The Positive Effects of Biased Self-Perceptions in Firms*

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Abstract

Many firms and organizations involve a team of agents in which the marginal productivity of any one agent increases with the effort of others. Because the effort of each agent is not observable to any other agent, the performance of the firm is negatively affected by a free-rider problem and a general lack of cooperation between agents. In this context, we show that an agent who mistakenly overestimates her own marginal productivity works harder, thereby increasing the marginal productivity of her colleagues who then work harder as well. This not only enhances firm performance and value but may also make all agents better off, including the biased ones. Indeed, although biased agents overwork, they benefit from the positive externality that other agents working harder generates. The presence of a leader improves coordination and firm value, but self-perception biases can never be Pareto-improving when they affect the leader. Self-perception biases are also shown to affect the likelihood and value of mergers, job assignments within firms, and compensation contracts.

1. Introduction

Cooperation, coordination and synergies are sources of value in many economic situations. For example, the success and viability of integrated firms (Grossman and Hart, 1986; Scharfstein and Stein, 2000), partnerships (Farrell and Scotchmer, 1988; Levin and Tadelis, 2005), strategic alliances (Jensen and Meckling, 1995; Holmström and Roberts, 1998), and joint ventures (Alchian and Woodward, 1987; Kamien, Muller and Zang, 1992; Aghion and Tirole, 1994) are affected by the efficiency with which various entities and agents interact with each other. In fact, the view that firms form endogenously as a way to gather and take advantage of complementary activities dates back to Alchian and Demsetz (1972). More generally, economic development and the efficient provision of public goods often require the concerted efforts of several people, businesses and organizations. Campbell (1986) even argues that cooperation is a healthy component of evolutionary behavior.

Even if synergistic factors are gathered under the roof of one firm, the realization of their potential value is not automatic. Indeed, as pointed out by Knight (1921) and Coase (1937), properly organizing the firm's factors of production is key to the firm's success.[1] So, just like a principal must find ways to motivate an agent to work, the firm must find ways to promote cooperation amongst its agents if teamwork makes the firm more productive and more valuable. A natural approach to studying the incentives of an organization's members in the presence of cooperation and coordination issues has been Holmström's (1982) model of a team. A general insight from this model is that moral hazard and free-rider problems are prevalent in teams when the effort decisions of the teams' agents are unobservable. Because agents make decisions that are in their best self-interest, their unmonitored actions often fail to conform to their organization's objectives, unless proper incentives are provided to them. These problems are exacerbated when complementarities exist across agents, as any one agent does not fully internalize the impact that her decisions have on the decisions of others. Our paper takes these problems as given in the context of a firm, and uses insights from psychology to study the impact that agent biases will have on these problems and their solutions.

A large body of the psychology literature shows that individuals tend to overestimate their own skills. For example, Langer and Roth (1975), and Taylor and Brown (1988) document that individuals tend to perceive themselves as having more ability than is warranted. According to Kunda (1987), they also tend to believe in theories that imply that their own attributes cause desirable outcomes. Similarly, Fischhoff, Slovic and Lichtenstein (1977), and Alpert and Raiffa (1982) find that individuals tend to overestimate the precision of their information. In business settings,

[1] Marshall (1920) even refers to organization as the fourth factor of production (after land, labor and capital).

Larwood and Whittaker (1977) find that managers tend to believe that they are superior to the average manager, and Cooper, Woo and Dunkelberg (1988) find that entrepreneurs perceive their own chance for success as being higher than that of their peers. We incorporate such self-perception biases into the problem of a firm that must hire two agents for production. In particular, we assume that one agent, which we refer to as an *overconfident* agent, overestimates the degree to which her effort contributes to firm success (i.e., the marginal product of her effort). We show that the bias can not only make the firm more valuable by naturally overcoming the usual free-rider and effort coordination problems in teams, but it can also make both agents, including the overconfident one, better off.

The idea is that agents who overestimate their own marginal product tend to work harder. In particular, an agent who has an inflated opinion of her ability to create value can sometimes justify making a costly effort when an otherwise identical but rational agent would not. This extra effort reduces the free-rider problem directly, but it does more than that in the presence of complementarities which, as argued by Alchian and Demsetz (1972), are likely to drive the formation of firms in the first place. Indeed, when complementarities exist across the firm's agents, the effort of an agent increases the marginal productivity of the other agent, and as a result she too finds herself facing a situation in which her effort is more valuable. As a result, this other agent also exerts more effort, making the firm even more productive. When the production synergies between the two agents are large, even the biased agent ends up benefitting from her overinvestment in effort, as she shares the benefits of her colleague's increased effort (but still suffers the cost of her overinvestment in effort). Thus, overconfidence can generate a Pareto improvement in our setting. Interestingly, this result holds even when compensation contracts are endogenously chosen by the firm to maximize value. The firm then compensates the overconfident agent more than the other agent as the incentive for her to overinvest in effort provision is the force that renders synergistic production possible and valuable.

Other authors have considered the effects of behavioral traits in effort coordination problems amongst agents. For example, Rotemberg (1994) analyzes the effect of altruism on cooperation within a team. He shows that when complementarities between the team's agents exist, the presence of some altruistic agents can generate Pareto improvements, just like altruism can benefit all members of a family (not just the selfish ones), as argued by Becker (1974). Eshel, Samuelson and Shaked (1998) further show that altruistic teams are more likely to survive in the long run. Another example of behavioral considerations is found in the work of Kandel and Lazear (1992), who show that effort coordination problems can be overcome when there is peer pressure among members of an organization. In effect, peer pressure imposes an extra cost on agents that do not make the appropriate effort. These authors also discuss how peer pressure can emerge endogenously.

Interestingly, it is not the concern for others or of others that improves cooperation in our model. Instead, it is the extreme self-perception of some agents that does. Biased agents simply think that their contribution is large enough to justify their costly effort, without any consideration for their colleagues or teammates. The externalities associated with their effort matter little to overconfident agents but do foster cooperation within the organization. That is, their flattering views of themselves combine with their self-interest to generate externalities on others. So agents cooperate not because they want to, but because cooperation comes with being skilled (as they think they are) and working. In that sense, our model is closer to that of Kelsey and Spanjers (2004) who show how the ambiguity aversion of some agents leads them to use personal effort as insurance for the effort of others, alleviating the free-rider problem in the process. Also related is the work of Gaynor and Kleindorfer (1991) who show that misperceptions about the firm's production function can have positive effects.

Our paper also discusses how self-perception biases interact with some of the other mechanisms that have been proposed in the literature for alleviating the collective-action problem in organizations. One approach to incentive problems in these contexts has concentrated on the organizational structure of the firm. An example of this approach is found in the work of Hermalin (1998) who discusses the role of leadership in fostering coordinated effort. We incorporate leadership into our two-agent framework by letting the firm appoint a leader whose effort choice is made public before the other agent makes her effort choice. This structure naturally increases the extent of cooperation as it creates an incentive for the leader to work harder knowing that her effort choice will affect the effort exerted by the follower. In essence, as in Hermalin (1998), one agent leads the other by example. In contrast to Hermalin (1998) however, the leader's influence on the follower's action is due to complementarities between the two agents, and not to the leader's superior information. Using this framework, we study how self-perception biases and leadership interact in contributing to agents' welfare and to firm value. We find that self-perception biases can generate Pareto improvements only when the rational agent is the leader. We also show that firm value is maximized with a rational leader when complementarities are strong, but with a biased leader otherwise. This set of results on the optimal organizational design of a firm in the presence of behavioral biases is, to our knowledge, new in the literature. A related result is that of Rotemberg and Saloner (2000) who show that CEOs with a vision can enhance the incentives of other workers to engage in innovative activities.

It is worth noting that the positive ex ante effects of ex post biased behavior have been studied in other contexts. A prominent example is the work by Fershtman and Judd (1987), which shows that a firm in Cournot competition may choose to commit to an ex post inefficient strategy in order to affect the actions of the other firm. Similarly, Kyle and Wang (1997) show that the presence of a biased manager creates an analogous commitment for a money management firm. Finally, Heifetz, Shannon and Spiegel (2004) argue that biased agents may be better equipped to survive in the long run because of the effects that their biases can have on the behavior of other agents. Our paper differs from these papers in terms of its context (that of a firm), its scope (the interaction of self-perception biases with compensation contracts and organizational design) and its results (the possibility that biases lead to Pareto improvements, not just increases in value).

More generally, our paper adds to the growing literature in corporate finance that studies the behavioral biases of managers and CEOs in firms. In recent years, several papers in this area have provided evidence suggesting that many corporate managers are overconfident and that this affects the decisions they make for their firms. For example, Malmendier and Tate (2005, 2006) use the tendency of CEOs to hold stock options too long to proxy for their overconfidence. Others like Ben-David, Graham and Harvey (2006) and Sautner and Weber (2006) use survey evidence to estimate the overconfidence of top executives. Finally, Puri and Robinson (2006, 2007) use the Survey of Consumer Finance to establish a link between optimism, work ethic and the propensity to become an entrepreneur.

In this context, the contribution of our paper is twofold. First, we show that overconfidence can benefit all parties — including the biased agent herself, her colleagues, and the firm — and thus can generate Pareto improvements. We believe that this result is important for overconfidence to persist within the firm, as suboptimal behavior is likely to be eliminated by economic forces over time. Indeed, if the firm loses from the biases of its employees, as is often implicitly assumed in studies of managerial biases, biased agents will be replaced. Similarly, if agents lose as a result of their own biases, they are likely to realize their mistakes and leave. Our result that self-perception biases can increase economic surplus and benefit all agents provides a justification for the presence and survival of biases in firms.

Second, our paper points out that studying the effects of managerial biases without considering the endogenous contractual incentives that managers face as well as the organizational structure imposed on them may not be appropriate. Indeed, as we show, the bias of an agent can not only be realigned by a contract that negates its effect, but it can also be harnessed by organizations that provide this agent with the kinds of interactions that render her bias valuable to her and those around her. In fact, our analysis demonstrates that the bias of an agent cannot be studied in isolation either, as the presence of biased agents alters the behavior of other agents in the firm. That is, the relation between biases and decisions does not depend solely on the direct effect of the bias; rather, the changes in contracts and in the behavior of other agents contribute to the overall effect on decisions.

The rest of the paper is organized as follows. In section 2, we set up and solve our main model in which a firm hires two agents, one rational and one biased, for production. Section 3 looks at the effects of making one of the firm's members the leader. More precisely, the rational-leader and biased-leader scenarios are compared in terms of individual welfare and firm value. Various extensions of the model are considered in section 4. Finally, section 5 provides some empirical implications of our model, discusses a number of applications, and concludes. All proofs are contained in the appendix.

2. The Main Model

A. The Firm

Consider an all-equity firm, owned by risk-neutral shareholders (the *principal*), requiring the effort of two agents (indexed by $i = 1, 2$) for its production. For simplicity, we assume that the firm's existing assets are worth zero (any non-negative constant would do) and that its production comes from a single one-period project, which can either succeed or fail with probabilities π and $1 - \pi$ respectively. Without loss of generality, we assume that the project's risk is idiosyncratic and that the appropriate discount rate for the project, the riskless rate of return, is zero so that the project's value is simply its expected value. The project generates $\sigma > 0$ dollars at the end of the period if it succeeds, and it generates zero if it fails. Thus the firm's end-of-period cash flow is given by

$$\tilde{v} \equiv \begin{cases} \sigma & \text{prob. } \pi \\ 0 & \text{prob. } 1 - \pi. \end{cases} \tag{1}$$

The probability of success π is endogenous; it depends on the choice of effort made by each agent i. Each agent i can choose to work ($e_i = 1$) or not ($e_i = 0$). To capture the idea that production is enhanced by the cooperation of the two agents, we assume that the probability of success is given by

$$\pi = ae_1 + ae_2 + se_1e_2, \tag{2}$$

where $a \geq 0$, $s > 0$, and $2a + s < 1$. Parameter a measures the direct effect of an agent's effort on the probability of success. It can be interpreted as the ability level of the agents. Parameter s captures the effect of the interaction between the two agents on the probability of success. In assuming that $s > 0$, we are considering a situation in which the interaction is synergistic, that is, the two agents create positive externalities on each other. This assumption is consistent with

Alchian and Demsetz's (1972) view that firms form to take advantage of positive externalities or complementarities. For reasons that will become clearer later, we also assume that $\sigma \leq 2$, as this will allow us to concentrate our analysis on interior solutions.

Agents are risk-neutral, have zero wealth, and are protected by limited liability. Their effort decisions are made simultaneously and each agent's decision is unobservable to the other agent and the firm, rendering effort decisions non-contractible. As such, because only two project outcomes (i.e., end-of-period states of the world) are possible, their compensation contracts must specify how much each receives for a successful project ($\tilde{v} = \sigma$) and how much each receives for a failed project ($\tilde{v} = 0$). These contracts are chosen by the firm, knowing that agents choose their effort to maximize their expected utility and bear a private utility cost of effort. Effort costs, denoted by $\tilde{c}_i$ for agent i, are not known by anyone at the outset, but are known to be uniformly distributed between 0 and 1, and independent across agents.[2] Each agent privately observes her own cost, without observing the other's, before making her effort decision. This describes, for example, a situation in which agents learn the constraints they face (e.g., time, other commitments, task difficulty, etc.) after joining the firm, while not being able to infer the constraints of others.

With a risk-neutral principal and agents, compensation for failed projects decreases the motivation for effort and does not improve risk-sharing. As such, it is never optimal for the firm to reward its agents for failed projects,[3] and all compensation is paid only when $\tilde{v} = \sigma$. We denote agent i's compensation in that state by w_i. Given a compensation contract w_i, we can therefore denote the utility of agent i at the end of the period by

$$\tilde{U}_i = w_i \mathbf{1}_{\{\tilde{v}=\sigma\}} - \tilde{c}_i e_i, \tag{3}$$

where $\mathbf{1}_{\{E\}}$ denotes an indicator function for event E.

B. Introducing Self-Perception Biases

We assume that agent 2 suffers from a self-perception bias in that she thinks that she is more skilled than she really is, and so she thinks that the contribution of her effort to the project's chance of success is greater than her actual contribution. Specifically, she thinks her ability is $A > a$, although it is truly only a. This departure from true ability to perceived ability represents agent 2's bias. We

[2] These distributional assumptions about $\tilde{c}_i$ are made purely for analytical convenience. The only required assumption is that effort costs are not perfectly correlated.

