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7/20/06
10:00 AM

Measuring Aggregate Productivity Growth Using Plant-level Data

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First Version: December, 2003
Current Version: April, 2006

Abstract. We define productivity growth as the change in welfare that arises from additional output holding primary inputs constant. Using this traditional growth-accounting definition, we show that gains may arise because of plant-level technology shocks, and, in imperfectly competitive settings, from the reallocation of inputs across plants with differing markups and/or shadow values of primary inputs. With plant-level data, the alternative and most popular definition of productivity growth looks at the difference in the first moments of the productivity distribution. We show that this definition adds two terms to the growth-accounting measure, including one volatile term that has been called "reallocation." We show there is a very weak relationship between the two indexes in almost every 3-digit manufacturing industry in both Chile from 1987-1996 and Colombia from 1981-1991 - 49 in total - primarily because of this "reallocation" term. We explore the theoretical reasons for this sharp divergence, in the process uncovering a number of previously unnoticed and unattractive features of the first-moment definition. For example, it is not tethered to any theoretical model and it can report positive productivity growth when welfare has fallen.

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1. Introduction

Proper measurement of productivity growth is at the heart of a wide range of fields in economics. With the increasing availability of plant-level data, there is a large and rapidly growing body of research that estimates plant-level productivity and then aggregates.[1] Typically, plant-level productivity is measured using the total factor productivity (TFP) residual ($ln\omega$), computed as (log) output (lnQ) minus the contribution of inputs ($\beta' lnX$), or

$$ln\omega = lnQ - \beta' lnX, \tag{1}$$

where (1) represents the gross-output production function. In this paper, we ask (and answer) "Can we aggregate these residuals in a way that yields a measure akin to welfare change?"

In many settings the answer is "yes" if we define aggregate productivity growth in historical, growth-accounting terms. Let i index the N plants in the economy, P_i denote i's output price, and

This paper grew out of initial work in "When Industries Become More Productive, Do Firms?: Investigating Productivity Dynamics" (NBER Working Paper 6893), and appeared in an earlier form as "On the Micro-Foundations of Productivity Growth." We are grateful to the Russell Sage Foundation and the University of Chicago GSB for support and for helpful discussions from Bert Balk, Susanto Basu, Eric Bartelsman, Jeff Campbell, Steve Davis, Erwin Diewert, John Fernald, John Haltiwanger, Charles Hulten, Mitsukuni Nishida, Wendy Petropoulos, Mark Roberts, Johannes Van Biesebroeck, Alwyn Young, the NBER Productivity meetings, the Chicago Applied Micro Lunch, the Society for Economic Dynamics and the Econometric Society Meetings.

[1] The data sets used include U.S. data from the Longitudinal Research Database (LRD) of the U.S. Census, French data from the Declarations Annuelles des Salaires (DAS) collected by INSEE (Institut National de la Statistique et des Etudes Economiques), and several plant-level manufacturing censuses from developing countries, to name but a few. Examples of papers using the U.S. data include Bailey, Hulten, and Campbell (1992), Olley and Pakes (1996), Bernard and Jensen (1999), Bernard, Eaton, Jensen, and Kortum (2003), and Foster, Haltiwanger, and Krizan (2001). Abowd, Kramarz, and Margolis (1999) and Eaton, Kortum, and Kramarz (2004) use the French data while the several papers in Roberts and Tybout (1996) use data from developing countries. A careful bibliography would include dozens of papers.

Y_i equal the amount of i's output going to final demand.[2] Then aggregate productivity growth is the sum of "Domar-weighted" gross-ouput productivity residuals:

$$d\Omega = \sum_{i=1}^{N} \frac{P_i Q_i}{\sum_{i=1}^{N} P_i Y_i} dln\omega_i = \sum_{i=1}^{N} D_i dln\omega_i, \tag{2}$$

where $D_i = \frac{P_i Q_i}{\sum_{i=1}^{N} P_i Y_i}$ (Domar (1961)). This can be rewritten as

$$d\Omega = \sum_{i=1}^{N} \frac{P_i Q_i}{\sum_{i=1}^{N} P_i Y_i} \frac{d\omega_i}{Q_i} = \sum_{i=1}^{N} \frac{P_i d\omega_i}{\sum_{i=1}^{N} P_i Y_i},$$

where $d\omega_i$ is defined as the level of "net output at i," output remaining after the contribution of primary and intermediate inputs has been deducted at i, and P_i up-weights this additional output by its market value.[3]

In competitive settings $d\Omega$ exactly equals the change in society's ability to consume and invest holding primary inputs constant (Hulten (1978)). In an imperfectly competitive world, it also reflects shifts in inputs from "low-value" to "high-value" firms (Basu and Fernald (2002)). The welfare interpretation makes this measure useful for cost-benefit/policy analysis, and for the meta-analysis aggregating many studies, as it is readily comparable across time, industries, and countries.[4]

We observe data discretely so some approximation for period $T-1$ to T is needed. Tornqvist (1936), Theil (1967), Hulten (1973), Diewert (1976), Star and Hall (1976), Trivedi (1981), and Balk (2005), among others, have argued for multiplying the growth rate of productivity from $T-1$ to T, estimated using $\Delta ln\omega_{iT} = ln\omega_{iT} - ln\omega_{i,T-1}$, by the average of the weights from the beginning and ending period:

$$\widehat{d\Omega_T} = \sum_{i} \frac{D_{iT} + D_{i,T-1}}{2} * \Delta ln\omega_{iT}. \tag{5}$$

This is widely known as a Tornqvist-Divisia quantity index.

[2] Intermediate deliveries are given as $Q_i - Y_i$.

[3] If conditions from Bruno (1978) hold, one can use the value added production function residuals with its numerically equivalent representation

$$d\Omega = \sum_{i=1}^{N} s_{v_i} dln\omega_i^v, \tag{3}$$

where $dln\omega_i^v$ denotes the instantaneous growth rate in the residual from the value-added production function and s_{v_i} denotes the plant's share of aggregate value added.

[4] Hulten (2001) provides a thoughtful and detailed history of this index.

The second approach to aggregating productivity residuals - the one that is almost exclusively used - is some variant of the Bailey, Hulten, and Campbell (1992) index (the "BHC Index"):

$$BHC_T = \sum_i s_{G_{iT}} ln\omega_{iT} - \sum_i s_{G_{i,T-1}} ln\omega_{i,T-1}. \tag{6}$$

where $s_{G_{iT}}$ the gross-output share of the plant i in the economy at time T.[5] This index is the difference in first moments between period $T-1$ and T, with the "level" of aggregate productivity defined as the share-weighted average of log-level productivities.[6] The widespread use of this index and many suggested variants can be found in the extensive literature review in Foster et al. (2001).

Our main empirical result is that these two measures differ significantly in their assessment of productivity growth at the plant-level and in the aggregate. Using panel data for the 49 biggest manufacturing industries from Chile and Colombia, we show that industry-by-industry regressions of the growth-accounting index on the BHC index yield R^2s that range from between 0.2 to 0.4, indicating that these indexes are not linear transformations of one another.[7]

We explore the theoretical reasons for this sharp divergence. For firms existing in period $T-1$ and T BHC can be decomposed as

$$BHC_T = \widehat{d\Omega_T} + \sum_i Err_{iT} + \sum_i \overline{ln\omega_{iT}} * \Delta s_{G_{iT}}, \tag{7}$$

where Err_{iT} is the error arising from use of the wrong share weight

$$Err_{iT} = \frac{(s_{G_{iT}} - D_{iT}) + (s_{G_{i,T-1}} - D_{i,T-1})}{2} * \Delta ln\omega_{iT}, \tag{8}$$

and

$$\overline{ln\omega_{iT}} * \Delta s_{G_{iT}} = \frac{(ln\omega_{iT} + ln\omega_{i,T-1})}{2} * (s_{G_{iT}} - s_{G_{i,T-1}}),$$

with $\overline{ln\omega_i}$ the average of $ln\omega_{i\cdot}$ across periods.[8] Thus, BHC is equal to the growth-accounting

[5] Typically, BHC variants use industry shares. We assume here, to the benefit of the BHC index, that it is the share of gross output in the aggregate economy (this is a scaling issue).

[6] Olley and Pakes (1996) use exponentiated log-levels of productivity.

[7] Overall, these findings are robust to a number of different methods of estimating plant-level TFP, including ordinary least squares, Solow's approach, and the Levinsohn and Petrin (2003) proxy approach.

[8] See Griliches and Regev (1995), Fox (2003), and Diewert and Fox (2005), all of whom use Bennet (1920). The main variant of the BHC decomposition is given by

$$\sum_i lns_{G_{i,T-1}} * (ln\omega_{iT} - ln\omega_{i,T-1}) + \sum_i ln\omega_{iT} * (s_{G_{iT}} - s_{G_{i,T-1}}).$$

which is similar to (7), and for this reason it will share the problems of (7) (plus one additional problem because it is not (7)).

measure *plus* an error term arising because the D_i weights are not used, *plus* the average log-level of productivity multiplied by the change in gross-output share.

There are theoretical reasons to believe that the bias arising from these errors may be large. Using the gross output share weight s_{G_i} introduces error when $P_i Q_i \neq P_i Y_i$ for any firm i, that is, when there are intermediate deliveries between plants in the economy. The greater the share of intermediate deliveries in the economy, the larger the error. In most economies, plant deliveries to other plants are a significant proportion of gross output.

Practitioners typically characterize the second error, $\sum_i \overline{ln\omega_i} * \Delta s_{G_{iT}}$, as reallocation's role in productivity growth. To our knowledge it has never been derived from any theory model. Empirically, as in our data, reallocation is often large and volatile, dominating the BHC productivity index.

