The Economics of Constitutional Rights and Voting Rules

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Abstract

Constituions typically specify that some laws require greater levels of support to pass than others. Laws that overturn protected constitutional rights, for example, are much harder to pass than are most other laws. This paper analyzes how the characteristics of a law influence how much support the law should have in order to pass. It shows that while the expected total benefit from a law should not affect the optimal vote share required for passage, the dispersion in gains and losses should. The fraction of winners from a law has a non-monotonic effect on the optimal vote share. These results can help one understand what ”rights” deserve constitutional protection and what ”rights” do not.

1 Introduction

In the United States, if the President wants to declare war, this requires only simple majority support in both houses of Congress. By contrast, if a district attorney wants to try a criminal before a judge, this requires two-thirds majority support in both houses of Congress and the consent of three-fourths of the state legislatures. The reason for the difference is the right to trial by jury guaranteed in the United States Constitution whereas declarations of war are simply part of the normal business of Congress. This paper explores the question of why some laws should be especially hard (or easy) to pass whereas other laws should simply be decided by majority rule. More specifically, it analyzes what is the optimal voting rule (or optimal vote share) for any given potential law and how various characteristics of a law should affect (or not affect) the optimal voting rule for that law.

To analyze these issues, I develop a simple model with a constitutional phase and legislation phase. In the constitution phase, a social planner (or all individuals behind a veil of ignorance) decide on the vote share required to pass any given law. In the legislation phase, the population votes on the law and the law passes if and only if the fraction of the population supporting the law exceeds the vote share required in the constitution. Since I assume that people vote for a law if and only if the law gives them a greater payoff than the absence of the law (whose payoff I normalize to zero), the action in the model occurs at the first stage.[1] The social planner chooses a vote share that maximizes expected social welfare (defined simply as the expectation of the sum of the payoffs of all individuals in the population) given that people will vote for the law if and only it gives them a positive payoff. An individual's payoff from a law consists of an idiosyncratic component and a common component. There are two groups in the population, whom I call winners and losers. The idiosyncratic payoffs of winners and losers are is a random variable drawn different uniform distributions and all winners have a greater idiosyncratic payoff than do all losers. These distributions and the fraction of the population that are winners is known in the constitution phase (although people do not know their individual payoffs). The common payoff component is a random variable in the first stage. This can be thought of as how well the law matches with the state of the world which affects everyone equally. Because of the this uncertain common component, in the constitutional phase one does not know the total benefit of the law or what fraction of the

[1] I assume that bargaining and transfers are not feasible in the voting stage–that is, people cannot be paid to vote for a law which gives them a negative payoff.

population will support it. The ideal outcome for the social planner (or all individuals deciding behind a veil of ignorance) is to choose a vote threshold such that the law will pass if and only if it provides a positive total benefit to the population.

The main results of the paper are as follows. First, majority rule is not generally optimal. This explains why it makes sense to have some types of laws that are harder (or easier) to pass than others.[2] Second, the optimal vote share required to pass a law depends only on the fraction of winners from the law and the ratio of a measure of the dispersion of gains of winners (or a measure of dispersion of losses of losers) over the sum of the mean gains of winners and the mean losses of losers. The mean benefit of the law itself has no direct effect on the optimal vote share because if the law is likely to be a good one because it will have a high mean benefit then it should also be the case that many people should support it. Third, the larger is the dispersion of gains of winners (relative to the sum of the mean gain for winners plus the mean loss for losers) the easier it should be to pass the law. Similarly, the larger is the dispersion of losses of losers (relative to the sum of the mean gain for winners plus the mean loss for losers) the harder it should be to pass the law. The reason for this is that if the dispersion of gains (for example) is large, it can more easily be the case that most people lose from the law but the total benefit from the law is still positive. Fourth, the fraction of winners from the law has a non-monotonic effect on the optimal vote share. If all the population will be winners (or losers) then the optimal vote share is one-half. This means that for any law in which everyone's expected benefit from the law is drawn from the same (uniform) distribution, majority rule is optimal. But, as the fraction of winners increases from zero, the optimal vote share initially declines below one-half. At some point it then begins to increase above one-half but then declines back to one half at the point where everyone in the population is a winner.

These results can help one think about what laws should be deemed "rights" protected by the constitution (and thus very difficult to change) and what laws should be left to be decided by simple majority rule. Consider, for example, the issue of whether criminal defendants should have the option to choose a jury or bench trial. Those who will benefit from restricting this option will primarily benefit from increased deterrence through making it easier to convict guilty defendants that juries are more likely to acquit than judges. This deterrence is likely to benefit people fairly

[2] Of course, we do not generally see provisions in which a given law can pass with less than majority support. Notice, however, that if it is optimal to allow law X to pass if say thirty percent of the law support it, this is equivalent to making X the default provision and requiring $Not - X$ to have seventy percent support to pass.

equally. That is, the dispersion in benefits among the winners from a law eliminating the option to choose a jury trial is likely to be fairly small. By contrast, the dispersion in losses among losers is likely to be quite large. While anyone who is charged with a crime benefits from having the option to choose a jury trial, this option is likely to be worth much more in some situations than in others. Furthermore, since most people are never charged with a crime and don't face a significant risk of being charged with a crime, the fraction of winners from a law eliminating the right to a jury trial is probably quite large, but still clearly less than one. This is exactly the situation in which the model predicts that it should be quite hard to pass a law eliminating the right to a jury trial, suggesting that giving such a right constitutional protection makes sense.

This paper also has applications for the formations of new "constitutions" either for states (such as Iraq), international organizations (such as the European Union), or private organizations (such as a charitable organization). Any organization that expects there to be conflict of interest among its members in the future must establish rules for resolving that conflict.[3] The results in this paper suggest how to establish voting rules for different issues in a way that will maximize the ex ante expected utility of the members of the organization based on the characteristics of the different possible issues the organization may face in the future.

