag{1.31}$$

Let us now study the problem of the young household. If he consumes C_{1t} in the first period, he must acquire a quantity of money $P_t C_{1t}$ to satisfy his cash in advance constraint, and therefore borrow $P_t C_{1t}$ from the central bank, so that he holds a quantity of money and bonds:

$$M_{1t} = P_t C_{1t} \qquad B_{1t} = -P_t C_{1t} \tag{1.32}$$

As a consequence he will hold at the end of period t (and transfer to period $t+1$) a quantity of financial assets equal to:

$$\Omega_{t+1} = M_{1t} + (1+i_t) B_{1t} + P_t Y_t - P_t T_t - P_t C_{1t}$$

$$= P_t Y_t - P_t T_t - (1+i_t) P_t C_{1t} \tag{1.33}$$

Combining (31) and (33) we find that second period consumption is equal to:

$$C_{2t+1} = \frac{\Omega_{t+1}}{P_{t+1}} = \frac{P_t Y_t - P_t T_t - (1+i_t) P_t C_{1t}}{P_{t+1}} \tag{1.34}$$

Inserting this into the utility function (28), we find that the young household will choose his first period consumption C_{1t} so as to maximize:

$$\alpha \log C_{1t} + \log \left[P_t Y_t - P_t T_t - (1+i_t) P_t C_{1t} \right] \tag{1.35}$$

The first order condition for C_{1t} is:

$$\frac{\alpha}{C_{1t}} = \frac{(1+i_t) P_t}{P_t Y_t - P_t T_t - (1+i_t) P_t C_{1t}} \tag{1.36}$$

which yields the first period consumption function:

$$C_{1t} = \frac{\alpha}{1+\alpha} \frac{Y_t - T_t}{1+i_t} \tag{1.37}$$

Combining with (31) we obtain total consumption:

$$C_t = C_{1t} + C_{2t} = \frac{\alpha}{1+\alpha} \frac{Y_t - T_t}{1+i_t} + \frac{\Omega_t}{P_t} \tag{1.38}$$

Now the equation of equilibrium on the goods market is $C_t = Y_t$, which yields:

$$Y_t = \frac{\alpha}{1+\alpha} \frac{Y_t - T_t}{1+i_t} + \frac{\Omega_t}{P_t} \tag{1.39}$$

We have also a second equilibrium equation saying that, in view of the cash in advance constraint (29), the total quantity of money M_t is equal to $P_t C_t$, i.e.:

$$M_t = P_t Y_t \tag{1.40}$$

Note that the two equations (39) and (40) somehow correspond to traditional IS and LM equations.

1.4.3 The liquidity effect

Let us now assume that the quantity of money is exogenous. Let us combine equations (39) and (40) and solve for the interest rate. We find[2]:

$$1 + i_t = \frac{\alpha}{1+\alpha} \frac{Y_t - T_t}{Y_t} \frac{M_t}{M_t - \Omega_t} \tag{1.41}$$

There is clearly a liquidity effect since:

$$\frac{\partial i_t}{\partial M_t} = -\frac{\alpha}{1+\alpha} \frac{Y_t - T_t}{Y_t} \frac{\Omega_t}{(M_t - \Omega_t)^2} < 0 \tag{1.42}$$

1.4.4 Interest pegging and price determinacy

Let us consider again the issue of price determinacy under interest rate pegging, and assume that the nominal interest rate is exogenously given at the level i_t in period t. We can solve equation (39) for the price level:

$$P_t = \frac{(1+\alpha)(1+i_t)\Omega_t}{(1+i_t+\alpha i_t) Y_t + \alpha T_t} \tag{1.43}$$

We see that the price level is fully determinate. We further see that:

$$\frac{\partial P_t}{\partial i_t} = \frac{-\alpha(1+\alpha)\Omega_t (Y_t - T_t)}{[(1+i_t+\alpha i_t) Y_t + \alpha T_t]^2} < 0 \tag{1.44}$$

so that the price depends negatively on the interest rate, as is usually expected.

1.4.5 Fiscal policy and determinacy

We shall now consider exactly the same fiscal policy that we examined in section 3.3:

$$P_t T_t = i_t B_t + (1-\gamma)\Omega_t + \delta P_t Y_t \qquad \gamma \geq 0 \qquad \delta \geq 0 \tag{1.45}$$

Let us combine it with the identity $\Omega_t = M_t + B_t$ and the two equilibrium equations (39) and (40). We obtain:

[2]Note that $M_t - \Omega_t$ in the denominator of the last fraction is always positive since it is equal to $P_t C_{1t}$.

$$P_t Y_t = \frac{1 + i_0 + \alpha\gamma}{1 + i_0 + \alpha\delta} \tag{1.46}$$

We see that, unlike with the FTPL, there is no need to have a high γ to obtain price determinacy.

1.4.6 A summary

We have just seen that the two models we presented display strikingly different properties:

(a) A positive shock on money leads to a nominal interest rate increase in the Ricardian model, to a decrease in the OLG model.

(b) In the case of a nominal interest rate peg, the Ricardian model displays nominal indeterminacy whereas in the OLG model the price level is fully determinate.

(c) In the Ricardian model the FTPL associates price determinacy with fiscal policies that make government liabilities explosive, whereas no such policies are required for price determinacy in the OLG model.

We shall now argue that these important differences are related to the presence of a Pigou effect in the OLG model, and its absence in the Ricardian model.

1.5 The Pigou effect

We shall say that there is a Pigou effect if the amount of financial assets Ω_t is considered, at least partly, as wealth by the households. In particular it will have a positive influence on consumption. We see from equation (38) that there is indeed such a Pigou effect in the overlapping generations model. Furthermore it is clear that the results in sections 4.3, 4.4 and 4.5 are due to this presence of Ω_t in the central equilibrium equation (39).

On the other hand we saw that in the Ricardian model, although Ω_t appears in budget constraints such as (5), it disappears in the consumption function (equation 14) or in the equilibrium equations (equation 16), which leads to the bizarre properties of the Ricardian model. We shall now investigate why this happens.

1.5.1 The intertemporal budget constraint

We want to explain why in the Ricardian model, although nominal assets appear in the period by period budget constraints, they do not seem to play any role in the end. Let us indeed recall the budget constraint for period t:

$$\Omega_{t+1} = (1 + i_t)\,\Omega_t + P_t Y_t - P_t T_t - (1 + i_t)\,P_t C_t \tag{1.47}$$

We see that Ω_t still appears at this stage. In order to show why it disappears in the end, we shall now derive the intertemporal budget constraint of the household. Let us recall the discount factors:

$$D_t = \prod_{s=0}^{t-1} \frac{1}{1 + i_s} \qquad D_0 = 1 \tag{1.48}$$

Consider the household's budget equation (47) for period s. Multiplying it by D_{s+1} it becomes:

$$D_{s+1}\Omega_{s+1} = D_s\Omega_s + D_{s+1}P_sY_s - D_{s+1}P_sT_s - D_sP_sC_s \tag{1.49}$$

If we now sum all discounted budget constraints (49) from time t to infinity, and assume that $D_s\Omega_s$ goes to zero as s goes to infinity (a usual transversality condition), we obtain the intertemporal budget constraint of the household:

$$\sum_{s=t}^{\infty} D_sP_sC_s = D_t\Omega_t + \sum_{s=t}^{\infty} D_{s+1}P_sY_s - \sum_{s=t}^{\infty} D_{s+1}P_sT_s \tag{1.50}$$

Aggregate financial wealth Ω_t is still present on the right hand side, which might lead us to believe that nominal assets can play a role. But this reasoning is misleading because it treats initial financial wealth Ω_t and the sequence of taxes T_s, $s \geq t$ as independent. As we shall now see, they are closely linked through the government's intertemporal budget constraint. We have to evaluate the discounted value of taxes, and for that let us consider the government's budget constraint (equation 6) for period s:

$$\Omega_{s+1} = (1 + i_s)\,\Omega_s - i_sM_s - P_sT_s \tag{1.51}$$

Let us multiply it by D_{s+1}, and use $M_s = P_sC_s = P_sY_s$:

$$D_{s+1}\Omega_{s+1} = D_s\Omega_s - (D_s - D_{s+1})\,P_sY_s - D_{s+1}P_sT_s \tag{1.52}$$

Let us sum all these equalities from time t to infinity:

$$\sum_{s=t}^{\infty} D_{s+1}P_sT_s = D_t\Omega_t - \sum_{s=t}^{\infty} (D_s - D_{s+1})\,P_sY_s \tag{1.53}$$

We can already note that any increase in the value of nominal assets Ω_t is matched by an identical increase in the discounted value of taxes. Let us now

insert this value of discounted taxes (53) into the household's intertemporal budget constraint (50). We find:

$$\sum_{s=t}^{\infty} D_s P_s C_s = \sum_{s=t}^{\infty} D_s P_s Y_s \tag{1.54}$$

We see that the financial assets have totally disappeared from the intertemporal budget constraint! Now let us maximize the utility function (1) subject to this budget constraint. We obtain the consumption function:

$$D_t P_t C_t = (1-\beta) \sum_{s=t}^{\infty} D_s P_s Y_s \tag{1.55}$$

We see that Ω_t appears neither in the budget constraint (54) nor in the consumption function (55). There is no Pigou effect in the Ricardian model, contrary to what happens in the OLG model.

So what we have found is that the Ricardian intuition, which was usually applied to show that real bonds are not real wealth (Barro, 1974), applies to financial assets as well (Weil 1991).

1.6 Conclusions

We studied in this chapter two polar monetary models, a Ricardian one (the Ramsey-Sidrauski-Brock model) and a non-Ricardian one (the overlapping generations model), and we found some striking differences.

Considering first the effect of a monetary expansion on the nominal interest rate, we found that this led to an increase of the nominal interest rate in the Ricardian model, a decrease in the OLG model.

We then studied the issue of price determinacy under interest rate pegging, and we found that there is nominal indeterminacy in the Ricardian model, full determinacy in the OLG model.

Finally we considered the fiscal theory of the price level (FTPL), and found that the adventurous policy prescriptions that appeared in the Ricardian version of the model become irrelevant in the OLG version. As we shall see in later chapters, these are still more important differences of this type.

We also saw that an importance difference between the two models was the presence of a Pigou effect in the OLG model, its absence in the Ricardian model.

Now a problem in this comparison is that we cannot go continuously from one model to the other. This is why we shall in the next chapter describe a model due to Weil (1987, 1991), which is a non-Ricardian model

with properties similar to the OLG model, but which includes the above Ricardian model as a particular case.

1.7 References

The Pigou effect appears in Pigou (1943). The importance of its role has notably been emphasized by Patinkin (1956) under the name of "real balance effect". It has been analyzed in intertemporal maximizing models by Weil (1987, 1991).

The Ricardian model with the infinitely lived consumer derives notably from the seminal works of Ramsey (1928), Sidrauski (1967) and Brock (1974, 1975).

The overlapping generations model was pioneered by Allais (1947), Samuelson (1958) and Diamond (1965).

The difference between Ricardian and non Ricardian models, notably from the point of view of fiscal policy, has been studied by Barro (1974), who notably asked whether real bonds are net wealth for agents in an overlapping generations model.

The cash in advance constraint is due to Clower (1967). In the original version the consumer had to carry cash from the previous period in order to consume. The timing we use (Helpman, 1981, Lucas, 1982), allows newborn agents to have a positive consumption, a useful feature since the models we shall use in this book will have newborn agents.

The price indeterminacy issue was uncovered by Sargent and Wallace (1975). A useful taxonomy of various cases of indeterminacy is found in McCallum (1986).

The liquidity effect dates back to Keynes (1936) and Hicks (1937). A recent appraisal is Christiano, Eichenbaum and Evans (1997).

Chapter 2

Pigou Reconstructed: The Weil Model

2.1 Introduction

We have seen in the previous chapter how the distinction between Ricardian and non-Ricardian economies, and whether the Pigou effect is present or not, are important to obtain sensible answers to important questions of monetary theory.

Now the Ricardian and OLG models that we used for the comparison give extreme opposite results and are quite far apart. It would thus be extremely useful to have a non-Ricardian monetary model that, unlike the OLG model, would "nest" the Ricardian model as a special case. Such a model has been actually developed by Weil (1987, 1991). So we shall describe in the next section a simple version of this model, which we shall use in chapters 3 to 6. We shall notably emphasize in this chapter how a Pigou effect is generated in such a model, and derive some useful dynamic equations.

2.2 The model

In the Weil model, as in the Sidrauski-Brock Ricardian model, households never die and live an infinite number of periods. But, as in the OLG model, new "generations" of households are born each period. Call N_t the number of households alive at time t. Since nobody dies, we have $N_{t+1} \geq N_t$. We will actually mainly work below with the case where the population grows at the constant rate $n \geq 0$, so that $N_t = (1+n)^t$.

2.2.1 Households

Consider a household j (i.e. a household born in period j). We denote by c_{jt} and m_{jt} his consumption and money holdings at time $t \geq j$. This household receives in periods $t \geq j$ an endowment y_{jt} and maximizes the following utility function:

$$U_{jt} = \sum_{s=t}^{\infty} \beta^{s-t} Log\, c_{js} \tag{2.1}$$

He is submitted in period t to a "cash in advance" constraint:

$$P_t c_{jt} \leq m_{jt} \tag{2.2}$$

Household j enters period t with a financial wealth ω_{jt}. Transactions occur in two steps. First the bond market opens, and the household lends an amount b_{jt} at the nominal interest rate i_t (of course b_{jt} can be negative if the household borrows to obtain liquidity). The rest is kept under the form of money m_{jt}, so that:

$$\omega_{jt} = m_{jt} + b_{jt} \tag{2.3}$$

Then the goods market opens, and the household sells his endowment y_{jt}, pays taxes τ_{jt} in real terms and consumes c_{jt}, subject to the cash in advance constraint (2). Consequently, the budget constraint for household j is:

$$\omega_{jt+1} = (1 + i_t)\, \omega_{jt} - i_t m_{jt} + P_t y_{jt} - P_t \tau_{jt} - P_t c_{jt} \tag{2.4}$$

2.2.2 Aggregation

Aggregate quantities are obtained by summing the various individual variables. Since there are $N_j - N_{j-1}$ agents in generation j, these aggregates are equal to:

$$Y_t = \sum_{j \leq t} (N_j - N_{j-1})\, y_{jt} \qquad C_t = \sum_{j \leq t} (N_j - N_{j-1})\, c_{jt} \tag{2.5}$$

$$T_t = \sum_{j \leq t} (N_j - N_{j-1})\, \tau_{jt} \qquad \Omega_t = \sum_{j \leq t} (N_j - N_{j-1})\, \omega_{jt} \tag{2.6}$$

$$M_t = \sum_{j \leq t} (N_j - N_{j-1})\, m_{jt} \qquad B_t = \sum_{j \leq t} (N_j - N_{j-1})\, b_{jt} \tag{2.7}$$

2.2.3 Endowments and taxes

To be complete we have to describe how endowments and taxes are distributed among households. We assume for the time being that all households have the same income and taxes, so that:

$$y_{jt} = y_t = \frac{Y_t}{N_t} \qquad \tau_{jt} = \tau_t = \frac{T_t}{N_t} \tag{2.8}$$

A more general scheme will be considered in section 2.6 below. We shall also assume that endowments per head grow at the rate ζ, so that:

$$\frac{y_{t+1}}{y_t} = \zeta \qquad \frac{Y_{t+1}}{Y_t} = (1+n)\,\zeta \tag{2.9}$$

2.2.4 Government

The households' aggregate financial wealth Ω_t has as a counterpart an identical amount Ω_t of financial liabilities of the government. The evolution of these liabilities is described by the government's budget constraint, which is the same as in chapter 1:

$$\Omega_{t+1} = (1+i_t)\,\Omega_t - i_t M_t - P_t T_t \tag{2.10}$$

2.3 The dynamics of the economy

We shall derive here a number of dynamic relations in this non Ricardian model, and show at the same time how and why a Pigou effect develops because of population growth.

