The Identity of the Generator in the Problem of Social Cost

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Abstract: One of Coase's central insights is that distinguishing between the generator and the recipient of an externality is of limited value because externality problems are reciprocal. We reconsider whether the identity of the generator matters in a model with transactions costs in the form of ex-ante non-contractible investments but frictionless bargaining over the externality and other variables ex post. In this framework, parties have a strategic incentive to invest. We provide natural definitions of the generator and the recipient of an externality which imply that these parties have differing strategic investment incentives. We show that socially optimal property rights for a generator should be weaker than those of a recipient. In particular, if the rights holder is the generator of the externality, there are cases in which it is more efficient to give the generator damage rights than (stronger) injunction rights. If the rights holder is the recipient it is never optimal to give it damage rights. Further, we show that the recipient rights holder will always strategically invest to make itself more vulnerable to harm, and that the generator rights holder will always strategically invest to produce more harm.

JEL Codes: K11, D23, C78, H23

1. Introduction

One of the most important insights in Coase's "Problem of Social Cost" is his emphasis on the reciprocal nature of problems where one party is perceived as imposing harm (such as smoke pollution) on another:

> The question is commonly thought of as one in which A inflicts harm on B and what has to be decided is: how should we restrain A? But this is wrong. We are dealing with a problem of a reciprocal nature. To avoid the harm to B would inflict harm on A. The real question that has to be decided is: should A be allowed to harm B or should B be allowed to harm A? The problem is to avoid the more serious harm. (Coase 1960, p. 2)

Viewed in this way, the concept of "the generator" of an externality is not useful for the economic analysis of bilateral externality problems. The term *generator* could equally be applied to a factory belching smoke or to the party who would adversely affect the factory by insisting on reduced smoke pollution. According to Coase, in a world with zero transactions costs, no special effort should be taken to restrain a polluting factory, impose damages, or to remove it from a particular location. Indeed, the famous Coase Theorem states that social surplus will be unchanged regardless of whether rights are allocated to a polluting factory, or to the party damaged by this pollution.

Below we present a model of bilateral externalities with transactions costs that arise because one party makes an investment choice before the other arrives. Such ex-ante invest-ment is therefore non-contractible, [1] though we assume that there is frictionless bargaining over the choice of externality and other variables ex-post, when the other party arrives. Our contribution is to show that in this world (which we feel to be quite descriptive of many real

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[1] i.e. the parties cannot write a contract on such investment if they have never met

situations) the concept of a generator of an externality is economically meaningful, because property rights should optimally change depending on the identity of the generator of an externality.

When ex-ante investment by the first party is non-contractible, different property rights lead to different strategic incentives depending on the characteristics of the first party. Our results support the general notion that a generator's rights should optimally be made *weaker* than a recipient's rights: Specifically, we show that if the first party rights holder is the generator, then if the impact on that party's total and marginal benefit from the externality is small, social welfare is lower when the generator is given the right to choose the level of the externality (i.e. an injunction right), than when it is allowed the weaker right to claim only damages. However, if the first-party rights holder is the recipient, then social welfare is *always* higher if the recipient holds injunction rather than damage rights. Here, there is also an interesting (seemingly counter-intuitive) effect in that the recipient rights-holder always strategically decides to make itself more vulnerable to pollution. It will under-invest if investment reduces its vulnerability to pollution (e.g. a resteraunt next to a disco chooses not sound proof its walls), and over-invest if investment increases its vulnerability to pollution (e.g. the resteraunt expands its premises, exposing more business to noise). In contrast, the generator will always threaten to pollute more: it over-invests if investment is in a 'dirty' technology, and under-invests if investment is in a 'clean' technology.

The paper provides a clear and rigorous definition of what is meant by the generator of an externality. The striking feature of our analysis is that the definition of generator we find to be economically meaningful is exactly the popular conception of the term, i.e. that the generator of an externality is a party who disrupts the natural state of things by imposing

some physical change on a commonly-shared resource such as air or water. Smoke courses through the air and into the shirts cleaned by a laundry. Polluted water poisons fish that a fisher tries to catch. Vibrations from a candy factory prevent a doctor from being able to hear a patient's heart-beat. The generator of an externalilty is an *active* party, who, if left to its own devices, would choose a positive level of pollution; an interior solution. In contrast, the recipient is passive in the sense that its activities are harmed by a physical transmission from the generator. The recipient's optimal *stand alone* level of pollution is zero; a corner solution.

The "activity" of the generator and the "passivity" of the recipient turn out to be crucial distinguishing features in our model in which transactions costs take the form of noncontractible ex ante investments. Parties make their ex ante investments strategically, with an eye toward their bargaining positions in ex post negotiations over the externality. A party obtains a larger share of the joint surplus from these negotiations the better its threat point and the worse the other party's threat point. Parties have an incentive to distort their investments relative to the social optimum to increase the harm with which they can credibly threaten the other party. If the recipient is given rights over the externality, it does not need to distort its investment to impose maximal harm on the generator. The recipient can credibly threaten to require the externality level to be zero because this is its ideal externality level, independent of how much the recipient has invested. The situation is different if the generator is given rights over the externality. The generator's ex ante investment affects its ideal externality level, and thus affects the harm it can threaten to cause the recipient.

These different strategic incentives underly the reasons why property rights will optimally vary according to the identity of the generator. Consider the example of a factory and a

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laundry. Assume that rights to pollute can be given to either party. These rights will allow the holder to threaten the other party with the holder's most preferred, i.e. *stand alone* or *threat-point*, choice of pollution. Suppose that the factory can choose either a clean or a dirty technology, and the laundry can choose either a small or a large operation. Assume that if the factory selects the dirty technology, its stand-alone choice of pollution will be higher than if it chooses the clean technology. The choice of dirty technology harms the laundry more than the choice of clean technology. In contrast, regardless of whether the laundry chooses a large or a small plant, its stand-alone choice is zero pollution. The laundry's choice does not alter the degree of harm that the factory suffers. In this illustrative example, the first-party generators rights are sometimes optimally weakened (damages rather than injunctions) in order to reduce its incentive to invest in the dirty technology. However, such a weakening of rights is never needed when the first party rights-holder is the recipient.