Our view is the fundamental root of all of the problems with BHC-type indexes is its lack of a link to some model. It is easy to construct examples where the BHC index is *negatively* correlated with welfare, or where the BHC index reports productivity growth from reallocation when in fact there is none. There is a wide debate in this literature about whether the labor share or gross output share is most appropriate. Our paper shows this debate is immediately resolved in the growth-accounting framework, and that neither of these proposed weights are appropriate if welfare is the objective.[9] In summary, we believe that researchers should motivate their productivity measure using a model that makes its interpretation clear.[10] This is especially true if one wants to undertake comparisons of the contribution of "real productivity" vs. "reallocation" to aggregate productivity growth, as is often done in the literature using the BHC index.

Real Productivity versus Reallocation Revisited

We revisit the question of what reallocation is if welfare enhancing productivity growth is our index. We focus on the case in which there are non-constant returns to scale, with firms taking (possibly different) input prices as given, and with a fixed set of final goods. This setting nests the most common theoretical models of reallocation.[11] In a competitive setting, there is no increase in welfare that arises from simply reshuffling inputs across firms with different productivity levels, as

[9] Bartelsman and Gray (1996) is the only paper of which we are aware that uses Domar weights in this literature.

[10] See Olley and Pakes (1996) for a measure based on efficient allocation of inputs across plants.

[11] See the review of the theoretical literature in Foster et al. (2001).

marginal revenue products of inputs are aligned across firms. Here again, the BHC index fails, as its reallocation term will generally not be zero in the base case of perfect competition.

Hall (1990) and Basu and Fernald (2002) show that reallocation effects only arise as we move away from a frictionless/perfectly-competitive world. Specifically, under imperfect competition, distortions exist that lead to input allocations that are not welfare maximizing. For example, an input's marginal revenue product (MRP) may differ across producers. In this setting changes in welfare can occur as inputs are reallocated from the "low MRP" firms to the "high MRP" firms.

If firms minimize costs and take input prices as given,[12] Lemma 3 shows that aggregate productivity growth can be decomposed into four terms, one derived from the plant-level technology shocks, and three other terms arising from the reallocation of inputs across plants with different markups and/or shadow values of primary inputs. In the competitive limit, these distortions are eliminated, and there is no welfare gain from reallocation.

We suggest also looking at the *change* in the productivity growth rate to further illuminate the underlying dynamics of reallocation and real productivity growth. If productivity growth is measured as in (5), we show that the change in growth decomposes into three terms. One term represents the change in the aggregate growth rate from changes in plant-level productivity growth. A second term represents the change in the growth rate from the reallocation of output across plants with differing productivity levels. The final term represents a net entry effect. In our data, the reallocation effect is almost always present, economically important, reasonably stable, and almost always works to increase the growth rate in aggregate productivity, even in the instances where the aggregate growth rate in productivity is falling.

Entry and Exit

We also revisit the issue of entry and exit, which is often given as the motivation for using the BHC index. We show that another attractive feature of the growth-accounting measure from (2) is that conceptually neither entry nor exit raises any difficulties. We argue that no model has been put forward to motivate the BHC index treatment of entry and exit. We compare the treatment of entry and exit between the BHC measure and the growth-accounting measure and show that the two differ dramatically.

A missing values problem arises because we do not observe the exact time between $T-1$ and T at which entry/exit occurs, nor do we observe the path of the entrant/exiter's productivity growth

[12] These conditions are satisfied by a large class of theoretical models that permit reallocation effects.

rate.[13] We bound the error that arises from this missing information, and we provide estimators for the actual productivity contribution from entrants and exiters. In the process we show that the error in the aggregate from the missing data is small if these plants contribute only a small fraction to value added or have stagnant productivity growth– a proviso that is true in our annually observed data.

Sections 2-4 develop the economic model and the measurement methods. Section 5 addresses entry and exit and Section 6 introduces the change in productivity growth index. Readers interested only in the discussion of the widely used BHC index (and problems with it) as well as the associated empirical results can skip directly to Sections 7-9.

2. Aggregate Productivity Growth: Perfect Competition

We start in a competitive setting, with no frictions, constant returns to scale, and a fixed set of final goods. This canonical case is a starting point for the more realistic models that allow for imperfections, non-constant returns to scale, and/or frictions. The competitive case provides three messages. First, Lemma 1 shows that (2) exactly measures the change in welfare that arises as society's ability to consume and invest increases in response to plant-level technology shocks (Hulten (1978)). Second, the result provides the appropriate aggregation weights for measuring welfare change in both the competitive and imperfectly competitive case. Finally, Lemma 1 shows that even when inputs are reallocated, there is no "reallocation" effect when marginal revenue products for inputs are equal across firms.

Let the constant returns-to-scale technology be given by $Q_i = F^i(M^i, X^i, t)$, where $(M^i = M_1^i, \ldots, M_N^i)$ and $(X^i = X_1^i, \ldots, X_K^i)$ denote respectively intermediate and primary inputs used at firm i. We suppress the t index for notational convenience. Let P_i denote the output price and $Y_i = Q_i - \sum_{j=1}^N M_i^j$ equal final deliveries of good i. Then P_iY_i is i's contribution to final demand, and aggregate final demand is given by

$$\sum_{i=1}^N P_iY_i = \sum_{i=1}^N P_i(Q_i - \sum_{j=1}^N M_i^j) = \sum_{i=1}^N P_iQ_i - \sum_i P_i(\sum_{j=1}^N M_i^j),$$

the difference between the aggregate value of gross output and the aggregate expenditure on intermediate inputs.

[13] For example, the Tornqvist-Divisia approximation requires observing data at both $T-1$ and T.

From Solow (1957), total differentiation of lnQ_i and optimization together yield

$$dlnQ_i = \sum_{j=1}^{N} \beta_j^i dlnM_j^i + \sum_{k=1}^{K} \beta_k^i dlnX_k^i + dln\omega_i \quad = \sum_{j=1}^{N} s_{M_j^i} dlnM_j^i + \sum_{k=1}^{K} s_{W_k^i} dlnX_k^i + dln\omega_i \quad i = 1, \ldots, N, \tag{9}$$

where $\beta_j^i = s_{M_j^i} = \frac{P_j M_j^i}{P_i Q_i}$ and $\beta_k^i = s_{W_k} = \frac{W_k X_k^i}{P_i Q_i}$; the elasticities of output with respect to the intermediate and primary inputs are equal to the revenue shares under perfect competition. Plant-level technical efficiency is

$$dln\omega_i = \frac{\partial F^i / \partial t}{F^i},$$

the well-known Solow residual. Lemma 1 shows that aggregate productivity growth $d\Omega$ is given by:

$$d\Omega = \sum_{i=1}^{N} \frac{P_i Q_i}{\sum_i P_i Y_i} dln\omega_i. \tag{10}$$

Domar (1961) first proposed these aggregation weights.[14] In his words, they are defined in order to allow one to:

> be free to take the economy apart, to aggregate one industry with another, to integrate final products with their inputs, and to reassemble the economy once more and possibly over different time units without affecting the magnitude of the Residual.

This greatly facilitates policy analysis because results are comparable across studies. Subsets of plants in the economy - such as industries or recent entrants/exiters - can be considered separately, and their relationship to the aggregate is directly known. Hulten (1978) proves (10) for sector-level production functions (the type of data available in the 1970's) and Lemma 1 restates his argument when i indexes plants instead of sectors.

[14] The plant-level efficiency shocks must be grossed up by the ratio of plant gross output to aggregate value added. The story is as follows. When i's output is part intermediate input, some of plant i's output is used as input at other plants, so $dln\omega_i > 0$ leads both to an increase in final demand for i and to an increase in i's intermediate deliveries. These new intermediate input deliveries are used in production at the plants to which they are delivered, where they increase output. This new output will both increase final demand and fulfill more intermediate deliveries elsewhere. The process continues. In the end, the greater the role of intermediate inputs used in the economy, the larger $\frac{\sum_{i=1}^{N} P_i Q_i}{\sum_i P_i Y_i}$ is, and the larger the impact of increases in plant-level technical efficiency on final demand.

Lemma 1

Let $(Y = Y_1, \ldots, Y_N)$, $(M = M_1, \ldots, M_N)$, and $(P = 1, \ldots, P_N)$ denote vectors of real final demand, intermediate input, and normalized output prices for the N plants in the economy. Let $(X = X_1, \ldots, X_K)$ and $(W = W_1, \ldots, W_K)$ denote vectors of primary inputs and their factor prices. Define the social production possibilities frontier implicitly using $F(Y, X, t) = 0$. Assume 1) $F(\cdot)$ is continuously differentiable and homogeneous of degree zero in (Y, X), 2) that the economy is in a competitive equilibrium, and 3) the technology of each firm is characterized by constant returns to scale $Q_i = F^i(M^i, X^i, t)$ $i = 1, \ldots, N$, with $(M^i = M^i_1, \ldots, M^i_N)$ and $(X^i = X^i_1, \ldots, X^i_K)$. Then the instantaneous shift in the social production possibilities frontier is the difference between the Divisia index of aggregate final demand and the Divisia index of total primary inputs:

$$d\Omega = \sum_{i=1}^{N} \frac{P_i Y_i}{\sum_{i=1}^{N} P_i Y_i} \frac{dY_i}{Y_i} - \sum_{k=1}^{K} \frac{W_k X_k}{\sum_{k=1}^{K} W_k X_k} \frac{dX_k}{X_k}, \tag{11}$$

and (11) is equal (10).

The proof follows Hulten directly and is provided in Appendix A. The growth rate in aggregate final demand is given by the sum across plants of their net output share times their growth rate in final demand. To hold primary input use constant, aggregate productivity growth deducts the aggregate growth rate of primary inputs, which is given by the sum across the primary inputs of the aggregate primary input share times the growth rate the aggregated primary input. Lemma 1 shows (10) and (11) are equal.

Equation (10) can be directly expressed in terms of the share of firm value added, denoted $s_{v_i} = \frac{P_i Y_i}{\sum_i P_i Y_i}$, and the residual from the value-added production technology. Let $s_{M^i} = \frac{\sum_{j=1}^{N} P_j M^i_j}{P_i Q_i}$ be the revenue share of intermediate inputs.