2.3.1 Households' intertemporal budget constraints

We shall continue to aggregate discounted values with the nominal discount rates:

$$D_t = \prod_{s=0}^{t-1} \frac{1}{1+i_s} \qquad D_0 = 1 \tag{2.11}$$

Consider the household's budget equation (4). We assume that i_t is strictly positive. Then the household always satisfies the "cash in advance" constraint (2) exactly, so that $m_{jt} = P_t c_{jt}$ and thus the budget constraint for period s is written:

$$\omega_{js+1} = (1 + i_s)\,\omega_{js} + P_s y_{js} - P_s \tau_{js} - (1 + i_s)\,P_s c_{js} \tag{2.12}$$

Applying the discount rate D_{s+1} to this budget constraint, it becomes:

$$D_{s+1}\omega_{js+1} = D_s\omega_{js} + D_{s+1}\,(P_s y_{js} - P_s \tau_{js}) - D_s P_s c_{js} \tag{2.13}$$

If we aggregate all budget constraints (13) from time t to infinity, and assume that $D_s\omega_{js}$ goes to zero as s goes to infinity (the transversality condition), we obtain the intertemporal budget constraint of the household:

$$\sum_{s=t}^{\infty} D_s P_s c_{js} = D_t\omega_{jt} + \sum_{s=t}^{\infty} D_{s+1} P_s\,(y_{js} - \tau_{js}) \tag{2.14}$$

2.3.2 The consumption function

Maximizing the utility function (1) subject to the intertemporal budget constraint (14) yields the first order conditions:

$$D_{s+1} P_{s+1} c_{js+1} = \beta D_s P_s c_{js} \tag{2.15}$$

Combining these first order conditions and the intertemporal budget constraint (14) yields the following consumption function for a household j:

$$D_t P_t c_{jt} = (1 - \beta)\left[D_t\omega_{jt} + \sum_{s=t}^{\infty} D_{s+1} P_s\,(y_{js} - \tau_{js})\right] \tag{2.16}$$

Let us now insert into (16) the above assumption (equation 8) that $y_{js} = y_s$ and $\tau_{js} = \tau_s$. Summing individual consumption functions (16) across the N_t agents alive in period t, we obtain the aggregate consumption C_t:

$$D_t P_t C_t = (1 - \beta)\left[D_t\Omega_t + N_t \sum_{s=t}^{\infty} D_{s+1} P_s\,(y_s - \tau_s)\right] \tag{2.17}$$

We noted in chapter 1 that the presence of Ω_t in the consumption function at this stage did not necessarily mean that a Pigou effect would arise in the end, because the value of Ω_t was cancelled by an identical value of discounted taxes, so we now study the government's budget constraint.

2.3.3 The government's intertemporal budget constraint

Let us consider the government's budget constraint (10) in period s, multiplied by D_{s+1}:

$$D_{s+1}\Omega_{s+1} = D_s\Omega_s - D_{s+1}P_sT_s - D_{s+1}i_sM_s \tag{2.18}$$

Now let us define "total taxes" in period s, $\mathcal{T}_s$, as:

$$P_s\mathcal{T}_s = P_sT_s + i_sM_s \tag{2.19}$$

Total nominal taxes consist of proper taxes P_sT_s and the money economized by the state because of the cash in advance constraint i_sM_s, the "money tax". Using this definition, (18) can be rewritten:

$$D_{s+1}\Omega_{s+1} = D_s\Omega_s - D_{s+1}\mathcal{T}_s \tag{2.20}$$

Summing from time t to infinity, and assuming that $D_s\Omega_s$ goes to zero when s goes to infinity, we get:

$$D_t\Omega_t = \sum_{s=t}^{\infty} D_{s+1}\mathcal{T}_s \tag{2.21}$$

We see that, as in the Ricardian model, every single dollar of financial wealth is matched by discounted current and future taxes. But the difference is that, whereas in the Ricardian model the currently alive generation will pay hundred percent of these taxes in the future, here some future generations will pay part of the taxes. This will yield a Pigou effect, as we will now demonstrate.

2.4 The Pigou effect

As we indicated in the introduction, an important part of the story is the Pigou effect, through which financial wealth influences consumption and the dynamic equations. We shall now see how it arises.

2.4.1 Consumption and financial assets

Consider the government's budget constraint (18), replacing M_s by P_sY_s and i_s by $(D_s - D_{s+1})/D_{s+1}$:

$$D_{s+1}\Omega_{s+1} = D_s\Omega_s - (D_s - D_{s+1})P_sY_s - D_{s+1}P_sT_s \tag{2.22}$$

Dividing by N_s this is rewritten:

$$D_{s+1}P_s(y_s - \tau_s) = D_sP_sy_s + \frac{D_{s+1}\Omega_{s+1} - D_s\Omega_s}{N_s} \tag{2.23}$$

Inserting (23) into (17) we obtain:

$$D_t P_t C_t = (1-\beta)\left[N_t \sum_{s=t}^{\infty} D_s P_s y_s + D_t \Omega_t + N_t \sum_{s=t}^{\infty} \frac{D_{s+1}\Omega_{s+1} - D_s \Omega_s}{N_s}\right] \tag{2.24}$$

which yields after rearranging the terms in Ω_s:

$$D_t P_t C_t = (1-\beta) N_t \left[\sum_{s=t}^{\infty} D_s P_s y_s + \sum_{s=t}^{\infty} D_{s+1}\Omega_{s+1}\left(\frac{1}{N_s} - \frac{1}{N_{s+1}}\right)\right] \tag{2.25}$$

The first sum inside the brackets is the usual sum of discounted incomes. But we see that, as soon as some new generations appear in the future, terms representing nominal wealth appear in the consumption function. This is the Pigou effect (Pigou, 1943) or "real balance effect" (Patinkin, 1956). Formula (25) shows in a crystalclear manner how this effect disappears when population is constant.

2.4.2 The extent of the Pigou effect

Now we may wonder what part of financial wealth will be considered as "real wealth" by the currently alive generations. Consider the case where by an adequate policy (such a policy will be made explicit, for example, in chapter 3) Ω_t remains constant in time and equal to Ω_0. In such a case the "supplementary wealth" beyond discounted incomes is, from (25), equal to:

$$\left[\frac{N_t}{D_t}\sum_{s=t}^{\infty} D_{s+1}\left(\frac{1}{N_s} - \frac{1}{N_{s+1}}\right)\right]\Omega_0 = \varpi_t \Omega_0 \tag{2.26}$$

It is easy to see that ϖ_t is between 0 and 1 and, other things equal, larger when the rate of increase of the population is greater. As an example, assume that the population's growth rate and the nominal interest rate are constant over time:

$$\frac{N_{t+1}}{N_t} = 1 + n \qquad i_t = i_0 \tag{2.27}$$

Then:

$$\varpi_t = \varpi = \frac{n}{n + i_0 + n i_0} \tag{2.28}$$

and this is increasing in n if $i_0 > 0$.

2.4.3 Taxes and the Pigou effect

We have just seen that part of financial assets, now and in the future, represents actual purchasing power in the non-Ricardian model, whereas it does not in the Ricardian model. As we already hinted at, the reason is the following: some of the future taxes that are the counterpart of current nominal wealth will not be paid by the currently alive agents, but by future, yet unborn, generations, so that this part of Ω_t represents actual purchasing power for the currently alive agents. We shall now make this intuition more formal.

Using the expression of "total taxes" $\mathcal{T}_s$ in equation (20), the consumption function (24) can be rewritten:

$$D_t P_t C_t = (1-\beta)\left[N_t \sum_{s=t}^{\infty} D_s P_s y_s + D_t \Omega_t - N_t \sum_{s=t}^{\infty} \frac{D_{s+1}\mathcal{T}_s}{N_s}\right] \tag{2.29}$$

We see that generations alive in t will pay at time $s > t$ only a fraction $N_t/N_s < 1$ of total taxes. From this the Pigou effect will arise, as we shall now see. Combining the two equations (21) and (29), we rewrite the consumption function as:

$$D_t P_t C_t = (1-\beta)\left[N_t \sum_{s=t}^{\infty} D_s P_s y_s + \sum_{s=t}^{\infty} \frac{N_s - N_t}{N_s} D_{s+1}\mathcal{T}_s\right] \tag{2.30}$$

Note that $N_s - N_t$ is the number of agents alive in period s, but yet unborn at period t. So the wealth of agents currently alive consists of two terms: (a) the discounted sum of their incomes, and (b) the part of taxes (including the "money tax") that will be paid by future generations in order to "reimburse" the current financial wealth. This second term is what creates the Pigou effect, and we see that an essential ingredient of it is that there will be future, yet unborn, generations that will share the burden of future taxes that are the counterpart of current financial wealth.

2.5 Intertemporal equilibrium and a dynamic equation

So far we have given a number of equilibrium equations emphasizing the intertemporal structure of the model, notably the intertemporal budget constraints. In the chapters that follow it will be very useful to have a simple

dynamic equation relating some central variables at times t and $t+1$. This is done through the following proposition:

Proposition 2.1 *The dynamics of the model is characterized by the following dynamic relations:*

$$P_{t+1}Y_{t+1} = \beta\frac{N_{t+1}}{N_t}\left(1+i_t\right)P_tY_t - \left(1-\beta\right)\left(\frac{N_{t+1}}{N_t}-1\right)\Omega_{t+1} \tag{2.31}$$

or, if $N_{t+1}/N_t = 1+n$:

$$P_{t+1}Y_{t+1} = \beta\left(1+n\right)\left(1+i_t\right)P_tY_t - \left(1-\beta\right)n\Omega_{t+1} \tag{2.32}$$

Proof: Insert the condition $C_t = Y_t$ into (25) and divide by N_t:

$$D_tP_ty_t = \left(1-\beta\right)\left[\sum_{s=t}^{\infty}D_sP_sy_s + \sum_{s=t}^{\infty}D_{s+1}\Omega_{s+1}\left(\frac{1}{N_s}-\frac{1}{N_{s+1}}\right)\right] \tag{2.33}$$

Let us rewrite (33) for $t+1$:

$$D_{t+1}P_{t+1}y_{t+1} = \left(1-\beta\right)\left[\sum_{s=t+1}^{\infty}D_sP_sy_s + \sum_{s=t+1}^{\infty}D_{s+1}\Omega_{s+1}\left(\frac{1}{N_s}-\frac{1}{N_{s+1}}\right)\right] \tag{2.34}$$

Subtract (33) from (34):

$$D_{t+1}P_{t+1}y_{t+1} = \beta D_tP_ty_t - \left(1-\beta\right)\left(\frac{1}{N_t}-\frac{1}{N_{t+1}}\right)D_{t+1}\Omega_{t+1} \tag{2.35}$$

Multiplying by N_{t+1}/D_{t+1}, we obtain (31). Replacing N_{t+1}/N_t by $1+n$ we obtain (32). Q.E.D.

We may note that equations (31) and (32) have a strong resemblance with the traditional "Euler" equations. A main difference is the presence of the last terms in equations (31) and (32), which introduce nominal wealth and are a consequence of the Pigou effect. As we shall see in the next chapters, this creates a number of striking and actually more intuitive results.

2.6 A generalization: decreasing resources

In order to simplify the exposition we made so far the particular assumption that agents in all generations have exactly the same endowment and taxes (equation 8), and this is an assumption we shall keep for simplicity in many developments that follow. But it turns out that, in order to explain within this model some properties of the OLG model in the first chapter, like price determinacy or liquidity effects (these will be studied in chapter 3 to 6), it will be useful to make a simple further generalization. Namely we shall assume that relative endowments and taxes decrease with age as follows:

$$y_{jt} = \psi^{t-j} y_t \qquad \tau_{jt} = \psi^{t-j} \tau_t \qquad \psi \leq 1 \qquad j \leq t \tag{2.36}$$

where y_t and τ_t are the income and taxes of a newborn agent in period t. The assumption in equation (8) corresponds to $\psi = 1$.

Under the more general hypothesis (36) proposition 2.1 is replaced by:

Proposition 2.2 *Assume the population grows at the rate $n > 0$. Under hypothesis* (36) *the dynamics of the model is characterized by the following relation:*

$$\psi P_{t+1} Y_{t+1} = \beta (1+n)(1+i_t) P_t Y_t - (1-\beta)(1+n-\psi)\Omega_{t+1} \tag{2.37}$$

Proof: Appendix. Q.E.D.

2.7 The autarkic interest rate

In this article we shall be essentially interested in equilibria (and dynamic paths) where financial assets are positively valued and create an operative link between the various generations. But, since Samuelson (1958) and Gale (1973), we know that in OLG models there are also equilibria where each generation somehow lives in "autarky". As it turns out, it will appear in subsequent chapters that a similar phenomenon appears in this model for some equilibria. Moreover the real interest rate that prevails in such "autarkic" equilibria will play an important role in the determinacy conditions that we will find in chapters 4 to 6, so it is useful to characterize it now. We shall consider the more general model of section 2.6. Let us recall the utility functions:

$$U_{jt} = \sum_{s=t}^{\infty} \beta^{s-t} \log c_{js} \tag{2.38}$$

and the endowments:

$$y_{jt} = \psi^{t-j} y_t \tag{2.39}$$

$$y_{t+1} = \zeta y_t \tag{2.40}$$

To make things simple, we assume that government spending and taxes are zero, so that the assumption that we are in autarkic equilibrium translates into:

$$c_{jt} = y_{jt} \qquad \forall j, t \tag{2.41}$$

Now maximization of utility under the intertemporal budget constraint yields the first order condition:

$$P_{t+1} c_{jt+1} = \beta \left(1 + i_t\right) P_t c_{jt} \tag{2.42}$$

Recall the definition of the gross real interest rate R_t:

$$R_t = 1 + r_t = \frac{\left(1 + i_t\right) P_t}{P_{t+1}} \tag{2.43}$$

We shall define the autarkic interest rate as the real rate of interest that would prevail under an "autarkic" situation, characterized by equalities (41). We have:

Proposition 2.3 *Under hypotheses* (39) *and* (40) *the autarkic gross real interest rate, which we shall denote as* ξ*, is equal to:*

$$\xi = \frac{\psi \zeta}{\beta} \tag{2.44}$$

Proof: Combining equations (39) to (43) we obtain (44). Q.E.D.

We may note that the quantity $\psi\zeta$ in the numerator of (44) is the rate of increase of individual endowments since, combining (39) and (40), we obtain:

$$y_{jt+1} = \psi \zeta y_{jt} \tag{2.45}$$

We shall see in the next chapters that this autarkic interest rate ξ will play a central role in a number of determinacy conditions.

2.8 Conclusions

We described in this chapter the model of Weil (1987, 1991), which is somehow an intermediate between the Ricardian and the OLG models. As in the Ricardian model all agents are infinitely lived, but as in the OLG model new agents enter the economy over time. The Ricardian model appears as a particular limit case, when the rate of birth is zero. We saw that a Pigou effect naturally appears in this model (Weil 1991). The reason is the following. In a model with a single family of agents, every single cent of financial wealth today is compensated by the same amount of discounted taxes in the future, so that it does not represent any real wealth now, as we already saw in chapter 1. In the non-Ricardian economy part of these future taxes will be paid by yet unborn agents, and this part that will be paid by unborn agents represents real wealth to currently alive agents (formula 30).

We shall now see in the next chapters that this brings major changes for the study of many important monetary issues.

2.9 References

The model in this chapter is due to Weil (1987, 1991), who showed notably that financial assets are net wealth in these models (the Pigou effect).

The first model with a demographic structure similar to that in this chapter is due to Blanchard (1985). The emphasis was put on the stochastic death rate of households, which could be handled elegantly using a life insurance scheme due to Yaari (1965).

Later Weil (1989) showed that the important results in such models could be obtained in a model with only births and no deaths. This result was confirmed by Buiter (1988) who built a model with different rates of death and birth. In particular if there is no birth but a constant rate of death, Ricardian equivalence still prevails. So, since the important non-Ricardian results are due to the birth rate and not to the death rate, we use for simplicity a model with birth only.

The structure of long term equilibria in OLG models was notably studied by Gale (1973).

