

# International Portfolios with supply, demand and redistributive shocks

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**Abstract:** The objective of this paper is to understand three stylized facts observed in industrialized countries: 1) portfolios are biased on local equity; 2) international portfolios are long in foreign currency and short in domestic currency; 3) valuation effects are such that an exchange rate depreciation is associated to a positive transfer of wealth. We build a two-country, two-good model with stocks and bonds where uncertainty is not only due to productivity shocks but also due to shocks on the distribution of income between labor and capital and to demand shocks. We show that, in this case, optimal portfolios are broadly consistent with the above stylized facts. We perform this analysis both in situations of complete and incomplete markets.

**Keywords :** International risk sharing, Home bias in portfolio, Valuation effects

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# 1 Introduction

In the past twenty years, both gross and net capital flows have increased to unprecedented levels (see Lane and Milesi-Ferretti, 2006b). In industrialized countries, this has been the case for both stocks and bonds. One consequence of this period of financial integration is that gross financial positions now exceed 100% of GDP in several industrialized countries so that differences in returns on foreign assets held by domestic agents and on domestic assets held by foreign agents can generate sizable wealth transfers between countries. Lane and Milesi-Ferretti (2003 and 2006 a, b) and Tille (2003) have recently shown the extent of these valuation effects. Another salient feature of the financial integration process at work in the past twenty years is that it has not eliminated the financial home bias in stocks. It remains large in all industrialized countries even though most legal and technological impediments to international trade in assets have been eliminated.

In this paper, we study jointly the composition of equity portfolios and the valuation effects in a two-country two-good general equilibrium model with portfolio choice on stocks and bonds. We analyze them in the context of the stylized fact of the home bias in consumption which has remained strong even of the presence of increasing trade flows. A recent literature has revived the interest in the link between the financial and real home biases. In particular Obstfeld and Rogoff (2000), have argued that trade costs for goods can explain the puzzle of the home bias in portfolios.

The objective of this paper is in part to reproduce the following stylized facts for industrialized countries: first, as already mentioned, home bias in stocks is pervasive despite the process of financial integration (see table 1 in the appendix). Second, international portfolios are long in foreign currency and short in domestic currency (Lane and Milesi-Ferretti (2006 b), Tille (2003) and table 2 in the appendix). Third, a depreciation of the exchange rate generates a transfer of wealth towards the home country (Gourinchas and Rey (2005), Tille (2003), Lane and Milesi-Ferretti (2006 a, b) among others).

We argue that usual models that incorporate international portfolio choice are generally unable to generate these three broad facts simultaneously. One of the main reason is that the existing literature has focused on supply shocks. Indeed, a very general result is that when faced with a negative domestic supply shock - which triggers a real exchange rate appreciation - the optimal portfolio should generate a transfer of wealth from abroad to finance imports and stabilize home consumption. However, in practice,

we observe that countries that experience an exchange rate appreciation suffer capital losses on their external assets. We show that the introduction of demand shocks helps to reproduce plausible valuation effects with respect to exchange rate movements for a wide range of parameter values.

Contrary to most of the existing literature, we allow for domestic and foreign stocks as well as for domestic and foreign bonds. We do this both in the case of complete and incomplete markets. We find that a combination of shocks that affect the distribution of income between capital and labor on the one hand and that affect relative demand for home and foreign goods on the other hand can qualitatively replicate both a home bias in stocks and gross foreign asset (stocks plus bonds) positions that produce realistic valuation effects. The intuition of this result is the following: a shock that increases dividends of domestic firms while reducing income of domestic agents can be hedged by holding stocks of domestic firms, the home bias observed in the data. The simplest shock that has this property is one that redistributes income from labor to dividends. Given this home bias in stocks, in order to insure against shocks that decrease both relative dividends and terms of trade, agents will want to hold foreign currency assets (stocks and bonds). Such a portfolio will produce a wealth transfer for the country that experiences a depreciation of the exchange rate. One shock that has this property is what we call an iPod shock. That shock changes world relative demand of Home versus Foreign goods: it may reflect a change in preferences or in the quality or number of varieties of the Home goods. In the quantitative section of the paper, we show that in a situation of incomplete markets a plausible combination of productivity, demand and redistributive shocks produces realistic foreign asset positions, and that it captures the valuation effects observed in industrialized countries.

Our model is the first which, to our knowledge, analyzes, in the context of incomplete markets, the home bias in stocks in the presence of two bonds, Home and Foreign. Interestingly, the introduction of these bonds helps to explain the home bias in stocks in our case. The reason is that movements in the terms of trade can be hedged by holding bonds. In the existing literature, the hedging of the real exchange rate is achieved by holding foreign stocks hence generating a counterfactual foreign bias.

In the next section, we review the related literature and point to the difference with the present analysis. In section 3, we provide a brief description of the set-up of the model. Section 4 solves the optimal portfolio in a situation of complete markets restricting the number of shocks to equal the number

of available assets. In section 5, we analyze the robustness of the complete markets results by considering situations where markets are incomplete, due a larger array of shocks; we provide closed-form solutions for equilibrium portfolios under incomplete markets, using the solution method developed by Devereux and Sutherland (2006). In the last section, we simulate our model to analyze its quantitative properties.

## 2 Related literature

Since the well-known paper of French and Poterba (1991) that documented equity home bias, various form of heterogeneity among investors have been analyzed, in order to explain international portfolio holdings. Indeed, without cross-country heterogeneity of investors, all investors would, in equilibrium, hold the same portfolio of worldwide assets, and thus no bias towards local assets would exist (see Lewis (1999) for a survey). In the present paper, we abstract from barriers to international capital movements and assume that any investor can purchase any securities without transaction costs. In other words, each investor face the same investment opportunity set; this is, admittedly, a strong assumption but our result would be reinforced if we assumed a financial friction to buying foreign securities<sup>1</sup>. However, we here assume that consumers have a greater preference for the locally produced good than for imported goods (consumption home bias). This is a realistic assumption, as the local content of aggregate consumption far exceeds imported goods (see Kollmann (2005)). Consumption home bias implies that the (consumption-based) real exchange rate fluctuates in response to supply and demand shocks.

Uppal (1993), Coeurdacier (2005), Kollmann (2005) and Obstfeld (2006) study portfolio choice in models with consumption home bias; in those settings, productivity shocks are the only disturbance, and the only traded assets are domestic and foreign stocks. It appears that those models can only generate equity home bias when the substitution elasticity between domestic and imported goods is (roughly speaking) smaller than unity. Intuitively, a country that receives a negative output shock experiences an improvement of its terms of trade; when the substitution elasticity between local and imported goods is low, then the terms of trade improvement is so strong that the return on local equity rises compared to the return on foreign equity; thus, local equity has a high (relative) return in states of the world in which the country's income is low; this makes holding local equity attractive, and induces investors to

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<sup>1</sup>See Martin and Rey (2004) and (2006), Heathcote and Perri (2004), Coeurdacier and Guibaud (2005), Coeurdacier (2005), Tille and Van Wincoop (2006).

mainly invest their wealth in local stocks. By contrast, when the substitution elasticity exceeds unity, then the relative return on local equity drops, when local output falls, and hence foreign equity is a better hedge for output fluctuations. Hence, a model with just productivity shocks only generates equity home bias under the condition that a negative local output shock raises the relative return on local equity. Essentially, in such a model, equity home bias only arises when a country's relative equity return is highly positively correlated with its terms of trade (and, hence, with its real exchange rate). Yet, empirically, the correlation between relative equity returns and real exchange rate changes is close to zero (see Warnock and van Wincoop (2006)). Our model here reproduces this low correlation, yet it also generates realistic equity home bias. This is due to the fact that our model assumes trade in stocks and in two differentiated bonds, denominated in the home and the foreign good, respectively. In our setting, bonds will essentially be used to hedge real exchange rate risk, a feature that we consider as realistic. Moreover, we assume the existence of relative demand (iPod) shocks, and redistributive shocks in addition to the more standard productivity shocks. Those new shocks break the close link between terms of trade movements and relative equity returns.