Lemma 2

$$d\Omega = \sum_{i=1}^{N} s_{v_i} dln\omega^v_i, \tag{12}$$

where $dln\omega^v_i = \frac{dln\omega_i}{(1 - s_{M^i})}$.

See Appendix A for proof.

No Reallocation under Perfect Competition

If an input has the same marginal revenue product across producers, we say there is no welfare gain from reallocating this input. More generally, if all inputs satisfy this condition, then there are no welfare gains from the reallocation of any inputs. We say an index satisfies the *Zero-Reallocation Criterion* when, in this case, it reports zero productivity growth from input reallocation. This is true in the competitive case, as an input's marginal revenue product is equal across producers, so primary inputs can be reallocated with no productivity/welfare reallocation effect. None of the BHC-type indexes satisfies the *Zero-Reallocation Criterion*.

3. Aggregate Productivity Growth: Imperfect Competition

Let $K = \sum_i K_i$, $L = \sum_i L_i$, and let aggregate final demand be given as $Y = \sum_i P_i Y_i$. With possibly quasi-fixed inputs let P_{L^i} and P_{K^i} reflect the shadow values of labor and capital respectively at each firm i.[15] Basu and Fernald (2002) use Hall (1990) to link the change in aggregate productivity, defined as

$$d\Omega = dlnY - \frac{\sum_i P_{K^i} K_i}{Y} dlnK - \frac{\sum_i P_{L^i} L_i}{Y} dlnL, \tag{13}$$

to changes in plant-level technical efficiencies and their effect on input reallocation across plants.[16] They also show that $d\Omega$ provides a first order approximation to the change in welfare from productivity shocks when a representative agent model reasonably approximates the demand side. We summarize these results in Lemmas 3 and 4 respectively.

The Micro Link to the Aggregate

Define the input index for plant i, I_i, as

$$dlnI_i = s_{K^i} dlnK_i + s_{L^i} dlnL_i + \sum_{j=1}^{N} s_{M_j^i} dlnM_j^i,$$

where s_{L^i}, s_{K^i}, and s_{M^i} denote the revenue shares, and let the markup be given as

$$\mu_i = P_i / MC_i,$$

with MC_i denoting marginal cost. Let $P_L = \frac{\sum_i P_{L^i} L_i}{\sum_i L_i}$ and P_K analogously, as the "average" shadow values of primary inputs across firms. Lemma 3 links (13) to plant-level data. It does not require profit-maximization, making the measure robust to any type of static or dynamic price-setting behavior.

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15 Berndt and Fuss (1986) show that productivity must be measured using the shadow rental price of the input as opposed to the unit price.In a frictionless setting these two prices are equal.

16 Hall (1990) shows that the productivity residual under imperfect competition reflects more than just the effects of technical progress.

Lemma 3

If firms minimize costs and take input prices as given, then plant-level gross output can be written as

$$dlnQ_i = \mu_i dlnI_i + dln\omega_i. \qquad (14)$$

The measured plant-level productivity residual is given as

$$dlnQ_i - dlnI_i = (\mu_i - 1)dlnI_i + dln\omega_i. \qquad (15)$$

Aggregate productivity growth is equal to

$$\begin{aligned} d\Omega &= \frac{1}{Y} * \sum_{i=1}^{N} \left[D_i * (dlnQ_i - dlnI_i) + L_i * (P_{L^i} - P_L)dlnL_i + K_i * (P_{K^i} - P_K)dlnK_i\right] \\ &= \frac{1}{Y} * \left[\sum_{i=1}^{N} D_i * dln\omega_i + \sum_{i=1}^{N} D_i * (\mu_i - 1)dlnI_i + \sum_{i=1}^{N} L_i * (P_{L^i} - P_L)dlnL_i + \sum_{i=1}^{N} K_i * (P_{K^i} - P_K)dlnK_i.\right] \end{aligned} \qquad (16)$$

See Appendix A for proof. (16) shows that aggregate productivity growth is additively separable in four terms, with changes in technical efficiency being only one component under imperfect competition. The other three terms reflect gains in productivity arising from the reallocation of inputs across plants and we review each of these in turn.

The first reallocation term in (16) arises because only $dlnI_i$ is deducted from $dlnQ_i$ (instead of $\mu_i dlnI_i$). If primary or intermediate inputs are reshuffled to plants with higher markups this term serves to increase aggregate productivity growth. The final two terms relate to shadow values of labor and capital. Aggregate productivity growth increases if there is a reallocation of inputs from plants with lower shadow values to plants with higher shadow values. Overall, any reallocation of inputs towards firms with higher markups and/or shadow values of capital or labor cause both productivity and welfare to increase.

In the case when markets are competitive, μ_i equals one for all i, and all firms share a common shadow value for each of the inputs, so all reallocation terms in this case are zero. Welfare change is equal to $\sum D_i dln\omega_i$, the rate of productivity growth under perfect competition. Thus aggregate productivity growth in the imperfectly competitive case is in the limit - when prices converge to marginal cost and shadow values converge to unit prices - aggregate productivity growth under perfect competition.

Welfare

For the welfare result provided by Lemma 4 we must define a general equilibrium framework. Suppose there are $N-1$ consumption goods and an Nth investment good. Consumption at time t of good j is denoted $C_{t,j}$. $\overline{L}$ is the per-period endowment of labor, A_t are assets, P_{It} is the price of the investment good, P_{Lt} and P_{Kt} are the wage and rental price of capital, δ is the depreciation rate of capital, K, and Π_t are pure profits, which are rebated to consumers.[17]

Lemma 4

Suppose a large number of identical households choose consumption, investment, and labor to maximize lifetime utility:

$$Max\ U = \sum_{s=0}^{\infty} \beta^s u(C_{1,t+s}, C_{2,t+s}, \ldots, C_{N-1,t+s}, \overline{L} - L_{t+s})$$

such that

$$A_{t+1} = A_t + P_{Lt}L_t + P_{Kt}K_t + (1-\delta) * (P_{I,t+1} - P_{It})K_t + \Pi_t - \sum_{i=1}^{N-1} P_{i,t}C_{i,t}.$$

Let a temporary one-period shock change assets, consumption, and/or labor supply. Then a first-order approximation to the change in utility is proportional to

$$d\Omega_t = dlnY_t - \frac{P_{Kt}K_t}{Y_t} dlnK_t - \frac{P_{Lt}L_t}{Y_t} dlnL_t. \tag{17}$$

See Appendix.

Basu and Fernald (2002) provide a proof that allows for much more general settings. Lemma 4 is provided to illustrate the intuition, and it shows that the change in welfare is proportional to the change in aggregate productivity, defined as the aggregate growth rate of value added minus the weighted aggregate growth rates of primary inputs, where the weights are their revenue share of value added in the aggregate.

The result is a direct consequence of the envelope theorem. Consumers are optimized prior to the shock, so their marginal rates of substitution between goods are equal to relative market prices, including the marginal rate of substitution between labor and leisure.[18] The change in output net of inputs that arises from the shock - valued at market prices - provides a first order approximation to welfare (see Appendix A for a simple example).

[17] For simplicity we ignore private bonds, which in equilibrium will equal zero because arbitrage provides for an interest rate leading households to be indifferent between holding a bond and holding capital.

[18] The marginal rates of substitution are not equal to the marginal rates of transformation because of markups.

4. Estimating Productivity Growth

In this section we focus on estimation of aggregate productivity growth from period $T-1$ to period T, as defined by

$$\Omega_{[T-1,T]} = \int_{T-1}^{T} \sum_i D_{it} dln\omega_{it}. \tag{18}$$

This is a Divisia index, and is equal to the integral of (2) over the period $T-1$ to T. There are two steps to estimating (18). Plant-level productivity residuals must be recovered. Then they must be aggregated. We discuss each in turn.

We show that our findings are robust to two prominent and complementary methods that are frequently used to estimate plant-level productivity residuals. One method begins with an assumed functional form for the production function and then directly estimates its parameters.[19] The second approach uses the Solow (1957) insight that optimizing behavior implies observed revenue shares are equal to the elasticity of output with respect to the inputs.

Data are observed discretely, requiring some discrete time approximation to the integral in (18). For productivity measurement the most popular discrete-time approximations are given by log-change indexes.[20] In this case, $\Delta ln\omega_{iT} = ln\omega_{iT} - ln\omega_{i,T-1}$ is used as the approximation to the average growth rate between $T-1$ and T. In the approximation to (18), a weight α_{iT} is typically applied to the estimated average growth rate at each firm, and then aggregated across plants, yielding:

$$\sum_i \alpha_{iT} \Delta ln\omega_{iT}.$$

For α_{iT}, Tornqvist (1936) advocates averaging beginning and ending period values. This yields

$$\alpha_{iT} = 1/2 * [D_{iT} + D_{i,T-1}] = 1/2 * \left[\frac{P_{iT}Q_{iT}}{\sum_i P_{iT}Y_{iT}} + \frac{P_{i,T-1}Q_{i,T-1}}{\sum_i P_{i,T-1}Y_{i,T-1}} \right].$$

In the context of the value-added residual $dln\omega_i^v$, the Tornqvist weight is given by

$$\alpha_{iT} = \frac{s_{v_{iT}} + s_{v_{i,T-1}}}{2}.$$

Theil (1967), Hulten (1973), Diewert (1976), Star and Hall (1976), Trivedi (1981), and Balk (2005), among others, have all argued for this approximation.[21]

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[19] One can use ordinary least squares or one of many alternatives that attempt to address the simultaneity of input choices and productivity raised by Marschak and Andrews (1944). Alternatives include (for example) instrumental variables, fixed effects, and the proxy methods of Olley and Pakes (1996) and Levinsohn and Petrin (2003).

[20] Sato (1987) in New Palgrave describes the properties of these approximations.