2.10 Appendix: Proof of proposition 2.2

The derivation of the consumption function for generation j is essentially the same as for $\psi = 1$, except for the fact that we must take into account the fact that income y_{jt} and taxes τ_{jt} now explicitly depend on the date j the household was born. So the consumption function of generation j is, combining (16) and (36):

$$\begin{aligned} D_t P_t c_{jt} &= (1-\beta)\left[D_t\omega_{jt} + \sum_{s=t}^{\infty} D_{s+1}P_s\left(y_{js} - \tau_{js}\right)\right] \\ &= (1-\beta)\left[D_t\omega_{jt} + \sum_{s=t}^{\infty} D_{s+1}P_s\psi^{s-j}\left(y_s - \tau_s\right)\right] \end{aligned} \tag{2.46}$$

There are $N_j - N_{j-1}$ households in generation j. So, summing over all generations $j \leq t$, we obtain total consumption C_t:

$$D_t P_t C_t = (1-\beta)\left[D_t\Omega_t + \sum_{j=-\infty}^{t}\left(N_j - N_{j-1}\right)\sum_{s=t}^{\infty} D_{s+1}P_s\psi^{s-j}\left(y_s - \tau_s\right)\right]$$

$$= (1-\beta)\left[D_t\Omega_t + \sum_{j=-\infty}^{t}\psi^{t-j}\left(N_j - N_{j-1}\right)\sum_{s=t}^{\infty} D_{s+1}P_s\psi^{s-t}\left(y_s - \tau_s\right)\right] \tag{2.47}$$

Now let us call:

$$\mathcal{N}_t = \sum_{j=-\infty}^{t}\psi^{t-j}\left(N_j - N_{j-1}\right) \tag{2.48}$$

If $N_t = (1+n)^t$, $n > 0$, then:

$$\mathcal{N}_t = \frac{nN_t}{1+n-\psi} \tag{2.49}$$

We have:

$$y_t = \frac{Y_t}{\mathcal{N}_t} \qquad \tau_t = \frac{T_t}{\mathcal{N}_t} \tag{2.50}$$

Aggregate consumption (equation 88) is therefore now given by:

$$D_t P_t C_t = (1-\beta)\left[D_t\Omega_t + \mathcal{N}_t\sum_{s=t}^{\infty} D_{s+1}P_s\psi^{s-t}\left(y_s - \tau_s\right)\right] \tag{2.51}$$

The equilibrium equation is obtained by inserting $C_t = Y_t$ into (92):

$$D_t P_t Y_t = (1-\beta)\left[D_t\Omega_t + \mathcal{N}_t \sum_{s=t}^{\infty} D_{s+1} P_s \psi^{s-t}\left(y_s - \tau_s\right)\right] \tag{2.52}$$

Divide both sides by $\mathcal{N}_t$ and use $Y_t = \mathcal{N}_t y_t$:

$$D_t P_t y_t = (1-\beta)\left[\frac{D_t\Omega_t}{\mathcal{N}_t} + \sum_{s=t}^{\infty} D_{s+1} P_s \psi^{s-t}\left(y_s - \tau_s\right)\right] \tag{2.53}$$

Let us rewrite this equation for $t+1$:

$$D_{t+1} P_{t+1} y_{t+1} = (1-\beta)\left[\frac{D_{t+1}\Omega_{t+1}}{\mathcal{N}_{t+1}} + \sum_{s=t+1}^{\infty} D_{s+1} P_s \psi^{s-t-1}\left(y_s - \tau_s\right)\right] \tag{2.54}$$

Let us now multiply (95) by ψ and subtract (94) from it:

$$\psi D_{t+1} P_{t+1} y_{t+1} - D_t P_t y_t = (1-\beta)\left[\frac{\psi D_{t+1}\Omega_{t+1}}{\mathcal{N}_{t+1}} - \frac{D_t\Omega_t}{\mathcal{N}_t} - D_{t+1} P_t\left(y_t - \tau_t\right)\right] \tag{2.55}$$

Multiply the government's budget equation (10) by $D_{t+1}/\mathcal{N}_t$:

$$\frac{D_t\Omega_t}{\mathcal{N}_t} = \frac{D_{t+1}\Omega_{t+1}}{\mathcal{N}_t} + D_{t+1} P_t \tau_t + \left(D_t - D_{t+1}\right) P_t y_t \tag{2.56}$$

Insert (97) into (96):

$$\psi D_{t+1} P_{t+1} y_{t+1} = \beta D_t P_t y_t + (1-\beta)\left(\frac{\psi}{\mathcal{N}_{t+1}} - \frac{1}{\mathcal{N}_t}\right) D_{t+1}\Omega_{t+1} \tag{2.57}$$

Multiply (98) by $\mathcal{N}_{t+1}/D_{t+1}$:

$$\psi P_{t+1} Y_{t+1} = \beta \frac{\mathcal{N}_{t+1}}{\mathcal{N}_t}\left(1+i_t\right) P_t Y_t - (1-\beta)\left(\frac{\mathcal{N}_{t+1}}{\mathcal{N}_t} - \psi\right)\Omega_{t+1} \tag{2.58}$$

Taking finally from (90) $\mathcal{N}_{t+1}/\mathcal{N}_t = N_{t+1}/N_t = 1+n$, we obtain:

$$\psi P_{t+1} Y_{t+1} = \beta\left(1+n\right)\left(1+i_t\right) P_t Y_t - (1-\beta)\left(1+n-\psi\right)\Omega_{t+1} \tag{2.59}$$

which is equation (37).

Chapter 3

Liquidity Effects

3.1 Introduction

We saw in chapter 1 that the liquidity effect, i.e. a negative response of the nominal interest rate to monetary injections, is difficult to obtain in the Ricardian monetary DSGE models. The reason is the "inflationary expectations effect", which was described formally in chapter 1, and which raises the nominal interest rate in response to a monetary injection. We saw also that a liquidity effect was present in a non-Ricardian OLG model.

What we want to do here is to show that a liquidity effect naturally appears in a non-Ricardian environment. In a nutshell, the channel is the following: we have already seen that in a non-Ricardian economy a "Pigou effect" appears. And this Pigou effect produces a liquidity effect, as we shall demonstrate formally below. But before going to a fully rigorous model, we shall give a brief intuitive argument based on a traditional IS-LM model.

3.2 Liquidity effects in a simple IS-LM model

To guide our intuition as to why the Pigou effect leads to a liquidity effect, let us consider a simple traditional IS-LM model augmented with such a Pigou effect. To make the exposition particularly simple we write this model in loglinear form:

$$y = -a\left(i - \pi^{e}\right) + b\left(\omega - p\right) + cy \qquad IS \tag{3.1}$$

$$m - p = -di + ey \qquad LM \tag{3.2}$$

$$y = y_0 \tag{3.3}$$

where π^e is the expected rate of inflation, $\omega = Log\,\Omega$, $p = LogP$ and:

$$a > 0 \qquad b > 0 \qquad c > 0 \qquad d > 0 \qquad e > 0 \tag{3.4}$$

Equation (3) expresses market clearing (which will be assumed throughout this chapter). The IS equation (1) says that output is equal to demand, which itself depends negatively on the real interest rate $i - \pi^e$, and positively on real wealth $\omega - p$. Note that the presence of this last term with $b > 0$, which corresponds to a Pigou effect, is specific of the non Ricardian framework. Now we can solve for the nominal interest rate i and price level p. Omitting irrelevant constants this yields:

$$i = \frac{a\pi^e - bm}{a + bd} \tag{3.5}$$

$$p = \frac{ad\pi^e + am}{a + bd} \tag{3.6}$$

We first see, differentiating the expression of i in (5), that:

$$\frac{\partial i}{\partial m} = \frac{1}{a + bd}\left(a\frac{\partial \pi^e}{\partial m} - b\right) \tag{3.7}$$

We recognize two effects: The first term in the parenthesis corresponds to the "inflationary expectations effect", which is positive if a positive money shock raises inflationary expectations ($\partial\pi^e/\partial m > 0$). Secondly there is a negative "liquidity effect", itself due to the Pigou effect ($b > 0$).

Now the underlying mechanism for the liquidity effect is the following: an increase in money creates a price increase (equation 6). This price increase decreases demand because of the Pigou effect (the second term in the right hand side of equation 1). To maintain total demand at the market clearing level, the first term in (1) must increase, i.e. the real rate of interest must decrease. This decrease in the real interest rate creates the liquidity effect.

3.3 The model and monetary policy

We shall now develop the above argument in the framework of a rigorous non-Ricardian model. We shall use the Weil model, already described in chapter 2, section 2. We must now be a little more specific on monetary policy.

3.3.1 Monetary policy

There are actually several ways to model monetary policy, i.e. how governement intervenes on the bonds market. For example the government can target the quantity of money, the interest rate, or any intermediate objective.

In this chapter, as in all studies on the "liquidity effect", we shall assume that the government uses the quantity of money M_t as the policy variable, and that consequently the nominal interest rate i_t is endogenously determined through the equilibrium in the bonds market.

Let us recall that at the beginning of period t agents go to the bonds market and allocate their aggregate financial wealth Ω_t between money and bonds, so that:

$$M_t + B_t = \Omega_t \tag{3.8}$$

The government aims at choosing directly the value of M_t. A positive shock on money M_t corresponds to a purchase of bonds (against money) by the government. Following the literature we shall make the assumption that M_t is a stochastic process. As an example, it is often assumed that money increases are autocorrelated in time:

$$\log\left(\frac{M_t}{M_{t-1}}\right) = \frac{\varepsilon_t}{1-\rho\mathcal{L}} \qquad 0 \leq \rho < 1 \tag{3.9}$$

where ε_t is an i.i.d. stochastic variable and $\mathcal{L}$ is the lag operator.

As indicated above, we shall say there is a liquidity effect if a positive shock on M_t leads to a decrease in i_t.

3.4 Dynamic equilibrium

We shall now derive the dynamics of the model. The central equation, which describes the dynamics of nominal income, turns out to be a stochastic version of equation (32) in chapter 2:

Proposition 3.1 *The dynamics of nominal income is given by:*

$$E_t\left(P_{t+1}Y_{t+1}\right) = \beta\left(1+n\right)\left(1+i_t\right)P_tY_t - \left(1-\beta\right)n\Omega_{t+1} \tag{3.10}$$

Proof: Bénassy (2006b, 2007).

Now since the model is non Ricardian, the complete dynamics will depend on the actual tax policy. Let us recall the equation of evolution of Ω_t:

$$\Omega_{t+1} = (1 + i_t)\,\Omega_t - i_t M_t - P_t T_t \tag{3.11}$$

Since our emphasis is on monetary policy, and in order to simplify the dynamics below, we shall choose the simplest tax policy, and assume that the government balances its budget period by period. Taxes will thus cover exactly interest payments on bonds:

$$P_t T_t = i_t B_t \tag{3.12}$$

We may immediately note, using (8) and (11), that under the balanced budget policy (12) total financial wealth will remain constant:

$$\Omega_t = \Omega_0 \qquad \text{for all } t \tag{3.13}$$

The dynamic equation (10) then becomes:

$$E_t\left(P_{t+1}Y_{t+1}\right) = \beta\,(1+n)\,(1+i_t)\,P_t Y_t - (1-\beta)\,n\Omega_0 \tag{3.14}$$

Now since $M_t = P_t C_t = P_t Y_t$ this can be rewritten:

$$E_t M_{t+1} = \beta\,(1+n)\,(1+i_t)\,M_t - (1-\beta)\,n\Omega_0 \tag{3.15}$$

3.5 Liquidity effects

We shall now see that the non-Ricardian character of the economy, i.e. the fact that $n > 0$, will produce a liquidity effect.

3.5.1 The nominal interest rate

We can actually solve explicitly equation (15) for the nominal interest rate:

$$1 + i_t = \frac{1}{\beta\,(1+n)} E_t\left(\frac{M_{t+1}}{M_t}\right) + \frac{(1-\beta)\,n\Omega_0}{\beta\,(1+n)\,M_t} \tag{3.16}$$

We see that the first term, which is present even if $n = 0$, displays the "inflationary expectations effect": indeed, the nominal interest rate will rise if a positive monetary shock announces future money growth, i.e. if:

$$\frac{\partial}{\partial M_t}\left[E_t\left(\frac{M_{t+1}}{M_t}\right)\right] > 0 \tag{3.17}$$

which is what is generally found empirically. In the example above (equation 9) this will occur if $\rho > 0$. We shall assume in what follows that the money process satisfies condition (17).

Now the second term, which appears only if $n > 0$, i.e. if we are in a non-Ricardian framework, clearly introduces a liquidity effect, since an increase in money directly decreases the nominal interest rate. The higher n, the stronger this effect.

We can give an even simpler expression. Assume that money M_t is stationary around the value M_0. From (15), the corresponding stationary value of the interest rate, i_0, is related to M_0 and Ω_0 by:

$$M_0 = \beta (1+n)(1+i_0) M_0 - (1-\beta) n \Omega_0 \tag{3.18}$$

Let us define the composite parameter:

$$\theta = \beta (1+n)(1+i_0) \tag{3.19}$$

If we want to have a Pigou effect, net financial assets must be positive, i.e. $\Omega_0 > 0$. As a consequence, from (18) the parameter θ must satisfy:

$$\theta > 1 \tag{3.20}$$

Now combining (16), (18) and (19), we obtain:

$$\frac{1+i_t}{1+i_0} = \frac{1}{\theta} E_t \left(\frac{M_{t+1}}{M_t} \right) + \left(1 - \frac{1}{\theta} \right) \frac{M_0}{M_t} \tag{3.21}$$

We see that formula (21) gives a balanced view between the new non-Ricardian liquidity effect and the traditional inflationary expectations effect. We can note that the higher θ (and thus notably the higher n), the stronger the liquidity effect will be.

3.5.2 The real interest rate

Recall the definition of the gross real interest rate R_t:

$$R_t = 1 + r_t = (1+i_t) \frac{P_t}{P_{t+1}} \tag{3.22}$$

Now $Y_{t+1}/Y_t = (1+n)\zeta$. Combining this with $M_t = P_t Y_t$, and equations (18), (19) and (21), we obtain:

$$\frac{1}{R_t} = \frac{\beta}{\zeta} \frac{M_{t+1}/M_t}{E_t (M_{t+1}/M_t) + (\theta - 1)(M_0/M_t)} \tag{3.23}$$

and:

$$E_t \left(\frac{1}{R_t} \right) = \frac{\beta}{\zeta} \frac{E_t (M_{t+1}/M_t)}{E_t (M_{t+1}/M_t) + (\theta - 1)(M_0/M_t)} \tag{3.24}$$

In view of assumption (17) we see that the real interest rate will react negatively to a positive money shock, in the sense that:

$$\frac{\partial}{\partial M_t}\left[E_t\left(\frac{1}{R_t}\right)\right] > 0 \tag{3.25}$$

This real interest rate effect will counteract the inflationary expectations effect, and is at the basis of the liquidity effect.

3.6 A stronger liquidity effect

We shall now see that the liquidity effect is strengthened if we consider the more general case, seen in chapter 2, section 2.6, where households' resources evolve over time as follows:

$$y_{jt} = \psi^{t-j} y_t \qquad \tau_{jt} = \psi^{t-j}\tau_t \qquad \psi \leq 1 \tag{3.26}$$

where y_t and τ_t are the income and taxes of a newborn agent in period t. Now we have the following proposition, which generalizes proposition 3.1:

Proposition 3.2 *Consider a non-Ricardian economy* $(n > 0)$ *where endowments and taxes evolve according to* (26). *Then the dynamics of nominal income is given by:*

$$\psi E_t\left(P_{t+1}Y_{t+1}\right) = \beta\left(1+n\right)\left(1+i_t\right)P_tY_t - \left(1-\beta\right)\left(1+n-\psi\right)\Omega_{t+1} \tag{3.27}$$

Proof: Bénassy (2007).

Now let us use $M_t = P_tC_t = P_tY_t$ and the fact that, under fiscal policy (12), $\Omega_t = \Omega_0$ for all t. Equation (27) becomes:

$$\psi E_t M_{t+1} = \beta\left(1+n\right)\left(1+i_t\right)M_t - \left(1-\beta\right)\left(1+n-\psi\right)\Omega_0 \tag{3.28}$$

We solve for the nominal interest rate:

$$1 + i_t = \frac{\psi}{\beta\left(1+n\right)}E_t\left(\frac{M_{t+1}}{M_t}\right) + \frac{\left(1-\beta\right)\left(1+n-\psi\right)\Omega_0}{\beta\left(1+n\right)M_t} \tag{3.29}$$

in which we see that the second term does indeed produce a liquidity effect. Now from (28) the stationary values of M_0, Ω_0 and i_0 are related by:

$$\psi M_0 = \beta (1+n)(1+i_0) M_0 - (1-\beta)(1+n-\psi)\Omega_0 \tag{3.30}$$

Let us now give a more general definition of the parameter θ:

$$\theta = \frac{\beta (1+n)(1+i_0)}{\psi} \tag{3.31}$$

Combining (29), (30) and (31) we find that the nominal interest rate is given by:

$$\frac{1+i_t}{1+i_0} = \frac{1}{\theta} E_t \left(\frac{M_{t+1}}{M_t} \right) + \left(1 - \frac{1}{\theta} \right) \frac{M_0}{M_t} \tag{3.32}$$

This is exactly the same expression as (21), but the expression of θ has been generalized from (19) to (31). We see that a lower value of ψ increases the value of θ and therefore enhances the non-Ricardian liquidity effect. In the extreme case where $\psi = 0$ (i.e. when agents have all their income in the first period of their life), θ is infinite and the liquidity effect totally dominates.

We may note that the above result gives us an explanation of why the liquidity effect always dominated in the OLG model in chapter 1. Indeed households in that model have no resources in the second period, which makes that OLG model similar to the model of this section with $\psi = 0$, i.e. the case where, as we just saw, the liquidity effect fully dominates.

3.7 The persistence of the liquidity effect

We shall now show that we have not only a liquidity effect, but that this effect can be quite persistent. Let us loglinearize equations (21) or (32), which yields:

$$\frac{i_t - i_0}{1+i_0} = \frac{1}{\theta} (E_t m_{t+1} - m_t) - \left(1 - \frac{1}{\theta} \right) (m_t - m_0) \tag{3.33}$$

where the Ricardian particular case is obtained by taking $\theta = 1$. Let us consider the following stationary money process:

$$m_t - m_0 = \frac{\varepsilon_t}{(1-\rho\mathcal{L})(1-\mu\mathcal{L})} \qquad 0 < \rho < 1 \quad 0 < \mu < 1 \tag{3.34}$$

where ε_t is i.i.d.. Then:

$$E_t m_{t+1} - m_t = \frac{(\mu + \rho - 1 - \mu\rho\mathcal{L})\varepsilon_t}{(1-\rho\mathcal{L})(1-\mu\mathcal{L})} \tag{3.35}$$

If $\mu + \rho > 1$, then a positive monetary innovation $\varepsilon_t > 0$ creates the expectation of a monetary increase next period, which is the assumption traditionally associated with the "inflationary expectations effect" (equation 17). We shall assume $\mu + \rho > 1$ so as to have this effect.

