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Risk Taking and Gender in Hierarchies

Suzanne Scotchmer[1]

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Department of Economics, University of California, Berkeley
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Abstract: If promotion in a hierarchy is based on a random signal of ability, rates of promotion are affected by risk taking. Further, the statistical properties of the surviving populations of risk takers and non-risk takers will be different, and will be changing throughout the hierarchy. I define promotion hierarchies with and without memory, where memory means that promotion depends on the entire history of success. In both types of hierarchies, surviving risk takers have lower average ability than surviving non risk takers at any stage where they have a higher probability of survival. However, that will not apply in the limit. With a common set of promotion standards, risk takers will survive with lower probability than non risk takers, and will have higher average ability. I give several interpretations for how these theorems relate to affirmative action, in light of considerable evidence that males are more risk taking than females.

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1 Introduction

In this paper, I study the effect of risk taking on promotion in hierarchies, where promotion at each stage depends on a signal of ability. The motivation comes from a substantial body of evidence that males are more risk taking than females, and from the continuing controversy about why males and females have different patterns of success in labor markets. Granting the premise that the genders differ in risk taking, does this have explanatory power for labor markets? The answer is mixed, partly because the theorems below can be applied to labor markets in different ways.

The theorems proved below compare promotions drawn from two subpopulations, one of which generates accurate signals of ability and the other of which generates noisy signals of ability. The premise is that true abilities have the same distribution in both populations, at least initially, but that agents in one population give a noisy signal to the decision maker. This is a reduced-form hypothesis that might follow from preferences and optimizing behavior, or might reflect behavior that is hard-wired. This distinction does not matter for the theorems that I prove, although it may matter for the interpretation.

The objective of the paper is to understand how the statistical properties of surviving populations change in a hierarchy with a large (infinite) number of stages, under various assumptions about the promotion standards.

I introduce two types of promotion hierarchies: those with memory and those without memory. In a hierarchy without memory, promotion at stage t depends only on the signal of ability generated in stage t. With memory, promotion can depend on the entire history of signals. Hierarchies such as sports tournaments do not have memory, since survival depends only on winning the current match. Hierarchies such as academic labor markets have memory, although promotion would typically depend more heavily on current performance than on past performance. To maximize the difference between hierarchies with memory and those without, I assume for the case of memory that all past signals are used symmetrically. There is no extra weight given to recent performance.

In section 2, I discuss some of the evidence that males are more risk taking than females, and draw out some contradictions among the promotion objectives of (a) promoting according to gender-blind standards, (b) promoting equal numbers, and (c) promoting populations with equal average abilities. There is no promotion policy that equalizes both the numbers of survivors and their average abilities.

In sections 3 and 4, I develop formal results about hierarchies with and without memory. In both cases, if the objective is to equalize abilities, then more of the non risk takers must be promoted than risk takers. This is because a surfeit in the number of risk takers coincides with a deficit in their ability. This is true at any stage of the hierarchy, and regardless of how the standards are chosen.

The main contribution of this paper is to show that the statistical properties of the surviving populations can be reversed as the hierarchy progresses. For example, if the standards are gender blind and relatively stringent at the beginning, the surviving risk takers at early stages may be more numerous and less able than the surviving non risk takers. However, this is reversed at the end. The ratio of surviving risk takers to surviving non risk takers declines until there are fewer risk takers, but they have higher ability. To remedy these discrepancies – to equalize numbers or abilities – the risk takers (males) will need an affirmative action boost at the end, even though the non risk takers (non risk takers) may need an affirmative action boost at the beginning.

Many labor markets have some of the features described in this paper, such as markets for lawyers, academics, and corporate executives. However, none has an infinite number of promotions stages. Nevertheless, it is useful to study the infinite hierarchy because that is where we see the fundamental forces at work, leading to reversals.

2 Risk taking and Promotion in a Hierarchy

One motivation for this inquiry is the considerable evidence that males are more risk taking than females. For example, Eckel and Grossman (2002) show that males and females have different gambling behavior. In other experiments (2005b) they show not only that females are more risk averse, but that other agents (not just researchers on gender) perceive this to be true. In their recent review (2005a), they argue that the evidence is especially strong in “field studies” (natural experiments such as observing behavior in placing bets), but less conclusive in “contextual environmental” experiments such as experiments involving insurance choices. One of the most interesting risk taking contexts is investment. By observing investment portfolios, Jianakoplos and Bernasek (1998) found that males have much higher risk tolerance than females. (See also Bajtesmit and Bernasek, 1996.) There is also evidence from psychologists. For example, Ginsburg and Miller (1982) gathered data on children’s behavior at a zoo, where the children could choose to engage in risky activities or not. Young boys were much more inclined to put themselves at risk than young girls. Males and females even differ in their exam-taking behavior (Espinosa and Gardeazabal, 2005).

Accepting the hypothesis that males take more risks than females, scholars have suggested evolutionary arguments for why it should be so. For example, Dekel and Scotchmer (1999) postulated that males play “winner-take-all” games, and explored a precise sense in which such games do (or do not) lead to riskier behavior. The premise in that paper, which is also the easiest interpretation of the model below, is that risk taking is genetically coded.

This paper is concerned with the consequences of risk taking, and not with an explanation of it. The model below compares labor market outcomes of agents who give noisy signals in the labor market with those who give accurate signals. A central question, however, is whether a propensity to take risks leads to noisy signals in the labor market. Some evidence suggests that it does. The same behavior that would lead females to behave more conservatively on exams might lead them to take more conservative actions in the labor market. For example, in the academic sphere, a risk taker might work on new and untried topics, with the risk of not finding an audience, or even a publishable result, while a more conservative approach would be to extend the work of others. If agents have different tolerances for risk, both choices could be rational.

On the other hand, noisiness of labor market signals might be due to the *amount* of information generated more than to its quality. If, for sociological reasons, males are given more opportunity to perform, or are monitored more closely than females, then the signals they generate are less noisy because there is more information about them.

Finally, there can be unobservable confounding factors that overturn any intrinsic difference in risk taking. Becker and Eagly (2004) found that females were at least as likely as males to put themselves at risk in protecting Jews in the Holocaust, and females are considerably more likely to put themselves at risk by donating kidneys to relatives in need. However, the authors hypothesize that such behavior might be rooted in a greater willingness of females to care for others, or to heed an ethical calling. Females may be motivated by objectives that overcome, and therefore obscure, an aversion to risk.

With these reservations in mind, I will nevertheless adopt the hypothesis that risk taking leads to noisy labor market signals, and will often refer to risk takers and non risk takers as “male” and “female.” I return to this issue in section 5.

Before turning to the hierarchy, I use figure 1 and a single round of promotion to show how risk taking creates conflicts among three natural objectives of labor policy:

- equal promotion standards
- equal numbers of promotions
- promotion of a pool of agents with equal average or marginal ability

In figure 1, the distribution of true ability a is shown by density g. The distribution of true ability is assumed to be the same in both populations, a risk taking population (say, males) and a risk-averse population (say, females). The density $\tilde{g}$ represents the distribution of signals that the risk taking population will generate, when their true ability a is confounded by noise.

