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A Low-Pass Filtered Panel Model For Estimating Production Function Parameters: The Substitution Elasticity And Growth Theory

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Preliminary -- Comments Welcome

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Abstract

This paper presents a new estimation strategy that combines low-pass filters with standard panel data techniques to obtain estimates of production function parameters. We are particularly concerned with the value of the elasticity of substitution between labor and capital because of its key role in models of long-run growth. A host of important issues, including the possibility of perpetual growth or decline, depend on the precise positive value of σ. This paper examines the role of σ in the neoclassical growth model and estimates σ by combining a low-pass filter with standard panel data techniques to better measure the theoretical constructs appropriate to production function estimation. Our approach is in the spirit of Friedman's permanent income theory of consumption and Eisner's related permanent income theory of investment. While their approaches and ours are similar in relying on permanent components, we extract these components with spectral methods that are more powerful and general for identifying these unobservables. We transform the data with the Baxter-King low-pass filter that depends on two parameters, the critical periodicity defining the long-run frequencies and a window for the number of lags approximating the ideal low-pass filter. Based on an analysis of the spectrum of the transformed series, we confirm that our choices of the critical periodicity and window emphasize long-run variation.

The empirical results are based on the comprehensive panel industry data constructed by Dale Jorgenson and his research associates. Our estimate of σ is 0.288 for the baseline values of the critical periodicity of eight years and window of three years. This result is robust to variations in the window. As the periodicity declines from eight to the minimum value of two, the elasticity declines owing to the distorting effects of transitory variation.

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> A "crucial" assumption is one on which the conclusions do depend sensitively, and it is important that crucial assumptions be reasonably realistic.
>
> Solow (1956, p. 65)

> The relation between the theoretical constructs used in consumption research and the observable magnitudes regarded as approximating them has, I believe, received inadequate attention.
>
> Friedman (1957, p. 7)

1. Introduction

The essential innovation contained in the neoclassical growth model was the modification of the steady-state equilibrium condition. Prior to Solow (1956), growth models determined the steady-state with a set of independent parameters, and equilibrium was achieved in only the most unlikely of circumstances. Solow's innovation was to introduce a variable capital/output ratio in place of the Harrod-Domar fixed parameter. This tour de force resolved the pressing analytic problem of the knife-edge solution inherent with independent parameters by relaxing the "crucial" assumption that production occurs with fixed factor proportions. However, this innovation pushed the key issues of growth onto other "crucial" assumptions embedded in the neoclassical production function. Key among these is the elasticity of substitution between labor and capital, σ, a parameter that has received too little notice in the growth literature.

This paper examines the role of σ in the neoclassical growth model and estimates this parameter by combining a low-pass filter with standard panel data techniques to better measure the long-run relations appropriate to production function estimation. We begin in Section 2 with a discussion of the key role played by σ in a variety of issues in growth theory -- the possibility of perpetual growth or decline, the level of steady-state income per capital, the speed of convergence, the rate of return on capital, the role of biased technical change, and the allocation of per capita income between factors of production and the efficiency with which they are utilized. In one sense, Solow's fundamental innovation replaces the Harrod-Domar assumption that $\sigma = 0$ with the more general assumption that $\sigma > 0$. Our discussion highlights that a host of important growth issues depend on the precise positive value of σ.

Section 3 develops three results concerning the estimation of the aggregate σ. In aggregate time series data, long-run relations have been identified by cointegration properties. However, our first result demonstrates that aggregate data will be uninformative about the value of σ. Consequently, we are lead to focus on disaggregate data at the industry (or firm) level. Since we are ultimately interest resides with an aggregate substitution elasticity, a mapping is required from parameters estimated on industry data to the aggregate parameter of interest for growth theory. Our second result develops a formula that generates such an aggregate estimate that recognizes substitution effects within an industry and reallocation effects across industries. Our third result shows that, if factor shares are independent of demand elasticities at the industry level, then reallocation effects are absent and the aggregate σ is a simple weighted average of the industry σ_i's.

Section 4 develops strategy for estimating industry σ_i's utilizing low-pass filters defined in the frequency domain and analyzed with spectral methods. [1] Production function parameters are recovered by focusing on the long-run relations between arguments appearing in the first-order condition for capital. Our approach is in the spirit of Friedman's (1957) permanent income theory of consumption and Eisner's (1967) related permanent income theory of investment. Friedman observed that the fundamental relation between consumption and income obtained between their permanent components and then identified the permanent components in terms of geometric distributed lags of past values. Eisner also emphasized the distinction between the transitory and permanent components of variables affecting investment demand and isolated the effect of the latter by grouping firms by industry and then using the group means as the data used in estimation. While our approach also relies on permanent components, we extract these components with spectral methods that are more powerful and general for identifying these unobservable variables. We transform the data with a low-pass filter developed by Baxter and King (1999) that depend on two parameters, the critical periodicity ($p^{\#}$) defining long-run frequencies and a window (q) for the number of lags and leads used to approximate the ideal low-pass filter. The transformed data reflect long-run variation and closely match production function concepts. The theoretical properties of the spectral representation of the low-pass filter are then examined. We compute the spectra associated with our estimator to assess the extent to which our choices of the critical periodicity and window are successful in emphasizing long-run variation. We also vary the key periodicity

[1] Engle and Foley (1975) also use spectral methods to study capital formation. They estimate a model relating investment spending to an equity price series (approximately a Brainard-Tobin's Q variable) and use a band-pass filter to emphasize middle frequencies centered at two years.

($p^{\#}$) and window (q) parameters to assess the sensitivity of the allocation of variance across the low and high frequencies.

Section 5 contains empirical results based on the comprehensive panel of U.S. industry data constructed by Dale Jorgenson and his research associates. Our econometric model relates the long-run capital/output ratio to the long-run relative price of capital. The benchmark estimate of σ is 0.288 for our baseline values of the critical periodicity of eight years ($p^{\#} = 8$) and a window of three years ($q = 3$). This result is robust to variations in the window. Moreover, there is great value in using the spectral methods to extract permanent components. As the periodicity declines from eight to the minimum value of two, the estimated σ declines by one-third owing to the distorting effects of transitory variation. At $p^{\#} = 2$, the low-pass filter is neutral, the raw data are not transformed, low frequency variation is not emphasized, and σ reaches a lower bound (relative to other values of $p^{\#}$) of 0.206.

Section 5 also contains three alternative sets of estimates of σ that serve as robustness checks. First, the prior estimates are based on the assumption of exogenous prices, and hence OLS is the appropriate estimation technique. Nonetheless, the assumption may not hold strictly. Instrumental variables estimate confirm the prior OLS results. We also document that the instrumental variables are relevant. Second, the benchmark estimates are based on the assumption of a constant σ across time; we document that our estimate of σ is temporally stable. Third, we examine alternative specifications for estimating σ. The first-order conditions yield two additional estimating equations that contain the labor/output ratio or the labor/capital ratio as the dependent variable and a relative price multiplying σ. However, the neoclassical growth model implies that neither series is stationary, a prediction consistent with the stylized facts of growth highlighted by Kaldor (1961) and King and Rebelo (1999, pp. 940-941). Hence, the low-pass filter used in this study is not applicable because spectral methods require stationary data. We nonetheless examine the estimates of σ derived from these models given the important study of Berndt (1976) that reported a disturbing wide range of estimates.

(Section 6 presents one estimate from the unconstrained model that allows the $\tilde{\sigma}$'s to vary across industries. This result is robust in comparison to the constrained estimate.)

Section 7 concludes.

2. The Implications of σ for Growth Theory

The elasticity of substitution was introduced by Hicks (1932) to analyze changes in the shares of capital and labor in a growing economy. His key insight was that the impact of the capital/labor ratio on the distribution of income (given output) could be completely characterized by the curvature of the isoquant (Blackorby and Russell, 1989, p. 882). It is well known that the elasticity of substitution (σ) is important in, among other areas, the analysis of trade and factor returns (Jones and Ruffin, 2003) and tax policies (Chirinko, 2002). Less well known is the critical role played by σ in models of economic growth. Several of the prominent issues are discussed in this section.

2.1. From The Harrod-Domar Knife-Edge to the Solow Interval

The neoclassical revolution in growth theory places the burden of equilibrium on the properties of the production function. When σ equals unity, the capital/labor ratio (KL) converges to a positive, finite value because, as KL moves towards its limiting values of 0 or ∞, the marginal product of capital (MPK[KL : σ]) and average product of capital (APK[KL : σ]) tend to ∞ or 0, respectively. Thus the Inada conditions are satisfied and, as determined by Solow's fundamental equation of motion for k, capital accumulation converges to zero. However, when σ departs from unity, some interesting possibilities arise, and the capital stock and per capita income can exhibit perpetual decline or perpetual growth. Whether these outcomes obtain depends on the relation of σ to two critical values that depend on other parameters of the neoclassical growth model. These relations are portrayed in Figure 1. Values of σ greater than (less than) a critical value, $\sigma_H^{\#}$ ($\sigma_L^{\#}$), lead to perpetual growth (perpetual decline) in the capital stock. More standard behavior occurs for values of σ between $\sigma_H^{\#}$ and $\sigma_L^{\#}$; in this no growth case, capital accumulation converges to zero and k to a positive, finite value. The neoclassical model replaces the Harrod-Domar knife-edge with the Solow interval defined by the two critical σ's.