Now combining (33), (34) and (35) we can compute the full effect of monetary shocks on the interest rate:

$$\frac{i_t - i_0}{1 + i_0} = \frac{(\mu + \rho - \theta - \mu\rho\mathcal{L})\,\varepsilon_t}{\theta\,(1 - \rho\mathcal{L})\,(1 - \mu\mathcal{L})} \tag{3.36}$$

We first see that if $\mu + \rho > 1$ the Ricardian version of the model ($\theta = 1$) always delivers an increase in interest rates on impact in response to monetary injections. We thus obtain the traditional "inflationary expectations effect".

Let us now move to the non-Ricardian case $\theta > 1$. Looking at formula (36), we see that the first period impact $\mu + \rho - \theta$ is negative as soon as:

$$\theta > \mu + \rho \tag{3.37}$$

We shall further see that this liquidity effect is persistent, and that condition (37) is actually sufficient for a monetary injection to have a negative effect on the interest rate, not only in the current period, but in all subsequent periods as well. Formula (36) can indeed be rewritten as:

$$\frac{i_t - i_0}{1 + i_0} = \frac{1}{\theta\,(\mu - \rho)}\left[\frac{\rho\,(\theta - \rho)\,\varepsilon_t}{1 - \rho\mathcal{L}} - \frac{\mu\,(\theta - \mu)\,\varepsilon_t}{1 - \mu\mathcal{L}}\right] \tag{3.38}$$

This can be expressed as a distributed lag of all past innovations in money ε_{t-j}, $j \geq 0$:

$$\frac{i_t - i_0}{1 + i_0} = \sum_{j=0}^{\infty} \varkappa_j \varepsilon_{t-j} \tag{3.39}$$

with:

$$\varkappa_j = \frac{\rho^{j+1}\,(\theta - \rho) - \mu^{j+1}\,(\theta - \mu)}{\theta\,(\mu - \rho)} \tag{3.40}$$

We want to show now that condition (37) is a sufficient condition for $\varkappa_j < 0$ for all j. This is done simply by rewriting (40) as:

$$\varkappa_j = \frac{\mu + \rho - \theta}{\theta}\left(\frac{\mu^{j+1} - \rho^{j+1}}{\mu - \rho}\right) - \frac{\mu\rho}{\theta}\left(\frac{\mu^j - \rho^j}{\mu - \rho}\right) \tag{3.41}$$

The second term is always negative or zero. The first term is negative if $\theta > \mu + \rho$. So condition (37) is sufficient for the non-Ricardian liquidity effect

to dominate the usual inflationary expectations effect, not only on impact, but for all subsequent periods as well.

3.8 Conclusions

We developed in this chapter a new mechanism through which liquidity effects are introduced into dynamic monetary models.

The basic channel is the following: (a) in a non Ricardian economy, accumulated financial assets represent, at least partly, real wealth to the generations alive (the Pigou effect) and: (b) this Pigou effect gives rise to a liquidity effect as follows: An increase in money raises prices, which decreases the real value of financial wealth. Because of the wealth effect this reduces aggregate demand. In order to maintain aggregate demand at the market clearing level the real interest rate goes down. This creates, ceteris paribus, the liquidity effect.

3.9 References

This chapter is adapted from Bénassy (2006b).

The liquidity effect dates back to Keynes (1936) and Hicks (1937). Some evidence is provided, for example, by Christiano and Eichenbaum (1992).

One can find in the earlier literature a few DSGE models which produce a liquidity effect with different mechanisms. Two prominent ones are:

- Models of limited participation (Lucas, 1990, Christiano and Eichenbaum, 1992, Fuerst, 1992), where households cannot adapt immediately their financial portfolios when a monetary policy shock occurs.

- Models of sticky prices (Jeanne, 1994, Christiano, Eichenbaum and Evans, 1997), where prices are preset in advance. The liquidity effect occurs if the intertemporal elasticity of substitution in consumption is sufficiently low.

Chapter 4

Interest Rate Rules and Price Determinacy

4.1 Introduction

We continue our investigation of monetary issues in non-Ricardian economies with a topic that has been largely debated in recent years, that of price determinacy under interest rate rules where the nominal interest rate reacts to various endogenous variables, and particularly to the rate of inflation. Indeed following Taylor's (1993) seminal article, there has been recently a very strong renewal of interest in the study such interest rate rules. We shall in this chapter study this issue in the framework of dynamic non Ricardian models. We shall notably scrutinize two particularly famous results:

- The first one, which originates with the article by Sargent and Wallace (1975), basically says that, under a pure nominal interest rate peg, there is nominal indeterminacy, as we saw in chapter 1.

- The second one is often referred to as the "Taylor principle"[1]. The basic idea is that, in order to make prices determinate the central bank should respond "aggressively" to inflation. If interest rates respond only to inflation, a classic result is that, in order to have determinate prices, nominal interest rates should respond more than hundred percent to inflation.

As we will see, these two results turn out to be true in rigorous models of "Ricardian" economies populated with a single dynasty of consumers. But, as we saw in chapter 1, these economies have, as far as monetary issues

[1]It should be noted that, although Taylor (1993) recommends a strong response of interest rates to inflation, this is more for optimality reasons than to ensure price determinacy as in this chapter. Optimality aspects of the Taylor principle are studied in Bénassy (2006a, 2007).

are concerned, a number of peculiar properties. So we shall in this and the next chapters extend the analysis of interest rate rules to non Ricardian economies where new agents enter in each period, and see whether this makes a difference or not for the analysis. We shall see that it does.

We shall actually see that considering non-Ricardian instead of Ricardian economies dramatically modifies the answers to the two above questions. Notably:

- A pure interest rate peg is fully consistent with local price determinacy, provided the interest rate satisfies a natural "rate of return" condition.

- Prices can be determinate even if the nominal interest rate responds less than hundred percent to inflation.

4.2 The model and policy

The model is exactly the same as the model of chapter 2 (section 2). We shall be, however, a little more specific about government and policy. The government includes a fiscal authority (which sets taxes) and a monetary authority (which sets nominal interest rates).

4.2.1 Monetary policies

Unlike in the previous chapter, where the quantity of money was the instrument of monetary policy, we shall consider interest rate rules where the nominal interest rate is the central instrument of monetary policy.

As indicated in the introduction, we shall study two types of monetary policies. The first is interest rate pegging, which consists in setting the nominal interest rate i_t exogenously. In most of what follows we shall, for the simplicity of exposition, take the particular case where the interest rate is pegged at a constant value:

$$i_t = i_0 \qquad \forall t \tag{4.1}$$

The second type of policy we shall consider consists of "Taylor rules" (Taylor, 1993), through which the nominal interest rate responds to inflation. Let us denote the inflation rate as:

$$\pi_t = \log \Pi_t = \log \left(\frac{P_t}{P_{t-1}} \right) \tag{4.2}$$

A typical Taylor rule will be written in loglinear way[2]:

$$i_t - i_0 = \phi(\pi_t - \pi_0) \qquad \phi \geq 0 \tag{4.3}$$

where π_0 is the long run rate of inflation and i_0 a target interest rate. The "Taylor principle" suggests that, for prices to be determinate, the coefficient ϕ should be greater than 1.

4.2.2 Fiscal policy

Since our focus is not on fiscal policy, in order to simplify the dynamics below, we shall in a first step assume that the tax policy of the government consists in balancing the budget period by period[3]. Taxes will thus cover exactly interest payments on bonds:

$$P_t T_t = i_t B_t \tag{4.4}$$

4.3 The dynamic equilibrium

We have seen in chapter 2 (proposition 2.1) that the following dynamic equation holds:

$$P_{t+1}Y_{t+1} = \beta(1+n)(1+i_t)P_tY_t - (1-\beta)n\Omega_{t+1} \tag{4.5}$$

Now let us recall from the previous chapter the government budget constraint:

$$\Omega_{t+1} = (1+i_t)\Omega_t - i_t M_t - P_t T_t \tag{4.6}$$

Combining (4), (6) and $\Omega_t = M_t + B_t$ we immediately see that under the balanced budget policy (4) total financial wealth will remain constant:

$$\Omega_t = \Omega_0 \qquad \text{for all } t \tag{4.7}$$

The dynamic equation (5) then becomes:

$$P_{t+1}Y_{t+1} = \beta(1+n)(1+i_t)P_tY_t - (1-\beta)n\Omega_0 \tag{4.8}$$

[2]We can use a loglinear approximation because in this chapter we will study local determinacy only. Global determinacy is studied in the next chapter.

[3]A more general policy is considered in section 4.9.

4.4 Ricardian economies and the Taylor principle

We shall now briefly review some traditional results on price determinacy under interest rate rules in the Ricardian setting. In the Ricardian model $n = 0$, and equation (5) becomes:

$$P_{t+1}Y_{t+1} = \beta (1 + i_t) P_t Y_t \tag{4.9}$$

which is the traditional aggregate dynamic equation. Using this equation and the intertemporal budget constraint we already saw in chapter 1 that exogenously pegging the nominal interest rate i_t was leading to nominal indeterminacy, as was pointed out by Sargent and Wallace (1975).

4.4.1 The Taylor principle

Let us now consider more general interest rate rules of the form (equation 3):

$$i_t - i_0 = \phi (\pi_t - \pi_0) \qquad \phi \geq 0 \tag{4.10}$$

and loglinearize equation (9), using $Y_{t+1}/Y_t = \zeta$:

$$\pi_{t+1} = i_t + Log (\beta/\zeta) - n \tag{4.11}$$

Inserting (10) into (11), we obtain:

$$\pi_{t+1} = \phi (\pi_t - \pi_0) + i_0 + Log (\beta/\zeta) - n \tag{4.12}$$

which can be rewritten as:

$$\pi_t = \frac{\pi_{t+1}}{\phi} + \frac{\phi\pi_0 - i_0 - Log (\beta/\zeta) + n}{\phi} \tag{4.13}$$

Clearly the inflation rate will be determinate if $\phi > 1$ (the Taylor principle). Since the past price is predetermined, a determinate inflation rate also means a determinate price. So the Taylor principle holds in this Ricardian framework, at least for local determinacy.

4.5 Determinacy under an interest rate peg

We now revert to the more general non-Ricardian framework, and consider the first problem we mentioned, that of a pure interest rate peg. We shall

first study the Walrasian version of the model. To simplify the exposition we will consider here the particular case where the pegged interest rate is constant in time:

$$i_t = i_0 \qquad \forall t \tag{4.14}$$

The case of a variable pegged interest rate is treated in appendix 4.1. With (14) the dynamic equation (8) is written:

$$P_{t+1}Y_{t+1} = \beta\,(1+n)\,(1+i_0)\,P_tY_t - (1-\beta)\,n\Omega_0 \tag{4.15}$$

In what follows it will be convenient to use nominal income $\mathcal{Y}_t$ as our working variable:

$$\mathcal{Y}_t = P_tY_t \tag{4.16}$$

so that (15) is rewritten:

$$\mathcal{Y}_{t+1} = \beta\,(1+n)\,(1+i_0)\,\mathcal{Y}_t - (1-\beta)\,n\Omega_0 \tag{4.17}$$

We see that there is a locally determinate solution in $\mathcal{Y}_t$ provided that:

$$\theta = \beta\,(1+n)\,(1+i_0) > 1 \tag{4.18}$$

and this solution is given by:

$$\mathcal{Y}_t = \mathcal{Y}_0 = \frac{(1-\beta)\,n\Omega_0}{\beta\,(1+n)\,(1+i_0) - 1} \tag{4.19}$$

4.6 Taylor rules

Let us continue with the non-Ricardian model and turn to the more general Taylor rules (3). To see whether the Taylor principle still holds, we loglinearize equation (8), and obtain the following equation:

$$p_{t+1} + y_{t+1} = \theta\,(p_t + y_t) + \varrho\,(i_t - i_0) \tag{4.20}$$

with:

$$\theta = \beta\,(1+n)\,(1+i_0) \qquad \varrho = \beta\,(1+n) \tag{4.21}$$

Combining with the equation giving the interest rate we obtain, omitting irrelevant constant terms:

$$p_{t+1} = \theta p_t + \varrho\phi\pi_t \tag{4.22}$$

This can actually be rewritten as a two dimensional dynamic system in inflation and the price level:

$$p_t = \pi_t + p_{t-1} \tag{4.23}$$

$$\pi_{t+1} = (\theta - 1)\, p_t + \phi\varrho\pi_t = (\theta - 1 + \varrho\phi)\, \pi_t + (\theta - 1)\, p_{t-1} \tag{4.24}$$

or in matrix form, lagging variables one period:

$$\begin{bmatrix} \pi_t \\ p_{t-1} \end{bmatrix} = \begin{bmatrix} \theta - 1 + \varrho\phi & \theta - 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ p_{t-2} \end{bmatrix} \tag{4.25}$$

The characteristic polynomial is:

$$\Psi(\lambda) = \phi\varrho(1 - \lambda) + \lambda(\lambda - \theta) = \lambda^2 - (\theta + \phi\varrho)\lambda + \phi\varrho \tag{4.26}$$

We have one predetermined variable (the past price) and a non predetermined one (inflation). So, applying the Blanchard-Kahn (1980) conditions, there will be a determinate solution if the polynomial $\Psi(\lambda)$ has one root of modulus smaller than 1, and the other greater than 1. So we compute:

$$\Psi(0) = \varrho\phi \geq 0 \tag{4.27}$$

$$\Psi(1) = 1 - \theta \tag{4.28}$$

Since furthermore $\Psi(\lambda)$ goes to infinity when λ goes to infinity, we see that, if $\theta > 1$, we have one root between zero and one, and the other greater than one. So $\theta > 1$ is again a sufficient condition for local determinacy, and whether ϕ is above or below 1 is not important anymore.

4.7 Economic interpretations

We just found that prices will be determined if $n > 0$ and condition (18), i.e. $\theta > 1$, is satisfied. This holds both for an interest rate peg and for a Taylor rule like (3). This is a very substantial change, so it is time to give some economic interpretations.

4.7.1 The Pigou, or real balance effect

When one looks at the dynamic equations (5) and (8), it appears clearly that a feature that drives most of the results is the presence of financial assets Ω_t in the dynamic equations. This is indeed a "nominal anchor", which is intrumental in tying down the value of prices. We already mentioned that this presence of financial assets in various behavioral equations has a history in the literature under the names of "Pigou effect" (Pigou, 1943) or "real balance effect" (Patinkin, 1956), and its importance appears again here.

4.7.2 Determinacy and the return on financial assets

Now $n > 0$ creates a Pigou effect. But this not the end of the story. Clearly this effect will be really operative only if the agents actually want to hold money and financial assets. And this is where the central condition (18) comes in. In order to interpret it, let us rewrite (18) under the following form:

$$\zeta (1+n)(1+i_0) > \frac{\zeta}{\beta} \tag{4.29}$$

The left hand side is the real rate of return on bonds. Indeed since $P_t Y_t = \mathcal{Y}_t$ is constant, and real resources grow at the rate $\zeta(1+n)$, in the steady state prices decrease at the rate $\zeta(1+n)$, and therefore the real rate of interest is $\zeta(1+n)(1+i_0)$.

Now ζ/β on the right hand side of (29) is the "autarkic" gross real interest rate ξ, i.e. the real rate of return that would prevail if agents of each generation traded only between themselves, in total autarky from the other generations (chapter 2, section 2.7).

So conditions (18) or (29) essentially say that the real rate of return of bonds must be superior to the autarkic rate of return. We see that the above condition is very much similar to that found by Wallace (1980) for the viability of money in the traditional Samuelsonian (1958) overlapping generations model. There is a difference, though: in Wallace (1980) the only financial store of value is money, so the rate of return condition concerns the return on money. Here this condition concerns the return on bonds, and accordingly the nominal interest rate plays an important role.

We shall call this condition the "financial dominance" (FD) criterion. A more general version will be given in the next chapter when we study global determinacy.

4.8 The Taylor principle with a Phillips curve

So far we have studied the issue of price determinacy under the assumption of full market clearing. But the issue of price determinacy under interest rate rules has been very often studied in models with non clearing markets where output is demand determined and prices adjust partially according to a forward looking "Phillips curve" of the type:

$$\pi_t = \frac{1}{f} E_t \pi_{t+1} + g y_t \qquad f > 1 \qquad g > 0 \tag{4.30}$$

We want to show now that the results we obtained above in a Walrasian economy extend to this framework as well. Clearly the rigorous derivation of such a Phillips curve in our setting would take us a bit too far, notably with an infinity of households, all with different marginal utilities of income. So we shall simply take the Phillips curve (30) as given, and show that going from a Ricardian to a non-Ricardian framework leads again to major changes.

