"A dynamic model of female labor force participation rate and human capital investment."

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Abstract:

In this paper we develop a dynamic model of human capital investment and labor participation decision with the purpose of explaining several stylized facts found in the data. The main empirical fact we are concerned with is that married women's labor force participation rate is u-shaped over the course of development. We also analyze the behavior of the relative education levels of men and women, the relative education levels of women in the labor force and outside of it, as well as the magnitude and sign of income and wage effects, as an economy develops. The underlying assumption of the model is the existence of economies of scope in the traditional sector of the economy, between household production and farm production. This creates a tradeoff between remaining in traditional production where farm work and child rearing can be undertaken simultaneously, or moving to modern production (the "city") where income is higher, given a high enough level of education.

JEL Classification: J16, J22, J24, J43, O11, O40
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"*Thus the participation of married women in the labor force may well be somewhat U-shaped over the course of economic development. Participation was initially high in family-farm regions and in cities with small retail establishments and substantial boarding. With the growth of the market, the progressive separation of home and work, and the movement of families from farms to cities, the participation of married women fell. Eventually, the trend reversed, and the participation of married women gradually rose with increased white-collar employment, education and changes in other factors …The U-shape in female labor force participation over the course of economic development is evident in several countries and is most apparent in those undergoing substantial economic development and a shift in the locus of production from the home to the market.*" Claudia Goldin pg. 45

1. Introduction

This paper presents a dynamic model of female labor force participation and human capital investment with the aim of accounting for several phenomena that have been documented empirically. Its main purpose is to provide an explanation for the U-shaped female labor force participation rate found in cross section and historical data. The model, however, goes further and also matches several other stylized facts. These include:

1. For developing countries, female labor force participation is negatively related to urbanization.
2. Early on, the educational attainment of women in the labor force is lower than that of women in general but as development progresses this is reversed.
3. The ratio of women's education to men's education is less then one and initially drops. Eventually it begins to increase and approaches one.
4. In the beginning the income effect is negative and relatively large in absolute value. It then approaches and becomes zero over time.
5. The wage effect is positive and initially small but increases with time.

Furthermore a possible connection is made between the U-shape, number two above and the puzzling finding from growth regressions that female educational attainment is negatively correlated with observed economic growth.

The underlying assumption of the model, which drives the results, is a simple one: in the traditional sector of the economy there are economies of scope in the production of household goods (particularly child rearing) and consumption goods (which here we identify with "market work"). The tradeoff that emerges then is between taking advantage of these economies of scope in the traditional sector, or giving them up and moving into the modern sector, where wages for market work are higher. Consequently the optimal choice depends on the level of human capital of the members of the household[1].

This paper is part of a broader research project aimed at understanding the role and experience of women in the process of economic development using both contemporary cross section and historical time series data. The purpose is to look at both how economic growth and development affect women's lives – are they truly "engines of liberation" for women? – and in turn to better document the contributions of women to the process itself. (Szulga 2004, 2006). It also has several important implications for development programs, particularly those which focus on women's education and the contributions of women both within the home and as workers in the formal market place. One policy relevant conclusion suggested by this paper is that increasing women's education may not result in immediate increases in output or the rate of economic growth. This is true if a significant portion of women in the economy who get educated never enter the labor force (or enter for only a brief period of time before marriage). However it would be naïve to conclude based on these types of estimates that female education makes little or no contribution to the economy. At early and middle stages of development, when the majority of female education is undertaken for the purposes of household production – rearing high quality children – the benefits of increased female education will not be discernible. In fact, there's likely to be a one generation lag; a time period which is generally longer than the standard five or ten year panels typically used in growth empirics.

The model presented in this paper is a bit unorthodox in several respects. In particular we assume that a lot of functions, which are usually taken to be curved, are linear. In fact we assume that instantaneous utility is linear in consumption (it is consumption) and that the household good and the market good are perfect substitutes. This is done for two reasons. The first, as is often the case with simplifying assumptions, is that linearity of instantaneous utility bequeaths tractability and allows for closed form solutions for many variables of interest. The basic intuition of the model however survives, as do most of the results, if convexity is substituted instead in the usual places. The model cannot be solved explicitly in that case however. The second reason is perhaps more substantial. We assume that the household good and the consumption goods are perfect substitutes. This is because we are interested in

[1] From now on we use "education" and "human capital" interchangeably. All the usual caveats apply.

female labor force **participation** and not hours worked. Labor force participation is essentially a binary variable and hence it makes a lot of sense to focus on a case that produces corner solutions. This is in contrast to some of the previous work on the subject, which, very casually, tends to treat labor force participation and hours worked as synonymous concepts. With these stipulations in mind we now turn to a discussion of the said previous work.

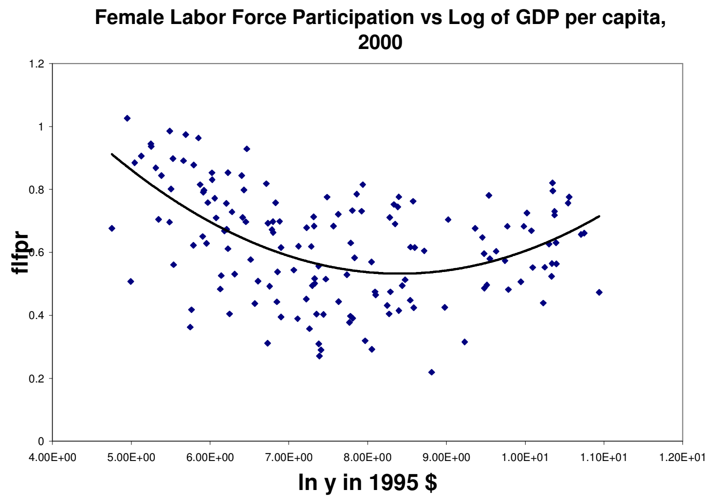
2. Literature Review

2.1 The Stylized Facts

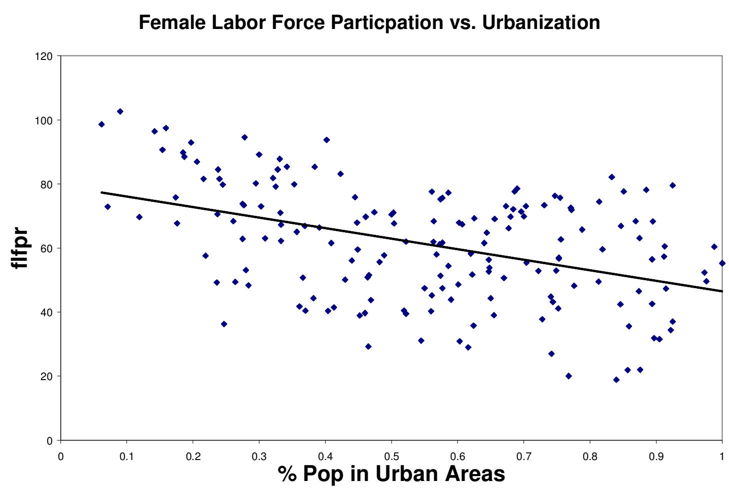
The stylized facts mentioned above have been well documented in the literature. The most notable work is Claudia Goldin's book "Understanding the Gender Gap" (1992). The theoretical model found below can be seen as providing a theoretical basis for the patterns documented therein. However, this paper utilizes an alternative assumption to explain the U-shaped path of female labor force participation – the existence of economies of scope on the farm rather then that of a social stigma associated with female market work which is the basis of Goldin's results. Of course, the two assumptions are not mutually exclusive.