[21] Readers (albeit perhaps a select few) may recognize the use of the average of the starting and ending share as Newton's trapezoid rule for approximating the area under a continuous curve.

Lemma 5

The Tornqvist approximation to the Divisia index is superlative (or without error) if the underlying function has the homogeneous translog form. With constant returns to scale the Tornqvist approximation to the Divisia index is uniquely superlative.

See Diewert (1976), who uses the quadratic approximation lemma to single out the Tornqvist approximation as most preferred. When the underlying function is not translog, the Tornqvist approximation remains attractive because the translog form provides a second-order approximation.

Trivedi develops approximation results for a more general class of functions.

Lemma 6

The error in the Tornqvist approximation to the Divisia index is on the order of the square of the length of the time interval.

See Trivedi (1981). Note that Paasche and/or Laspeyres log change approximations use $\alpha_{iT} = D_{iT}$, the ending period share (Paasche), or $\alpha_{iT} = D_{i,T-1}$, the beginning period share (Laspeyres). They will generally have more approximation error because they both ignore one of the two available pieces of information on the path from $T-1$ to T of D_{it}. Lemmas 5 and 6 are the reasons we prefer the Tornqvist approximation to the Divisia index.[22]

5. Accounting for Entry and Exit

One important issue that does arise with the move from aggregate to plant-level data is that plants enter and exit (while, e.g., industries do not). Neither entry nor exit raises any conceptual difficulties for the aggregate productivity growth measure proposed in Lemma 1 when the setting is competitive and we operate in continuous time. However, new goods that enter at prices below their reservation value do raise the standard questions for welfare measurement, as discussed in Petrin (2002) and Goolsbee and Petrin (2004).[23]

Let $1_{i,Continue}$, $1_{i,Enter}$, and $1_{i,Exit}$ be indicator variables for whether firm i is a continuing firm, an entrant, or an exiter respectively for the period $T-1$ to T. Firm i's contribution to aggregate

[22] Before moving on we note that we have not addressed a more fundamental (although perhaps less well-known) question. (18) is a line integral, so we must either observe the path followed between $T-1$ and T, which we do not, or we must make do with observations at $T-1$ and T, in which case the value of the line integral must be independent of the path taken. Hulten (1973) explores the necessary and sufficient conditions for (18) to satisfy path independence. Petrin (2005) discusses the implications of these conditions for using plant-level data. See also Balk (2003).

[23] Petrin (2005) addresses this question when plant-level data are observed and the change in welfare is the object of interest.

productivity growth from period $T-1$ to T is then given as

$$1_{i,Continue}\int_{T-1}^{T} D_{it}\ dln\omega_{it} + 1_{i,Exit}\int_{T-1}^{t^*} D_{it}\ dln\omega_{it} + 1_{i,Enter}\int_{t^*}^{T} D_{it}\ dln\omega_{it}, \tag{19}$$

where t^* denotes the time of entry or exit and $D_{it^*} = 0$. Specifically, a continuing firm contributes $\int_{T-1}^{T} D_{it}\ dln\omega_{it}$, an exiting firm contributes $\int_{T-1}^{t^*} D_{it}\ dln\omega_{it}$, and an entrant contributes $\int_{t^*}^{T} D_{it}\ dln\omega_{it}$.

Since observations on plants are only made at discrete intervals, for plants that enter and exit the choice of discrete approximation must be revisited because we do not have enough information to calculate the Tornqvist-Divisia quantity. In particular, while we know that $D_{it^*} = 0$ at the time of entry/exit and from that time backward/forward, we do not observe the exact time between $T-1$ and T at which entry/exit occurs, nor do we observe i's productivity level at this time. This presents a problem for any measure that is based on following the path of plant-level productivity.

An advantage of the growth-accounting framework is that it tells us exactly how much error is added, and it suggests both bounds on the error and approximations to it. Before turning to these measurements, we note that many economic questions relate to the performance of entrants or exiters. Even in the worst case scenario in which all entrants (exiters) are truncated from the productivity calculation in their entering (exiting) year, one can compute these plants' contribution to aggregate productivity growth in the other years of their existence. Thus, for annual data, categories for recent entrants and/or upcoming exiters can be defined separate from continuing plants, their fraction of productivity growth being then directly comparable to continuing plants. *In fact, a principle feature of the growth-accounting measure is that we are free to group plants in any subaggregates that we desire without affecting the measure of aggregate productivity growth.*

Using (19), it is possible to bound the error that arises from this missing information. Let the contribution of all continuing plants to aggregate productivity growth be given by:

$$C = \sum_{i=1}^{N} 1_{i,Continue}\int_{T-1}^{T} D_{it}\ dln\omega_{it}.$$

Similarly, let exiters' contribution be

$$EX = \sum_{i=1}^{N} 1_{i,Exit}\int_{T-1}^{t^*} D_{it}\ dln\omega_{it} \tag{20}$$

and entrants

$$EN = \sum_{i=1}^{N} 1_{i,Enter}\int_{t^*}^{T} D_{it}\ dln\omega_{it}. \tag{21}$$

Then the true value for aggregate productivity growth over the period [T-1,T] is bounded:

$$\Omega_{[T-1,T]} \in [C - |EX| - |EN|, C + |EX| + |EN|]. \tag{22}$$

The error in aggregate productivity growth from truncating entrants and exiters will be small if $|EX|$ and $|EN|$ are small. $|EX|$ and $|EN|$ are likely to be small when these plants contribute only a small fraction to value added or have stagnant productivity growth.

The fraction of value added that is attributable to these plants can be directly calculated from the data. In the two annual censuses from Chile and Colombia that we use in the empirical work, continuing plants account for between 94% and 98% of industry value added. When data are observed at 5 year intervals, the fraction of value added accounted for by entrants and exiters ranges from 10% to 40%. Thus, with annual data no additional correction may be necessary, but with data collected at 5 year intervals some approximation to the contribution to growth from entrants and exits may be desirable. We turn to this question next.

Approximations

We develop the approximations using the Tornqvist-Divisia measure as our estimation objective. There are two related issues for any approximation to an individual plant's contribution to $|EX|$ or $|EN|$: what is the appropriate estimator for i) the weight; and ii) the growth rate of productivity. We suggest estimators for these quantities.[24]

We first focus on the correct weight and consider the case of exit; the case of entry is symmetric. Our goal is an estimate of the average Domar weight between time $T-1$ and time T. We know for exiters that the weight decreases (possibly monotonically) from the observed share at time $T-1$ to zero some time after $T-1$, and remains at zero until T. We propose using $\frac{D_{i,T-1}}{2}$, which is an upper bound to the "average-Domar-weight" that is used in (5). The bound is also helpful because it can be used in a resampling procedure we describe momentarily that estimates the variability in the estimated values of $|EX|$ or $|EN|$.

Next we consider possible estimators of the average growth rate of productivity. For the exit case we want an estimate of $\Delta ln\omega_{it^*} = ln\omega_{it^*} - ln\omega_{i,T-1}$. The missing piece of information is $ln\omega_{it^*}$, the productivity level at the time of exit. We suggest forecasting $ln\omega_{iT}$ using past values of $ln\omega$ and any other relevant state variables. Given an estimate $\widehat{ln\omega_{iT}}$, one can use $\Delta ln\omega_{iT} = \widehat{ln\omega_{iT}} - ln\omega_{i,T-1}$ as an approximation to $ln\omega_{it^*} - ln\omega_{i,T-1}$.

[24] A complete model of industry evolution may provide additional structure useful in inferring the unobserved paths of technical efficiency and value added for entrants and exiters.

Some examples may be helpful. If productivity is posited as a first-order Markov process, a regression of $\ln \omega_{iT}$ on previous log-levels of productivity provides an estimate of

$$E[\ln \omega_{iT} \,|\, ln\omega_{i,T-1}].$$

A second-order Markov process would include $ln\omega_{i,T-2}$ in the conditioning set. If the other state variables are co-determined with productivity, then the regression might include these state variables. For example, a first order Markov process with state variables productivity and capital k would point to

$$E[\ln \omega_{iT} \,|\, ln\omega_{i,T-1}, k_{i,T-1}]$$

as the key equation of interest in forecasting the missing productivity level. In the case of entry, the conditional mean will use future outcomes to project the current state at the time of entry.

An important question in the construction of the estimator is what set of plants to use when estimating this process. If entrants and exiters are assumed to be similar to continuing plants, all plants can be pooled in an estimation routine. Alternatively, if the process for recent entrants and recent exiters is thought to be different, we could select on the appropriate set of plants and estimate on this subset.

Robustness analysis will be important, especially as the time interval between plant-level observations grows. Many resampling schemes are available that will illustrate the variability in $|\widehat{EX}|$ and $|\widehat{EN}|$. In the case of entry, random draws can be taken from the (conditional) distribution of $ln\omega_{iT}$ and from the observed interval $[0, \frac{D_{i,T-1}}{2}]$ to generate a range of possible estimates of the contribution to aggregate productivity growth of the entering firm i in period T. Repeating this with all entrants and aggregating provides an estimate of the variability introduced into aggregate productivity from not observing the path of the entrant. The estimator is easily modified for the exiters. Other sources of sampling error, such as that in the estimated equation for productivity, are easily incorporated into this resampling scheme in the usual way.

6. Decomposing Changes in Aggregate Productivity Growth Rates

In this section we show how the *change* in the productivity growth rate provides further information on the underlying dynamics of real productivity growth and reallocation. We suggest an approximation to the change in the growth rate that uses data from three juxtaposed time periods, which we denote as T, $T+1$, and $T+2$. Our estimated change in the growth rate is given by the difference in two "average-log-change" indexes:

$$\sum_{i=1}^{N_2} \frac{D_{i,T+2} + D_{i,T+1}}{2} \Delta ln\omega_{i,T+2} - \sum_{i=1}^{N_1} \frac{D_{i,T+1} + D_{iT}}{2} \Delta ln\omega_{i,T+1} \tag{23}$$

where (for example) $\Delta ln\omega_{i,T+2} = ln\omega_{i,T+2} - ln\omega_{i,T+1}$. Equation (23) decomposes into a "productivity" term, a "reallocation" term, and a "net entry" term. The set of plants that exist in T, $T+1$, and $T+2$ is denoted C (for continuers), and each of these plants contributes a productivity component and a reallocation component to changes in the aggregate productivity growth rate. Plants that exist only in T and $T+1$, or only in $T+1$ and $T+2$, contribute to the overall change in (23), but only through the "net entry" term described below.