As before the monetary authority uses an interest rate rule of the Taylor type:

$$i_t - i_0 = \phi \left(\pi_t - \pi_0 \right) \tag{4.31}$$

In order to better highlight the differences, let us now begin with the Ricardian version of the model.

4.8.1 The Ricardian case

Output is now endogenous, and assumed to be demand determined, so equation (8) is still valid. Loglinearizing it we obtain:

$$y_{t+1} = y_t + Log\beta + (i_t - \pi_{t+1}) \tag{4.32}$$

Combining this with the interest rule (31) yields:

$$y_{t+1} = y_t + Log\beta + i_0 + \phi \left(\pi_t - \pi_0 \right) - \pi_{t+1} \tag{4.33}$$

Equations (30) and (33) are rewritten, replacing $E_t \pi_{t+1}$ by π_{t+1}, since the model is deterministic, and omitting constants:

$$\pi_{t+1} = f \left(\pi_t - g y_t \right) \tag{4.34}$$

$$y_{t+1} = (1 + fg) y_t + (\phi - f) \pi_t \tag{4.35}$$

This is written under matrix form:

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1+fg & \phi - f \\ -fg & f \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \end{bmatrix} \tag{4.36}$$

The characteristic polynomial is:

$$\Psi(\lambda) = \lambda^2 - (1 + f + fg)\lambda + f(1 + g\phi) \tag{4.37}$$

$$\Psi(0) = f(1 + g\phi) > 0 \tag{4.38}$$

$$\Psi(1) = fg(\phi - 1) \tag{4.39}$$

If $\phi < 1$, we have one root between 0 and 1. Since neither y_t and π_t are predetermined, this means that we have indeterminacy. On the other hand, if $\phi > 1$ the two roots have modulus greater than 1, and we have determinacy. We thus find again that the Taylor principle holds in this Ricardian framework.

4.8.2 The non-Ricardian case

Let us now move to the non-Ricardian economy. Equation (8) still holds. Loglinearizing it, we find that output, inflation and prices are linked by the following equation:

$$y_{t+1} + p_{t+1} = \theta(y_t + p_t) + \varrho(i_t - i_0) \tag{4.40}$$

where the values of θ and ϱ are given in equation (21). We now express y_{t+1}, π_{t+1} and p_t as a function of the corresponding lagged variables:

$$p_t = \pi_t + p_{t-1} \tag{4.41}$$

$$\pi_{t+1} = f(\pi_t - gy_t) \tag{4.42}$$

$$y_{t+1} = (\theta + fg)y_t + (\theta - 1 + \phi\varrho - f)\pi_t + (\theta - 1)p_{t-1} \tag{4.43}$$

or in matrix form (omitting the constants):

$$\begin{bmatrix} y_t \\ \pi_t \\ p_{t-1} \end{bmatrix} = \begin{bmatrix} \theta + fg & \theta - 1 + \phi\varrho - f & \theta - 1 \\ -fg & f & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ p_{t-2} \end{bmatrix} \tag{4.44}$$

The characteristic polynomial is:

$$\Psi(\lambda) = (1-\lambda)(f-\lambda)(\theta-\lambda) + fg\varrho\phi(1-\lambda) + fg\lambda(\lambda-\theta) \tag{4.45}$$

We shall now show that $\theta > 1$ is again a sufficient condition for determinacy. There is one predetermined variable (the past price) and two non predetermined ones (output and inflation). So there will be a determinate solution if the polynomial $\Psi(\lambda)$ has one root of modulus smaller than 1, and two roots of modulus greater than 1. Let us compute:

$$\Psi(0) = f(\theta + g\varrho\phi) > 0 \tag{4.46}$$

$$\Psi(1) = fg(1-\theta) \tag{4.47}$$

So there is, assuming $\theta > 1$, one root between zero and one. Now the product of the three roots is $\Psi(0) = f(\theta + g\varrho\phi) > 1$. So the only possible case where the two remaining roots would not be of modulus greater than 1 would be that where we have two negative roots, one smaller than -1, one greater. In that case we would have $\Psi(-1) < 0$. This means that, together with $\theta > 1$, $\Psi(-1) > 0$ is a sufficient condition for determinacy. So we compute:

$$\Psi(-1) = 2(1+f)(1+\theta) + 2fg\varrho\phi + fg(1+\theta) > 0 \tag{4.48}$$

To summarize, if $\theta > 1$, we have one root between zero and one, and two roots of modulus greater than one, so that the inflation rate is determinate, and thus so is the price level.

4.9 Generalizations

We shall now study two generalizations of the model we have studied so far. The first one replaces the hypothesis of budget balance by the possibility of constant growth of government liabilities. The second one introduces decreasing resources over time, as in chapter 2, section 2.6. This last generalization will allow us notably to explain why there was no problem of determinacy in the OLG model of chapter 1.

4.9.1 Variable government liabilities

We shall now study a generalization of the fiscal policy (4) and assume that, instead of balancing the budget, the government engineers through taxes

proportional expansions (or reductions) of its financial liabilities Ω_t (such an experiment was studied in Wallace, 1980), and we shall see how this affects the conditions for determinacy. More precisely we shall assume taxes of the form:

$$P_t T_t = i_t B_t + (1-\gamma)\,\Omega_t \qquad \gamma > 0 \tag{4.49}$$

As a result the evolution of Ω_t is given by, combining (6), (49) and $\Omega_t = M_t + B_t$:

$$\Omega_{t+1} = \gamma \Omega_t \tag{4.50}$$

Most of the analysis seen previously is still valid, and in particular equation (5) which we recall here:

$$P_{t+1}Y_{t+1} = \beta\,(1+n)\,(1+i_t)\,P_t Y_t - (1-\beta)\,n\Omega_{t+1} \tag{4.51}$$

The dynamic system consists of equations (50) and (51). Dividing (51) by (50) we obtain:

$$\frac{P_{t+1}Y_{t+1}}{\Omega_{t+1}} = \frac{\beta\,(1+n)\,(1+i_t)}{\gamma}\frac{P_t Y_t}{\Omega_t} - (1-\beta)\,n \tag{4.52}$$

We can first study the determinacy conditions for a pure interest rate peg $i_t = i_0$. Inserting this into (52), we see that the condition for determinacy is:

$$\beta\,(1+n)\,(1+i_0) > \gamma \tag{4.53}$$

or $\theta > \gamma$. We may first note that this equation has an interpretation very similar to that of equation (18) that we saw before. Indeed it can be rewritten:

$$\frac{\zeta\,(1+n)\,(1+i_0)}{\gamma} > \frac{\zeta}{\beta} \tag{4.54}$$

Since nominal assets are growing at the rate γ, the long run rate of inflation is $\gamma/\zeta\,(1+n)$, so that the left hand side is the real rate of return on financial assets, and the rest of the intuition given in section 4.7 continues to hold.

We shall now see that the "expanded" condition (53) is actually sufficient for determinacy in all the non-Ricardian cases we have been considering in sections 4.6 and 4.8. Let us indeed loglinearize equation (52). We obtain:

$$p_{t+1} + y_{t+1} - \omega_{t+1} = \frac{\theta}{\gamma}\,(p_t + y_t - \omega_t) + \frac{\varrho\phi}{\gamma}\,(\pi_t - \pi_0) \tag{4.55}$$

where θ and ϱ are the same as in (21). We see that all the analysis we carried in the previous sections will be valid provided we replace p_t by $p_t - \omega_t$ and the parameters θ and ϱ by θ/γ and ϱ/γ respectively.

We should note that condition (53) shows most clearly the tradeoffs faced by the government on fiscal and monetary policy. Indeed a stricter fiscal policy (low γ) allows to lead a less rigorous monetary policy (low i_0), and conversely a stricter monetary policy (high i_0) allows to lead a less rigorous fiscal policy (high γ).

4.9.2 Decreasing resources

We shall now consider a second generalization, and assume that relative endowments and taxes decrease in time at the rate $\psi \leq 1$ as follows:

$$y_{jt} = \psi^{t-j} y_t \qquad \tau_{jt} = \psi^{t-j} \tau_t \qquad j \leq t \tag{4.56}$$

where y_t and τ_t are the income and taxes of a newborn agent in period t. We saw in chapter 2 (proposition 2.2) that under this more general hypothesis the dynamic equation (5) is replaced by:

$$\psi P_{t+1} Y_{t+1} = \beta (1+n)(1+i_t) P_t Y_t - (1-\beta)(1+n-\psi)\Omega_{t+1} \tag{4.57}$$

Let us consider again the case of a nominal interest rate peg $i_t = i_0$ with the balanced budget fiscal policy (4). Then (57) is rewritten:

$$\psi P_{t+1} Y_{t+1} = \beta (1+n)(1+i_0) P_t Y_t - (1-\beta)(1+n-\psi)\Omega_0 \tag{4.58}$$

and the condition for determinacy becomes:

$$\frac{\beta (1+n)(1+i_0)}{\psi} > 1 \tag{4.59}$$

We see that with a low value of ψ this determinacy condition becomes easier to satisfy.

In chapter 1 we had studied an overlapping generations model where, unlike in the Ricardian model, price determinacy was always ensured. We shall now see that we can mimick this result with the model of this section. Households in the OLG model of chapter 1 had no resources in the second period of their lives. This would correspond here to the value $\psi = 0$. Now if we insert this value into (58), we immediately find:

$$P_t Y_t = \frac{(1-\beta)\,\Omega_0}{\beta\,(1+i_0)} \tag{4.60}$$

We see that we also have full price determinacy.

4.10 Conclusions

We have seen that going from a Ricardian to a non Ricardian framework changes dramatically the conditions for local price determinacy under interest rate rules. It is usually found in a Ricardian framework that interest rate pegging leads to nominal indeterminacy, and that a more than one to one response of interest rates to inflation (the Taylor principle) leads to price determinacy.

We found instead that a strong response of the interest rate rule to inflation is not necessary for price determinacy, which can be achieved even under an interest rate peg. We identified sufficient conditions for determinacy (conditions 18 or 53), which express that the real rate of return on nominal assets must be superior to the "autarkic" real rate of return that would prevail if each generation had no trade with other generations. This condition ensures that agents will be actually willing to hold money and financial assets in the long run, obviously a critical condition if one wants money to have value, and prices to be determinate.

Now all the determinacy conditions we derived in this chapter are local determinacy conditions. In the next chapter we shall pose the more demanding question of global determinacy.

4.11 References

This chapter is adapted from Bénassy (2005). Early contributions to the problem of price determinacy under an interest peg in non Ricardian economies are found in Bénassy (2000) and Cushing (1999).

Early rigorous results on price determinacy in a Ricardian model are found in Leeper (1991). The Taylor principle was initiated in Taylor (1993, 1998). Surveys on interest rate rules can be found for example in McCallum (1999), Taylor (1999).

The financial dominance criterion appears in Wallace (1980) in an OLG model with money as a single store of value and Bénassy (2005) in a model with both money and interest bearing assets.

A few contributions have sought to modify the "traditional" results on interest pegging or the Taylor principle. For example Benhabib, Schmitt-Grohé and Uribe (2001b) show the importance of how money enters the utility and production functions. McCallum (1981) advocates linking directly interest rates to a "nominal anchor" like the price level. Roisland (2003) shows that capital income taxation modifies the Taylor principle.

The Phillips curve initiates in Phillips (1958). Forward looking Phillips curves such as (30) are most often derived from a framework of contracts à la Calvo (1983). They can be also derived from a model with convex costs of changing prices (Rotemberg, 1982a,b). See Rotemberg (1987) for an early derivation under both interpretations.

4.12 Appendix 4.1: Interest rate pegging with variable interest rates

We shall consider here the case where the pegged interest rate can vary in time. Equation (17) is replaced by:

$$\mathcal{Y}_{t+1} = \beta\left(1+n\right)\left(1+i_t\right)\mathcal{Y}_t - \left(1-\beta\right)n\Omega_0 \tag{4.61}$$

This can be rewritten as:

$$\mathcal{Y}_t = \frac{\mathcal{Y}_{t+1} + \left(1-\beta\right)n\Omega_0}{\beta\left(1+n\right)\left(1+i_t\right)} \tag{4.62}$$

A sufficient condition for determinacy is:

$$\beta\left(1+n\right)\left(1+i_t\right) > 1 \qquad \forall t \tag{4.63}$$

Let us use again the discount factors:

$$D_t = \prod_{s=0}^{t-1} \frac{1}{1+i_s} \tag{4.64}$$

Using the discount factors (64), equation (62) can be rewritten:

$$\mathcal{Y}_t = \frac{D_{t+1}}{D_t}\frac{\mathcal{Y}_{t+1} + \left(1-\beta\right)n\Omega_0}{\beta\left(1+n\right)} \tag{4.65}$$

If condition (63) is satisfied, this can be integrated forward:

$$\mathcal{Y}_t = \frac{\left(1-\beta\right)n\Omega}{D_t}\sum_{i=1}^{\infty}\frac{D_{t+i}}{\beta^i\left(1+n\right)^i} \tag{4.66}$$

Chapter 5

Global Determinacy

5.1 Introduction

We have seen in the previous chapter that moving from a Ricardian to a non Ricardian framework brought major changes to the conditions of price determinacy in response to interest rate rules. In particular we saw that, although the Taylor principle is rightly considered as a condition for price determinacy (at least a local one) in Ricardian economies, in non Ricardian economies an other criterion, the financial dominance criterion, emerged as a relevant alternative.

Now the analysis of the previous chapter is about local determinacy, and we shall study in this chapter the same issue from the point of view of global determinacy. We will find again that in a non-Ricardian framework the Taylor principle is not anymore the central determinacy condition. On the other hand the "financial dominance" (FD) criterion, which we began studying in the previous chapter, appears to be also essential not only for local determinacy, but for global determinacy as well.

5.2 The model

We shall use the same model as in the previous chapter (it is described in chapter 2, section 2). In particular this model is characterized by the two following dynamic equations:

$$\Omega_{t+1} = (1 + i_t)\,\Omega_t - i_t M_t - P_t T_t \tag{5.1}$$

$$P_{t+1}Y_{t+1} = \beta\,(1 + n)\,(1 + i_t)\,P_t Y_t - (1 - \beta)\,n\Omega_{t+1} \tag{5.2}$$

We shall now further examine more precisely the two governmental policies, monetary and fiscal policies.

5.2.1 Monetary policy

In the preceding chapter we studied local determinacy, and accordingly used a loglinearized version of the interest rate rule. Here we shall assume that monetary policy takes the form of "Taylor rules" linking the value of the nominal interest rate to inflation, but this time under the more general form[1]:

$$1 + i_t = \Phi(\Pi_t) \tag{5.3}$$

with:

$$\Pi_t = \frac{P_t}{P_{t-1}} \tag{5.4}$$

This interest rate rule must respect the zero lower bound on the nominal interest rate:

$$\Phi(\Pi_t) \geq 1 \qquad \forall \Pi_t \tag{5.5}$$

We shall further assume:

$$\Phi'(\Pi_t) \geq 0 \tag{5.6}$$

An important parameter is the elasticity ϕ of the function Φ:

$$\phi(\Pi_t) = \frac{\Pi_t \Phi(\Pi_t)}{\Phi(\Pi_t)} \tag{5.7}$$

As we already indicated, the "Taylor principle" says that this elasticity should be greater than 1. Note that, because of the constraint that the nominal interest rate must be greater than zero, the "Taylor principle" cannot be verified for all values of Π_t. In particular $\phi(0) = 0$.

5.2.2 Fiscal policy

Since the object of our study is principally monetary policy, we want to take the simplest possible fiscal policies. If the budget was balanced, taxes would be equal to interest payments on bonds $i_t B_t$, so that one would have:

$$P_t T_t = i_t B_t \tag{5.8}$$

[1]The function Φ gives $1 + i_t$, not i_t, as a function of Π_t because this is the term that appears in the intertemporal maximization conditions.

Because the rate of expansion of government liabilities will actually play a substantial role below, we shall consider, as in the previous chapter, a more general class of policies, of the form:

$$P_t T_t = i_t B_t + (1-\gamma)\,\Omega_t \qquad \gamma > 0 \tag{5.9}$$

As compared to the balanced budget policy (8), the term $(1-\gamma)\,\Omega_t$ has been added. It says that the government may want to withdraw a fraction $1-\gamma$ of its outstanding financial liabilities. If γ is greater than 1, this actually corresponds to an expansion of government liabilities.

5.2.3 Dynamics

Putting together equations (1), (9) and the definition $\Omega_t = M_t + B_t$ we first find the equation of evolution of Ω_t:

$$\Omega_{t+1} = \gamma \Omega_t \tag{5.10}$$

Secondly combining (2) and (3) we obtain:

$$P_{t+1} Y_{t+1} = \beta\,(1+n)\,\Phi\,(\Pi_t)\,P_t Y_t - (1-\beta)\,n\Omega_{t+1} \tag{5.11}$$

Equations (10) and (11) are the basic dynamic equations of our model.