Consider the first round of promotions. Suppose that the promotion standard for males is $\bar{c}$. That is, every male who generates a signal above $\bar{c}$ is promoted. The other promotion standards are for females: The promotion standard f^e will ensure that females are promoted with the same probability as males, and the promotion standard f^a will ensure that the expected ability of promoted females is the same as that of promoted males.

If the promotion policy is gender blind, then females are also promoted according to the standard $\bar{c}$. In the example of figure 1, where more males than females are promoted (because $\bar{c}$ is above the mean), the promoted females have higher expected ability than promoted males. Further, the promotion policy is inefficient. Given the number of promotions, the total ability of promoted agents could be increased by substituting a non risk taker for a risk taker. The expected ability of the marginal non risk taker is $\bar{c}$, but is less than $\bar{c}$ for the risk taker. (The latter uses symmetry of the distributions and the fact that $\bar{c}$ is above the mean. See section 5.)

In an intuitive sense, it is because risk takers are overpromoted that their average ability must be lower. To promote more of them, it is necessary to reach further down into the ability distribution. In addition, some of those promotions are mistakes. This insight is formalized in Lemmas 1 and 6 below. At every stage of the hierarchy, surviving females have higher expected ability whenever the expected number of surviving males is at least as large, regardless of what proportion of the total pool is promoted.

Since the gender blind promotion policy is inefficient and also “inequitable” in the sense that more risk takers (males) are promoted than non risk takers, we might

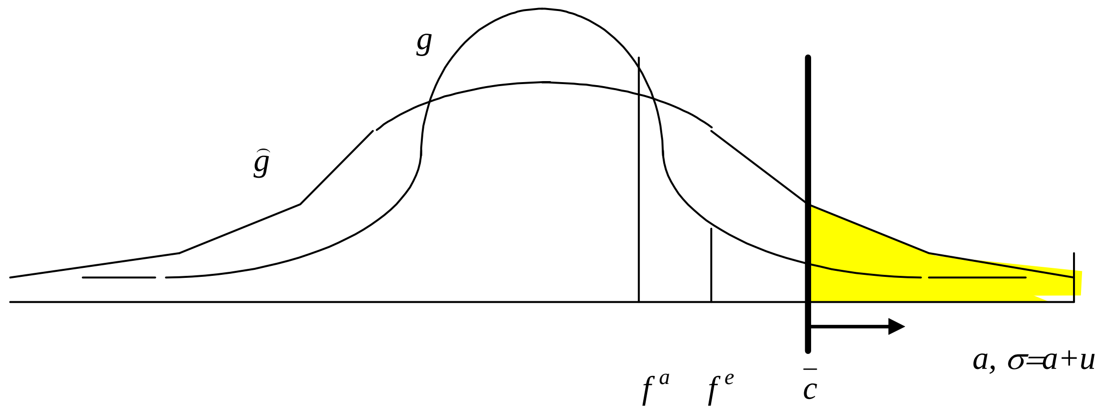

Figure 1: First Stage of a Hierarchy

consider other rules for promotion. Suppose, instead, that the objective is to promote equal numbers, as indicated by the promotion standard f^e for females in figure 1. Then

- the promoted risk takers (males) still have lower ability than the promoted non risk takers (females); and
- the promotion standard for non risk takers (females) is lower than for risk takers (males), provided fewer than half are promoted at stage 1, and otherwise higher.

Another critierion might be to promote agents with equal ability rather than equal numbers. This criterion could not be a legal rule, since ability is not observable. Qualitatively, though, one can see that the promotion standard for females would have to be even lower than the one that ensures equal numbers, such as f^a in figure 1. If pools of agents with equal average ability are promoted, then

- fewer males than females are promoted; and
- the standard for female promotion is even lower than the one that equalizes numbers.

Figure 1 illustrates why "affirmative action" in labor markets is such a vexed issue. To know whether affirmative action serves a social purpose, we must first identify the purpose. It is not possible to equalize numbers, equalize abilities, and also satisfy the procedural objective of having gender-blind standards. Moreover, what is show below is that, for any of these objectives, the nature of the asymmetry in the treatment of risk takers and non risk takers must invert at some point in the hierarchy.

3 The Hierarchy without Memory

An agent's ability A is a random variable with distribution G and density g, finite variance, and support equal to the real line. Males and females (risk takers and non risk takers) have the same distributions of abilities. A risk taking agent also generates a sequence of random errors $U_1, U_2, ...U_t, ...$, which are distributed independently according to a cumulative distribution function Φ with a bounded density function ϕ, mean zero, finite variance, and support equal to the real line.[2] The random draw A and the sequence of errors $U_1, U_2, ...U_t, ..$ generate a sequence of random signals $Z_1, Z_2, ...Z_t, ...$where $Z_t = A + U_t$ is also a random variable.

Promotion standards are a sequence of real numbers, $c = c_1, c_2, ...c_t...$ We will say that the sequence is *bounded* if there exists $\underline{c}, \bar{c}$ such that $\underline{c} < c_t < \bar{c}$ for each t.

We say that a risk taker *survives* to t if $Z_d \geq c_d, d = 1, ..., t$. Define an indicator function with values $\mathbf{1}_t^c(Z) \in \{0, 1\}$, such that the value is 1 if $Z_d \geq c_d, d = 1, ..., t$.. The expected value of the indicator function is the probability of survival to t. Conditioning on $A = a$, we denote the expected value of the indicator function by $\mathcal{S}_t^M(a, c)$, interpreted as the probability that a risk taker with ability a survives to t.

$$\mathcal{S}_t^M(a, c) := E_Z[\mathbf{1}_t^c(Z) : A = a] = \Pi_{d=1}^t (1 - \Phi(c_d - a))$$

[2]Full support is not required for all the results below, but I make the assumption here to avoid technical assumptions in the formal statements. Boundedness of the density function is convenient in the proof of Lemma 2(3), but not necessary.

The probability that a random risk taker survives to t is therefore

$$E_A\left[\mathcal{S}_t^M(A,c)\right] = \int_{-\infty}^{\infty} \mathcal{S}_t^M(a,c)\;\; g(a)\,da = \int_{-\infty}^{\infty} g(a)\;\Pi_{d=1}^t\left[1-\Phi(m_d-a)\right]da$$

and the expected ability of a random risk taking survivor at stage t is

$$E_A^M\left[A|c,t\right] := \frac{E_A\left[A\mathcal{S}_t^M(A,c)\right]}{E_A\left[\mathcal{S}_t^M(A,c)\right]} = \int_{-\infty}^{\infty} a\;\frac{g(a)\Pi_{d=1}^t\left[1-\Phi(m_d-a)\right]}{\int_{-\infty}^{\infty} g(a)\Pi_{d=1}^t\left[1-\Phi(m_d-a)\right]da}da \qquad (1)$$

For concreteness, I will often refer to risk takers as males and non risk takers as females (hence the superscripts M and F) even though I point out in section 5 that these interpretations can be reversed.