To examine the role of σ in generating non-standard equilbria, we need to consider the Solow's equation of motion for k and the limiting behavior of the average product of capital.[2] The well-known equation of motion in the neoclassical growth model is as follows,

$$\dot{KL}/KL = s * APK[KL;\sigma] - (n + \delta), \tag{1}$$

where s, n, and δ are the rates of saving, population growth, and depreciation, respectively. The APK[KL : σ] is derived from the following intensive form of the CES production function (ignoring in this section the role of technical change),

$$f[KL:\sigma] = \{\phi KL^{((\sigma-1)/\sigma)} + (1-\phi)\}^{(\sigma/(\sigma-1))} \qquad 0 \le \phi \le 1, \tag{2}$$

where f[KL:σ] is a per capita neoclassical production function depending on σ and ϕ, the capital distribution parameter. The MPK[KL: σ] and APK[KL: σ] are as follows,

$$MPK[KL:\sigma] \equiv f_k[KL:\sigma] = \phi\{\phi + (1-\phi)KL^{((1-\sigma)/\sigma)}\}^{(1/(\sigma-1))}, \tag{3}$$

[2] The analysis is this sub-section draws on the presentations of the neoclassical growth model in Barro and Sala-i-Martin (1995, Section 1.3.3), de La Grandville (1989), Klump and Preissler (2000), Klump and de La Grandville (2000), and de La Grandville and Solow (2004). The latter paper also discusses how increases in σ expand production possibilities in a manner similar to exogenous technical progress.

$$\mathrm{APK}[\mathrm{KL}:\sigma] \equiv \mathrm{f}[\mathrm{KL}:\sigma]/\mathrm{KL} = \{\phi + (1-\phi)\mathrm{KL}^{((1-\sigma)/\sigma)}\}^{(\sigma/(\sigma-1))}. \tag{4}$$

The two non-standard cases arise because 1) MPK[KL: σ] fails to satisfy one of the Inada conditions and 2) this positive, finite limit affects the APK[KL: σ] so that no root exists for equation (1). In the first case when σ > 1, the limits for MPK and APK as KL → ∞ are as follows,

$$\lim_{\mathrm{KL}\to\infty} \mathrm{MPK}[\mathrm{KL}:\sigma] = \lim_{\mathrm{KL}\to\infty} \mathrm{APK}[\mathrm{KL}:\sigma] = \phi^{(\sigma/(\sigma-1))} \qquad \sigma > 1 \tag{5}$$

Perpetual growth arises when the limit in equation (5) is above the value of the APK[KL: σ] that sets $\dot{\mathrm{KL}}/\mathrm{KL} = 0$ in equation (1). This critical value of the APK[KL: σ] can be stated in terms of a critical value of σ, $\sigma_{\mathrm{H}}^{\#}$, determined by setting equation (1) to zero and solving for $\sigma_{\mathrm{H}}^{\#}$ in terms of four other model parameters collected in $\Gamma_{\mathrm{H}} = \{\delta, \phi, \mathrm{n}, \mathrm{s}_{\mathrm{H}} : \mathrm{s}_{\mathrm{H}} > \mathrm{n} + \delta\}$,

$$\begin{aligned} \sigma > \sigma_{\mathrm{H}}^{\#} &\equiv \mathrm{g}[\Gamma_{\mathrm{H}}] \equiv \mathrm{g}[\delta, \phi, \mathrm{n}, \mathrm{s}_{\mathrm{H}} : \mathrm{s}_{\mathrm{H}}\phi > \mathrm{n} + \delta] \\ &= \log[\mathrm{s}_{\mathrm{H}}/(\mathrm{n}+\delta)]/\log[(\phi \mathrm{s}_{\mathrm{H}})/(\mathrm{n}+\delta)] > 1, \end{aligned} \tag{6}$$

where Γ_{H} represents the collection of four parameters such that $\mathrm{g}[\Gamma_{\mathrm{H}}] > 1$. When σ is high and substitution is relatively easy, the decrement to the marginal and average products of capital is modest as KL goes to infinity. If σ exceeds the critical value defined in equation (6), perpetual accumulation of capital and perpetual growth in per capita income are possible even in the absence of technical change. While receiving some sporadic attention over the past 50 years,[3] the distinct possibility of perpetual growth in the neoclassical model has gone largely unnoticed, being eclipsed by the popularity of endogenous growth models.

To gain some intuition for this result, note that the limits in equation (5) are increasing in any positive, finite value of σ. The higher is σ, the greater the "similarity" between capital and labor in the production function (Brown, 1968, p. 50). Assume that the increase in the capital/labor ratio represents an increment to capital with labor held fixed. When σ is high, the incremental capital is easily substituted for labor, resulting in a nearly equiproportionate increase in both factors. Under constant returns to scale, diminishing returns sets-in very slowly, and the marginal and average products of capital can remain above the critical value so that capital accumulation is always positive.

In the second case when σ < 1, the other Inada condition fails,

$$\lim_{KL\to 0} \mathrm{MPK}[KL:\sigma] = \lim_{KL\to 0} \mathrm{APK}[KL:\sigma] = \phi^{(\sigma/(\sigma-1))} \qquad \sigma<1, \tag{7}$$

and this limit is below a critical value,

$$\begin{aligned} \sigma < \sigma_L^{\#} &\equiv g[\Gamma_L] \equiv g[\delta,\varphi,n,s_L : s_L < n+\delta] \\ &= \log[s_L/(n+\delta)]/\log[(\phi s_L)/(n+\delta)] < 1. \end{aligned} \tag{8}$$

When σ is low, capital and labor are "dissimilar" productive factors. With limited substitution possibilities, reductions in capital have little positive impact on marginal productivity. In an effort to raise the marginal product, capital

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[3] Solow (1956, pp. 77-78) and Pitchford (1960) were the first to note the possibility of perpetual growth. See the papers cited in fn. 2 for more recent statements.

accumulation remains negative and, for a value of σ below the $\sigma_L^{\#}$ defined in equation (8), KL declines perpetually.

In their textbook on economic growth, Burmeister and Dobell (1970, p. 34) refer to situations where $\sigma \neq 1$ as "troublesome cases" because they do not yield balanced growth paths. It is far from clear why the requirements for balanced growth paths in a particular theoretical model should dictate the shape of the production function, especially when $\sigma = 1$ is a sufficient but not necessary condition for a balanced growth path (cf. Acemoglu (2003) discussed below). To treat σ as a free parameter determined by the theory runs dangerously close to the fallacy of affirming the consequent. An alternative approach interprets cases where $\sigma \neq 1$ as quite interesting, suggesting needed modifications to the standard neoclassical growth model and highlighting the key role played by σ.

2.2. Per Capita Income

The value of σ is linked to per capita income and growth. Klump and de La Grandville (2000) show that, for two countries with identical initial conditions (in terms of k, n, and s), the country with a higher value of σ experiences higher per capita income at any stage of development, including the steady state (if it exists).[4] De La Grandville (1989) argues theoretically that a relative price change (e.g., a decrease in the price of capital) leads to relatively more output the higher the value of σ. (He also notes a second channel depending on σ -- a higher substitution elasticity permits a greater flow of resources between sectors with different factor

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[4] A caveat has been advanced by Miyagiwa and Papageorgiou (2003), who demonstrate that a monotonic relationship between σ and growth does not exist in the Diamond overlapping-generations model.

intensities.) Yuhn (1991) finds empirical support of this hypothesis in the case of South Korea.

2.3. Speed of Convergence

The speed of convergence to the steady-state depends on σ through capital accumulation. Turnovsky's (2002, pp. 1776-1777) calibrated neoclassical growth model indicates that the rate of convergence is sensitive to and decreasing in σ. For a given productivity shock, the speed of convergence is 45.3% (per year) when σ equals 0.1, but drops markedly to 12.2% when σ equals 0.8. The speed of convergence falls further to 8.9%, 6.4%, and 3.5% as σ is increased to 1.0, 1.2, and 1.5, respectively.[5]

Several papers have shown that the influence of σ on the speed of convergence interacts with other parameters in the model. In Ramanathan (1975), the speed of convergence is negatively related to the share of capital. The larger the capital share, the less rapidly the average product of capital declines and, since the APK is positively related to σ (cf. equation (4)), larger values of σ slow convergence. Mankiw (1995, p. 291) reports that an increase in the capital share from one-third to two-thirds reduces the speed of convergence by one-half. In the Klump and Preissler (2000, p. 50) model, the Ramanathan/Mankiw result holds, and the speed of convergence also depends on the relation between the initial and steady-state capital intensities.

[5] These figures are based on an intratemporal elasticity of substitution between consumption and leisure of 1.0 and an intertemporal elasticity of substitution for the composite consumption good of 0.4. The pattern of results is robust to variations in the latter parameter.

2.4. Other Issues in Growth Models

The value of σ can play an important role in assessing the plausibility of the neoclassical growth model. King and Rebelo (1993, Section IV) show that, in a Cass-Koopmans model with endogenous saving, the rate of return on capital (R) is sensitive to σ and is implausibly high when some part of growth is due to transitional dynamics. When transitional dynamics account for 25% of growth, R decreases modestly in σ. However, when transitional dynamics are more important, R increases dramatically in σ. Mankiw (1995, p. 287) also investigates the relation between σ and R but in terms of the following formula,

$$dR/R = -((1-\mu)/(\mu\sigma))\,(dYL/YL), \tag{9}$$

where μ is the capital factor share and YL is per capita income. Equation (9) approximates the difference in rates of return between poor and rich countries with the income differential represented by dYL/YL. For example, if σ = 4.0, the difference in the rate of return is only about 3 percent.[6] But if σ falls to 1.0 or 0.5, the above differential becomes implausibly large, increasing to 100 and 10,000 respectively. The relation between σ and R appears to be model dependent, but extant results suggest that the neoclassical model may not be correctly specified.

The importance of technical change in growth models is sensitive to σ. Acemoglu (2003) examines the tension between fluctuations in income shares, σ = 1, and balanced growth. He develops a model in which technical change is both labor-augmenting and capital-augmenting and shows that, along the balanced growth path, all technical change will be labor-augmenting. If σ < 1, technical

[6] These computations are based on $\phi = 0.33$ and an income level in rich countries that is 10 times larger than in poor countries.

change stabilizes income shares, and the balanced growth path is stable and unique. In his review of developmental accounting (which assesses how much of cross-country differences in per capita income are attributable to factors of production and the efficiency with which they are utilized), Caselli (forthcoming, Section 7) shows that the relative roles assigned to factors of production and efficiency is very sensitive to σ. When σ is near 0.5, variation in factors of production accounts for almost 100% of the variation in per capita income across countries. The percentage is decreasing in σ and drops to 40% for $\sigma = 1.0$ (the Cobb-Douglas case) and 25% for $\sigma = 1.5$. Caselli concludes (end of Section 7) that "gathering more information on this elasticity is a high priority for development accounting."

3. Estimation Strategy: Aggregate σ

Given the importance of the value of the aggregate σ, it would seem natural to obtain estimates with aggregate data. However, this approach proves problematic. The first result presented in this section shows that, if long-run relations are identified by cointegration properties, then aggregate data will be uninformative about the value of σ. This negative result suggests that we need to focus on disaggregate data at the industry (or firm) level. Assuming that econometrically sound estimates of σ_i can be obtained, a mapping is required from these micro parameters to the macro parameter of interest for growth theory that recognizes substitution effects within an industry and reallocation effects across industries.[7] Based on Hicks' formula for the derived demand of a factor of production, we develop a mapping from industry σ_i's to the aggregate σ. Our third result shows that, if factor shares are independent of demand elasticities at the industry level, then the aggregate σ is a simple weighted average of the industry σ_i's.