Another strand of literature related to our paper analyzes the impact of non-tradable labour incomes on equity home bias. According to this literature, the presence of labour income either worsens the home bias in equities puzzle (Cole (1988), Brainard and Tobin (1992), Baxter and Jermann (1997)) or helps explaining it (Bottazzi, Pesenti and van Wincoop (1996), Palacios-Huerta (2001), Julliard (2002 and 2004), Engel and Matsumoto (2006)). In the models discussed by these authors, the composition of equity portfolios hinges on the sign (and the magnitude) of the correlation between physical capital returns and returns to human capital: as workers/investors seek to hedge human capital risk, they hold local equity assets if local equity returns are negatively correlated with labor income (Bottazzi, Pesenti and van Wincoop (1996), Engel and Matsumoto (2006))<sup>2</sup>. In our paper, labor income and equity returns are partially disconnected due to the presence of redistributive shocks from labour to capital (or dividends). There are two main differences that set our paper apart from the existing theoretical literature on the role of labor income in international portfolio choice. First, as already mentioned, we allow for the presence of two differentiated (tradable) goods and two differentiated bonds<sup>3</sup>. Due to the

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<sup>2</sup>Lustig and van Nieuwerburgh (2006) argue that, empirically, physical and human capital returns are negatively correlated

<sup>3</sup>Engel and Matsumoto (2006) are a notable exception regarding on this aspect.

use of home and foreign bonds, the direction of the equity bias is not pinned down by the covariance structure of wages and equity returns. This can produce drastically different equity portfolios compared to the existing literature: the intuition of our result can be simply exposed in a situation where only productivity (endowment) shocks and redistributive shocks are present. In that case, there is full equity home bias for any covariance structure of the shocks and whatever the magnitude of the redistributive shocks. Because redistributive shocks only affect how income is shared between dividends and wages, leaving bond returns unchanged, home investors hold the entire local equity to insure themselves against these redistributive shocks. Output shocks will be hedged using positions in home and foreign bonds: indeed, productivity shocks affect bond returns at home and abroad (by altering terms-of-trade) and consequently bonds will be an efficient tool to hedge these shocks.

A second difference is that while most existing theoretical models assume that an equity portfolio exists that perfectly mimics wage returns (a case of perfect spanning), we relax this assumption within a model featuring market incompleteness. Market incompleteness is an important novelty with regard to the previous literature. Indeed, in most papers where equity portfolios are solved endogenously, markets are assumed to be complete. This leaves a model where portfolios might be consistent with observed portfolios (and the home bias in equities) but at the expense of the counterfactual equality between the ratio of marginal utilities of consumption across countries and the real exchange rate (the well-known "consumption-real exchange rate anomaly"; see Kollmann (1991, 1995, 1996), Backus and Smith (1993)): as recently reformulated by Corsetti, Dedola, Leduc (2007), this equality is clearly counterfactual since the correlation between relative consumption and the real exchange is low in the data (see also Chari, Kehoe, McGrattan (2002)). While Corsetti, Dedola, Leduc (2007) (or Kollmann (1995,1996)) tackle the puzzle by restricting the menu of assets, we adopt here a different strategy by increasing the number of shocks to obtain imperfect spanning and incomplete markets. Our model is then a first attempt to provide a portfolio-balance model featuring incomplete financial markets. In terms of solution strategy, our paper uses the new techniques provided by Devereux and Sutherland (2006).

The theoretical literature on valuation effects is more recent and has focused on their impact on current account adjustments. This is for example the case of Tille (2005), Blanchard, Giavazzi and Sa (2005) and Ghironi, Lee and Rebucci (2006). Ghironi, Lee and Rebucci (2006) have a richer dynamic

business cycle model (with endogenous labour and production) than the present paper. However, they assume that international financial transactions are costly and restricted to stocks. In their model, steady state equity portfolios are pinned down by costs to holding foreign stocks. By contrast, our model assumes trade in stocks and bonds in a frictionless financial market.

Although our paper is certainly connected with previous papers on international risk-sharing, our model differs mostly by the nature of the shocks considered (in particular the iPod shocks), the presence of differentiated bonds at home and abroad and the presence of imperfect spanning (incomplete markets). In particular, to our knowledge, this paper is the first to endogenize gross equity and gross bond positions in a two-country/two-good model with incomplete markets<sup>4</sup>.

### 3 Set-up of the model

#### 3.1 Goods and preferences

We consider a two periods ( $t = 0, 1$ ) endowment-economy. There are two symmetric countries, home ( $H$ ) and foreign ( $F$ ). Each country produces one good. There is no output (and no consumption) at  $t = 0$  and agents trade claims (stocks and bonds) in period 0. In period 1, production takes place and the country  $i$  receives an exogenous endowment  $y_i$  of good  $i$ . We assume that for both countries  $E_0(y_i) = 1$  where  $E_0$  is the conditional expectations operator, given date  $t = 0$  information. Once stochastic endowments are realized, agents consume using their revenues from their portfolio chosen in period 0 and from their labour income.

We introduce the aggregate consumption index in country  $i$  in period 1 which is defined as the aggregator of home and foreign goods:

$$C_H = \left[ a^{1/\phi} (\Psi_H c_H^H)^{(\phi-1)/\phi} + (1-a)^{1/\phi} (\Psi_F c_F^H)^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)} \quad (1)$$

$$C_{Ft} = \left[ a^{1/\phi} (\Psi_F c_F^F)^{(\phi-1)/\phi} + (1-a)^{1/\phi} (\Psi_H c_H^F)^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)} \quad (2)$$

where  $\phi$  is the elasticity of substitution between home and foreign goods, and  $c_j^i$  is the consumption of goods from country  $j$  by country  $i$ ,  $i = H, F$  in period 1.  $\Psi_i$ ,  $i = H, F$  are demand or preference shocks

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<sup>4</sup>Rigobon and Pavlova (2004) also have differentiated bonds but they do not focus on portfolios and have complete markets

with  $E_0(\Psi_i) = 1$ . They reflect a change at the world level in the preference for the good of country  $H$  or  $F$ . To be more illustrative we call them “iPod shocks”. Note however that this shock can have a more supply oriented interpretation. It could be interpreted as a quality shock or, in a model with love for variety of the Dixit-Stiglitz type, as an increase in the number of varieties of the good produced by the country. Broda and Weinstein (2007) report that electronics, records and tapes is the product group that has the largest quality/new goods bias in the price index. Hence, our choice of name.

We also allow for a preference bias for home goods by assuming  $a > \frac{1}{2}$ . When  $a = \frac{1}{2}$ , agents would have identical preferences. Note that in the “Cobb-Douglas” case ( $(\phi - 1)/\phi = 0$ ),  $a$  is the share of revenues dedicated to home goods.

Each country has a representative agent with these preferences:

$$U_i = E_0 \left[ \frac{C_i^{1-\sigma}}{1-\sigma} \right] \quad (3)$$

where  $C_i$  is consumption in country  $i = H, F$ . As in most of the literature, we take the coefficient of relative risk aversion  $\sigma$  to be equal or superior to one.

The welfare based price indices that correspond to the above specification of preferences are:

$$\begin{aligned} P_H &= \left[ a (p_H/\Psi_H)^{1-\phi} + (1-a) (p_F/\Psi_F)^{1-\phi} \right]^{1/(1-\phi)} \\ P_F &= \left[ (1-a) (p_H/\Psi_H)^{1-\phi} + a (p_F/\Psi_F)^{1-\phi} \right]^{1/(1-\phi)} \end{aligned} \quad (4)$$

The resource constraints are given by:

$$c_H^H + c_H^F = y_H \quad (5)$$

$$c_F^F + c_F^H = y_F \quad (6)$$

where  $y_i$  is the exogenous level of output of country  $i$  in period 1.

We introduce  $q$  the relative price of home goods over foreign goods (the terms-of-trade) as:

$$q \equiv \frac{p_H}{p_F} \quad (7)$$

## 3.2 Financial markets

As mentioned above, financial assets are traded in period 0. We will consider stocks and bonds. Each stock represents a share in one of the future endowments  $y_i$ . The supply of each type of share is normalized



at unity. An exogenous fraction  $k_i$  of the country  $i$  endowment accrues to share holders, while a fraction  $(1 - k_i)$  is received by the local household ( $(1 - k_i)y_i$  can be interpreted as ‘labour’ income). We assume that  $E_0(k_i) = \bar{k}$  ( $\bar{k}$  is interpreted as the steady-state capital share, identical in both countries). Agents can also issue/purchase two bonds, a Home bond and a Foreign bond. Buying one unit of the Home (respectively Foreign) bond in period 0 gives one unit of the Home (respectively Foreign) good at  $t = 1$ . We interpret these two bonds as debt securities paying in different currencies. The two bonds are in net zero-supply.