We say that a non risk taker (female) agent with ability a *survives* to stage t if $a \geq c_d$, $d = 1,2,...t$. We use $\mathcal{S}_t^F(a,f)$ directly as the indicator function:

$$\mathcal{S}_t^F(a,f) = \begin{array}{ll} 0 & \text{if } a < \max_{i=1.,,,t}\{c_i\} \\ 1 & \text{if } a \geq \max_{i=1.,,,t}\{c_i\} \end{array}$$

The probability that a random non risk taker (female) survives to stage t is $E_A\left[\mathcal{S}_t^F(A,f)\right] = 1 - G\left(\max_{i=1.,,,t}\{c_i\}\right)$, and the expected ability of a random female survivor is

$$E_A^F\left[A|c,t\right] := \frac{E_A\left[A\mathcal{S}_t^F(A,c)\right]}{E_A\left[\mathcal{S}_t^F(A,c)\right]} = \int_{\max_{i=1.,,,t}\{c_i\}}^{\infty} a\frac{g(a)}{1-G\left(\max_{i=1.,,,t}\{c_i\}\right)}da \qquad (2)$$

We say that the promotion standards are *gender-blind* if all agents face the same promotion standards. When we do not assume gender-blind promotion standards, we will refer to the males' (risk takers') promotion standards by $m = m_1, m_2, ..., m_t, ...$ and to the females' (non risk takers') promotion standards by $f = f_1, f_2, ..., f_t, ...$.For females, we can assume without loss of generality that the promotion standards are nondecreasing. If at any point a higher cutoff is followed by a lower cutoff, that is, $f_{t+1} < f_t$, then f_{t+1} can be replaced by f_t with no consequence. If f is nondecreasing, a female survives to stage t if $a \geq f_t$ and does not survive otherwise.

We begin with two lemmas. The intuition for the first lemma is that the promoted males include mistakes in both directions. Lower-ability males are promoted by mistake, and higher-ability males are excluded by mistake. Since no mistakes are made in promoting females, the only way to ensure that promoted males have as high ability as females is to promote fewer of them.

Lemma 1 *Let* m, f *be promotion standards for risk takers and non risk takers in a hierarchy without memory. Suppose that the probability of survival of a random risk taker is no smaller than the probability of survival of a random non risk taker at time* $t: E_A\left[\mathcal{S}_t^M(A,m)\right] \geq E_A\left[\mathcal{S}_t^F(A,f)\right]$. *Then the expected ability of a random surviving risk taker is smaller than the expected ability of a random surviving non risk taker:*

$$\frac{E_A\left[A\mathcal{S}_t^M(A,m)\right]}{E_A\left[\mathcal{S}_t^M(A,m)\right]} < \frac{E_A\left[A\mathcal{S}_t^F(A,f)\right]}{E_A\left[\mathcal{S}_t^F(A,f)\right]} \tag{3}$$

Proof: (a) For each a in the support of G, it holds that

$$\begin{aligned} 0 &< \mathcal{S}_t^M(a,m) < 1 \\ \mathcal{S}_t^F(a,f) &= \begin{array}{ll} 0 & \text{if } a < f_t \\ 1 & \text{if } a \geq f_t \end{array} \end{aligned}$$

The strict inequalities in the first line follow from the assumption that ϕ has full support on the real line. As a consequence, every male has a positive probability of being promoted at every stage, but promotion is never guaranteed. Therefore $\mathcal{S}_t^F(a,f) - \mathcal{S}_t^M(a,m) > 0$ for all a such that $(a - f_t) \geq 0$, and $\mathcal{S}_t^F(a,f) - \mathcal{S}_t^M(a,m) < 0$ for all a such that $(a - f_t) < 0$. It follows that

$$E_A\left[\left(\mathcal{S}_t^F(A,f) - \mathcal{S}_t^M(A,m)\right)(A - f_t)\right] > 0$$

This implies that

$$E_A\left[A\mathcal{S}_t^F(A,f)\right] - E_A\left[A\mathcal{S}_t^M(A,m)\right] > f_t \left[E_A\left[\mathcal{S}_t^F(A,m)\right] - E_A\left[\mathcal{S}_t^M(A,f)\right]\right] \tag{4}$$

Then $E_A\left[\mathcal{S}_t^M(A,m)\right] = E_A\left[\mathcal{S}_t^F(A,f)\right]$ only if $E_A\left[A\mathcal{S}_t^F(A,f)\right] > E_A\left[A\mathcal{S}_t^M(A,m)\right]$, which implies (3). If $E_A\left[\mathcal{S}_t^M(A,m)\right] > E_A\left[\mathcal{S}_t^F(A,f)\right]$, the same result holds, since the expected number of risk takers can only be increased by lowering their promotion standards and including agents with lower ability. □

In the next lemma, the first part reflects the fact that, regardless of the promotion standards, each male has positive probability of being eliminated at each stage. Since excluded agents cannot re-enter the pool, almost no males survive in the long run.

The second part reflects the fact that, regardless of the promotion standards, only the males with very high ability are likely to survive many opportunities to be eliminated. Thus, in the "long run", it does not matter very much what the promotion standards are, as long as there is a possibility to be eliminated at each stage. Males who survive will likely have very high ability. In contrast, a female survives with probability one if her ability is above the maximum promotion standard. This means that more females survive in the long run even without extraordinary ability.

The third part concerns the marginal risk takers who would be excluded by increasing the standard. The risk takers who would be eliminated would be those with signal $Z_t = A + U_t = c_t$. Define the expected ability of the marginal surviving risk taker at stage t as the expected ability of agents who generate signal c_t :

$$e_A^M [A|c,t] := \int_{-\infty}^{\infty} a \, \frac{g(a) \, \phi(c_t - a) \, \mathcal{S}_{t-1}^M (a,c)}{\int_{-\infty}^{\infty} g(a) \, \phi(c_t - a) \, \mathcal{S}_{t-1}^M (a,c) \, da} da \tag{5}$$

Lemma 2 *Let $m = m_1, m_2, ...m_t, ...$ be bounded promotion standards for a hierarchy without memory. Then*
(a) Given $\varepsilon > 0$, there exists $\tilde{t}$ such that for $t > \tilde{t}$, the probability that a risk taker survives to stage t is less than ε;
(b) Given $x > 0$, there exists $\tilde{t}$ such that for $t > \tilde{t}$, the expected ability of a random surviving risk taker is larger than x.
(c) Given $x > 0$, there exists $\hat{t}$ such that for $t > \hat{t}$, the expected ability (5) of the marginal surviving risk taker is larger than x.

The proof is in the appendix.