Our model is related to the incomplete-contracts literature begun in Grossman and Hart (1986) and Hart and Moore (1990) to explain ownership in a theory of the firm. As is standard in the incomplete-contracts literature, we assume that there are noncontractible investments ex ante, but bargaining is efficient ex post. The present paper is closest to our earlier work in Pitchford and Snyder (2003). Our earlier work focused on sequential location as a source of transactions costs and on the queston of whether it is more efficient for the court to assign property rights to the first mover or the second mover into an area. The efficiency of property rights did not depend on the identity of the generator in our earlier work because of assumptions in the model ensuring the generator's ideal externality level was not a function of its ex ante investment. In particular, we assumed that the externality was constrained to lie in a bounded set $[0, \bar{e}]$; the recipient's ideal externality level was at one corner, zero, and

the generator's ideal externality level was at the other corner, $\bar{e}$. In the present paper, we adopt the more natural assumption that the externality level is unbounded above, though it continues to be bounded below by zero. Hence, the generator's ideal externality level is an interior solution that in general depends on its ex ante investment. The assumptions in our earlier work simplified the analysis, but forced us to abstract from the strategic effects that are the focus of the present paper. The possibility that injunction rights may lead a party to distort its investment to increase the harm it can threaten another party in externality problems was noted informally by Mumey (1971), and is related to the extensive literature on blackmail (see, for early work, e.g., Landes and Posner 1975, Epstein 1983, Lindgren 1984, and for more recent work, e.g., Helmholz 2001, Posner 2001, Gomez and Ganuza 2002). A by-product of our analysis is a comparison of damages versus injunction rights, the topic of a large literature begun by Calabresi and Melamed (1972) and including Ayers and Talley (1995), Kaplow and Shavell (1995, 1996), and the *Yale Law Journal* symposium (Sherwin 1997).

2. Model

The model has two periods, an ex ante and an ex post period, two players $i = 1, 2$, and a court, which specifies and enforces a property-rights rule. In the ex ante period, the court specifies a property rights regime. Then player 1 becomes aware of an opportunity to sink investment expenditure $x_1 \in [0, \infty)$ in a specific location. The land on which player 1 invests is assumed to have been purchased in a competitive market at a price normalized to zero.[2]

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[2]The surplus functions in the ensuing discussion can be thought of as surplus net of the price of land. It is important to note that the competitive land price in a market with heterogenous buyers will be strictly less than the purchaser's valuation in most auction models.

Player 2 arrives in the ex post period. He has the opportunity to invest $x_2 \in [0, \infty)$ at a location near player 1. Location in the nearby area leads to a negative externality $e \in [0, \infty)$ between the players. We assume the players can engage in costless bargaining over x_2 and e, so that they end up choosing the levels which maximize their joint payoff. The sole transaction cost in the model is that players cannot bargain over x_1 this follows directly from our assumption of ex ante anonymity, i.e., that the identity of player 2 is unknown to player 1 when 1 makes its ex ante investment decision.

Let $u_i(x_i, e)$ be the gross surplus for player $i = 1, 2$. We will assume the following regularity conditions hold.

Assumption 1. $u_i(x_i, e)$ *is continuously differentiable in both arguments.*

Assumption 2. $u_i(x_i, e)$ *is strictly concave.*

Assumption 3. $u_i(x_i, e)$ *satisfies an Inada condition in x_i; i.e., $\partial u_i(0, e)/\partial x_i = \infty$ for all $e \in [0, \infty)$.*

Assumption 4. *The net surplus function $u_i(x_i, e) - x_i$ is coercive; i.e.,*

$$\lim_{\|x_i, e\| \to \infty} [u_i(x_i, e) - x_i] = -\infty,$$

where $\|x_i, e\| = \sqrt{x_i^2 + e^2}$ is the distance norm.

Assumptions 1 and 2 are standard. Assumptions 3 and 4 will ensure that the privately and socially optimal investment levels are in the interior of $[0, \infty)$. This is not essential for the results, but will allow us to state our propositions more elegantly with strict inequalities eliminate a number of economically uninteresting cases. Note that the assumption of coerciveness implies that both players' net surpluses become very negative if either the investment or externality level grow without bound.

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The notion of generator and recipient of a negative externality can now be made precise.

Definition. *Player i is the* recipient *of the negative externality e if $\partial u_i(x_i, e)/\partial e < 0$ for all $(x_i, e) \in [0, \infty)^2$.*

Definition. *Player i is the* generator *of the negative externality e if, for each $x_i \in [0, \infty)$, there exists a neighborhood around 0 such that $\partial u_i(x_i, e)/\partial e > 0$ for all e in the intersection of this neighborhood and $[0, \infty)$.*

To ensure that the privately and socially externality levels are in the interior of $[0, \infty)$, we

will make the following assumption.

Assumption 5. *If i is the generator, $u_i(x_i, e)$ satisfies an Inada condition in e: $\partial u_i(x_i, 0)/\partial e = \infty$ for all $x_i \in [0, \infty)$.*

Again, this assumption not essential but will simplify the analysis as described above.

The definitions imply that the recipient is always harmed by the externality. Its preferred

externality level is zero. Assumptions 2, 4, and 5 together imply that the generator's surplus

is initially increasing in e, reaching an interior optimum, and then declining for larger e.