Each household fully owns the local stock, at birth, and has zero initial foreign assets. Then the country ( $i$ ) household faces the following budget constraint, at  $t = 0$ :

$$p_S S_i^i + p_S S_j^i + b_i^i + b_j^i = p_S \quad (8)$$

where  $S_j^i$  is the number of shares of stock  $j$  held by country  $i$ , at the end of period 0, while  $b_j^i$  represents claims (held by  $i$ ) to future unconditional payments of good  $j$  (bond in  $j$ -currency).  $p_S$  is the stock price that is identical in both countries due to the symmetric structure of the two countries<sup>5</sup>.

The symmetry of preferences and shock distributions also implies symmetric equilibrium portfolios such that :  $S_H^H = S_F^F$ ,  $S_F^H = S_H^F$ ,  $b_H^H = b_F^F$  and  $b_F^H = b_H^F$ . Market clearing conditions in asset markets requires:  $S_H^H + S_H^F = S_F^F + S_F^H = 1$  and  $b_H^H + b_H^F = b_F^F + b_F^H = 0$ . In what follows, we denote a country’s holding of the local stock by  $S$ , and its holdings of bonds denominated in the country’s local good by  $b$ . Then,  $S > \frac{1}{2}$  means that the country exhibit some home bias in equity and  $b < 0$  means that the country is borrowing in its own currency in the bond market and lending  $b$  in foreign currency. In the symmetric model, the pair  $\{S; b\}$  fully describes international portfolios.

Finally, we define a country’s net foreign currency position (FCP) as the gross asset position denominated in foreign currency (foreign good units) net of liabilities in foreign currency. Up to a first-order approximation, one can show that  $S = \bar{k}$ . Hence, FCP is the sum of Foreign currency claims in stocks and bonds:

$$FCP = \bar{k}(1 - S) - b \quad (9)$$

As explained in the introduction,  $FCP$  is positive for industrialized countries in the data. Note that,  $FCP$  is the expected payment of foreign currency from the portfolio in period 1. Of course, in this

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<sup>5</sup>Bond prices are also identical due to symmetry.

symmetric model, if the country is long in foreign currency ( $FCP > 0$ ), the country is short in its own currency by the same amount. A positive  $FCP$  means that, ceteris paribus, a depreciation ( $\hat{q}$ ) of the exchange rate generates a positive transfer towards the country equal to ( $\hat{q}.FCP$ ).

## 4 Characterization of world equilibrium with complete markets

To build up intuition, we first characterize the equilibrium with (effectively) complete markets. As shown below, markets are complete when the number of shocks equals the number of assets.

### 4.1 Efficient consumption and terms-of-trade

When markets are complete, the symmetric equilibrium allocation is Pareto efficient, i.e. it corresponds to the allocation chosen by a social planner who maximizes the sum of countries utilities with equal weights subject to the resource constraints (5) and (6):

$$\max_{\{c_H^H, c_F^H, c_H^F, c_F^F\}} \frac{C_H^{1-\sigma}}{1-\sigma} + \frac{C_F^{1-\sigma}}{1-\sigma} \quad (10)$$

The competitive equilibrium prices are identified with the Lagrange multipliers associated with the resource constraints. The multiplier on (5) is the price of one unit of the Home good. Similarly, the multiplier on (6) is the price of one unit of the Foreign good. We obtain the following first order conditions for the intratemporal allocation of consumption:

$$\begin{aligned} c_H^H &= a\Psi_H^{\phi-1}p_H^{-\phi}C_H^{1-\sigma\phi} & c_H^F &= (1-a)\Psi_H^{\phi-1}p_H^{-\phi}C_F^{1-\sigma\phi} \\ c_F^H &= (1-a)\Psi_F^{\phi-1}p_F^{-\phi}C_H^{1-\sigma\phi} & c_F^F &= a\Psi_F^{\phi-1}p_F^{-\phi}C_F^{1-\sigma\phi} \end{aligned} \quad (11)$$

Using the market clearing condition (5 and 6), we get:

$$c_H^H + c_H^F = \Psi_H^{\phi-1}p_H^{-\phi} \left[ aC_H^{1-\sigma\phi} + (1-a)C_F^{1-\sigma\phi} \right] = y_H \quad (12)$$

$$c_F^F + c_F^H = \Psi_F^{\phi-1}p_F^{-\phi} \left[ aC_F^{1-\sigma\phi} + (1-a)C_H^{1-\sigma\phi} \right] = y_F \quad (13)$$

Taking the ratio and defining  $\Omega(x) = \frac{1+x(\frac{1-a}{a})}{x+(\frac{1-a}{a})}$  we get:

$$\left( \frac{\Psi_H}{\Psi_F} \right)^{\phi-1} q^{-\phi} \Omega \left[ \left( \frac{C_F}{C_H} \right)^{1-\sigma\phi} \right] = \frac{y_H}{y_F} \quad (14)$$

When markets are complete and countries are symmetric, the ratio of Home to Foreign marginal utility of consumption is linked to the real exchange rate by the following, familiar condition:

$$\left(\frac{C_H}{C_F}\right)^{-\sigma} = \frac{P_H}{P_F} \quad (15)$$

so that because of perfect insurance, any change in parameter values that raises domestic consumption relative to foreign consumption must be associated with a real depreciation. Then, the goods market equilibrium can be summarized by:

$$q^{-\phi} \left(\frac{\Psi_H}{\Psi_F}\right)^{\phi-1} \Omega \left[\left(\frac{P_H}{P_F}\right)^{\frac{1}{\sigma}-\phi}\right] = \frac{y_H}{y_F} \quad (16)$$

## 4.2 Budget constraints

Remember that  $S$  is the number of Home shares that Home agents hold and  $1 - S$ , the number of Foreign shares that Home agents hold while  $b$  are the holdings of Home bonds by Home agents. The period 1 budget constraints of countries  $H$  and  $F$  are respectively:

$$P_H C_H = S k_H p_H y_H + (1 - S) k_F p_F y_F + p_H b - p_F b + (1 - k_H) p_H y_H \quad (17)$$

$$P_F C_F = (1 - S) k_H p_H y_H + S k_F p_F y_F + p_F b - p_H b + (1 - k_F) p_F y_F \quad (18)$$

$k_i = H, F$  is the share of output that goes to capital owners in the form of dividends such that  $(1 - k_i) p_i y_i$  is the 'wage' of the household.

These budget constraints imply:

$$P_H C_H - P_F C_F = (2S - 1) (p_H y_H k_H - p_F y_F k_F) + 2b(p_H - p_F) + (1 - k_H) p_H y_H - (1 - k_F) p_F y_F \quad (19)$$

which says that the difference between countries' expenditures equals the difference between their incomes. The latter depends on portfolio and labor incomes.

We will focus on three possible sources of disturbances in this economy. The first is a productivity shock which affects the ratio:

$$y \equiv \frac{y_H}{y_F}$$

We also introduce a source of uncertainty in the form of an exogenous change in the share of output that goes to dividends and the share that goes to labor.

$$k \equiv \frac{k_H}{k_F}$$

The relative iPod shock is:

$$\Psi \equiv \frac{\Psi_H}{\Psi_F}$$

### 4.3 Log-linearization of the model

We log-linearize expression (16) around the symmetric steady-state (see Coeurdacier (2005)) where the steady state values of  $q$ ,  $y$  and  $k$  are equal to one. We denote with a hat the deviations of the variables from the symmetric steady state values which are denoted with a bar  $\hat{x} = \log(x/\bar{x})$ . The log-linearization of the welfare-based real exchange rate which we denote  $RE\widehat{R}_{WB}$  gives:

$$RE\widehat{R}_{WB} = \frac{\widehat{P}_H}{P_F} = (2a - 1) (\widehat{q} - \widehat{\Psi}) \quad (20)$$

As usual, real exchange rate fluctuations are driven by the presence of a home bias in consumption ( $a > 1/2$ ). The welfare-based real exchange rate increases with the terms-of-trade and decreases with iPod shocks (adjustment for the quality or preference changes). Here it is important to notice that the real exchange rate observed by the statistician might be different from the welfare-based real exchange rate since quality or preference changes are very imperfectly measured. We denote by  $\widehat{RER}$  a real exchange rate measure derived from CPI measures that do not capture quality or taste changes. We have  $\widehat{RER} = (2a - 1) \widehat{q}$ .