5.3 Ricardian economies and the Taylor principle

We begin our investigation with the traditional Ricardian version of the model. For that it is enough to take $n = 0$. Equation (11) then simplifies as:

$$P_{t+1} Y_{t+1} = \beta \Phi\,(\Pi_t)\,P_t Y_t \tag{5.12}$$

which, since $Y_{t+1}/Y_t = \zeta$, is rewritten as:

$$\Pi_{t+1} = \frac{\beta}{\zeta}\Phi\,(\Pi_t) = \frac{\Phi\,(\Pi_t)}{\xi} \tag{5.13}$$

where ξ is the autarkic real interest rate that was defined in chapter 2, section 2.7. From (13) the potential steady state values of Π_t, denoted as Π, are solutions of the equation:

$$\frac{\Phi\,(\Pi)}{\Pi} = \xi \tag{5.14}$$

We shall denote as $\Pi^k, k = 1, ...K$, the solutions to this equation ranked in ascending order. Depending on the shape of the function Φ and the value of ξ, equation (14) can have potentially any number of solutions (including zero)[2]. Figure 5.1 represents the case of two solutions, Π^1 and Π^2. We may note that the Taylor principle is verified at Π^2, but not at Π^1.

Figure 5.1

5.3.1 Local determinacy

Let us first linearize equation (13) around a particular potential steady state Π. We find:

$$\Pi_{t+1} - \Pi = \frac{\Phi'(\Pi)}{\xi}(\Pi_t - \Pi) \tag{5.15}$$

The condition for local determinacy is thus:

$$\Phi'(\Pi) > \xi \tag{5.16}$$

or, combining with (14):

$$\frac{\Pi\Phi'(\Pi)}{\Phi(\Pi)} = \phi(\Pi) > 1 \tag{5.17}$$

So, if the elasticity of the function Φ is greater than 1, the inflation rate and the price level are locally determinate. This is the Taylor principle. But of course we must go further and inquire under which conditions global determinacy holds.

5.3.2 Global determinacy

Consider the case represented in figure 5.1, with two potential equilibria. As we can see, if there is an equilibrium where $\phi(\Pi^2) > 1$, because of the zero lower bound on the nominal interest rate, there must be another equilibrium Π^1 where the Taylor principle is not satisfied, i.e. where $\phi(\Pi^1) < 1$, and which is locally indeterminate. The corresponding dynamics is depicted in figure 5.1, which represents equation (13). Dynamic paths initiating between the two equilibria Π^1 and Π^2 converge towards the indeterminate equilibrium Π^1.

[2] We may note that the condition $\phi'(\Pi_t) \geq 0$ is actually sufficient to have no more than two solutions.

To ensure global determinacy we must find additional conditions that will ensure that such paths are actually not feasible. Continuing with the example of figure 5.1 it can be shown that a sufficient condition for global determinacy of Π^2 in that case is that, besides the Taylor principle $\phi(\Pi^2) > 1$:

$$\Pi^1 < \Pi^* < \Pi^2 \tag{5.18}$$

where:

$$\Pi^* = \frac{\gamma}{\zeta} \tag{5.19}$$

More generally if there are K solutions $\Pi^1, ..., \Pi^k, ..., \Pi^K$ to the equation $\Phi(\Pi) = \xi\Pi$, then Π^K is globally determinate if[3]:

$$\phi\left(\Pi^K\right) > 1 \tag{5.20}$$

$$\Pi^{K-1} < \Pi^* < \Pi^K \tag{5.21}$$

Condition (21) is also a sufficient condition for the equilibrium Π^K to satisfy the transversality conditions.

So we see that the Taylor principle (20) is still part of the determinacy conditions. It is supplemented, however, with condition (21) ensuring that only the equilibrium corresponding to the highest inflation Π^K is acceptable.

We shall now see that these determinacy conditions are substantially modified when one moves to a non-Ricardian situation.

5.4 Non Ricardian economies: dynamics and steady states

We shall now move to non-Ricardian economies, assuming that $n > 0$.

5.4.1 The dynamic system

Let us recall the dynamic system (10), (11):

$$\Omega_{t+1} = \gamma\Omega_t \tag{5.22}$$

$$P_{t+1}Y_{t+1} = \beta(1+n)\Phi(\Pi_t)P_tY_t - (1-\beta)n\Omega_{t+1} \tag{5.23}$$

[3]See for example Woodford (1999, 2003), Benhabib, Schmitt-Grohe and Uribe (2001a,b), Bénassy and Guillard (2005) and Bénassy (2007).

It will actually be convenient in what follows to use as working variables inflation Π_t and the predetermined variable X_t defined as[4]:

$$X_t = \frac{\Omega_t}{P_{t-1}Y_{t-1}} \tag{5.24}$$

Then the dynamic system (22), (23) is rewritten:

$$X_{t+1} = \Pi^* \frac{X_t}{\Pi_t} \tag{5.25}$$

$$\Pi_{t+1} = \frac{\Phi(\Pi_t)}{\xi} - \nu\Pi^* \frac{X_t}{\Pi_t} \tag{5.26}$$

with:

$$\Pi^* = \frac{\gamma}{\zeta(1+n)} \qquad \nu = \frac{(1-\beta)n}{(1+n)\zeta} \tag{5.27}$$

5.4.2 The two types of steady states

From (25) and (26) potential steady states Π and X are solutions of the set of equations:

$$X = \Pi^* \frac{X}{\Pi} \tag{5.28}$$

$$\Pi = \frac{\Phi(\Pi)}{\xi} - \nu X \tag{5.29}$$

We see that there are two types of steady states, that we will call respectively "Ricardian" and "non Ricardian":

Definition 5.1 *Ricardian equilibria, or equilibria of type $\mathcal{R}$, are the solutions to the system* (28) *and* (29) *characterized by:*

$$X^k = 0 \qquad \frac{\Phi\left(\Pi^k\right)}{\Pi^k} = \xi \qquad k = 1, ..., K \tag{5.30}$$

A non-Ricardian equilibrium, or equilibrium of type $\mathcal{NR}$, is the solution to the system (28) *and* (29) *characterized by:*

$$\Pi = \Pi^* = \frac{\gamma}{\zeta(1+n)} \qquad X = X^* = \frac{1}{\nu}\left[\frac{\Phi(\Pi^*)}{\xi} - \Pi^*\right] \tag{5.31}$$

[4]This representation is borrowed from Guillard (2004).

Steady states of type $\mathcal{R}$ ("Ricardian") are similar to the steady states in Ricardian economies (14), with a supplementary condition for the stationary value of X, which here is equal to zero. In both cases the potential equilibrium rates of inflation are given by equations (14) or (30). The real gross rate of interest R_t is equal to $\xi = \zeta/\beta$, i.e. the autarkic rate, whatever the value of the inflation rate.

The (unique) steady state of type $\mathcal{NR}$ ("non Ricardian") is more specific to the non-Ricardian environment. The inflation rate Π^* is not given anymore by the properties of the Taylor rule, but is equal to the rate of growth of government liabilities γ divided by the rate of growth of output $\zeta(1+n)$, a most traditional formula. The real (gross) rate of interest, noted R^*, is not equal to ξ anymore. For example we can deduce it from the inflation rate by:

$$R^* = \frac{\Phi(\Pi^*)}{\Pi^*} \tag{5.32}$$

5.5 The financial dominance criterion

We have already seen in the previous chapter that the Taylor principle was replaced, as far as local determinacy was concerned, by a new criterion that was called the "financial dominance" criterion. This criterion was expressed in a "local" way. Since we will need a more global approach, and this criterion will play an important role in this and the next chapter, we now give a more general definition.

Definition 5.2 *The "financial dominance" (FD) criterion is satisfied for the value of inflation* Π *if:*

$$\frac{\Phi(\Pi)}{\Pi} > \xi \tag{5.33}$$

or, in words, the gross real interest rate $\Phi(\Pi)/\Pi$ *generated by the interest rate rule is above the autarkic rate* ξ*. In order to characterize financial dominance with a simple compact parameter, let us define:*

$$\kappa(\Pi) = \frac{1}{\xi}\frac{\Phi(\Pi)}{\Pi} \tag{5.34}$$

The financial dominance (FD) criterion holds if:

$$\kappa(\Pi) > 1 \tag{5.35}$$

We shall now see that this criterion will play a central role for both local and global determinacy

5.6 Local determinacy and financial dominance

In order to show the respective relevance of the financial dominance and the Taylor principle criteria, we shall now first study the local determinacy of our potential equilibria. Linearizing the system (25), (26) around a steady state (X, Π) we find, using (28) and (34):

$$\begin{bmatrix} \Pi_{t+1} - \Pi \\ X_{t+1} - X \end{bmatrix} = \begin{bmatrix} \phi\kappa + \nu X/\Pi & -\nu\Pi^*/\Pi \\ -X/\Pi & \Pi^*/\Pi \end{bmatrix} \begin{bmatrix} \Pi_t - \Pi \\ X_t - X \end{bmatrix} \tag{5.36}$$

with:

$$\phi = \phi(\Pi) \qquad \kappa = \kappa(\Pi) \qquad \Pi^* = \frac{\gamma}{(1+n)\zeta} \tag{5.37}$$

The characteristic polynomial corresponding to this linearized dynamic system is given by:

$$\Psi(\lambda) = (\lambda - \phi\kappa)\left(\lambda - \frac{\Pi^*}{\Pi}\right) - \frac{\lambda\nu X}{\Pi} \tag{5.38}$$

which, using (29) and (34), can be rewritten:

$$\Psi(\lambda) = (\lambda - \phi\kappa)\left(\lambda - \frac{\Pi^*}{\Pi}\right) + (1-\kappa)\frac{\Pi^*}{\Pi} \tag{5.39}$$

The determinacy conditions are actually quite different depending on whether the equilibrium is of type $\mathcal{R}$ or $\mathcal{NR}$, as defined in section 5.4, so we study them in turn.

5.6.1 Equilibria of type $\mathcal{R}$

Consider a steady state Π^k of type $\mathcal{R}$ (definition 5.1). In that case we have $\kappa = 1$, so that the corresponding characteristic polynomial is:

$$\Psi_{\mathcal{R}}(\lambda) = \left(\lambda - \phi^k\right)\left(\lambda - \frac{\Pi^*}{\Pi^k}\right) \tag{5.40}$$

This characteristic polynomial has two roots:

$$\lambda_1 = \phi^k \qquad \lambda_2 = \frac{\Pi^*}{\Pi^k} \tag{5.41}$$

There is local determinacy if one of these roots is of modulus greater than 1, the other smaller than 1, that is if:

$$\phi^k < 1 \qquad \text{and} \qquad \Pi^k < \Pi^* \tag{5.42}$$

or:

$$\phi^k > 1 \qquad \text{and} \qquad \Pi^k > \Pi^* \tag{5.43}$$

We see that the position of ϕ^k with respect to 1, and thus the Taylor principle, still plays a central role for these equilibria.

5.6.2 Equilibria of type $\mathcal{NR}$

Now for the steady state of type $\mathcal{NR}$ (definition 5.1) we have $\Pi = \Pi^*$, so the associated characteristic polynomial is:

$$\Psi_{\mathcal{NR}}(\lambda) = (\lambda - \phi\kappa)(\lambda - 1) + (1 - \kappa)\lambda \tag{5.44}$$

We can compute:

$$\Psi_{\mathcal{NR}}(0) = \phi\kappa > 0 \tag{5.45}$$

$$\Psi_{\mathcal{NR}}(1) = 1 - \kappa \tag{5.46}$$

We see that the condition for local determinacy is:

$$\kappa > 1 \tag{5.47}$$

We recognize the "financial dominance" (FD) condition that we described above (definition 5.2). The Taylor principle $\phi > 1$ is not the relevant criterion anymore.

5.7 Non Ricardian dynamics: a graphical representation

We shall next study under which conditions global determinacy holds in a non-Ricardian environment. We shall see that the Taylor principle will be almost completely replaced by the "financial dominance" criterion defined in section 5.5, at least for the non-Ricardian equilibria.

We shall make extensive use below of graphical representations to represent the dynamic equations (25) and (26), so we begin with it.

From (25) the locus $X_{t+1} = X_t$ has actually two branches whose equations are:

$$X_t = 0 \qquad \text{and} \qquad \Pi_t = \Pi^* \qquad \Pi^* = \frac{\gamma}{\zeta(1+n)} \tag{5.48}$$

From (26) the curve $\Pi_{t+1} = \Pi_t$ can be written:

$$X_t = H(\Pi_t) \tag{5.49}$$

with:

$$H(\Pi_t) = \frac{\Pi_t}{\nu \Pi^*} \left[\frac{\Phi(\Pi_t)}{\xi} - \Pi_t \right] \tag{5.50}$$

We may further note that the dynamic evolutions of X_t and Π_t are characterized by (we restrict ourselves to $X_t \geq 0$ in what follows):

$$X_{t+1} > X_t \qquad \text{if :} \qquad \Pi_t < \Pi^* \tag{5.51}$$

$$\Pi_{t+1} > \Pi_t \qquad \text{if :} \qquad X_t < H(\Pi_t) \tag{5.52}$$

5.8 Global financial dominance

We shall now see how the financial dominance criterion can ensure global determinacy. We shall first consider in the following proposition the case where financial dominance holds for all values of the inflation rate.

Proposition 5.1 *If the financial dominance criterion holds for all values of the inflation rate, i.e. if:*

$$\frac{\Phi(\Pi_t)}{\Pi_t} > \xi \qquad \forall \Pi_t \tag{5.53}$$

then there is a single globally determinate equilibrium of type $\mathcal{NR}$.

Proof: We first see that under condition (53) there cannot be an equilibrium of type $\mathcal{R}$ since these equilibria are all characterized by $\Phi(\Pi) = \xi \Pi$. So there remains only the unique equilibrium Π^* of type $\mathcal{NR}$. Now since $\Phi(\Pi^*) > \xi \Pi^*$, we know from the results of section 5.6.2 that this equilibrium is characterized by saddle-point dynamics and is locally determinate.

Figure 5.2 depicts the two curves $X_{t+1} = X_t$ and $\Pi_{t+1} = \Pi_t$, as well as the dynamics of the economy given by (51) and (52), in the case corresponding to condition (53). It is easy to see from the dynamics depicted in figure 5.2 that this equilibrium is globally determinate. Q.E.D.

We may note that the preceding result does not depend on the elasticity of the function Φ as long as condition (53) applies. In other words the Taylor

principle is irrelevant for local or global determinacy when condition (53) holds.

Figure 5.2

5.9 Partial financial dominance

We shall now consider cases where the financial dominance criterion is not satisfied for all values of the inflation rate, and see that nevertheless this criterion plays a central role in achieving global determinacy. Again we study in the next proposition equilibria of type $\mathcal{NR}$ (definition 5.1).

Proposition 5.2 *Consider an equilibrium of type $\mathcal{NR}$, characterized notably by an inflation rate Π^* such that:*

$$\frac{\Phi(\Pi^*)}{\Pi^*} > \xi \tag{5.54}$$

Then this equilibrium will be globally determinate if and only if:

$$\frac{\Phi(\Pi_t)}{\Pi_t} > \xi \qquad \forall \Pi_t \geq \Pi^* \tag{5.55}$$

Proof: Bénassy and Guillard (2005), Bénassy (2007). Q.E.D.

What proposition 5.2 tells us is that what is important is that the FD condition be satisfied for "high" values of inflation. So if there is a danger that the real value of financial assets might be driven to zero by high inflation, this condition will ensure that agents will actually want to hold these financial assets because their return is attractive, and this will prevent a "collapse" of money and financial assets.

So we have proved that for achieving global determinacy of non-Ricardian equilibria the Taylor principle should be replaced by the financial dominance criterion. However this could be an empty result if there was no function $\Phi(\Pi_t)$ such that the financial dominance criterion holds, while the Taylor principle does not. We shall thus now check that there are functions $\Phi(\Pi_t)$ such that the Taylor principle is not satisfied, and nevertheless condition (54) is satisfied[5].

[5] In such a case the weaker condition (55) is a fortiori satisfied.

5.10 Global determinacy: an example

We will now give a simple example of interest rate rules where the Taylor principle does not hold, and nevertheless global determinacy can obtain because the financial dominance criterion is satisfied.

We shall indeed study simple linear interest rate rules:

$$\Phi(\Pi_t) = A\Pi_t + B \qquad A > 0 \quad B > 1 \tag{5.56}$$

Consider the function:

$$\Phi(\Pi_t) - \xi\Pi_t = A\Pi_t + B - \xi\Pi_t \tag{5.57}$$

If this function is positive for all values of Π_t, in view of proposition 5.1 there will be global determinacy. A sufficient condition for this is that:

$$A > \xi \tag{5.58}$$

Now let us compute the elasticity of this interest rate rule:

$$\phi(\Pi_t) = \frac{\partial \log\Phi(\Pi_t)}{\partial \log\Pi_t} = \frac{\partial \log(A\Pi_t + B)}{\partial \log\Pi_t} = \frac{A\Pi_t}{A\Pi_t + B} < 1 \tag{5.59}$$

This elasticity is always smaller than 1. If the parameter A satisfies (59) global determinacy will be achieved with $\phi < 1$.