I now use these lemmas to characterize the populations that survive gender-blind promotion standards. Together with figure 1, the following proposition shows that there is a reversal in the relative numbers and relative abilities of the surviving populations of risk takers and non risk takers. At the beginning, under the reasonable conditions of figure 1, gender blind strategies lead to a higher survival rate among risk takers, but lower ability. At the end, this is reversed. According to Proposition 3, there is eventually a higher survival rate among non risk takers, but they have lower ability.

Proposition 3 (Gender Blind Promotions) *Let $c = c_1, c_2, \ldots$ be bounded, gender-blind promotion standards for a hierarchy without memory. Then there exists $\tilde{t}$ such that for $t > \tilde{t}$ the survival probability of a random risk taker is smaller than the survival probability of a random non risk taker, but the expected ability of surviving risk takers is larger than the expected ability of surviving non risk takers.*

Proof: The first part follows directly from Lemma 2 by choosing $\varepsilon > 0$ such that $(1 - G(c_t)) > \varepsilon$ for all t. The second part follows from Lemma 1. $\square$

We now turn to alternative policy goals. We first consider the goal of equalizing the probabilities of promotion at each stage, and then consider the goal of equalizing the average ability of the survivors at each stage.

It follows directly from Lemma 2(a) that if the promotion standards m, f are bounded, the survival rates of males and females in the limit are different. Proposition 4 says this in a different way: If survival rates are the same, the males' promotion standards cannot be bounded below. In particular, it is impossible to support equal promotions with the most natural hierarchy in which standards are increasing.

Proposition 4 (Promoting Equal Numbers) *Let m, f be promotion standards in a hierarchy without memory such that risk takers and non risk takers have the same probability of survival at each stage t. If the sequence f converges to a finite limit, then the sequence m is not bounded below.*

Proof: The sequence $f = f_1, f_2, ...$ is nondecreasing and converges. The sequence of female survival rates $E_A\left[\mathcal{S}_t^F(A, f)\right]$, $t = 1, 2, ...$, also converges, and, by hypothesis, the sequence of male survival rates $E_A\left[\mathcal{S}_t^M(A, m)\right]$, $t = 1, 2, ...$, converges to the same limit, say L. Choose an $\varepsilon > 0$ such that $\varepsilon < L$. Suppose, contrary to the proposition, that the sequence m is bounded below by $\underline{m}$. The male survival rate at stage t satisfies

$$E_A\left[\mathcal{S}_t^M(A, m)\right] = \int_{-\infty}^{\infty} g(a)\mathcal{S}_t^M(a, m)da \leq \int_{-\infty}^{\infty} g(a)(1 - \Phi(\underline{m} - a))^t da \tag{6}$$

Choose $\tilde{a}, \hat{a}$ such that $\hat{a} < \tilde{a}$ and

$$\begin{aligned} 1 - G(\tilde{a}) &< \varepsilon/3 \\ G(\hat{a}) &< \varepsilon/3 \end{aligned}$$

Choose $\hat{t}$ such that $(1 - \Phi(\underline{m} - \tilde{a}))^{\hat{t}} < \varepsilon/3$. Then if $t > \hat{t}$, the upper bound on the male survival rate at stage t, (6), can be written

$$\int_{-\infty}^{\hat{a}} g(a)(1 - \Phi(\underline{m} - a))^t da + \int_{\hat{a}}^{\tilde{a}} g(a)(1 - \Phi(\underline{m} - a))^t da + \int_{\tilde{a}}^{\infty} g(a)(1 - \Phi(\underline{m} - a))^t da$$

$$\begin{aligned} &< \int_{-\infty}^{\hat{a}} g(a)da + [G(\tilde{a}) - G(\hat{a})](1 - \Phi(\underline{m} - \tilde{a}))^t + \int_{\tilde{a}}^{\infty} g(a)da \\ &< \varepsilon/3 + (1 - \Phi(\underline{m} - \tilde{a}))^t + \varepsilon/3 < \varepsilon < L \end{aligned}$$

This is a contradiction. □

Proposition 5 (Promoting Equal Average Ability) *(a) Suppose that the expected abilities of surviving males and females are the same at stage $\hat{t}$ under the promotion standards m, f in a hierarchy without memory. Then the survival rate of females at stage $\hat{t}$ is greater than that of males. (b) In a hierarchy without memory, there are no bounded promotion standards m, f for which promoted males have the same average ability as promoted females at each t.*

Proof: Part (a) follows from Lemma 1, which would otherwise be contradicted. Part (b) follows from Lemma 2(b), which says that, for any bounded sequences, the average ability of surviving males is higher than the average ability of surviving females for late stages of the hierarchy (large t). □

4 The Hierarchy with Memory

Say that the hierarchy has *memory* if promotion depends on the performance in all periods up to the promotion date. In the hierarchy without memory, promotion at stage t depends only on having survived the last promotion, and on the performance afterwards, but not on the margin with which promotion to $t-1$ was achieved.

I will study the special case in which promotion depends symmetrically on the signals generated in the entire history to date, through their average. For risk takers, survival depends on a different set of random events than before. For the random sequence $Z_1, Z_2, ...Z_t, ..,$ define the sequence of sample means $\bar{Z}_1, \bar{Z}_2, ...\bar{Z}_t, ..,$ where $\bar{Z}_t = \frac{1}{t}\sum_{k=1}^{t} Z_k$ for each t. We again describe survival with an indicator function, where $\bar{\mathbf{1}}_t^c(Z) \in \{0,1\}$ takes value 1 if $\bar{Z}_d \geq c_d, d = 1, ..., t$. The expected value of the indicator function is the probability of survival to t. When $A = a$, we denote this expected value by $\tilde{\mathcal{S}}_t^M(a,c)$:

$$\tilde{\mathcal{S}}_t^M(a,c) := E_Z\left[\bar{\mathbf{1}}_t^c(Z) : A = a\right]$$

The survival function $\tilde{\mathcal{S}}_t^M(\cdot, c)$ is continuous and increasing, and can be written

$$\tilde{\mathcal{S}}_t^M(a,c) = \int_{c_1-a}^{\infty} \phi(u_1) \int_{2(c_2-a)-u_1}^{\infty} \phi(u_2) ... \int_{t(c_t-a)-\sum_{i=1}^{t-1} u_t}^{\infty} \phi(u_t)\, du_t ... du_2\, du_1$$

At each a, the probability of survival $\tilde{\mathcal{S}}_t(a,c)$ is decreasing with t, and bounded below by zero. Hence the sequence converges at each a. Let

$$\tilde{\mathcal{S}}^M(a,c) = \lim_{t\to\infty} \tilde{\mathcal{S}}_t^M(a,c) \text{ for each } a \in \mathbf{R}$$

The limiting expected ability of surviving risk takers is the following, provided the probability of survival in the limit (the denominator) is positive.

$$\tilde{E}_A^M[A|c] = \int_{-\infty}^{\infty} a \frac{\tilde{\mathcal{S}}^M(a,c)\, g(a)}{\int_{-\infty}^{\infty} \tilde{\mathcal{S}}^M(a,c)\, g(a)\, da} da \tag{7}$$

For hierarchies with memory, Lemma 6 is the analog of Lemma 1, and is proved analogously.