(16) and (20) imply:

$$\widehat{y} = - \left[ \phi \left( 1 - (2a - 1)^2 \right) + (2a - 1)^2 \frac{1}{\sigma} \right] (\widehat{q} - \widehat{\Psi}) - \widehat{\Psi} \equiv -\lambda \widehat{q} + (\lambda - 1) \widehat{\Psi} \quad (21)$$

where  $\lambda \equiv \phi(1 - (2a - 1)^2) + \frac{(2a-1)^2}{\sigma}$ . Note that  $\lambda > 0$  as  $1 \geq a > 1/2$ .

It follows from (21), that the equilibrium relative price is:

$$\widehat{q} = -\frac{1}{\lambda} \widehat{y} + \frac{\lambda - 1}{\lambda} \widehat{\Psi} \quad (22)$$

As expected, terms-of-trade are decreasing in the relative productivity shocks  $\widehat{y}$  (with an elasticity  $-\frac{1}{\lambda}$ ). Terms of trade are increasing in the relative demand (iPod) shock  $\widehat{\Psi}$ , if and only if  $\lambda > 1$  (this is the case for an elasticity of substitution sufficiently high, roughly speaking larger than one). The reason why the sign of the response of terms of trade to the iPod shock  $\widehat{\Psi}$  depends on the substitution elasticity is the following: the relative supply of good  $H$  in “efficiency” units is  $\Psi.y$ , while the relative price of one

efficiency unit of good  $H$  (in terms of efficiency units of good  $F$ ) is  $q/\Psi$ . Hence an iPod shock both induces an increase in demand and an increase in the supply of efficiency units of the good (the quality adjusted supply). While the first effect tends to increase the relative price (not adjusted for quality) with an elasticity of one, the second effect reduces the relative price with an elasticity of  $\frac{1}{\lambda}$ . When demand is sufficiently elastic ( $\phi$  and therefore  $\lambda$  are high enough) then the demand effect dominates and relative prices (not adjusted for quality) increase with the iPod shock.

We next log-linearize equation (19) around the symmetric steady-state and use the log-linearization of the real exchange rate (20) and the complete markets implication on the link between consumption and real exchange rates given by (15). Note that the difference in returns between the Home and Foreign bonds is given by the change in the terms of trade  $\hat{q}$ . We obtain:

$$P_H \widehat{C}_H - P_F \widehat{C}_F = (1 - \frac{1}{\sigma}) \underbrace{(2a - 1)}_{RER_{WB}} (\hat{q} - \widehat{\Psi}) = \bar{k} (2S - 1) (\hat{q} + \hat{k} + \hat{y}) + 2b\hat{q} + (1 - \bar{k})(\hat{q} + \hat{y} - \frac{\bar{k}}{1 - \bar{k}} \hat{k}) \quad (23)$$

The first equality shows the Pareto optimal reaction of relative expenditures to an appreciation of the welfare based real exchange rate. This reaction depends on the coefficient of relative risk aversion. In a Pareto efficient equilibrium, a shock that appreciates the (welfare based) real exchange rate of country  $H$ , induces a reduction in country  $H$  relative consumption spending when  $\sigma \geq 1$ . The risk-sharing condition (15) shows that when the (welfare based) real exchange rate appreciates by 1%, then relative aggregate country  $H$  consumption ( $\frac{C_H}{C_F}$ ) drops by  $1/\sigma$  % in an efficient equilibrium; hence efficient relative consumption spending by  $H$  ( $\frac{P_H C_H}{P_F C_F}$ ) changes by  $(1 - \frac{1}{\sigma})\%$ ; relative spending by  $H$  drops thus, when  $\sigma > 1$ . The expression to the right of the second equal sign in (23) shows the change in country  $H$  relative income (compared to the income of  $F$ ). When  $\sigma > 1$ , the portfolio that replicates the Pareto optimal allocation is such that a real appreciation (welfare based) should be associated with a reduction in relative expenditures and incomes.

The asset market is effectively complete (up to a first order approximation) when there exists a pair  $(S, b)$  such that (22) and the second equation in (23) hold for arbitrary realizations of the shocks  $\hat{y}, \hat{k}, \widehat{\Psi}$ .

Clearly, the market can only be complete when there are (at most) two relative shocks. Strictly speaking, markets are complete only up to the first order approximation.

## 4.4 Complete markets examples

We now sequentially solve for equilibrium portfolios in economies with just two (relative) shocks.

### 4.4.1 Productivity and iPod shocks

We start with a situation with productivity shocks  $\hat{y}$  and iPod shocks  $\widehat{\Psi}$ . The following portfolio  $(S, b)$  ensures that (23) and (22) hold for arbitrary realizations of  $\hat{y}$  and  $\widehat{\Psi}$ :

$$S = 1/2 \left[ 1 - \frac{(2a-1)(1-1/\sigma)}{\bar{k}} - \frac{1-\bar{k}}{\bar{k}} \right]; \quad b = 0 \quad (24)$$

The local equity share depends on three terms: the first term,  $1/2$ , represents the diversification motive in a single-good world with zero labor income (as analyzed by Lucas (1982)): in such a world equity portfolios are fully diversified. The second term,  $\frac{1}{2} \frac{(2a-1)(1-1/\sigma)}{\bar{k}}$ , represents the hedging of real exchange rate risk as analyzed in Coeurdacier (2005), Kollmann (2005), Obstfeld (2007) and van Wincoop and Warnock (2006). This term tends to generate foreign bias (turns negative), for a sufficiently high elasticity of substitution between goods (which insures  $\lambda > 1$ ) and sufficiently high risk aversion ( $\sigma > 1$ ). In the absence of a bias in preferences ( $a = 1/2$ ), the real exchange rate is constant and the second term disappears. The third term,  $\frac{1}{2} \frac{1-\bar{k}}{\bar{k}}$ , represents the equilibrium portfolio in a single-good world with labor income, in which the labor share is fixed (so that labor and capital incomes are perfectly positively correlated). It is identical to the one that appears in Baxter and Jerman (1995). In such a world, a foreign equity bias ( $S < 0$ ) emerges, as foreign equity returns are less closely correlated with labor income than domestic equity returns. With productivity and iPod shocks, bonds are not useful for the hedging of terms of trade risk (the latter can be hedged using stocks).

Hence, this case shares some of the usual difficulties of the literature to explain the home bias in stocks. It, however makes clear that in order to get more plausible portfolios, one needs a shock that both eliminates the perfect correlation between dividends and the real exchange rate as well as the perfect correlation of labor and capital income. A shock on the share of income that goes to capital and labor is exactly such a shock that we now analyze.

#### 4.4.2 Output and capital share shocks

We now analyze a situation with output shocks  $\hat{y}$  and capital share shocks  $\hat{k}$ . It follows from (23) and (22) that, in this case, the equilibrium portfolio is:

$$S = 1 \quad ; \quad b = \frac{1}{2} [(2a - 1) (1 - 1/\sigma) + \lambda - 1] \quad (25)$$

Hence, complete (100%) equity home bias appears ( $S = 1$ ) for any preference parameter and any stochastic structure of the two shocks. This full Home bias is in sharp contrast with the results obtained in the previous literature. that also studied international portfolio choice in models with a variable capital share; see, e.g., Baxter and Jermann (1995) and Botazzi et al. (1996). In contrast to most of that literature, we here consider a world with two goods and the possibility to hedge risk using bonds.

The predicted full equity home bias,  $S = 1$ , generated by the model is induced by capital share shocks. Full local equity ownership insulates household disposable income from shocks to the capital share; this is important, as the efficient consumption allocation does not depend on the capital share. Intuitively, capital share shocks entail that domestic equity returns are high when domestic labor income is low (and vice-versa); this makes holding local equity attractive. This mechanism operates even when the unconditional correlation between labor and capital income is positive. The unconditional correlation is positive when output shocks are sufficiently volatile, compared to capital share shocks.

Once capital share shocks have been hedged by holding equity, the remaining output risk can be hedged using the bond portfolio; this is so because output shocks induce terms of trade movements that affect the difference between the returns on domestic and foreign-good bonds.

When  $S = 1$  holds, then a country's net imports equal the net bond payment received by the country (as can readily seen from the budget constraints (17) and (18)). The country  $H$  budget constraint is then given by:  $(P_H/p_F)C_H - qy_H = (q - 1)b$ . In an efficient equilibrium, a Home output increase always worsens the  $H$  terms of trade ( $q$ ). When the elasticity of substitution between  $H$  and  $F$  goods is large, then Home net imports fall, in an efficient equilibrium. For low substitution elasticities, by contrast, net imports rise<sup>6</sup>.