The question of which "rights" deserve special protection and which do not is a very old one. Rousseau (1762) was one of the first to explicitly discuss this question in the context of how the optimal vote share necessary to pass a law should vary with the importance of the question. That said, formal economic analysis of constitutional design issues such as this is quite recent. Buchanan and Tullock (1962) were one of the first to raise the issue of optimal voting rules for constitutional questions, but they do not directly address it. More recent work on the size of majority voting often focuses on the stability of social decisions and the prevention of electoral cycles (see Caplin and Nalebuff 1988; Dasgupta and Maskin 1998; and Barbera and Jackson 2004 among others). More closely related to this paper are the normative papers on constitutional design. Aghion, Alesina, and Trebbi (2004) provide an economic analysis of the optimal amount of discretion to give to a leader. Their basic model is similar to the model in this paper with two important differences. First, the idiosyncratic benefits from the law (or reform, in their terminology) are drawn from the same uniform distribution for all members of the population. Thus, they find that simple majority rule is optimal if the leader is never corrupt (this is exactly the same result that

[3]This paper is less applicable to corporate charters since those who write the charter (the stockholders) are likely to have common aims, maximizing shareholder value, in the future.

occurs in this paper if the fraction of winners from a law is zero or one, so that all members of the population draw from the same uniform distribution). Second, they include the possibility that a corrupt leader will propose a reform that is in fact simply an expropriation, which is not part of the model in this paper. Also related to this paper is Mueller's work on constitutional design with uncertainty (2001) and on constitutional rights (1991).

In the next section, I present the basic model of the paper. Subsection 2.1 analyses the model in the case where the vote share required to pass a law is large enough that at least some losers must benefit from the law in order for it to pass. Subsection 2.2 analyses the opposite case in which a law can pass even if some winners do not vote for it. Subsection 2.3 examines the intermediate case in which the optimal vote share under either condition does not satisfy that condition, so that the optimal vote share is approximately the fraction of winners. Section 3 presents the main results of the paper and provides some graphs of the optimal vote share for various parameter values. Section 4 briefly discusses an extension of the model to the case where the fraction of winners is a random variable at the constitutional stage. Section 5 concludes. Proofs not in the text are contained in the appendix.

2 Model

Consider a simple two period model. Period zero is the constitutional phase and period one is the legislation phase. There are a continuum of people of size one, all of whom are alive in period zero and period one. In period zero, these players have to decide whether or not to enshrine a given law into the constitution and if they do so, they have to decide how hard it will be to overturn this law in period one. That is, they have to decide what fraction of the population, $\hat{z}$, must vote to pass this law in period one in order to implement it. All players are risk neutral. All payoffs occur in period one. In period zero, all players are identical. That is, we assume that in period zero all players are behind a veil of ignorance such that they do not know what their type will be in period one. Thus, in period zero, the players make decisions to maximize expected total welfare in period one. Thus, this can be thought of as a model of constitution formation by a benevolent social planner. In period 1, each member of the population will be either a winner or a loser from the law. The probability that a person is a winner is given by z. Winners have an idiosyncratic payoff from the law that is distributed uniformly between $[w, w+W]$. Losers have an idiosyncratic payoff from the law that is distributed uniformly between $[-l-L, -l]$. $W, L, w+l \geq 0$. That is, while I assume that all winners have a higher idiosyncratic payoff than all losers, it is possible that some losers have a positive idiosyncratic payoff or that some winners have a negative idiosyncratic payoff. A person's total payoff from the law is given by her idiosyncratic payoff plus a common component, x. x reflects new information about the desirability of the law given the state of the world in period one. x is distributed uniformly between $[-b/2, b/2]$.

Thus, total welfare from the law, if it is in place, is given by

$$z(w + W/2) - (1 - z)(l + L/2) + x$$

The z winners receive an average payoff of $w + W/2$ and the $1 - z$ losers receive an average loss of $(l + L/2)$, and all people receive x from the law. The payoff from not passing the law is normalized to zero. Of course, what really matters for social welfare is the total welfare from the law when it passes. To determine that, there are two cases to consider. If $\hat{z} > z$, then the law can only pass if all winners vote for the law and some losers do as well. Since people vote for the law if and only if it gives them a positive payoff, this means the law passes if and only if $z + (1 - z)(\frac{x-l}{L}) > \hat{z}$. That is, x must be large enough to induce enough losers to vote for the law that, together with the winners, they make up at least $\hat{z}$ of the population. Alternatively, if $\hat{z} < z$, then the law can pass without support from any losers and with support from only some of the winners. That is, the law passes if and only if $z(\frac{W+w+x}{W}) > \hat{z}$.

2.1 Optimal $\hat{z}$ if $\hat{z} > z$

If $\hat{z} > z$, then the law passes if and only if x, the common benefit from the law is large enough to give enough losers a positive payoff from the law. That is, the law passes if and only if $x > \frac{l(1-z)+L(\hat{z}-z)}{1-z}$. If at least $\hat{z}$ of the population must vote for the law, then expected total welfare as a function of $\hat{z}$ is (recall that if the law does not pass, then total welfare is zero):

$$\int_{\frac{l(1-z)+L(\hat{z}-z)}{1-z}}^{b/2} (\frac{1}{b}(z(w + W/2) - (1 - z)(l + L/2) + x))dx$$