5.11 Conclusions

We have examined in this chapter conditions under which interest rate rules achieve global determinacy in non-Ricardian economies. We identified two types of equilibria, each with distinct determinacy conditions.

The first type of equilibrium (type $\mathcal{R}$) was called "Ricardian". These equilibria look very much like equilibria in a Ricardian economy. In such equilibria the global determinacy conditions are similar to those in a pure Ricardian economy, i.e. the Taylor principle supplemented with conditions on the growth of assets ensuring that only the potential equilibrium with the highest inflation rate is feasible.

Things change radically when one considers the second type of equilibria, non-Ricardian (type $\mathcal{NR}$) equilibria. There financial assets have real value, and the real interest rate is different from the autarkic one. In such equilibria a central global determinacy condition appears to be the "financial dominance" criterion, which basically says that, through the nominal interest

rate rule, the real interest rate should be maintained at a value superior to the autarkic real interest rate. In that way households will be willing to hold financial assets, and the total value of these assets will give the "nominal anchor" that will pin down the price level.

5.12 References

This chapter is adapted from Bénassy and Guillard (2005).

The issue of global determinacy under interest rate rules in Ricardian economies has been notably studied in Benhabib, Schmitt-Grohe and Uribe (2001a, 2001b, 2002) and Woodford (1999, 2003).

The financial dominance criterion appears initially in Wallace (1980) for an OLG economy where money is the single store of value. It is extended in Bénassy (2005) for economies with both money and bonds.

Chapter 6

Fiscal Policy and Determinacy

6.1 Introduction

In chapter 1 we outlined some aspects of the fiscal theory of the price level, according to which, when nominal interest rates are not reactive enough, price determinacy can be regained through adequate fiscal policies. As we pointed out, however, these fiscal policies are rather dangerous since they imply that government liabilities will be explosive in most circumstances. These controversial policy implications have led to numerous contributions and a heated debate.

What we want to show in this chapter is that the controversial policy implications are actually due to the particular "Ricardian" framework whithin which the results were derived, and we will show that moving to a "non-Ricardian" framework yields much less controversial results. In particular price determinacy, whether local or global, is consistent with much more reasonable fiscal policies.

6.2 The model

We will use again the model described in chapter 2, section 2, but have to be more specific on monetary and fiscal policies.

6.2.1 Monetary policy

In what follows we shall study two types of monetary policies. First, and since we want to concentrate on the effects of fiscal policy, we shall consider again a simple policy of interest rate pegging, which is the typical situation where the FTPL holds. To simplify the exposition we shall assume that the

pegged interest rate is constant in time, so that:

$$i_t = i_0 \qquad \forall t \tag{6.1}$$

We shall then also study more general policies where, as in the previous chapter, the nominal interest rate responds to inflation as follows:

$$1 + i_t = \Phi(\Pi_t) \qquad \Phi(\Pi_t) \geq 1 \tag{6.2}$$

with $\Pi_t = P_t/P_{t-1}$.

6.2.2 Fiscal policy

If the budget was balanced, taxes would be equal to interest payments on bonds:

$$P_t T_t = i_t B_t \tag{6.3}$$

Clearly since fiscal policy will be the object of this chapter we want to consider more general fiscal policies, and so we shall assume that the government has policies of the form (already discussed in chapter 1):

$$P_t T_t = i_t B_t + (1-\gamma)\Omega_t + \delta P_t Y_t \qquad \gamma \geq 0 \qquad \delta \geq 0 \tag{6.4}$$

6.3 The dynamic equations

Let us recall the government budget equation:

$$\Omega_{t+1} = (1+i_t)\Omega_t - i_t M_t - P_t T_t \tag{6.5}$$

Combining (4) and (5) with $\Omega_t = M_t + B_t$, we find:

$$\Omega_{t+1} = \gamma\Omega_t - \delta P_t Y_t \tag{6.6}$$

Turning now to nominal income $P_t Y_t$, it was shown in proposition 2.1, chapter 2, that, assuming $N_{t+1}/N_t = 1+n$, its dynamics is given by:

$$P_{t+1}Y_{t+1} = \beta(1+n)(1+i_t)P_t Y_t - (1-\beta)n\Omega_{t+1} \tag{6.7}$$

Combining this with equation (2) we obtain:

$$P_{t+1}Y_{t+1} = \beta(1+n)P_t Y_t \Phi(\Pi_t) - (1-\beta)n\Omega_{t+1} \tag{6.8}$$

Equations (6) and (8) are the basic dynamic equations of our model. Now, in order to contrast the results with what will follow, we shall first examine as a benchmark some determinacy conditions in the traditional Ricardian case.

6.4 Ricardian economies and the FTPL

6.4.1 Dynamics

We begin our investigation with the traditional Ricardian version of the model. For that it is enough to take $n = 0$. Equation (8) is then rewritten as:

$$P_{t+1}Y_{t+1} = \beta P_t Y_t \Phi(\Pi_t) \tag{6.9}$$

Taking, as in chapter 5, Π_t and the predetermined variable $X_t = \Omega_t / P_{t-1}Y_{t-1}$ as our working variables, the dynamic system (6), (9) is rewritten as:

$$\Pi_{t+1} = \frac{\Phi(\Pi_t)}{\xi} \tag{6.10}$$

$$X_{t+1} = \frac{\gamma X_t}{\zeta \Pi_t} - \delta \tag{6.11}$$

Steady states (Π, X) of this system (when they exist) are characterized by:

$$\Pi = \frac{\Phi(\Pi)}{\xi} \tag{6.12}$$

$$X = \frac{\gamma X}{\zeta \Pi} - \delta \tag{6.13}$$

We shall assume that the system (12), (13) admits at least one steady state. Linearizing (10) and (11) around it we find:

$$\begin{bmatrix} \Pi_{t+1} - \Pi \\ X_{t+1} - X \end{bmatrix} = \begin{bmatrix} \phi & 0 \\ -\gamma X/\zeta \Pi^2 & \gamma/\zeta \Pi \end{bmatrix} \begin{bmatrix} \Pi_t - \Pi \\ X_t - X \end{bmatrix} \tag{6.14}$$

with:

$$\phi = \phi(\Pi) = \frac{\Pi \Phi'(\Pi)}{\Phi(\Pi)} \tag{6.15}$$

The two roots are thus ϕ and $\gamma/\zeta\Pi$.

6.4.2 Fiscal policies and local determinacy

The variable X_t is predetermined, whereas Π_t is not. So local determinacy will be obtained if one root has modulus greater than 1. This gives us two possibilities for local determinacy. The first is:

$$\phi > 1 \qquad \gamma < \zeta\Pi \tag{6.16}$$

The first inequality in (16) is the Taylor principle, which we have already studied. We may note that this Taylor principle is combined with a "prudent" fiscal policy which puts an upper bound on γ, the coefficient of expansion of government liabilities.

But we see that with the more general tax function appears a new possibility for local determinacy, which corresponds to the FTPL, i.e.:

$$\phi < 1 \qquad \gamma > \zeta\Pi \tag{6.17}$$

The condition $\phi < 1$ says that the Taylor principle is not satisfied. The condition $\gamma > \zeta\Pi$ means that the coefficient γ, which somehow measures the "target" expansion of government liabilities, must be higher than $\zeta\Pi$. Since $\zeta\Pi$ is the rate of growth of nominal income, this will entail in particular that the ratio of government liabilities to income can be explosive, obviously a not very reasonable fiscal policy.

6.5 Local determinacy in the non-Ricardian case

We will now move to a non-Ricardian framework, and we want to show that, at least for equilibia of the $\mathcal{NR}$ type, adventurous fiscal policies like (17) are not necessary anymore for determinacy. More precisely we will study a case which admits equilibria of both types identified in the previous chapter, $\mathcal{R}$ and $\mathcal{NR}$. We shall see that, although the determinacy conditions of equilibria of type $\mathcal{R}$ are similar to those of the FTPL, the conditions for equilibria of type $\mathcal{NR}$ imply much more reasonable fiscal policies.

We shall study in this section local determinacy, leaving global determinacy to the next section. In order to make the comparison with the Ricardian case particularly transparent, we shall continue to concentrate here on the case of an interest rate peg $\Phi(\Pi_t) = 1 + i_0$. We shall further asssume $\delta = 0$. The dynamic system (6) and (8) becomes:

$$\Omega_{t+1} = \gamma\Omega_t \tag{6.18}$$

$$P_{t+1}Y_{t+1} = \beta\,(1+n)\,(1+i_0)\,P_tY_t - (1-\beta)\,n\Omega_{t+1} \tag{6.19}$$

or in terms of the variables Π_t and X_t:

$$X_{t+1} = \Pi^* \frac{X_t}{\Pi_t} \tag{6.20}$$

$$\Pi_{t+1} = \frac{1+i_0}{\xi} - \nu\Pi^* \frac{X_t}{\Pi_t} \tag{6.21}$$

with:

$$\Pi^* = \frac{\gamma}{\zeta\,(1+n)} \qquad \nu = \frac{(1-\beta)\,n}{(1+n)\,\zeta} \tag{6.22}$$

Steady states of the system (Π, X) are given by:

$$X = \Pi^* \frac{X}{\Pi} \tag{6.23}$$

$$\Pi = \frac{1+i_0}{\xi} - \nu\Pi^* \frac{X}{\Pi} \tag{6.24}$$

We see that there are two steady states, one of type $\mathcal{R}$ (Π^1) and one of type $\mathcal{NR}$ (Π^*):

$$X^1 = 0 \qquad \Pi^1 = \frac{1+i_0}{\xi} \qquad \text{Type } \mathcal{R} \tag{6.25}$$

$$\Pi = \Pi^* \qquad X^* = \frac{1}{\nu}\left(\frac{1+i_0}{\xi} - \Pi^*\right) \qquad \text{Type } \mathcal{NR} \tag{6.26}$$

Let us linearize (20) and (21) around these steady states. We obtain:

$$\begin{bmatrix} \Pi_{t+1} - \Pi \\ X_{t+1} - X \end{bmatrix} = \begin{bmatrix} \nu X/\Pi & -\nu\Pi^*/\Pi \\ -X/\Pi & \Pi^*/\Pi \end{bmatrix} \begin{bmatrix} \Pi_t - \Pi \\ X_t - X \end{bmatrix} \tag{6.27}$$

The characteristic polynomial is:

$$\Psi(\lambda) = \lambda^2 - \left(\frac{\nu X}{\Pi} + \frac{\Pi^*}{\Pi}\right)\lambda \tag{6.28}$$

The roots are 0 and $(\nu X + \Pi^*)/\Pi$, so the condition for local determinacy is:

$$\frac{\nu X}{\Pi} + \frac{\Pi^*}{\Pi} > 1 \tag{6.29}$$

Let us now investigate successively this condition for the two types of equilibria.

For the equilibrium of type $\mathcal{R}$, $X = 0$, so condition (29) boils down to:

$$\Pi^* > \Pi^1 \tag{6.30}$$

or, in view of (22) and (25), and since $\xi = \zeta/\beta$:

$$\gamma > \beta (1+n)(1+i_0) = \theta \tag{6.31}$$

We see that this condition is very similar to the "FTPL" condition we already saw above (equation 26 in chapter 1), and similarly calls for a "large" expansion of government's liabilities.

Let us now consider the equilibrium of type $\mathcal{NR}$. There $\Pi = \Pi^*$, so condition (29) boils down to:

$$X > 0 \tag{6.32}$$

or, in view of (26) and the definition of Π^* (equation 22):

$$\gamma < \beta (1+n)(1+i_0) = \theta \tag{6.33}$$

We see that this is the opposite of (31)! This time the condition for local stability is that the coefficient of fiscal expansion be *lower* than a given value, not higher, as in equation (31).

We can represent the dynamics in the (Π_t, X_t) plane (figure 6.1). We see that condition (33) implies that the vertical $\Pi_t = \Pi^*$ will be on the left of the intersection of the curve $\Pi_{t+1} = \Pi_t$ with the horizontal axis, so that the dynamic system looks indeed as in figure 6.1.

Figure 6.1

Now, although we have just found that the non-Ricardian equilibrium could be locally determinate under reasonable fiscal policies, the system as a whole is indeterminate (figure 6.1). So we shall now move to the problem of global determinacy.

6.6 Global determinacy

We shall now show that we can achieve not only local, but also global determinacy without having to implement adventurous fiscal policies. Let us recall from section 6.4 that in the Ricardian framework there are two alternative conditions for price determinacy, the Taylor principle and the FTPL.

What we want to show is that in a non-Ricardian world it is possible to obtain global determinacy eventhough none of these two conditions is satisfied. We actually already treated implicitly the case $\delta = 0$ in the previous chapter (propositions 5.1 and 5.2), so we shall now investigate sufficient conditions for global determinacy when $\delta > 0$.

Proposition 6.1 *Assume $\delta > 0$, and that the monetary policy satisfies:*

$$\phi(\Pi_t) < 1 \tag{6.34}$$

$$\frac{\Phi(\Pi_t)}{\Pi_t} > \xi \tag{6.35}$$

then there is a single globally determinate equilibrium of type $\mathcal{NR}$.

Proof: From (6) and (8) we deduce the dynamic system in Π_t and X_t:

$$X_{t+1} = \Pi^* \frac{X_t}{\Pi_t} - \delta \tag{6.36}$$

$$\Pi_{t+1} = \frac{\Phi(\Pi_t)}{\xi} + \delta\nu - \nu\Pi^* \frac{X_t}{\Pi_t} \tag{6.37}$$

where the expressions of Π^* and ν have been given in equation (22).

The curve $X_{t+1} = X_t$ has for expression:

$$X_t = \frac{\delta\Pi_t}{\Pi^* - \Pi_t} \tag{6.38}$$

The curve $\Pi_{t+1} = \Pi_t$ has for expression:

$$X_t = \frac{\Pi_t}{\nu\Pi^*}\left[\frac{\Phi(\Pi_t)}{\xi} - \Pi_t + \delta\nu\right] \tag{6.39}$$

Note first that under condition (35) (the financial dominance criterion) the $\Pi_{t+1} = \Pi_t$ curve is entirely above the horizontal axis. Secondly we may note that the derivatives at the origin for the curves $X_{t+1} = X_t$ and $\Pi_{t+1} = \Pi_t$ are respectively:

$$\frac{\delta}{\Pi^*} \qquad \text{and} \qquad \frac{\delta}{\Pi^*} + \frac{\Phi(0)}{\nu\xi\Pi^*} \tag{6.40}$$

so that the at the origin the curve $\Pi_{t+1} = \Pi_t$ is always above the curve $X_{t+1} = X_t$ as in figure 6.2.

Figure 6.2

Now in order to have global determinacy we first want to check that, as in figure 6.3, the two curves $X_{t+1} = X_t$ and $\Pi_{t+1} = \Pi_t$ have a unique intersection. In view of (38) and (39) the potential intersections are given by the solutions to the equation:

$$\frac{\delta\Pi_t}{\Pi^* - \Pi_t} = \frac{\Pi_t}{\nu\Pi^*}\left[\frac{\Phi(\Pi_t)}{\xi} - \Pi_t + \delta\nu\right] \tag{6.41}$$

Dividing by Π_t and subtracting δ/Π^* from both sides, (41) becomes:

$$\frac{\delta\Pi_t}{\Pi^* - \Pi_t} = \frac{1}{\nu}\left[\frac{\Phi(\Pi_t)}{\xi} - \Pi_t\right] \tag{6.42}$$

The left and right hand sides of (42) are represented in figure 6.3.

Figure 6.3

A sufficient condition for a unique intersection is that the derivative of the left hand side of (42) be always larger than the derivative of the right hand side at a potential intersection point. The condition is thus:

$$\frac{\delta\Pi^*}{(\Pi^* - \Pi_t)^2} > \frac{1}{\nu}\left[\frac{\Phi'(\Pi_t)}{\xi} - 1\right] \tag{6.43}$$

which, taking into account (42) (which is verified at an intersection point), becomes:

$$\frac{\Pi^*}{\Pi^* - \Pi_t}\left[\frac{\Phi(\Pi_t)}{\xi} - \Pi_t\right] > \Pi_t\left[\frac{\Phi'(\Pi_t)}{\xi} - 1\right] \tag{6.44}$$

Since the first fraction of the left hand side is bigger than one, a sufficient condition is:

$$\frac{\Phi(\Pi_t)}{\xi} - \Pi_t > \Pi_t\left[\frac{\Phi'(\Pi_t)}{\xi} - 1\right] \tag{6.45}$$

or:

$$\frac{\Pi_t\Phi'(\Pi_t)}{\Phi'(\Pi_t)} < 1 \tag{6.46}$$

which is condition (34). So we see that, if the interest rate rule does not satisfy the Taylor principle, the intersection is unique as in figures 6.2 and 6.3. Now figure 6.2 depicts the global dynamics of the system, and we see that the unique steady state is globally determinate. Q.E.D.