The "change-in-productivity" term is written as

$$\sum_{i \in C} \frac{D_{i,T+2} + 2 * D_{i,T+1} + D_{iT}}{4} * (\Delta ln\omega_{i,T+2} - \Delta ln\omega_{i,T+1})$$

Each continuer contributes the change in the rate of their productivity growth (the term on the right). The weight in the aggregation to the industry level is an average over the three time periods of the plant's Domar weight, where period $T+1$ gets twice the weight of period T and period $T+2$.

The second term is the reallocation term, given by:

$$\sum_{i \in C} (\Delta ln\omega_{i,T+2} + \Delta ln\omega_{i,T+1})/2 * (D_{i,T+2} - D_{iT})/2.$$

The term on the right gives the change in the Domar weight from period T to period $T+2$. This multiplies the average rate of productivity growth over the periods for the plant (i.e. $ln(\frac{\omega_{i,T+2}}{\omega_{iT}})/2$).

The third term is the net entry term and is given as

$$\sum_{i \in T+1, T+2, not\ T} \frac{D_{i,T+2} + D_{i,T+1}}{2} * \Delta ln\omega_{i,T+2} - \sum_{i \in T, T+1, not\ T+2} \frac{D_{i,T+1} + D_{iT}}{2} * \Delta ln\omega_{i,T+1}.$$

It gives the difference in growth rates between those plants absent in period T but present in $T+1$ and $T+2$ and those plants absent in period $T+2$ but present in T and $T+1$. Since the weights on either side of the difference do not sum to one, this term can be driven both by the prevalent growth rates of entrants and exiters and by their mass in the population.

The sum of the real productivity, reallocation, and net entry terms is equal to (23). Ignoring the net entry term, if there is no change in the growth rate of productivity at any plant, the productivity term is zero. In this case, the aggregate rate of productivity growth can only increase (decrease) if there is reallocation of gross output from period T to period $T+2$ towards (away from) plants with higher average productivity growth rates. Similarly, if there is no change in the Domar weight from the start (T) to the end ($T+2$), the reallocation term is zero, and the aggregate productivity growth rate can only increase if the average growth rate of productivity increases. Net entry contributes a term that is positive (negative) if the Domar weighted sum of the growth rates of entrants exceeds (falls below) the Domar weighted sum of the growth rates of the exiters.

7. The Bailey-Hulten-Campbell Index and Decomposition

In the introduction we summarized the empirical results from Section 9, which show that there is a stark divergence in the data between the BHC index and the traditional growth-accounting measure. The divergence arises because the BHC index and all of its variants have not been linked to welfare. Below we demonstrate that the BHC index can be negatively correlated with welfare, uses weights that are incompatible with welfare measurement, and approximates the welfare change arising from entry and exit in a manner that is difficult to rationalize.

BHC define aggregate productivity growth as the difference in share-weighted log-levels of productivity:

$$BHC_T = \sum_i s_{G_{iT}} ln\omega_{iT} - \sum_i s_{G_{i,T-1}} ln\omega_{i,T-1}. \tag{24}$$

BHC show that (24) can be expressed in terms of those plants that are present in both periods (continuers), and the plants that enter and exit, where the indicators denote these three types:

$$\begin{aligned} BHC_T =& \sum_i 1_{i,Continue} * (s_{G_{iT}} ln\omega_{iT} - s_{G_{i,T-1}} ln\omega_{i,T-1}) \\ &+ \sum_i 1_{i,Enter} * s_{G_{it}} ln\omega_{iT} - \sum_i 1_{i,Exit} * s_{G_{i,T-1}} ln\omega_{i,T-1}. \end{aligned} \tag{25}$$

We use continuing firms to illustrate most of our points.

For a continuing firm i, its contribution to productivity growth can be decomposed as

$$\begin{aligned} BHC_{iT} =& \overline{D_{iT}} * \Delta ln\omega_{iT} + Err_{iT} + \overline{ln\omega_{iT}} * \Delta s_{G_{iT}} \\ =& \frac{D_{iT} + D_{i,T-1}}{2} * \Delta ln\omega_{iT} + \frac{(s_{G_{iT}} - D_{iT}) + (s_{G_{i,T-1}} - D_{i,T-1})}{2} * \Delta ln\omega_{iT} + \frac{ln\omega_{iT} + ln\omega_{i,T-1}}{2} * \Delta s_{G_i} \end{aligned} \tag{26}$$

The first term is the growth-accounting term. It is tightly linked to welfare measurement via Lemmas 1-6 in perfectly and many imperfectly competitive settings. Indeed, those Lemmas demonstrate that this term properly measures an important part of the change in welfare arising from productivity growth in virtually every theoretical model cited in Foster et al. (2001).[25] Thus, the growth-accounting component of the BHC index *is* well-founded in economic theory.

[25] This includes models of creative destruction and vintage capital like Aghion and Howitt (1992), Caballero and Hammour (1996), Campbell (1998), and many others.

Choice of Weight

There is an extensive discussion on the advantages and disadvantages of using labor share vs. gross-output share when comparing variants of the BHC index. From a welfare perspective, this dialog has overlooked a critical point. As Lemmas 1 and 2 show, if the desired measurement is welfare, then the ambiguity is immediately resolved. The choice of the weight is entirely determined by which productivity residual - gross output or value added - is used in the calculation. If i's gross output residual is used, then i's revenue divided by aggregate value added is the correct weight. If the value-added residual is employed, then the share of value added is the correct weight. In neither one of these cases is the labor share or the gross output share the correct weight. The error Err_{it} reflects this discrepancy.

BHC Reallocation Has No Theoretical Link to Welfare

Practitioners call

$$\frac{ln\omega_{iT} + ln\omega_{i,T-1}}{2} * \Delta s_{G_{iT}}$$

the productivity growth arising from "reallocation". *Not one* of the models used to motivate the importance of reallocation from Foster et al. (2001) point to this term as having any precise link to welfare measurement. Some consequences of this term having no theoretical tethering follow.

BHC May Be Negatively Correlated with Welfare

It is easy to construct examples where the BHC index is negatively correlated with welfare. We provide one example that is illustrative of the problems of the BHC reallocation term when measuring the change in welfare is the objective. Let there be two firms in a competitive economy producing the same good in periods 1 and 2. One firm has higher productivity, but both firms produce because of either decreasing returns to scale or transport costs and geographically disbursed consumers. Suppose there is no change in technical efficiency for either firm from period 1 to period 2, but a distortion arises that causes the share of gross output to increase at the more productive plant. For example, a tax on the low productivity firm's output may be imposed. No change in technical efficiency means no contribution to welfare along this dimension. *Welfare unambiguously falls* as the distortion moves the economy away from the optimal point on the production possibilities frontier. The *BHC index is positive* because the reallocation term is positive; the weight at the more productive plant (in levels) has increased.

Entry and Exit

As noted in Section 5, the contribution to the change in welfare from entrants and exiters in a growth-accounting setting is well-defined: for an entrant it is given as $\int_{t^*}^{T} D_{it}\ dln\omega_{it}$ and for an exiter it is $\int_{T-1}^{t^*} D_{it}\ dln\omega_{it}$, where t^* denotes the time of entry or exit and $D_{it^*} = 0$.

The aggregate contribution to productivity growth from entrants and exiters in the BHC index is given in the last line of (25). The contribution of an entrant is given by $D_{iT} ln\omega_{iT}$, its share at the end time T multiplied by its log-level of productivity. Similarly, the contribution of an exiter to productivity growth in the BHC index is given by $D_{i,T-1} ln\omega_{i,T-1}$. Thus the BHC index multiplies the log-level of productivity times a weight to calculate the entrant/exiter contribution.

8. Data and Estimation

We turn to two manufacturing censuses to explore the empirical issues that we raise. One census is from Chile's Instituto Nacional de Estadistica (INE), and the second is from Colombia's Departamento Administrativo Nacional de Estadistica (DANE.) The Chilean data span the period 1987 through 1996 and the Colombian data span the years 1981-1991. We focus on 3-digit industries with more than 200 observations, of which there are 23 in Chile and 26 in Colombia. Here, we provide a brief overview of these data. They have been used in numerous other productivity studies, and we refer the interested reader to those papers for a more detailed data description.[26]

The data are unbalanced panels and cover all manufacturing plants with at least ten employees. Plants are observed annually and they include a measure of output, two types of labor, capital, and intermediate inputs. Because of the way our plant-level data are reported, we treat plants as firms, although there are probably multi-plant firms in the sample. Real value added is nominal value added adjusted by the 3-digit industry price index. Labor is the number of man-years hired for production, and plants distinguish between their blue- and white- collar workers (we include two labor types in the production function). The method for constructing the real value of capital is documented in Liu (1991) for the Chilean data, and a similar approach is adopted for the Colombian data.[27] A data problem for the Chilean census is that approximately 3% of the plant-year observations appear to be "missing"; a plant id number is present in year $t-1$, absent in year

[26] See Liu (1991), Liu (1993), Liu and Tybout (1996), Tybout, de Melo, and Corbo (1991), Pavcnik (2002), Levinsohn (1999), and Levinsohn and Petrin (2003).