The key feature of the recipient/generator distinction is that the recipient's preferred level

of the externality does not vary with its investment. Referring to Figure 1, an increase in

the recipient's investment from x_r' to x_r'' does not affect its preferred externality level, which

is a corner solution at zero. In contrast, an increase in the generator's investment will affect

its preferred externality level. The effect depends on the sign of the cross partial derivative

$\partial^2 u_i/\partial x_i \partial e$. In what is perhaps the leading case, $\partial^2 u_i/\partial x_i \partial e > 0$, implying that an increase

in the generator's investment from x_g' to x_g'' increases its marginal benefit from an additional

unit of e, in turn implying that its preferred externality level increases. (This case would

arise, for example, if investment increases the size of the generator's facility; the larger the

Figure 1: Gross surpluses as a function of the externality

what is perhaps the leading case, $\partial^2 u_i / \partial x_i \partial e > 0$, implying that an increase in the generator's investment from x'_g to x''_g increases its marginal benefit from an additional unit of e, in turn implying that its preferred externality level increases. (This case would arise, for example, if investment increases 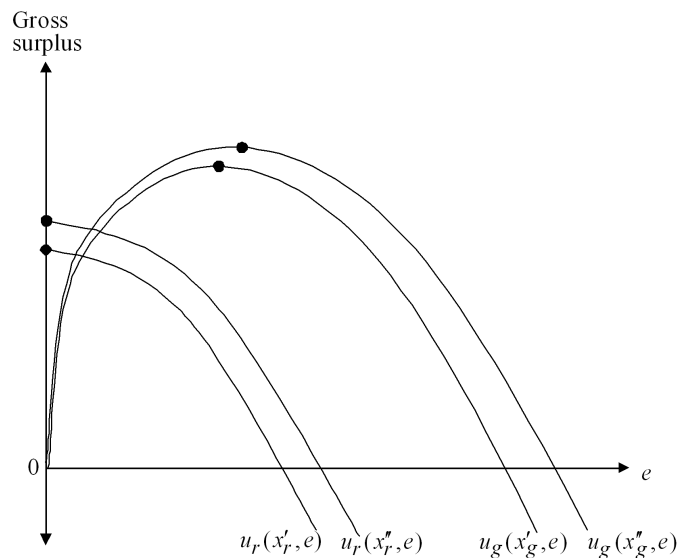the size of the generator's facility; the larger the facility, the more pollution generated when the facility is run at optimal capacity.) In this case, by investing more, the generator can increase the harm it can credibly threaten to inflict on the recipient. The absence of this strategic effect with the recipient, and its presence with the generator, is the fundamental asymmetry that leads to all of our subsequent results.

Let $v_1(x_1, e) = u_1(x_1, e) - x_1$ be 1's surplus net of investment. Let

$$v_2(e) = \max_{x_2 \in [0, \infty)} \{u_2(x_2, e) - x_2\}$$

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mally. Define

$$e_1^*(x_1) = \underset{e \in [0,\infty)}{\mathrm{argmax}}\{v_1(x_1, e)\}$$

$$e_2^* = \underset{e \in [0,\infty)}{\mathrm{argmax}}\{v_2(e)\}$$

$$e_J^*(x_1) = \underset{e \in [0,\infty)}{\mathrm{argmax}}\{v_1(x_1, e) + v_2(e)\}.$$

In words, $e_1^*(x_1)$ and e_2^* are the privately optimal externality levels in the players' stand-alone

problems, and $e_J^*(x_1)$ is the joint optimum.

Player 1's ex ante choice of x_1 affects both players' equilibrium allocations through the

bargain that takes place between players ex post. As mentioned, we assume efficient bar-

gaining, in particular the version of Nash (1950) bargaining in Binmore, Rubinstein, and

Wolinsky (1986) involving an exogenous probability of breakdown ex post. Let $\alpha \in (0, 1)$

be player 1's share of the gains from Nash bargaining and $1 - \alpha$ be 2's share. If bargain-

ing breaks down, the default or threat-point outcome is determined by the property-rights

regime specified by the court ex ante. That is, a breakdown in bargaining leaves the players

to select x_2 and e according to the property-rights specified by the court. Let $t_i(x_1)$ be player

i's threat-point payoff. As we will see, the threat-point payoffs are potentially functions of

x_1 for both players, for player 1 because player 1's surplus is a direct function of x_1, for

player 2 indirectly through the ex post choice of e. Since default choices of e are typically

inefficient, the parties will bargain to the ex-post efficient choice of e. Let $s(x_1)$ denote the

resulting maximized joint surplus:

$$s(x_1) \quad = \quad \max_{e \in [0,\infty)} \{v_1(x_1, e) + v_2(e)\} \tag{1}$$

$$= \quad v_1(x_1, e_J^*(x_1)) + v_2(e_J^*(x_1)). \tag{2}$$

Player 1's equilibrium surplus from Nash bargaining is the sum of its threat point $t_1(x_1)$ and α times the gains from bargaining $s(x_1) - t_2(x_1) - t_2(x_1)$, which upon rearranging equals

$$(1 - \alpha)t_1(x_1) + \alpha s(x_1) - \alpha t_2(x_1). \tag{3}$$

Our accounting convention nets out the cost of player 1's ex ante investment in expression (3), so (3) already reflects player 1's surplus from an ex ante perspective. It is the relevant objective function player 1 maximizes when choosing x_1. We only need to specify player 1's ex ante payoff function because 1's choice of x_1 is the only welfare-relevant one in the model. All other variables (x_2 and e) are chosen optimally ex post conditional on x_1 due to efficient bargaining.

3. Property Rights

The court sets the property-rights regime ex ante. Property rights enter the model through the threat points $t_i(x_i)$. The threat points enter into 1's objective function (3), which in turn determines x_1. In this section we will define various property-rights regimes. In the next section we will analyze the efficiency of these regimes as indexed by the ex ante investment level x_1.