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<sup>6</sup>For example, it is easy to see that when the substitution elasticity is infinite (so that the two goods are perfect substitutes) then the country that receives a higher output ships a fraction of the additional output to the other country, when there is efficient risk sharing; hence the country that receives the higher output lowers its net imports. When the two goods are substitutes, then term of trade worsen, which dampens the fall in net imports (the relative price of imports rises).

There exists a threshold value of  $\phi$  for which net imports are unaffected by output shocks; that threshold is given by the value of  $\phi$  for which the right-hand side of (25) is zero<sup>7</sup>. The right-hand side of (25) is positive (negative) when a domestic output increase raises (lowers) net imports.

For large values of  $\phi$  (such that the right-hand side of (25) is positive, i.e. roughly speaking values of  $\phi$  in excess of unity), countries therefore go long in local-good bonds (and short in foreign-good bonds): this ensures that an increase in the local endowment (which lowers the country's terms of trade) triggers a capital loss on the country's bond portfolio. This induces the country to lower its net imports, as prescribed by efficient international risk sharing. By contrast, for low values of  $\phi$ , countries go short in local good bonds; the terms of trade worsening that results from a positive domestic output shock then leads to a capital gain that allows the country to finance the efficient increase in its net imports.

More generally, any shock or combination of shocks that takes away resources from consumers and redistributes them to private Home firms would have the same effect on portfolios as the capital share shocks discussed in this Section. The appendix shows that a model with output shocks and shocks to government generates the same portfolio as the model with output shocks and capital share shocks.

The main conclusion here is therefore that the introduction of redistributive shocks provides an incentive to have a large home bias in stocks. We find it interesting that the introduction of such reasonable shocks provides a strong incentive to hold Home stocks. This result however comes here at the price of a position on bonds that is realistic only for  $\lambda < 1$ . Indeed in the data, industrialized countries have a positive gross position in foreign currency (*FCP*). Here, we have:

$$FCP \equiv \bar{k}(1 - S) - b = -b \tag{26}$$

For  $\lambda > 1$  (an elasticity of substitution between goods greater than unity),  $b > 0$  obtains, i.e. investors are short in Foreign bonds and long in local bonds; so that a terms of trade depreciation induces a transfer of wealth from the Home to the Foreign country. This is in contrast with the valuation effects that have been described by a recent literature (Lane and Milesi-Ferretti (2003 and 2006a,b), Tille (2003) and Gourinchas and Rey (2005)).

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<sup>7</sup>When  $\sigma = 1$  the threshold value of is:  $\phi = 1$ .



### 4.4.3 iPod and capital share shocks

We now show that iPod  $\hat{\Psi}$  and capital share shocks  $\hat{k}$  are sufficient to obtain both a strong home bias in stocks, and a positive foreign currency position (FCP). When there are just these two types of shocks, the equilibrium portfolio is:

$$S = 1 \quad ; \quad b = -\frac{1}{2} - \frac{1}{2} \frac{(2a-1)(1-1/\sigma)}{\lambda-1} \quad (27)$$

As before, capital share shocks are hedged by holding local equity  $S = 1$  ; iPod risk is hedged using the bond portfolio. The bond position is structured in such a manner that external capital gains/losses track changes in efficient consumption spending induced by iPod shocks. As discussed above, a negative good  $H$  iPod shock (a decrease in  $\Psi = \Psi_H/\Psi_F$ ) increases the (welfare based) real exchange rate. When the coefficient of relative risk aversion exceeds unity ( $\sigma > 1$ ), this induces an increase in relative country  $H$  consumption spending (see (23)). Country  $H$  terms of trade deteriorate when  $\lambda > 1$ , i.e. when the substitution elasticity is (roughly speaking) greater than unity.(see (22)). When  $\sigma > 1$  and  $\lambda > 1$  hold, country  $H$  hence experiences an increase in its efficient relative consumption spending, in states of the world in which its terms of trade deteriorate; in order to finance efficient spending and receive a transfer in these states of the world, the country thus goes short in local-good bonds ( $b < 0$ ).

Hence, a combination of demand and redistributive shocks helps to reproduce the broad facts presented in the introduction: home bias in stocks, long position in foreign currency (short position in domestic currency) and qualitatively consequently plausible valuation effects, and that for a wide range of parameters values.

## 5 Characterization of world equilibrium with incomplete markets

Complete markets provide a simple and useful benchmark for analyzing some elements of international portfolio behavior. However, the complete markets assumption has some unrealistic implications; for example, it implies that ratios of Home to Foreign marginal utilities of consumption are perfectly correlated with real exchange rates; that prediction is often violated in aggregate data (e.g., Kollmann (1991, 1995, 1996), Backus-Smith (1993)). We thus discuss now a model variant with incomplete markets; as before two stocks and two bonds are traded internationally. We now assume that the world economy is

subjected to the three types of shocks simultaneously. In that setting, the asset markets structure does not support the Pareto efficient allocation.

The solution methods developed by Coeurdacier (2005), Devereux and Sutherland (2006) and van Wincoop and Tille (2006) allow to solve for equilibrium portfolios, in economies with incomplete markets. Those methods solve for portfolio that satisfy a second order accurate approximation of household Euler equations for the four assets:

$$0 = E_0 [m_i (p_F - p_H)] = E_0 \left[ m_i \left( \frac{k_j p_j y_j}{p_S} - p_H \right) \right] \text{ for } j = \{H, F\} \quad (28)$$

where  $m_i = \frac{U'(C_i)}{P_i}$  is the marginal utility of household  $i$  divided by its consumption price index (stochastic discount factor for agent  $i$ ).

If we rewrite (28) in relative terms, we get:

$$0 = E_0 \left[ (m_i - m_j) \left( \frac{k_j p_j y_j}{p_S} - p_H \right) \right] \text{ for } j = \{H, F\} \quad (29)$$

$$0 = E_0 [(m_i - m_j) (p_F - p_H)] \quad (30)$$

(29) can be stated as:  $E_o(m \cdot ER) = 0$ , where  $m = m_i - m_j$  is the cross-country difference of stochastic discount factors, while  $ER$  is the vector of excess returns on the two stocks and the Foreign bond (relative to the return of the Home bond):

$$ER = \begin{pmatrix} \frac{k_H p_H y_H}{p_S} - p_H \\ \frac{k_F p_F y_F}{p_S} - p_H \\ p_F - p_H \end{pmatrix}$$

A second-order accurate approximation of this condition is given by  $E_o(\widehat{m} \widehat{ER}) = 0$ , where  $\widehat{m}$  and  $\widehat{ER}$  are first order accurate<sup>8</sup>.

The equilibrium portfolio is computed in a two-step approach:

(i) For given values of  $S$  and  $b$ , the budget constraint (17), the intratemporal allocation across goods, and the goods market clearing conditions (5) and (6) are solved for  $m$  and  $ER$ . A linear approximation gives:  $\widehat{m} = A(S, b)\widehat{\xi}$  and  $\widehat{ER} = B(S, b)\widehat{\xi}$ , where  $\xi = [y_H, y_F, k_H, k_F, \Psi_H, \Psi_F]'$  is the vector of exogenous variables.  $A(S, b)$  and  $B(S, b)$  are vectors/matrices (of dimensions  $(1 \times 6)$  and  $(3 \times 6)$ , respectively) that are functions of equilibrium portfolios  $S$  and  $b$ .

<sup>8</sup>All approximations are taken around the equilibrium of a deterministic economy in which the exogenous variables are set at the mean values assumed in the stochastic model; note that  $m = 0$  and  $r_X = 0$  in the deterministic economy.

(ii) Determination of the values of  $S$  and  $b$  for which  $E_o(\widehat{m} \widehat{ER}) = 0$  holds, i.e.  $B(S, b)\Sigma A(S, b) = 0$ , where  $\Sigma = E_o \begin{bmatrix} \widehat{\xi} & \widehat{\xi} \end{bmatrix}$  is the covariance matrix of exogenous disturbances.

