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Optimal Growth Through Product Innovation*

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Abstract

In Lentz and Mortensen (2005), we formulate and estimate a market equilibrium model of endogenous growth through product innovation. Based on our parameter estimates, we provide quantitative equilibrium solutions to the model and compare them with a social planner's solution. We find that the socially optimal growth rate is much larger than the market equilibrium growth rate when firm differences in the ability to create productive products are highly persistent as a consequence of the "business stealing" externality present in the model. However, if an incumbent can buy the rights to a new product that threatens its market at its value to the innovator, then the optimal growth rate is only slightly larger than the market equilibrium growth rate. We also find that differences growth rates across the specifications are smaller when innovative ability is less persistent.

JEL Classification: E22, E24, J23, J24, L11, L25.

Keywords: Optimal growth, planner's problem, product innovation, innovation spill overs, creative-destruction externality.

1 Introduction

In Lentz and Mortensen (2005), we formulate and estimate a structural market equilibrium model of growth through product innovation. The model is an extended version of that proposed by Klette and Kortum (2004) originally designed to explain the link between innovation investment and the size distribution of firms. Their framework in turn is an elaboration of the Grossman and Helpman (1991) model of endogenous growth through creative-destruction. In our version of the model, firms differ with respect to the quality of the intermediate products they create through investment in research and development (R&D). We find that heterogeneity in this sense is needed to explain the size distribution of firms and the distribution of labor productivity observed in our panel data of Danish firms. One implication of this form of heterogeneity is that labor reallocation from slower growing firms to faster growing firms that create more profitable higher quality products plays an important role in determining the aggregate growth rate.

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The purpose of this paper is to explore the quantitative welfare implications of generalizations of the model estimated by Lentz and Mortensen. Namely, we formulate and compute the socially optimal R&D strategy for the modelled environments and compare it with alternative specifications of a market equilibrium outcome.

The market equilibrium solution to the model need not be socially optimal for three different reasons spelled out in Grossman and Helpman (1991). First, the most recent product innovator has monopoly power and use it to set prices above the marginal cost of production. Second, every innovation replaces an older version of some product and by doing so truncates the stream of quasi rents accruing to its previous innovator. Finally, because each new improvement builds on those of previous innovators, innovation has a positive "spill-over" effect on future productivity which is not fully captured by the innovator in a market equilibrium. The net deviation of the equilibrium growth rate from that which is socially optimal is unclear. One of the contributions of a quantitative equilibrium model is its ability to reflect light on the relative magnitudes of these effects.

We begin by formulating and characterizing the market solutions associated with different specifications of the economic environment and the corresponding planner's problem in section 2 and 3 of the paper. We then use the parameter estimates obtained by Lentz and Mortensen (2005) to compute the implications of each market solution for aggregate growth and then compare these implications with the outcome of the planner's problem for the same environment. The alternative market specifications considered differ with respect to whether or not the rights to innovations can be traded. If firm differences with respect to creative ability are highly persistent, then the planner's optimal innovation investment strategy yields a growth rate which is much larger than that obtained in market equilibrium without buyouts. However, if a threatened incumbent is able to buy the rights to a new product at its value to the innovator, the optimal growth rate is only slightly larger. The differences between market equilibria and optimal growth rates are much less if firm heterogeneity is not persistent.

2 The Model

Firms come in an amazing range of shapes and sizes. This fact cannot be ignored in any analysis of the relationship between firm size and factor productivity. Furthermore, an adequate theory must account for entry, exit, and firm evolution in order to explain observed size distributions. Klette and Kortum (2004) construct a stochastic model of firm product innovation and growth that is consistent with stylized facts regarding the firm size evolution and distribution. In Lentz and

Mortensen (2005), we find that an extension of their model that allows for cross firm heterogeneity in the quality of innovations is needed to explain our Danish data.

2.1 Preferences and Technology

The utility of the representative household at time t is given by

$$U_t = \int_t^\infty \ln C_s e^{-\rho(s-t)} ds \tag{1}$$

where $\ln C_t$ denotes the instantaneous utility of the single consumption good at date t and ρ represents the pure rate of time discount. Each household is free to borrow or lend at interest rate r_t. Nominal household expenditure at date t is $E_t = P_t C_t$. Optimal consumption expenditure must solve the differential equation $\dot{E}/E = r_t - \rho$. Following Grossman and Helpman (1991), we choose the numeraire so that $E_t = 1$ for all t without loss of generality, which implies $r_t = r = \rho$ for all t. Note that this choice of the numeraire also implies that price of the consumption good, P_t, falls over time at a rate equal to the rate of growth in consumption.

The quantity of the consumption produced is determined by the quantity and quality of the economy's intermediate inputs. Specifically, there is a unit continuum of intermediate good inputs and consumption is determined by the production function

$$\ln C_t = \int_0^1 \ln(A_t(j)x_t(j))dj = \ln A_t + \int_0^1 \ln x_t(j)dj \tag{2}$$

where $x_t(j)$ is the quantity of input $j \in [0,1]$ at time t, $A_t(j)$ is the productivity of input j at time t, and A_t represent aggregate productivity. The level of productivity of each input and aggregate productivity are determined by the number of technical improvements made in the past. Specifically,

$$A_t(j) = \Pi_{i=1}^{J_t(j)} q_i(j) \text{ and } \ln A_t \equiv \int_0^1 \ln A_t(j)dj. \tag{3}$$

where $J_t(j)$ is the number of innovations made in intermediate input j up to date t and $q_i(j) > 1$ denotes the quantitative improvement (step size) in productivity attributable to the i^{th} innovation in product j. Innovations arrive at rate δ which is endogenous in market equilibrium and the same for all intermediate products.

2.2 The Behavior of a Firm

Because intermediate goods of the same type are perfect substitutes, the creator of the latest most productive version is the sole supplier at any point in time. The price charged is limited by the ability of suppliers of previous versions to supply one of equal value. In Nash-Bertrand equilibrium, any successful innovator takes over the market for its good type by setting the price just below that at which final good producers are indifferent between the new more productive product supplied by the innovator and the alternative supplied by the previous provider. The price charged is the product of the relative productivity of the innovation and the previous producer's marginal cost of production. Given the symmetry of demands for the different good types and the assumption that future innovation likelihoods are independent of the type of good, one can drop the good subscript without confusion. Given stationary of quantities along the equilibrium growth path, the time subscript can be dropped as well.