Property rights are multidimensional, depending on the variables the holder is allowed to choose, the penalty for infringement, and the identity of the holder. For example, property rights can be conditioned on the period in which the players show up. Property rights are often allocated to the first mover into a location, following the so-called "coming to the nuisance" doctrine. In theory, however, property rights could also be allocated to the second mover. To simplify the analysis by reducing the number of different regimes we need to consider, we will restrict attention throughout the paper to the case in which property rights are allocated to the first mover (player 1 in our model). As argued in Pitchford and Snyder (2003), allocating property rights to the second mover may not be efficient because it can lead to underinvestment in x_1 and can lead to an wasteful delay game.

We will further restrict attention to two commonly-studied property-rights regimes, injunctions and damages. An injunction regime gives the holder the right to set e if bargaining breaks down. If player 1 is the injunction-rights holder, it would set e to maximize its stand-alone payoff, i.e., it would choose externality level $e_1^*(x_1)$. The threat-point payoffs are $t_1(x_1) = v(x_1, e_1^*(x_1))$ and $t_2(x_1) = v_2(e_1^*(x_1))$.

We formalize damage rights in the following way. The damage-rights holder does not have the right to set e—the other player does—but has the right to extract a payment equal to the difference between its surplus if the externality level were set at its preferred level less its realized surplus. More concretely, if player 1 is the damage-rights holder, player 2 has the right to set e but must pay player 1 $u_1(x_1, e_1^*(x_1)) - u_1(x_1, e)$. Player 1's threat-point payoff equals its realized surplus $u_1(x_1, e) - x_1$ plus the damage payment, which upon rearranging, equals $t_1(x_1) = v_1(x_1, e_1^*(x_1))$. Note that $t_1(x_1)$ is independent of the level of e that player 2 chooses if bargaining breaks down, so $t_1(x_1)$ can be computed without explicitly solving for

Table 1: Threat-Point Payoffs for the Property-Rights Regimes under Consideration

First-mover property-rights regime	Player 1's threat-point payoff $t_1(x_1)$	Player 2's threat-point payoff $t_2(x_1)$
Injunction rights	$v_1(x_1, e_1^*(x_1))$	$v_2(e_1^*(x_1))$
Damage rights	$v_1(x_1, e_1^*(x_1))$	$s(x_1) - v_1(x_1, e_1^*(x_1))$

this e. This is not true of player 2's threat point; to compute $t_2(x_1)$, we need to solve for 2's optimal choice of e if bargaining breaks down. This choice maximizes 2's surplus minus the damage payment, which upon rearranging equals

$$u_1(x_1, e) + v_2(e) - u_1(x_1, e_1^*(x_1)). \tag{4}$$

It is straightforward to see that expression (4) is maximized by setting e to the joint optimum $e_J^*(x_1)$. Substituting $e_J^*(x_1)$) for e in (4) and rearranging, we have $t_2(x_1) = s(x_1) - v_1(x_1, e_1^*(x_1))$.

Table 1 lists the threat points for reference. Note that $t_1(x_1)$ is equal across the two rights regimes; the rights regimes only differ in the specification of $t_2(x_1)$. Throughout the remainder of the paper, we will refer to a first-mover injunction regime simply as "injunctions" and a first-mover damages regime simply as "damages".

4. Analysis

The goal of our paper is to show how the generator of an externality differs, in an economically meaningful way, from the recipient of an externality. To do this, we consider the social ranking of rights regimes in the case in which player 1 is the recipient (Proposition 2) and compare this ranking with the case in which player 1 is the generator (Proposition 3). Before turning to Propositions 2 and 3, we prove Proposition 1 as a preliminary result. Proposition 1 verifies that the recipients preferred level of the externality is zero and that the socially preferred level lies strictly between the recipients and generators preferred choices. The results in Proposition 1 will be used in the proofs of the subsequent propositions.

Proposition 1. *(a) If player 1 is the generator and player 2 is the recipient, then* $0 = e_2^* < e_J^*(x_1) < e_1^*(x_1)$. *(b) If player 1 is the recipient and player 2 is the generator, then* $0 = e_1^*(x_1) < e_J^*(x_1) < e_2^*$.

The next proposition states that if player 1 is the recipient, injunctions are socially more efficient than damages because player 1's ex ante investment is closer to the first best. Whether injunctions are more efficient because they lead to less underinvestment or less over-investment depends on the interaction between the recipient's investment and the externality, which in formal terms depends on the sign of the second cross partial $\partial^2 u_1(x_1, e)/\partial e \partial x_1$. A series of definitions related to the sign of this cross partial will allow us to state Proposition 2 in succinct terms.

Definition. *Investment* increases recipient i's vulnerability *if an increase in* x_i *increases* i's *marginal harm from the externality; i.e.,* $\partial^2 u_i(x_i, e)/\partial e \partial x_i < 0$.

For an example of this definition, consider a case in which the recipient is a laundry which dries clothes outside on a clothesline. An increase in the size of its business potentially

exposes more laundry to harm from smoke pollution. We refer to this sort of investment as increasing the laundry's vulnerability.

Alternatively, the recipient could invest in a way that makes it less vulnerable. For example, consider a case of a laundry that can invest in modernizing its drying technology from outdoor clotheslines to indoor electric dryers. As the following definition states, we refer to this sort of investment as decreasing the laundry's vulnerability.

Definition. *Investment* reduces recipient i's vulnerability *if an increase in x_i reduces i's marginal harm from the externality; i.e., $\partial^2 u_i(x_i, e)/\partial e \partial x_i > 0$.*

Let x_1^D be player 1's equilibrium ex ante investment under a damage-rights regime, x_1^I that under an injunction-rights regime, and x_1^F in the first best. We have the following proposition.