[3] Of course, because the firm's existing assets are assumed to be worth zero, it is technically impossible for the firm to offer any compensation to the agents when the project yields zero. Even if the firm had any value when the project fails, however, it would still be suboptimal for it to compensate the agents in that state.

denote this quantity by $b \equiv A - a \in [0, 1 - 2a - s)$, and refer to it as her *self-perception bias* or *level of overconfidence*. We assume that agent 2's overconfidence boils down to a disagreement between the two agents about her skill. In particular, we assume that agent 1 knows that agent 2 is biased, and that agent 2 knows that agent 1 thinks that way, but disagrees with her. In essence, therefore, the two agents agree to disagree as in Morris' (1996) work, but the object of this disagreement is agent 2's skill.[4] The assumption that agent 1 realizes that agent 2 is biased is important for some, but not all, of our results. In particular, it does affect our welfare analysis as it pertains to agent 2. This is because our welfare results depend on whether other agents change their behavior when teamed with a biased agent. Still, as long as agent 1 assigns a positive probability to the possibility that agent 2 is biased, our welfare results will go through.[5]

Our characterization of agent 2's bias is consistent with a behavioral characteristic of individuals that has been extensively documented in the psychology literature. In particular, Langer and Roth (1975), and Taylor and Brown (1988) document the fact that people tend to overestimate their own skills, and Larwood and Whittaker (1977) show that business managers suffer from the same bias. Similarly, Greenwald (1980) documents that people's self-evaluations tend to be unrealistically positive. Moreover, Dunning, Meyerowitz and Holzberg (1989) find that such biases are more pronounced when the definition of competence is ambiguous, which is likely to be the case in many economic contexts. In a group context, Caruso, Epley and Bazerman (2006) provide evidence that individuals believe their contribution to group output to be greater than it really is. As we show next, such self-perception biases can have useful properties when agents must cooperate within a firm.

C. Equilibrium with Exogenous Compensation Contracts

To study the role played by the second agent's bias in the model's equilibrium, we start by exogenously fixing the compensation contracts of the two agents to w_1 and w_2 respectively. Section 2E endogenizes these compensation contracts by letting the firm pick w_1 and w_2 to maximize its value. Of course, it will then be the case that the firm never chooses contracts that promise its agents more total compensation than the firm's profits, as it would be more beneficial for it to close its doors instead. As such, we assume that $w_1 + w_2 \leq \sigma$ even for this section.

At the time each agent makes her effort decision, she does not know whether the other agent

[4] Our results hold under the alternative assumption that agent 2 thinks that agent 1 agrees with her about her skill. The derivation and proofs of these results are available from the authors upon request.

[5] In fact, we show that this is the case in section 4D.

will exert effort or even the cost of that effort. Instead, she must anticipate the expected level of effort from the other agent. In equilibrium, because utility is decreasing in effort cost, it will be the case that agent i works if and only if her cost of effort does not exceed some threshold that we denote by $k_i \in [0,1]$. That is, if it is optimal for an agent to work when the cost of effort is $\tilde{c}_i = k_i$, then she will also find it optimal to work when $\tilde{c}_i < k_i$. Solving for the equilibrium involves finding the equilibrium k_i for each agent. Because $\tilde{c}_i$ is uniformly distributed on the unit interval, k_i also represents the frequency with which agent i is expected to exert effort in equilibrium. For this reason, a higher k_i also means that agent i works harder and, as such, we sometimes refer to it as her effort level.

Let us take the position of the rational agent, agent 1, after she observes that her effort will cost $\tilde{c}_1 = c_1$. She anticipates the second agent to work if $\tilde{c}_2 \leq k_2$, and so she anticipates her to work with probability k_2. Thus agent 1 seeks to solve the following maximization problem:

$$\begin{aligned}\max_{e_1 \in \{0,1\}} \mathrm{E}\left[\tilde{U}_1 \mid \tilde{c}_1 = c_1\right] &= w_1 \mathrm{E}[\pi] - c_1 e_1 \\ &= w_1\big(a e_1 + a\mathrm{E}[e_2] + s e_1 \mathrm{E}[e_2]\big) - c_1 e_1 \\ &= w_1\big(a e_1 + a k_2 + s e_1 k_2\big) - c_1 e_1. \end{aligned} \tag{4}$$

From this, it is easy to show that agent 1 works ($e_1 = 1$) if and only if $\tilde{c}_1 \leq w_1(a + s k_2)$, and so

$$k_1 = w_1(a + s k_2). \tag{5}$$

Agent 1 works harder when she gets compensated more (large w_1), when she is more skilled (higher a), when agent 2 works harder (large k_2), and when the synergies with her colleague are larger (large s). Agent 2 is biased and so, after she observes $\tilde{c}_2 = c_2$, she seeks to solve the following problem (in which the "B" subscript denotes the fact that she is biased):

$$\begin{aligned}\max_{e_2 \in \{0,1\}} \mathrm{E}_{\mathrm{B}}\left[\tilde{U}_2 \mid \tilde{c}_2 = c_2\right] &= w_2 \mathrm{E}_{\mathrm{B}}[\pi] - c_2 e_2 \\ &= w_2\Big(a\mathrm{E}_{\mathrm{B}}[e_1] + (a+b)e_2 + s\,\mathrm{E}_{\mathrm{B}}[e_1]\,e_2\Big) - c_2 e_2 \\ &= w_2\big(a k_1 + (a+b)e_2 + s k_1 e_2\big) - c_2 e_2. \end{aligned} \tag{6}$$

Thus agent 2 works ($e_2 = 1$) if and only if $\tilde{c}_2 \leq w_2(a + b + s k_1)$, and we have

$$k_2 = w_2(a + b + s k_1). \tag{7}$$

In addition to the comparative statics in w_2, a, s and k_1, the second agent works harder as the opinion she has of her own skill gets more inflated (as b gets larger). In other words, because the biased agent thinks that her effort is more productive than it really is, she is less reluctant to invest in effort than her rational counterpart.

The result that skilled (and biased) agents work harder is a direct product of the assumption that the marginal productivity of effort is increasing in a. The same result would obtain if we were to assume that the marginal disutility of effort is lower at all effort levels for higher ability agents. Such an assumption is in fact made in several papers in which there is skill heterogeneity across agents, whether the models concentrate on signaling (e.g., Spence, 1973), rank-order tournaments (e.g., Lazear and Rosen, 1981), screening (e.g., Garen, 1985), or multi-period contracting (Lewis and Sappington, 1997). Admittedly however, there is no economic theory justifying any assumption that implies a positive relationship between skill and effort. Indeed, one can easily imagine contexts in which a highly skilled agent simply scales back on effort, as her lower but more productive effort achieves the same result as the more sustained effort of lower skilled agents and allows her to enjoy more leisure utility. Interestingly, Schor (1993) documents that workers do exactly the opposite: they allocate the hours that suddenly become available for leisure to extra work. So it appears that individuals benefit from leisure up to a certain point, but derive more utility from work once they have achieved a certain minimum leisure utility. Their behavior even prompts Simon (1991, p. 33) to ask "why do employees not substitute leisure for work more consistently than they do?" In this light, the decision of agents in our model should be interpreted as an allocation of effort to activities that are perceived as more productive from the agent's perspective, and the overconfident agent simply overestimates (by b) how productive she can make these activities. Consistent with Felson's (1984) evidence, such agents with large self-appraisals tend to work harder.

The resulting equilibrium effort level of the two agents is derived in the following lemma.

Lemma 1 *In equilibrium, agent 1 makes an effort if and only if her cost of effort does not exceed*

$$k_1 = \frac{\left[a + (a+b)sw_2\right]w_1}{1 - s^2 w_1 w_2}, \tag{8}$$

and agent 2 makes an effort if and only if her cost of effort does not exceed

$$k_2 = \frac{(a + b + asw_1)\,w_2}{1 - s^2 w_1 w_2}. \tag{9}$$

It is easy to verify that the effort levels of the two agents in equilibrium are both increasing in w_1, w_2, a, s and b. The last two are crucial to our analysis. As b increases, agent 2's perception of her own ability and productivity increases. From her perspective, this increased productivity is enough to warrant an effort, that is, her effort does not require as much of an effort on the part of agent 1 as before. Agent 1 also works harder as b increases. This is due to the fact that, because agent 1 knows that agent 2 works harder, she knows that the potential synergistic gains, through s, from their combined effort are likely larger than before. This makes her effort more valuable, and so she is more willing to pay its cost. In other words, when the efforts of the agents are synergistic, the marginal productivity of one increases in the other one's effort, and so the higher effort of one increases the effort of the other. Of course, if s were negative or even zero, this result would disappear. This may in fact be an avenue for potential tests of our model. Indeed, later in the paper, we argue that the increase in effort due to overconfidence should make it more likely for the firm to succeed. If complementarities are important for this to occur, then we should observe more overconfident individuals working in firms, organizations and industries that benefit more from synergies among workers.

D. Firm Value and Agent Welfare

Before we turn to the principal's problem of optimally choosing w_1 and w_2 in section 2E, it is useful to understand how overconfidence affects the value of the firm and the welfare of the two agents (i.e., their expected utility from working at this firm) when contracts are exogenous. The intuition developed here applies not only to our later results with endogenous contracts, but also to situations in which compensation cannot be contracted. For example, exogenous contracts often describe how the various parties will benefit from joint ventures between firms, from the reputation that their joint success creates, and from the provision of a public good.

Firm value is simply the portion of the profits that is not distributed as compensation to the agents. With compensation contracts w_1 and w_2, it is given by

$$F \equiv (\sigma - w_1 - w_2)\mathrm{E}[\pi] = (\sigma - w_1 - w_2)(ak_1 + ak_2 + sk_1k_2). \tag{10}$$

Because agent 2 is overconfident, her welfare can be assessed from two perspectives. First, we could calculate her expected utility as she perceives it ex ante, that is, assuming that her effort contributes an additional probability of success of b. This, we think, is uninteresting as agent 2 will not experience this utility on average ex post. In fact, as discussed by Gervais, Heaton and Odean (2007), solving any principal-agent model with such a measure of welfare for the overconfident agent amounts to the agent willingly allowing the firm to steal economic surplus from her.

A more appropriate measure of welfare for this agent is the utility that she will experience on average at the end of the period. This measure takes into account the fact that the agent overinvests in effort provision but is calculated using the true probability of each outcome under this behavior. Besides keeping our analysis more disciplined,[6] such a measure also has the advantage of being a

[6] It is easy to make overconfident agents happy ex ante, as they think their actions make good outcomes more

better measure of how agent 2 will feel, on average, at the end of the period. As such, and as discussed by Gervais, Heaton and Odean (2007), it is more likely to determine whether the agent will be tempted to stay with the firm or move to other firms or activities that provide her with her reservation utility, although we do not consider these issues per se in our model. For these reasons, we take this to be the measure of welfare in what follows and in the rest of the paper when we calculate the expected utility of the overconfident agent. We start with the following proposition.

Proposition 1 *For the equilibrium described in Lemma 1,*

(i) firm value is always increasing in b;

(ii) the welfare of agent 1 is always increasing in b;

(iii) the welfare of agent 2 is increasing in b if and only if

$$\big(1 - 2s^2 w_1 w_2\big) b < as(1 + sw_2)w_1. \tag{11}$$

Parts (i) and (ii) of the proposition show that an increase in the level of overconfidence of agent 2 always increases firm value and always improves the welfare of agent 1. This is not surprising. When agent 2 overestimates her skill, she works harder by picking an effort level k_2 above that which an otherwise identical but rational agent would pick as an optimal response to k_1 by the first agent. Since agent 1 benefits from the extra output that agent 2's extra effort creates and does not pay the cost of that effort, she is trivially better off. Because both agents increase their effort level as b increases, the firm benefits from a higher likelihood of project success since it generates net profits of $\sigma - w_1 - w_2$ more often.

More interesting is part (iii) of the proposition which shows that agent 2 is sometimes made better off by an increase in her own bias, i.e., when (11) is satisfied. This condition can be satisfied for two reasons. It is satisfied when $s^2 w_1 w_2 > \frac{1}{2}$, regardless of the values for a and b. In this case, it is easy to verify that both agents gain so much from their synergies that both agents would be better off if they could both commit to work all the time, i.e., commit to $k_1 = k_2 = 1$. Because increasing b increases the equilibrium level of effort of both agents and brings them closer to this maximal effort, they are both better off. When $s^2 w_1 w_2 < \frac{1}{2}$, synergistic payoffs are not large enough to warrant full effort even if the two agents could coordinate on k_1 and k_2, and so it is not always the case that increasing the bias of agent 2 is beneficial to her. Instead, she is better off only if

$$b < \frac{as(1 + sw_2)w_1}{1 - 2s^2 w_1 w_2}, \tag{12}$$

likely than they are in reality.

that is, as long as her bias is not too extreme. Intuitively, this result comes from the tradeoff between agent 2's overinvestment in effort and the synergistic feedback effect of agent 1's increased effort. More precisely, even though agent 2 does not properly choose her own effort given her true ability and the level of effort of agent 1 (this is the cost), she benefits from the fact that agent 1 works harder as a response to her increased effort as seen in (5) (this is the benefit). When b (and k_2) is small, a marginal increase in k_2 creates a synergistic gain that more than outweighs the increased cost of effort. When b (and k_2) gets larger however, the marginal cost of effort becomes larger,[7] and agent 2 ends up hurting herself through her effort decisions.

Notice that the right-hand side of (12) is increasing in w_1, w_2, a and s. As the actual marginal productivity, individual or joint, of the two agents increases, the larger effort cost associated with the bias of agent 2 becomes more worthwhile. Interestingly, although agent 2's overconfidence can make her better off when w_2 is arbitrarily small, w_1 must be large enough for this bias to pay off. Otherwise, the knowledge that agent 2 overinvests in effort does not alter agent 1's effort choice enough for the synergistic feedback effect to benefit agent 2. Taken together, the three parts of Proposition 1 imply that the overconfidence of agent 2 creates a Pareto improvement for the firm and the two agents when (11) holds.