[27] It is a weighted average of the peso value of depreciated buildings, machinery, and vehicles, each of which is assumed to have a depreciation rate of 5%, 10%, and 20% respectively. No initial capital stock is reported for some plants, although investment is recorded. When possible, we used a capital series that was reported for a subsequent base year. For a small number of plants, capital stock is not reported in any year. We estimated a projected initial capital stock based on other reported plant observables for these plants. We then used the investment data to fill out the capital stock data.

t, and then present again in year $t+1$. We impute the values for these observations using $t-1$ and $t+1$ information (see the Appendix).

We estimate value-added production function parameters for each of the 3-digit industries and use the parameters to estimate the plant-level TFP residuals. For any industry, the production function coefficients are assumed to be constant over time and across plants, although our findings are robust to loosening this assumption. We employ three different approaches to estimating the coefficients: ordinary least squares, revenue shares, and the proxy method from Levinsohn and Petrin (2003), which includes controls to address the correlation of productivity with input choices. For the revenue shares, for each industry we use the average over plants and time of the shares. Our intent here is *not* to compare estimators, as that has been done (again and again) elsewhere. Rather we wish to investigate whether our empirical results on measuring productivity are robust to these common methods of estimation.

9. Results

A necessary condition for the growth-accounting index and the BHC index to agree is that they yield similar results for continuing firms, which make up on average 95% of plant observations in the industries in our data. For these firms we compare the growth-accounting measure with the BHC index for 49 3-digit manufacturing industries from Chile and Colombia. Readers interested in the calculation of the additional terms suggested by Lemmas 3 and 4 are referred to Nishida and Petrin (2004).

In an attempt to keep the analysis manageable, we start with a detailed description of results for the largest Chilean manufacturing industries. We then describe how these findings generalize. The main result is that the micro patterns observed in the largest industries in Chile are indicative of the findings for the entire 49 3-digit industries from both countries.

Our approach is to compare the indexes using the value-added representation of productivity growth, so we use the value-added residual and apply the value-added share weight. We apply the same weight in the BHC index calculation, thus abstracting from the usual error in welfare introduced by using the wrong aggregation weight. We normalize to industry value added, which is not correct for exact welfare measurement, but which is easily translated to the exact contribution via multiplication.[28]

Table 1 reports the annual estimates of growth rates in productivity for the two measures for ISIC 311, the Food Products industry (the largest in Chile). The production function coefficients

[28] The rescaling factor is economy final demand divided by industry value-added.

are estimated using ordinary least squares, and the productivity calculations use only plants that exist in period $t-1$ and period t (the continuers), which in this industry account for 94.4% of plant-year observations and 96.4% of industry value added over the sample period. Column 1 is the growth in real industry value added for 1988 to 1996, column 2 is the growth-accounting measure of productivity, column 3 is the BHC productivity measure, and column 4 is the difference between these two terms, which is equal to the BHC "reallocation" term described earlier when the correct aggregation weight is used (here value-added share).

The growth-accounting measure averages 3.64% per annum when scaled by industry value-added, with standard deviation of 4.72% across the nine years. On average it accounts for slightly less than one-half of the growth rate in value added, which is consistent with the findings reported in Basu and Fernald (2002).[29] The BHC index averages –2.93% per annum with a standard deviation of 13.48%. The divergence in these summary statistics arises because the two indexes themselves are widely divergent, as is evident from a comparison of columns 2 and 3. Column 4 is the difference and has a mean of -6.57%, with a standard deviation of 10.74%. Its volatility across the sample period is consistent with the general findings in the literature that "reallocation" can be large and volatile.

For ISIC 311, table 2 compares estimates of productivity growth across different estimators for production function parameters. The top half of the table is the growth-accounting measure and the bottom half is the BHC index. For the top half, column 2 is the same as column 2 from table 1, which uses ordinary least squares to obtain production function estimates. Column 3 uses the proxy approach from Levinsohn and Petrin (2003), and column 4 uses revenue shares. While the production function estimates (not reported here) do differ somewhat across the three approaches, the productivity growth numbers using the growth-accounting index are reasonably similar across the three estimators. Only in 1995 does there seem to be substantial divergence between the revenue share estimate and the two alternatives.

For the BHC index, the signs tend to be common across the three sets of production function estimates, but the magnitudes are quite different, with the LP and OLS estimates systematically the largest in absolute value terms. The main reason for the volatility is reflected in the "reallocation" terms, which are systematically more volatile for LP and OLS relative to revenue shares.

Table 3 summarizes the relationship between the growth-accounting index and the BHC index for the eight largest industries in Chile. The industries (along with their ISIC codes) are Food

[29] Their approach differs principally in their use of U.S. data that has been aggregated to the industry level.

Products (311), Metals (381), Textiles (321), Wood Products (331), Apparel (322), Plastics (356), Non-electric Machinery (382), and Other Chemicals (352). The index numbers use only plants that exist in $t-1$ and t, which account for between 94% and 98% of industry value added (in this annual data, truncation due to entry and exit is not a severe problem). For each industry and each estimator of production function coefficients, the appropriate row reports the intercept, slope, and r-squared from a regression of the 9 annual growth rates using the BHC index as the independent variable and the growth-accounting index as the dependent variable. For example, the first row for ISIC 311 is the regression of column 2 on column 3 from table 1. An r-squared of 1 and an intercept of 0 and slope of 1 would indicate that the measures are identical. An r-squared of 1 would indicate that the BHC measure is a linear transformation of the growth-accounting index.

For industry 311, the r-squared's range from 0.22 to 0.47. The intercept is significantly different from zero and the slope is significantly different from one.[30] Overall, these results suggest that the BHC index is a poor proxy for the growth-accounting measure for ISIC 311.

These results are similar across all eight industries. R-squareds are typically low, ranging between 0.2 and 0.4. Intercept and slope coefficients are significantly different from zero and one respectively.

The messages that come out of the results from tables 1-3 are confirmed by the other 15 manufacturing industries in Chile and the 26 manufacturing industries from Colombia. Overall, for Chile, using OLS only 4 of 23 industries had r-squareds above 0.5, using revenue shares only 5 of 23 had r-squareds above 0.5, and using the proxy approach only 1 in 23 industries had an r-squared over 0.5. Most slope coefficients across estimators and industries varied between 0.1 and 0.3.[31] For Colombia, using OLS only 4 of 26 industries had r-squareds above 0.5, using revenue shares 8 of 26 had r-squareds above 0.5, and using the proxy approach only 5 of 26 industries had an r-squared over 0.5. Slope coefficients across estimators and industries also varied between 0.1 and 0.3. In summary, the results demonstrate that the BHC index adds a "reallocation" term to the growth-accounting index that is large and volatile, making it a very noisy indicator of the growth-accounting index. This raises questions about the large discussion in the literature that revolves around BHC-type indexes of productivity growth.

[30] These results do not correct for error in the parameter estimates. For this industry, with almost 10,000 observations, the parameter estimates are very precisely estimated.

[31] The industries that had higher r-squareds were also the industries where the slope coefficients tended to be larger.

Decomposing the Change in Productivity Growth Rates

We now turn to the decomposition of the change in productivity growth rates from Section 6. There we showed how to decompose the change in the growth rate of productivity into a term that is increasing if growth rates at the firm level are increasing (the "real productivity" term), and a term that is increasing when plants with higher average growth rates gain larger shares of value added (the "reallocation" term).

Table 4 provides this decomposition for ISIC 311 in Chile. The change in the growth rate from year to year is reported in column 2. Note that it does not exactly equal the change in the growth rate reported in column 1; the difference is due to the entry/exit plants, (those plants that do not exist in T, $T+1$, and $T+2$, e.g.), and it is equal to the change in the growth rate that is attributable to entry/exit. As defined in Section 6, for those plants that exist in the three periods, the change in column 2 can be decomposed into the "real productivity" term, column 3, and the "reallocation" term, column 4. The real productivity component is quite volatile, averaging –5.95% with a standard deviation of 7.33%, with both positive and negative outcomes. The reallocation term is positive, very stable, and always contributes to increases in the rate of productivity growth, even in periods when the overall growth rate falls.

Overall, the reallocation series for ISIC 311 in Chile is remarkably similar in spirit to the reallocation series from the other 48 manufacturing industries. Regardless of the estimator for production function coefficients, the industry, or the country, the annual reallocation terms are almost universally positive, and within an industry over time they vary very little, with a typical standard deviation less than 1.[32] Thus, these results suggest that reallocation effects - when defined in terms of changes in productivity growth rates - are very stable and almost always contribute positively to industry growth, even when the overall growth rate is falling.

10. Conclusions

In this paper we define productivity growth as the change in welfare that arises from additional output holding primary inputs constant. We show that this definition provides practitioners with a measurement useful for cost-benefit/policy analysis that is readily comparable across time, industries, countries, and empirical studies. Using traditional growth-accounting results, we show that gains may arise because of plant-level technology shocks, and, in imperfectly competitive settings, from the reallocation of inputs across plants with differing markups and/or shadow values of primary inputs.

[32] They do vary across industries, with most industries falling in between 3% to 8%.

The most popular index for measuring productivity growth with plant-level data uses the difference in the "first moments" of the productivity distribution, where a variety of first moments have been proposed. We show this definition adds additional terms to the growth-accounting measure, one of which has been called "reallocation." Our empirical results indicate that there is a very weak relationship between the two aggregate productivity indexes in almost every 3-digit manufacturing industry in both Chile from 1987-1996 and Colombia from 1981-1991 - 49 in total - primarily because this "reallocation" term is large and volatile. These findings are robust to different estimation approaches for plant-level productivity.

We explore the theoretical reasons for this sharp divergence, in the process uncovering a number of previously unnoticed and unattractive features of the first-moment definition: it is not linked tightly to welfare in any theoretical model of which we are aware; it can report positive productivity growth when welfare has fallen; and it uses labor or gross-output as share weights, neither of which is correct for welfare measurement. In the end, our results call into question the literature's interpretation of the "reallocation" term as productivity growth. Our findings also suggest that researchers should regularly include the traditional growth-accounting measure in their results for both comparability and ease of interpretation.