Labor and capital, in fixed proportions, are used in the production of intermediate inputs to the final goods production process. Labor productivity is the same across all intermediate products and is set equal to unity. The required capital expressed in units of output, a constant κ, is also the same for all products. The operating profit per unit obtained from supplying an intermediate product is $p(1-\kappa)-w$ which implies that the lowest price that the previous supplier is willing to charge, that which yields no profit, is $w/(1-\kappa)$. The quality leader will charge $p = qw/(1-\kappa)$ because consumers are exactly indifferent between buying from the quality leader at this price and the zero profit price of the previous supplier. As profit maximizing by the competitive suppliers of the consumption good requires

$$P_t \frac{\partial C_t}{\partial x_t(j)} = \frac{P_t C_t}{x_t(j)} = p_t(j)$$

and $P_t C_t = E_t = 1$ by choice of the numeraire, the demand function for any product and the labor requirement for its production is

$$x = \frac{1}{p} = \frac{1-\kappa}{wq}. \tag{4}$$

Hence, the gross profit associated with supplying the good of quality q is

$$\pi(q) = p(1-\kappa)x - wx = (1-\kappa)\left(1-q^{-1}\right). \tag{5}$$

Following Klette and Kortum (2004), the discrete number of products supplied by a firm, denoted as k, is defined on the integers and its value evolves over time as a birth-death process reflecting product creation and destruction. In their interpretation, k reflects the firm's past successes in the product innovation process as well as current firm size. New products are generated by R&D investment. The firm's R&D investment flow generates new product arrivals at frequency γk. The total R&D investment cost is $wc(\gamma)k$ where $c(\gamma)k$ represents the labor input required in the research and development process. The function $c(\gamma)$ is assumed to be strictly increasing and convex. According to the authors, the implied assumption that the total cost of R&D investment is linearly homogenous in the new product arrival rate and the number of existing product, "captures the idea that a firm's knowledge capital facilitates innovation." In any case, this cost structure is needed to obtain firm growth rates that are independent of size as typically observed in the data.

The market for any version of an intermediate product currently supplied is destroyed by the creation of a new more productive alternative by some other firm, which occurs at the rate δ. Below we refer to γ as the firm's product innovation rate and to δ as the common creative-destruction rate faced by all firms. The firm chooses γ to maximize the expected present value of its future net profit flow. Of course, the R&D strategies of all incumbent firms and potential entrants determine the equilibrium value of δ.

There are persistent but generally temporary differences with respect to the improvement in productivity (the quality of an innovation q) offered by the products that a firm creates. For simplicity, we assume that the a firm's identity is is determined by a Markov chain defined on two states, denoted as $j = 0$ and 1. Without loss of generality, the quality of a product created by a firm in state $j = 1$ is larger. Formally, a firm creates higher quality product is state 1 in the sense that $q_1 > q_0$. Finally, the given exogenous transition rate out of type state j is λ_j. Because a firm's creativity type is transitory, the firm may be supplying products of both qualities at any point in time.

2.3 The Value Function Without Buyouts

Given that the innovator always takes over the market for the new version of the product it has created, the value of a firm that currently supplies k_0 products of low quality and k_1 high quality products is the solution to the asset pricing equation

$$\begin{aligned} rV_i(k_0,k_1) &= \max_{\gamma\geq 0}\Big\{k_0\pi_0 + k_1\pi_1 - wc(\gamma)k + \gamma k\big[V_0(k_0+1,k_1) - V_0(k_0,k_1)\big] \\ &\quad +\delta\big[k_0V_0(k_0-1,k_1) + k_1V_0(k_0,k_1-1) - kV_0(k_0,k_1)\big] \\ &\quad +\lambda_0\big[V_1(k_0,k_1) - V_0(k_0,k_1)\big]\Big\} \qquad (6) \\ rV_1(k_0,k_1) &= \max_{\gamma\geq 0}\Big\{k_0\pi_0 + k_1\pi_1 - wc(\gamma)k + \gamma k\big[V_1(k_0+1,k_1) - V_1(k_0,k_1)\big] \\ &\quad +\delta\big[k_0V_1(k_0-1,k_1) + k_1V_1(k_0,k_1-1) - kV_1(k_0,k_1)\big] \\ &\quad +\lambda_1\big[V_0(k_0,k_1) - V_1(k_0,k_1)\big]\Big\} \qquad (7) \end{aligned}$$

where $k \equiv k_0 + k_1$ is the total number supplied and the subscript represent's the firm's current type. In each case, the first three terms on the right sides of (6) and (7) represent the cash flow obtained by supplying the portfolio of current products net of the current expenditure on R&D. The next three terms are respectively the expected charge in value attributable to the arrival of a new product line, the destruction of the existing product line, and a transition to the other creativity state.

The unique solution to (6) and (7) takes the form

$$\begin{aligned} V_i\left(k_0, k_1\right) &= \frac{k_0\pi_0 + k_1\pi_1}{r+\delta} + k\Psi_i, \ i = 0, 1 \text{ where} \\ \Psi_i &= \max_{\gamma\geq 0}\left\{\frac{\gamma\left(\frac{\pi_i}{r+\delta} + \Psi_i\right) - wc\left(\gamma\right) + \lambda_i\Psi_j}{r+\delta+\lambda_i}\right\}, \ j = \left|i-1\right|, \end{aligned} \tag{8}$$

as one can verify by substitution. Obviously, the first term on the right side of (8) represents the expected present value of the future profit streams generated by the firm's current products. The second term is the value of the firm R&D operation. One can think of there being k research operations, each associated with an existing product line, and regard Ψ_i as the asset price assigned to each operation when in creativity state i. That price is the expected present value of the net profit generated by the choice of the current innovation frequency γ.

The fact that a type 1 innovation yields a higher profit rate implies that the value of R&D is higher when the firm is high creativity state.

Proposition 1 $\pi_1 > \pi_0$ *implies* $\Psi_1 > \Psi_0$.

Proof. To the contrary, suppose that $\Psi_0 - \Psi_1 \geq 0$. Under the supposition, the equations of (8) and $\pi_1 > \pi_0$ imply the following contradiction,

$$\begin{aligned} \Psi_1 &= \max_{\gamma\geq 0}\left\{\frac{\gamma\frac{\pi_1}{r+\delta} - wc\left(\gamma\right) + \lambda_1\left(\Psi_0 - \Psi_1\right)}{r+\delta-\gamma}\right\} \\ &\geq \max_{\gamma\geq 0}\left\{\frac{\gamma\frac{\pi_1}{r+\delta} - wc\left(\gamma\right)}{r+\delta-\gamma}\right\} > \max_{\gamma\geq 0}\left\{\frac{\gamma\frac{\pi_0}{r+\delta} - wc\left(\gamma\right)}{r+\delta-\gamma}\right\} \\ &\geq \max_{\gamma\geq 0}\left\{\frac{\gamma\frac{\pi_0}{r+\delta} - wc\left(\gamma\right) + \lambda_0\left(\Psi_1 - \Psi_0\right)}{r+\delta-\gamma}\right\} = \Psi_0. \end{aligned}$$

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As any positive choice of the state contingent innovation rate satisfies,

$$wc'(\gamma_i) = v_i \equiv \frac{\pi_i}{r+\delta} + \Psi_i, \tag{9}$$

the sufficient second order condition ($c''(\gamma) > 0$) and Proposition 1 imply that firm's innovate more frequently while in the high creativity state. In other words, $\gamma_1 > \gamma_0$.