The optimal constitution will then choose $\hat{z}$ to maximize expected total welfare. The first order condition, under the assumption that $\hat{z} > z$, is given by[4]:

$$\frac{L\{L(1+z^2-2\hat{z})-(2(w+l)+W)(1-z)z\}}{2b(1-z)^2}=0$$

Solving this first order condition for $\hat{z}$ gives the optimal $\hat{z}$, provided $\hat{z} > z$:

$$\hat{z}=\frac{L(1+z^2)-(2(w+l)+W)(1-z)z}{2L}$$

This expression becomes easier to interpret with the following change of variables. Let $\mu_w = w + W/2$ and $\mu_l = l + L/2$, so that these represent the mean idiosyncratic gain that winners have from the law and the mean idiosyncratic loss that losers have from the law. Then, I will also write L, the dispersion in the possible idiosyncratic losses, in terms of the sum of the mean gain and loss. That is, $L = \alpha_L(\mu_w + \mu_l)$. With these change of variables, one can write $\hat{z}$ as follows:

$$\hat{z}=\frac{1}{2}+\frac{z(2z-2+\alpha_L)}{2\alpha_L}$$

Since this solution is derived under the assumption that $\hat{z} > z$, it is valid if and only if $z < \frac{\alpha_L}{2}$. Right away, one can see that majority rule is not generally optimal. For $\hat{z} > z$, majority rule is only optimal if either there will be no winners from the law or $z = 1 - \frac{\alpha_L}{2}$. The following lemma summarizes the comparative statics for the optimal vote threshold in this case.

Lemma 1 If the fraction of winners is less than the vote share required for the law to pass, $\hat{z} > z$, then the optimal vote share required to pass the law is $\hat{z} = \frac{1}{2} + \frac{z(2z-2+\alpha_L)}{2\alpha_L}$. The optimal vote share required for passage is increasing in the dispersion of the possible idiosyncratic losses (measured relative to the sum of the mean gain and loss from the law), α_L. The optimal vote share is decreasing in the fraction of winners from the law if and only if $z < \frac{1}{2} - \frac{\alpha_L}{4}$.

Proof. The optimal $\hat{z}$ is derived above. $\frac{d}{d\alpha_L}(\frac{1}{2}+\frac{z(2z-2+\alpha_L)}{2\alpha_L}) = \frac{(1-z)z}{(\alpha_L)^2} > 0$. $\frac{d}{dz}(\frac{1}{2}+\frac{z(2z-2+\alpha_L)}{2\alpha_L}) = \frac{-2+4z+\alpha_L}{2\alpha_L}$, this is negative if and only if $z < \frac{1}{2} - \frac{\alpha_L}{4}$. Q.E.D.

Lemma one says that provided some losers will need to vote for the law in order for it to pass, increasing the dispersion in the idiosyncratic losses (faced by losers) should make the law harder to

[4] It is easy to see that expected social welfare is concave in $\hat{z}$, so the first order condition gives the unique maximum (again, so long as this maximum satisfies $\hat{z} > z$).

pass. The reason for this is that when the dispersion of losses is large, having some losers benefit from the law does not rule out the possibility that the average amount that losers lose from law is still large. With large dispersion in losses, losers as a group could suffer greatly even though there are some losers willing to vote for the law. Thus, a higher vote threshold is necessary. The reason the dispersion only matters in relation to the sum of the mean gain (for winners) and mean loss (for losers) is that the role dispersion in losses plays is to suggest that even though a given fraction of the population voted for the law, the law still could reduce social welfare. For that to be the case, the losses from the big losers must outweigh the gain from the winners, measured by their mean gain, and the positive common shock which makes some losers gain from the law, which must be bigger in magnitude the larger the mean loss of losers. This means that, contrary to what one might expect, not only do larger mean gains for winners suggest the law should be easier to pass but larger mean losses for losers also suggest the law should be easier to pass. Of course, these comparative statics are only valid if α_L is not so small (greater than $2z$) that $z \geq \hat{z}$.

The other comparative static result from the lemma is that the optimal vote share is initially declining in the fraction of winners, but is increasing in the fraction of winners when z gets large enough. For small dispersion ($\alpha_L < 2/3$), the optimal vote share is declining in z for any $z < \hat{z}$, but for large dispersion z can be large enough (and still satisfy $z < \hat{z}$) to make the optimal vote share increasing in z. This means that when there will be very few winners from the law, the law should be easier to pass the more winners there are, but if the fraction of winners gets large enough, increasing the fraction of winners should make the law harder to pass.

Lastly, it is important to note what does not matter in selecting the optimal vote share. When all the winners must vote for a law in order for it to pass, the dispersion in benefits among the winners does not affect the optimal vote share. Furthermore, notice that the overall expected benefit from the law plays no direct role. In fact, even the winner's mean gain and the loser's mean loss do not matter independently, but only matter through their sum (*not* their difference) and this sum only matters in relation to the dispersion in the losses.

2.2 Optimal $\hat{z}$ if $\hat{z} < z$

If $\hat{z} < z$, then the law can pass without support from any losers so long as the common benefit from the law is large enough that $\hat{z}/z$ winners benefit from the law. That is, the law passes if and only if $x > \frac{W(\hat{z}-z)-wz}{z}$. Once again, if at least $\hat{z}$ of the population must vote for the law, then expected total welfare as a function of $\hat{z}$ is (recall that if the law does not pass, total welfare is zero):

$$\int_{\frac{W(\hat{z}-z)-wz}{z}}^{b/2} (\frac{1}{b}(z(w+W/2)-(1-z)(l+L/2)+x))dx$$