In innovative papers, Caballero (1994) and Schaller (forthcoming) measure long-run values by exploiting the cointegration relations between the capital/output ratio and the relative price (or user cost) of capital. The cointegrating model can be written as follows (equation numbers in this section are prefixed with P),

$$ky_t - a + b * uc_t = e_t, \tag{P1}$$

where ky_t is the logarithm of the capital/output ratio, a and b are estimated parameters, uc_t is the logarithm of the user cost of capital (which equals the

[7] We thank Robert Solow and an anonymous referee for highlighting the importance or reallocation effects in inferring an aggregate σ from disaggregate estimates.

logarithm of the price of capital (p_t^K) less the logarithm of the price of output (p_t^Y)), and e_t is an error term. The ky_t and uc_t series are assumed to be I(1) and cointegrated; thus long-run movements in ky_t and uc_t dominate. (If the two series are I(0), then this approach no longer measures long-run values.) The cointegrating vector is (1, b). The estimated value of b equals the price elasticity of capital; under a CES technology with constant returns, this elasticity equals the aggregate σ (this equality relation will be made explicit in Section 4).

While it provides an elegant solution to the problem of estimating the substitution elasticity from data subject to short-run deviations from long-run values, the cointegrating model will not be informative in an important case. A well-accepted stylized fact since at least Klein and Kosobud (1961) is that factor shares are constant in the long-run. Evaluating the relation between ky_t and p_t^K in light of this fact, we demonstrate the following result,

RESULT I: *If* ky_t *and* p_t^K *are I(1) and cointegrated and if factor shares are constant, then the cointegrating vector will be [1, 1] independent of the production technology.*

We begin with the definition of the logarithm of capital's factor share (μ_t^K),

$$\mu_t^K \equiv \left(k_t + p_t^K\right) - \left(y_t + p_t^Y\right) = c + \upsilon_t \tag{P2}$$

where k_t and y_t are the logarithms of capital and output, respectively, c is a constant, and υ_t a mean zero error term. Equation (P2) can be rearranged in terms of the capital/output ratio and the user cost,

$$\left(k_t - y_t\right) + \left(p_t^K - p_t^Y\right) = ky_t + uc_t = c + \upsilon_t \tag{P3}$$

which is a special case of the cointegrating model with a=0 and b=1. Thus, the combination of cointegrating properties and constant long-run factor shares necessarily yields a b coefficient of unity.[8] While the Cobb-Douglas technology is a sufficient condition for constant factor shares, it is not necessary (cf. Acemoglu, 2003). Hence, the cointegrating model will not be informative about b and the related technology parameters.

There is no small number of other models using aggregate data to infer σ. However, as mentioned in Section 1, these models need to solve the challenging problem of measuring long-run values relevant for production function estimation. Extant solutions involve explicit modeling of dynamics, an approach that will be particularly challenging given the complexity of the underlying dynamics.

We are thus lead to consider estimating aggregate σ from industry data based on long-run values, an approach that will be developed in Section 4.[9] Assuming that econometrically sound estimates of industry σ_i's can be obtained, a mapping is required from these industry elasticities to an aggregate elasticity that recognizes substitution and reallocation effects. We label the aggregate σ based on a mapping of industry σ_i's as $\tilde{\sigma}$.

We wish to obtain a mapping that clearly delineates the effects of substitution and reallocation effects, the latter defined as the difference between

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[8] This property of cointegrated relations explains a puzzle noted by Fisher (1971), who found in simulation studies that, when the factor share was nearly constant, an aggregate Cobb-Douglas technology fit the data well independent of the underlying technology. Similar results were obtained by Fisher, Solow, and Kearl (1977) when the study was extended to include CES production functions.

[9] The cointegration model requires long spans of data that may not be available at the industry level.

scale effects computed at the industry and aggregate levels. We begin with Hicks' formula for the derived demand of a factor of production,

$$\tilde{\lambda} = \tilde{\sigma} - \tilde{\sigma}\tilde{\mu}^{K} + \tilde{\eta}\tilde{\mu}^{K}, \tag{P4}$$

where $\tilde{\lambda}$ is the aggregate own price elasticity of capital, $\tilde{\sigma}$ is the aggregate substitution elasticity, $\tilde{\mu}^{K}$ is capital's factor share in the aggregate, and $\tilde{\eta}$ is the price elasticity of output.

In equation (P4), the three terms in parentheses capture in a succinct manner the substitution and scale effects associated with a change in the user cost of capital. The first term captures the direct substitution effect holding output price and output constant. The second term represents an additional indirect substitution effect driven by the lower marginal cost of production. Under competitive conditions, the decline in marginal cost translates into a decline in the output price. The extent of this decline is determined by the relative importance of capital in production represented by $\tilde{\mu}^{K}$. Since output price enters the denominator of the user cost, the decline in output price raises the relative price of and lowers the demand for capital. The third effect occurs because the lower factor price allows the firm to slide down the product demand curve and increase output. This scale effect is represented by the product of $\tilde{\mu}^{K}$ and the price elasticity of output ($\tilde{\eta}^{K}$) in the third term of equation (P4). It is interesting to that reallocation effects, which are central to our exercise, were explicitly assumed away by Hicks (1963, p. 241)

Equation (P4) can be solved for the aggregate substitution elasticity,

$$\tilde{\sigma} = \left(\tilde{\lambda} - \tilde{\eta}\tilde{\mu}^{K}\right)\tilde{\Omega}, \tag{P5}$$

where $\tilde{\Omega} = (1 - \tilde{\mu}^K)^{-1}$. In equation (P5), the aggregate substitution elasticity depends on the aggregate price elasticity less a subtraction for the scale effect. As capital's share becomes vanishingly small, $\tilde{\mu}^K \to 0$ and $\tilde{\Omega} \to 1$, and the scale effect disappears.

Equation (P5) is related to the industry parameters (per Basu and Fernald, 1997, Section III) by defining the aggregate own price elasticity as a weighted-average of the industry own price elasticity,

$$\tilde{\lambda} = \sum_i \omega_i \lambda_i \,, \tag{P6a}$$

$$\lambda_i = \sigma_i - \sigma_i \mu_i^K + \eta_i \mu_i^K \,, \tag{P6b}$$

where the ω_i's are weights reflecting the size of industry i (e.g., the percentage of total capital stock in industry i). Substituting equations (P6) into (P5) and rearranging terms, we obtain the following equation and key result,

RESULT II: *The mapping between the aggregate and industry substitution elasticities is given by the following formula that is the sum of substitution and reallocation effects,*

$$\tilde{\sigma} = \left\{ \sum_i \omega_i \, \sigma_i \left(1 - \mu_i^K \right) \right\} \tilde{\Omega} + \left\{ \sum_i \omega_i \left(\eta_i \mu_i^K - \tilde{\eta} \, \tilde{\mu}^K \right) \right\} \tilde{\Omega} \,. \tag{P7}$$

The substitution effect is represented by the first summation and equals the direct and indirect substitution effects at the industry level ($\sigma_i(1-\mu_i^K)$) weighted by industry size and adjusted by $\tilde{\Omega}$. The reallocation effect is represented by the second summation containing the difference in scale effects at the industry and aggregate levels. This difference captures the important characteristic that reallocation is measured by the industry scale effect relative to the aggregate scale effect. The response of industry output to a change in the price of capital will be larger than the average aggregate value if industry output is very price sensitive or the industry price falls substantially because capital plays an important role in this industry's cost structure. As before, the differential is weighted by industry size and adjusted by $\tilde{\Omega}$.

While Result II presents the general formula for mapping industry and aggregate elasticities, its usefulness is compromised to some degree because it depends on price elasticities of output that are not readily available. However, the presence of scale effects does not necessarily imply that reallocations affect the mapping from industry to aggregate substitution elasticities.

RESULT III: *If, at the industry level, the price elasticity of output is independent of capital's factor share, then the reallocation effect vanishes and the mapping from industry to aggregate parameters is given by the following equation,*

$$\tilde{\sigma} = \left\{ \sum_i \omega_i \, \sigma_i \left(1 - \mu_i^K \right) \right\} \tilde{\Omega} = \left\{ \sum_i \omega_i \, \sigma_i \left(1 - \mu_i^K \right) / \left(1 - \tilde{\mu}^K \right) \right\}. \qquad \textit{(P8)}$$

The result follows straightforwardly from equation (P7). Under the independence assumption stated in Result III, $\sum_i \omega_i \eta_i \mu_i^K = \sum_i \omega_i \eta_i \sum_i \omega_i \mu_i^K = \tilde{\eta} \tilde{\mu}^K$. Thus, the aggregate substitution elasticity is the weighted-average of the industry substitution elasticities adjusted by the labor's relative factor share.

4. Estimation Strategy: Industry σ_i's

4.1. The First-Order Condition

Our approach focuses on long-run production relations and low-frequency variation in the model variables. The long-run is defined by the vector of output and inputs consistent with profit maximization when all inputs can be adjusted without incurring costly frictions. This focus allows us to ignore short-run adjustment issues that are difficult to model and may bias estimates. Production for industry i at time t is characterized by the following Constant Elasticity of Substitution (CES) technology that depends on long-run values denoted by *,

$$\begin{aligned} Y^*_{i,t} &= Y[K^*_{i,t}, L^*_{i,t}, A^*_{i,t}, B^{K*}_t, B^{L*}_t] \qquad (10) \\ &= A^*_{i,t}\left\{\phi(B^{K*}_t K^*_{i,t})^{[(\sigma-1)/\sigma]} + (1-\phi)(B^{L*}_t L^*_{i,t})^{[(\sigma-1)/\sigma]}\right\}^{[\sigma/(\sigma-1)]}, \end{aligned}$$

where $Y^*_{i,t}$ is long-run real output, $K^*_{i,t}$ is the long-run real capital stock, $L^*_{i,t}$ is the long-run level of labor input, ϕ is the capital distribution parameter, and σ is the elasticity of substitution between labor and capital. Technical progress is both neutral ($A^*_{i,t}$), and biased for capital and labor (B^{K*}_t and B^{L*}_t, respectively). Neutral technical change can have both industry and aggregate effects, and biased technical change, since it affects capital goods available to all industries, has an aggregate effect. Equation (10) is homogeneous of degree one in $K^*_{i,t}$ and $L^*_{i,t}$ and has three desirable features for the purposes of this study. First, this production function depends on only two parameters -- ϕ representing the distribution of factor returns and, most importantly, σ representing substitution possibilities between the factors of production. Second, the CES function is strongly separable and thus can be expanded to include many additional factors of production (e.g., intangible capital) without affecting the estimating equation derived below. This feature gives the CES specification an important advantage relative to other production functions that allow for a more general pattern of substitution possibilities (e.g., the translog, minflex-Laurent). Third, the Cobb-Douglas production function is a special case of the CES; as $\sigma \rightarrow 1$ and biased technical change disappears ($B_t^{K*} = 1 = B_t^{L*}$,), equation (10) becomes $Y_{i,t}^{*} = A_{i,t}^{*} \left\{ K_{i,t}^{*\,[\phi]} \; L_{i,t}^{*\,[1-\phi]} \right\}$.