Lemma 6 *Let m, f be promotion standards for risk takers and non risk takers in a hierarchy with memory. Suppose that the probability of survival of a random risk taker is no smaller than the probability of survival of a random non risk taker at stage t : $E_A\left[\mathcal{S}_t^M(A,m)\right] \geq E_A\left[\mathcal{S}_t^F(A,f)\right]$. Then the expected ability of a random surviving risk taker is smaller than the expected ability of a random surviving non risk taker:*

$$\frac{E_A\left[A\mathcal{S}_t^M(A,m)\right]}{E_A\left[\mathcal{S}_t^M(A,m)\right]} < \frac{E_A\left[A\mathcal{S}_t^F(A,f)\right]}{E_A\left[\mathcal{S}_t^F(A,f)\right]}$$

For hierarchies without memory, we showed in Lemma 2 and Proposition 3 that most risk takers will eventually be eliminated, provided the standards are bounded below. Each risk taker has infinitely many opportunities to throw himself out of the pool, and if any risk takers survive, it is only those with exceptional ability. As long as the promotion standards for males and females are bounded, the expected ability of surviving risk takers is arbitrarily large for large enough t, and in particular is higher in the limit than for surviving non risk takers.

I now show that, with memory, risk takers survive in the limit with positive probability. Nevertheless, it is still true, as in hierarchies without memory, that surviving risk takers will be less numerous than surviving non risk takers, and will have higher average ability. This must be proved in a different manner than Proposition 3, since the analog to Lemma 2 does not hold. Further, Proposition 8 only holds for promotion standards that are nondecreasing. With decreasing promotion standards, all non-risk takers with ability above c_1 would survive, and none would be eliminated after stage one. This is not true of risk takers. If the performance standards decrease rapidly, risk takers with abilities lower than c_1 might survive in large numbers, and the limiting expected ability of risk takers could be lower than that of non risk takers.

In any case, increasing promotion standards are the more natural case.

Proposition 8 follows from the shape of the limiting survival function $\tilde{\mathcal{S}}^M(\cdot, c)$, described in Lemma 7 and shown in figure 2.

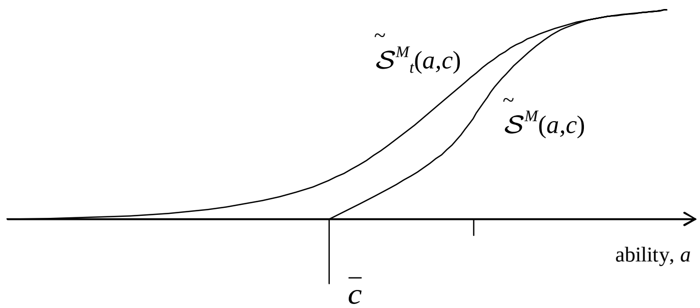

Figure 2: Survival of Risk Takers in a Hierarchy with Memory

Lemma 7 *Suppose that the promotion standards $c_1, c_2 ... c_t ..$ are nondecreasing and converge to $\bar{c}$. For each $a \leq \bar{c}$, $\tilde{\mathcal{S}}^M(a, c) = 0$. For each $a > c$, $\tilde{\mathcal{S}}^M(a, c) > 0$. The limit function $\tilde{\mathcal{S}}^M$ is nondecreasing.*

The proof is in the appendix.

Proposition 8 (Gender Blind Promotions with Memory) *Let $c = c_1, c_2., , , c_t, ...$ be a nondecreasing sequence of gender blind promotion standards that converge to $\bar{c}$ in a hierarchy with memory. Then there exists $\hat{t}$ such that for $t > \hat{t}$, the survival probability of a random risk taker is smaller than the survival probability of a random non risk taker, but the surviving risk takers have higher expected ability.*

Proof: First, fewer risk takers than non risk takers survive in the limit. For $a < \bar{c}$, neither risk takers nor non risk takers survive. For $a > \bar{c}$, the probability that a non risk taker survives is one, while, for risk takers, the survival probability is less than one: $\mathcal{S}(a, c) < 1 - \Phi(c_1 - a) < 1$.

Because the risk takers' limit probabilities of survival are nondecreasing with a, the limit distribution of their abilities first-order dominates the limit distribution of non risk takers' abilities. Thus, the expected ability of surviving risk takers is no smaller than that of surviving non risk takers. But since $\tilde{\mathcal{S}}(a, c) > \tilde{\mathcal{S}}(\bar{c}, c)$ for some $a > c$, the limiting expected ability of surviving risk takers is strictly greater than that of surviving non risk takers. $\square$

For hierarchies with memory, there is no analog to Proposition 4, but the following is the analog to Proposition 5.

Proposition 9 (Promoting Equal Average Ability with Memory) *Suppose that the expected abilities of surviving males and females are the same at stage $\hat{t}$ under the promotion standards m, f in a hierarchy with memory. Then the survival rate of females at stage $\hat{t}$ is greater than that of males.*

Proof: By Lemma 6, if the expected number of surviving risk takers at stage t is greater than or equal to the surviving non risk takers, then the average ability of surviving risk takers is lower than that of surviving females. The proposition follows from an equivalent statement: If the average ability of surviving males is as great or greater than the average ability of surviving females, then the expected number of surviving males is lower. $\square$

For completeness, the following proposition gives some insight into how the promotion standards must differ with and without memory, in order to equalize the number of survivors.

Proposition 10 *Let $\hat{c}^a$ and c be promotion standards in hierarchies with and without memory, respectively, which yield the same probabilities of survival at each t for a risk taking agent with ability a. Then it holds that $\hat{c}^a_1 = c_1$ and $\hat{c}^a_t > (1/t)\sum_{d=1}^{t} c_d$ for each* $t > 1$.

Proof: A risk taking agent with ability a will have a sequence of random errors in his signal, $\{U_t\}$. With and without memory, respectively, the agent survives the first stage in the events

$$\{U_1 \geq c_1 - a\}, \ \ \{U_1 \geq \hat{c}_1^a - a\}$$

so $c_1 = \hat{c}_1^a$. Without memory, the agent survives two stages in the event

$$\{U_1 \geq c_1 - a \text{ and } U_2 \geq c_2 - a\} \tag{8}$$

With memory the agent survives two stages in the event

$$\{U_1 \geq c_1 - a \text{ and } U_2 \geq c_2 - a + (c_1 - a - U_1)\} \tag{9}$$

Since $0 \geq (c_1 - a - U_1)$, the event (8) implies the event (9), but not vice versa. Thus, the probability of the event (8) is lower than the probability of the event (9). There exists $\tilde{c}_2 > c_2$ such that the probabilities of survival are equalized at the first two stages, when $\hat{c}_1^a = c_1$ and $\hat{c}_2^a = (1/2)(c_1 + \tilde{c}_2) > (1/2)(c_1 + c_2)$:

$$\Pr\{U_1 \geq c_1 - a \text{ and } U_2 \geq c_2 - a\} = \Pr\{U_1 \geq c_1 - a \text{ and } U_2 \geq \tilde{c}_2 - a + (c_1 - a - U_1)\}$$