The optimal constitution will then choose $\hat{z}$ to maximize expected total welfare. The first order condition, under the assumption that $\hat{z} < z$, is given by:

$$\frac{W\{W(-z^2-2(\hat{z}-z))+(2(w+l)+L)(1-z)z\}}{2bz^2}=0$$

Solving this first order condition for $\hat{z}$ gives the optimal $\hat{z}$, provided $\hat{z} > z$:

$$\hat{z}=\frac{Wz+(2(w+l)+W+L)(1-z)z}{2W}$$

As above, using the change of variables $\mu_w = w + W/2$ and $\mu_l = l + L/2$ along with $W = \alpha_W(\mu_w + \mu_l)$ makes this expression easier to interpret. With these change of variables, one can write $\hat{z}$ as follows:

$$\hat{z}=\frac{1}{2}+\frac{(1-z)(2z-\alpha_W)}{2\alpha_W}$$

Since this solution is derived under the assumption that $\hat{z} < z$, it is valid if and only if $z > 1 - \frac{\alpha_W}{2}$. Right away, one can see that majority rule is not generally optimal. For $\hat{z} > z$, majority rule is only optimal if either there will be no losers from the law or $z = \frac{\alpha_W}{2}$. The following lemma summarizes the comparative statics for the optimal vote threshold in this case.

Lemma 2 If the fraction of winners is more than the vote share required for the law to pass, $\hat{z} < z$, then the optimal vote share required to pass the law is $\hat{z} = \frac{1}{2}+\frac{(1-z)(2z-\alpha_W)}{2\alpha_W}$. The optimal vote share required for passage is decreasing in the dispersion of the possible idiosyncratic gains (measured relative to the sum of the mean gain and loss from the law), α_W. The optimal vote share is increasing in the fraction of winners from the law if and only if $z < \frac{1}{2} + \frac{\alpha_W}{4}$.

Proof. The optimal $\hat{z}$ is derived above. $\frac{d}{d\alpha_W}(\frac{1}{2}+\frac{(1-z)(2z-\alpha_W)}{2\alpha_W}) = -\frac{(1-z)z}{(\alpha_W)^2} < 0$. $\frac{d}{dz}(\frac{1}{2}+\frac{(1-z)(2z-\alpha_W)}{2\alpha_W}) = \frac{2-4z+\alpha_W}{2\alpha_W}$, this is positive if and only if $z < \frac{1}{2}+\frac{\alpha_W}{4}$. Q.E.D.

Lemma two says that provided the law can pass without support from any losers (and with support from only some of the winners), increasing the dispersion in the idiosyncratic gains (experienced by winners) should make the law easier to pass. The reason for this is that the larger the dispersion of gains, the larger are the total the gains from the biggest winners, those who vote for the law. This increases the total welfare from the law for any given vote threshold. The reason the dispersion only matters in relation to the sum of the mean gain (for winners) and mean loss (for losers) is that the role dispersion in gains plays is to suggest that even though only some of the winners (and none of the losers) voted for the law, the law still could increase social welfare. For that to be the case, the gains from the big winners must outweigh the losses from the losers, measured by their mean loss, and the negative common shock which makes some winners lose from the law, which must be bigger in magnitude the larger the mean gain for winners. This means that, contrary to what one might expect, not only do larger mean losses for losers suggest the law should be harder to pass but larger mean gains for winners also suggest the law should be harder to pass. Of course, these comparative statics are only valid if α_W is not so small (greater than $2(1-z)$) that $z \leq \hat{z}$.

The other comparative static result from the lemma is that the optimal vote share is initially increasing in the fraction of winners (when $\hat{z} < z$), but is decreasing in the fraction of winners when z gets large enough. For small dispersion ($\alpha_W < 2/3$), the optimal vote share is decreasing in z for any $z > \hat{z}$, but for large dispersion, z can be small enough (and still satisfy $z > \hat{z}$) to make the optimal vote share increasing in z. This means that if the fraction of winners from the law is large, the law should be easier to pass the more winners there are, but if the fraction of winners is not quite so large (but still greater than $\hat{z}$), increasing the fraction of winners should make the law harder to pass.

Lastly, it is again important to note what does not matter in selecting the optimal vote share. When a law can pass without support of any losers, the dispersion in benefits among the losers does not affect the optimal vote share. Furthermore, notice that, just as in the case above, the overall expected benefit from the law plays no direct role, the winner's mean gain and the loser's mean loss do not matter independently, but only matter through their sum (*not* their difference) and this sum only matters in relation to the dispersion in the gains.

2.3 $\hat{z} \simeq z$

The first subsection found the optimal vote share under the assumption that this vote share exceeded the fraction of winners from the law. As a result, this optimal vote share was only valid if $z < \frac{\alpha_L}{2}$, otherwise the optimal vote share derived would not exceed the number of winners from the law. Similarly, the second subsection found the optimal vote share under the assumption that this vote share was less than the fraction of winners from law. This optimal vote share was only valid if

$z > 1 - \frac{\alpha_W}{2}$. Notice, however, that this leaves a region in which neither formula for the optimal vote share is valid if $1 - \frac{\alpha_W}{2} - \frac{\alpha_L}{2} \geq 0$ or $\alpha_W + \alpha_L \leq 2$. Since $W + L = (\alpha_W + \alpha_L)(\mu_W + \mu_L) = (\alpha_W + \alpha_L)(w + l) + (\alpha_W + \alpha_L)(W + L)/2$ and $w + l \geq 0$, this means that $\alpha_W + \alpha_L \leq 2$. Thus, there will be values of z for which the optimal vote share should either equal to z or just above it or just below it.