Constrained by the CES production function (10), a profit-maximizing firm chooses capital so that its marginal product equals the Jorgensonian user cost of capital, $P_{i,t}^{K*}$, which combines interest, depreciation, and tax rates and the relative price of capital goods. (The firm also sets the marginal product of labor equal to the real wage rate, $P_{i,t}^{L*}$; this condition will be discussed in Section VI.C.) Differentiating equation (10) with respect to capital and rearranging terms (as detailed in the Appendix), we obtain the following factor demand equation for the long-run capital/output ratio,

$$(K_{i,t}^{*} / Y_{i,t}^{*}) = \phi^{\sigma} \; (P_{i,t}^{K*})^{-\sigma} \; U_{i,t}^{K*}, \tag{11a}$$

$$U_{i,t}^{K*\,[1/\sigma]} \equiv A_{i,t}^{*[\sigma-1]} \; B_t^{K*[\sigma-1]}. \tag{11b}$$

To capture fixed industry and aggregate effects, we assume that the error term follows a two-way error component model,

$$U_{i,t}^{K*} = \exp[u_i^{K*} + u_t^{K*} + u_{i,t}^{K*}], \tag{12}$$

where $u_{i,t}^{K*}$ may have a non-zero mean. Taking logs of the first-order condition in equation (11a),

$$\ln(K^*_{i,t} / Y^*_{i,t}) = \sigma \ln(\phi) - \sigma \ln(P^{K*}_{i,t}) + \ln(U^{K*}_{i,t}), \tag{13}$$

removing fixed industry effects by first-differencing equation (13), and defining $ky^*_{i,t} \equiv \ln(K_{i,t} / Y_{i,t})^*$, $p^{K*}_{i,t} = \ln(P^{K*}_{i,t})$, $\tau^K_t \equiv \Delta u^{K*}_t$, and $e^K_{i,t} \equiv \Delta u^{K*}_{i,t}$, we obtain the following estimating equation,

$$\Delta ky^*_{i,t} = \zeta^K - \sigma \Delta p^{K*}_{i,t} + \tau^K_t + e^{K*}_{i,t}, \tag{14}$$

where τ^K_t are aggregate fixed time effects and ζ^K is a constant term (included in place of one of the τ^K_t's).

Conditional on observing $ky^*_{i,t}$ and $p^{K*}{}_{i,t}$, equation (14) provides a rather straightforward framework for estimating σ. Consistent estimates are obtained because the industry-level factor prices are exogenous. (This assumption is relaxed in Section 5.2, which contains instrumental variables estimates.) Importantly, in light of the recent critique and evidence by Antrás (2004), our estimates of σ are immune to biased technical change.[10] The key unresolved issue is that the long-run values denoted by *'s are not observable, an issue to which we now turn.

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[10] See his equation (1'), which is comparable to our equation (14) multiplied by minus one. The effects of biased technical change are removed by time effects in our framework based on panel data and by a linear time trend in Antrás' framework based on aggregate data. If we adopt Antrás' specification of biased technical change, $B^{K*}_t = \exp[\lambda t]$. In this case, λ is absorbed in the constant in equation (14).

4.2. Low-Pass Filters and Long-Run Values

Estimation of equation (14) is made difficult because the long-run values are not observed. Previous research generally addresses this problem in one of three ways. One approach estimates an investment equation that links changes in the observed capital stock to the unobserved long-run capital stock by assuming that 1) the change in the observed capital stock is measured by investment spending, 2) that the change in the long-run capital stock is measured by changes in output and the price of capital, and 3) that these changes are distributed over time due to various short-run frictions (Chirinko and von Kalckreuth, 2004, Appendix B). Relying on investment data replaces the unobservability problem with a set of difficult issues concerning dynamics and the specification of investment equations.[11] A second approach assumes that the observed capital stock, output, and price of capital approximate the long-run values and estimates variants of equations (13) or (14). This procedure effectively removes the *'s from these equations, an assumption that seems unwarranted. Third, as noted above, Caballero (1994) and Schaller (forthcoming) measure long-run values in an equation similar to (13) by exploiting the cointegration relation between the capital/output ratio and the relative price (or user cost) of capital. Deviations between the long-run and observed values are accounted for with the Stock and Watson (1993) correction, which has a substantial influence on the estimated σ's.

Our approach also focuses on the first-order condition that holds in the long-run but uses the Baxter-King (1999) low-pass filter (LPF) defined in the frequency domain to measure the long-run values of variables denoted by *'s. A LPF allows frequencies lower than some critical frequency, $\omega^{\#}$, to pass through to the

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[11] Chirinko, Fazzari, and Meyer (2004) document that, relative to the approach pursued in this paper, investment equations based on firm-level panel data impart a downward bias on estimates of σ.

transformed series but excludes frequencies higher than $\omega^{\#}$. Baxter and King present two important results regarding LPF's for the purpose of the current study. They derive the formulas that translate restrictions from the frequency domain into the time domain. For an input series, x_t, the ideal LPF for a critical value $\omega^{\#}$ produces the transformed series, $x_t^*[\omega^{\#},q]$, for an infinite lag and lead lengths, $q \rightarrow \infty$,

$$x_t^*[\omega^{\#},q] = \lim_{q\to\infty} \sum_{h=-q}^{q} d_h[\omega^{\#}]x_{t-h}, \tag{15a}$$

$$d_h[\omega^{\#}] = d'_h[\omega^{\#}] + \theta[\omega^{\#},q], \tag{15b}$$

$$d'_h[\omega^{\#}] = \omega^{\#}/\pi, \qquad h = 0, \tag{15c}$$

$$d'_h[\omega^{\#}] = \sin[|h|\,\omega^{\#}]/(|h|\,\pi), \qquad h = \underline{+}1, \underline{+}2, \ldots, q, \tag{15d}$$

$$\theta[\omega^{\#},q] = \lim_{q\to\infty}\left(1 - \sum_{h=-q}^{q} d'_h[\omega^{\#}]\right) \Big/ (2q+1), \tag{15e}$$

$$\omega^{\#} = 2\pi/p^{\#} = F[p^{\#}], \qquad p^{\#} = [2, \infty), \tag{15f}$$

where the $d_h[\omega^{\#}]$'s are weights defined as the sum of two terms – a provisional set of weights denoted by a prime (the $d'_h[\omega^{\#}]$'s in equations (15c) and (15d)) and a frequently imposed normalization that the $d_h[\omega^{\#}]$'s sum to 1 (per the constant $\theta[\omega^{\#},q]$ computed in equation (15e)). Equation (15g) defines the inverse relation between the critical frequency ($\omega^{\#}$) and the critical periodicity ($p^{\#}$), the latter defined as the length of time required for the series to repeat a complete cycle.

Since periodicities are relatively easy to interpret, hereafter we focus on $p^{\#}$ in place of $\omega^{\#}$.

A difficulty with implementing equations (15) is that the *ideal* LPF requires an infinite amount of data. Baxter and King's second key result is that the *optimal approximate* LPF for a window (i.e., the length of the leads and lags) of finite length q truncates the symmetric moving average at q. Thus, for $|h| \leq q$, the $d_h[p^{\#}]$'s

are given in equations (15); for $|h| > q$, $d_h[p^{\#}] = 0$. The optimal approximate LPF for the critical periodicity $p^{\#}$ and lead and lag length q, $LPF[p^{\#}, q]$, is given by equations (15) for any finite q.

4.3. Spectral Properties of the Low-Pass Filter Panel Model

Our estimation strategy is designed to emphasize long-run variation and, in this sub-section, we use spectral analysis to assess the extent to which choices of the critical periodicity and window are successful.[12] The estimating equation is derived in three steps: a) define long-run values with the $LPF[p^{\#}, q]$ (equations (15)); b) insert these long-run values into the first-order condition for optimal capital accumulation and take logarithms (equation (13)); c) first-difference this logarithmic equation to remove industry fixed effects (equation (14)). Each of these steps impacts the spectrum of the transformed data and hence the relative weights given to long-run variation in the variables ultimately entering the estimating equation. To compute the spectrum of a transformed series, we rely on the fundamental result from spectral analysis linking the spectrum of an output

[12] See Hamilton (1994, Chapter 6) or Sargent (1987, Chapter IX) for discussions of spectral analysis.

series to the product of the spectrum of an input series and a scalar that may be a function of ω, $p^{\#}$, or q. To understand the impact of each step, we need only compute the scalar associated with each transformation.

In analyzing the spectral properties of our estimator, it is convenient to recast the LPF transformation (for a finite q), the logarithmic transformation, and the first-difference transformation as follows,

$$x_t^*[p^{\#},q] = \sum_{h=-q}^{q} d_h[p^{\#}]\ x_{t-h}, \tag{16a}$$

$$y_t^*[p^{\#},q] = \ln[x_t^*[p^{\#},q]], \tag{16b}$$

$$z_t^*[p^{\#},q] = \Delta y_t^*[p^{\#},q], \tag{16c}$$

where x_t represents the raw data series, either $(K_{i,t} / Y_{i,t})$ or $p_{i,t}^K$. The spectra corresponding to the $x_t^*[.]$, $y_t^*[.]$, and $z_t^*[.]$ output series in equations (16) are defined over the interval ω = [0, π] as the product of the spectrum for the input series and a scalar that is nonnegative, real, and may be depend on ω, $p^{\#}$, or q,

$$g_{x*}[e^{-i\omega}] = a_\omega[p^{\#},q]\ g_x[e^{-i\omega}], \tag{17a}$$

$$g_{y*}[e^{-i\omega}] = b\ g_{x*}[e^{-i\omega}], \tag{17b}$$

$$g_{z*}[e^{-i\omega}] = c_\omega\ g_{y*}[e^{-i\omega}], \tag{17c}$$

where $g_x[e^{-i\omega}]$ is the spectrum for the raw series and the scalars are defined as follows,

$$a_\omega[p^\#,q] = \alpha[p^\#,q] \left\{ \begin{aligned} &(2/p^\#) + 2\sum_{h=1}^{q} \cos[h\omega]\, d'_h[p^\#] \\ &+ \theta[p^\#,q]\, \{(1-\cos[\omega(2q+1)])/(1-\cos[\omega])\}^{1/2} \end{aligned} \right\}^2 \tag{17d}$$

$$b \;= \beta\, (\mu_{x*})^{-2}, \tag{17e}$$

$$c_\omega = \gamma\; 2\; (1-\cos[\omega])\,, \tag{17f}$$

where μ_{x*} equals the unconditional expectation of $x_t^*[.]$. To ensure comparability in the analyses to follow that vary $p^\#$ and q, the three spectra are normalized by an appropriate choice of a constant (α, β, or γ) so that the integrals for equations (17a), (17b), and (17c) evaluated from 0 to π equal 1.0.