Similarly, at stage $t = 3$,

$$\begin{aligned} & \Pr\{U_1 \geq c_1 - a \text{ and } U_2 \geq c_2 - a \text{ and } U_3 \geq c_3 - a\} \\ < \ & \Pr\left\{ \begin{array}{c} U_1 \geq c_1 - a \text{ and } U_2 \geq \tilde{c}_2 - a + (c_1 - a - U_1) \\ \text{and } U_3 \geq c_3 - a + (c_1 + \tilde{c}_2 - 2a - U_2 - U_1) \end{array} \right\} \end{aligned}$$

since $0 \geq (c_1 + \tilde{c}_2 - 2a - U_2 - U_1)$. Thus, there exists $\tilde{c}_3 > c_3$ such that

$$\begin{aligned} & \Pr\{U_1 \geq c_1 - a \text{ and } U_2 \geq c_2 - a \text{ and } U_3 \geq c_3 - a\} \\ = \ & \Pr\left\{ \begin{array}{c} U_1 \geq c_1 - a \text{ and } U_2 \geq \tilde{c}_2 - a + (c_1 - a - U_1) \\ \text{and } U_3 \geq \tilde{c}_3 - a + (c_1 + \tilde{c}_2 - 2a - U_2 - U_1) \end{array} \right\} \end{aligned}$$

Thus, there exists a sequence $\tilde{c}_1, \tilde{c}_2, ...$ such that $\tilde{c}_1 = c_1$, $\tilde{c}_t > c_t$ for $t > 1$, and for each t,

$$\Pr\{U_d \geq (c_d - a) \text{ for all } d \leq t\} = \Pr\left\{\sum_{i=1}^{d} U_i \geq \sum_{i=1}^{d} (\tilde{c}_i - a) \text{ for all } d \leq t\right\}$$

Thus, the promotion standards with memory $\hat{c}^a$ defined by $\hat{c}_1^a = c_1$, $\hat{c}_t^a = \frac{1}{t}\sum_{i=1}^{t} \tilde{c}_i$ for $t > 1$, yield the same probabilities of survival at each t as the promotion standards without memory c, and $\hat{c}_t^a > (1/d)\sum_{d=1}^{t} c_d$ for all $t > 1$. $\square$

Proposition 10 does not assert that the promotion standards $\hat{c}^a$ are the same for agents with different abilities. However it does imply that to maintain the same overall promotion rate with and without memory, the standards must satisfy $\hat{c}_t > (1/d)\sum_{d=1}^{t} c_d$ for each $t > 1$, since otherwise the promotion rate would be higher at some t for *every* a.

5 Interpretations

So far, the strategy of this paper has been to make primitive hypotheses about promotion standards, and then to study the consequences when some agents generate noisy signals of ability and others do not. The reduced-form nature of the inquiry is intentional. The labor-market effects that follow from the hypotheses on promotion standards do not require an explanation for why one population generates noisier signals than another, but only that such differences exist. Nevertheless, in this section I say more about why labor market signals might be noisy, and whether the hypotheses on promotion standards might be justified from primitive objectives.

In section 2, I cited evidence that males are more risk-taking than females. However, that does not necessarily imply that the signals relevant to promotion are noisier. In fact, as I already hinted, the hypothesis about which gender gives noiser signals can be inverted.

There are formal and informal ways of accumulating evidence. Formal evidence such as testing may simulate the laboratory environments where risk-taking emerges as noise in the signal. But informal signals, such as accrue through casual interactions, will be less precise for females than for males if females have less opportunity to perform. Males may be more closely observed – their papers may be read instead of shelved, or they may be approached more often for conversation, help or collaboration. Under this interpretation, a shift from formal to informal standards in a gender-blind hierarchy will increase the promotion rate of females at early stages, but decrease it at later stages when more evidence has accumulated despite these sociological effects.

I have investigated two plausible constraints on promotions: that the standards must be gender blind, or that promotion rates must be equal. These are of interest because they reflect what is observable to a court.

However, we can alternatively ask whether the constraints we have studied follow from a more primitive objective. The obvious objective is efficiency, although efficiency is hard to define in a partial model of a labor market such as this. See Holzer and Neumark (2000) for an overview of this multifaceted subject, and Lundberg and Startz (1983), Lundberg (1991), Milgrom and Oster (1987) for some specific efficiency effects. I will consider the particularly simple objective of trying to promote the most able agents.

Say that promotions standards m, f are *efficient at stage t* if m_t, f_t solve the following problem:

$$\text{maximize} \quad \int_{-\infty}^{\infty} a\mathcal{S}_t(a,m)\, g(a)\, da + \int_{f_t}^{\infty} a g(a)\, da$$

$$\text{subject to} \quad \int_{-\infty}^{\infty} \mathcal{S}_t(a,m)\, g(a)\, da + \int_{f_t}^{\infty} g(a)\, da \leq N$$

The optimum entails that the marginally promoted risk taker has expected ability equal to the marginally promoted non risk taker: $e_A^M[A|m,t] = f_t$. A court could not enforce this rule, since it cannot observe ability, but we can still say something about the efficiency of gender-blind promotion standards or standards that lead to equal promotion rates. It is convenient for this purpose to assume that the distributions G and Φ are symmetric, as in figure 1, and also that G is single peaked.

The following remark implies that gender blind standards are not efficient. At the beginning of the hierarchy, efficiency could be improved by trading some risk takers for more non risk takers. The ability of the marginal non risk takers who would thus be included is c_1, which is larger than the ability of the marginal risk takers who would be excluded. The trade must be reversed at later stages. At large t, efficiency can be improved by trading some non risk takers for more risk takers. It is only in the hierarchy with memory that the marginal risk takers and marginal non risk takers have the same expected ability in the limit, even though the average ability of risk takers is higher than that of non risk takers.

Proposition 11 *Suppose that the distributions G and Φ are symmetric and centered at zero, that the density g is single peaked as well as symmetric, that the standards c are gender blind, and that $c_1 > 0$. Then*
(a) The expected ability of the marginal risk taker at stage 1 is smaller than c_1.
(b) When the hierarchy does not have memory, there exists $\tilde{t}$ such that for $t > \tilde{t}$, the expected ability of the marginal risk taker is higher than c_t.
(c) When the hierarchy has memory, the expected ability of the marginal risk takers in the limit distribution of survivors is the same as the expected ability of the marginal non risk takers.