Proposition 2. *Suppose player 1 is the recipient and player 2 is the generator. Injunctions yield higher social surplus than damages. Social welfare is strictly less than in the first best in both. If player 1's investment reduces its vulnerability, then there is underinvestment in both regimes relative to the first best, with $x_1^D < x_1^I < x_1^F$. If player 1's investment increases its vulnerability, then there is overinvestment in both regimes relative to the first best, with $x_1^F < x_1^I < x_1^D$.*

The intuition for the efficiency component of this result can be seen by comparing the marginal benefits from investment that the recipient faces under each regime with the first best. Under damages, substituting the threat points from Table 1 into player 1's surplus from Nash bargaining in expression (3), the recipient's net payoff after bargaining equals $v_1(x_1, 0)$. Differentiating with respect to x_1 yields the marginal benefit $\partial v_1(x_1, 0)/\partial x_1$. This differs from the marginal social benefit from investment $\partial s(x_1)/\partial x_1 = \partial v_1(x_1, e_J^*(x_1))/\partial x_1$, and so investment is inefficient under damages. Under damages, the recipient is always fully compensated. Consequently it does not account for the fact that the externality will be set at

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the joint surplus maximizing level $e_J^*(x_1)$. Under injunctions, the recipient's net payoff after bargaining is $(1-\alpha)v_1(x_1,0)+\alpha s(x_1)$. Differentiating with respect to x_1 yields the marginal benefit from investment $(1-\alpha)\partial v_1(x_1,0)/\partial x_1 + \alpha\partial s(x_1)/\partial x_1$. This is a weighted-average of the first-order condition under damages and in the first best, and so yields a solution closer to the first best than under damages.

The choice of investment by the recipient is somewhat counterintuitive: the proposition says that under both injunctions and damages, if player 1 is the recipient, it distorts its investment in the direction of making itself more vulnerable than in the first best. One's first thought might be that the distortion would be in the direction of excessive protection against the externality. The reason player 1's investment is distorted in the other direction is that being the rights holder insulates player 1 somewhat from harm from the externality. Under damages, the recipient is fully insulated since it is compensated for any harm from the externality. Under injunctions, it is partially insulated since its equilibrium surplus depends in part on its threat point, and in its threat point it has the authority to set the externality level. Figures 2 and 3 illustrate the mathematics behind the result. Consider the case of damages. As mentioned above, player 1's marginal private benefit under damages is $\partial v_1(x_1,0)/\partial x_1$. If investment reduces its vulnerability, as in Figure 2, then the function $\partial v_1(x_1,e)/\partial x_1$ is decreasing in e. Therefore, the marginal social benefit from investment, $\partial v_1(x_1,e_J^*(x_1))/\partial x_1$, lies below player 1's marginal private benefit under damages, and there is overinvestment. If investment increases player 1's vulnerability as in Figure 3, then the marginal social benfit lies above player 1's marginal private benefit, and there is overinvestment. When player 1 is the recipient, the marginal benefit for the injunctions case always lies between the damages case and the first best, as shown in the figures.
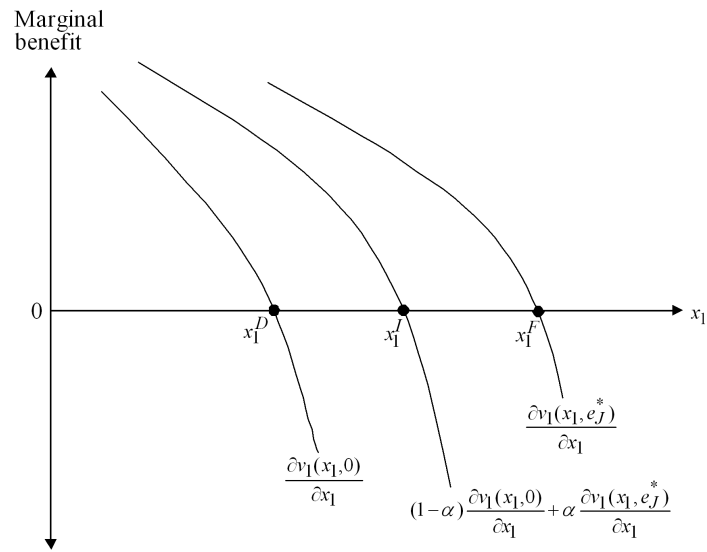
Marginal
benefit

0   x_1^D x_1^I x_1^F x_1

$\dfrac{\partial v_1(x_1, e_J^*)}{\partial x_1}$

$\dfrac{\partial v_1(x_1, 0)}{\partial x_1}$ $(1-\alpha)\dfrac{\partial v_1(x_1, 0)}{\partial x_1} + \alpha\dfrac{\partial v_1(x_1, e_J^*)}{\partial x_1}$

Figure 2: Underinvestment by the recipient when investment decreases its vulnerability