The three scalars $a_\omega[p^\#,q]$, b, and c_ω correspond to the LPF, logarithmic, and first-difference transformations, respectively, and are derived as follows. The $a_\omega[p^\#,q]$ scalar is based on Sargent (1987, Chapter XI, equation (33)),

$$a_\omega[p^\#,q] = \alpha[p^\#,q]\; \left\{ \sum_{h=-q}^{q} e^{-ih\omega}\, d_h[p^\#] \right\} \left\{ \sum_{h=-q}^{q} e^{ih\omega}\, d_h[p^\#] \right\}. \tag{18}$$

The two-sided summations are symmetric about zero and only differ by the minus sign in the exponential terms. Hence, the two sums in braces are identical. The

$d_h[.]$'s appearing in the summations are separated into $\theta[.]$ and the $d'_h[.]$'s (cf. equations (15)). For the latter terms, a further distinction is made between the term at h=0 and the remaining terms (h=±1,±q) that are symmetric about h=0. Equation (18) can be written as follows,

$$a_\omega[p^\#,q] = \alpha[p^\#,q] \left\{ \begin{array}{l} \theta[p^\#,q] \sum_{h=-q}^{q} e^{ih\omega} \\ +(2/p^\#) + \sum_{h=1}^{q} (e^{-ih\omega} + e^{ih\omega})\, d'_h[p^\#] \end{array} \right\}^2 . \tag{19}$$

The first sum of exponential terms is evaluated based on Sargent (1987, p. 275),

$$\begin{aligned} \sum_{h=-q}^{q} e^{ih\omega} &= \left\{ \left(\sum_{h=-q}^{q} e^{ih\omega} \right)^2 \right\}^{1/2} \\ &= \{((1-\cos[(2q+1)\omega]/(1-\cos[\omega])\}^{1/2} . \end{aligned} \tag{20}$$

The second sum of exponential terms is evaluated with the Euler relations, $e^{\pm ih\omega} = \cos[h\omega] \pm \sin[h\omega]$,

$$\sum_{h=1}^{q} (e^{-ih\omega} + e^{ih\omega})\, d'_h[p^\#] = 2 \sum_{h=1}^{q} \cos[h\omega]\, d'_h[p^\#]. \tag{21}$$

The b scalar is based on the approximation in Granger (1964, p. 48, equation 3.7.6), which states that the approximation will be accurate if the mean is much larger than the standard deviation of the input series ($x_t^*[.]$). The c_ω scalar is based on the well-known formula for the first-difference transformation (Hamilton, 1994, equation 6.4.8). Note that b and c_{ω} are independent of $p^{\#}$ and q. The importance of the above analytical results is that the combined effects of the three transformations are captured by three scalars that multiply the spectrum of the raw series,

$$g_{Z*}[e^{-i\omega}] = \left\{a_{\omega}[p^{\#},q] * b * c_{\omega}\right\} * g_{x}[e^{-i\omega}]. \tag{22}$$

With equation (22), we are now in a position to examine the extent to which our estimation strategy based on the definition of the long-run ($p^{\#} = 8$) and the window (q=3) approximates the ideal low-pass filter and hence is successful in emphasizing long-run variation in the data. Since the spectra for the raw series ($g_{x}[e^{-i\omega}]$) and the scalars associated with the logarithmic and first-difference transformations (b and c_{ω}, respectively) do not depend on $p^{\#}$ or q, they can be ignored in drawing relative comparisons among estimators. Alternative values of $p^{\#}$ or q will only affect the $LPF[p^{\#},q]$ and the associated frequency response scalar, $a_{\omega}[p^{\#},q]$.

Our first set of analyses holds the window fixed at q = 3 and examines different values of the critical periodicity, $p^{\#}$, that determines which frequencies are passed-through in the $LPF[p^{\#},q]$. Four values of $p^{\#}$ are considered in Figure 2. We begin with the minimum value of the critical frequency, $p^{\#} = 2$, which corresponds to the standard investment equation that does not transform the raw data (other than the logarithmic and differencing operations). The frequency response for the standard investment model ($a_{\omega}[p^{\#} = 2, q = 3]$) is flat, indicating that this estimator does not reweight the variances across frequencies of the raw series. By contrast, our benchmark model represented by $a_\omega[p^{\#} = 8, q = 3]$ effects a substantial reweighting. With $p^{\#} = 8$, the benchmark model emphasizes the variances from periodicities greater than or equal to 8 years (which corresponds to $\omega^{\#} \leq 0.79$ on the horizontal axis), thus allocating a substantial amount of weight to those frequencies that we believe will yield better estimates of production function parameters. The remaining entries in Figure 2 are for the intermediate cases, $a_\omega[p^{\#} = 4, q = 3]$ and $a_\omega[p^{\#} = 6, q = 3]$.

The benchmark model is based on the assumption that periodicities greater than or equal to 8 years contain useful information for the parameter estimates. We now explore how much additional reweighting occurs when we increase the critical periodicity above 8; specifically, for values of $p^{\#}$ equal to 10, 20 and, in the limit, ∞. The $a_\omega[.]$'s corresponding to these critical values are graphed in Figure 3. Comparing the frequency responses for these higher periodicities indicates that they weight the lower frequencies in a manner very similar to the benchmark case of $p^{\#} = 8$.

This analysis suggests two conclusions concerning our choice of the critical periodicity. First, our estimation strategy based on $p^{\#} = 8$ appears to be reasonably successful in emphasizing long-run variation. This critical value appears to be a well-accepted standard for separating long-run frequencies from short-run and medium-run frequencies.[13] Second, results in Figure 3 suggest that parameter estimates are likely to be insensitive to the critical periodicity for values of $p^{\#} > 8$.

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[13] A critical value of $p^{\#} = 8$ is used by Baxter and King (1999, p. 575), Levy and Dezhbakhsh (2003, p. 1502), Prescott (1986, p. 14), and Stock and Watson, 1999, p. 11). Burns and Mitchell (1946) report that the duration of the typical business cycle in the U.S. is less than 8 years.

We can also use the spectral formulas to assess the impact of variations in the window, q, in approximating the ideal low-pass filter. Recall that the ideal LPF is based on the limiting behavior as q --> ∞. Since the span of our data are limited, this procedure is not feasible, and our empirical work relies on the optimal approximation based on a finite number of q leads and lags. This approximation introduces error into the analysis because variances associated with frequencies other than those desired enter into the transformation of the model variables. However, increasing q is costly in terms of lost degrees of freedom.

The tradeoff between approximation error and degrees of freedom is assessed in Figure 4, which plots $a_{\omega}[p^{\#} = 8, q]$ and values of q equal to 1, 3 and 5. The ideal LPF is also plotted as a rectangle that takes on a constant positive values for ω's corresponding to $p^{\#} \geq 8$, and 0 for all other ω's.[14] When q = 1, the approximation error is substantial, and the LPF is extensively contaminated by the variances associated with frequencies above the critical value. This contamination can not be totally eliminated with finite data but, when q = 3, it is reduced substantially. When q = 5, the LPF moves closer to the ideal LPF, but the improvement relative to q=3 is modest. Since using a window of q = 5 is costly in terms of degrees of freedom and the reduction in approximation error appears small, we will adopt q=3 as our preferred window, though we will experiment with q=1 and q=5 to examine robustness.[15]

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[14] This constant positive value is chosen so that the area under the ideal LPF is 1.0. A critical value of $p^{\#} = 8$ implies $\omega^{\#} = \pi / 4 = 0.785$, and hence the positive constant equals $(0.785)^{-1}$ = 1.273.

[15] Baxter and King (1999, pp. 581-582) reached a similar conclusion based on their analysis of band-pass filters.

5. Empirical Results: Homogenous Industry σ_i's

This section estimates σ using our low-pass filter model defined with various critical periodicities and windows. Data are obtained from the webpage of Dale Jorgenson (http://post.economics.harvard.edu/faculty/jorgenson/data/35klem.html), and represent output, inputs, and prices for 35 industries for the period 1959-1996. This section imposes the restriction that the σ_i's are identical across all 35 industries; this restriction will be relaxed in Section 6.

5.1. OLS Estimates

In our model, σ is identified by the correlation between the growth rates of the capital/output ratio and the relative price of capital, conditional on fixed industry and aggregate effects (see equation (14)). The OLS results from our low-pass filtered model based on $p^{\#} = 8$ and $q = 3$ are as follows,

$$\begin{gathered} \Delta ky^{*}_{i,t} = \underset{(0.001)}{0.007} - \underset{(0.017)}{0.308}\ \Delta p^{K*}_{i,t} + \tau^{K}_{t} + e^{K*}_{i,t}, \\ \tilde{\sigma} = \underset{(0.004)}{0.288} \qquad R^2 = 0.404 \end{gathered} \tag{23}$$

where $\tilde{\sigma}$ is computed according to equation (P8) with ω-weights based on capital stocks (the results are very robust to defining the ω-weights by real output, nominal output, or 1/35).[16] The point estimate for $\tilde{\sigma}$ is 0.288 with a very small standard error of 0.004, and the R^2 is 0.404. As we shall, see subsequent results very rarely depart in a meaningful way from the benchmark estimates presented in

[16] The standard error of $\tilde{\sigma}$ is computed with the following formula, $\left[\sum_i \omega_i^2\, \mathrm{VAR}[\sigma_i]\right]^{0.5}$.

equation (23) and, when meaningful differences occur, they are due to the presence of high frequency variation in the model variables.

Table 1 examines the sensitivity of estimates of σ to variations in the window (q) and the critical periodicity ($p^{\#}$). For a given $p^{\#}$, the σ's are very robust to variations in q. For example, when $p^{\#} = 8$, the σ's are 0.275, 0.308, and 0.292 for q of 1, 3, and 5, respectively. The standard errors rise as the window is increased and more data are used in computing the filters and less data are available for estimation. Nonetheless, the standard errors for the σ's remain less than 0.02 for all entries. (Note that the R^2's are not strictly comparable across cells because the dependent variable depends on $p^{\#}$ and q.) These results suggest that little is gained by increasing the size of the window and compromising degrees of freedom above
$q = 3$.

Table 1 also allows us to assess the robustness of variations in $p^{\#}$ for a given q by reading down the columns. For $q = 3$, as $p^{\#}$ increase from 8 to ∞ in column 2, the estimates of σ hardly change. Consistent with the theoretical analysis in Figures 2 and 3, this robustness confirms that the relevant information about the long-run has been captured at $p^{\#} = 8$. When $p^{\#}$ is set to its minimum value of 2, the low-pass filter is neutral, and the raw data are transformed only by logarithmic and first-difference operations (cf. equation (22)). In this case, σ drops by one-third relative to the benchmark value (0.193 vs. 0.288). Thus, high frequency and presumably transitory variation affects the estimates of σ and, as has been frequently noted in the permanent income literature, transitory variation attenuates point estimates.