Szulga (2004) examines the determinants of female labor force participation using contemporary cross section data. The U-shape relationship between income and female labor force participation is presented in panel (a) of Figure 1 below. Stylized fact 1 on participation and urbanization, is presented in panel (b). Note that both these results are robust to the inclusion of various control variables; fertility, male education, age dependency ratio, as well as cultural and institutional variables, As with historical data, the relationship between per capita income and female labor force participation is U-shaped, but cultural and institutional factors shift the curve.

Figure 1 - The U-shape and stylized fact #1



a. The estimated equation is flfpr=2.54-.48*ln y+.028*$\ln y^2$. The t-statistics are 6.34 and 5.98 respectively.



b. The estimated equation is flfpr=79.37-.33*%Urban. The t-statistic is 6.3

A good portion of the literature on female labor force participation rate has focused on the post WW2 period. However, because it concentrates on the post WW2 period it is mostly concerned with the upward sloping portion of the U-shape. Greenwood, Seshadri and Yorukoglu (2005) posit that the increase in female work since the 1950's is due to the increased productivity of physical capital used in household production, which served as a substitute for female labor in the household, releasing it for occupation in the market. The same idea can be found in Greenwood and Seshadri (2005). The introduction of household appliances economized on the time needed to produce a given level of the household good, allowing women to enter the labor force. In the model of this paper this would correspond to a *decrease* in the utility value of female work. As can be seen below the implication is that this would bring more women into the labor force. Hence, once again, the explanation is complementary to the one found in this paper.

Unfortunately this focus on post WW2 period still leaves unexplained what has caused married women to exit the labor force in large number over the course of early nineteenth century. Furthermore, given that technological progress in household production likely predates WW2, the question that arises is why women did not exit the household and enter the market earlier.

Henry Tam (2003) offers another explanation for the patterns in female labor force participation based on a distinction between physical human capital and mental human capital. One advantage of his model is that it also incorporates a fertility decision. Similarly to this paper, he also uses two sectors of. However, unlike in the present work, there is a one time switch from a primitive form of production to the modern. This is because there is no heterogeneity among households. In contrast, in our model households switch to the modern sector one by one so that at any given time a portion of the economy may be "modern" and a portion "traditional". In Tam's paper when the economy is pre-modern physical human capital, with which only males are endowed, plays the dominant role. At the same time, through "physical physical capital" (machines) accumulation, wages rise. This has the effect of increasing fertility and decreasing female labor force participation. At a critical point a switch to the modern form of production ensues and now mental human capital becomes more important. Once enough mental human capital has been accumulated through education, female labor force participation rises and fertility declines.

The second shortcoming of the literature is that it tends to treat female labor force participation and hours worked by women as synonymous concepts. While this may be a useful assumption during the modeling process, it should be recognized that it potentially obscures many underlying important economic phenomenon. An increase in work intensity by women who are already in the labor force is not the same as an influx of new female workers into the labor pool. This is particularly true if the female labor force is heterogeneous and variant over time. This fact is brought out strongly by Goldin's analysis of different cohorts of workingwomen over time (Goldin, pg. 154-157). It seems that the changes in female labor force participation rate after WW2 had a lot more to do with the composition of women who were entered the labor force rather than the number of hours worked (part time vs. full time) of women who were already there.

Hence, any model which makes tries to account for the observed patterns in female labor force participation rate and female education, should integrate the trends before WW2 and prior to it (i.e. explain the upward sloping as well as the downward sloping portion of the U) and also the heterogeneity found in female workers and non-workers over time (why at a given time some women work and some do not - which implies hours worked are not identical with labor force participation rates). We believe the model below makes a contribution along these lines incorporating a level of generality lacking in prior theoretical research.

2.2 Definition of household work and "for market work" and labor force participation

As has often been noted, the line between "household" work and "market" work is blurry at best. It becomes even more so the further back in time one goes, turning into a big arbitrary smudge when one considers an economy composed solely of subsistence farmer households. There's also a potential for circularity here as in some historical cases of data collection, explicitly or implicitly, the work done by women was defined as "household work" and men's work "an occupation". Hence a woman in the labor force (more precisely a "gainful worker", with the definition used by the US Census before 1940) was for all intents and purposes a woman who did what was at the time considered men's work.

Presently, the World Bank includes subsistence farmers, of either gender, in its definition of a labor force. This leads to two possible definitions of the labor force. Under a narrow definition, only work for a wage or with intent of a sale in a market is counted as labor force participation. A broader definition would include subsistence farmers, sporadic work such as a proprietorship of a boardinghouse, and manufacturing work done by women in the home where the husband sells the final output in a market. Hence Goldin argues that, in addition to female agricultural laborers, female boardinghouse keepers and manufacturing workers in homes and factories were systematically undercounted in the Censuses prior to 1910 (pg.58)[2]. The distinction between the two definitions of a labor force has implications for both empirical and theoretical dimensions of the subject (see also Horrell and Humphries (1995) for further discussion of possible biases in historical data on women's work).

Empirically, for the narrow definition in the US, female labor force participation rose slowly over the course of the nineteenth century and early twentieth, increasing substantially in the 1950's. Hence the trend is increasing over the whole period and in fact, most of the previous theoretical work on the subject has focused on explaining precisely this increase. However, using the broad definition, female labor force participation decreased from the beginning of nineteenth century until the 1890's, at least among married women. In this case we get a U-shaped female labor force participation rate, which is the main subject of this paper.

The theoretical model that follows is in the spirit of the broader definition of the labor force. This is by the virtue of the fact that women who are in traditional sector are considered to be in the labor force – they work on a farm or in cottage production – while simultaneously carrying out household activities such as child rearing. Simply redefining these women as excluded from the measure of the labor force would produce the ever-increasing trend in accord with the narrow definition. The model, hence, works either way. However, excluding female farm workers would obscure the economic forces at work, which engender the changing role of women in the development process.

2.2 Other work

The existence of economies of scope on the farm or in the traditional sector, to our knowledge, has not received wide or explicit attention in either the economic or history literature. At the same time, it seems like a natural assumption and many accounts of life during and prior to nineteenth century imply their presence. The quote at the beginning of this paper – particularly the reference to "progressive separation of work and home" - definitely seems to hint at their existence. Similarly, although the purpose of her research is different, Jane Adams (1998) in "Decoupling of Farm and Household" provides a description of nineteenth and early twentieth century farm life. She writes: "Prior to World War II, the household was inextricably integrated with the entire farm operation". Furthermore: "Most of this labor (children) was borne and

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[2] Indeed, the attempt at correcting the omission of these categories led to the rejection of female employment figures in the 1910 US Census.

raised on the farm by the wife. Large families were the rule: many women spent ten to twenty of their productive years being pregnant and nursing babies, *while* managing the complex activities needed to provision the household…" (emphasis added). Similarly, Winstanley (1996) quotes a women farmer from Lanceshire in the 1890's as saying "I work harder now than when I was a farm servant. The work is far rougher, for I and my daughter go out and help in the fields, to *save expense in labour*" (emphasis added)

Finally work in growth empirics has uncovered a bit of a puzzle. It appears that in many specifications female education is negatively correlated with subsequent economic growth. In Barro and Sala-i-Martin, (1995 – first edition) this appears as a fairly robust result. Lorgelly and Owens (1998) and Lorgelly et. al (1999) have questioned this result as arising from presence of outliers or some form of econometric misspecification. Barro and Sala-I-Martin themselves attribute it to the fact that a low female to male education ratio is a proxy for low level of development and hence is picking up standard convergence effects. Szulga (2004) however argues that the negative correlation between female education and growth arises from the interaction of female labor force participation, household work and education. The present paper to some extent embodies this idea. Although in our model female education and market income increase concurrently, for early and middle stages of the development process, the rise in market income is solely due to the increases in male education. At the same time investment in female education eats up a portion of income with nothing to show for it in terms of productivity until much later (i.e. parents invest in their own education for the sake of their children). Hence an econometric specification which estimates the effect of female education on growth, conditional on the level of male education would, if the model is true, find at best no correlation between female education and growth, at least for low and middle income countries.