Marginal
benefit

0   x_1^F x_1^I x_1^D x_1

$\dfrac{\partial v_1(x_1, 0)}{\partial x_1}$

$\dfrac{\partial v_1(x_1, e_J^*)}{\partial x_1}$ $(1-\alpha)\dfrac{\partial v_1(x_1, 0)}{\partial x_1} + \alpha\dfrac{\partial v_1(x_1, e_J^*)}{\partial x_1}$
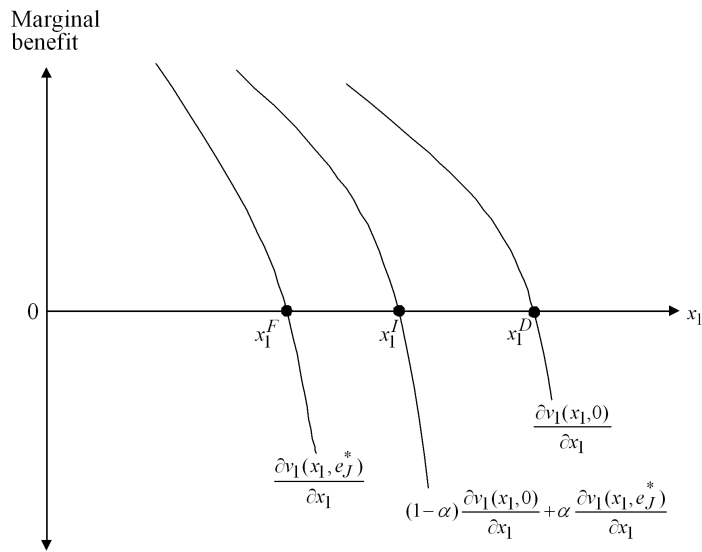
Figure 3: Overinvestment by the recipient when investment increases its vulnerability

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The next proposition addresses the case in which the first party is the generator. The next

two definitions concern the type of technology that the generator can adopt. An investment is

said to be dirty if higher levels increase the generator's marginal benefit from the externality.

For example consider a factory that produces smoke pollution in fixed proportions to its

output. If it expands the scale of its operation, then it will naturally increase the benefit it

receives from an extra unit of pollution. Alternatively, an investment is called clean if an

increase in investment reduces the marginal benefit from pollution. This could occur, for

example, investment involves more modern technologies that are less polluting. Formally,

we have the following definitions.

Definition. *Generator i's investment is* clean *if an increase in x_i reduces i's marginal benefit from the externality; i.e., $\partial^2 u_i(x_i, e)/\partial e \partial x_i < 0$.*

Definition. *Generator i's investment is* dirty *if an increase in x_i increases i's marginal benefit from the externality; i.e., $\partial^2 u_i(x_i, e)/\partial e \partial x_i > 0$.*

Proposition 3. *Suppose player 1 is the generator and player 2 is the recipient. Suppose $v_1(x_1, e) = g(x_1) + \gamma h(x_1, e)$ for some $g(x_1)$ satisfying Assumptions 1–4; for some $h(x_1, e)$ satisfying Assumptions 1–5; and for $\gamma > 0$. If player 1's investment is dirty (with the preceding functional forms implying $\partial h(x_1, e)/\partial e \partial x_1 > 0$), there exists $\tilde{\gamma}$ such that for all $\gamma < \tilde{\gamma}$, $x_1^F < x_1^D < x_1^I$, and social welfare is higher with damages than with an injunction. If player 1's investment is clean (with the preceding functional forms implying $\partial h(x_1, e)/\partial e \partial x_1 < 0$), there exists $\tilde{\tilde{\gamma}}$ such that for all $\gamma < \tilde{\tilde{\gamma}}$, $x_1^I < x_1^D < x_1^F$, and, again, social welfare is higher with damages than with an injunction.*

Given the functional form for player 1's surplus, $v_1(x_1, e) = g(x_1) + \gamma h(x_1, e)$, as γ becomes

small, the impact of the externality on its total and marginal payoff becomes negligible,

whereas the choice of externality by the generator is unaffected by γ since $e_1^*(x_1)$ solves

$\partial h(x_1, e_1^*(x_1))/\partial e \equiv 0$. In other words, player 1 does not benefit much from polluting, but

since the benefit is positive, its ideal pollution level can remain relatively high and under

an in injunction it can continue credibly to threaten the other party with substantial harm from the externality. Thus, the strategic incentive that the generator has to harm the recipient through its choice of externality does not vanish. Under damages, as γ becomes small, player 1's marginal investment incentives, $\partial g(x_1)/\partial x_1 + \gamma \partial h(x_1, e_1^*(x_1))/\partial x_1$ approach $\partial g(x_1)/\partial x_1$; i.e., marginal investment incentives are negligibly influenced by the externality. In the first best, marginal social investment incentives, $\partial g(x_1)/\partial x_1 + \gamma \partial h(x_1, e_J^*(x_1))/\partial x_1$, also approach $\partial g(x_1)/\partial x_1$. Therefore, social welfare under damages approaches the first best.

5. Conclusion

The central results of the paper are Propositions 2 and 3. Proposition 2 states that if player 1 is the recipient of the externality and is the rights holder, it is socially more efficient to give it injunction rights than damage rights. If player 2 is the generator, a new strategic effect arises with injunction rights in that player 1 will have an incentive to distort its investment to alter its ideal externality level and threaten more harm to the other player. This strategic effect does not arise with damages because the externality is set by player 2 (in return for compensation for harm). Proposition 3 states that there are limiting cases in which investment under damage rights approaches the first best, but player 1's incentive to distort its investment in order to threaten harm to player 2 does not vanish under injunctions. Taken together, Propositions 2 and 3 imply that social welfare can be improved by conditioning rights on the identity of the generator. In this sense, the distinction between the generator and the recipient can be economically meaningful in externality problems.