Our result about the welfare of agent 2 is related to the work of Bénabou and Tirole (2002) who show how some behavioral biases can enhance personal motivation and welfare. In their work, the individual is studied in isolation: self-deception improves welfare when the motivation gains from ignoring negative signals outweigh the losses from ignoring positive ones. In contrast, our model revolves around the interactions of biased individuals with others. In particular, the gains from the biased decisions of some individuals (their mis-allocation of effort) are not the result of improved self-motivation. Instead, they come from the effect they have on the motivation of others. In a related paper, Bénabou and Tirole (2003) study the role of motivation in a decision setting involving two individuals. However, the emphasis of their work is different from ours, as they concentrate on the role played by ego-bashing when private benefits are associated with the adoption of one's idea.

To finish this section, it is worth noting that a special case of the firm considered so far is one in which $w_1 + w_2 = \sigma$. In this case, the entire profits of the firm are distributed as compensation to the two agents. Because there is no residual claimant, this could describe the case of a partnership in which the two partners share the joint product of their efforts.[8] In this context, synergies may

[7] To be precise, the marginal effect on expected effort cost from an increase in k_2 is $\frac{\partial}{\partial k_2}\left(\frac{k_2^2}{2}\right) = k_2$.

[8] In fact, the case with $w_1 = w_2 = \frac{\sigma}{2}$ is the analogue to Holmström's (1982) team problem in which agents share the team's output equally. The only difference is that we allow for a synergistic term in the team's production function.

include the central role played by firm reputation, as in Tirole (1996), especially in firms that provide human-capital-intensive services. In such firms, the effort of a partner contributes to the firm's reputation, and this increases the productivity of effort of other partners. For example, a lawyer who expects his peers to shirk realizes that the reputation of the firm is likely to deteriorate, and thus that his effort will have very little effect on the overall value.[9] In this light, condition (12) indicates that overconfidence in partnerships will be beneficial to all only if synergies exist between the partners and if every partner receives a large enough fraction of profits.

E. The Firm's Choice of Contracts

Our analysis so far has concentrated on exogenously specified compensation contracts w_1 and w_2. Clearly however, knowing how the two agents react to their and their colleague's compensation contracts, the firm will seek to maximize its value by properly tailoring the contracts to its agents' characteristics and how they interact with each other. The firm's problem is to choose the pair of contracts $\{w_1, w_2\}$ that maximizes its profits net of compensation as given in (10), knowing that its agents will pick effort levels of k_1 and k_2 as derived in Lemma 1.[10]

To solve for the optimal contract, we make one further assumption about the firm's production function. In particular, we assume that $a = 0$, so that the firm's production comes exclusively from the cooperation of its two agents. This simplification is necessary for us to get closed-form solutions for the firm's choice of contracts. Since Proposition 1 shows that the benefits of overconfidence in this two-agent model can only come from synergies, this additional assumption does not affect the main point of our analysis. Also, because the firm's existing assets are assumed to be worth zero, the value of the firm then corresponds to the value of this cooperation between the two agents. That is, all other sources of firm value, which are unimportant for our results, are simply left out. Still, our results with $a = 0$ are followed by some numerical analysis that looks at the effect of a positive a.

Lemma 2 *Suppose that production is only generated through the cooperation of the two agents (i.e., $a = 0$). To maximize its value, the firm sets the contracts of the two agents to*

$$w_1 = \frac{2\sigma}{8 - s^2\sigma^2} \quad \textit{and} \quad w_2 = \frac{\sigma}{2}. \tag{13}$$

[9]The importance of reputation in the context of partnerships is in fact emphasized in two recent papers by Morrison and Wilhelm (2004) and by Levin and Tadelis (2005).

[10]Implicitly, we have assumed that the reservation utility of the two agents is low enough that their participation constraint never binds.

It is easy to verify that w_2 is greater than w_1. That is, the firm realizes that the overconfidence of agent 2 makes the cost of her effort appear cheap relative to what she expects to gain from it. That is, any increase in w_2 has a bigger impact on the overconfident agent's effort level than the same increase in w_1 has on the rational agent's effort level. Indeed, as long as $w_2 < 1$ (which is the case in (13) since $\sigma < 2$),

$$\frac{\partial k_2}{\partial w_2} = \frac{b}{\left(1 - 2s^2 w_1 w_2\right)^2} > \frac{b w_2}{\left(1 - 2s^2 w_1 w_2\right)^2} = \frac{\partial k_1}{\partial w_1}.$$

Thus the firm is always more valuable when it pays the overconfident agent more than the rational agent. Of course, since firm value comes exclusively from cooperation, this does not mean that all compensation goes to the overconfident agent. To capture this value, the firm must increase the rational agent's compensation and it does so more when the synergies between the agents are large (i.e., w_1 is increasing in s). Also, the benefit of increasing the overconfident agent's compensation is limited as the cost of doing so eventually outweighs the effort benefit.

With endogenous contracts, the firm takes full advantage of the potential production value of overconfidence. In what follows, we analyze the impact this has on agent welfare. In models that include overconfident agents (or, more generally, irrational agents), it is generally the case that these agents systematically lose to other more rational parties in the economy, once these other parties optimize their behavior. Indeed, it is often the case that the interactions of biased agents with rational agents or value-maximizing firms result in a simple transfer of economic surplus from the irrational agents to their more rational counterparties. In essence, irrational agents unknowingly leave money on the table, which others are more than happy to take. As we show next, this need not be the case when the biased actions of some agents commit them and, through synergies, their colleagues to the firm.

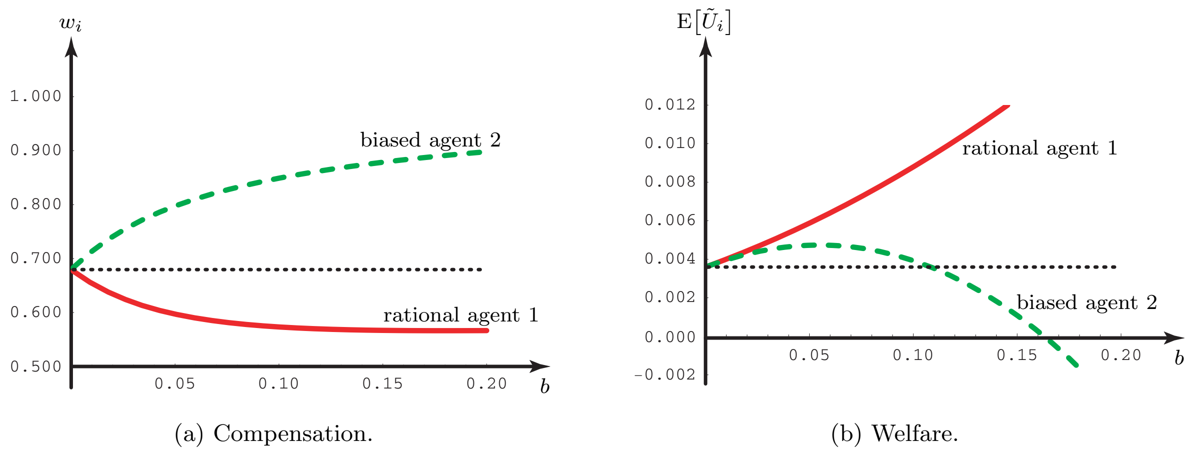
Proposition 2 *With the value-maximizing contracts of Lemma 2,*

(i) firm value is increasing in b;

(ii) the welfare of agent 1 is increasing in b;

(iii) the welfare of agent 2 is increasing in b if and only if $s^2\sigma^2 > \frac{8}{3}$.

It is not surprising to see in parts (i) and (ii) that rational agents and firms benefit from the presence of biased agents, as these agents tend to work harder than they should. More surprising is the fact in part (iii) that overconfident agents are sometimes better off as a result of their bias, *even when everyone else is optimizing their behavior*, that is, even when compensation contracts are endogenous. For this to be the case, the effort feedback caused by the biased agent's overinvestment



(a) Compensation.

(b) Welfare.

Figure 1: The above graphs show the equilibrium compensation and welfare of the rational agent (agent 1, continuous line) and the overconfident agent (agent 2, dashed line). The dotted line shows the equilibrium compensation and welfare that would result if the two agents were rational (i.e., if b were zero). In both graphs, $a = 0.05$, $s = 0.6$, and $\sigma = 2$ were used.

in effort must be large enough. This happens when the synergies between the two agents are large, and when the firm has a lot to gain from agent cooperation as it can then afford more incentive compensation for both agents.

As noted above, these results are obtained under the assumption that $a = 0$. This is because we cannot get closed-form solutions with a positive a. To confirm that the results in Proposition 2 do not depend on the assumption that $a = 0$, we solve the model numerically with positive values for a. The effect of a positive a is illustrated in the two graphs in Figure 1. First, Figure 1(a) shows the equilibrium compensation contracts for the rational (solid line) and biased (dashed line) agents as a function of the bias b, and contrasts these quantities with what the two agents would get in a model where both are rational (dotted line). Consistent with our results above, the graph shows that the biased agent is always paid more than the rational agent, regardless of the value of b. In fact, the biased (rational) agent always gets more (less) than what she would get were the two agents rational. Interestingly, the results under a positive a show that the compensation of the biased (rational) agent increases (decreases) in b. This is because the potential contribution of synergies to firm value becomes relatively more important (than that from a) as the bias of agent 2 increases. As a result, the contracts slowly converge to the values derived in Lemma 2 as b increases.

Figure 1(b) shows the welfare of the two agents and compares it with the welfare of a rational agent paired with another rational agent (dotted line). Clearly, the rational agent is always better off with a biased colleague. In fact, her utility is increasing in the degree of the bias. As in Proposition 2, it is the case that Pareto improvements are possible even when compensation contracts are chosen by the firm. Indeed, as shown in Figure 1(b), this result is not driven by our earlier assumption that $a = 0$. Also, as in Proposition 1, agent 2 is better off as a result of her bias as long as the bias is not too extreme. That is, despite the fact that agent 2 gets more compensation than agent 1, her overinvestment in effort provision eventually outweighs her gain from synergies. Agent 1 is still better off: she benefits more from her colleague's overinvestment in effort than she loses from her reduced wage.

3. Leadership

As shown in section 2, the presence of an overconfident agent can increase a firm's value by increasing the equilibrium levels of effort. That is, some behavioral traits of agents can naturally make them valuable teammates and their teams valuable to firms. Of course, firms not only control the compensation contracts they give to their agents, but they can also change the way agents interact. In this section, we look at the possibility that a firm calls upon one of the two agents to lead production. A natural question arises: do rational or overconfident agents make better leaders? To answer it, we alter the setup of section 2 in order to accommodate the presence of a leader.

A. Introducing a Leader

Because the unobservability of effort is partially responsible for the lack of cooperation between the two agents, it is natural to expect public effort choices to reduce the extent of these problems. Indeed, if an agent knows more about the effort made by her colleague when she makes her own effort decision, she is more likely to cooperate with her colleague by also working. The notion of leadership that we explore in this section captures this sequential aspect in effort choices. In particular, we assume that the firm can organize in such a way that the effort choice of one agent, the *leader*, is observed by the other agent, the *follower*, who then decides whether or not to exert an effort. We assume that the firm cannot observe the leader's effort, so that optimal compensation still involves the firm making payments to the two agents when the project is successful. We denote these payments by w_{L} for the leader and by w_{F} for the follower. Also, as in our main model, we start by assuming that $a \geq 0$.

The model with a leader is solved in essentially the same way as the no-leader model of section 2, with the exception that the follower can make her effort choice based on the observable effort of the leader. This, of course, means that the follower will choose a different effort level depending on whether the leader exerts an effort or not. In terms of notation, we denote the effort level of the leader by k_{L} and those of the follower by k_{F1} (after the leader exerts an effort) and k_{F0} (after the leader chooses not to work). As before, we first derive the equilibrium with exogenous contracts w_{L} and w_{F} before turning our attention to the firm's problem of choosing contracts and its leader.

Lemma 3 *With a biased leader (and a rational follower), the equilibrium effort levels are given by* $k_{\mathrm{L}} = w_{\mathrm{L}}\big[a+b+sw_{\mathrm{F}}(2a+s)\big]$*,* $k_{\mathrm{F1}} = w_{\mathrm{F}}(a+s)$*, and* $k_{\mathrm{F0}} = w_{\mathrm{F}}a$*. With a rational leader (and a biased follower), the equilibrium effort levels are given by* $k_{\mathrm{L}} = w_{\mathrm{L}}\big[a+sw_{\mathrm{F}}(2a+b+s)\big]$*,* $k_{\mathrm{F1}} = w_{\mathrm{F}}(a+b+s)$*, and* $k_{\mathrm{F0}} = w_{\mathrm{F}}(a+b)$*.*

When she follows, the biased agent always overinvests in effort compared to the rational agent. In particular, her effort level exceeds that of the rational follower by bw_{F}, whether or not the leader works. This is because the biased agent thinks that her effort contributes an additional probability b of project success and so she is willing to invest more effort into the project to the extent that she is compensated for it. Interestingly however, the biased leader does not always work harder than the rational leader. To see this, notice that the difference between the effort level of the biased leader and that of the rational leader is equal to $bw_{\mathrm{L}}(1 - sw_{\mathrm{F}})$. Thus the rational leader works harder than the biased leader when the synergies between the two agents (s) and the compensation incentives for the follower to exert effort (w_{F}) are large enough. When this is the case (i.e., when $sw_{\mathrm{F}} > 1$), firm value is clearly larger with a rational leader and a biased follower, as switching the role of two agents unequivocally reduces effort.

Importantly, the effort levels (k_{F1} and k_{F0}) of the rational follower do not depend on b. Only the frequency with which k_{F1} and k_{F0} are used is affected by b. This is because she knows exactly when the biased agent made an effort, and thus does not need to work harder in anticipation of potential synergies. When the rational agent leads, however, she anticipates the greater effort that the biased follower will exert, and internalizes this in her effort choice. In particular, the synergies between the two agents make her increase her effort level by $sbw_{\mathrm{L}}w_{\mathrm{F}}$ relative to the level that she would choose if she were paired with a rational follower instead, that is, sw_{L} times the extra effort of the follower that is due to her bias, bw_{F}.

In our setting, the leader uses her public choice of effort to influence the effort decision of the follower. In particular, the leader can internalize the externalities that her effort choice has on her colleague, as her actions affect her colleague's actions. In that sense, she leads by example. This notion of leadership is similar to that developed by Hermalin (1998). In his work, the leader is endowed with some information about the profitability of a project, and uses her public effort choice to boost the credibility of her attempt to signal it to the other agents. Our model differs from Hermalin's in that our leader does not have any informational advantage about the project. In particular, her leadership role is limited to the fact that she moves first and her effort is publicly observable. Another difference is that we allow a biased agent to lead a rational follower or to follow a rational leader. Our goal is to compare these two organizational structures and study the effects of overconfidence in each.