3. The Model

There are two goods, two factors, and two production sites. The two goods are a "market" good (food or income) and a "household" good (child rearing). Utility is linear.

The two production sites are a "traditional" production site (the farm) and a "modern" production site (the city). Each household makes a location decision as to where its economic activities will be undertaken. The trade off involved is that while the "market" wages may be higher in the city, there are economies of scope on the farm, which allow female labor to be supplied simultaneously to household and market production. The intuitive example is that of much of agricultural work, or traditional "cottage" production, which can be carried out while the children are present. This in turn means that the children can be reared at the same time as work for income is being done. This generally is not an option for factory or office work.

The two factors are male and female labor. Male labor can only be used in market production. Female labor can be used to produce either good. Since utility is linear, if the household is located in the city, female labor will only be supplied to the one sector where it's productivity is higher. Hence the labor supply choice for the households in the city is a binary one; whether the woman will work in the house or in the market.

3.1 Static location decision

Let the production function for market work on the farm be given by: $Y_F = A_F L_F$ where A_F is a productivity parameter and L_F is labor employed in the market good sector on the farm. We assume perfect competition in the labor and goods market. Hence labor gets its marginal product and the wage rate on the farm is just $w_F = A_F$ (implicitly we are normalizing the price of the market good to 1). Analogously we have $Y_C = A_C L_C$ and $w_C = A_C$ in the city, where we assume $A_C > A_F$.

Households are differentiated by their endowment of human capital. For simplicity (this assumption can be easily relaxed) we assume that the initial endowment of male human capital is the same as that of female human capital within each household. We denote this endowment as $h^M = h^F = h \in \{h_{\min}, h_{\max}\}$. (Once we introduce investment in human capital into the model, each h will change over time and female and male human capital levels will diverge). The role of human capital is that it determines the effective labor endowment of each household. We can think of each household as being endowed with 1 unit of male and 1 unit of female labor and the h being a quality adjustment. Hence, in essence, household j has $h^M + h^F$ units of effective labor to supply.

Finally we assume that production of the household good does not depend on female human capital. Since utility is linear, either 0 or 1 unit of female labor will be used in household production. We write the utility value of the household good produced with 1 unit of labor as P.

Thus, we can write the one period utility, without the cost of investment, as:

$$m_1(t) = 2A_F + P$$
$$m_2(t) = h^M(t)A_C + P$$
$$m_3(t) = h^M(t)A_C + h^F(t)A_C$$

Hence $m_s(t)$ is the utility in period t the household enjoys at location s, without considering the cost of investment in human capital.

We assume the cost of investment is a quadratic function. The "net" per period utility is then

$$u_s(t) = m_s(t) - \frac{\phi}{2}i^F(t)^2 - \frac{\phi}{2}i^M(t)^2$$

where $i^F(t)$ is investment in female education, $i^M(t)$ investment in male education, ϕ is a parameter affecting the marginal cost of investment and $s \in \{1,2,3\}$. Note again s is a choice variable (hence a function of time), albeit a discrete one.

When a household chooses s we say that it is in "state s" or, in reference to its levels of female and male human capital, in "Area s". Hence if:
s=1 household is on the farm and both spouses work
s=2 household is in the city and only the male works (single worker city household)
s=3 household is in the city and both spouses work (double worker city household)

The costs of investing in human capital are independent of the state the household is in. Therefore the household will choose the s which maximizes its current utility, and choose the i(t)'s to maximize its remaining lifetime utility. This means that the household will choose s according to its position in h^M / h^F space. We can find the loci of points in this space which indicate combinations of male and female human capital at which a household would be indifferent between it's various location/labor supply options:

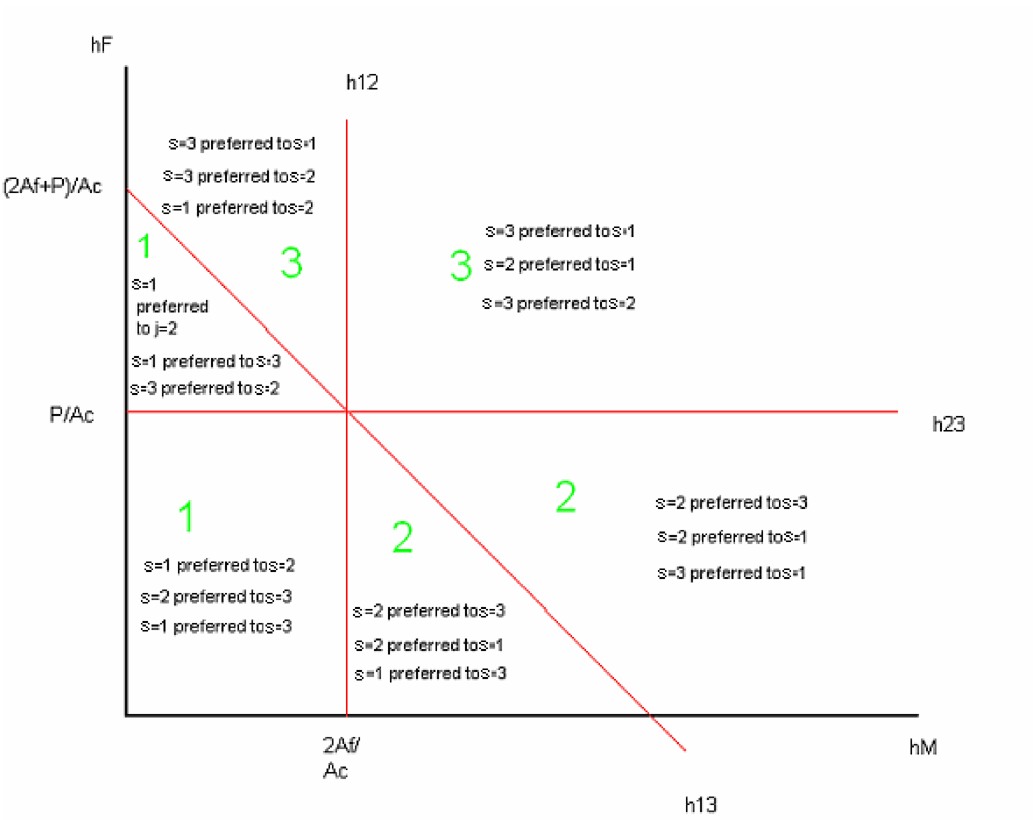
$$h_{12}^{M} = \frac{2A_F}{A_C} \Leftrightarrow m_1 = m_2$$

$$h_{23}^{F} = \frac{P}{A_C} \Leftrightarrow m_2 = m_3$$

$$h_{13}^{F} = \frac{2A_F + P}{A_C} - h_{13}^{M} \Leftrightarrow m_1 = m_3$$

These three boundary lines are illustrated below in Figure 1, where the large numbers indicate the optimal "s" for the relevant region.

Figure 2 – Static location decision



Since we assumed that $h_i^M = h_i^F = h_i \in \{h_{\min}, h_{\max}\}$ we can think of different households as initially lying in the h^M / h^F space along a 45 degree ray from origin.