The "business stealing" externality present when new products replace old ones is offset somewhat if the rights to an innovation can be traded. Indeed, Proposition 1 implies that an incumbent currently in more creative state has an incentive to buy the rights to a new innovation of low quality, which only has value v_0, even if the incumbent cannot exploit the technology it embodies.[1] Formally, the capital loss that the firm would otherwise suffer exceeds the value to the creator,

$$\begin{aligned} V_1(k_0,k_1) - V_1(k_0,k_1-1) &= \frac{\pi_1}{r+\delta} + \Psi_1 > \\ \frac{\pi_0}{r+\delta} + \Psi_1 &= V_1(k_0,k_1) - V_1(k_0-1,k_1) \\ &> \frac{\pi_0}{r+\delta} + \Psi_0 \equiv v_0, \end{aligned}$$

as a corollary of Proposition 1 and the definition of v_0 in equation (9). In the next section, we allow buyouts of this kind.

2.4 Value Function with Buyout

Suppose an incumbent can make a take-it-or-leave-it offer for the rights to a market threatening innovation. Since an offer equal to its value to the innovator, v_0, will be made and accepted in this case the creativity state contingent value functions are,

$$\begin{aligned} rV_0(k_0,k_1) &= \max_{\gamma\geq 0}\Big\{k_0\pi_0 + k_1\pi_1 - wc(\gamma)k + \gamma k\big[V_0(k_0+1,k_1) - V_0(k_0,k_1)\big] \\ &\quad +\delta\big[k_0V_0(k_0-1,k_1) + k_1V_0(k_0,k_1-1) - kV_0(k_0,k_1)\big] \\ &\quad +\lambda_0\big[V_1(k_0,k_1) - V_0(k_0,k_1)\big]\Big\} \\ rV_1(k_0,k_1) &= \max_{\gamma\geq 0}\Big\{k_0\pi_0 + k_1\pi_1 - wc(\gamma)k + \gamma k\big[V_1(k_0+1,k_1) - V_1(k_0,k_1)\big] \\ &\quad +\delta\left(1-\beta\right)\big[k_0V_1(k_0-1,k_1) + k_1V_1(k_0,k_1-1) - kV_1\left(k_0,k_1\right)\big] \\ &\quad -\delta\beta k v_0 + \lambda_1\big[V_0(k_0,k_1) - V_1(k_0,k_1)\big]\Big\}, \end{aligned}$$

where β is the fraction of new innovations that are of low quality. Here, one can verify that the solution is

$$\begin{aligned} V_0\left(k_0,k_1\right) &= \frac{k_0\pi_0 + k_1\pi_1}{r+\delta} + k\Psi_0 \\ V_1\left(k_0,k_1\right) &= \frac{k_0\left(\pi_0 - \delta\beta v_0\right) + k_1\left(\pi_1 - \delta\beta v_0\right)}{r+\delta\left(1-\beta\right)} + k\Psi_1. \end{aligned} \tag{10}$$

In this case

$$\Psi_0 = \max_{\gamma\geq 0}\left\{\frac{\gamma\left[\frac{\pi_0}{r+\delta} + \Psi_0\right] - wc\left(\gamma\right) + \lambda_0\Psi_1}{r+\delta+\lambda_0}\right\}$$

[1]Indeed, Microsoft has been accused of doing precisely this in order to protect the quasi rents accruing to existing product lines.

is the same as above because the creator of a low quality product is indifferent between introducing the innovation and the sale of the right to do so at it value v_0. However,

$$\Psi_1 = \max_{\gamma \geq 0} \left\{ \frac{\gamma \left[\frac{\pi_1 - \delta \beta v_0}{r + \delta(1-\beta)} + \Psi_1 \right] - wc(\gamma) + \lambda_1 \Psi_0}{r + \delta (1 - \beta) + \lambda_1} \right\}. \tag{11}$$

The function form here reflects the fact that a buyout prevents destruction of a product line but is costly. In this case, the first order conditions for the positive optimal innovation rate choices are

$$wc'(\gamma_0) = v_0 \equiv \frac{\pi_0}{r + \delta} + \Psi_0 \tag{12}$$

$$wc'(\gamma_1) = v_1 \equiv \frac{\pi_1 - \delta \beta v_0}{r + \delta (1 - \beta)} + \Psi_1. \tag{13}$$

At the other extreme bargaining outcome, the innovator has the power to make the take it or leave it offer. As before, a buyout only occurs if the innovation is of low quality and threatens a market supplied by a firm in the high creative state. In this case, the innovator extracts all of the incumbent's rent. If the incumbent's product is of low quality then the innovator obtains $v_{01} = \pi_0 / (r + \delta) + \Psi_1$. If the product is of high quality, the innovator extracts $v_{11} = \pi_1 / (r + \delta) + \Psi_1$. Hence, the expected buyout value is $\hat{v}_1 = \frac{K_{01}}{K_1} v_{01} + \frac{K_{11}}{K_1} v_{11}$, where $K_1 \equiv K_{01} + K_{11}$ and K_{ij} is the mass of type i products supplied by type j incumbents. Now the value functions are,

$$\begin{aligned} rV_0(k_0, k_1) &= \max_{\gamma \geq 0} \Big\{ k_0 \pi_0 + k_1 \pi_1 - wc(\gamma) k \\ &\quad + \gamma k \big[(1 - K_1) \big[V_0(k_0 + 1, k_1) - V_0(k_0, k_1) \big] + K_1 \hat{v}_1 \big] \\ &\quad + \delta \big[k_0 V_0(k_0 - 1, k_1) + k_1 V_0(k_0, k_1 - 1) - k V_0(k_0, k_1) \big] \\ &\quad + \lambda_0 \big[V_1(k_0, k_1) - V_0(k_0, k_1) \big] \Big\} \\ rV_1(k_0, k_1) &= \max_{\gamma \geq 0} \Big\{ k_0 \pi_0 + k_1 \pi_1 - wc(\gamma) k + \gamma k \big[V_1(k_0 + 1, k_1) - V_1(k_0, k_1) \big] \\ &\quad + \delta \big[k_0 V_1(k_0 - 1, k_1) + k_1 V_1(k_0, k_1 - 1) - k V_1(k_0, k_1) \big] \\ &\quad + \lambda_1 \big[V_0(k_0, k_1) - V_1(k_0, k_1) \big] \Big\}. \end{aligned}$$