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Appendix A

Proof of Lemma 1

The proof mimics Hulten. $F(\cdot)$ is differentiable and homogeneous of degree zero in (Y, X), which implies

$$\sum_{i=1}^{N} \frac{\partial F}{\partial Y_i} Y_i = -\sum_{k=1}^{K} \frac{\partial F}{\partial X_k} X_k. \tag{27}$$

By definition $F(Y, X, t) = 0$, so total differentiation of $F(\cdot)$ yields

$$-dF = \sum_{i=1}^{N} \frac{\partial F}{\partial Y_i} dY_i + \sum_{k=1}^{K} \frac{\partial F}{\partial X_k} dX_k.$$

where dF, dY, and dX denote the instantaneous change with respect to time. Divide through by the left and right hand side of (27) and substitute in the competitive equilibrium conditions $\frac{\partial F/\partial Y_i}{\partial F/\partial Y_1} = P_i$, $i = 2, \ldots, N$ and $-\frac{\partial F/\partial X_k}{\partial F/\partial Y_1} = W_k$, $k = 1, \ldots, K$, Then, the shift in the social PPF holding primary inputs constant is given as the rate of change of $F(\cdot)$ divided by $\sum_{i=1}^{N} \frac{\partial F}{\partial Y_i} Y_i$:

$$\frac{-dF}{\sum_{i=1}^{N} \frac{\partial F}{\partial Y_i} Y_i} = \sum_{i=1}^{N} \frac{P_i Y_i}{\sum_i P_i Y_i} \frac{dY_i}{Y_i} - \sum_{k=1}^{K} \frac{W_k X_k}{\sum_k W_k X_k} \frac{dX_k}{X_k}.$$

This establishes the first claim.

The second claim of the Lemma asserts that this quantity is equal to $\sum_{i=1}^{N} \frac{P_i Q_i}{\sum_i P_i Y_i} dlnF^i$, where $Q_i = F^i(M^i, X^i, t)$ and $dlnF^i = dln\omega_i$ from (10) (we write it in this way to be consistent with the notation here). To show this, note that in equilibrium supply and demand in the product and factor markets are equated:

$$Q_i = Y_i + \sum_{j=1}^{N} M_i^j \quad i = 1, \ldots, N$$

and

$$X_k = \sum_{j=1}^{J} X_k^j \quad k = 1, \ldots, K.$$

Total differentiation of these equations gives

$$\frac{dQ_i}{Q_i} = \frac{dY_i}{Q_i} + \frac{\sum_{j=1}^{N} dM_i^j}{Q_i} = \frac{P_i Y_i}{P_i Q_i} \frac{dY_i}{Y_i} + \sum_{j=1}^{N} \frac{P_i M_i^j}{P_i Q_i} \frac{dM_i^j}{M_i^j} \quad i = 1, \ldots, N \tag{28}$$

and

$$\frac{dX_k}{X_k} = \sum_{i=1}^{N} \frac{W_k X_k^i}{W_k X_k} \frac{dX_k^i}{X_k^i} \quad k = 1, \ldots, K. \tag{29}$$

Necessary conditions for the equilibrium are: $\frac{\partial Q_i}{\partial M_j^i} = \frac{P_j}{P_i}$ and $\frac{\partial Q_i}{\partial X_k^i} = \frac{W_k}{P_i}$, $i, j = 1, \ldots, N$ and $k = 1, \ldots, K$. Substituting these equalities into the logarithmic derivative of $Q_i = F^i(M^i, X^i, t)$ and dividing through by Q_i yields

$$\frac{dQ_i}{Q_i} = \sum_{j=1}^{N} \frac{P_j M_j^i}{P_i Q_i} \frac{dM_j^i}{M_j^i} + \sum_{k=1}^{K} \frac{W_k X_k^i}{P_i Q_i} \frac{dX_k^i}{X_k^i} + dlnF^i \quad i = 1, \ldots, N, \tag{30}$$

where $dlnF^i$ is the well-known Solow residual. Solving (28) for $\frac{P_i Y_i}{P_i Q_i} \frac{dY_i}{Y_i}$ and plugging in (30) yields

$$\frac{P_i Y_i}{P_i Q_i} \frac{dY_i}{Y_i} = \sum_{j=1}^{N} \frac{P_j M_j^i}{P_i Q_i} \frac{dM_j^i}{M_j^i} + \sum_{k=1}^{K} \frac{W_k X_k^i}{P_i Q_i} \frac{dX_k^i}{X_k^i} + dlnF^i - \sum_{j=1}^{N} \frac{P_i M_i^j}{P_i Q_i} \frac{dM_i^j}{M_i^j} \quad i = 1, \ldots, N \tag{31}$$

Multiplying (31) through by $\frac{P_i Q_i}{\sum_{i=1}^N P_i Y_i}$ and noting from the accounting identity $\sum_{i=1}^N P_i Y_i = \sum_{k=1}^K W_k X_k$ we have

$$\frac{P_i Y_i}{\sum_i P_i Y_i} \frac{dY_i}{Y_i} = \sum_{j=1}^{N} \frac{P_j M_j^i}{\sum_i P_i Y_i} \frac{dM_j^i}{M_j^i} + \sum_{k=1}^{K} \frac{W_k X_k^i}{\sum_k W_k X_k} \frac{dX_k^i}{X_k^i} + \frac{P_i Q_i}{\sum_i P_i Y_i} dlnF^i - \sum_{j=1}^{N} \frac{P_i M_i^j}{\sum_i P_i Y_i} \frac{dM_i^j}{M_i^j}. \tag{32}$$

Aggregating across i gives

$$\sum_{i=1}^{N} \frac{P_i Y_i}{\sum_i P_i Y_i} \frac{dY_i}{Y_i} = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{P_j M_j^i}{\sum_i P_i Y_i} \frac{dM_j^i}{M_j^i} + \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{W_k X_k^i}{\sum_k W_k X_k} \frac{dX_k^i}{X_k^i} + \sum_{i=1}^{N} \frac{P_i Q_i}{\sum_i P_i Y_i} dlnF^i - \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{P_i M_i^j}{\sum_i P_i Y_i} \frac{dM_i^j}{M_i^j}. \tag{33}$$

The intermediate inputs cancel out because

$$\sum_{i=1}^{N} \sum_{j=1}^{N} P_j dM_j^i = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i dM_i^j.$$

Rearranging (33) then yields

$$\sum_{i=1}^{N} \frac{P_i Q_i}{\sum_i P_i Y_i} dlnF^i = \sum_{i=1}^{N} \frac{P_i Y_i}{\sum_i P_i Y_i} \frac{dY_i}{Y_i} - \sum_{k=1}^{K} \frac{W_k X_k}{\sum_k W_k X_k} \frac{dX_k}{X_k}$$

This establishes the second claim. #

Proof of Lemma 2

Let $s_{M^i} = \frac{\sum_{j=1}^{N} P_j M_j^i}{P_i Q_i}$, the payments to intermediates divided by gross output. The residual from the value-added production function is given by $dlnF^{iV} \equiv \frac{dlnF^i}{(1-s_{M^i})}$, the change in plant-level efficiency from the gross output production function $dlnF^i$ grossed up by $\frac{1}{(1-s_{M^i})}$. Denote VA_i as value added for firm i. Because $VA_i = P_i Q_i - \sum_j P_j M_j^i$, and $\sum_i VA_i = \sum_i P_i Y_i$, it follows directly that

$$\sum_{i=1}^{N} s_{v_i} dlnF^{iV} = \sum_{i=1}^{N} \frac{VA_i}{\sum_{i=1}^{N} VA_i} \frac{dlnF^i}{(1-s_{M^i})} = \sum_{i=1}^{N} \frac{P_i Q_i - \sum_j P_j M_j^i}{\sum_{i=1}^{N} P_i Y_i} \frac{dlnF^i}{\frac{(P_i Q_i - \sum_j P_j M_j^i)}{P_i Q_i}} = \sum_{i=1}^{N} \frac{P_i Q_i}{\sum_i P_i Y_i} dlnF^i.$$

#

Proof of Lemma 3

Let Z_j denote a generic input with P_j^Z its price, and let $Z = (Z_1, \ldots, Z_J)$ equal the full vector of inputs. Firm i minimizes cost while achieving the target output Q^*:

$$Min_{Z_1, Z_2, \ldots, Z_J} \sum_j P_j^Z Z_j$$

such that

$$F^i(Z) \geq Q^*.$$

The Lagrangian is

$$L = \sum_j P_j^Z Z_j - \lambda[F^i(Z) - Q^*]$$

with first order conditions (FOC):

$$\frac{\partial L}{\partial Z_j} = P_j^Z - \lambda \frac{\partial F^i(Z)}{\partial Z_j}$$

for $j = 1, \ldots, J$, and

$$\frac{\partial L}{\partial Q^*} = \lambda.$$

With $\lambda = MC_i$, the input FOCs can be rewritten as

$$\frac{P_i}{P_i} \frac{P_j^Z}{MC_i} \frac{Z_j}{F^i(Z)} = \frac{\partial F^i(Z)}{\partial Z_j} \frac{Z_j}{F(Z)}.$$

Reexpressed we have

$$\mu_i s_j = \epsilon_{Z_j},$$

for $j = 1, \ldots, J$, where optimizing firms equate - for each input - the product of the markup and revenue share with the elasticity of output with respect to the input. With three inputs labor, capital, and materials production can be written as

$$dlnQ_i = \sum_i \mu_i (s_{L^i} dlnL^i + s_{K^i} dlnK^i + s_{M^i} dlnM^i) + dln\omega_i, \tag{34}$$

a generalization of Solow's formulation to the case of imperfect competition. This establishes (14), and (15) and (16) follow directly.