Hence the set of households, which are in the city with both members working, is:

$$S_{C2} = \left\{ (h_j^M, h_j^F) : h_j^M > \frac{2A_F}{A_C}, h_j^F > \frac{2A_F + P}{A_C} - h_j^M \right\}.$$

Likewise,

$$S_{C1} = \left\{ (h_j^M, h_j^F) : h_j^M > \frac{2A_F}{A_C}, h_j^F < \frac{P}{A_C} \right\}$$

$$S_F = \left\{ (h_j^M, h_j^F) : h_j^M < \frac{2A_F}{A_C}, h_j^F < \frac{2A_F + P}{A_C} \right\}$$

are the sets of households, indexed by j, in the city with only the male working and on the farm with both members working, respectively.

In the following sections we analyze what happens to the households as their position changes through time due to investment in human capital and how the relative size of the above sets changes accordingly.

3.2 Investment decision and dynamics of a single household

We assume that each household starts with initial levels of h^M and h^F low enough such that state 1 is initially optimal. That is $h^M(0) < \frac{2A_F}{A_C}$ and $h^F(0) < \frac{P}{A_C}$. Furthermore we also assume that $h^M(0) = h^F(0)$ (no initial difference in the market ability of the two genders, although in principle we could easily relax this assumption - for example a difference in the initial ability could correspond to differences in primary schooling of men and women). Finally, as drawn above, we assume $2A_F < P$, that is, farm productivity is substantially lower then household productivity, which is plausible for economies at very low level of development (Note that if "the farm" represents the "traditional sector" it's not much of a stretch to assume this holds for all economies, since, almost by definition the "traditional sector" implies very low market productivity). This assumption is also crucial for the model to generate the U-shaped female labor force participation rate found in the data since otherwise at least some households would go straight from Area 1 (farm, both work) to Area 3 (city, both work). It is relevant to note at this point that when constructing labor force participation figures subsistence farmers and other agricultural workers are counted as part of the labor force in the World Bank dataset, although this is not necessarily the case with historical data (see discussion above)

Lifetime utility is linear in period specific utility

$$U = \int_0^\infty u_s(t) e^{-\beta t} dt$$

Laws of motion for human capital are:

$$\frac{\partial h^M(t)}{\partial t} = i^M(t) \quad \text{and} \quad \frac{\partial h^F(t)}{\partial t} = i^F(t)$$ [3]

To solve the dynamic optimization problem,

$$\underset{s,i^F,i^M}{Max}\ U = \int_0^\infty u_s(t)e^{-\beta t}dt \quad \text{subject to} \quad \dot{h}^M(t) = i^M(t) \quad \text{and} \quad \dot{h}^F(t) = i^F(t)$$

we first rely on intuition[4]. There are three possible steady states, in terms of the choice of both i(t)'s and s's and corresponding three possible dynamic paths.

Possible steady state/path 1: $\{s(t) = 1, i^F(t) = 0, i^M(t) = 0 \forall t\}$

- Never invest, stay on farm forever.

Possible steady state/path 2: $Initially \{s(t) = 1, i^F(t) = 0, i^M(t) > 0\}$
$then \{s(t) = 2, i^F(t) = 0, i^M(t) > 0\}$

- Invest only in male education, initially stay on farm and then when h^M is high enough move to the city and stay a one-earner household forever.

Possible steady state/path3: $Initially \{s(t) = 1, i^F(t) > 0, i^M(t) > 0\}$
$then \{s(t) = 2, i^F(t) > 0, i^M(t) > 0\}$
$then \{s(t) = 3, i^F(t) > 0, i^M(t) > 0\}$

- Invest in both male and female education while on the farm, move northwestward until $h^M(t) = h^M_{12}$ then switch to single earner city household state, keep moving northwest and switch to two-earner city household when $h^F(t) = h^F_{23}$, keep investing in both.

Note that the path which involves investing only in female education, moving straight

[3] Note that including depreciation does not alter the results of the model.
[4] Note also that for we are implicitly allowing the per period utility, which is linear in consumption and investment, to be negative. Also, this dynamic optimization problem can be solved in the usual manner with Hamiltonians.

north and switching to state 3 after crossing the line $h_{13}^{F}(t) = \frac{2A_F}{A_C} - h_{13}^{M}(t)$ and then investing in both h's is ruled out by the fact that $2A_F < P$.

As we will see, for plausible parameter values possibilities 1 and 2 are dominated by possibility 3. Hence, for now we concentrate on solving the maximization problem involving this particular steady state. In this case we can write the lifetime utility as a sum of three parts – utility while in state 1, utility in state 2, utility in state 3, and subtract the present value cost of investing in both types of human capital.

$$U = \int_0^{t_1} (2A_F + P)e^{-\beta t}dt + \int_{t_1}^{t_2} (h^M(t)A_C + P)e^{-\beta t} + \int_{t_2}^{\infty} (h^M(t)A_C + h^F(t)A_C)e^{-\beta t}$$
$$-\frac{\phi}{2}\int_0^{\infty} i^F(t)^2 e^{-\beta t} - \frac{\phi}{2}\int_0^{\infty} i^M(t)^2 e^{-\beta t}$$

First note that t1 and t2, which denote the times at which the household switches from state 1 to state 2 and from state 2 to state 3 respectively, are determined by the values of h^M and h^F. Hence the optimal "switching time" is implied by particular paths of $i^M(t)$ and $i^F(t)$. This means we can consider these as given when maximizing with respect to $i^M(t)$ and $i^F(t)$, find the optimal investment values, plug them into the equation above and then optimize U with respect to t1 and t2. (Alternatively we can calculate how long, as a function of investment, it will take the household to move from its initial position to the boundaries h_{12} and h_{23} – this will pin down the values for t1 and t2. Of course, either way we should get the same answer.)

Consider a household which has invested enough in h^M and h^F in the past so that it is about to enter area 3 – its optimal choice of state is s=3. Since this is a steady state it plans on staying in this state forever more. In this case the marginal benefit (in present value terms) of a unit of investment in either type of human capital is $\frac{A_C}{\beta}$, while the marginal cost, with the assumed quadratic form, is just the amount of investment times phi. Hence if the household is optimizing at this stage, it means that investment is $\frac{A_C}{\phi\beta}$ for both female and male education. That is

$$i^M(t) = i^F(t) = \frac{A_C}{\phi\beta} \Leftrightarrow t > t_2 .$$

Now consider the choice of optimal investment before the household enters Area 3 (i.e. before time t2). Here we have to consider investment in male and female education separately. Specifically, before entering Area 3, the household will be in Area 2 where s=2 is optimal. At this point it will receive the benefit from having the male work in the market – the benefit from ability and any investment in male education occurs concurrently. Hence its marginal benefit and cost from investment in male education is same as in Area 3. Therefore the optimal value of $i^M(t)$ in this portion of the lifetime is still $\frac{A_C}{\phi\beta}$. However, things are different for investment in female education. While in state 2, the household receives no utility from female human capital. However it anticipates that in the future, once h^F is high enough, the female will also work, hence it pays to invest in female education right now. This benefit however has to be discounted by the time it will take the household to move from its current position inside area 2 to the boundary between area 2 and area 3. In other words the marginal benefit of investment in female education at this stage is given by $\frac{A_C}{\phi\beta}e^{-\beta(t_2-t)}$ which will be the level of investment in female education for t<t2. Note that this reasoning applies to investment in female education while in Area 1 as well. Similarly, while in area 1 investment in men's education has to be discounted by $e^{-\beta(t_1-t)}$. This implies the following paths for investment:

$$i^M(t) = \begin{Bmatrix} \frac{A_C}{\phi\beta} \Leftrightarrow t \geq t_1 \\ \frac{A_C}{\phi\beta}e^{-\beta(t_1-t)} \Leftrightarrow t \leq t_1 \end{Bmatrix}$$

and

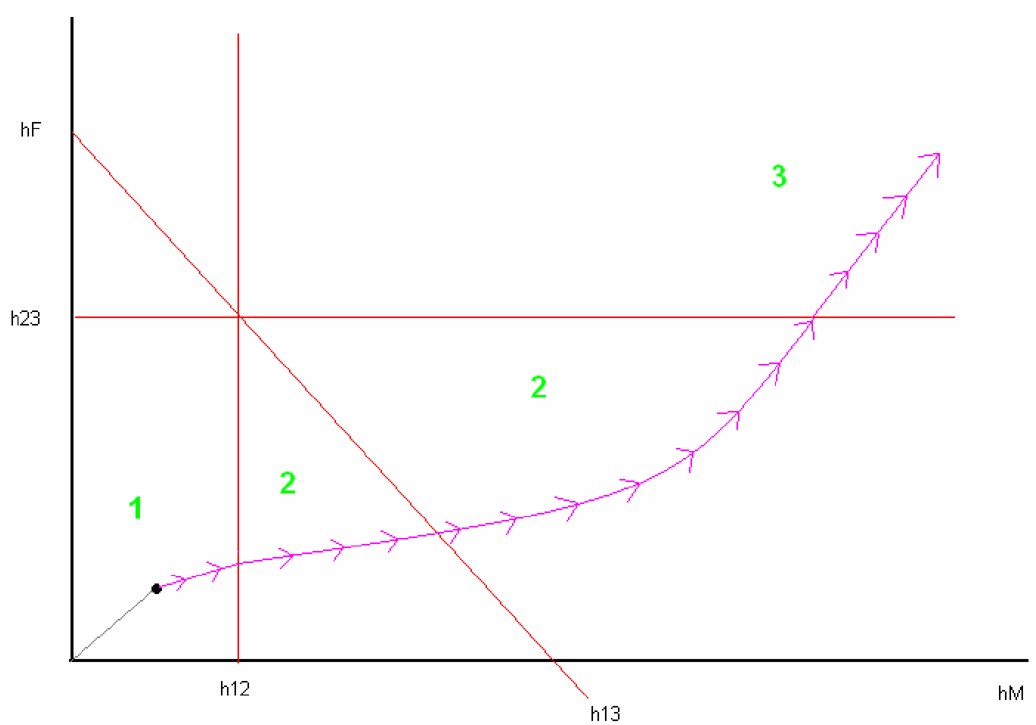
$$i^F(t) = \begin{Bmatrix} \frac{A_C}{\phi\beta} \Leftrightarrow t \geq t_2 \\ \frac{A_C}{\phi\beta}e^{-\beta(t_2-t)} \Leftrightarrow t \leq t_2 \end{Bmatrix}$$

This means the path of the ratio of male investment in education to female investment in education is given by

$$\frac{i^M(t)}{i^F(t)} = \begin{Bmatrix} e^{-\beta(t_1-t_2)} \Leftrightarrow 0 \le t \le t_1 \\ e^{-\beta(t-t_2)} \Leftrightarrow t_1 \le t \le t_2 \\ 1 \Leftrightarrow t \ge t_2 \end{Bmatrix}$$

which is illustrated below.

Figure 3 – Dynamic path in hF/hM space



This is also the slope of the dynamic path in h^M / h^F space. Obviously the slope of this path is positive (the household moves northwest), but it can also be easily verified that it is concave (investment in male education exceeds investment in female education, hence h^M grows faster than h^F) with the slope that approaches and becomes 1 as the household enters Area 3.

This aspect of the model is just Stylized Fact 3. Over the course of development male education begins to rise first and only later does female education begin catching up with it. The underlying assumption which generates this result, like almost all implications of this model, is the existence of economies of scope while on the farm – the fact that female labor can be simultaneously used in household production and "market" (agricultural) work. In fact, it is quite surprising how many of the phenomenon in the real world relating to development, female education, and female labor force participation can be explained by this simple assumption alone.

Knowing the investment at each t allows us to rewrite utility in terms of parameters and t1 and t2 alone and maximize it with respect to t1 and t2. Alternatively, we know that t1 will occur when the household moves from its initial position (here $h^M(0)$) to the point at which it wants to switch to state 2 (here h_{23}^M). Mathematically:

$$t_1 = \frac{DISTANCE \quad TO \quad BOUNDARY}{AVERAGE \quad SPEED} = \left\{\frac{2A_F}{A_C} - h^M(0)\right\}\frac{1}{Avg.i^M{}_{untilt1}} = \left\{\frac{2A_F}{A_C} - h^M(0)\right\}\frac{1}{(\int_0^{t_1} i^M(t))/t_1}$$

$$\Rightarrow \int_0^{t_1} i^M(t) = \left\{\frac{2A_F}{A_C} - h^M(0)\right\}$$

$$\Rightarrow \int_0^{t_1} \frac{A_C}{\phi\beta} e^{-\beta(t_1 - t)} = \frac{A_C}{\phi\beta^2}(1 - e^{-\beta t_1}) = \left\{\frac{2A_F}{A_C} - h^M(0)\right\}$$

Solving the above for optimal time to switch from state 1 to state 2 we have

$$t_1 = -\frac{1}{\beta}\ln\left\{1 - \frac{\phi\beta^2}{A_C}\left(\frac{2A_F}{A_C} - h^M(0)\right)\right\}$$

which can also be obtained from maximizing U with respect to t1. Here also we can see the necessary condition for path 3 to dominate paths 1 and 2; $t_1 > 0$.
We can use similar reasoning to derive t2:

$$t_2 = -\frac{1}{\beta}\ln\left\{1 - \frac{\phi\beta^2}{A_C}\left(\frac{P}{A_C} - h^F(0)\right)\right\}$$

In both cases we need h(0) to be less then the indifference value but not so small (alternatively A_F and P big enough) that the above expressions become undefined. Formally we need that

$$\frac{2A_F}{A_C} > h^M(0) > \frac{2A_F}{A_C} - \frac{A_C}{\phi\beta^2}$$

and

$$\frac{P}{A_C} > h^F(0) > \frac{P}{A_C} - \frac{A_C}{\phi\beta^2}$$

These are also the conditions needed to ensure that Possible Path/Steady Steady 3 dominates the other 2.