In this case, the solutions take the form

$$rV_0(k_0, k_1) = \frac{k_0 (\pi_0 + \gamma_0 K_1 \hat{v}_1) + k_1 (\pi_1 + \gamma_0 K_1 \hat{v}_1)}{r + \delta} + k \Psi_0 \tag{14}$$

$$rV_1(k_0, k_1) = \frac{k_0 \pi_0 + k_1 \pi_1}{r + \delta} + k \Psi_1.$$

where

$$\Psi_0 = \max_{\gamma\geq 0} \frac{\gamma\left[K_1\hat{v}_1 + (1-K_1)\left[\frac{\pi_0+\gamma_0 K_1\hat{v}_1}{r+\delta} + \Psi_0\right]\right] - wc(\gamma) + \lambda_0\Psi_1}{r+\delta+\lambda_0}$$

$$\Psi_1 = \max_{\gamma\geq 0} \frac{\gamma\left(\frac{\pi_1}{r+\delta} + \Psi_1\right) - wc\left(\gamma\right) + \lambda_1\Psi_0}{r+\delta+\lambda_1}$$

Finally, the first order conditions for the optimal state contingent innovation rate choices are

$$\begin{aligned} wc'\left(\gamma_0\right) &= v_0 \equiv K_1\hat{v}_1 + (1-K_1)\left[\frac{\pi_0+\gamma_0 K_1\hat{v}_1}{r+\delta} + \Psi_0\right] \\ wc'\left(\gamma_1\right) &= v_1 \equiv \frac{\pi_1}{r+\delta} + \Psi_1. \end{aligned} \tag{15}$$

2.5 Firm Entry and Labor Market Clearing

The entry of a new firm requires a successful innovation. Suppose that there is a constant measure m of identical potential entrants. The rate at which any one of them generates a new product is γ and the total cost is $wc(\gamma)$ where the cost function is the same as that faced by an incumbent. The firm's type is unknown ex ante but is realized immediately after the arrival of an innovation. Since the aggregate entry rate is $\eta = m\gamma$, the entry rate satisfies the following free entry condition

$$wc'\left(\frac{\eta}{m}\right) = V_0(0,1)\phi_0 + V_1(1,0)\phi_1 = v_0\phi_0 + v_1\phi_1 \tag{16}$$

where ϕ_i is the probability that the entrant will turn out to be of creativity type i initially. Obviously, in this formulation learning one's type takes no time, which is unrealistic but a useful abstraction for the purposes of this paper.

There is a fixed measure of available workers, denoted by ℓ, seeking employment at any positive wage. In equilibrium, these are allocated across production and R&D activities, those performed by both incumbent firms and potential entrants. The number of workers required per product of type $i = 0, 1$ is $x_i = 1/p_i = (1-\kappa)/q_i w$ from equations (4). The number of R&D workers employed per product by incumbent firms of type $j = 0, 1$ is $\ell_R(j) = c(\gamma_j)$. Because each potential entrant innovates at frequency η/m, the aggregate number of workers engaged by all m in R&D is $\ell_E = mc(\eta/m)$. Hence, the equilibrium wage satisfies the labor market clearing condition

$$\ell = \frac{1-\kappa}{w}\left(\frac{K^0}{q_0} + \frac{K^1}{q_1}\right) + K_0 c(\gamma_0) + K_1 c(\gamma_1) + mc(\eta/m) \tag{17}$$

where $K^i \equiv K_{i0} + K_{i1}$ represents the fraction of products of productivity q_i and $K_j \equiv K_{0j} + K_{1j}$ denotes the fraction supplied by firms in state j.

2.6 Aggregate Dynamics

The overall state of the aggregate economy can be represented by the joint distribution of products over product types and firm states. Let K_{ij} represent the number of products of type $i \in \{0,1\}$ supplied by firms in creativity state $j \in \{0,1\}$. If innovator and incumbent are of the same type or the innovator is more creative, then the incumbent is priced out of the market because the value to the innovator exceeds the lost value to the incumbent. However, if the innovation is of low quality and the incumbent is in the high creativity state, then the innovation is bought out by the incumbent independent of how the surplus is divided between innovator and incumbent. Finally, because every firm leaves state i at rate λ_i, the follow laws of motion hold.

$$\begin{aligned}
\dot{K}_{11} &= \lambda_0 K_{10} + \eta\phi_1 + \gamma_1 K_{11} - (\delta(1-\beta) + \lambda_1) K_{11} && (18)\\
\dot{K}_{01} &= \lambda_0 K_{00} + \gamma_1 K_{01} - (\delta(1-\beta) + \lambda_1) K_{01} && (19)\\
\dot{K}_{10} &= \lambda_1 K_{11} + \gamma_0 K_{10} K_0 - (\delta + \lambda_0) K_{10} && (20)\\
\dot{K}_{00} &= \lambda_1 K_{01} + (\eta\phi_0 + \gamma_0 K_{00}) K_0 - (\delta + \lambda_0) K_{00} && (21)
\end{aligned}$$

where β is the fraction of the innovation flow created by firms in the low productivity state, η is the entry rate and ϕ_j is the fraction of firms that enter in creativity state $j = 0, 1$. By implication,

$$\begin{aligned}
\dot{K}_1 &= \dot{K}_{01} + \dot{K}_{11} = \lambda_0 K_0 + \eta\phi_1 + \gamma_1 K_1 - (\delta(1-\beta) + \lambda_1) K_1 && (22)\\
\dot{K}_0 &= \dot{K}_{00} + \dot{K}_{10} = \lambda_1 K_1 + (\eta\phi_0 + \gamma_0 K_0) K_0 - (\delta + \lambda_0) K_0. && (23)
\end{aligned}$$

There are also two identities. First, the flow of creations by low creativity firms that are bought out is equal to the flow of products supplied by high creative types that are not destroyed by innovation and the fractions of the goods supplied by both firm types, K_0 and K_1, must sum to unity. Formally, these conditions

$$\delta\beta K_1 \equiv (\eta\phi_0 + \gamma_0 K_0)(1 - K_0) = 0 \text{ and } K_0 + K_1 \equiv 1$$

imply that

$$\beta = \frac{\eta\phi_0 + \gamma_0 K_0}{\delta}. \qquad (24)$$

In turn this fact and $\dot{K}_0 + \dot{K}_1 = 0$ at every date require that the innovation rate equal the sum of the innovations of entrants plus those the average rate of incumbents,

$$\delta = \eta + \gamma_1 K_1 + \gamma_0 K_0. \qquad (25)$$

In other words, δ is the aggregate creation rate and β is the fraction that are created by firm in the low creativity state.

To find the steady state solution, use equation (25) to substitute out δ in equation of (23). The result is

$$\dot{K}_0 = \lambda_1 K_1 - K_0 \left[\gamma_1 K_1 + \eta\phi_1 + \lambda_0\right] = 0$$

As this expression defines a strictly increasing relationship between K_0 and K_1 that passes through the origin, the fact that $K_0 + K_1 = 1$ implies that a unique positive steady state solution for the pair (K_0, K_1) exists for any set of innovation and transitions rates.