Devereux and Sutherland (2006) provide a closed form solution for the equilibrium portfolio. One can show that in the presence of the three types of shocks (productivity, iPod and redistributive shocks), the optimal position for stocks is given by:

$$\begin{aligned} S &= 1 - \frac{1}{2\bar{k}} \frac{2a(\phi - 1)\sigma_y^2\sigma_\Psi^2 [(2a - 1)(1 - 1/\sigma) + \lambda - 1]}{2a(\phi - 1)(\lambda - 1)(\sigma_y^2 + \sigma_k^2)\sigma_\Psi^2 + \sigma_y^2\sigma_k^2} \\ b &= \frac{1}{2} \frac{[(2a - 1)(1 - 1/\sigma) + \lambda - 1] [\sigma_y^2 + 2a(1 - \phi)\sigma_\Psi^2]}{2a(\phi - 1)(\lambda - 1)(\sigma_y^2 + \sigma_k^2)\sigma_\Psi^2 + \sigma_y^2\sigma_k^2} \end{aligned} \quad (31)$$

where  $\sigma_y = std(\widehat{y})$ ,  $\sigma_\Psi = std(\widehat{\Psi})$  and  $\sigma_k = std(\widehat{k})$  are the standard deviations of the (relative) productivity, iPod and redistributive shocks respectively. We assume that shocks are uncorrelated. Note that eliminating one of the shocks (setting one of the variances to zero) brings us back to the complete markets situation analyzed in the preceding section. The portfolio of local stocks in equation (31) can be rewritten as:

$$\begin{aligned} S &= \frac{1}{2} - \frac{1}{2\bar{k}} \frac{2a(2a - 1)(\phi - 1)(1 - 1/\sigma)\sigma_y^2\sigma_\Psi^2}{2a(\phi - 1)(\lambda - 1)(\sigma_y^2 + \sigma_k^2)\sigma_\Psi^2 + \sigma_y^2\sigma_k^2} \\ &\quad + \frac{1}{2\bar{k}} \frac{[2a(\phi - 1)(\lambda - 1)\sigma_\Psi^2 + \sigma_y^2]\sigma_k^2}{2a(\phi - 1)(\lambda - 1)(\sigma_y^2 + \sigma_k^2)\sigma_\Psi^2 + \sigma_y^2\sigma_k^2} - \frac{1 - \bar{k}}{2\bar{k}} \end{aligned} \quad (32)$$

The equity portfolio depends on four terms. The first term reflects the diversification motive in a single-good world with zero labor income; the second term represents a hedge portfolio for real exchange rate risk (see above discussion); real exchange rate hedging is now more complex than in the model variant discussed before, because of the presence of a larger number of shocks<sup>9</sup>. The second term tends to generate a foreign equity bias for sufficiently high substitution elasticities between goods and sufficiently high risk aversion. Again, in the absence of trade barriers ( $a = 1/2$ ), the real exchange rate is constant and this term disappears. The third term comes from the presence of the redistribution shock and as in the complete market case induces a Home bias in stocks. This means that when redistributive shocks are sufficiently large (more precisely  $\sigma_k^2$  is large relative to  $\sigma_\Psi^2$ ), a home bias in stocks always exists. The last term, again, represents the equilibrium portfolio in a single-good world with labor income, in which the labor share is fixed (so that labor and capital incomes are perfectly positively correlated).

<sup>9</sup>Note that only part of real exchange rate movements are hedged through stocks, i.e the RER uncertainty that has not been hedged through bonds.

Note also that with an elasticity of substitution of unity ( $\phi = 1$ ), full Home bias ( $S = 1$ ) is obtained for all configurations of parameters as long as the variance of the distribution shock is not zero (an extension of Cole and Obstfeld (1991)).

The gross position in bonds is given by:

$$b = -\bar{k}(1 - S) \frac{\sigma_k^2}{\sigma_\Psi^2} \left[ \frac{\sigma_\Psi^2}{\sigma_y^2} - \frac{1}{2a(\phi - 1)} \right] \quad (33)$$

where  $S$  is given by (32).

(32) shows the role of the relative importance of the demand and productivity shocks for the equity portfolio. Agents will want to go long on foreign-good bonds when iPod shocks are large enough relative to productivity shocks. The reason is that a negative iPod shock that deteriorates terms of trade also deteriorates relative income and therefore can be hedged by holding Foreign bonds (whose relative return rises when domestic terms of trade worsen). By contrast, negative productivity shocks that improve terms of trade and deteriorate relative income can be hedged by holding domestic bonds that are valued more in these states of nature.

In the economy with the three simultaneous types of shocks, the foreign currency position is:

$$FCP = (1 - S) \bar{k} - b = (1 - S) \bar{k} \left\{ 1 + \frac{\sigma_k^2}{\sigma_\Psi^2} \left[ \frac{\sigma_\Psi^2}{\sigma_y^2} - \frac{1}{2a(\phi - 1)} \right] \right\} \quad (34)$$

which is positive as long as there is no full home bias ( $S < 1$ ) in stocks and productivity shocks are not too large with respect to the iPod shocks<sup>10</sup>.

## 6 Quantitative analysis with incomplete markets

### 6.1 Calibration

In this Section, we calibrate the incomplete markets model with output shocks, capital share shocks and iPod shocks<sup>11</sup>. We computed standard deviations of annual rates of change for real GDP growth and capital shares, for each G7 countries (1972-2003)<sup>12</sup>. Across the G7 countries, the mean standard

<sup>10</sup>Note that for  $\phi < 1$ , the positive sign of  $FCP$  is non-ambiguous for interior portfolios.

<sup>11</sup>We have checked that the addition of government expenditures shocks, that are analyzed in the appendix in the complete markets solution, does not change quantitatively the structure of portfolios.

<sup>12</sup>We measure a countries capital share as 1-(compensation of employees)/(GDP-indirect taxes), using annual data from OECD National Accounts.

deviations of (rates of change of) real GDP and the capital share are 1.91% and 2.34%, respectively. The mean capital share (across G7 countries) is 40%. In the quantitative model, we hence set  $std(\hat{y}_i) = 1.91\%$ ,  $std(\hat{k}_i) = 2.34\%$   $i = H, F$  and  $\bar{k} = 0.4$ .

Equilibrium portfolios under incomplete markets depend on the standard deviations of the two countries' relative endowments,  $y \equiv \frac{y_H}{y_F}$  and relative capital shares  $k \equiv \frac{k_H}{k_F}$ . We computed a country's relative real GDP and capital share, compared to a geometric average of the remaining G7 countries' GDP and the capital shares<sup>13</sup>. Relative outputs and capital shares undergo highly persistent fluctuations. The mean values (across G7 countries) of the standard deviation of growth rate of relative GDPs is 1.59%. For relative capital shares the corresponding mean standard deviation is 2.39%<sup>14</sup>. For all countries, the relative capital share is more volatile than relative GDP. In the calibrated model, we therefore set  $\sigma_y = std(\hat{y}) = 1.59\%$ ,  $\sigma_k = std(\hat{k}) = 2.39\%$ .

As mentioned before, the iPod shock that changes the demand for goods of a country can have several interpretations and its quantification is less easy than for the other two shocks. One is that it reflects quality changes and/or changes in the number of varieties produced by a country. Recent evidence at very disaggregated level by Broda and Weinstein (2007) suggests that this is an important phenomenon. In the model here, portfolios and other endogenous variables only depend on the relative iPod shock. We experiment two value for the volatility of relative "iPod shocks" ( $\Psi \equiv \frac{\Psi_H}{\Psi_F}$ ),  $\sigma_\Psi = std(\hat{\Psi}) = 1\%$ , and  $std(\hat{\Psi}) = 2\%$ , i.e. in one case where the iPod shock is less volatile than relative output, while in the other case, it is slightly more volatile than relative output. We also report results for the case where  $std(\hat{\Psi}) = 0$ . In that case, there are only productivity and capital share shocks.

Across G7 countries, the mean value in 2004 (weighted by relative country size) of the imports/GDP ratio is 20%<sup>15</sup>. Hence, we set  $a = 0.8$ .

The substitution elasticity  $\phi$  equals the price elasticity of foreign trade flows. A wide range of empirical estimates has been reported. Hooper and Marquez (1995) survey a large number of time-series studies that estimated (long run) price elasticities of aggregate trade flows, for the US, Japan, Germany, the UK

<sup>13</sup>The weights are based on countries' time-averaged share in G7 nominal GDP.