If $z = \hat{z}$, notice that with positive probability exactly $\hat{z}$ of the population will vote for the law whenever $w + l > 0$. That is because there will be a region, larger than a single point, for the value of the common shock for which all winners will vote for the law and all losers will vote against it. In this situation, I assume the law passes with probability one-half. Thus, expected social welfare with $z = \hat{z}$ is:

$$\int_l^{b/2} \left(\frac{1}{b}(z(w+W/2) - (1-z)(l+L/2) + x)\right)dx + (1/2)\int_{-w}^{l} \left(\frac{1}{b}(z(w+W/2) - (1-z)(l+L/2) + x)\right)dx$$

I then compare this to social welfare when $\hat{z} = z + \varepsilon$ (the law only passes if the common shock, x, is greater than l) and $\hat{z} = z - \varepsilon$ (the law only passes as long as the common shock, x, is greater than $-w$). Thus, the difference between these three thresholds is determined entirely by expected social welfare given that $x \in [-w, l]$, that is, by the sign of $\int_{-w}^{l}(\frac{1}{b}(z(w+W/2) - (1-z)(l+L/2) + x))dx$. This is positive if and only if $z > \frac{1}{4}(2 - \alpha_W + \alpha_L)$. That is, for $z \in [\frac{\alpha_L}{2}, 1 - \frac{\alpha_W}{2}]$, the optimal vote share is just greater than if $z < \frac{1}{4}(2 - \alpha_W + \alpha_L)$ and is just less than z if $z > \frac{1}{4}(2 - \alpha_W + \alpha_L)$.

3 Results

The analysis from the last section allows one to describe the optimal vote-share for all possible values of z. The first proposition describes the result.

Proposition 1 If $z < \frac{\alpha_L}{2}$, the optimal vote share is $\hat{z} = \frac{1}{2} + \frac{z(2z-2+\alpha_L)}{2\alpha_L}$. For $z \in [\frac{\alpha_L}{2}, \frac{1}{4}(2 - \alpha_W + \alpha_L))$, the optimal vote share is $\hat{z} = z + \varepsilon$. For $z = \frac{1}{4}(2 - \alpha_W + \alpha_L)$, the optimal vote share is $\hat{z} = z$. For $z \in (\frac{1}{4}(2 - \alpha_W + \alpha_L), 1 - \frac{\alpha_W}{2}]$, the optimal vote share is $\hat{z} = z - \varepsilon$. For $z > 1 - \frac{\alpha_W}{2}$, the optimal vote share is $\hat{z} = \frac{1}{2} + \frac{(1-z)(2z-\alpha_W)}{2\alpha_W}$.

Proof. Except for the $z = \frac{1}{4}(2 - \alpha_W + \alpha_L)$ case, this follows from the first two lemmas and the analysis just prior to this proposition. For $z = \frac{1}{4}(2 - \alpha_W + \alpha_L)$, $\int_{-w}^{l}(\frac{1}{b}(z(w+W/2) - (1-z)(l + L/2) + x))dx = 0$. Since $\varepsilon > 0$, setting $\hat{z} \neq z$ will result in some welfare loss either from passing the law when $x \in (-w - W\varepsilon/z, -w)$ or from not passing the law when $x \in (l, l + L\varepsilon/(1-z))$. Q.E.D.

When a law will have no winners or no losers, then the optimal vote threshold is exactly one half. If there will be a small number of winners, then the optimal vote threshold is strictly less than one half. That is, when fraction of the winners is quite small, the law not require a majority to pass. On the other hand, if the fraction of winners is very large, though less than one, the law should require a super-majority to pass. In general, there are only three levels of z for which majority rule is optimal, no winners, no losers, and one point in the middle. Unless there is substantial asymmetry between the gains of winners and the losses from losers (either α_W or α_L is greater than one), this point in the middle is at $z = 1/2$. That is, majority rule is often optimal if half the population is likely to be a winner from the law.

The following proposition collects the comparative statics results from the three cases to see how the optimal vote threshold depends on various features of the environment.

Proposition 2 (i) The optimal vote threshold is increasing in the dispersion of the possible idiosyncratic losses (measured relative to the sum of the mean gain and loss from the law), α_L, if $z < \frac{\alpha_L}{2}$. Otherwise, the optimal vote threshold is independent of α_L. (ii) The optimal threshold is decreasing in the dispersion of the possible idiosyncratic gains (measured relative to the sum of the mean gain and loss from the law), α_W, if $z > 1 - \frac{\alpha_W}{2}$. Otherwise, the optimal vote threshold is independent of α_W. (iii) The optimal vote threshold is decreasing in the fraction of winners, z, if and only if $z < Min\{\frac{1}{2} - \frac{\alpha_L}{4}, \frac{\alpha_L}{2}\}$ or $z > Max\{\frac{1}{2} + \frac{\alpha_W}{4}, 1 - \frac{\alpha_W}{2}\}$.

Proof. This follows directly from the lemmas and the first proposition. Q.E.D.

Parts (i) and (ii) of the proposition are essentially identical to the results from the first two lemmas. The only additional fact to note is that the region where the dispersion of losses (gains) matter occurs when the fraction of winners is small (large) and is greater the larger is this dispersion of losses (gains). Since $\alpha_W + \alpha_L \leq 2$ (and this is strict unless $w + l = 0$), unless $\alpha_W + \alpha_L = 2$, there is some region of z for which the dispersion of losses (or gains) does not matter, and, in fact, there is a region where neither matter.