We note in passing that the dynamic in the no buyouts case is simply

$$\dot{K}_1 = \lambda_0 K_0 + \eta\phi_1 + \gamma_1 K_1 - (\delta + \lambda_1) K_1 \tag{26}$$

$$\dot{K}_0 = \lambda_1 K_1 + \eta\phi_0 + \gamma_0 K_0 - (\delta + \lambda_0) K_0. \tag{27}$$

2.7 The Growth Rate

For equations (2) and (3), the contribution of any innovation to the aggregate rate of consumption growth in the natural log of its improvement in productivity over the previous version. Since an innovation is introduced if it is either a high quality product or it is a low quality product replacing a product supplied by a firm in the low creativity state, the growth rate is consumption is

$$g = \frac{\dot{C}}{C} = \eta(\phi_0 \ln(q_0) K_0 + \phi_1 \ln(q_1)) + \gamma_0 \ln(q_0) K_0^2 + \gamma_1 \ln(q_1) K_1. \tag{28}$$

Without buyouts, the growth rate is,

$$g = \frac{\dot{C}}{C} = \eta(\phi_0 \ln(q_0) + \phi_1 \ln(q_1)) + \gamma_0 \ln(q_0) K_0 + \gamma_1 \ln(q_1) K_1. \tag{29}$$

2.8 Market Equilibrium

A steady state *market equilibrium* is a triple composed of a labor market clearing wage w, entry rate η, and creative destruction rate δ together with an optimal creation rate for each firm type, γ_j, $j \in (0, 1)$ and a distribution of products across product and firm types, K_{ij}, $(i, j) \in (0, 1)^2$, that satisfy equations (16), (17), (**??**), (15), and the steady state conditions implied by equations (18)-(21).

3 The Social Planner's Problem

3.1 Formulation

The social planner chooses a non-negative time paths for the production rate of both product types, x_i, the rate of new product creation for each firm creativity state, γ_j, and the rate of product innovation by potential entrants, γ_e to maximize the present discounted utility of the representative household's consumption subject to the fact that there are a fixed number of intermediate products, a labor resource constraint, and laws of motion for the state variables. Under complete and symmetric information, both the firm and the planner observes the realized productivity of any innovation.

The planner may also find it in her interest to screen innovations before adoption. The adoption decision impacts the evolution of the product distribution and the current growth rate as we have already demonstrated. The planner's adoption decision is a trade-off between the gain from an innovation's quality improvement and the potential loss associated with the increased scale of the innovator and the reduced scale of the incumbent as reflected in the number of products supplied, k. The latter is a loss if the incumbent is more creative. However, if the incumbent is not more creative than the innovator, then the change in scale of the two is always yields a weak gain in welfare. Hence, the adoption decision is reduced to a determination of whether or not to adopt low quality product for a product currently supplied by more creative incumbent. Let the choice indicator, denoted as Φ, equal unity if the innovation is adopted in this case.

The planner's strategy, the choice of $(x_0, x_1, \gamma_0, \gamma_1, \gamma_e, \Phi)$ that maximizes the expected present value of the representative consumer's utility stream subject to a set of constraints. Formally, the criterion is

$$\int_0^\infty \ln C_t e^{-rt} dt = \int_0^\infty \left[\ln A_t + \sum_{i=0}^{1} \sum_{j=0}^{1} \ln \left(x_i(t) \right) K_{ij}(t) \right] e^{-rt} dt$$

where A represents aggregate productivity and K_{ij} is the fraction of product of type i currently supplied by incumbent firms in creativity state j. The constraints follow: Employment cannot exceed the available labor supply,

$$\ell \geq \sum_{i=0}^{1} \sum_{j=0}^{1} \left[x_i + c(\gamma_j) \right] K_{ij} + mc\left(\gamma_e\right).$$

The potential entry rate is $\eta = m\gamma_e$. As a consequence, the law of motion for aggregate productivity can be written

$$\begin{aligned}\frac{d\ln A}{dt} &\equiv g = m\gamma_e\left[\phi_0\left(1-(1-\Phi)K_1\right)\ln q_0+\phi_1\ln(q_1)\right]\\ &\quad+\gamma_0K_0\left(1-(1-\Phi)K_1\right)\ln q_0+\gamma_1K_1\ln q_1.\end{aligned}$$

where $K_j = K_{0j}+K_{1j}$ is the fraction of products supplied by firms in creativity state j and, because a low quality innovation threatens the market of a creative firm with probability $1-K_0 = K_1$, the distribution of products across firm types evolves according to

$$\begin{aligned}\dot{K}_0 &= \lambda_1K_1+\left[m\gamma_e\phi_0+\gamma_0K_0\right]\left(1-(1-\Phi)K_1\right)\\ &\quad-\left(m\gamma_e\phi_0+\gamma_0K_0+m\gamma_e\phi_1+\gamma_1K_1+\lambda_0\right)K_0\end{aligned}$$

$$\begin{aligned}\dot{K}_1 &= \lambda_0K_0+m\gamma_e\phi_1+\gamma_1K_1\\ &\quad-\left(\left[m\gamma_e\phi_0+\gamma_0K_0\right]\Phi+m\gamma_e\phi_1+\gamma_1K_1+\lambda_1\right)K_1.\end{aligned}$$

Although the complete planner's problem includes laws of motion for every K_{ij}, only K_0 and K_1 are decision relevant state variables.

The planner's problem is a relatively standard one in dynamic control. The constraint augmented present value Hamiltonian for the problem can be written as,

$$\begin{aligned}H &= \ln A+\sum_i\sum_j\ln\left(x_i(t)\right)K_{ij}(t)\\ &\quad+\omega\left(\ell-\sum_i\sum_j\left[x_i+c(\gamma_j)\right]K_{ij}-mc\left(\gamma_e\right)\right)\\ &\quad+\Lambda\left[\begin{array}{c} m\gamma_e\left[\phi_0\left(1-(1-\Phi)K_1\right)\ln q_0+\phi_1\ln(q_1)\right]\\ +\gamma_0K_0\left(1-(1-\Phi)K_1\right)\ln q_0+\gamma_1K_1\ln q_1\end{array}\right]\\ &\quad+\upsilon_0\left[\begin{array}{c}\lambda_1K_1+\left[m\gamma_e\phi_0+\gamma_0K_0\right]\left(1-(1-\Phi)K_1\right)\\ -\left(m\gamma_e\phi_0+\gamma_0K_0+m\gamma_e\phi_1+\gamma_1K_1+\lambda_0\right)K_0\end{array}\right]\\ &\quad+\upsilon_1\left[\begin{array}{c}\lambda_0K_0+m\gamma_e\phi_1+\gamma_1K_1\\ -\left(\left[m\gamma_e\phi_0+\gamma_0K_0\right]\Phi+m\gamma_e\phi_1+\gamma_1K_1+\lambda_1\right)K_1\end{array}\right]\end{aligned}$$

where ω is the labor supply constraint multiplier, Λ is the shadow price of the state variable $\ln A$ and υ_j is the shadow price or co-state variable associated with K_j, $j\in(0,1)$.