It is easy to show that both agents work harder in the presence of a leader than without one. This is true whether the leader is the rational agent or the biased agent. The mechanism is simple: the fact that her action is observed by the follower commits the leader to exert more effort. Because of complementarities, the higher effort exerted by the leader creates an incentive for the follower to work harder, on average, as well. In fact, this is another difference between our model and that of Hermalin (1998): because our model's leader does not have any information about the project's fundamentals, she can only commit the follower to working harder when synergies exist between them (i.e., when $s > 0$).

B. Firm Value and Agent Welfare with Exogenous Contracts

For the balance of this section, we explore how the firm's organizational structure and the self-perception biases of its agents combine to affect individual welfare and firm value. As before, we start our analysis with exogenously specified contracts w_{L} and w_{F}, where $w_{\text{L}} + w_{\text{F}} \leq \sigma$. Our first result is the analogue to Proposition 1 when either of the two agents is appointed as the leader.

Proposition 3 *With a biased leader, the expected utility of the (biased) leader is decreasing in b, while the expected utility of the (rational) follower is increasing in b. With a rational leader, the expected utility of the (rational) leader is increasing in b, while the expected utility of the (biased) follower is increasing in b if and only if $b < sw_{\text{L}}\left[a + \frac{1}{2}(2a+s)sw_{\text{F}}\right]$. Firm value is increasing in b with a biased leader or a rational leader.*

As before, the rational agent always benefits from an increase in the bias of her teammate. This is not surprising, as the rational agent shares the product of the biased agent's overinvestment in effort but not the cost, not to mention the fact that she also optimally adjusts her response to her teammate's behavior. More interesting is the result that an increase in the biased agent's misperceptions can only be Pareto-improving if the leader is rational. That is, the biased agent can benefit from her own misperceptions only when she is the follower, not when she is the leader. This is because the overconfident agent can only potentially benefit from her bias when this bias acts as a commitment device for her to exert more effort than rationally optimal. In our model, leadership is also a commitment device for the leader, but it leaves no additional commitment value for the bias of the overconfident leader. Instead, only the costs of the overconfident agent's overinvestment in effort subsist, and so her bias always makes her worse off when she leads. However, overconfidence does have some commitment value when the biased agent is the follower, as shown by the fact that the rational leader's effort level is increasing in b. Still, when her bias is too extreme, she overinvests in effort and makes herself worse off in the process. Only a small enough bias creates a Pareto improvement.[11]

C. The Firm's Decisions

We finish this section by analyzing the decisions of the firm when it can appoint one of the two agents as the leader. This means solving for the optimal contracts that maximize firm value when either of the two agents leads. In addition, to further improve its performance, the firm can choose to appoint the rational agent or the overconfident agent as its leader. As in section 2E, we must assume that $a = 0$ in order to get closed-form solutions. Again, because it is the cooperation between the two agents that lies at the heart of our paper, this assumption does not affect our message. With this assumption in hand, the following lemma derives the optimal compensation contracts under both leadership scenarios.

Lemma 4 *When the firm is led by a biased agent, the compensation contracts that maximize firm value are*

$$w_{\mathrm{L}} = \frac{\sigma - w_{\mathrm{F}}}{2} \in \left(\frac{\sigma}{4}, \frac{\sigma}{3}\right) \quad \textit{and} \quad w_{\mathrm{F}} = \frac{2s^2\sigma - 3b + \sqrt{4s^4\sigma^2 + 4s^2\sigma b + 9b^2}}{8s^2} \in \left(\frac{\sigma}{3}, \frac{\sigma}{2}\right). \tag{14}$$

When the firm is led by a rational agent, the compensation contracts that maximize firm value are

$$w_{\mathrm{L}} = \frac{\sigma}{4} \quad \textit{and} \quad w_{\mathrm{F}} = \frac{\sigma}{2}. \tag{15}$$

Interestingly, in both regimes, the follower gets paid more than the leader, i.e., $w_{\mathrm{F}} > w_{\mathrm{L}}$. To see the intuition, it is useful to analyze the effects of w_{L} and w_{F} on the probability of success. The probability of success in both regimes is the product of the effort level of the leader, k_{L}, and the effort level of the follower after the leader exerts an effort, k_{F1}. While the wage of the leader (w_{L}) affects only k_{L}, the wage of the follower (w_{F}) affects both k_{F1} and k_{L}. This is because the leader takes into account the anticipated effort of the follower, while the follower already knows whether the leader exerted effort or not. As a result, the effect of w_{L} on the probability of success is $\frac{\partial k_{\mathrm{L}}}{\partial w_{\mathrm{L}}} \cdot k_{\mathrm{F1}}$, and the effect of w_{F} on the probability of success is $\frac{\partial k_{\mathrm{L}}}{\partial w_{\mathrm{F}}} \cdot k_{\mathrm{F1}} + \frac{\partial k_{\mathrm{F1}}}{\partial w_{\mathrm{F}}} \cdot k_{\mathrm{L}}$. From Lemma 3, we know

[11] We would like to thank our referee for pointing out the intuition in this paragraph.

that $\frac{\partial k_{\mathrm{L}}}{\partial w_{\mathrm{L}}} = \frac{k_{\mathrm{L}}}{w_{\mathrm{L}}}$, and $\frac{\partial k_{\mathrm{F1}}}{\partial w_{\mathrm{F}}} = \frac{k_{\mathrm{F1}}}{w_{\mathrm{F}}}$. Thus, when $w_{\mathrm{F}} = w_{\mathrm{L}}$, we have $\frac{\partial k_{\mathrm{L}}}{\partial w_{\mathrm{L}}} \cdot k_{\mathrm{F1}} = \frac{\partial k_{\mathrm{F1}}}{\partial w_{\mathrm{F}}} \cdot k_{\mathrm{L}}$, and so the effect of increasing w_{F} on the probability of success is greater than that of increasing w_{L}. This explains why the firm sets w_{F} above w_{L}.[12]

Having endogenized the compensation contracts under both firm structures, we can now turn our attention to the firm's problem of choosing a leader.

Proposition 4 *As the overconfidence of the biased agent (i.e., b) increases from zero, the firm is more valuable if the rational agent is appointed as the leader when $s\sigma > 1$, and if the biased agent is appointed as the leader otherwise.*

The rational agent makes a better leader when synergies are high and when successful projects yield large cash flows. To gain some intuition, it is useful to revisit the firm's value in both regimes given exogenous w_{L} and w_{F}. This value is the product of the project's payoff net of compensation when it succeeds, $(\sigma - w_{\mathrm{L}} - w_{\mathrm{F}})$, and the probability of success, $k_{\mathrm{L}} \cdot k_{\mathrm{F1}}$. With a biased leader, this value is given by $(\sigma - w_{\mathrm{L}} - w_{\mathrm{F}}) \cdot \left(w_{\mathrm{L}} b + w_{\mathrm{L}} w_{\mathrm{F}} s^2\right) \cdot (w_{\mathrm{F}} s)$, and the contribution of overconfidence to value (i.e., the value that a positive b adds) is

$$\Delta_{\mathrm{BL}} \equiv (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}})\, w_{\mathrm{L}} w_{\mathrm{F}} b s. \tag{16}$$

With a rational leader, firm value is $(\sigma - w_{\mathrm{L}} - w_{\mathrm{F}}) \cdot \left(w_{\mathrm{L}} w_{\mathrm{F}} b s + w_{\mathrm{L}} w_{\mathrm{F}} s^2\right) \cdot (w_{\mathrm{F}} b + w_{\mathrm{F}} s)$, and overconfidence contributes

$$\begin{aligned} \Delta_{\mathrm{RL}} &\equiv (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}}) \left[w_{\mathrm{L}} w_{\mathrm{F}} b s \cdot (w_{\mathrm{F}} b + w_{\mathrm{F}} s) + \left(w_{\mathrm{L}} w_{\mathrm{F}} b s + w_{\mathrm{L}} w_{\mathrm{F}} s^2\right) \cdot w_{\mathrm{F}} b\right] \\ &= (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}})\, w_{\mathrm{L}} w_{\mathrm{F}} b s \cdot 2 w_{\mathrm{F}} (b + s) \end{aligned} \tag{17}$$

to this value. As before, overconfidence affects only the effort of the leader when the biased agent leads, whereas it affects the effort level of both agents when the rational agent leads. As a result, the contribution of overconfidence to firm value contains an additional term when the leader is rational. A simple comparison of Δ_{BL} and Δ_{RL} in (16) and (17) shows that large synergies (i.e., large s) and the firm's ability to compensate the biased follower well (i.e., large w_{F} in (17)) will make the rational-leader scenario better for the firm. Also, the firm can only sufficiently increase the biased follower's compensation when project success yields large benefits, i.e., when σ is large. Of course,

[12] While our result that the leader is paid less than the follower seems counter-factual, it is important to note that it is obtained in a model with one leader and one follower. In reality, it is likely that the leader's actions influence those of several followers and generate multiple layers of synergies. As such, proper incentives at the top echelons of the firm will justify greater compensation. These issues, although interesting in their own right, are beyond the scope of our paper.

the firm has an additional degree of freedom, namely the option to channel more compensation towards the biased agent. Still, as shown in Proposition 4, the benefits of a rational leader still obtain when s and σ are large.

4. Extensions

In this section, we consider a number of alternative specifications of our main model. These extensions allow us to further extend our predictions about the effects of overconfidence in firms. In order to emphasize the role played by effort complementarities between the agents, we keep assuming that $a = 0$ in all these extensions.

A. Mergers and Acquisitions

Alchian and Demsetz (1972) argue that firms will take advantage of production synergies by acquiring the inputs to production that are more valuable when pooled. In our model so far, we assume that the firm has already attracted its synergistic labor force and looks for the optimal way to motivate it for production purposes. In this section, we step back and ask how the potentially synergistic effort of an outside agent or firm can be acquired into a merged firm that can then be more valuable than the sum of its parts.[13] Again, our emphasis is on the role that overconfidence can play in this acquisition.

Suppose that firm 1 is owned by a principal who currently operates with a single rational agent. We still assume without loss of generality that the firm's assets in place are worth zero. With $a = 0$ and no synergies coming from a second agent, this firm is trivially worth zero.[14] A second firm, firm 2, is privately owned by its only agent, an *entrepreneur*. We assume for simplicity that this second firm's production function is the same as the first firm's and that its assets in place are also worth zero. The principal realizes that pooling the labor force of the two firms would create valuable synergies for his firm, in the form of $\tilde{v}$ as specified in (1). In particular, the principal contemplates offering the entrepreneur w_2 if the joint project is successful (i.e., if $\tilde{v}_1 = \sigma$) in return for her participation in the project. In what follows, we look at the possibility that such an acquisition can be made, and assess the value that it then creates.

Because the entrepreneur owns the second firm, she values it according to her own beliefs. This means that she knows it is worth zero if she is rational, but she thinks it is worth more if she is

[13]For more on the role of synergies in mergers, see Hietala, Kaplan and Robinson (2003).

[14]The value of zero is meant to be a normalized value. We could have assumed that the assets in place have a positive value and that the agent performs some (non-synergistic) tasks that also contribute value.

overconfident (i.e., if $b > 0$). Indeed, in that case, she thinks that her effort creates σb and, since it costs her $\tilde{c}_2$ in utility, she works if and only if $\tilde{c}_2 \leq \sigma b \equiv k_2$. Her expected utility from owning this firm is therefore

$$\mathrm{E}_{\mathrm{B}}\big[\tilde{U}_2\big] = \sigma \Pr{}_{\mathrm{B}}\big\{\tilde{v}_2 = \sigma\big\} - \mathrm{E}\big[\tilde{c}_2 \mid \tilde{c}_2 \leq \sigma b\big] = \frac{\sigma^2 b^2}{2}. \tag{18}$$

This quantity, which is increasing in the entrepreneur's bias, becomes her (endogenous) reservation utility when the first firm attempts to acquire her production. That is, she must expect at least this much in utility if she is to join forces with the first firm.

Given that the first firm's agent is rational, the principal realizes that the merger creates value only if the entrepreneur is overconfident, as otherwise Lemma 1 tells us that both agents will shirk in equilibrium once the two agents work for the same merged firm. So overconfidence now has both a positive and a negative role. On the one hand, it makes cooperation between the two agents possible and valuable once the firms merge. On the other hand, it also makes the entrepreneurial firm more expensive to acquire as the owner thinks that she can get more out of it than she really can. That is, in her decision to accept firm 1's offer, she trades off a share of her own private (but overestimated) contribution to value in exchange for a share of the merged firm's synergies. The following proposition characterizes the terms of the deal when one is indeed possible.

Proposition 5 *A deal to merge the two firms is possible if and only if $s\sigma > 1$. If $s^2\sigma^2 \geq \frac{8}{3}$, then firm 1 offers $w_2 = \frac{\sigma}{2}$ to the entrepreneur and pays agent 1 $w_1 = \frac{2\sigma}{8-s^2\sigma^2}$. If instead $s^2\sigma^2 \in \left(1, \frac{8}{3}\right)$, then the firm offers*

$$w_2 = \frac{4}{s\left(s\sigma + \sqrt{s^2\sigma^2 + 8}\right)} \tag{19}$$

to the entrepreneur and pays agent 1

$$w_1 = \frac{s^2\sigma^2 + s\sigma\sqrt{s^2\sigma^2 + 8} - 4}{4s^2\sigma}. \tag{20}$$

When $s^2\sigma^2 \geq \frac{8}{3}$, firm 1 offers the agents the contracts derived in Lemma 2. In this case, the synergistic benefits of pooling the production of the two agents are so large that the entrepreneur expects more than (18) in utility from his contract w_2. That is, it is worthwhile for the firm to strongly motivate agent 2 with more compensation and, as a result, her utility constraint to participate in the merger is not binding, as she gets more from her share of the production synergies than she loses from the share of private production that she gives up. When $s^2\sigma^2 \in \left(1, \frac{8}{3}\right)$ however, the value-maximizing contract of Lemma 2 is not enough to convince the entrepreneur to merge with firm 1. As a result, firm 1 must offer a larger w_2 (i.e., $w_2 > \frac{\sigma}{2}$) to make the deal successful. In effect, the entrepreneur's overconfidence renders her threat to turn down the firm's offer credible and so improves her negotiating power. So, while the value of the merged firm is the same as in (the proof of) Proposition 2 when $s^2\sigma^2 \geq \frac{8}{3}$, it is less than that when $s^2\sigma^2 \in \left(1, \frac{8}{3}\right)$, as the overconfident entrepreneur captures some of the surplus that her own bias creates in the first place. Finally, when $s\sigma \leq 1$, the synergies created from pooled resources are so small that the firm is unable to offer a contract that the entrepreneur values more than her own private firm. From her standpoint, sharing her private production is not worth any share of the small synergies that merging would create.