Appendix

Proof of Proposition 1: We will prove part (a). The proof of part (b) is similar and thus omitted. Suppose player 1 is the generator and player 2 is the recipient. Then $u_2(x_2, e)$ is strictly decreasing in e by definition of the recipient, implying $v_2(e)$ is strictly decreasing in e. Thus $e_2^* = \text{argmax}_{e \in [0, \infty)} \{v_2(e)\} = 0$. Assumption 5 implies $e_J^*(x_1) > 0$. The proof is completed by showing $e_J^*(x_1) < e_1^*(x_1)$. Consider the nested objective function

$$u_1(x_1, e) + v_2(e) - \theta v_2(e), \tag{5}$$

where $\theta = 0$ yields the objective function for $e_J^*(x_1)$ and $\theta = 1$ that for $e_1^*(x_1)$. We proceed by verifying the conditions required for Strict Monotonicity Theorem 1 of Edlin and Shannon (1998) hold for expression (5). Expression (5) is continuously differentiable because the individual terms are continuously differentiable by Assumption 1. Assumptions 4 and 5 imply $e_1^*(x_1)$ is an interior solution. The second cross partial of expression (5) with respect to e and θ equals $-v_2'(e) > 0$. Hence, (5) exhibits increasing marginal returns. Thus, Strict Monotonicity Theorem 1 of Edlin and Shannon (1998) applies, implying $e_J^*(x_1) < e_1^*(x_1)$. $\square$

Proof of Proposition 2: Suppose player 1 is the recipient. We will prove the proposition for the case in which player 1's investment reduces its vulnerability, i.e., $\partial^2 u_1(x_1, e)/\partial e \partial x_1 > 0$. The proof for the case in which player 1's investment increases its vulnerability is similar and thus omitted.

We will first prove $x_1^D < x_1^I$. Substituting the threat points associated with a damages regime (see Table 1) into expression (3) implies that player 1's ex ante equilibrium surplus under damages equals

$$v_1(x_1, e_1^*(x_1)). \tag{6}$$

Similarly, it can be shown that player 1's ex ante equilibrium surplus under an injunction equals

$$(1 - \alpha) v_1(x_1, e_1^*(x_1)) + \alpha s(x_1) - \alpha v_2(e_1^*(x_1)). \tag{7}$$

Nesting (6) and (7), player 1's objective function, determining its ex ante investment, can be written

$$v_1(x_1, e_1^*(x_1)) + \theta[s(x_1) - v_1(x_1, e_1^*(x_1)) - v_2(e_1^*(x_1))], \tag{8}$$

where $\theta = 0$ under damages and $\theta = \alpha$ under an injunction. We proceed by verifying the conditions required for Strict Monotonicity Theorem 1 of Edlin and Shannon (1998) hold for (8). Expression (8) is continuously differentiable since the separate terms are continuously differentiable by Assumption 1. Assumptions 3 and 4 imply that x_1^D is an interior solution.

The second cross partial with respect to x_1 and θ equals

$$s'(x_1) - \frac{dv_1(x_1, e_1^*(x_1))}{dx_1} - \frac{dv_2(e_1^*(x_1))}{dx_1} \tag{9}$$

$$= s'(x_1) - \frac{\partial v_1(x_1, 0)}{\partial x_1} \tag{10}$$

$$= \frac{\partial v_1(x_1, e_J^*(x_1))}{\partial x_1} - \frac{\partial v_1(x_1, 0)}{\partial x_1} \tag{11}$$

$$= \int_0^{e_J^*(x_1)} \frac{\partial v_1^2(x_1, e)}{\partial x_1 \partial e} de. \tag{12}$$

Equation (10) holds since player 1 is the recipient, so $e_1^*(x_1) = 0$ by part (b) of Proposition 1. Since $e_1^*(x_1)$ is a constant, the derivative in the third term of (9) is zero. Equation (11) holds by applying the Envelope Theorem to find the derivative $s'(x_1)$ using the definition of $s(x_1)$ in equation (2). Equation (12) is positive since $\partial^2 u_1(x_1, e)/\partial e \partial x_1 > 0$ by maintained assumption, implying $\partial^2 v_1(x_1, e)/\partial e \partial x_1 > 0$, and since $e_J^*(x_1) > 0$ by Proposition 1. Hence, expression (8) exhibits increasing marginal returns in x_1 and θ. Strict Monotonicity Theorem 1 applies to (8), implying $x_1^D < x_1^I$.

Next, we will show $x_1^I < x_1^F$. The objective function in the first best is $s(x_1)$. This can be nested with the objective function under an injunction in expression (7) as follows:

$$s(x_1) + \theta \left[s(x_1) - v_1(x_1, e_1^*(x_1)) + \left(\frac{\alpha}{1-\alpha}\right) v_2(e_1^*(x_1)) \right], \tag{13}$$

where $\theta = -(1-\alpha)$ under injunctions and $\theta = 0$ in the first best. Arguments paralleling those in the preceding paragraph can be used to show that the Strict Monotonicity Theorem 1 applies to expression (13), implying $x_1^I < x_1^F$.

To complete the proof, we need to translate the investment ranking into a social-welfare ranking. By Assumption 2, $u_2(x_2, e) - x_2$ is strictly concave. Furthermore it is maximized over a convex set $x_2 \in [0, \infty)$. By the Maximum Theorem under Convexity (see, e.g., Sundaram 1996, Theorem 9.17.3), the associated value function $v_2(e)$ is also strictly concave. By Assumption 2, $u_1(x_1, e)$ is strictly concave, implying $v_1(x_1, e)$ is strictly concave. The sum of strictly concave functions $v_1(x_1, e) + v_2(e)$ is strictly concave. By the Maximum Theorem under Convexity, the associated value function $s(x_1)$ is strictly concave. Therefore, the ranking $x_1^D < x_1^I < x_1^F$ implies damages are strictly less efficient than an injunction, which in turn is less efficient than the first best. $\square$

Proof of Proposition 3: Suppose player 1 is the generator. Suppose $v_1(x_1, e) = g(x_1) + \gamma h(x_1, e)$ for some $g(x_1)$ satisfying Assumptions 1–4; for some $h(x_1, e)$ satisfying Assumptions 1–5; and for $\gamma > 0$. We will prove the proposition for the case in which player 1's investment is dirty, implying $\partial^2 h(x_1, e)/\partial e \partial x_1 > 0$ given our functional form assumption. The proof for the case in which player 1's investment is clean is similar and thus omitted.