Table 2 explores robustness with respect to the constant term and fixed aggregate effects with $p^{\#} = 8$ and $q = 3$. Column 1 contains our benchmark results. When we remove the time dummies in column 2, σ rises from 0.308 to 0.350, and the R^2 falls from 0.404 to 0.335. Columns 3 and 4 remove the constant term, and the results mirror those in columns 1 and 2. Thus, estimates of σ are robust to including a constant and fixed time effects. Both are included in the estimating equation for the subsequent results.

5.2. IV Estimates

The estimates reported in Section 5.1 are based on the assumption of exogenous prices, and hence OLS is the appropriate estimation technique. Since the exogeneity assumption necessary for consistent estimation may not hold strictly, instrumental variable estimates provide a useful robustness check.

Table 3 contains the instrumental variable results using $p^{K*}_{i,t-2}$ as the instrument for the same range of values for $p^{\#}$ and q that appeared in Table 1. For values of $p^{\#}$ equal to or greater than 6, the IV estimates of σ are approximately 0.10 higher than their OLS counterparts in Table 1. All of these σ's are statistically significant at the 1% level. The discrepancy widens for $p^{\#} = 4$. However, for $p^{\#} = 2$, the results become nonsensical. Thus, for those estimates emphasizing long-run variation, the IV estimates of σ are greater than the comparable OLS estimates, but still far from σ equal to unity.

Recent work with instrumental variables has raised concerns about weak instruments and biased estimates (Nelson and Startz, 1990).[17] Instrumental relevance is assessed with the test statistic proposed by Stock, Wright, and Yogo (2002), which involves an auxiliary regression of the model variable on the instrument and a comparison of the F-statistic for the goodness of fit to a critical value of 8.96 (reported in their Table 1). As shown below, the σ's in Table 3, only

[17] Note that Hansen-Sargen test of instrument validity is not useful in this just identified model.

the instrument used for the $p^{\#} = 2$ results is weak, and hence these estimates are unreliable. This result suggests a reason for the implausible estimates for $p^{\#} = 2$. All of the other estimates are based on strong (relevant) instruments. For $p^{\#} \geq 4$, the estimated σ's range from 0.304 to 0.423.

5.3. Split-Sample Estimates

To further assess the robustness of our results, Table 4 contains estimates from the first and second halves of the sample, 1958-1977 and 1978-1996, respectively. The results closely follow those reported previously. For example, for our preferred specification with $p^{\#} = 8$ and $q = 3$, the estimates of σ from the first and second halves of the sample are 0.261 and 0.362, respectively. These estimates bracket our preferred estimate from the full sample of 0.308.

5.4. Other Estimating Equations

The first-order conditions for profit maximization yield two additional estimating equations that contain the labor/output ratio or the labor/capital ratio as the dependent variable. However, the neoclassical growth model implies that neither series is stationary, an implication consistent with the first two of the stylized facts of growth advanced by Kaldor (1961). Hence, the low-pass filter used in this study is not strictly applicable because spectral methods require stationary data. Murray (2003) and Cogley and Nason (1995) document the problems that can arise when band pass filters are applied to nonstationary data and Mallick (in process) is exploring the effects of nonstationary data on the variety of estimates of σ appearing in the literature. With this important caveat noted, we nonetheless examine the estimates of σ derived from the equations with labor/output (ℓy^*) or the capital/labor
ratio ($k\ell^*$) as the dependent variable, and estimate the following equations,

$$\Delta\ell y^{*}_{i,t} = \zeta^{L} - \sigma\,\Delta p^{L*}_{i,t} + \tau^{L}_{t} + e^{L*}_{i,t}\,, \tag{24}$$

$$\Delta k\ell^{*}_{i,t} = \zeta^{KL} - \sigma\,\Delta(p^{K*}_{i,t} - p^{L*}_{i,t}) + \tau^{KL}_{t} + e^{KL*}_{i,t}\,. \tag{25}$$

Table 5 contains σ's and R^2's for q = 3 and the usual range of $p^{\#}$'s. Column 1 contains the previously reported estimates for ky* and columns 2 and 3 the results for **ℓ**y* and k**ℓ***, respectively. Relative to the results with ky*, the σ's estimated with the **ℓ**y* equation are higher for all critical periodicities (save $p^{\#}$ = 4); those for k**ℓ*** are lower. The maximal difference among the σ's across the three specifications is 0.17. While this difference is not negligible, it must be kept in mind that the estimates based on **ℓ**y* and k**ℓ*** are not on firm statistical footing. Nonetheless, this array of estimates of σ is bounded above by 0.40.

In a well-known study, Berndt (1976) estimated these three first-order conditions, and uncovered a disturbingly wide range of results. Part of this dispersion was due to different definitions of factor prices. But he also found that the labor/output equation delivered higher values of σ. With the exception of the $p^{\#}$ = 4 results, Table 5 confirms the Berndt finding.

6. Empirical Results: Heterogeneous Industry σ_i's

This section relaxes the constraint that $\sigma_i = \sigma$ and permits separate σ_i's to be estimated for each industry. A full set of results are not available for this heterogeneous industry σ_i's model. The initial indication is that the results are robust. For example, when the constraint is imposed across industries, $\tilde{\sigma}$ is 0.288 (0.004). In the unconstrained model, $\tilde{\sigma}$ is lower and equals 0.170 (0.022). The standard error rises in the unconstrained model, though the substitution elasticity is still precisely estimated. While the two aggregate estimates are statistically different, the difference is not economically meaningful.

7. Summary and Conclusions

The elasticity of substitution between labor and capital (σ) is a crucial parameter in growth theory. Solow's fundamental innovation can be cast in terms of σ, where the Harrod-Domar assumption that $\sigma = 0$ is replaced with the more general assumption that $\sigma > 0$. Our discussion highlights that a host of important growth issues depend on the precise positive value of σ. It affects the possibility of perpetual growth or decline, the level of steady-state income per capital, the speed of convergence, the rate of return on capital, the role of biased technical change, and the allocation of per capita income to factors of production and the efficiency with which they are utilized.

This key production function parameter is estimated by focusing on the long-run relations using a low-pass filter defined in the frequency domain. Our preferred point estimate is 0.288, and it proves robust to variations in several directions. Our review of the growth literature, suggests that σ is not an engine of growth. This estimate is well below the critical value needed for perpetual growth in the neoclassical growth model. Moreover, the empirical results suggest that the dynamic macroeconomics in general and the growth literature in particular need to move away from the convenient but inaccurate assumption of σ equal to unity. Such a departure from a Cobb-Douglas production function will force an expansion of the neoclassical growth model to include, among other factors, a central role for biased technical change in influencing factor shares and balanced growth.

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Appendix:
Specifying the Marginal Product of Capital
With Neutral and Biased Technical Change

This appendix presents the details of the derivation of the marginal product of capital when there is both neutral and biased technical change. We assume that production possibilities are described by the following CES technology that relates output ($Y^*_{i,t}$) to capital ($K^*_{i,t}$), labor ($L^*_{i,t}$), neutral technical progress ($A^*_{i,t}$), and biased technical progress on capital and labor (B^{K*}_t and B^{L*}_t, respectively) for firm i at time t,

$$\begin{aligned} Y^*_{i,t} &= Y[K^*_{i,t}, L^*_{i,t}, A^*_{i,t}, B^{K*}_t, B^{L*}_t], \\ &= A^*_{i,t}\left\{\phi(B^{K*}_t K^*_{i,t})^{[(\sigma-1)/\sigma]} + (1-\phi)(B^{L*}_t L^*_{i,t})^{[(\sigma-1)/\sigma]}\right\}^{[\sigma/(\sigma-1)]} \end{aligned} \tag{A1}$$

where ϕ is the capital distribution parameter and σ is the elasticity of substitution between labor and capital.

The derivative of $Y^*_{i,t}$ with respect to $K^*_{i,t}$ is computed from equation (A1) as follows,

$$\begin{aligned} Y'^*_{i,t} = {} & [\sigma/(\sigma-1)]A^*_{i,t}\left\{\phi(B^{K*}_t K^*_{i,t})^{[(\sigma-1)/\sigma]} + (1-\phi)(B^{L*}_t L^*_{i,t})^{[(\sigma-1)/\sigma]}\right\}^{[[\sigma/(\sigma-1)]-1]} \\ & *[(\sigma-1)/\sigma]\phi(B^{K*}_t K^*_{i,t})^{[[(\sigma-1)/\sigma]-1]} B^{K*}_t. \end{aligned} \tag{A2}$$

Noting that the set of parameters in the exponent of $(B^{K*}_t K^*_{i,t})$ on the second line of equation (A2) can be rewritten,

$$((\sigma-1)/\sigma)-1=-1/\sigma, \tag{A3}$$

we rearrange equation (A2) as follows,

$$\begin{aligned} Y'^{*}_{i,t} = \phi\ & K^{*\,[-1/\sigma]}_{i,t} \\ & A^{*}_{i,t}\left\{\phi(B^{K*}_{t}K^{*}_{i,t})^{[(\sigma-1)/\sigma]}+(1-\phi)(B^{L*}_{t}L^{*}_{i,t})^{[(\sigma-1)/\sigma]}\right\}^{[\sigma/(\sigma-1)]} \\ & \left\{\phi(B^{K*}_{t}K^{*}_{i,t})^{[(\sigma-1)/\sigma]}+(1-\phi)(B^{L*}_{t}L^{*}_{i,t})^{[(\sigma-1)/\sigma]}\right\}^{-1} \\ & B^{K*[(\sigma-1)/\sigma]}_{t}. \end{aligned} \tag{A4}$$

In equation (A4), the second line equals $Y^{*}_{i,t}$ per equation (A1), and the third line equals the product of $Y^{*}_{i,t}$ and $A^{*}_{i,t}$ raised to the appropriate powers,

$$\begin{aligned} Y'^{*}_{i,t} = \phi\ & K^{*\,[-1/\sigma]}_{i,t} \\ & Y^{*}_{i,t} \\ & Y^{*[(1-\sigma)/\sigma]}_{i,t}\ A^{*[(\sigma-1)/\sigma]}_{i,t} \\ & B^{K*[(\sigma-1)/\sigma]}_{t}, \end{aligned} \tag{A5}$$

which can be rewritten as follows,

$$Y'^{*}_{i,t} = \phi\ K^{*\,[-1/\sigma]}_{i,t}\, Y^{*[1/\sigma]}_{i,t}\, A^{*[(\sigma-1)/\sigma]}_{i,t}\ B^{K*\,[(\sigma-1)/\sigma]}_{t}\ . \tag{A6a}$$

$$= \phi\ (Y^{*}_{i,t}/K^{*}_{i,t})^{[1/\sigma]}\, U^{K*\,[1/\sigma]}_{i,t}, \tag{A6b}$$

$$U^{K*\,[1/\sigma]}_{i,t} \equiv A^{*[\sigma-1]}_{i,t}\ B^{K*[\sigma-1]}_{t}. \tag{A6c}$$

We assume that the marginal product of capital in equation (A6a) is equated to the price of capital, $P_{i,t}^{K*}$,

$$P_{i,t}^{K*} = \phi\ (Y_{i,t}^{*} / K_{i,t}^{*})^{[1/\sigma]}\ U_{i,t}^{K*[1/\sigma]}. \tag{A7}$$

Equation (A7) can be rearranged to isolate the capital/output ratio on the left-side,

$$(K_{i,t}^{*} / Y_{i,t}^{*}) = \phi^{\sigma}\ (P_{i,t}^{K*})^{-\sigma}\ U_{i,t}^{K*}, \tag{A8}$$

which is equation (11a) in the text.