As the optimal controls maximize the Hamiltonian given state and co-state variables, the first order necessary conditions for all the continuous choice variables are

$$\begin{aligned}
\frac{\partial H}{\partial x} &= \left(\frac{1}{x}-\omega\right) K_i \leq 0 \text{ (with } = \text{ if } x_{ij}>0\text{)}, \; i=0,1 \text{ and } j=0,1. \qquad (30)\\
\frac{\partial H}{\partial \gamma_0} &= \left[\begin{array}{c} (\Lambda \ln q_0 + \upsilon_0)\left(1-(1-\Phi)K_1\right) \\ -\upsilon_0 K_0 - \upsilon_1 K_1 \Phi - \omega c'(\gamma_0) \end{array}\right] K_0 \leq 0 \text{ (with = if } \gamma_0>0\text{)}.\\
\frac{\partial H}{\partial \gamma_1} &= \left[\Lambda \ln q_1 + \upsilon_1 - \upsilon_1 K_1 - \upsilon_0 K_0 - \omega c'(\gamma_1)\right] K_1 \leq 0 \text{ (with = if } \gamma_1>0\text{)}.\\
\frac{\partial H}{\partial \gamma_e} &= \left[\begin{array}{c} \Lambda\left[\left(1-(1-\Phi)K_1\right)\phi_0 \ln q_0 + \phi_1 \ln(q_1)\right] \\ +\upsilon_0\left[\left(1-(1-\Phi)K_1-K_0\right)\phi_0 - K_0\phi_1\right] \\ +\upsilon_1\left[(1-K_1)\phi_1 - K_1\phi_0\Phi\right] - \omega c'(\gamma_e) \end{array}\right] m \leq 0 \text{ (with = if } \gamma_e>0\text{)}.
\end{aligned}$$

The assumption that the cost of R&D is convex in the innovation rate is sufficient to guarantee that the second order necessary conditions are satisfied. The optimality condition for the discrete choice of Φ can be represented as

$$\begin{aligned}
\Phi &= 1 \text{ if and only if} \qquad (31)\\
\frac{\partial H}{\partial \Phi} &= (m\gamma_e\phi_0 + \gamma_0 K_0)\left(\Lambda \ln(q_0) + \upsilon_0(1-K_0) - \upsilon_1 K_1\right) K_1 \geq 0.
\end{aligned}$$

Because they enter symmetrically in the production of the consumption good and they cost the same at the margin, the first equation of (30) implies that all intermediate products should be supplied at the same rate. Formally,

$$x_j = x \equiv \frac{1}{\omega}, \; j \in (0,1). \qquad (32)$$

Given this result, the co-state (Euler) equations are

$$\begin{aligned}
\frac{\partial H}{\partial \ln A} &= 1 = r\Lambda - \dot{\Lambda} \qquad (33)\\
\frac{\partial H}{\partial K_0} &= \ln(x) - \omega[x + c(\gamma_0)] + \Lambda\gamma_0(1-(1-\Phi)K_1)\ln q_0 \\
&\quad +\upsilon_1(\lambda_0 - \gamma_0\Phi K_1) + \upsilon_0\left[\begin{array}{c} \gamma_0(1-K_0-(1-\Phi)K_1) \\ -\left(m\gamma_e\phi_0 + \gamma_0 K_0 + m\gamma_e\phi_1 + \gamma_1 K_1 + \lambda_0\right) \end{array}\right] \\
&= r\upsilon_0 - \dot{\upsilon}_0 \\
\frac{\partial H}{\partial K_0} &= \ln(x) - \omega[x + c(\gamma_0)] + \Lambda\left[\gamma_1 \ln q_1 - (m\gamma_e\phi_0 + \gamma_0 K_0)(1-\Phi)\ln q_0\right] \\
&\quad +\upsilon_0\left[\lambda_1 - (m\gamma_e\phi_0 + \gamma_0 K_0)(1-\Phi) - \gamma_1 K_0\right] \\
&\quad +\upsilon_1[\gamma_1(1-K_1) - ([m\gamma_e\phi_0 + \gamma_0 K_0]\Phi + m\gamma_e\phi_1 + \gamma_1 K_1 + \lambda_1)] \\
&= r\upsilon_1 - \dot{\upsilon}_1
\end{aligned}$$

Of course, υ_i represents the social value of creating a high quality product. Finally, the necessary transversality conditions require that Λe^{-rt} and $\upsilon_j e^{-rt}$ converge to zero as $t \to \infty$.

The transversality condition requires that the shadow price of log productivity equal the inverse of the discount rate at all dates,

$$\Lambda = \frac{1}{r}. \tag{34}$$

As a consequence of (31) and (34), the optimal adoption strategy has the following reservation property:

$$\Phi = 1 \text{ if and only if } \frac{\ln q_0}{r} + \upsilon_0 \geq \upsilon_0 K_0 + \upsilon_1 K_1 \tag{35}$$

In other words, a low quality innovation is adopted were it to replace a product supplied by a more creative firm type if and only if the present utility value of an innovation's spill over on aggregate future productivity plus the value of the product to the innovator exceeds the expected value of the product it will replace.

The remaining equations of (30) can be rewritten as

$$\begin{aligned} \omega c'(\gamma_0) &= \left(\frac{\ln q_0}{r} + \upsilon_0\right)(1 - (1-\Phi) K_1) - \upsilon_0 K_0 - \upsilon_1 K_1 \Phi \\ \omega c'(\gamma_1) &= \frac{\ln q_1}{r} + \upsilon_1 - \upsilon_1 K_1 - \upsilon_0 K_0 \\ \omega c'(\gamma_e) &= \phi_0 \left(\left(\frac{\ln q_0}{r} + \upsilon_0\right)(1 - (1-\Phi) K_1) - \upsilon_0 K_0 - \upsilon_1 K_1 \Phi\right) \\ &\quad + \phi_1 \left(\frac{\ln (q_1)}{r} + \upsilon_1 - \upsilon_1 K_1 - \upsilon_0 K_0\right). \end{aligned} \tag{36}$$

In all cases, the marginal cost of innovation is equal to the expected social return. For a firm in the more creative state ($j = 1$), the social return is the the sum of the value of the spill-over plus the difference between the value of a high quality product and the expected value of the product it will replace. If low quality produces were always adopted, the expected return in the case of a firm in the less creative state has a similar interpretation. However, if a low quality product is not adopted were it to replace a product supplied by a firm in the more creative state ($\Phi = 1$), then the expected return is simply the value of the spill-over since $K_0 = 1 - K_1$. Note that if one were to set $\Phi = 1$ when condition (31) fails to hold, then the return to R&D for a firm in the low creativity state is negative. Finally, the expected return to entry is the unconditional mean, the expected return prior to the realization of the firm's future type.