B. Task Difficulty

Suppose that the role of each agent in the firm's production is not symmetric. In particular, assume that the two tasks that must be performed by the agents have a different degree of difficulty. We capture this possibility by assuming that the effort cost $\tilde{c}_i$ of agent i is uniformly distributed on $\left[0, \bar{c}_i\right]$, where a large $\bar{c}_i$ corresponds to a difficult task.[15]

Proposition 6 *When $\tilde{c}_1 \sim \mathrm{U}\left[0, \bar{c}_1\right]$ and $\tilde{c}_2 \sim \mathrm{U}\left[0, \bar{c}_2\right]$, the firm maximizes its value by setting the agents' contracts to*

$$w_1 = \frac{2\sigma\bar{c}_1\bar{c}_2}{8\bar{c}_1\bar{c}_2 - s^2\sigma^2} \quad \text{and} \quad w_2 = \frac{\sigma}{2}. \tag{21}$$

The overconfidence of agent 2 makes the firm more valuable and both agents better off as long as $s^2\sigma^2 > \frac{8\bar{c}_1\bar{c}_2}{3}$. The value of the firm is higher when the more difficult task is assigned to the rational agent.

As in Proposition 2, the firm offers the larger compensation to the biased agent, and the Pareto optimality of overconfidence requires that s and σ be large enough. When $\bar{c}_2$ is large however, her overinvestment in effort in response to any given effort level k_1 by agent 1 is more costly. Also, when $\bar{c}_1$ is large, the overinvestment in effort by the biased agent is not responded to as strongly by the rational agent. Both effects contribute to making overconfidence less likely to benefit everyone.

The last part of the proposition adds one more dimension to the firm's organizational choices. When tasks vary in difficulty, the firm benefits from assigning the more difficult ones to more rational agents. The intuition stems from the fact that production in our model is generated by synergies between the two agents. As a result, the effort level of the rational agent (k_1) is proportional to the biased agent's effort level (k_2), that is, the rational agent chooses her effort only based on the effort of the biased agent. Thus, the stifling effects that increasing $\bar{c}_1$ and increasing $\bar{c}_2$ have on the effort choice of the rational agent are similar. For the biased agent however, there is an additional

[15] We need to assume that $\sigma^2 < 4\bar{c}_1\bar{c}_2$ for interior solutions to obtain.

stifling effect of increasing $\bar{c}_2$: it also reduces the part of her effort that is not motivated by the other agent's effort but rather by her bias. In other words, the effort of the biased agent can be viewed as the 'engine' that gets the team of agents going. As such, it is critical for the principal not to slow the biased agent down, and this is done by assigning the easier tasks to her. To our knowledge, this conclusion that different jobs should be assigned to different employees based on these agents' self-perceptions is new to the job design literature (e.g., Holmström and Milgrom, 1991).

C. Reputation

In our model, the agents get their (positive) utility exclusively from compensation. If agents carry their team's performance and the reputation that comes with it as they move onto other tasks or jobs, they get extra utility from a project success. This would be especially true for younger agents whose early performance will affect their lifetime work prospects. In our one-period model, we can capture these reputation effects by assuming that agents get extra utility from project success and that this extra utility is costless for the firm. For example, this assumption would be consistent with the idea that agents have access to better jobs after they exhibit good performance. To this end, let us assume that agent i receives reputation utility of r_i along with her compensation when the project is successful.[16] The following proposition shows how this affects the results of Lemma 2 and Proposition 2.

Proposition 7 *When agents care about their reputation, the firm maximizes its value by setting their contracts to*

$$w_1 = \frac{2(\sigma + r_1 + r_2)}{8 - s^2(\sigma + r_1 + r_2)^2} - r_1 \quad \textit{and} \quad w_2 = \frac{r_1 - r_2 + \sigma}{2}. \tag{22}$$

The overconfidence of agent 2 makes the firm more valuable and both agents better off as long as $s^2(\sigma + r_1 + r_2)^2 > \frac{8}{3}$.

Again it is necessary that s be large enough and it is sufficient for both σ and s to be large enough for the overconfidence of the second agent to create a Pareto improvement. However, when agents also care about their reputation, σ does not need to be as large as in Proposition 2. Interestingly,

[16] We need to impose the restriction that $\sigma + r_1 + r_2 < 2$ and that r_1 and r_2 are small enough (smaller than $\frac{\sigma}{3}$ is sufficient) for interior solutions to obtain. Also note that our notion of reputation is different from the notion of collective reputation studied by Tirole (1996), as it is the individual agents, not the team, who carry the reputation that successful outcomes generate.

it does not matter whether it is the reputation utility of agent 1 or agent 2 that is large. Through s, the higher incentive for one agent to work harder translates into the other agent working harder too. That is, overconfidence is useful in any firm where some agents get large private benefits from the success of synergistic production. Finally, note that each agent's compensation decreases in her own reputation benefit and increases in the other agent's reputation benefit. This is because inducing an agent to exert effort is cheaper when that agent cares about her own reputation, but more incentive compensation is worthwhile when the other agent's concern for reputation increases the potential for synergies.

D. Unknown Types and Learning

Throughout the paper, we have done our analysis for a situation in which the overconfidence of agent 2 is known by agent 1 and the firm, and the rationality of agent 1 is known by agent 2 and the firm. As we show next, it is not necessary for the overconfidence of agents to be known for our results to obtain. The mere possibility that agents are overconfident is enough to create extra firm value and agent welfare from better cooperation.

Suppose that every agent i is either overconfident ($\tilde{b}_i = b$) with probability $\phi \in (0, 1)$ or rational ($\tilde{b}_i = 0$) with probability $1 - \phi$. Agents know their own type but others, including the firm, do not. Of course, overconfident agents misinterpret their own type: they think that they are more skilled than they really are. Because the firm does not know the type of the agents it hires, it will offer the same contract w to both agents.[17] Although agents know their own type, they know that the other agent will use a different effort level depending on her type. Let us denote the effort level of the rational type by k_{R} and that of the biased type by k_{B}.

After observing her own cost of effort $\tilde{c}_i = c_i$, a rational agent i knows that her effort will only contribute to the project's probability of success if the other agent also works. From her perspective, this other agent will exert effort with probability $\phi k_{\mathrm{B}} + (1 - \phi)k_{\mathrm{R}}$, and so her expected utility from exerting effort increases by $ws\big[\phi k_{\mathrm{B}} + (1 - \phi)k_{\mathrm{R}}\big] - c_i$. This implies that she will work if $c_i \leq k_{\mathrm{R}}$, where

$$k_{\mathrm{R}} = ws\big[\phi k_{\mathrm{B}} + (1 - \phi)k_{\mathrm{R}}\big]. \tag{23}$$

Similar reasoning leads a biased agent j with an effort cost of $\tilde{c}_j = c_j$ to exert effort if and only if $c_j \leq k_{\mathrm{B}}$, where

$$k_{\mathrm{B}} = w\Big(b + s\big[\phi k_{\mathrm{B}} + (1 - \phi)k_{\mathrm{R}}\big]\Big). \tag{24}$$

[17] Because compensation contracts are uni-dimensional, the firm cannot use a menu of contracts to screen agents either.

Solving for k_{R} and k_{B} in (23) and (24) yields the equilibrium effort level of the two types of agents:

$$k_{\mathrm{R}} = \frac{\phi b s w^2}{1 - sw} \quad \text{and} \quad k_{\mathrm{B}} = \frac{bw\big[1 - (1-\phi)sw\big]}{1 - sw}. \tag{25}$$

Note that it is no longer the presence of overconfidence that makes rational agents exert effort, but the potential presence of it. Indeed, $k_{\mathrm{R}} > 0$ if and only if $\phi > 0$. In fact, it is possible that the firm's two agents are both rational and they both exert effort, even though the two of them would shirk if they knew each other's type. The following proposition derives the contract that the firm will offer to both agents in equilibrium.

Proposition 8 *With unknown agent types, the firm maximizes its value by setting both agents's contracts to*

$$w = \frac{3 - \sqrt{9 - 4s\sigma}}{2s}. \tag{26}$$

The potential presence of overconfidence (i.e., the fact that $\phi > 0$) makes the firm more valuable and both types of agents better off as long as

$$s\sigma > \frac{5}{4} \quad \text{and} \quad \phi > \frac{3 + \sqrt{9 - 4s\sigma}}{2s\sigma} - 1. \tag{27}$$

It is again critical that s and σ be large enough for overconfidence to have a Pareto-improving effect. When the overconfidence of agents is unknown however, this is not sufficient. It must also be the case that ϕ is large, that is, it must be likely that agents are biased. Otherwise, the commitment value of overconfidence is small as agents cannot rely on the overinvestment in effort from their colleagues. As a result, agents who happen to be overconfident end up overworking and pay too much in effort costs.

Using (23) and (24), it is easy to verify that $k_{\mathrm{B}} = k_{\mathrm{R}} + bw$ so that, as before, overconfident agents work harder than they should given their colleague's effort level. This has important repercussions here, when types are unknown. Because overconfident agents work harder, it will be the case that project success will tend to be associated with agent overconfidence. This is stated more formally in the following proposition.

Proposition 9 *After a successful project ($\tilde{v} = \sigma$), the firm's agents assign a higher posterior probability that the other agent is biased. Similarly, the firm assigns a higher posterior probability that at least one agent is biased and a higher posterior probability that both agents are biased.*

Interestingly, even when the two agents happen to be rational, which is the case with probability $(1-\phi)^2$, a successful project makes these agents put more weight on the possibility that their colleague is overconfident. If these agents were to interact in a second period, the first-period success would therefore further enhance the impact of the potential presence of overconfidence. More generally, although our one-period model does not allow us to precisely describe the dynamics of overconfidence beliefs over time, it is clear that cooperation and project success should be followed by more cooperation and project success, regardless of the agents' types. In other words, overconfidence breeds success, and success leads to more overconfidence.

5. Conclusion

Teamwork synergies and cooperation have long been identified as important factors in firms' production. How firms extract the full value from these potential synergies is less clear. As shown by Holmström (1982), when agents share their team's output but their contribution to that output is unobservable, these agents have a tendency to free-ride. Indeed, because an agent pays the full cost of her effort but only gets a fraction of its benefit, she scales back on her own effort and instead tends to rely on the effort of others. In equilibrium, the team fails to generate its full first-best production. This problem is exacerbated by the presence of complementarities within the team: because agents do not fully account for the positive externalities that their effort creates, the team's level of cooperation is suboptimal and more value is lost. With both problems, mechanisms that increase the effort exerted by the team's agents recover some of the lost surplus.

This paper explores the role of biased self-perceptions in firms that face effort coordination problems. When agents overestimate their own skills, and thus overestimate the marginal product of their effort, they naturally tend to work harder as, for them, the extra cost of effort is worth the extra reward that they perceive. This of course reduces the extent of the free-rider problem. Such agents also care less about potential complementarities: their own marginal product warrants the extra cost of effort whether or not synergies are realized. Interestingly, this can make the firm and all agents, including the biased ones, better off. On the one hand, the overinvestment in effort by a biased agent costs her some utility. On the other hand, her increased effort creates a beneficial feedback effect, as the other agents react to the synergistic increase in their marginal product by working harder, thereby increasing the firm's output and thus the biased agent's share of that payoff.

Our model also generates an array of empirical predictions that can be taken to the data and guide future empirical work. First, we find that overconfidence is expected to benefit all parties when complementarities in the firm are sufficiently strong. This suggests that overconfidence is more likely to persist in firms that benefit more from synergistic teamwork. Second, we find that rational agents make better leaders when complementarities are large. This generates implications for the relation between the firm's organization, the bias of its workers, and the nature of production. Third, we find that overconfident agents will receive larger performance-based compensations. This is because, due to their bias, compensating them is particularly effective in increasing their team's productivity. Fourth, for similar reasons, the firm will choose to allocate easier tasks to overconfident agents, rather than to rational ones. Fifth, the effect of overconfidence in synergistic teamwork is enhanced when any of the team's agents care about reputation. Sixth, overconfidence makes mergers valuable but can also deter them. On the one hand, mergers are expected to take place when overconfidence can prompt the realization of sufficiently strong synergies. On the other hand, when the target firm is run by an overconfident agent, it is hard to convince her to join forces.

We believe that our model can also be applied to various other settings in corporate finance. One is the multi-division firm. The division managers of a multi-division firm typically face effort coordination problems. As in our model, division managers bear the full costs of their efforts, but share the gains with other division managers. This point has been discussed and demonstrated in several articles, including Boot and Schmeits (2000), and Scharfstein and Stein (2000). Moreover, complementarities, which are important in our model, may arise in this context as a simple result of production synergies. They may also be a product of financing spillovers that efficient internal capital markets render possible. Indeed, as shown by Stein (1997), the success of one division will provide more resources to the firm, and thus will enable other divisions to get more financing for their investments. This may increase the incentive of other division managers to be productive (although we should also point out that the competition for resources may have an opposite effect on endogenous incentives, as shown by Brusco and Panunzi (2005)).

Another setting to which our model naturally applies is venture capital. The idea that the venture capital function is plagued by a double-sided moral hazard problem between the venture capitalist (VC) and the entrepreneur can be found in Sahlman (1990), Lerner (1995), Hellmann and Puri (2002), and Kaplan and Strömberg (2004). These authors argue that, in addition to the contribution that the entrepreneur's effort is bound to have on the potential success of the company, the VC's effort towards monitoring, advising and organizing the company can also impact its eventual fate. As such, it is reasonable to think of the relationship between the entrepreneur and the VC as an effort coordination problem in which the effort of one benefits both, as in the model of venture capital by Casamatta (2003). In this context, complementarities between the entrepreneur and the VC are also likely to exist, as the dedication of one to the company can potentially make the other more dedicated as well. For example, a VC with limited human capital may choose to allocate more of it to a company in which the entrepreneur appears to be fully engaged. Likewise, the entrepreneur is less likely to turn his attention to alternative outside opportunities if she feels the committed support of the VC.