<sup>14</sup>The standard deviations of annual growth rates of relative GDP for the US, Japan, Germany, France, UK, Italy and Canada are 1.7%, 2.1%, 1.2%, 1.44%, 1.5%, 1.6% and 1.5%, respectively. The corresponding standard deviations of log growth rates of relative capital shares are 2.1%, 2.5%, 1.6%, 2.0%, 3.7%, 1.9%, 2.83%.

<sup>15</sup>Computing the mean import share for G7 countries gives 28% but this might be misleading since some G7 countries are relatively small. So we use the mean import share weighted by the relative size of countries.

and Canada; the median estimates (post-Bretton Woods era) of for those countries are 0.97, 0.80, 0.57, 0.6, and 1.01, respectively. The median estimate across the five countries is 0.88. We below consider a range of values of  $\phi$ . That range encompasses values of  $\phi$  that have been as assumed in recent quantitative macro/finance models; see, e.g. Kollmann (2005), Heathcote and Perri (2003), and Chari et al. (2002) who have set at 0.6, 0.9, 1.5 respectively. It is well known that in the trade literature, estimates of the elasticity of substitution between home and foreign goods is larger, between around 4 and 8. Hence, we will also report our results for a elasticity of 2 in Table 2 and for higher elasticities in a graph.

Estimates of  $\sigma$  in the range of 2 (or greater) are common for industrialized countries (e.g., Barrionuevo (1992)); in the quantitative experiments below, the risk aversion parameter is set at  $\sigma = 2$ .

Table 1 gives an overview of our parameter estimates:

Table 1: Parameter estimates

elasticity substitution	risk aversion	share domestic goods	mean capital share	std productivity	std capital share	std iPod
$\phi$	$\sigma$	$a$	$k$	$\sigma_y$	$\sigma_k$	$\sigma_\Psi$
0.6 to 2	2	0.8	0.4	1.59%	2.39%	1 to 2%

## 6.2 Numerical results

Table 2 reports numerical results. Columns (1)-(2) list the standard deviation of the relative iPod shock and the elasticity of substitution. Columns (3)-(8) show model predictions for the baseline structure with two stocks and two bonds.

For the baseline model, we report the local equity share ( $S$ ), local holdings of local-good bonds ( $b$ ), and the foreign currency position ( $FCP = (1 - S)\bar{k} - b$ ) (see columns (3)-(5)). In addition, we report three statistics that describe the behavior of equity returns and the real exchange rate (see columns (6)-(8)): the cross-country correlation of equity returns<sup>16</sup>; the covariance of the measured real exchange ( $\widehat{REER}$ ) with the equity return difference:  $\widehat{R}_H - \widehat{R}_F = k_H \widehat{p}_H y_H - k_F \widehat{p}_F y_F$  normalized by the variance of equity returns, i.e.  $\frac{cov(\widehat{R}_H - \widehat{R}_F, \widehat{REER})}{var(\widehat{R}_H - \widehat{R}_F)}$ ; the correlation between relative aggregate consumption and the (measured) real exchange rate. Note that the last two statistics are based on CPIs, real exchange rates and aggregate consumption measures that do not take into the account preference of quality changes ( $\widehat{\Psi}$ ); this is again motivated by the fact that empirical CPI and real consumption measures do not (or only very

<sup>16</sup>The correlation pertains to equity returns expressed in terms of the Home good.

partially) capture quality/variety changes (see Broda and Weinstein (2006)). Also, purely psychological demand shocks are not reflected in official consumption data.

Table 2: Numerical results

std iPod $\sigma_\Psi$	elasticity substitution $\phi$	Home bias stocks $S$	Position bonds $b$	Foreign currency pos. $FCP$	Correlation equity returns	$\frac{cov(\widehat{R}_H - \widehat{R}_F, RER)}{var(\widehat{R}_H - \widehat{R}_F)}$	Correlation (Cons.,RER)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	0.6	1.00	-0.08	0.06	0.67	0.28	-1.00
1%	0.6	0.98	-0.07	0.08	0.66	0.30	-0.79
2%	0.6	0.95	-0.08	0.10	0.62	0.35	-0.53
0	0.9	1.00	0.03	-0.03	0.69	0.10	-1.00
1%	0.9	1.00	0.03	-0.03	0.69	0.11	-0.96
2%	0.9	1.01	0.03	-0.03	0.68	0.13	-0.85
0	1.5	1.00	0.22	-0.22	0.69	0.07	-1.00
1%	1.5	0.92	0.14	-0.11	0.69	0.07	-0.60
2%	1.5	0.75	-0.05	0.14	0.69	0.07	-0.56
0	2	1.00	0.38	-0.38	0.68	-0.02	-1.00
1%	2	0.81	0.10	-0.02	0.69	-0.02	-0.43
2%	2	0.60	-0.21	0.37	0.70	-0.08	-0.53

In the baseline model with two stocks and two bonds, market are complete when there are no iPod shocks ( $\sigma_\Psi = 0$ ). A 100% equity home bias ( $S = 1$ ) is then obtained, and countries hold a long position in Home bonds if  $\phi > 1$ . In the model versions with  $\phi > 1$ , the long position in home currency is sizable (it amounts to 22% of expected output, when  $\phi = 1.5$ ). This is just a restatement of the puzzle presented before; with productivity shocks and redistributive shocks only, the model is unable to reproduce a long position in foreign currency for an elasticity of substitution larger than 1. Introducing iPod shocks reduces the equity home bias, and the home bond position, when the substitution elasticity exceeds unity. For example, when  $\phi = 1.5$ , the model with  $\sigma_\Psi = 2\%$  predicts that 75% of equity is locally owned, and that a country goes short in home bonds; the overall foreign good claim is positive,  $FCP = 0.14$ ; this implies that an exogenous unexpected 10% worsening of a country's terms of trade generates a capital gain that represents 1.4% ( $= 0.10 * 0.14$ ) of (expected) output.

Kollmann (2006) reports external equity liabilities (defined as the sum of portfolio equity and FDI liabilities) of OECD economies, in 2003 (data source: IMF international investment positions database). US foreign equity liabilities amounted to 37% of US GDP. Among G7 countries, foreign equity liabilities represented between 12% (Italy) and 78% (UK) of domestic GDP. Assuming that the domestic capital stock is about 3 times larger than annual GDP, these figures suggest that between 74% and 96% of the capital stock located in G7 countries was owned by local investors, in 2003. The predicted locally held equity share in the three-shocks model is thus broadly consistent with G7 data.

The correlation of equity returns given in Table 2 are between 0.6 and 0.7 for various choices of parameter values. The model thus matches the high correlation of stock returns across G7 countries, 0.63<sup>17</sup>. This high correlation is partly due to the positive correlation of output shocks across countries, but it also reflects terms-of-trade movements (Cole and Obstfeld (1993), Rigobon and Pavlova (2004)): a positive output shock at home raises the relative price of the foreign good—it, hence, raises stock returns, in both countries. Similarly, while a domestic positive "iPod shock" raises the demand for the local good (and its price), raising home returns, the effect is partially mitigated if consumers cannot easily substitute home and foreign consumption.

The theoretical literature on international portfolio choice has shown that real exchange rate fluctuations generate a hedging motive in the choice of assets. Warnock and van Wincoop (2006) discuss a partial equilibrium two-country model, without labor income, in which the only assets are Home and Foreign stocks. In that model, equity home bias is an increasing function of the regression coefficient of the real exchange rate  $\widehat{REER}$ , on the equity return differential  $\widehat{R}_H - \widehat{R}_F$ <sup>18</sup>; when this "covariance-variance ratio" is zero, then equity home bias is zero, i.e. equity portfolios are perfectly diversified:  $S = 1/2$ . . Warnock and van Wincoop (2006) document that, empirically the "covariance-variance ratio" is close to zero<sup>19</sup>; they thus conclude that the "portfolio home bias associated with hedging real exchange rate risk is essentially zero" (p.11). Simple general equilibrium models driven by productivity shocks (without labor income or demand shocks) generate equity home bias, if the implied covariance-variance ratio is

<sup>17</sup>For each G7 country, we computed the correlation between the Home real equity return and the "foreign" (rest-of G7) equity return, where (as in the theoretical model), all returns are expressed in units of Home GDP. The mean Home-Foreign equity return correlation is 0.63 (based on annual MSCI returns for 1972-1994).