3.2 Steady State Planner Solution

In steady state, $d \ln A_t / dt = g$ for all t. Consequently, $\ln A_t = gt + \ln A_0$, where $\ln A_0$ is the initial quality level. Furthermore, because the optimal production choice is the same for both product type; $x_i(t) = x$. Thus, the steady state social planner criterion is,

$$
\begin{aligned}
\max \int_0^\infty \ln C_t e^{-rt} dt &= \max \int_0^\infty \left[\ln A_t + \sum_i \sum_j \ln \left(x_{ij}(t) \right) K_{ij}(t) \right] e^{-rt} dt \\
&= \max \int_0^\infty \left[gt + \ln A_0 + \ln (x) \right] e^{-rt} dt \\
&= \max \left[\ln A_0 \int_0^\infty e^{(g-r)t} dt + \ln (x) \int_0^\infty e^{-rt} dt \right] \\
&= \max \left[g \int_0^\infty t e^{-rt} dt + \frac{\ln (x) + \ln A_0}{r} \right] \\
&= \max \left[\frac{g}{r^2} + \frac{\ln (x) + \ln A_0}{r} \right] \\
&= \frac{\ln A_0}{r} + r \max \left[\ln (x) + \frac{g}{r} \right].
\end{aligned}
$$

The full steady state planner problem reduces to a choice of the real vector $(\Phi, \gamma_0, \gamma_1, \gamma_e)$ that solves

$$
\begin{aligned}
\max \quad & \{ r \ln (x) + g \} \qquad (37) \\
st \quad : \quad & x = \ell - c(\gamma_0)(1 - K_1) - c(\gamma_1) K_1 - mc(\gamma_e) \\
& g = m\gamma_e \left[\phi_0 \ln (q_0) (1 - (1 - \Phi) K_1) + \phi_1 \ln (q_1) \right] \\
& + \gamma_0 (1 - K_1) \ln (q_0) (1 - (1 - \Phi) K_1) + \gamma_1 K_1 \ln (q_1) \\
& \lambda_1 K_1 + \left[m\gamma_e \phi_0 + \gamma_0 (1 - K_1) \right] (1 - (1 - \Phi) K_1) \\
= \quad & (m\gamma_e \phi_0 + \gamma_0 (1 - K_1) + m\gamma_e \phi_1 + \gamma_1 K_1 + \lambda_0)(1 - K_1).
\end{aligned}
$$

In the following section we will compare the market equilibrium outcomes with the social planner solution. Note that welfare in the market solution takes a similar form to the social planner except here products are not supplied at the same rates. Normalizing the initial quality level at unity, welfare in the steady state market equilibrium is given by,

$$
\begin{aligned}
U &= r \sum_i \ln \left(\frac{Z(1-\kappa)}{q_i w} \right) K^i + g \\
&= r \left[\ln \left(\frac{Z(1-\kappa)}{w} \right) - \ln (q_0) K^0 - \ln (q_1) K^1 \right] + g,
\end{aligned}
$$

where Z is the household demand per product and K^i is the aggregate supply of type i products in steady state.

4 Numerical Solutions

In this section, we use model parameter estimates reported by Lentz and Mortensen (2005) to compare the quantitative properties of the market and the planner's solutions. We conclude the

Table 1: Model Parameter Values

Cost scale parameter	c_0	9.2477
Cost curvature parameter	c_1	2.8890
Capital cost per product	κ	0.4269
Interest rate	r	0.0500
Labor supply	L	42.9639
Value added	Z	15,981.1723
Entrant mass	m	1.4404
Low type quality improvement	q_0	1.0150
High type quality improvement	q_1	2.0100
Low type entrant probability	ϕ_0	0.9090
High type entrant probability	ϕ_1	0.0910

section by computing the welfare gain that could be achieved by a switch to the optimal solution when the market steady state solution characterizes the initial conditions.

4.1 Model parameters

The parameter estimates obtained by Lentz and Mortensen (2005) derived from Danish firm data using a simulated methods of moments method are reported in Table 1. In the empirical version of the model estimated, the R&D cost function is assumed to take the power form $c(\gamma) = c_0\gamma^{1+c_1}$. The parameter Z is the average real value added per product line and L is the total labor force. Since real value added per product is normalized at unity and the total measure of products is set to unity as well in the model, the total labor supply per product line is $\ell = L/Z$. The estimated model is based on 3 types of firms where the type conditional quality realization is based on Weibull distributions. The model in this paper is simplified to two types and the type conditional quality realization distribution is degenerate. The 3 types in Lentz and Mortensen (2005) can be grouped into one high type and two low types. The parameter values in Table 1 are based on such a grouping. Specifically, the type conditional quality improvement is chosen so as to match $E\ln(q_i)$ across the two models. Finally, the model is estimated under the assumption that a firm's initial type realization is permanent ($\lambda_0 = \lambda_1 = 0$) and that no buyouts are possible.

The extreme right skew in the distribution of firm types at entry reflects the fact that very few innovations by potential entrants are of high quality. Indeed, over 90% of the entrants create innovation of only marginal value. These might be interpreted as simply "imitators."

Table 2: Market equilibria with high type persistence ($\lambda_0 = \lambda_1 = .001$)

	Planner	Buyout A	Buyout B	No buyout
w	–	138.1722	134.0601	188.9128
δ	0.1405	0.1373	0.1586	0.0866
η	0.0627	0.0549	0.0935	0.0490
γ_0	0.0221	0.0197	0.0649	0.0197
γ_1	0.0845	0.0831	0.0651	0.0725
K_0	0.0110	0.0112	0.0134	0.6607
K_1	0.9890	0.9888	0.9866	0.3393
K^0	0.0048	0.0049	0.0080	0.6999
K^1	0.9952	0.9951	0.9920	0.3001
v_0	–	0.0587	1.0050	0.0804
v_1	–	3.7572	1.7974	3.4654
Ψ_0	–	0.0135	0.4128	0.0184
Ψ_1	–	1.6788	0.4165	1.3568
L_X	32.4264	33.1379	34.2592	40.6731
L_R	10.5374	9.8260	8.7047	2.2907
g	0.0635	0.0609	0.0508	0.0208
U	0.2367	0.2358	0.2273	0.2039
τ	0.0000	0.0169	0.1701	0.4806

4.2 Comparison of planner and market outcomes

This section will present steady state market outcomes for the three variations of the model described in section 2; the model without buyouts, the model with buyouts where the incumbent has full bargaining power (Buyout Model A), and the model with buyout where the innovator has the full bargaining power (Buyout Model B). A market equilibrium is a pair (w, η) such that the labor market clears, the free entry condition is satisfied, and all innovation rates maximize expected firm value. The market equilibria outcomes are compared to the steady state planner solution. Table 2 presents outcomes for a high level of type persistence as assumed in the original estimation.