We conclude with a quote from Simon (1991, pp. 32-33): "[I]n most organizations, employees contribute much more to goal achievement than the minimum that could be extracted from them by supervisory enforcement of the (vague) terms of the employment contract." Our paper points to one potential reason for such apparent overinvestment in effort by employees. Still, a better understanding of the joint relationship between contracts, organization and the behavioral motives of agents appears to be a fruitful area for future research.

6. References

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7. Appendix

Proof of Lemma 1

In equilibrium it must be the case that k_1 is agent 1's optimal response to an effort level of k_2 by agent 2, and that k_2 is agent 2's optimal response to an effort level of k_1 by agent 1. That is, k_1 and k_2 must solve (5) and (7). It is easy to verify that (8) are (9) are the unique solution to this problem. ▮

Proof of Proposition 1

The firm's value is given by (10) using the equilibrium effort levels k_1 and k_2 derived in Lemma 1. From (8) and (9), we have

$$\frac{\partial k_1}{\partial b} = \frac{s w_1 w_2}{1 - s^2 w_1 w_2} \quad \text{and} \quad \frac{\partial k_2}{\partial b} = \frac{w_2}{1 - s^2 w_1 w_2}. \tag{28}$$

Since $s < 1$ and $w_1 w_2 \leq \frac{\sigma}{2} \cdot \frac{\sigma}{2} \leq 1$, both of these quantities are positive, and so

$$\frac{\partial F}{\partial b} = (\sigma - w_1 - w_2)\left(a\frac{\partial k_1}{\partial b} + a\frac{\partial k_2}{\partial b} + s k_2 \frac{\partial k_1}{\partial b} + s k_1 \frac{\partial k_2}{\partial b}\right) > 0.$$

For any given effort levels k_1 and k_2, we have

$$\mathrm{E}\big[\tilde{U}_1\big] = w_1 \mathrm{E}[\pi] - \mathrm{E}\big[\tilde{c}_1 \mid \tilde{c}_1 \leq k_1\big]\Pr\big\{\tilde{c}_1 \leq k_1\big\} = w_1\big(a k_1 + a k_2 + s k_1 k_2\big) - \frac{k_1^2}{2}, \tag{29}$$

so that, using (28),

$$\begin{aligned}
\frac{\partial \mathrm{E}\big[\tilde{U}_1\big]}{\partial b} &= w_1\left(a\frac{\partial k_1}{\partial b} + a\frac{\partial k_2}{\partial b} + s k_2 \frac{\partial k_2}{\partial b} + s k_1 \frac{\partial k_1}{\partial b}\right) - k_1 \frac{\partial k_1}{\partial b} \\
&= \frac{w_1\big(a s w_1 w_2 + a w_2 + s^2 k_2 w_1 w_2 + s k_1 w_2\big) - s k_1 w_1 w_2}{1 - s^2 w_1 w_2} \\
&= \frac{w_1 w_2 \big(a s w_1 + a + s^2 k_2 w_1\big)}{1 - s^2 w_1 w_2},
\end{aligned}$$

which is clearly positive. Similarly,

$$\mathrm{E}\big[\tilde{U}_2\big] = w_2 \mathrm{E}[\pi] - \mathrm{E}\big[\tilde{c}_2 \mid \tilde{c}_2 \leq k_2\big]\Pr\big\{\tilde{c}_2 \leq k_2\big\} = w_2\big(a k_1 + a k_2 + s k_1 k_2\big) - \frac{k_2^2}{2}, \tag{30}$$

so that, using (28),

$$\begin{aligned}
\frac{\partial \mathrm{E}\big[\tilde{U}_2\big]}{\partial b} &= w_2\left(a\frac{\partial k_1}{\partial b} + a\frac{\partial k_2}{\partial b} + s k_2 \frac{\partial k_2}{\partial b} + s k_1 \frac{\partial k_1}{\partial b}\right) - k_2 \frac{\partial k_2}{\partial b} \\
&= \frac{w_2\big(a s w_1 w_2 + a w_2 + s^2 k_2 w_1 w_2 + s k_1 w_2\big) - k_2 w_2}{1 - s^2 w_1 w_2} \\
&= \frac{w_2\big[a s w_1 w_2 + a w_2 + s k_1 w_2 - (1 - s^2 w_1 w_2) k_2\big]}{1 - s^2 w_1 w_2}.
\end{aligned}$$

This quantity is positive if and only if the expression in parentheses is positive. Using the fact that $(1 - s^2 w_1 w_2)k_2 = (a + b + asw_1)w_2$ from (9), this condition can be rewritten as

$$asw_1w_2 + aw_2 + sk_1w_2 - (a + b + asw_1)w_2 > 0,$$

which simplifies to $sk_1 - b > 0$. Using (8), this inequality can be rewritten as

$$\frac{\left[a + (a + b)sw_2\right]sw_1}{1 - s^2w_1w_2} > b,$$

which further simplifies to (11). ∎

Proof of Lemma 2

With $a = 0$, the value of the firm is

$$F = (\sigma - w_1 - w_2)\mathrm{E}[\pi] = (\sigma - w_1 - w_2)sk_1k_2,$$

where k_1 and k_2 are given by (8) and (9) with $a = 0$. That is,

$$F = \frac{(\sigma - w_1 - w_2)\, b^2 s^2 w_1 w_2^2}{(1 - s^2 w_1 w_2)^2}.$$

The firm chooses w_1 and w_2 to maximize this quantity. It is straightforward to verify that the first-order conditions for this maximization problem are equivalent to

$$\begin{aligned} &(\sigma - w_2)(1 + s^2w_1w_2) - 2w_1 = 0 \quad \text{and} \\ &2(\sigma - w_1) - w_2(3 - s^2w_1w_2) = 0. \end{aligned}$$

The only real values for w_1 and w_2 that solve these equations are given by (13), and the second-order conditions can be verified easily. ∎

Proof of Proposition 2

Because the optimal compensation contract in Lemma 2 does not vary with b, we can use the results of Proposition 1 to prove this proposition. Parts (i) and (ii) follow directly from parts (i) and (ii) of Proposition 1. When $a = 0$, condition (11) in part (iii) of Proposition 1 reduces to $s^2w_1w_2 > \frac{1}{2}$. After we replace w_1 and w_2 by their optimal values in Lemma 2, this inequality reduces to $s^2\sigma^2 > \frac{8}{3}$. ∎.

Proof of Lemma 3

Biased leader scenario. If the biased agent works, the rational agent's expected utility is $w_{\mathrm{F}}(2a+s)-\tilde{c}_{\mathrm{F}}$ if she works, and $w_{\mathrm{F}}a$ if she does not. Her effort choice is therefore $k_{\mathrm{F1}}=w_{\mathrm{F}}(a+s)$. If the biased agent does not work, the rational agent's expected utility is $w_{\mathrm{F}}a-\tilde{c}_{\mathrm{F}}$ if she works, and 0 if she does not. Thus her effort choice is $k_{\mathrm{F0}}=w_{\mathrm{F}}a$. Taking these subsequent effort choices into account, the biased agent's expected utility is $w_{\mathrm{L}}\big[a+b+w_{\mathrm{F}}(a+s)^2\big]-\tilde{c}_{\mathrm{L}}$ if she works, and $w_{\mathrm{L}}w_{\mathrm{F}}a^2$ if she does not work. Her effort choice is therefore $k_{\mathrm{L}}=w_{\mathrm{L}}\big[a+b+sw_{\mathrm{F}}(2a+s)\big]$.

Rational leader scenario. If the rational agent works, the biased agent's expected utility is $w_{\mathrm{F}}(2a+b+s)-\tilde{c}_{\mathrm{F}}$ if she works, and $w_{\mathrm{F}}a$ if she does not. Her effort choice is therefore $k_{\mathrm{F1}}=w_{\mathrm{F}}(a+b+s)$. If the rational agent does not work, the biased agent's expected utility is $w_{\mathrm{F}}(a+b)-\tilde{c}_{\mathrm{F}}$ if she works, and 0 if she does not. Thus her effort choice is $k_{\mathrm{F0}}=w_{\mathrm{F}}(a+b)$. Taking these subsequent effort choices into account, the rational agent's expected utility is $w_{\mathrm{L}}\big[a+w_{\mathrm{F}}(a+b+s)(a+s)\big]-\tilde{c}_{\mathrm{L}}$ if she works, and $w_{\mathrm{L}}w_{\mathrm{F}}(a+b)a$ if she does not work. Her effort choice is therefore $k_{\mathrm{L}}=w_{\mathrm{L}}\big[a+sw_{\mathrm{F}}(2a+b+s)\big]$. ∎

Proof of Proposition 3

With a leader, the probability that the project is successful is

$$\mathrm{E}[\pi]=ak_{\mathrm{L}}+a\big[k_{\mathrm{L}}k_{\mathrm{F1}}+(1-k_{\mathrm{L}})k_{\mathrm{F0}}\big]+sk_{\mathrm{L}}k_{\mathrm{F1}}, \tag{31}$$

and so the value of the firm is given by

$$F=(\sigma-w_{\mathrm{L}}-w_{\mathrm{F}})\mathrm{E}[\pi]=(\sigma-w_{\mathrm{L}}-w_{\mathrm{F}})\Big(ak_{\mathrm{L}}+a\big[k_{\mathrm{L}}k_{\mathrm{F1}}+(1-k_{\mathrm{L}})k_{\mathrm{F0}}\big]+sk_{\mathrm{L}}k_{\mathrm{F1}}\Big). \tag{32}$$

The expected effort cost of the leader is

$$\bar{C}_{\mathrm{L}}\equiv\mathrm{E}[\tilde{c}_{\mathrm{L}}\mid\tilde{c}_{\mathrm{L}}\le k_{\mathrm{L}}]=\frac{k_{\mathrm{L}}^2}{2}, \tag{33}$$

whereas that of the follower is

$$\bar{C}_{\mathrm{F}}\equiv\Pr\big\{e_{\mathrm{L}}=1\big\}\mathrm{E}\big[\tilde{c}_{\mathrm{F}}\mid\tilde{c}_{\mathrm{F}}\le k_{\mathrm{F1}}\big]+\Pr\big\{e_{\mathrm{L}}=0\big\}\mathrm{E}\big[\tilde{c}_{\mathrm{F}}\mid\tilde{c}_{\mathrm{F}}\le k_{\mathrm{F0}}\big]=k_{\mathrm{L}}\frac{k_{\mathrm{F1}}^2}{2}+(1-k_{\mathrm{L}})\frac{k_{\mathrm{F0}}^2}{2}. \tag{34}$$

Biased leader scenario. We can use (31), (33) and (34) to calculate the expected utility of the two agents. The biased leader's expected utility is given by

$$\begin{aligned}
\mathrm{E}\big[\tilde{U}_{\mathrm{L}}\big]=w_{\mathrm{L}}\mathrm{E}[\pi]-\bar{C}_{\mathrm{L}}&=w_{\mathrm{L}}\Big(ak_{\mathrm{L}}+a\big[k_{\mathrm{L}}k_{\mathrm{F1}}+(1-k_{\mathrm{L}})k_{\mathrm{F0}}\big]+sk_{\mathrm{L}}k_{\mathrm{F1}}\Big)-\frac{k_{\mathrm{L}}^2}{2}\\
&=w_{\mathrm{L}}\Big(ak_{\mathrm{L}}+a\big[k_{\mathrm{L}}w_{\mathrm{F}}(a+s)+(1-k_{\mathrm{L}})w_{\mathrm{F}}a\big]+sk_{\mathrm{L}}w_{\mathrm{F}}(a+s)\Big)-\frac{k_{\mathrm{L}}^2}{2}\\
&=w_{\mathrm{L}}k_{\mathrm{L}}\big[a+sw_{\mathrm{F}}(2a+s)\big]+w_{\mathrm{L}}w_{\mathrm{F}}a^2-\frac{k_{\mathrm{L}}^2}{2},
\end{aligned}$$

where we have used k_{F1} and k_{F0} derived Lemma 3 for the second equality. From the same lemma we know that $k_{\mathrm{L}} = w_{\mathrm{L}}\big[a + b + sw_{\mathrm{F}}(2a+s)\big]$, so that $\frac{\partial k_{\mathrm{L}}}{\partial b} = w_{\mathrm{L}}$. Thus

$$\frac{\partial \mathrm{E}\big[\tilde{U}_{\mathrm{L}}\big]}{\partial b} = w_{\mathrm{L}}^2\big[a + sw_{\mathrm{F}}(2a+s)\big] - k_{\mathrm{L}}w_{\mathrm{L}} = -bw_{\mathrm{L}}^2 < 0.$$

The rational follower's expected utility is given by

$$\begin{aligned}
\mathrm{E}\big[\tilde{U}_{\mathrm{F}}\big] &= w_{\mathrm{F}}\mathrm{E}[\pi] - \bar{C}_{\mathrm{F}} = w_{\mathrm{F}}\Big(ak_{\mathrm{L}} + a\big[k_{\mathrm{L}}k_{\mathrm{F1}} + (1-k_{\mathrm{L}})k_{\mathrm{F0}}\big] + sk_{\mathrm{L}}k_{\mathrm{F1}}\Big) - k_{\mathrm{L}}\frac{k_{\mathrm{F1}}^2}{2} - (1-k_{\mathrm{L}})\frac{k_{\mathrm{F0}}^2}{2} \\
&= w_{\mathrm{F}}\Big(ak_{\mathrm{L}} + a\big[k_{\mathrm{L}}w_{\mathrm{F}}(a+s) + (1-k_{\mathrm{L}})w_{\mathrm{F}}a\big] + sk_{\mathrm{L}}w_{\mathrm{F}}(a+s)\Big) - k_{\mathrm{L}}\frac{w_{\mathrm{F}}^2(a+s)^2}{2} - (1-k_{\mathrm{L}})\frac{w_{\mathrm{F}}^2a^2}{2} \\
&= w_{\mathrm{F}}k_{\mathrm{L}}\left[a + \frac{1}{2}w_{\mathrm{F}}(a+s)^2 - \frac{1}{2}w_{\mathrm{F}}a^2\right] + \frac{w_{\mathrm{F}}^2a^2}{2},
\end{aligned}$$

and therefore

$$\frac{\partial \mathrm{E}\big[\tilde{U}_{\mathrm{F}}\big]}{\partial b} = w_{\mathrm{F}}w_{\mathrm{L}}\left[a + \frac{1}{2}w_{\mathrm{F}}(a+s)^2 - \frac{1}{2}w_{\mathrm{F}}a^2\right] > 0.$$