Table 1: Benchmark Model
Ordinary Least Squares Estimates Of Equation (14)
Dependent Variable: Capital/Output Ratio
Various Critical Periodicities ($p^{\#}$) and Windows (q)

		q = 1	q = 3	q = 5
		(1)	(2)	(3)
$p^{\#} = 2$	σ { R^2}	0.206 {0.466}	0.206 {0.476}	0.218 {0.495}
$p^{\#} = 4$	σ { R^2}	0.275 {0.500}	0.278 {0.519}	0.281 {0.532}
$p^{\#} = 6$	σ { R^2}	0.276 {0.487}	0.296 {0.454}	0.274 {0.402}
$p^{\#} = 8$	σ { R^2}	0.275 {0.483}	**0.308 {0.404}**	0.292 {0.381}
$p^{\#} = 10$	σ { R^2}	0.274 {0.481}	0.311 {0.390}	0.314 {0.359}
$p^{\#} = 20$	σ { R^2}	0.274 {0.480}	0.304 {0.392}	0.327 {0.349}
$p^{\#} \rightarrow \infty$	σ { R^2}	0.273 {0.480}	0.302 {0.394}	0.313 {0.353}

Notes: Estimates of σ are based on panel data for 35 industries for the period 1959-1996. Standard errors are heteroscedastic consistent using the technique of White (1980), are less than 0.02 for all entries, and are not reported because all estimates of σ are statistically significant at the 1% level. A constant term and fixed time effects are included in the regression equation but are not reported. The R^2's are not comparable across cells because the dependent variable depends on $p^{\#}$ and q. Our preferred estimate is for the equation for which the Low-Pass Filter parameters are $p^{\#} = 8$ and q = 3.

Table 2: Benchmark Model
Ordinary Least Squares Estimates Of Equation (14)
Dependent Variable: Capital/Output Ratio
$p^{\#}$=8 and q=3
Combinations of Fixed Industry and Time Effects

	(1)	(2)	(3)	(4)
σ	0.308	0.350	0.308	0.352
	(0.017)	(0.018)	(0.020)	(0.018)
ζ^K	0.007	0.007	0.000	0.000
	(0.001)	(0.001)	(-----)	(-----)
τ^K_t	Yes	No	Yes	No
R^2	0.404	0.335	0.440	0.335

Notes: Estimates of σ are based on panel data for 35 industries for the period 1959-1996. Standard errors are heteroscedastic consistent using the technique of White (1980).

Table 3: Benchmark Model
Instrumental Variable Estimates Of Equation (14)
Dependent Variable: Capital/Output Ratio
Various Critical Periodicities ($p^{\#}$) and Windows (q)

		q = 1	q = 3	q = 5
		(1)	(2)	(3)
$p^{\#} = 2$	σ [F-stat]	-0.575[@] [6.88]	-0.605[@] [6.82]	1.560[@] [7.48]
$p^{\#} = 4$	σ [F-stat]	0.388 [25.33]	0.385 [25.73]	0.423 [21.00]
$p^{\#} = 6$	σ [F-stat]	0.389 [27.00]	0.342 [58.16]	0.304 [58.97]
$p^{\#} = 8$	σ [F-stat]	0.393 [25.90]	**0.331 [118.56]**	0.328 [74.97]
$p^{\#} = 10$	σ [F-stat]	0.396 [25.35]	0.340 [117.19]	0.338 [147.91]
$p^{\#} = 20$	σ [F-stat]	0.398 [24.78]	0.374 [57.04]	0.352 [129.11]
$p^{\#} \rightarrow \infty$	σ [F-stat]	0.399 [24.69]	0.382 [48.30]	0.362 [70.47]

Notes: Estimates of σ are based on panel data for 35 industries for the period 1959-1996. The instrument is $p^{K*}{}_{i,t-2}$. Standard errors are heteroscedastic consistent using the technique of White (1982) and are not reported because all estimates of σ are statistically significant at the 1% level with the exception of the σ's in row 1 marked with a [@], which are not significant at the 10% level. [F-stat] is the F-statistic for the first-stage regression of $\Delta p^{K*}{}_{i,t}$ on $p^{K*}{}_{i,t-2}$. The null hypothesis of a weak instrument is rejected at the 5% level for F-stat greater than or equal to 8.96 (Stock, Wright, and Yogo, 2002, Table 1). A constant term and fixed time effects are included in the regression equation but are not reported. Our preferred estimate is for the equation for which the Low-Pass Filter parameters are $p^{\#} = 8$ and q = 3.

Table 4: Benchmark Model
Ordinary Least Squares Estimates Of Equation (14)
Dependent Variable: Capital/Output Ratio
Various Critical Periodicities ($p^{\#}$) and q = 3
Split-Sample Results

	Period		q = 1	q = 3	q = 5
			(1)	(2)	(3)
$p^{\#} = 2$	1958-77	σ {R^2}	0.199 {0.536}	0.208 {0.517}	0.236 {0.527}
	1978-96	σ {R^2}	0.203 {0.413}	0.179 {0.401}	0.118 {0.364}
$p^{\#} = 4$	1958-77	σ {R^2}	0.243 {0.556}	0.250 {0.530}	0.270 {0.505}
	1978-96	σ {R^2}	0.291 {0.471}	0.268 {0.386}	0.216 {0.298}
$p^{\#} = 6$	1958-77	σ {R^2}	0.236 {0.540}	0.248 {0.436}	0.294 {0.434}
	1978-96	σ {R^2}	0.301 {0.472}	0.326 {0.367}	0.292 {0.321}
$p^{\#} = 8$	**1958-77**	σ {R^2}	0.232 {0.533}	**0.261 {0.410}**	0.315 {0.451}
	1978-96	σ {R^2}	0.302 {0.470}	**0.362 {0.357}**	0.300 {0.283}
$p^{\#} = 10$	1958-77	σ {R^2}	0.231 {0.531}	0.273 {0.423}	0.339 {0.415}
	1978-96	σ {R^2}	0.302 {0.470}	0.367 {0.349}	0.355@ {0.241}
$p^{\#} = 20$	1958-77	σ {R^2}	0.230 {0.528}	0.285 {0.458}	0.336 {0.399}
	1978-96	σ {R^2}	0.302 {0.469}	0.344 {0.337}	0.394@ {0.272}
$p^{\#} \to \infty$	1958-77	σ {R^2}	0.229 {0.528}	0.286 {0.465}	0.317 {0.403}
	1978-96	σ {R^2}	0.302 {0.469}	0.337 {0.335}	0.379@ {0.298}

Notes: Estimates of σ are based on panel data for 35 industries for the sub-samples indicated in the row headings. The data are filtered after the sub-sample is defined. Standard errors are heteroscedastic consistent using the technique of White (1980), are less than 0.02 for all entries with the exception of the σ's in column 3 marked with a @), and are not reported because all estimates of σ are statistically significant at the 1% level. A constant term and fixed time effects are included in the regression equation but are not reported. The R^2's are not comparable across cells because the dependent variable depends on $p^{\#}$ and q. The window is fixed at q = 3; our preferred estimate is for the equations for which the Low-Pass Filter parameter is $p^{\#} = 8$.

**Table 5: Benchmark And Alternative Models
Ordinary Least Squares Estimates Of
Equations (14), (24), and (25)
Alternative Dependent Variables
Various Critical Periodicities ($p^{\#}$) and q = 3**

		Dependent Variable		
		$\Delta ky^*_{i,t}$	$\Delta \ell y^*_{i,t}$	$\Delta k\ell^*_{i,t}$
		(1)	(2)	(3)
$p^{\#} = 2$	σ {R^2}	.206 {.476}	.233 {.214}	.109 {.417}
$p^{\#} = 4$	σ {R^2}	.278 {.519}	.239 {.249}	.168 {.419}
$p^{\#} = 6$	σ {R^2}	.296 {.454}	.298 {.276}	.207 {.393}
$p^{\#} = 8$	σ {R^2}	**.308 {.404}**	.353 {.310}	.245 {.387}
$p^{\#} = 10$	σ {R^2}	.311 {.390}	.379 {.328}	.255 {.384}
$p^{\#} = 20$	σ {R^2}	.304 {.392}	.398 {.341}	.239 {.377}
$p^{\#} \rightarrow \infty$	σ {R^2}	.302 {.394}	.399 {.340}	.233 {.375}

Notes: Estimates of σ are based on panel data for 35 industries for the period 1959-1996. Standard errors are heteroscedastic consistent using the technique of White (1980), are less than 0.02 for all entries, and are not reported because all estimates of σ are statistically significant at the 1% level. A constant term and fixed time effects are included in the regression equation, but are not reported. The R^2's are not comparable across cells because the dependent variable depends on $p^{\#}$ and q. The window is fixed at q = 3; our preferred estimate is for the equation in column (1) for which the Low-Pass Filter parameter is $p^{\#} = 8$.