Proof: (a) The expected ability of the marginal risk taker is

$$\begin{aligned} e_A\left[A|c,1\right] \quad : \quad &= \int_{-\infty}^{\infty} a \, \frac{g(a)\ \phi\left(c_1 - a\right)}{\int_{-\infty}^{\infty} g(a)\ \phi\left(c_1 - a\right) da} da = \int_{-\infty}^{\infty} \left(c_1 - x\right) \frac{g(c_1 - x)\ \phi\left(x\right)}{\int_{-\infty}^{\infty} g(a)\ \phi\left(c_1 - a\right) da} dx \\ &= c_1 - \int_{-\infty}^{\infty} x \frac{g(c_1 - x)\ \phi\left(x\right)}{\int_{-\infty}^{\infty} g(a)\ \phi\left(c_1 - a\right) da} dx < c_1 \end{aligned} \tag{10}$$

The inequality follows because the integral in (10) is positive. The denominator is positive, and the numerator can be written

$$\begin{aligned} &\int_0^{\infty} x g(c_1 - x)\ \phi\left(x\right) dx + \int_{-\infty}^{0} x g(c_1 - x)\ \phi\left(x\right) dx \\ = &\int_0^{\infty} y g(c_1 - y)\ \phi\left(y\right) dy - \int_0^{\infty} y g(c_1 + y)\ \phi\left(-y\right) dy \\ = &\int_0^{\infty} y\ \left[g(c_1 - y) - g\left(c_1 + y\right)\right]\ \phi\left(y\right) dy > 0 \end{aligned}$$

In the second line, $\phi(y) = \phi(-y)$ due to symmetry of ϕ and in the last line, $[g(c_1 - y) - g(c_1 + y)] > 0$ due to $c_1 > 0$, symmetry of g, and single-peakedness of g.

(b) is proved in Lemma 2(c).

(c) follows from figure 2, which shows that, in the limit, the marginal males have the same ability as the marginal females, even though there are negligibly few of them. □

6 Appendix

Proof of Lemma 2: Let $\underline{m} \leq m_t \leq \bar{m}$ for all $t = 1, 2, ...$ Since the distributions G and Φ have full support, the probability that any risk taker survives at any date t, conditional on having survived to $t-1$, is strictly less than one. That is, $1 - \Phi(m_t - a) < 1$ for every t and every $a \in \mathbf{R}$.

(a) Let $\varepsilon > 0$. Let $\tilde{a} > 0$ satisfy $0 < 1 - G(\tilde{a}) < \varepsilon/2$ and let $\tilde{t}$ satisfy $(1 - \Phi(\underline{m} - a))^{\tilde{t}} < \varepsilon/2$ for all $a \leq \tilde{a}$. Then for $t \geq \tilde{t}$,

$$\begin{aligned} & \int_{-\infty}^{\infty} g(a)\mathcal{S}_t^M(a, m)\, da \\ = \; & \int_{-\infty}^{\tilde{a}} g(a)\Pi_{d=1}^{t}(1 - \Phi(m_d - a))da + \int_{\tilde{a}}^{\infty} g(a)\Pi_{d=1}^{t}(1 - \Phi(m_d - a))da \\ \leq \; & \int_{-\infty}^{\tilde{a}} g(a)(1 - \Phi(\underline{m} - a))^t da + \int_{\tilde{a}}^{\infty} g(a)\Pi_{d=1}^{t}(1 - \Phi(m_d - a))da \\ < \; & G(\tilde{a})\varepsilon/2 + (1 - G(\tilde{a}) < \varepsilon \end{aligned}$$

(b) For given t, write the expected ability of surviving risk takers, $E_A^M[A|c, t]$, in two parts, restricting attention to those agents who have survived to t. The first term in (11) is the expected ability of survivors who satisfy $|a| > 4x$, times the probability of that event, and the second part is the expected ability of survivors who satisfy $|a| \leq 4x$, times the probability of that event.

$$E_A^M[A|c, t, |A| > 4x] \; \times \Pr[|A| > 4x] \; + \; E_A^M[A|c, t, |A| \leq 4x] \; \times \; \Pr[|A| \leq 4x] \quad (11)$$

or equivalently,

$$E_A^M[A|c,t] = \left[\frac{\int_{4x}^{\infty} ag(a)\,\mathcal{S}_t^M(a,m)\,da + \int_{-\infty}^{-4x} ag(a)\,\mathcal{S}_t^M(a,m)\,da}{\int_{-\infty}^{\infty} g(a)\,\mathcal{S}_t^M(a,m)\,da}\right] + \frac{\int_{-4x}^{4x} ag(a)\,\mathcal{S}_t^M(a,m)\,da}{\int_{-\infty}^{\infty} g(a)\,\mathcal{S}_t^M(a,m)\,da}$$

$$\begin{aligned}
&= \left[\frac{\int_{4x}^{\infty} ag(a)\,\mathcal{S}_t^M(a,m)\,da + \int_{-\infty}^{-4x} ag(a)\,\mathcal{S}_t^M(a,m)\,da}{\int_{4x}^{\infty} g(a)\,\mathcal{S}_t^M(a,m)\,da + \int_{-\infty}^{-4x} g(a)\,\mathcal{S}_t^M(a,m)\,da}\right] \\
&\times \left[\frac{\int_{4x}^{\infty} g(a)\,\mathcal{S}_t^M(a,m)\,da + \int_{-\infty}^{-4x} g(a)\,\mathcal{S}_t^M(a,m)\,da}{\int_{-\infty}^{\infty} g(a)\,\mathcal{S}_t^M(a,m)\,da}\right] \\
&+ \left[\frac{\int_{-4x}^{4x} ag(a)\,\mathcal{S}_t^M(a,m)\,da}{\int_{-4x}^{4x} g(a)\,\mathcal{S}_t^M(a,m)\,da}\right] \times \left[\frac{\int_{-4x}^{4x} g(a)\,\mathcal{S}_t^M(a,m)\,da}{\int_{-\infty}^{\infty} g(a)\,\mathcal{S}_t^M(a,m)\,da}\right]
\end{aligned}$$

Since $\Pr[|a| > 4x] + \Pr[|a \leq 4x|] = 1$, and $E_A^M[A|c,t,|A| \leq 4x] > -4x$, it will be enough to show that as t becomes large, $E_A^M[A|c,t,|A| > 4x]$ becomes large (larger than $2x$) and $\Pr[|a| \leq 4x]$ becomes small (smaller than $1/6$). Then $E_A^M[A|c,t] > (2x)(5/6) - (4x)(1/6) = x$.