We will first show $x_1^F < x_1^D$ for all $\gamma > 0$. Substituting our functional form for v_1 into the threat points associated with the damage regime listed in Table 1 and then substituting the resulting threat points into expression (3) yields the following expressio for player 1's ex

ante equilibrium surplus under damages:

$$g(x_1) + \gamma h(x_1, e_1^*(x_1)). \tag{14}$$

Substituting our functional form for v_1 into equation (2) yields the following expression for the social welfare function in the first best:

$$g(x_1) + \gamma h(x_1, e_J^*(x_1)) + v_2(e_J^*(x_1)). \tag{15}$$

Nesting the objective functions (14) and (15),

$$g(x_1) + \gamma h(x_1, e_J^*(x_1)) + v_2(e_J^*(x_1)) + \theta \left[\gamma h(x_1, e_1^*(x_1)) - \gamma h(x_1, e_J^*(x_1)) - v_2(e_J^*(x_1)) \right], \tag{16}$$

where $\theta = 0$ for the first best and $\theta = 1$ for the damages regime. We proceed by verifying that the conditions required for Strict Monotonicity Theorem 1 of Edlin and Shannon (1998) hold for (16). Since the component functions are continuously differentiable by Assumption 1, expression (16) is continuously differentiable. Assumptions 3 and 4 imply that $e_J^*(x_1)$ is an interior solution. The second cross partial of (16) with respect to x_1 and θ equals

$$\frac{d}{dx_1} \left[\gamma h(x_1, e_1^*(x_1)) \right] - \frac{d}{dx_1} \left[h(x_1, e_J^*(x_1)) + v_2(e_J^*(x_1)) \right] \tag{17}$$

$$= \gamma \left[\frac{\partial h(x_1, e_1^*(x_1))}{\partial x_1} \right] - \gamma \left[\frac{\partial h(x_1, e_J^*(x_1))}{\partial x_1} \right] \tag{18}$$

$$= \gamma \int_{e_J^*(x_1)}^{e_1^*(x_1)} \frac{\partial^2 h(x_1, e)}{\partial x_1 \partial e} de. \tag{19}$$

The first (respectively, second) term of equation (18) follows from differentiating the first (respectively, second) term in square brackets in (17) using the Envelope Theorem. Equation (19) is positive since $\partial^2 h(x_1, e)/\partial x_1 \partial e > 0$ by maintained assumption and since $e^* J(x_1) < e_1^*(x_1)$ by part (a) of Proposition 1. Hence, expression (16) exhibits increasing marginal returns in x_1 and θ. Strict Monotonicity Theorem 1 thus applies to (16), implying $x_1^F < x_1^D$.

Next, we show that there exists $\tilde{\gamma} > 0$ such that $x_1^D < x_1^I$ for all $\gamma \in (0, \tilde{\gamma})$. Substituting our functional form for v_1 into player 1's objective function under an injunction, expression (7), and nesting with its objective function under damages, expression (14), yields

$$g(x_1) + \gamma h(x_1, e_1^*(x_1, \gamma))$$
$$+ \theta \left[\gamma h(x_1, e_J^*(x_1, \gamma)) + v_2(e_J^*(x_1, \gamma)) - \gamma h(x_1, e_1^*(x_1, \gamma)) - v_2(e_1^*(x_1, \gamma)) \right], \tag{20}$$

where $\theta = 0$ under damages and $\theta = \alpha$ under an injunction. We have added an argument to $e_1^*(x_1, \gamma)$ and $e_J^*(x_1, \gamma)$ to reflect their dependence on parameter γ, which we will vary in the comparative statics exercise to follow. Using arguments similar to those in the preceding paragraph, we can verify that all the conditions required for Strict Monotonicity Theorem 1 hold for (20) hold for all $\gamma > 0$ except for increasing marginal returns. We will verify that increasing marginal returns in x_1 and θ hold for (20) for sufficiently small $\gamma > 0$ by verifying that the second cross partial of (20) with respect to x_1 and θ is positive in the limit as $\gamma \to 0$.

This second cross partial is

$$\frac{d}{dx_1}\left[\gamma h(x_1, e_J^*(x_1, \gamma)) + v_2(e_J^*(x_1, \gamma))\right] - \frac{d}{dx_1}\left[\gamma h(x_1, e_1^*(x_1, \gamma)) + v_2(e_1^*(x_1, \gamma))\right] \quad (21)$$

$$= \gamma\left[\frac{\partial h(x_1, e_J^*(x_1, \gamma))}{\partial x_1}\right] - \gamma\left[\frac{\partial h(x_1, e_1^*(x_1, \gamma))}{\partial x_1}\right] - v_2'(e_1^*(x_1, \gamma))\frac{\partial e_1^*(x_1, \gamma)}{\partial x_1}. \quad (22)$$

The first term in equation (22) comes from applying the Envelope Theorem to compute the derivative in the first term of (21). The second and third terms in equation (22) come from applying the Envelope Theorem to compute the derivative in the second term of (21). In the limit as $\gamma \to 0$, the first and second terms of (22) vanish, implying that the sign of (22) is determined by the sign of the third term. Since player 2 is the recipient, $v_2'(e_1^*(x_1, \gamma)) < 0$. Monotone comparative statics arguments similar to those used in the proof of Proposition 1 can be used to prove that, under the maintained assumption that player 1's investment is dirty, $\partial e_1^*(x_1, \gamma)/\partial x_1 > 0$. Hence the third term of (22) is positive, implying (22) is positive for sufficiently small $\gamma > 0$.

Using arguments paralleling those in the last paragraph of the proof of Proposition 2, we can show that the investment ranking translates into a social welfare ranking, so that damages are socially more efficient than an injunction for sufficiently small $\gamma > 0$. $\square$

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