Figure 1
Steady-State Relation Between
Growth in the Capital/Labor Ratio σ and ($\dot{\mathrm{KL}}/\mathrm{KL}$)
Equations (1), (5), (6), (7), and (8)

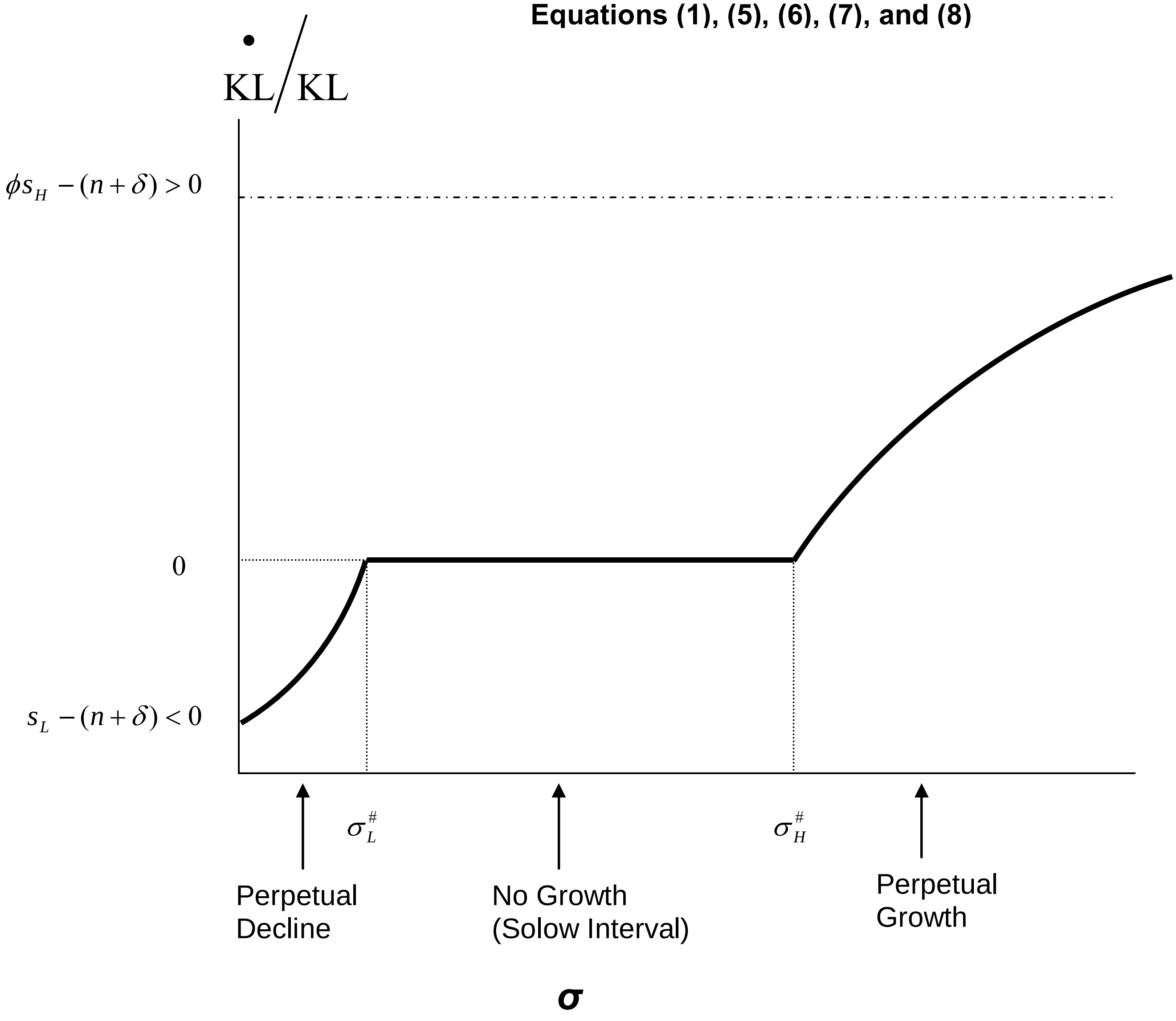

Figure 2
Frequency Response Of a[ω : p#, q]
Equation (17d)
Various Critical Periodicities (p#) With q=3

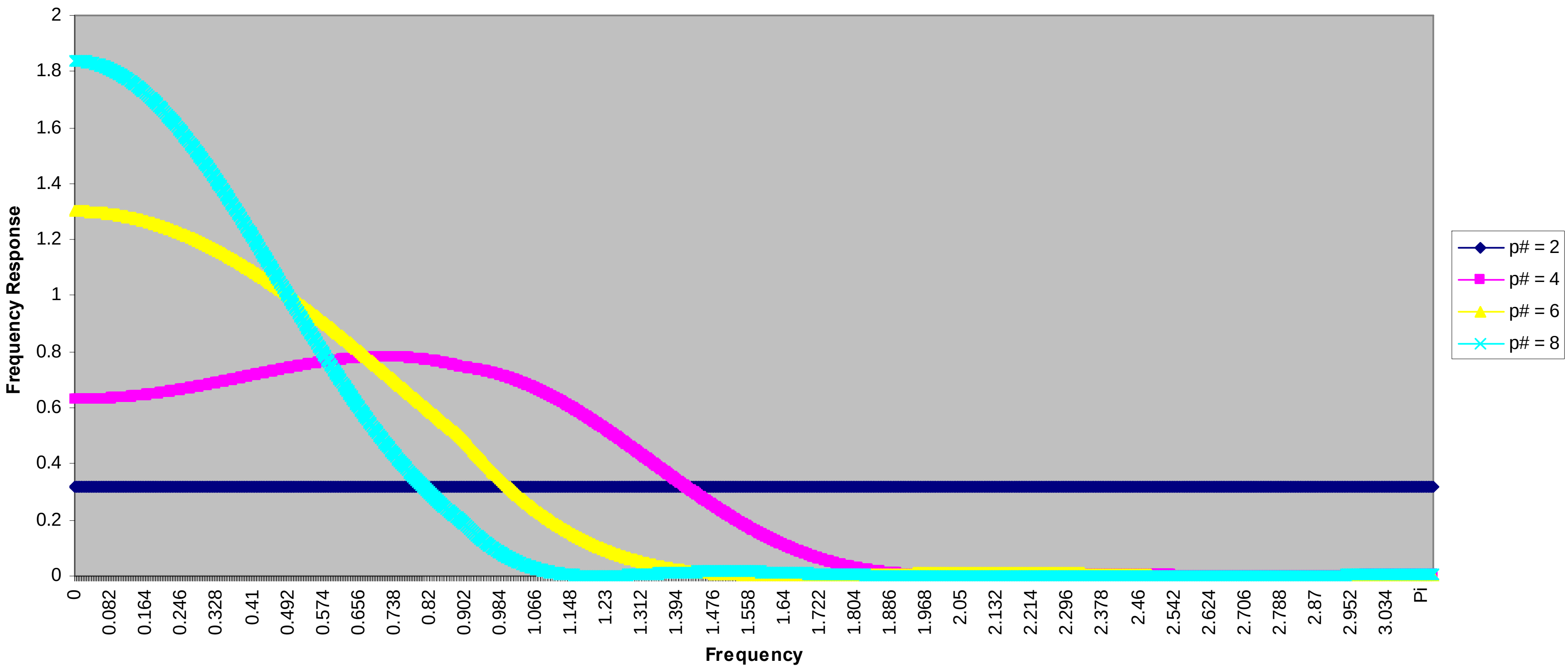

Figure 3
Frequency Response Of a[ω : p#, q]
Equation (17d)
For Various Critical Periodicities (p#) With q=3

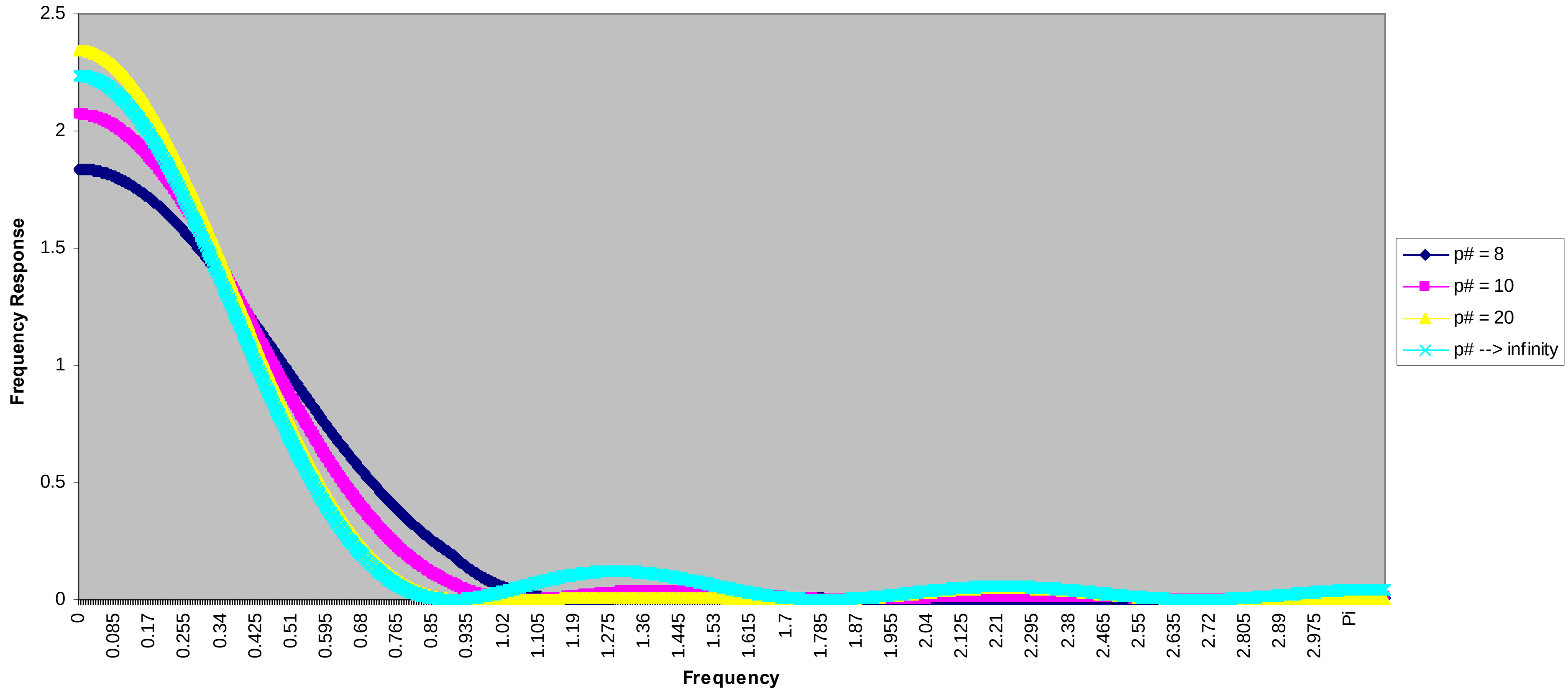

Figure 4
Frequency Response Of a[ω : p#, q]
Equation (17d)
For The Ideal LPF And Various Windows (q) With p# = 8