Of course we must check that there exist functions $\Phi(\Pi_t)$ such that the Taylor principle does not hold (condition 34) and the financial dominance

holds (condition 35). We have seen such functions in the preceding chapter, and notably the simple linear rules:

$$\Phi(\Pi_t) = A\Pi_t + B \qquad A \geq \xi \qquad B > 1 \tag{6.47}$$

6.7 Conclusions

We have seen that in the Ricardian case, if the Taylor principle is not satisfied, price determinacy can nevertheless obtain if the fiscal authority expands government liabilities at such a high rate that these liabilities become explosive (see for example condition 17), which is a central mechanism behind the fiscal theory of the price level.

This controversial prescription is not necessary anymore in a non-Ricardian world. We found indeed that in such a case an explosive expansion of government liabilities is not required for local or global price determinacy of non-Ricardian equilibria, and that price determinacy can be associated to very reasonable fiscal prescriptions (see for example condition 33). Finally we saw that global determinacy could be achieved with a combination of monetary and fiscal policies where monetary policy does not satisfy the "Taylor principle" and fiscal policies can be "reasonable" (proposition 6.1).

So it appears that the controversial policy prescriptions associated with the FTPL are linked with the Ricardian character of the economies in which they were derived. They are not necessary anymore when one moves to a (more realistic) non Ricardian framework.

6.8 References

This chapter is adapted from Bénassy (2004, 2007).

The fiscal theory of the price level comes notably from the works of Leeper (1991), Sims (1994) and Woodford (1994, 1995). Explicitations and criticisms are found notably in Buiter (2002), Kocherlakota and Phelan (1999), McCallum (2001), Niepelt (2004) and Weil (2002).

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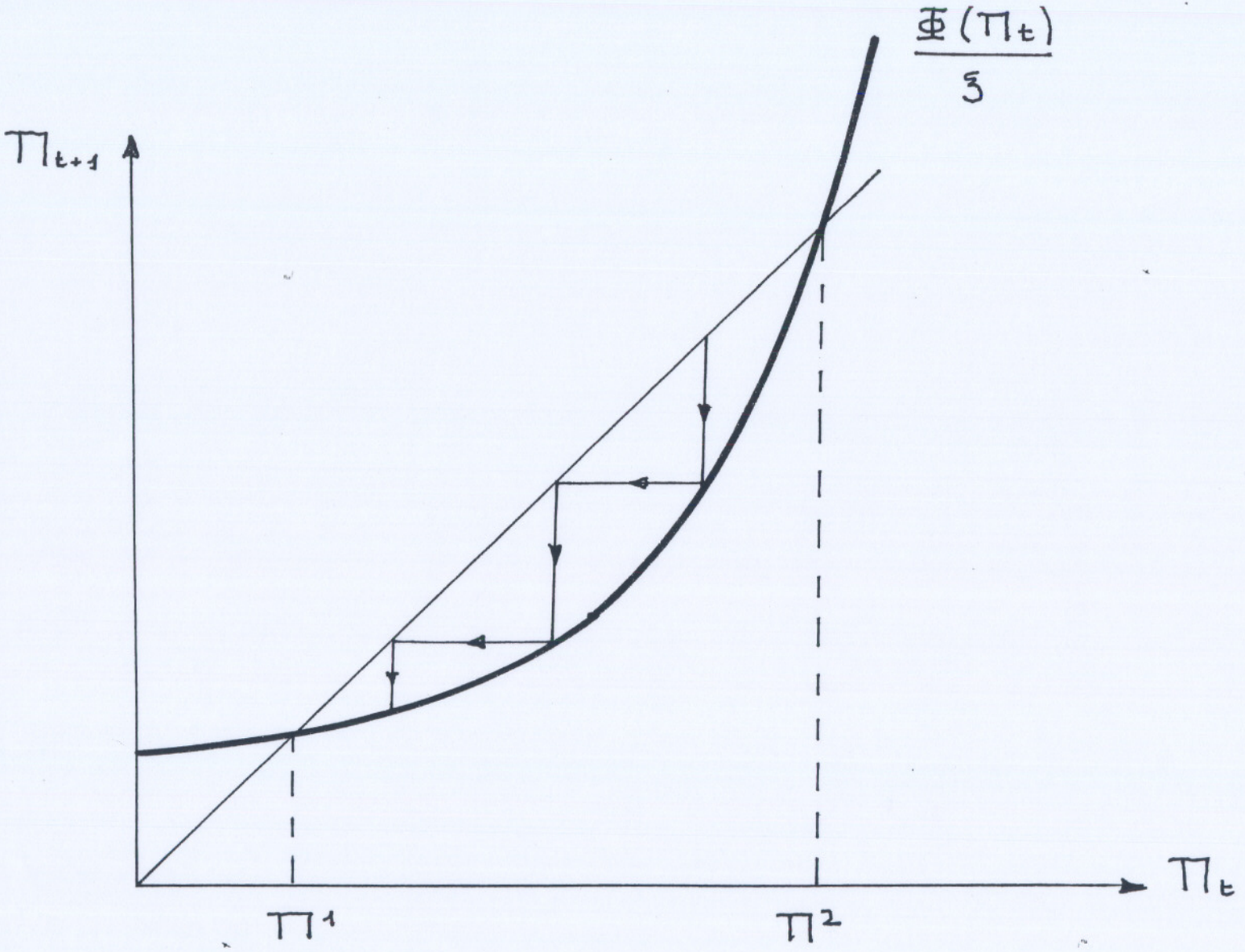

FIGURE 5.1

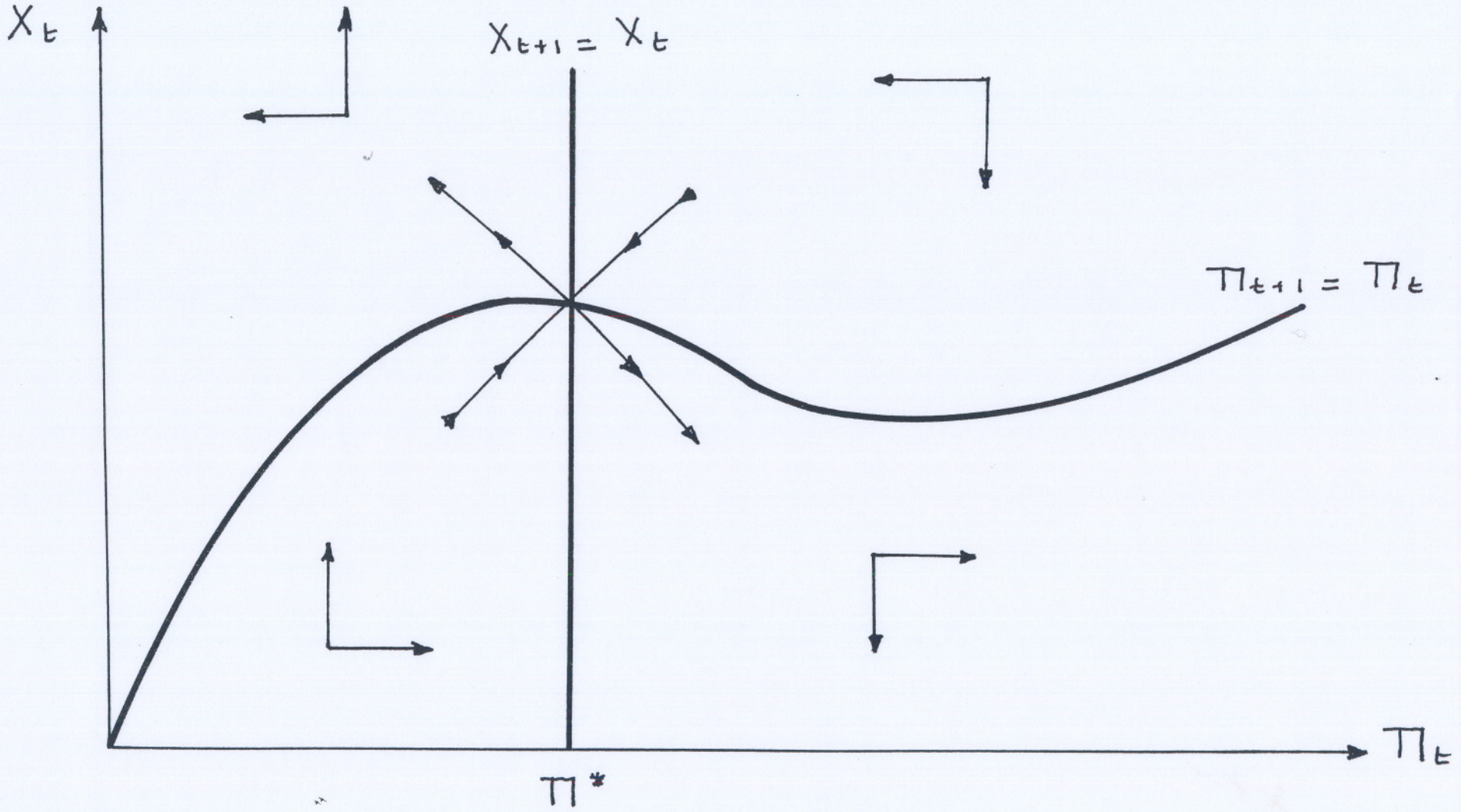

FIGURE 5.2

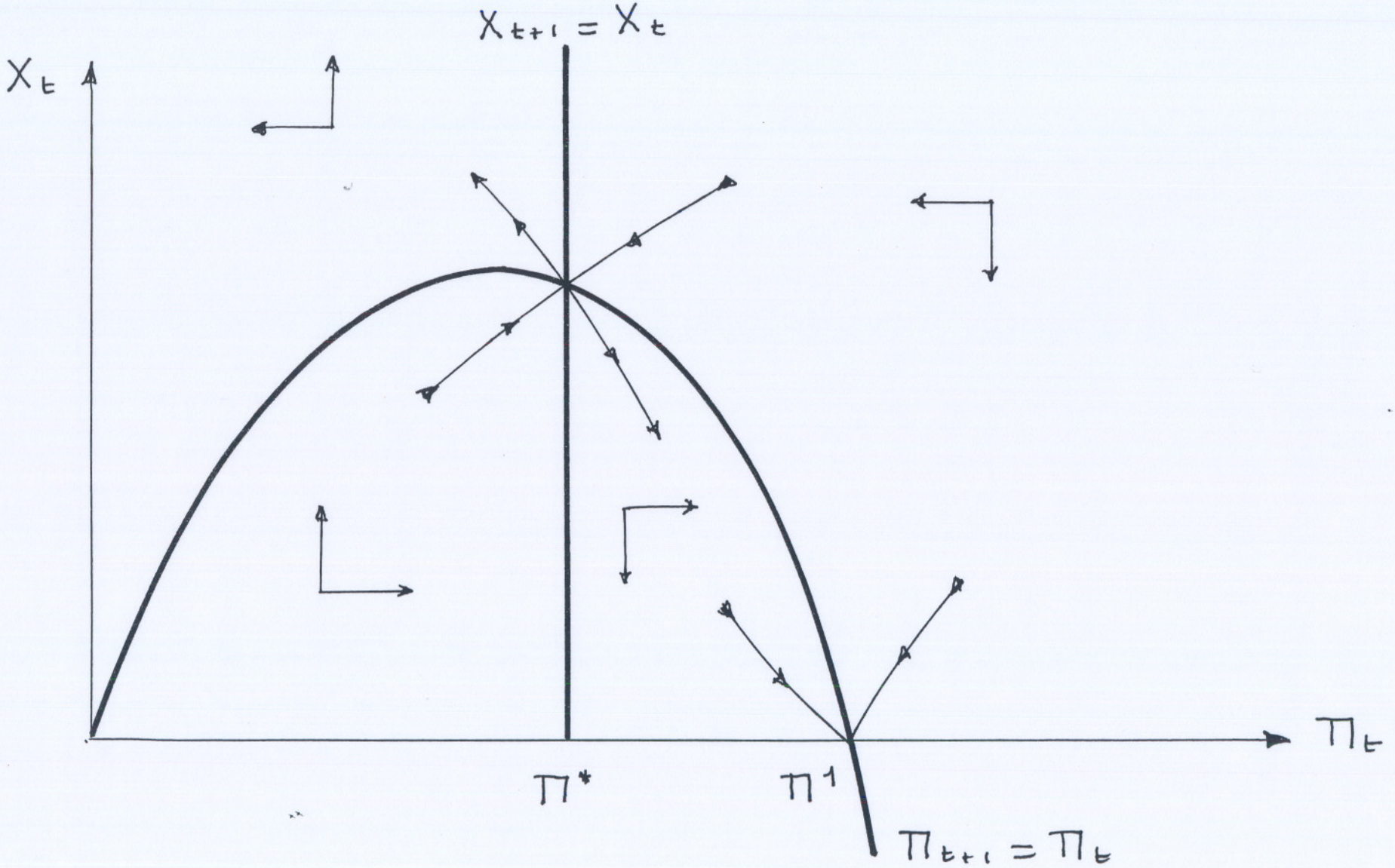

FIGURE 6.1

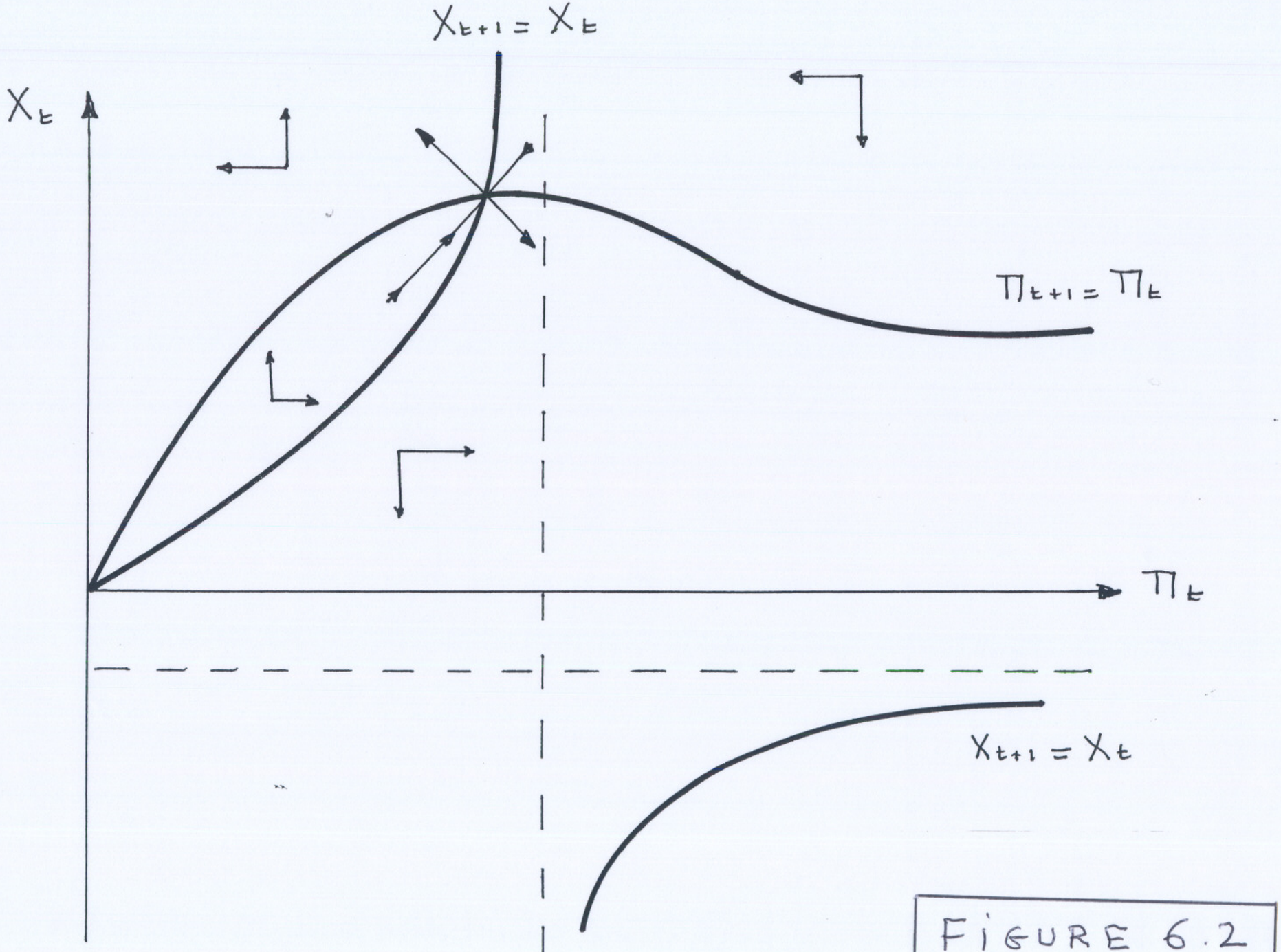

FIGURE 6.2

FIGURE 6.3