The above investment levels imply

$$h^M(t) = \begin{cases} h^M(0) + \dfrac{A_C}{\phi\beta^2} e^{-\beta t_1}(e^{\beta t} - 1) \Leftrightarrow t \le t_1 \\ \dfrac{2A_F}{A_C} + \dfrac{A_C}{\phi\beta}(t - t_1) \Leftrightarrow t \ge t_1 \end{cases}$$

and

$$h^F(t) = \begin{cases} h^F(0) + \dfrac{A_C}{\phi\beta^2} e^{-\beta t_2}(e^{\beta t} - 1) \Leftrightarrow t \le t_2 \\ \dfrac{P}{A_C} + \dfrac{A_C}{\phi\beta}(t - t_2) \Leftrightarrow t \ge t_2 \end{cases}$$

The lifetime utility as a function of t1 and t2 is:

$$U = \frac{1}{\beta}\left\{ 2A_F(1 - e^{-\beta t_1}) + P(1 - e^{-\beta t_2}) + A_C h^M(0) e^{-\beta t_1} + A_C h^F(0) e^{-\beta t_2} + \frac{1}{2}\frac{A_C^2}{\phi\beta^2}\Big((2 - e^{-\beta t_1})e^{-\beta t_1} + (2 - e^{-\beta t_2})e^{-\beta t_2}\Big) \right\}$$

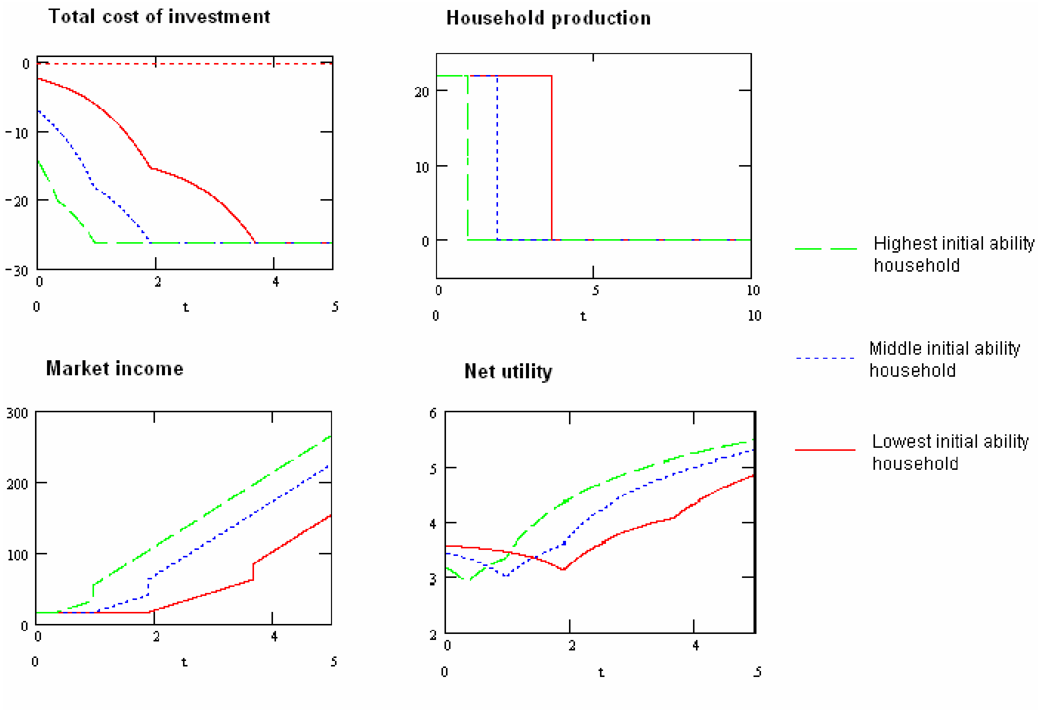
The first term represents the present value of market wages from agriculture for both members of the household up till t1. The second term is the present value of household production until time t2, when the female ceases to work at home, the third and fourth terms are the values of initial human capital for male and female appropriately discounted from the time when they first become utilitized and the last term is the present value of investment for both members of the household.

Maximizing the above U function with respect to t1 and t2, as can be easily verified, yields the values for t1 and t2 given above.

Below we show the paths of total cost of investment, household production, market income and net utility for the parameters:

$\{\beta = .5, A_F = 8, A_C = 12, P = 22, \phi = 22\}$

Figure 5 – Evolution of key variables over time



3.3 Distribution of households and female labor force participation over time

Next we want to consider how the distribution of households changes over time. We let j index both households and their initial position, so that $h^F(0) = h^M(0) = j$ and with only slight loss of generality (involving some additional but innocuous parameter restrictions to ensure that everyone starts in Area 1) we assume $j \in \{0,1\}$, hence automatically normalizing the measure of households to 1. Then, for example, $j = .9$ is the "90th percentile" household. We denote by $j1(t)$ the household, if any, which at time t finds itself on the boundary $h_{12}^M = \frac{2A_F}{A_C}$. Similarly we make $j2(t)$ be the household, if any, which at time t finds itself on the boundary $h_{23}^F = \frac{P}{A_C}$. As households begin their journey across the h^M / h^F space there are two possibilities for how they become allocated over time, depending on parameters (specifically the relative values of 1 (measure of households), P, and productivities).

First we have the simple case that at no time there are households in all three areas. Initially we'll have all households in Area 1, then some will begin crossing into Area 2. This assumption simply states that the lowest household (j=0) to enter Area 2, does so before the highest household (j=1) crosses into Area 3.

Alternatively it could be the case that the highest ability households start entering Area 3 while there are still some households in Area 1.

We analyze the simpler case first and normalize time so that at t=0 the highest ability household, j=0 is located right on the boundary $h_{12}^{M} = \frac{2A_F}{A_C}$. In the next instant of time additional households will begin exiting Area 1 and entering Area 2. The proportion of households in Area 1 will then be $j1(t)$ and those in Area 2 (1- $j1(t)$), up until time such that $j1(t)$ =0 and there are no households left in Area 1.
Given a time t>0 we want to find the household on the boundary, if any. We know that

$$h_{j1}^{M}(t) = \frac{2A_F}{A_C} = j1 + \frac{A_C}{\phi\beta^2} e^{-\beta t_1^{j1*}} (e^{\beta t} - 1)$$
$$= j1 + \frac{A_F}{\phi\beta^2}(e^{\beta t} - 1) - (\frac{2A_F}{A_C} - j1)(e^{\beta t} - 1)$$

Solving this for $i1(t)$ we have

$$j1(t) = \begin{Bmatrix} \frac{2A_F}{A_C} - \frac{A_C}{\phi\beta^2}\left(1 - e^{-\beta t}\right) \Leftrightarrow t \le t_1^0 \wedge t \ge t_1^1 \\ 0 \Leftrightarrow otherwise \end{Bmatrix}$$

where t_1^0 and t_1^1 are the times when the lowest and highest initial ability household each cross the first boundary $h_{12}^{M} = \frac{2A_F}{A_C}$, respectively. Hence we have that

$$\frac{\partial j1}{\partial t} = -\frac{A_C}{\phi e^{\beta t}} < 0$$
$$\frac{\partial^2 j1}{\partial t^2} = \frac{\beta A_C}{\phi e^{\beta t}} > 0$$

within the relevant range.

Hence the number of households on the farm is falling at a decreasing rate, while that of those in the city with only male working in the market is increasing at a decreasing rate.