Using k_{F1} and k_{F0} from Lemma 3 in (32), we have

$$F = (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}})\Big(ak_{\mathrm{L}} + a\big[k_{\mathrm{L}}w_{\mathrm{F}}(a+s) + (1-k_{\mathrm{L}})w_{\mathrm{F}}a\big] + sk_{\mathrm{L}}w_{\mathrm{F}}(a+s)\Big),$$

and so

$$\frac{\partial F}{\partial b} = (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}})\Big(aw_{\mathrm{L}} + a\big[w_{\mathrm{L}}w_{\mathrm{F}}(a+s) + (1-w_{\mathrm{L}})w_{\mathrm{F}}a\big] + sw_{\mathrm{L}}w_{\mathrm{F}}(a+s)\Big) > 0.$$

Rational leader scenario. In this case, the rational leader's expected utility is given by

$$\begin{aligned}
\mathrm{E}\big[\tilde{U}_{\mathrm{L}}\big] &= w_{\mathrm{L}}\mathrm{E}[\pi] - \bar{C}_{\mathrm{L}} = w_{\mathrm{L}}\Big(ak_{\mathrm{L}} + a\big[k_{\mathrm{L}}k_{\mathrm{F1}} + (1-k_{\mathrm{L}})k_{\mathrm{F0}}\big] + sk_{\mathrm{L}}k_{\mathrm{F1}}\Big) - \frac{k_{\mathrm{L}}^2}{2} \\
&= w_{\mathrm{L}}\Big(ak_{\mathrm{L}} + a\big[k_{\mathrm{L}}w_{\mathrm{F}}(a+b+s) + (1-k_{\mathrm{L}})w_{\mathrm{F}}(a+b)\big] + sk_{\mathrm{L}}w_{\mathrm{F}}(a+b+s)\Big) - \frac{k_{\mathrm{L}}^2}{2} \\
&= w_{\mathrm{L}}k_{\mathrm{L}}\big[a + sw_{\mathrm{F}}(2a+b+s)\big] + w_{\mathrm{L}}w_{\mathrm{F}}a(a+b) - \frac{k_{\mathrm{L}}^2}{2} \\
&= \frac{k_{\mathrm{L}}^2}{2} + w_{\mathrm{L}}w_{\mathrm{F}}a(a+b),
\end{aligned}$$

where we have used $k_{\mathrm{L}} = w_{\mathrm{L}}\big[a + sw_{\mathrm{F}}(2a+b+s)\big]$ from Lemma 3 for the last equality. Since $\frac{\partial k_{\mathrm{L}}}{\partial b} = sw_{\mathrm{L}}w_{\mathrm{F}}$, we have

$$\frac{\partial \mathrm{E}\big[\tilde{U}_{\mathrm{L}}\big]}{\partial b} = k_{\mathrm{L}}sw_{\mathrm{L}}w_{\mathrm{F}} + w_{\mathrm{L}}w_{\mathrm{F}}a > 0.$$

The biased follower's expected utility is given by

$$\begin{aligned}
\mathrm{E}\big[\tilde{U}_{\mathrm{F}}\big] &= w_{\mathrm{F}}\mathrm{E}[\pi] - \bar{C}_{\mathrm{F}} = w_{\mathrm{F}}\Big(ak_{\mathrm{L}} + a\big[k_{\mathrm{L}}k_{\mathrm{F1}} + (1-k_{\mathrm{L}})k_{\mathrm{F0}}\big] + sk_{\mathrm{L}}k_{\mathrm{F1}}\Big) - k_{\mathrm{L}}\frac{k_{\mathrm{F1}}^2}{2} - (1-k_{\mathrm{L}})\frac{k_{\mathrm{F0}}^2}{2} \\
&= w_{\mathrm{F}}\Big(ak_{\mathrm{L}} + a\big[k_{\mathrm{L}}w_{\mathrm{F}}(a+b+s) + (1-k_{\mathrm{L}})w_{\mathrm{F}}(a+b)\big] + sk_{\mathrm{L}}w_{\mathrm{F}}(a+b+s)\Big) \\
&\qquad\qquad\qquad\qquad - k_{\mathrm{L}}\frac{w_{\mathrm{F}}^2(a+b+s)^2}{2} - (1-k_{\mathrm{L}})\frac{w_{\mathrm{F}}^2(a+b)^2}{2} \\
&= w_{\mathrm{F}}k_{\mathrm{L}}\left[a + \frac{1}{2}w_{\mathrm{F}}s(2a+s)\right] + \frac{1}{2}w_{\mathrm{F}}^2(a^2-b^2),
\end{aligned}$$

and therefore

$$\frac{\partial \mathrm{E}\big[\tilde{U}_{\mathrm{F}}\big]}{\partial b} = w_{\mathrm{F}}sw_{\mathrm{L}}w_{\mathrm{F}}\left[a + \frac{1}{2}w_{\mathrm{F}}s(2a+s)\right] - bw_{\mathrm{F}}^2,$$

which is increasing if and only if $b < sw_{\mathrm{L}}\left[a + \frac{1}{2}(2a+s)sw_{\mathrm{F}}\right]$. Using k_{F1} and k_{F0} from Lemma 3 in (32), we have

$$\begin{aligned}
F &= (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}})\Big(ak_{\mathrm{L}} + a\big[k_{\mathrm{L}}w_{\mathrm{F}}(a+b+s) + (1-k_{\mathrm{L}})w_{\mathrm{F}}(a+b)\big] + sk_{\mathrm{L}}w_{\mathrm{F}}(a+b+s)\Big) \\
&= (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}})\Big(k_{\mathrm{L}}\big[a + sw_{\mathrm{F}}(2a+b+s)\big] + w_{\mathrm{F}}a(a+b)\Big) = (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}})\Big[k_{\mathrm{L}}^2 + w_{\mathrm{F}}a(a+b)\Big],
\end{aligned}$$

and so

$$\frac{\partial F}{\partial b} = (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}})(2k_{\mathrm{L}}sw_{\mathrm{L}}w_{\mathrm{F}} + w_{\mathrm{F}}a) > 0. \quad \blacksquare$$

Proof of Lemma 4

The firm's problem is to choose w_{L} and w_{F} in order to maximize

$$F = (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}})\mathrm{E}[\pi] = (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}})sk_{\mathrm{L}}k_{\mathrm{F1}},$$

as value only gets created when both agents work. When the leader is biased, we have $k_{\mathrm{L}} = w_{\mathrm{L}}\big(b + s^2w_{\mathrm{F}}\big)$ and $k_{\mathrm{F1}} = sw_{\mathrm{F}}$ from Lemma 3 (with $a = 0$). After simplifications, the first-order conditions (with respect to w_{L} and to w_{F} respectively) for this maximization problem are

$$0 = \sigma - 2w_{\mathrm{L}} - w_{\mathrm{F}}, \quad \text{and} \tag{35}$$

$$0 = (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}})\big(b + 2s^2w_{\mathrm{F}}\big) - w_{\mathrm{F}}\big(b + s^2w_{\mathrm{F}}\big). \tag{36}$$

The first of these conditions implies that $w_{\mathrm{L}} = \frac{\sigma - w_{\mathrm{F}}}{2}$. Using this in (36), the second condition reduces to

$$-4s^2w_{\mathrm{F}}^2 - \big(3b - 2s^2\sigma\big)w_{\mathrm{F}} + \sigma b = 0.$$

It is easy to verify that this quadratic equation has a unique positive root, and that this root is greater than $\frac{\sigma}{3}$ and less than $\frac{\sigma}{2}$, implying that $w_{\mathrm{L}} \in \big(\frac{\sigma}{4}, \frac{\sigma}{3}\big)$.

When the leader is rational, we have $k_{\mathrm{L}} = s w_{\mathrm{L}} w_{\mathrm{F}}(b+s)$ and $k_{\mathrm{F1}} = w_{\mathrm{F}}(b+s)$ from Lemma 3 (with $a = 0$). After simplifications, the first-order conditions (with respect to w_{L} and to w_{F} respectively) for this maximization problem are

$$0 = \sigma - 2w_{\mathrm{L}} - w_{\mathrm{F}}, \quad \text{and} \tag{37}$$

$$0 = (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}})2s(b+s) - s(b+s). \tag{38}$$

It is easy to verify that $w_{\mathrm{L}} = \frac{\sigma}{4}$ and $w_{\mathrm{F}} = \frac{\sigma}{2}$ uniquely solve these equations. ∎

Proof of Proposition 4

We can use the effort levels and contracts derived in Lemma 3 and Lemma 4 to calculate the value of the firm. With a biased leader, this value is given by

$$F = (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}}) s k_{\mathrm{L}} k_{\mathrm{F1}} = (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}}) s^2 w_{\mathrm{L}} w_{\mathrm{F}} \left(b + s^2 w_{\mathrm{F}}\right)$$

with w_{L} and w_{F} as given in (14). After replacing w_{L} and w_{F} and simplifying, this becomes

$$F = \frac{\left(6s^2\sigma + 3b - \Lambda\right)^2 \left(2s^2\sigma - 3b + \Lambda\right)\left(2s^2\sigma + 5b + \Lambda\right)}{16{,}384\, s^4},$$

where $\Lambda \equiv \sqrt{4s^4\sigma^2 + 4s^2\sigma b + 9b^2}$. Tedious but straightforward manipulations yield

$$\left(\frac{\partial F}{\partial b}\right)_{b=0} = \frac{s^2\sigma^3}{32}. \tag{39}$$

With a rational leader, the firm's value is given by

$$F = (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}}) s k_{\mathrm{L}} k_{\mathrm{F1}} = (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}}) s^2 w_{\mathrm{L}} w_{\mathrm{F}}^2 (b+s)^2$$

with w_{L} and w_{F} as given in (15). After replacing w_{L} and w_{F} and simplifying, this becomes

$$F = \frac{s^2\sigma^4(b+s)^2}{64}.$$

It is then straightforward to show that

$$\left(\frac{\partial F}{\partial b}\right)_{b=0} = \frac{s^3\sigma^4}{32}. \tag{40}$$

The firm will appoint the rational agent as its leader when (40) is greater than (39) or , equivalently, when $s\sigma > 1$. ∎

Proof of Proposition 5

We know from Lemma 2 that, once the two agents work for the same firm, firm value is maximized with $w_1 = \frac{2\sigma}{8-s^2\sigma^2}$ and $w_2 = \frac{\sigma}{2}$. With this contract, we know from Lemma 1 that the equilibrium effort levels of the two agents are given by

$$k_1 = \frac{bs\sigma^2}{2(4-s^2\sigma^2)} \quad \text{and} \quad k_2 = \frac{b\sigma(8-s^2\sigma^2)}{4(4-s^2\sigma^2)}.$$

The (biased) expected utility of agent 2 is then given by

$$\mathrm{E}_{\mathrm{B}}\big[\tilde{U}_2\big] = w_2\big(bk_2 + sk_1k_2\big) - \frac{k_2^2}{2} = \frac{b^2\sigma^2(8-s^2\sigma^2)^2}{32(4-s^2\sigma^2)^2}.$$

This contract meets the entrepreneur's (i.e., agent 2's) reservation utility is this quantity is at least $\frac{b^2\sigma^2}{2}$, that is, if $s^2\sigma^2 \geq \frac{8}{3}$. Otherwise (i.e., if $s^2\sigma^2 < \frac{8}{3}$), the entrepreneur must be offered more than $\frac{\sigma}{2}$ for her to give up her own firm. More precisely, given the equilibrium effort levels of the two agents in Lemma 1, the compensation contracts must satisfy

$$\frac{b^2\sigma^2}{2} \leq \mathrm{E}_{\mathrm{B}}\big[\tilde{U}_2\big] = w_2\big(bk_2 + sk_1k_2\big) - \frac{k_2^2}{2} = \frac{b^2w_2^2}{2(1-s^2w_1w_2)^2}$$

or equivalently, $w_2 \geq \frac{\sigma}{1+s^2\sigma w_1}$. Given that the firm gains nothing from offering the entrepreneur more than his reservation wage, its problem is to choose w_1 and w_2 to maximize

$$F = (\sigma - w_1 - w_2)\mathrm{E}[\pi] = (\sigma - w_1 - w_2)sk_1k_2 = \frac{(\sigma - w_1 - w_2)sb^2w_1w_2^2}{(1-s^2w_1w_2)^2} \tag{41}$$

subject to

$$w_2 = \frac{\sigma}{1+s^2\sigma w_1}. \tag{42}$$

Using (42) in (41) and simplifying, the firm's problem reduces to choosing w_1 to maximize

$$F = \left(\sigma - w_1 - \frac{\sigma}{1+s^2\sigma w_1}\right) b^2s^2\sigma^2w_1.$$

The first-order condition for this maximization problem is

$$\sigma - 2w_1 - \frac{\sigma}{(1+s^2\sigma w_1)^2} = 0,$$

which can be shown to be equivalent to

$$-2s^4\sigma^2w_1^2 - s^2\sigma(4-s^2\sigma^2)w_1 + 2(s^2\sigma^2-1) = 0.$$

Because the first two terms of this quadratic expression are negative for positive w_1, there is a unique positive root if and only if $s^2\sigma^2 > 1$ (and otherwise, there is no pair of compensation contracts that can attract the entrepreneur to the first firm). This root is given by (20) and, since $s^2\sigma^2 < \frac{8}{3}$, it can be shown to be smaller than $\frac{1}{s^2\sigma}$. Using (20) in (42), we get (19) which, given that $w_1 < \frac{1}{s^2\sigma}$, is greater than $\frac{\sigma}{2}$ and smaller than σ. ∎