<sup>18</sup>i.e. of the covariance between  $\widehat{R}_H - \widehat{R}_F$  and the real exchange rate  $\widehat{REER}$ , normalized by the variance of  $\widehat{R}_H - \widehat{R}_F$  :  $\frac{cov(\widehat{R}_H - \widehat{R}_F, \widehat{REER})}{var(\widehat{R}_H - \widehat{R}_F)}$ .

<sup>19</sup>The estimate for annual excess returns on US equity vis-à-vis 21 other OECD countries is 0.11.



much larger than that observed in the data. As discussed above, in the model here, capital share shocks create a powerful motive for holding local equity; those shocks induce equity return fluctuation that are disconnected from real exchange rate movements. Also, in the setting here, bonds can be used for hedging real exchange rate risk. This enables the present model to simultaneously generate a realistic equity home bias, *and* to match the low covariance-variance ratio seen in the data (see column (6) in Table 2). The model version with  $\phi = 1.5$  generates a covariance-variance ratio of about 0.10, consistent with the data. Warnock and van Wincoop (2006) also discuss a model with trade in stocks and in Home and Foreign bonds; when there are neither capital share nor iPod shocks, the degree of equity home bias depends on a covariance-variance ratio based on components of excess equity returns and of the real exchange rate that are orthogonal to the exchange rate movements; empirically, that conditional covariance-variance ratio is essentially zero. In the present model, the conditional covariance-variance ratio is exactly zero.

With complete markets and without iPod shocks ( $\sigma_\psi = 0$ ), the following risk sharing condition links relative aggregate consumption and the real exchange rate:  $\left(\frac{C_H}{C_F}\right)^{-\sigma} = \frac{P_H}{P_F}$ , which implies that relative consumption is perfectly negatively correlated with the real exchange rate (up to a linear approximation, see Kollmann (1991, 1995), Backus and Smith (1993)). Empirically, the consumption-real exchange correlation is close to zero. Among G7 countries, the mean correlation between the annual growth rates of relative consumption and the real exchange rate is 0.04 (1972-2003).

Incomplete markets break the perfect correlation between relative consumption and the real exchange rate, while iPod shocks weaken the link between measured relative consumption and the measured real exchange rate, when empirical CPI and real consumption measures do not (or only partially) capture quality/variety changes. However, in the model here, the predicted correlation between (measured) relative consumption and the (measured) RER remains too large (in absolute value), compared to the data (see Table 2, column (8)). For the specifications where we obtain realistic foreign asset position, the correlation is around -0.5 (for example, when  $\phi = 1.5$  and  $\sigma_\psi = 2\%$ , the consumption-real exchange rate correlation is -0.56). So even though we are going in the right direction, we cannot quantitatively reproduce the low consumption-real exchange rate correlation observed in the data.

We have also looked more in detail how our results on portfolios vary with the elasticity of substitution  $\phi$ . When the iPod shock, has an intermediate value of 1.5% for the standard deviation, our results are

quite robust to various estimates of this elasticity even in the range that international trade economists consider as plausible (4 or 5) and that macroeconomists would consider as too high. The equity home bias never goes below 60%. We have also checked that our results are quite insensitive to change in the coefficient of risk aversion. Intuitively, a higher coefficient of risk aversion increases slightly foreign stock and bond holdings.

## 7 Conclusion

This paper has shown that the combination of two simple sources of uncertainty, related to the distribution of income between capital and labor and to international demand conditions, can help understand the structure of international portfolios and associated valuation effects. This is in contrast to existing models that focus on productivity shocks as the main source of uncertainty. When a country is hit by a negative productivity shock, then its optimal portfolio should generate an external capital gain. As such a shock improves the country's terms-of-trade, this would imply that a real exchange rate appreciation is associated with an external capital gain. Empirically, however, exchange rate depreciations are associated with external capital gains. This paper has shown that the combination of two simple sources of uncertainty, related to the distribution of income between capital and labor and to relative demand for domestic versus foreign goods, can help understand the structure of international portfolios and associated valuation effects.

One can speculate that in a model with imperfect competition, shocks to the degree of competition and mark-ups would combine the properties of redistributive and relative demand shocks, and thus help to produce realistic international portfolios, as such shocks likewise redistribute income to profits and change terms of trade. We leave the analysis of such shocks for future research.

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## 8 Appendix

### 8.1 Stylized facts on international portfolios

Country	Country's share in world market capitalization (in%)	Share of domestic stocks in the aggregate portfolio (in%)	Share of domestic stocks in investment funds portfolios (in%)	Size of the home bias in portfolios = $\log\left(\frac{\text{column 3}}{\text{column 2}}\right)$
United-Sates	47.8	88.7	85.5	0.62
United-Kingdom	8.1	77	43.1	2.25
Japan	11.3	89.5	71.8	2.06
France	4.3	79.8	55.3	2.92
Germany	4.0	61.3	33.5	2.72
Canada	2.4	84	27.0	3.55
Italy	2.2	67.3	35.4	3.42
Switzerland	2.2	45.6	21.0	3.03
Netherlands	2.0	43.6	19.5	3.03
Spain	1.4	86	36.0	4.11
Australia	1.2	71.7	18.2	4.09

Table 1: Home bias in equity portfolios. Column 3 measures the share of domestic stocks in countries portfolios in 2001 for the biggest market capitalization. Sources: CPIS data. Column 4 measures the share of domestic stocks in a representative sample of mutual funds, averaged over the period 1999-2000, Source Chan *et al.* [2005].

	Net external position	Net domestic currency position	Net U.S. dollar position	Net other currencies position
China	12.5	-28.3	29.2	11.6
Euro Area	-15.0	-65.5	16.8	34.5
Japan	35.9	-26.9	38.5	21.9
United States	-21.5	(-74.8)	-74.8	53.4

Table 2: Currency exposure of international portfolios (in percent of GDP). Source Lane and Milesi-Feretti (2006).

## 8.2 The case of government expenditures shocks

In this appendix, we analyze another type of demand shock in the case of complete markets, namely a government expenditure shock. The other shock is the productivity shock. The resource constraints are now replaced by:

$$c_H^H + c_H^F = y_H - g_H \quad (35)$$

$$c_F^F + c_F^H = y_F - g_F \quad (36)$$

where  $g_i$  are government expenditures of country  $i$  which we assume fall entirely on the domestic good.

We denote the relative government spending shock as:

$$g = \frac{g_H}{g_F} \quad (37)$$

These government purchases  $g_i$  are financed by lump-sum taxes  $T_i$ .

Using the relation between the real exchange rate and consumption implied by complete markets:

$$q^{-\frac{1}{1-\rho}} \Omega \left[ \left( \frac{P_H}{P_F} \right)^{\frac{1}{\sigma} - \frac{1}{1-\rho}} \right] = \frac{y_H - g_H}{y_F - g_F}$$

We log-linearize this expression around the symmetric steady-state where the steady state value  $g$  is equal to one. We get:

$$(1 + s)\hat{y} - s\hat{g} = -\lambda\hat{q} \quad (38)$$

where  $s = \frac{\bar{g}_i/\bar{y}_i}{1-\bar{g}_i/\bar{y}_i}$ ,  $i = H, F$  is increasing in the steady state share of public spending over total production which we assume to be equal in both countries.



The equilibrium relative price is then:

$$\hat{q} = -\frac{1+s}{\lambda}\hat{y} + \frac{s}{\lambda}\hat{g} \quad (39)$$

Terms-of-trade are decreasing in the relative productivity shocks  $\hat{y}$  and increasing in the relative demand shocks  $\hat{g}$  (relative government spending).

We log-linearize the budget constraints and obtain:

$$\theta\hat{q}\left(1 - \frac{1}{\sigma}\right) + s(\hat{q} + \hat{g}) = (2S - 1)(1 + s)(\hat{q} + \hat{g}) + 2b\hat{q} \quad (40)$$

Portfolios that give the optimal allocation for all shocks are described by

$$S = 1 \quad ; \quad b = \frac{1}{2} [\theta(1 - 1/\sigma) + \lambda - 1] \quad (41)$$

so that a complete (100%) home bias appears in stocks ( $S = 1$ ) for any parameter value on preferences or trade costs just as in equation (25). The intuition is that a Home government expenditure shock will crowd out private income but also increase relative dividends through its effect appreciation effect on the terms of trade. From that point of view, a government expenditure shock has the same effect as a shock on the distribution of income between dividends and labor and it is not surprising therefore that it generates the same portfolio. In order to avoid crowding out of private consumption, the best strategy is indeed to hold domestic stocks that provide high dividends in these states of nature.