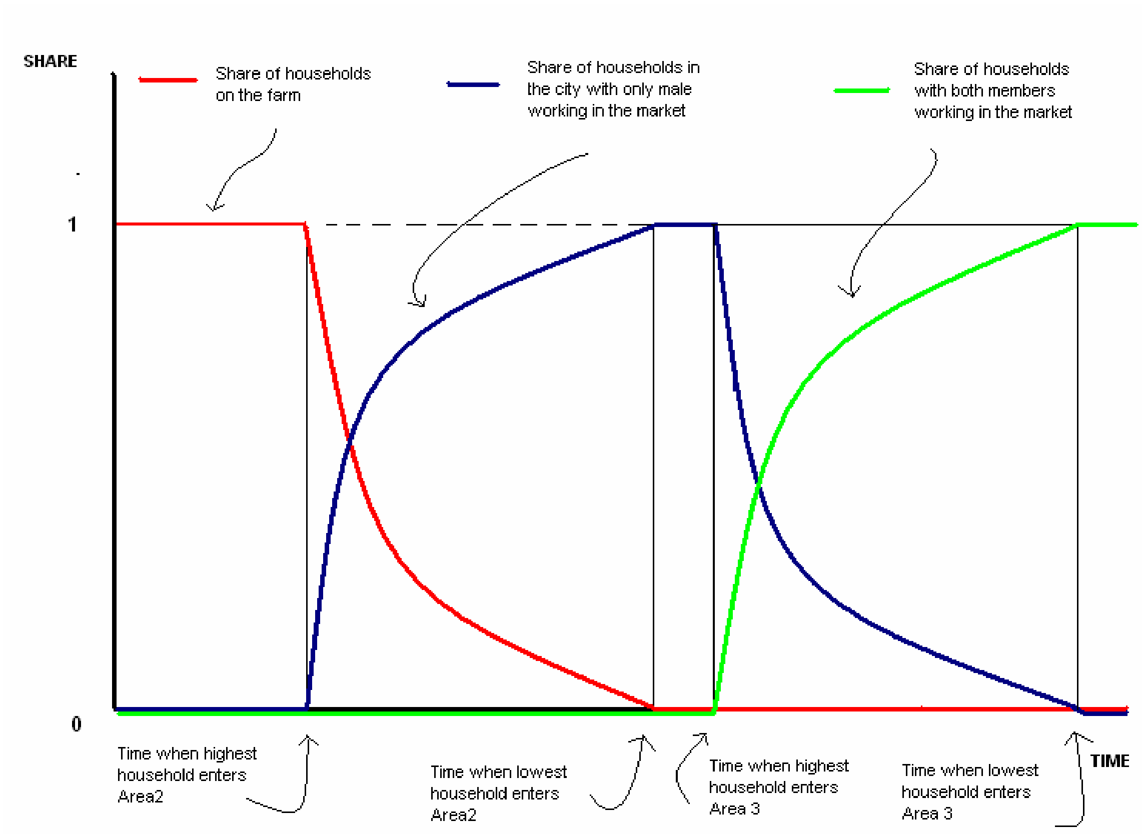
Eventually all households will pass into Area 2 and at some future time will begin entering Area 3. At that point the proportion of households in Area 2 will be given by $j2(t)$, and that of those in Area 3 by (1- $j2(t)$). Similarly to before we'll have:

$$j1(t)=\begin{Bmatrix} \frac{P}{A_C}-\frac{A_C}{\phi\beta^2}\left(1-e^{-\beta t}\right)\Leftrightarrow t\le t_2^0 \wedge t\ge t_2^1 \\ 0\Leftrightarrow otherwise \end{Bmatrix}$$

where t_2^0 and t_2^1 are defined analogously with respect to the boundary $h_{23}^F=\frac{P}{A_C}$.

Putting all this together we get the following paths for the share of the households in each area:

Figure 5 – Proportion of households in each state – the U-shape



The female labor force participation rate then is just the inverse of the share of households in Area 2. Note that it is negatively correlated with "urbanization" rate – the proportion of household in the city.

The analysis for the case where there is an overlap in time between households which are entering Area 2/leaving Area 1 and those entering Area 3/leaving Area 2 is similar, except that the female labor force participation rate never reaches zero. We present it in the section below.

As a final note of this section we wish to acknowledge that, obviously, in actuality female labor force participation never reaches 100%. Neither does the male participation rate for that matter. Of course there are many aspects outside the scope of this model, which exert influence on this variable. The implicit assumption is that these forces act in an analogous way on both male and female participation rates. The purpose of the model instead is to provide an explanation for the general nature of the relationship between female labor force participation and the level of development.

4. Simulation of the model and Stylized Facts

In this section we present several Propositions, which illustrate how the model replicates the Stylized Facts found in the introduction. Since in several cases the algebra can become intractable and cumbersome we relegate most of the proofs to the Appendix[5] and instead rely here on numerical simulations of the model. Note that the choice of parameters is largely dictated by the desire to present illustrative cases and the need to satisfy the necessary restrictions, rather than based on specific empirical estimates[6]

At this point it should also be noted what the proper interpretation of model time is. The usual justification for the infinitely lived household assumption is that of successive generations of households, each linked with future households via intergenerational altruism. This means two things. First the correct interpretation of a measure of time is that of a generation (so for example in the first case below, the time it takes for the economy to return to a “full” female labor force participation rate after first experiencing a decline – the width of the u-shape - is roughly three generations). Second the motivation for investment in human capital while on the farm (and in case of female education, while in the city with a single worker) is the anticipation that someday one’s children or grandchildren will leave the farm and work in the city. This assumes a production function for children where human capital is passed down from parent to child according to sex (female children inherit human capital from mothers, while male children, from their fathers). While this is unrealistic if taken literally, it can be a reasonable approximation if male and female human capital are not perfect substitutes[7].

First we present the unfolding of human capital as a function of time and initial position in the model. The graph below refers to male the path human capital. Female human capital looks the same, except that it lies below that of male human capital except at the initial position, by assumption.

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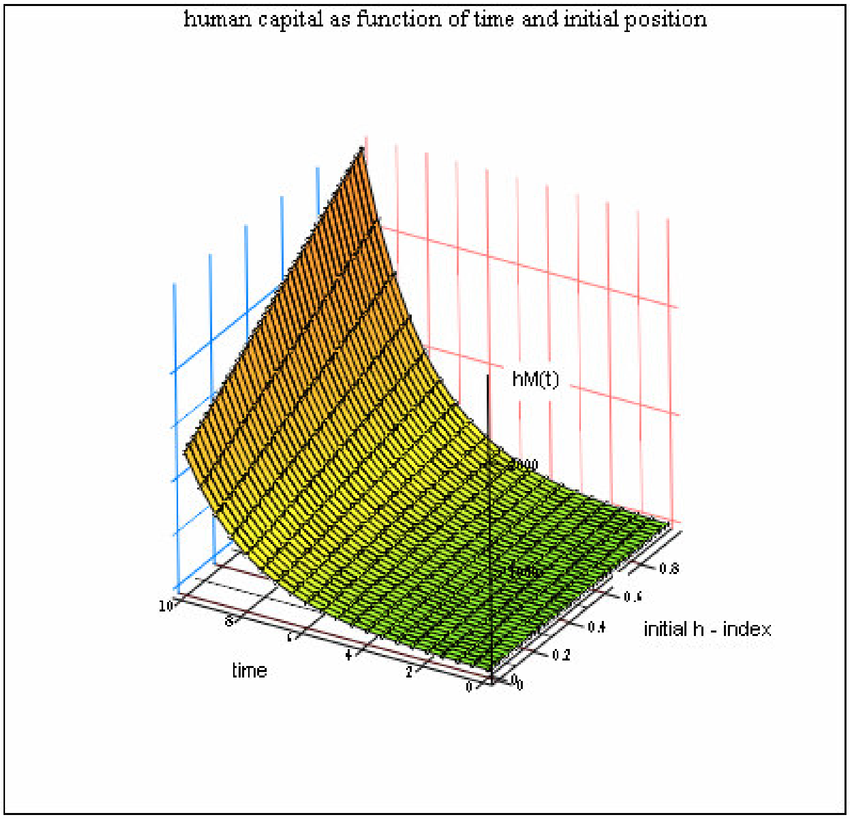
[5] At the moment available upon request.

[6] Of course in practice it is quite a challenge to obtain a satisfactory estimate of P and even farm productivity.