First show that $E_A^M[A|c,t,|A| > 4x] > 2x$ for large t.

$$\begin{aligned}
&E_A^M[A|c,t,|A| > 4x] \\
&= \frac{\int_{4x}^{\infty} ag(a)\,\mathcal{S}_t^M(a,m)\,da + \int_{-\infty}^{-4x} ag(a)\,\mathcal{S}_t^M(a,m)\,da}{\int_{4x}^{\infty} g(a)\,\mathcal{S}_t^M(a,m)\,da + \int_{-\infty}^{-4x} g(a)\,\mathcal{S}_t^M(a,m)\,da} \\
&> \frac{4x\int_{4x}^{\infty} g(a)\,\mathcal{S}_t^M(a,m)\,da + \int_{-\infty}^{-4x} ag(a)\,\mathcal{S}_t^M(a,m)\,da}{\int_{4x}^{\infty} g(a)\,\mathcal{S}_t^M(a,m)\,da + \mathcal{S}_t^M(-4x,m)\,G(-4x)} \\
&= \left[4x + \frac{\int_{-\infty}^{-4x} ag(a)\,\frac{\mathcal{S}_t^M(a,m)}{\mathcal{S}_t^M(-4x,m)}da}{\int_{4x}^{\infty} g(a)\,\frac{\mathcal{S}_t^M(a,m)}{\mathcal{S}_t^M(-4x,m)}da}\right] \Big/ \left[1 + \frac{G(-4x)}{\int_{4x}^{\infty} g(a)\,\frac{\mathcal{S}_t^M(a,m)}{\mathcal{S}_t^M(-4x,m)}da}\right] \qquad (12)
\end{aligned}$$

Using the boundedness of m, and the fact that Φ is strictly increasing on the entire real line, it holds as t gets large that

$$\frac{\mathcal{S}_t^M(a,m)}{\mathcal{S}_t^M(-4x,m)} = \prod_{d=1}^{t} \frac{[1-\Phi(m_d - a)]}{[1-\Phi(m_d + 4x)]} \to 0 \text{ for } a < -4x \qquad (13)$$

$$\frac{\mathcal{S}_t^M(a,m)}{\mathcal{S}_t^M(-4x,m)} = \prod_{d=1}^{t} \frac{[1-\Phi(m_d - a)]}{[1-\Phi(m_d + 4x)]} \to \infty \text{ for } a > 4x \qquad (14)$$

The value of (12) is less than $4x$, since the numerator adds a negative term to $4x$, and the denominator of (12) is greater than one. However, using the fact that $\int_{-\infty}^{-4x} ag(a)\,da$ is finite[3] and using (13),(14), it follows that the negative term in the numerator vanishes for large enough t. Using (14), the second term of the denominator of (12) vanishes as t becomes large, and we can therefore assert that $E_A^M[A|c,t,|A|>4x] > 2x$ for large enough t.

We now show that $\Pr[|a| \leq 4x] < 1/6$ for large enough t.

$$\begin{aligned}
\Pr[|a| \leq 4x] &= \frac{\int_{-4x}^{4x} g(a)\,\mathcal{S}_t^M(a,m)\,da}{\int_{-\infty}^{\infty} g(a)\,\mathcal{S}_t^M(a,m)\,da} < \frac{\int_{-4x}^{4x} g(a)\,\mathcal{S}_t^M(4x,m)\,da}{\int_{-\infty}^{\infty} g(a)\,\mathcal{S}_t^M(a,m)\,da} \\
&= \frac{\int_{-4x}^{4x} g(a)\,da}{\int_{-\infty}^{\infty} g(a)\,\frac{\mathcal{S}_t^M(a,m)}{\mathcal{S}_t^M(4x,m)}da} = \frac{\int_{-4x}^{4x} g(a)\,da}{\int_{-\infty}^{4x} g(a)\,\frac{\mathcal{S}_t^M(a,m)}{\mathcal{S}_t^M(4x,m)}da + \int_{4x}^{\infty} g(a)\,\frac{\mathcal{S}_t^M(a,m)}{\mathcal{S}_t^M(4x,m)}da}
\end{aligned}$$

The result follows because $\Pr[|A| > 4x] > 0$ and the following holds.

$$\frac{\mathcal{S}_t^M(a,m)}{\mathcal{S}_t^M(4x,m)} = \prod_{d=1}^{t} \frac{[1-\Phi(m_d-a)]}{[1-\Phi(m_d-4x)]} \to \infty \text{ for } a > 4x$$

(c) We omit the proof, which is essentially the same as for part (b), replace $E_A^M[A|c,t]$ with $e_A^M[A|c,t]$, and using the fact that ϕ is bounded. $\square$

Proof of Lemma 7: For the sequence of independent random variables $U_1, U_2, ...U_t, ..$, define the sequence of sample means $\bar{U}_1, \bar{U}_2, ...\bar{U}_t, ..$, where $\bar{U}_t = (1/t)\sum_{k=1}^{t} U_k$ for each t. That $\tilde{\mathcal{S}}_t^M(a,c) \to 0$ for $a < \bar{c}$ follows because $\bar{U}_t + a$ converges in probability to $a < c$. That $\tilde{\mathcal{S}}_t^M(\bar{c},c) \to 0$ follows because the limit distribution of $\sqrt{t}\bar{U}_t/v$ is normal, centered at 0, where v^2 is the variance of Φ. If $a = \bar{c}$, then for large t, $c_t - a = c_t - \bar{c}$ is close to 0. With positive probability it holds that $\sqrt{t}\bar{U}_t/v < c_t - \bar{c} < 0$. But since survival at t requires that $\bar{U}_t \geq 0$, this implies that the agent survives at each t with probability strictly less than one, so that the joint probability of survival at $t = 1, 2, ...$ is zero.

To show that $\tilde{\mathcal{S}}^M(a,c) > 0$ for $a > \bar{c}$, we argue instead that $\tilde{\mathcal{S}}^M(a,\{\bar{c},\bar{c},...\}) > 0$, since $\tilde{\mathcal{S}}^M(a,c) \geq \tilde{\mathcal{S}}^M(a,\{\bar{c},\bar{c},...\})$. An agent with random ability $A = a$ fails to survive if $\bar{U}_t < \bar{c} - a$ for some t. Using Lemma (6) of Dubins and Freedman (1965, p.

[3] Notice that $\int_{-\infty}^{\infty} a^2 g(a)\,da$ is finite because variance is finite.

801), if $b_1, b_2 > 0$,

$$\Pr\left[\bar{U}_t \leq -b_1 v^2 - \frac{b_2}{t} \text{ for some } t = 1, 2, ...\right] \leq \frac{1}{1 + b_1 b_2}$$

Thus,

$$\Pr\left[\bar{U}_t > -b_1 v^2 - \frac{b_2}{t} \text{ for all } t = 1, 2, ...\right] \geq 1 - \frac{1}{1 + b_1 b_2} > 0$$

Choose $b_1, b_2 > 0$ so that $-b_1 v^2 - b_2 = \bar{c} - a$. Then

$$\tilde{\mathcal{S}}^M (a, c) > \tilde{\mathcal{S}}^M (a, \{\bar{c}, \bar{c}, ...\}) = \Pr\left[\bar{U}_t \geq -b_1 v^2 - b_2 = \bar{c} - a \text{ for all } t = 1, 2, ...\right] >$$
$$\Pr\left[\bar{U}_t \geq -b_1 v^2 - b_2/t \text{ for all } t = 1, 2, ...\right] \geq 1 - \frac{1}{1 + b_1 b_2} > 0$$

$\square$

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