Part (iii) of the proposition explains how the optimal vote threshold varies with the fraction of winners over the entire unit interval. For small z, the optimal vote threshold is decreasing in z (from one-half at $z = 0$). This continues until either z is large enough that the optimal vote share formula for $z < \hat{z}$ reaches a local minimum or until z gets arbitrarily close to $\hat{z}$. At this point, the optimal vote threshold begins to increase in z from a point below one-half to a point above one-half. When z gets so large that the optimal $\hat{z}$ is determined by the interior solution

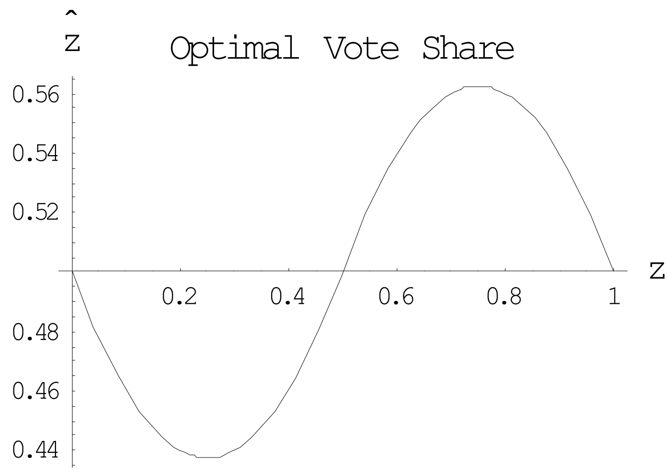

Figure 1: $\alpha_L = \alpha_W = 1$

under the assumption that $\hat{z} < z$ and this interior solution has reached a local maximum, then the optimal vote threshold begins to decrease in z once more. It continues to decrease until the optimal threshold once again reaches one-half at $z = 1$.

Taken together, the two propositions can shed light on the types of laws that should be easy to pass (less than majority support is optimal for most values of z) and what types of laws should be hard to pass (super-majorities should be required for most values of z). If the dispersion in losses is quite large (again, relative to the sum of the mean gain for winners and the mean loss for losers) and the dispersion in gains is small (also relative to the sum of the means), then the law should require a super-majority to pass unless there are likely to be very few winners from the law (in which case the optimal vote share is still not too much less than one-half). On the other hand, if the dispersion in losses is small and the dispersion in gains is large, the law should be able to pass with less than majority support unless the number of winners is likely to be quite large (in which case the optimal vote share will still not be much greater than one-half). The following pictures depict the optimal vote shares as a function of the fraction of winners for various values of the dispersion in gains and losses relative to the sum of the mean gains and losses.

In Figure 1, the dispersion in mean gains and losses are equal and sum to their maximum value (relative to the sum of the means). In this case, the optimal vote share is clustered closely around one-half and is anti-symmetric around $z = 1/2$. For large z the law should require a small super-majority to pass and for small z it should require slightly less than a majority. The picture is similar, though less tightly clustered around one-half if $\alpha_L = \alpha_W < 1$, as the next figure shows.

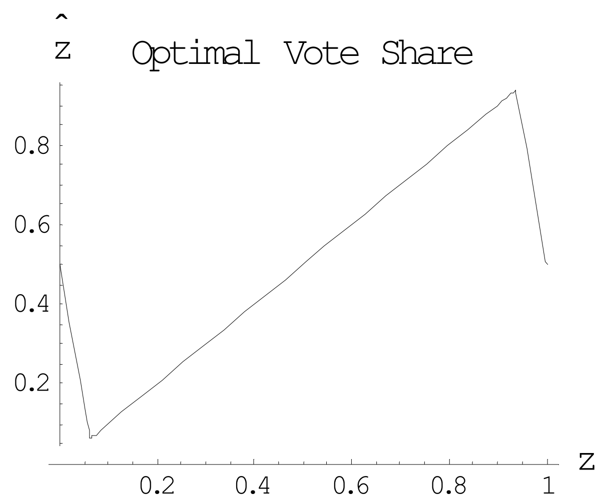

Figure 2: $\alpha_L = \alpha_W = 1/8$

Figure 2 shows that if the levels of dispersion are much smaller, though still equal for gains and losses, the optimal vote share is much more sensitive to the fraction of winners. The optimal vote share falls quickly from one-half at $z = 0$ until it reaches z, at which point it increases essentially one for one with z until it gets close to one, where it again drops quickly to back to one-half at $z = 1$. While the optimal vote share depends much more heavily on the fraction of winners, with equal dispersion for gains and losses, there is no reason to make the law easy or hard to pass if one has no reason to believe z is likely to be high or low. When the dispersion is very unequal, however, as the next picture shows, this is not necessarily the case.

In Figure 3, the dispersion of losses is very large compared to the dispersion of gains. As a result, the law should require a super-majority for all values of z except for those very close to zero. Furthermore, when the fraction of winners from the law gets fairly large, the optimal super-majority is quite large as well. The picture if the dispersion of gains is very large compared to the dispersion of losses is essentially exactly the opposite, as the next figure shows.

4 Extension: z Stochastic

In the main model of the paper, the fraction of winners from the law is known at the constitutional stage. This greatly simplifies the analysis and allows for more precise determinations of the optimal vote share. In many cases, however, the fraction of winners from a law may not be known at the time the constitution is being written. Thus, the optimal vote share must depend on the probability