#

Lemma 4: A Simple Example

The simple two-period general equilibrium model given in Basu and Fernald (2002) illustrates how the envelope theorem links productivity to welfare. Consider a competitive economy with optimizing agents: a representative consumer and firm, prices taken as given, a constant returns production function $Q_t = \omega_t F(K_t, L_t)$, and an initial endowment of capital K_1^*, and two independent productivity shocks (ω_1, ω_2). How much does welfare change in response to a shock $d\omega_1$?

This problem is isomorphic to the social planner's problem, who maximizes welfare:

$$Max_{C_1, C_2, L_1, L_2} \quad V(K_1^*, \omega_1, \omega_2) = E_1[U(C_1, 1 - L_1) + \beta U(C_2, 1 - L_2)]$$

subject to the budget constraints

$$C_1 = Q_1 - I_1 = \omega_1 F(K_1^*, L_1) - (K_2 - (1 - \delta) K_1^*)$$

and

$$C_2 = \omega_2 F(K_2, L_2) + (1 - \delta) K_2,$$

where I_1 denotes investment (savings) in period 1. Let $(C_1^*, C_2^*, L_1^*, L_2^*)$ denote the solution and $V^* = V(C_1^*, C_2^*, L_1^*, L_2^*; \omega_1)$.

The envelope condition says that the total change in maximal utility dV^* due to a small change in parameter $d\omega_1$ is equal to the partial effect evaluated at the initial optimum:

$$\frac{dV^*}{d\omega_1} = \frac{\partial V(C_1^*, C_2^*, L_1^*, L_2^*; \omega_1)}{\partial \omega_1}.$$

Using the Lagrange formulation for the constrained problem, it is straightforward to show that the marginal utility of wealth is equal to the partial derivative of utility with respect to C_1, when evaluated at the optimum:

$$\frac{dV^*}{dQ_1} = \frac{\partial U(C_1^*, L_1^*)}{\partial C_1} = \frac{\partial U^*}{\partial C_1}.$$

Inserting the above formulation for C_1 and C_2 into $V(\cdot)$ yields the unconstrained problem, and differentiating with respect to ω_1 gives

$$\frac{\partial V^*}{\partial \omega_1} = \frac{\partial U^*}{\partial C_1} * F(K_1^*, L_1^*).$$

The total change in utility is then:

$$dV^* = \frac{\partial U^*}{\partial C_1} * F(K_1^*, L_1^*) * d\omega_1 = \frac{\partial U^*}{\partial C_1} * Q_1^* * dln\omega_1.$$

The additional units of output available is given by the growth rate of productivity times the level of output, and the marginal utility of wealth maps this additional output into utility. The conclusion that the change in welfare is proportional to productivity growth follows directly.

Appendix B

Imputing Missing Values

Approximately 3% of the plant-year observations in Chile are "missing" according to the following definition: a plant id number is present in year $t-1$, absent in year t, and then present again in year $t+1$. We impute the values for these observations using $t-1$ and $t+1$ information and the structure of the estimated production function. We use the simple average of the $t-1$ and $t+1$ (log) productivity estimates for the period t productivity estimate. Similarly, we use the simple average of the $t-1$ and $t+1$ (log) input index estimates, where the weights in the index are the estimated production function parameters. All of our findings are robust to dropping these observations.

TABLE 1

Comparison of the Growth-Accounting

Index with the BHC Productivity Index

Ordinary Least Squares Estimates, ISIC 311, Chile

Year	Value Added	Growth-Accting Index $\sum_i \frac{(s_{it}+s_{i,t-1})}{2} ln(\frac{\omega_{it}}{\omega_{i,t-1}})$	BHC Index $\sum_i s_{it} ln\omega_{it} - \sum s_{i,t-1} ln\omega_{i,t-1}$	Difference (BHC "Reallocation" Term) $\sum_i \frac{(ln\omega_{it}+ln\omega_{i,t-1})}{2} * \Delta s_i$
	Rate of Growth in:			
1988	11.75	-3.12	-14.96	-11.84
1989	7.36	3.64	-7.45	-11.10
1990	4.59	-1.10	-10.52	-9.43
1991	13.82	7.36	-13.28	-20.63
1992	14.67	6.09	-8.11	-14.20
1993	8.98	5.09	-0.19	-5.28
1994	8.20	2.95	7.09	4.13
1995	7.06	-0.74	-7.24	-6.50
1996	-1.30	12.56	28.25	15.69
Average	8.35	3.64	-2.93	-6.57
Std. Dev.	4.89	4.72	13.48	10.74

The last column is the discrepancy between the BHC index and the growth-accounting index, which is equal to a reallocation-like term given by $\sum_i \overline{ln\omega_i} * \Delta s_i$. The comparison is done on firms that exist in period t and $t-1$, which account for 94.4% of the plant-year observations and 96.4% of industry value added. See text for details.

TABLE 2

Comparison of Productivity Indexes Across

OLS, Levinsohn-Petrin, and Revenue Share Productivity Estimates

ISIC 311, Chile

Year	Value Added	OLS	Levinsohn-Petrin	Revenue Shares
		Growth-Accting Index		
1988	11.75	-3.12	-0.04	0.77
1989	7.36	3.64	4.39	4.86
1990	4.59	-1.10	-1.26	-1.91
1991	13.82	7.36	7.68	8.98
1992	14.67	6.09	6.82	9.69
1993	8.98	5.09	5.87	4.41
1994	8.20	2.95	4.45	2.84
1995	7.06	0.74	1.41	-5.22
1996	-1.30	12.56	11.07	6.80
		BHC Index		
1988	11.75	-14.96	-19.16	-5.04
1989	7.36	-7.45	-11.94	-4.40
1990	4.59	-10.52	-20.43	-2.21
1991	13.82	-13.28	-24.42	-4.09
1992	14.67	-8.11	-15.54	-0.01
1993	8.98	-0.19	5.16	0.09
1994	8.20	7.09	4.81	4.33
1995	7.06	-7.24	-9.09	-10.40
1996	-1.30	28.25	39.66	1.07

Growth rates in productivity compared across methods used to estimate production function parameters. Comparison is done on firms that exist in period t and $t-1$.

TABLE 3

Regression of Growth-Accounting Productivity Index on BHC Productivity Index
Chilean Manufacturing, Industry by Industry, 1988-1996 (9 observations)

Industry Code	Coefficient Estimator	Intercept (Std. Err.)	Slope (Std. Err.)	R-squared
311	OLS	.043	.247	0.47
		(.012)	(.099)	
	LP	.051	.122	0.38
		(.011)	(.058)	
	Rev.	.036	.314	0.22
	Shares	(.015)	(.223)	
381	OLS	.031	.228	0.47
		(.023)	(.091)	
	LP	.028	.230	0.44
		(.024)	(.096)	
	Rev.	.008	.373	0.62
	Shares	(.021)	(.109)	
321	OLS	.010	.177	0.21
		(.023)	(.129)	
	LP	.017	.092	0.14
		(.020)	(.085)	
	Rev.	-.009	.503	0.38
	Shares	(.026)	(.026)	
331	OLS	.017	.207	0.25
		(.046)	(.132)	
	LP	.018	.146	0.20
		(.047)	(.108)	
	Rev.	.043	.501	0.57
	Shares	(.045)	(.163)	

Row one for ISIC 311 is the regression of column 2 on column 3 from table 1 (for example). An r-squared of 1 and an intercept of 0 and slope of 1 would indicate that the measures are identical. An r-squared of 1 would indicate that the BHC measure is a linear transformation of the growth-accounting index.

TABLE 3 (continued)
Regression of Growth-Accounting Index on BHC Productivity Index
Chilean Manufacturing, Industry by Industry, 1988-1996 (9 observations)

Industry Code	Coefficient Estimator	Intercept (Std. Err.)	Slope (Std. Err.)	R-squared
322	OLS	-.008	.223	0.22
		(.031)	(.154)	
	LP	-.008	.214	0.21
		(.033)	(.156)	
	Rev.	-.008	.482	0.44
	Shares	(.033)	(.201)	
356	OLS	-.047	.024	0.02
		(.029)	(.061)	
	LP	-.043	.028	0.02
		(.031)	(.066)	
	Rev.	-022	.165	0.15
	Shares	(.042)	(.143)	
382	OLS	.093	.068	0.14
		(.036)	(.063)	
	LP	.093	.058	0.11
		(.037)	(.062)	
	Rev.	.072	.180	0.19
	Shares	(.047)	(.137)	
352	OLS	.019	.274	0.27
		(.031)	(.168)	
	LP	.039	.156	0.17
		(.031)	(.128)	
	Rev.	.003	.416	0.50
	Shares	(.024)	(.155)	

Row one for ISIC 311 is the regression of column 2 on column 3 from table 1 (for example). An r-squared of 1 and an intercept of 0 and slope of 1 would indicate that the measures are identical. An r-squared of 1 would indicate that the BHC measure is a linear transformation of the growth-accounting index.

TABLE 4

Petrin and Levinsohn Decomposition of

the Change in Rate of Productivity Growth

"Real Productivity" vs. "Reallocation"

Ordinary Least Squares Estimates, ISIC 311, Chile

Year	Productivity Growth Rate	Change in Growth Rate	Real Productivity Component	Reallocation Component
1988	-3.12	—	—	—
1989	3.64	7.40	-1.79	9.18
1990	-1.10	-5.25	-14.83	9.59
1991	7.36	10.86	2.35	8.51
1992	6.09	-2.30	-10.64	8.34
1993	5.09	-0.66	-9.17	8.51
1994	2.95	-1.12	-9.72	8.60
1995	-0.74	-2.44	-10.08	7.64
1996	12.56	13.61	6.23	7.38
Average	3.64	2.51	-5.95	8.46
Std. Dev.	4.72	7.05	7.33	0.72

The first column is the rate of productivity growth from $t-1$ to t estimated using the growth-accouting index. The second column is the change in this growth rate for firms that exist in $t-2$, $t-1$, and t. The discrepancy between the change in column 1 and the level in column 2 is due to firms that do not exist in all three periods (the net entry term). The third and fourth column decompose column 2. See text for details.