In the table, L_X and L_R are respectively the amount labor engaged in product manufacturing and innovation. Welfare, U, in the market equilibria outcomes are compared to the planner problem by means of determining a tax, τ, on planner consumption such that agents are indifferent between the market outcome and the planner's solution,

$$r \ln \left(x^p \left(1 - \tau\right)\right) + g^p = \hat{U},$$

where $\hat{U}$ is the welfare in the market equilibrium in question and x^p and g^p are the planner consumption and growth levels, respectively.

A comparison of the planner's solution to the market equilibrium without buyouts reveals the fact that the optimal growth rate is over three times larger than the equilibrium growth rate. Although selection takes place in the sense that more creative firms innovate more frequently in equilibrium as well as in the planner's solution, more creative firms accounts for only 34% of the products supplied in steady state equilibrium as compared with almost 99% in the planner's solution. The difference is due to the fact that the planner will not adopt a low quality product if they were to otherwise replace a product line supplied by a more creative incumbent ($\Phi = 0$ is optimal.). As reflected in condition (31), this strategy is optimal because the contribution of the innovation to future aggregate productivity does not compensate for transfer of R&D capacity from the more to the less creative firm. Finally, the large overall differential in steady state welfare between the market equilibrium with no buyout possibility and the planner's solution represents 48% of planner consumption.

Given high firm type persistence, the option to buyout under either extreme bargaining outcome yields results that are more similar to those implied by the planner's solution than the market solution without buyouts. Specifically, more creative firms supply over 98% all products in steady state equilibrium in the case of both Model A and Model B. Clearly, more creative firm are exercising their option to buy the rights to low quality innovations when they threaten the market for a current product and doing so mimics the adoption policy that a planner would pursue. However, the two buyout models differ with respect to their implied growth rates and welfare levels.

In the case of Model A, where the incumbent has all the bargaining power, the market outcome closely approximates the planner's strategy. Optimal innovation rates by both types are only marginally smaller than those that the planner would choose. As a consequence, the growth rates are virtually the same and the welfare gain offered by the planner's solution represents on 1.7% of the planner's steady state consumption level.

Not surprisingly, the equilibrium in Model B, where the innovator has the bargaining power, is both less efficient and generates less growth because the less creative firm type has excessive incentive to innovate at the expense of more creative firms. Indeed, note that the equilibrium innovation rates are virtually identical. Even so, the compensating consumption tax is 17% in Model B as compared with 1.7% in Model A. This large difference is due to the "wasted" resources used by firms in the less creative state to capture rents from the more creative firms by innovating more frequently.

The buyout option, particularly when incumbent collect the rents, clearly goes a long way to eliminate the efficiency loss that occurs when firms in the less creative state expand innovation scale

Table 3: Market equilibria with low type persistence ($\lambda_0 = \lambda_1 = .2$)

	Planner	Buyout A	Buyout B	No buyout
w	–	206.2260	199.9882	210.1191
δ	0.1139	0.1213	0.1143	0.0993
η	0.0395	0.0643	0.0627	0.0533
γ_0	0.0230	0.0399	0.0410	0.0321
γ_1	0.0888	0.0709	0.0606	0.0618
K_0	0.4394	0.4495	0.4559	0.5318
K_1	0.5606	0.5505	0.5441	0.4682
K^0	0.7595	0.7694	0.7765	0.9034
K^1	0.2405	0.2306	0.2235	0.0966
v_0	–	0.6722	0.4702	0.3651
v_1	–	3.5501	2.1853	2.4242
Ψ_0	–	0.6228	0.2964	0.3083
Ψ_1	–	1.0567	0.4328	0.4950
L_X	35.3190	38.7639	40.1317	40.8953
L_R	7.6449	4.1999	2.8321	2.0685
g	0.0395	0.0319	0.0275	0.0241
U	0.2177	0.2129	0.2104	0.2088
τ	0.0000	0.0913	0.1368	0.1633

at the expense of a creative firm's capacity to innovate. However, even with buyouts, externalities exist. First, the attempted entry of the less creative firm into a more creative firm's market destroys it profit even if it does not actually destroy its product or its ability to create new ones. Second, the innovation itself, which would increase the quality level that others would build on in the future, is not adopted. Finally, the fact that intermediate good supplier price above marginal cost also generates allocative inefficiency. Still, a comparison of our quantitative solutions suggests that these considerations are of little net consequence.

In the low type persistence case reported in Table 3, the less creative type's negative innovation externality on the R&D capacity of the creative type firms is less dramatic. Therefore, allowing buyouts in the model has much less impact. In Table 3, the models are solved given $\lambda_0 = \lambda_1 = 0.2$, which implies that a firm switches type at a rate more than double that by which it develops new products. In this case, there is little selection effect in either the planner solution or any of the market outcomes. The low quality product remains a large part of the product mass with the no buyout equilibrium displaying almost the same proportion as that of the entry distribution. In the market equilibrium with buyout and in the planner solution, the selection effect is stronger and the fraction of high type products increases. Firm types exist in roughly equal proportion with increasing selection toward the high type as we move from the no buyout equilibrium towards the planner solution. But even in the planner solution, the more creative firms only supply 56% of the products.

The negative destruction or business stealing externality is less important when firm type is less persistent simply because firm type is a less meaningful distinction. Still, there are substantial differences in the growth rates and the levels of welfare attained across the various specifications. The buyout market equilibrium with full incumbent bargaining power, Model A, now exhibits considerably greater inefficiency relative to the planner solution. In particular, low quality products are significantly more expensive to buy out and, consequently, the return to innovation by firms in the more creative state is smaller. Because the innovation rate differential is smaller and, consequently, the equilibrium fraction of firms in the more creative state is substantially smaller, the growth rate is 0.75% lower than the optimal value and the compensating consumption tax is over 9%. Hence, substantial welfare loss relative to the planner's solution characterizes the best of the market equilibria considered.

5 Conclusion

We have shown that the "business stealing" externality in Lentz and Mortensen (2005) imposes a large efficiency loss on the order of almost 50% of consumption in the social planner solution. The unusually large magnitude of the negative externality is a particular feature of type heterogeneity in the model. Type selection can be an important contribution to growth and the efficiency loss is in large part caused by a diminished selection effect. We show that allowing incumbents to buy out innovators can restore the selection effect to almost that of the social planner. Consequently, the market equilibrium with the buyout option and the social planner economies are much closer to each other.

Type persistence is shown to have a large impact on the importance of the selection effect. With less type persistence, selection becomes less of a factor and the market equilibrium without buyout shows a smaller efficiency loss.

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