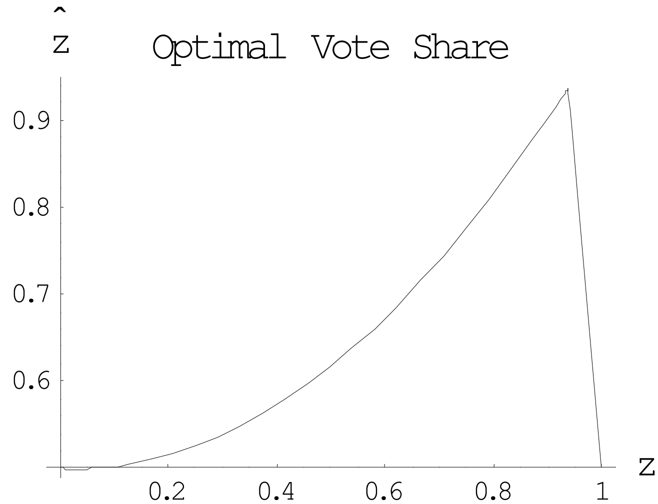

Figure 3: $\alpha_L = 15/8, \alpha_W = 1/8$

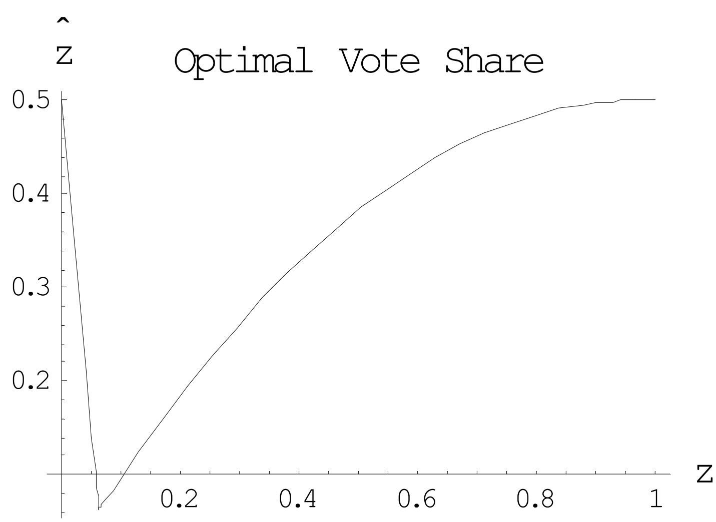

Figure 4: $\alpha_L = 1/8, \alpha_W = 15/8$

distribution for z rather than the actual value of z. Because the results above demonstrate the optimal vote share for all possible values of z, they are suggestive of what the optimal vote share may be given a particular probability distribution, but they are not definitive. For example, if $\alpha_L > \alpha_W$, the law should require a super-majority unless z is small (less than one-half). This suggests that unless one has reason to believe that z is likely to be small, some super-majority rule is optimal. It does not prove this, however, since it is always possible that the loss from having too large an threshold when z is small is substantially greater than the loss from having too small a threshold when z is large. Unfortunately, characterizing the optimal vote share for arbitrary distributions not tractable. It is possible to show, however, that if z is uniformly distributed between zero and one that the basic intuition from the last section continues to hold.

Proposition 3 If $z\tilde{}U(0,1)$, then the optimal vote share exceeds one-half if and only if $\alpha_L > \alpha_W$.

Proof. See Appendix

If, at the constitutional stage, z is equally likely to be anywhere between zero and one, then if the dispersion of losses to losers is larger than the dispersion of gains to winners, the law should be hard to pass (require a super-majority). On the other hand, if the dispersion of gains to winners exceeds the dispersion of losses to losers, the law should be easy to pass (require less than a majority). Once again, the mean gain for winners and mean loss for losers do not matter separately and their sum only matters in relation to the dispersion of gains and losses.

While this suggests that the results for fixed z are robust to stochastic z, it is important to note that Proposition 3 does not necessarily hold even for all distributions of z that are symmetric around one-half. For example, it is possible to show that if z can take only two values, z_0 and $1-z_0$ with equal probability, that for small enough z_0 and small α_L and α_W the optimal vote share exceeds one-half if and only if $\alpha_L < \alpha_W$. Thus, further study is needed to provide fully general results when z is stochastic.

5 Conclusion

This paper studies how a social planner should choose the vote share required to pass a law (or make any social decision) to maximize total welfare given that people will vote for a law if and only if doing so increases their own payoff. In particular, the paper demonstrates that the optimal voting rule for any social decision depends critically on certain characteristics of the decision, but there are some characteristics of a decision, such as the expected total benefit, which do not affect the optimal voting rule. This means that in many cases majority rule is not optimal (does not maximize expected total welfare of the population). Furthermore, the paper shows that the dispersion of the gains and losses from a law are a critical factor in determining how easy a law should be to pass. The larger are the dispersion of gains (losses), the easier (harder) the law should be to pass. The paper also demonstrates how the optimal vote share varies with the fraction of winners or losers from a law. Interestingly, this relationship is non-monotonic. That said, there are two special cases where majority rule is optimal: (1) the entire population draws its idiosyncratic benefit from a law from the same uniform distribution or (2) the expected number of winners is uniformly distributed between zero and one and the dispersion in gains equals the dispersion in losses.

The results in this paper can help one understand several important issues in constitutional design. First, the paper provides some explanation for what factors make a ”right” worthy of special constitutional protection (those factors that suggest a law violating that right should be hard to pass). Second, the paper can be useful in designing voting rules to be used in new constitutions or charters which describe future decision making processes for organizations. Most importantly, the results suggest that different issues will often call for very different voting rules. The characteristics of the decision at issue critically affect the optimal voting rule for that issue.

6 Appendix

Proof of Proposition 3. If $z\tilde{}U(0,1)$, then expected social welfare is given by:

$$\int_0^{\hat{z}}\int_{\frac{l(1-z)+L(\hat{z}-z)}{1-z}}^{b/2}(\frac{1}{b}(z(w+W/2)-(1-z)(l+L/2)+x))dxdz+\int_{\hat{z}}^{1}\int_{\frac{W(\hat{z}-z)-wz}{z}}^{b/2}(\frac{1}{b}(z(w+W/2)-(1-z)(l+L/2)+x))dxdz$$

Taking the derivative with respect to $\hat{z}$ and doing the standard change of variables gives the following:

$$\frac{(\mu_W+\mu_L)^2}{8b}\{4-(\alpha_L)^2-12\alpha_W-7(\alpha_W)^2+4\hat{z}(-2+3\alpha_L+2(\alpha_L)^2+3\alpha_W+2(\alpha_W)^2) \quad (1)$$
$$+4\alpha_L(2+\alpha_L)Log(1-\hat{z})-4\alpha_W(2+\alpha_W)Log(\hat{z})\}$$

Evaluating this at $\hat{z}=1/2$ yields:

$$\frac{(\mu_W+\mu_L)^2(\alpha_L-\alpha_W)(2+\alpha_L+\alpha_W)(3-Log(16))}{8b}$$

Since $Log(16)$ is approximately 2.77, this is positive if and only if $\alpha_L>\alpha_W$.