Proof of Proposition 6

As before, we look for an equilibrium in which agent i exerts effort if and only if $\tilde{c}_i \leq k_i$. Given that $\tilde{c}_i \sim \mathrm{U}\big[0, \bar{c}_i\big]$, agent i exerts effort with probability $\frac{k_i}{\bar{c}_i}$. Thus, after observing $\tilde{c}_1 = c_1$, agent 1's expected utility is $w_1 s \frac{k_2}{\bar{c}_2} - c_1$ if she exerts effort, and zero if she does not. This implies that she exerts effort if and only if

$$c_1 \leq w_1 s \frac{k_2}{\bar{c}_2} \equiv k_1. \tag{43}$$

Similarly, after observing $\tilde{c}_2 = c_2$, agent 2's (biased) expected utility is $w_2 \left(b + s\frac{k_1}{\bar{c}_1}\right) - c_2$ if she exerts effort, and zero if she does not. This implies that she exerts effort if and only if

$$c_2 \leq w_2 \left(b + s\frac{k_1}{\bar{c}_1}\right) \equiv k_2. \tag{44}$$

Solving for k_1 and k_2 in (43) and (44), we get

$$k_1 = \frac{\bar{c}_1 b s w_1 w_2}{\bar{c}_1\bar{c}_2 - s^2 w_1 w_2} \quad \text{and} \quad k_2 = \frac{\bar{c}_1\bar{c}_2 b w_2}{\bar{c}_1\bar{c}_2 - s^2 w_1 w_2}.$$

The firm's problem is to choose w_1 and w_2 to maximize

$$F = (\sigma - w_1 - w_2)\mathrm{E}[\pi] = (\sigma - w_1 - w_2) s \frac{k_1}{\bar{c}_1}\frac{k_2}{\bar{c}_2} = \frac{(\sigma - w_1 - w_2)\bar{c}_1 b^2 s^2 w_1 w_2^2}{(\bar{c}_1\bar{c}_2 - s^2 w_1 w_2)^2}.$$

It is easy to verify that the contracts in (21) solve this maximization problem. With these contracts, it is straightforward to verify that

$$F = \frac{b^2 s^2 \sigma^4}{16\bar{c}_2(4\bar{c}_1\bar{c}_2 - s^2\sigma^2)}, \tag{45}$$

$$\mathrm{E}\big[\tilde{U}_1\big] = \frac{\bar{c}_1 b^2 s^2 \sigma^4}{8(4\bar{c}_1\bar{c}_2 - s^2\sigma^2)^2}, \quad \text{and} \tag{46}$$

$$\mathrm{E}\big[\tilde{U}_2\big] = \frac{b^2\sigma^2(8\bar{c}_1\bar{c}_2 - s^2\sigma^2)(3s^2\sigma^2 - 8\bar{c}_1\bar{c}_2)}{32\bar{c}_2(4\bar{c}_1\bar{c}_2 - s^2\sigma^2)^2}. \tag{47}$$

Since $s^2\sigma^2 < 4\bar{c}_1\bar{c}_2$ (see footnote 15), it is clear that F and $\mathrm{E}\big[\tilde{U}_1\big]$ are increasing in b, whereas $\mathrm{E}\big[\tilde{U}_2\big]$ is increasing in b as long as $s^2\sigma^2 > \frac{8\bar{c}_1\bar{c}_2}{3}$. From (45), since $\bar{c}_1\bar{c}_2$ is unaffected when the costly task is assigned to one or the other agent, it is clear that firm value will be lower if the costly task is assigned to agent 2 (i.e., $\bar{c}_2 > \bar{c}_1$). ∎

Proof of Proposition 7

Again, we look for an equilibrium in which agent i exerts effort if and only if $\tilde{c}_i \leq k_i$. After observing $\tilde{c}_1 = c_1$, agent 1's expected utility is $(w_1 s + r_1)k_2 - c_1$ if she exerts effort, and zero if she does not. This implies that she exerts effort if and only if

$$c_1 \leq (w_1 + r_1)sk_2 \equiv k_1. \tag{48}$$

Similarly, after observing $\tilde{c}_2 = c_2$, agent 2's (biased) expected utility is $(w_2 + r_2)(b + sk_1) - c_2$ if she exerts effort, and zero if she does not. This implies that she exerts effort if and only if

$$c_2 \leq (w_2 + r_2)(b + sk_1) \equiv k_2. \tag{49}$$

Solving for k_1 and k_2 in (48) and (49), we get

$$k_1 = \frac{bs(w_1 + r_1)(w_2 + r_2)}{1 - s^2(w_1 + r_1)(w_2 + r_2)} \quad \text{and} \quad k_2 = \frac{b(w_2 + r_2)}{1 - s^2(w_1 + r_1)(w_2 + r_2)}.$$

The firm's problem is to choose w_1 and w_2 to maximize

$$F = (\sigma - w_1 - w_2)\mathrm{E}[\pi] = (\sigma - w_1 - w_2)sk_1k_2 = \frac{(\sigma - w_1 - w_2)b^2s^2(w_1 + r_1)(w_2 + r_2)^2}{\left[1 - s^2(w_1 + r_1)(w_2 + r_2)\right]^2}.$$

It is easy to verify that the contracts in (22) solve this maximization problem. The rest of the proof is similar to that of Proposition 6. ∎

Proof of Proposition 8

From the firm's perspective, the effort level of the two agents is a pair of random variables (independent of each other) with a mean of

$$\bar{k} \equiv \phi k_{\mathrm{B}} + (1 - \phi)k_{\mathrm{R}} = \frac{\phi bw}{1 - sw},$$

where the last equality is obtained by replacing k_{R} and k_{B} by their equilibrium values in (25) and by simplifying. As such, the firm's problem is to choose w in order to maximize

$$F = (\sigma - 2w)\mathrm{E}[\pi] = (\sigma - 2w)s\bar{k}^2 = \frac{(\sigma - 2w)s\phi^2b^2w^2}{(1 - sw)^2}. \tag{50}$$

The first-order condition for this maximization problem is

$$0 = \frac{dF}{dB} = s\phi^2b^2\frac{(2\sigma - 6w)w(1 - sw)^2 + 2s(\sigma - 2w)w^2(1 - sw)}{(1 - sw)^4} = \frac{2s\phi^2b^2w}{(1 - sw)^3}(\sigma - 3w + sw^2).$$

It is easy to verify that (26) is the unique $w \in \left(0, \frac{\sigma}{2}\right)$ that satisfies this condition and that the second-order condition is satisfied for this w. The fact that F is proportional to ϕ^2 in (50) and that the optimal w does not depend on ϕ implies that firm value is increasing in ϕ. The expected utility of rational agents is given by

$$\begin{aligned}\mathrm{E}\big[\tilde{U}_i\big] &= w \Pr\{\tilde{v} = \sigma\} - \mathrm{E}\big[\tilde{c}_i \mid \tilde{c}_i \le k_{\mathrm{R}}\big] = wsk_{\mathrm{R}}\bar{k} - \frac{k_{\mathrm{R}}^2}{2} \\ &= wsk_{\mathrm{R}}\big[\phi k_{\mathrm{B}} + (1-\phi)k_{\mathrm{R}}\big] - \frac{k_{\mathrm{R}}^2}{2}\end{aligned}$$

which, after replacing k_{R} and k_{B} by their equilibrium values in (25) and simplifying, reduces to

$$\mathrm{E}\big[\tilde{U}_i\big] = \frac{\phi^2 b^2 s^2 w^4}{2(1-sw)^2}.$$

Since the optimal w does not depend on ϕ, this quantity is clearly increasing in ϕ. The expected utility of biased agents is given by

$$\begin{aligned}\mathrm{E}\big[\tilde{U}_j\big] &= w \Pr\{\tilde{v} = \sigma\} - \mathrm{E}\big[\tilde{c}_j \mid \tilde{c}_j \le k_{\mathrm{B}}\big] = wsk_{\mathrm{B}}\bar{k} - \frac{k_{\mathrm{B}}^2}{2} \\ &= wsk_{\mathrm{B}}\big[\phi k_{\mathrm{B}} + (1-\phi)k_{\mathrm{R}}\big] - \frac{k_{\mathrm{B}}^2}{2}\end{aligned}$$

which, after replacing k_{R} and k_{B} by their equilibrium values in (25) and simplifying, reduces to

$$\mathrm{E}\big[\tilde{U}_j\big] = \frac{b^2 w^2\big[\phi^2 s^2 w^2 - (1-sw)^2\big]}{2(1-sw)^2} = \frac{b^2 w^2\big[1-(1-\phi)sw\big]\big[(1+\phi)sw - 1\big]}{2(1-sw)^2}.$$

It is easy to verify that w in (26) is greater than $\frac{1}{s}$ so that the first expression in brackets is positive. Thus the welfare of biased agents is increasing in ϕ if and only if $(1+\phi)sw - 1 > 0$, which is equivalent to

$$1 + \phi > \frac{1}{sw} = \frac{2}{3-\sqrt{9-4s\sigma}} = \frac{2}{3-\sqrt{9-4s\sigma}}\,\frac{3+\sqrt{9-4s\sigma}}{3+\sqrt{9-4s\sigma}} = \frac{3+\sqrt{9-4s\sigma}}{2s\sigma},$$

and in turn equivalent to the second inequality in (27). The first inequality in (27) ensures that $\frac{3+\sqrt{9-4s\sigma}}{2s\sigma} - 1 < 1$ since $\phi \in (0,1)$. ∎

Proof of Proposition 9

Let $\tilde{n}$ denote the number of overconfident agents working for the firm. Suppose that agent 1 is rational and that $e_1 = 1$ (otherwise project success is impossible). From her perspective,

$$\Pr\big\{\tilde{v} = \sigma \mid \tilde{b}_2 = b\big\} = sk_{\mathrm{B}} \quad \text{and} \quad \Pr\big\{\tilde{v} = \sigma \mid \tilde{b}_2 = 0\big\} = sk_{\mathrm{R}},$$

so that

$$\begin{aligned}
\Pr\{\tilde{n}=1 \mid \tilde{v}=\sigma\} &= \Pr\{\tilde{b}_2=b \mid \tilde{v}=\sigma\} = \frac{\Pr\{\tilde{v}=\sigma \mid \tilde{b}_2=b\}\Pr\{\tilde{b}_2=b\}}{\sum_{\beta\in\{0,b\}}\Pr\{\tilde{v}=\sigma \mid \tilde{b}_2=\beta\}\Pr\{\tilde{b}_2=\beta\}} \\
&= \frac{sk_{\mathrm{B}}\phi}{sk_{\mathrm{B}}\phi+sk_{\mathrm{R}}(1-\phi)} = \frac{k_{\mathrm{B}}\phi}{k_{\mathrm{B}}\phi+k_{\mathrm{R}}(1-\phi)}.
\end{aligned}$$

This quantity is greater than ϕ, the prior probability that a rational agent assigns to $\tilde{n}$ being equal to one, if $k_{\mathrm{B}} > k_{\mathrm{R}}$, which can be shown to be the case using (25). Now suppose that agent 1 is biased and that $e_1 = 1$. From her perspective,

$$\Pr_{\mathrm{B}}\{\tilde{v}=\sigma \mid \tilde{b}_2=b\} = b+sk_{\mathrm{B}} \quad \text{and} \quad \Pr_{\mathrm{B}}\{\tilde{v}=\sigma \mid \tilde{b}_2=0\} = b+sk_{\mathrm{R}},$$

so that[18]

$$\begin{aligned}
\Pr_{\mathrm{B}}\{\tilde{n}=1 \mid \tilde{v}=\sigma\} &= \Pr_{\mathrm{B}}\{\tilde{b}_2=b \mid \tilde{v}=\sigma\} = \frac{\Pr_{\mathrm{B}}\{\tilde{v}=\sigma \mid \tilde{b}_2=b\}\Pr\{\tilde{b}_2=b\}}{\sum_{\beta\in\{0,b\}}\Pr_{\mathrm{B}}\{\tilde{v}=\sigma \mid \tilde{b}_2=\beta\}\Pr\{\tilde{b}_2=\beta\}} \\
&= \frac{(b+sk_{\mathrm{B}})\phi}{(b+sk_{\mathrm{B}})\phi+(b+sk_{\mathrm{R}})(1-\phi)} = \frac{(b+sk_{\mathrm{B}})\phi}{b+s\big[k_{\mathrm{B}}\phi+k_{\mathrm{R}}(1-\phi)\big]}.
\end{aligned}$$

This quantity is greater than ϕ, the prior probability that an overconfident agent assigns to $\tilde{n}$ being equal to one, if $k_{\mathrm{B}} > k_{\mathrm{R}}$, which is the case. Finally, from the firm's perspective,

$$\begin{aligned}
&\Pr\{\tilde{v}=\sigma \mid \tilde{n}=2\} = \Pr\{\tilde{v}=\sigma \mid \tilde{b}_1=b, \tilde{b}_2=b\} = sk_{\mathrm{B}}^2, \\
&\Pr\{\tilde{v}=\sigma \mid \tilde{n}=1\} = \Pr\{\tilde{v}=\sigma \mid \tilde{b}_1=b, \tilde{b}_2=0\} = \Pr\{\tilde{v}=\sigma \mid \tilde{b}_1=0, \tilde{b}_2=b\} = sk_{\mathrm{B}}k_{\mathrm{R}}, \\
&\Pr\{\tilde{v}=\sigma \mid \tilde{n}=0\} = \Pr\{\tilde{v}=\sigma \mid \tilde{b}_1=0, \tilde{b}_2=0\} = sk_{\mathrm{R}}^2.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\Pr\{\tilde{n}=2 \mid \tilde{v}=\sigma\} &= \frac{\Pr\{\tilde{v}=\sigma \mid \tilde{n}=2\}\Pr\{\tilde{n}=2\}}{\sum_{n=0}^{2}\Pr\{\tilde{v}=\sigma \mid \tilde{n}=n\}\Pr\{\tilde{n}=n\}} \\
&= \frac{sk_{\mathrm{B}}^2\phi^2}{sk_{\mathrm{B}}^2\phi^2+2sk_{\mathrm{B}}k_{\mathrm{R}}\phi(1-\phi)+sk_{\mathrm{R}}^2(1-\phi)^2} = \left(\frac{k_{\mathrm{B}}\phi}{k_{\mathrm{B}}\phi+k_{\mathrm{R}}(1-\phi)}\right)^2,
\end{aligned}$$

which is greater than the firm's prior, $\Pr\{\tilde{n}=2\} = \phi^2$, if $k_{\mathrm{B}} > k_{\mathrm{R}}$, which is the case. We can show that $\Pr\{\tilde{n}=0 \mid \tilde{v}=\sigma\} < \Pr\{\tilde{n}=0\}$ in a similar way. ∎

[18]Note that the overconfident agent thinks that she is skilled, not overconfident.