This proves that there is a local maximum for expected social welfare at $\hat{z}>1/2$ if and only if $\alpha_L>\alpha_W$ and a local maximum of expected social welfare at $\hat{z}<1/2$ if and only if $\alpha_L<\alpha_W$. It remains to show that the global maximum also occurs at at $\hat{z}>1/2$ (or at $\hat{z}<1/2$ in the second case). I will prove this for the $\alpha_L>\alpha_W$ case, the proof for the reverse case is analogous. If $\alpha_L>\alpha_W$,the argument above shows that social welfare is increasing at $\hat{z}=1/2$. There can be at $\hat{z}<1/2$ that generates greater expected social welfare if either social welfare is concave or increasing in $\hat{z}$ for $\hat{z}<1/2$. I will show that for $\hat{z}\in(1/3,1/2]$ the social welfare function is concave in $\hat{z}$ and that for $\hat{z}\leq 1/3$ it is increasing in $\hat{z}$.

First, taking the derivative of (1) with respect to $\hat{z}$ gives:

$$\frac{(\mu_W+\mu_L)^2}{2b\hat{z}(1-\hat{z})}\{-2\hat{z}(1-\hat{z})+\hat{z}(1-3\hat{z})\alpha_L+\hat{z}(1-2\hat{z})(\alpha_L)^2-(1-\hat{z})(2-3\hat{z})\alpha_W-(1-\hat{z})(1-2\hat{z})(\alpha_W)^2\} \quad (2)$$

This has the sign of the term in the curly braces. For $\hat{z}<1/2$, this is decreasing in α_W and increasing in α_L So, for this to be positive it must be positive at $\alpha_W=0$ and $\alpha_L=2$. Evaluating the curly braces term of (2) at $\alpha_W=0$ and $\alpha_L=2$ yields $4\hat{z}(1-3\hat{z})$, which is clearly negative for $\hat{z}>1/3$.

So, if the first derivative is positive at $\hat{z}=1/2$ and the second derivative is negative for $\hat{z}\in$

$(1/3, 1/2]$, then there is no $\hat{z} \in (1/3, 1/2]$ which generates greater expected social welfare than some point $\hat{z} > 1/2$. To show that there is also no point $\hat{z} \leq 1/3$ that generates more social welfare, I now show that expected social welfare is increasing in $\hat{z}$ for $\hat{z} \leq 1/3$. Taking the second derivative of the curly braces term of (1) with respect to α_W gives $2(-7 + 8\hat{z} - 4Log(\hat{z}))$, which is always positive for $\hat{z} \leq 1/3$. Taking the first derivative of the curly braces term of (1) with respect to α_W at $\alpha_W = 0$ gives $4(-3 + 3\hat{z} - 2Log(\hat{z}))$. This is always positive for $\hat{z} \leq 1/3$. If the first derivative of (1) with respect to α_W at $\alpha_W = 0$ is always positive for $\hat{z} \leq 1/3$, the the first derivative of (1) with respect to α_W must be positive for any α_W if $\hat{z} \leq 1/3$ since the second derivative is positive. So, if (1) is ever negative for $\hat{z} \leq 1/3$ it must be negative at $\alpha_W = 0$. Evaluating the curly braces term of (1) at $\alpha_W = 0$ gives:

$$(2 + \alpha_L)[2 - \alpha_L + 4\hat{z}(2\alpha_L - 1) + 4\alpha_L Log(1 - \hat{z})]$$

This has the sign of the square bracket term, which is linear in α_L. So, if this is ever negative, it must be negative at the extreme values of α_L, which are zero and two. At $\alpha_L = 0$, this is $2 - 4\hat{z}$, which is positive if $\hat{z} \leq 1/3$. At $\alpha_L = 2$, this is $4(3\hat{z} + 2Log(1 - \hat{z}))$, which is also positive if $\hat{z} \leq 1/3$. Q.E.D.

References

[1] Barry Nalebuff Andrew Caplin. On 64 percent majority rule. *Econometrica*, LVI:787–814, 1988.

[2] Gordon Tullock James M. Buchanan. *The Calculus of Consent: Logical Foundations of Constitutional Democracy*. University of Michigan Press, Ann Arbor, MI, 1962.

[3] Dennis C. Mueller. Constitutional rights. *Journal of Law, Economics, and Organization*, 7(2):313–333, Autumn 1991.

[4] Dennis C. Mueller. The importance of uncertainty in a two-stage theory of constitutions. *Public Choice*, 108:233–258, 2001.

[5] Eric Maskin Partha Dasgupta. Robustness of majority rule. Mimeo, 1998.

[6] Francesco Trebbi Philippe Aghion, Alberto Alesina. Endogenous political institutions. *Quarterly Journal of Economics*, CXIX:565–612, 2004.

[7] Jean-Jacques Rousseau. *Du Contrat Social; Ou, Principes Du Droit Politique.* Garnier (reprinted 1966), Paris, France, 1762.

[8] Matthew O. Jackson Salvador Barbera. Choosing how to choose: Self-stable majority rules and constitutions. *Quarterly Journal of Economics*, CXIX:1011–1048, 2004.