[7] Re-specifying the model so that the children’s endowment of human capital prior to investment is the average of the parents involves a considerable analytical complication and is better handled in discrete time. We have considered such a model and intuitively the main implication of such a change appears to be to speed up the dynamics of the model for the females and slow them down for the males. Essentially, the time path of an individual household would be steeper in hf/hm. The restriction $P > 2A_C$ would still ensure that this path never crosses the 45 degree line however. The end result appears to be – although it is difficult to solve for closed form solutions – that the u-shape would be “narrower”)

(The parameters assumed below are $\{\beta = .5, A_F = 8, A_C = 12, P = 22, \phi = 22\}$)

Figure 6



4.1 Stylized Fact 1

The u-shaped female labor force participation rate is presented in Goldin (1996). As she notes, the u-shape is driven by variation in the cross section rather than across time. Goldin further points out in "Understanding the Gender Gap" (pg. 57) that the process can span as long as two centuries (depending on rate of income growth and possibly other factors). Consequently the only country, which exhibits a u-shape in its female labor force participation rate during the post war period, is Japan.

Proposition 1. The female labor force participation rate, broadly defined, is u-shaped.

This immediately follows from the signs of the first and second derivatives:

$$\frac{\partial j1(t)}{\partial t} = \frac{\partial j2(t)}{\partial t} = -\frac{A_C}{\phi e^{\beta t}} < 0$$

$$\frac{\partial^2 j1}{\partial t^2} = \frac{\partial^2 j2}{\partial t^2} = \frac{\beta A_C}{\phi e^{\beta t}} > 0$$

and the fact that female labor force participation is given by:

$$FLFPR(t) = \left\{ \begin{array}{c} \left\{ \begin{array}{c} 1 \Leftrightarrow t \leq t1(1) \\ jt1(t) \Leftrightarrow t \in [t1(1), t2(1)] \\ jt1(t) + 1 - jt2(t) \Leftrightarrow t \in [t2(1), t1(0)] \\ 1 - jt2(t) \Leftrightarrow t \in [t1(0), t2(0)] \\ 1 \Leftrightarrow t \geq t2(0) \end{array} \right\} \Leftrightarrow t2(1) < t1(0) \\ \\ \\ \left\{ \begin{array}{c} 1 \Leftrightarrow t \leq t1(1) \\ jt1(t) \Leftrightarrow t \in [t1(1), t1(0)] \\ 0 \Leftrightarrow t \in [t1(0), t2(1)] \\ 1 - jt2(t) \Leftrightarrow t \in [t2(1), t2(0)] \\ 1 \Leftrightarrow t \geq t2(0) \end{array} \right\} \Leftrightarrow t1(0) < t2(1) \end{array} \right\}$$

In the first case, the highest household reaches the boundary $\frac{P}{A_C}$ before the lowest household reaches the boundary $\frac{2A_F}{A_C}$, hence t2(1)<t1(0). In the lower case the reverse is true.

Below we present the two possible paths for female labor force participation rate.

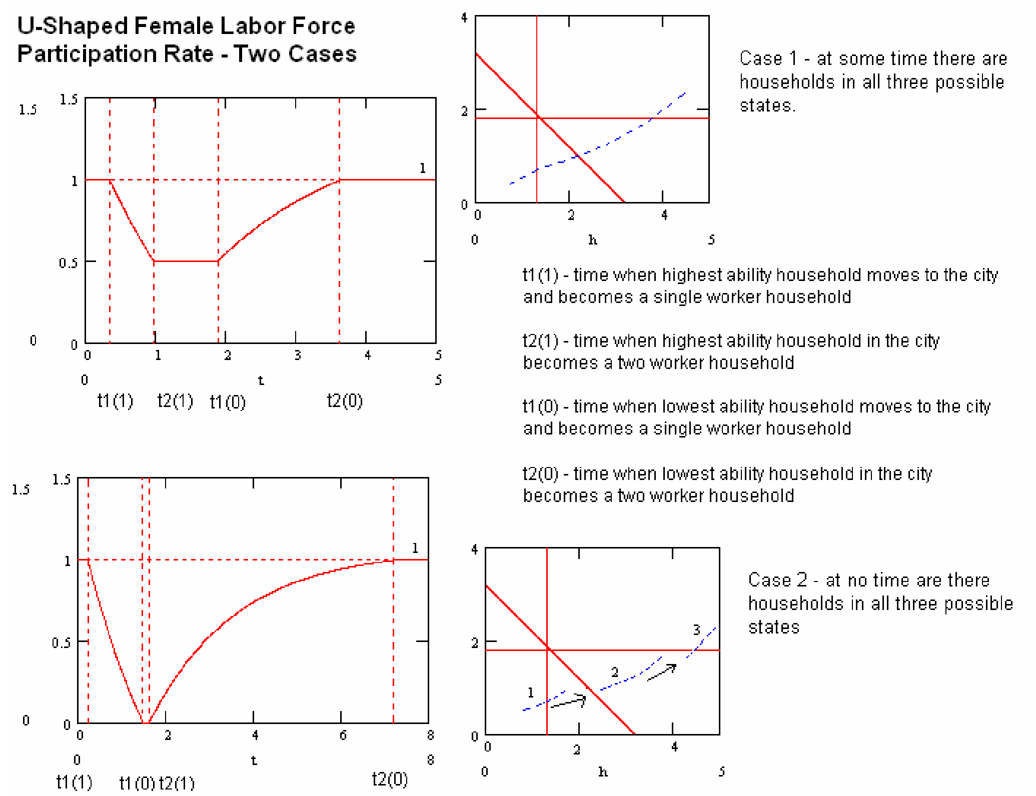
First, we have t2(1)<t1(0) and the parameters are:

$$\{\beta = .5, A_F = 8, A_C = 13, P = 30, \phi = 22\}$$

and second:

$$\{\beta = .5, A_F = 8, A_C = 12, P = 22, \phi = 22\}$$

Figure 7



In both cases the path of female labor force participation is "u-shaped" (v-shaped?)

In the remainder of this section we focus on the first case, which is of no general consequence for the propositions that follow.

4.2 Stylized Fact 2

Goldin (pg. 135) points out that although today women in the labor force have higher levels of education than women on average, this has not been true historically. In the first decade of the twentieth century the average years of schooling of a woman in the labor force was less then seven years, while that of an average women was eight years. By the time of World War II the average workingwoman had an extra year over the average woman. Hence there was a significant change in the educational composition of the female labor force vis a vis the female population in general. The model replicates this fact. If we believe that the trough of the u-shape occurred sometime around the turn of twentieth century (t between 1 and 2 in the model) then it is precisely at this time that the education of labor force participants would begin to catch up with that of women in general, as high ability households began turning into two worker families. It would be interesting to empirically examine the dynamics of the ratio of average education of the two groups in earlier years, however data limitations pose a serious challange in this respect[8].

Proposition 2 – At the beginning of the development process the average education of women in labor force is ***lower*** *than that of women in general, but this reverses itself over time.*

The two figures below illustrate this particular fact. While a precise proof, again relying on the signs of first and second derivatives, is messy, the intuition is straightforward. First, all women are on the farm, hence in the labor force. As human capital accumulation ensues, the highest ability households leave the farm and move to the city. At this point however the level of female education of women in the city is not high enough to justify market work. Hence the highest ability women exit the labor force and specialize in household work (because their husbands are relatively high skilled), while lower ability women remain on the farm. And they are within our broad definition of labor force. Eventually the human capital of the highest households reaches a level where it pays for the city women to enter the labor market and work for a wage, while at the same time, the low ability women leave the farm (and the labor force) and move to the city where they become "housewives".

[8] Goldin also stresses the extent to which changes in married women's educational attainment undermined the existence of the social stigma associated with women's work.

Figure 8

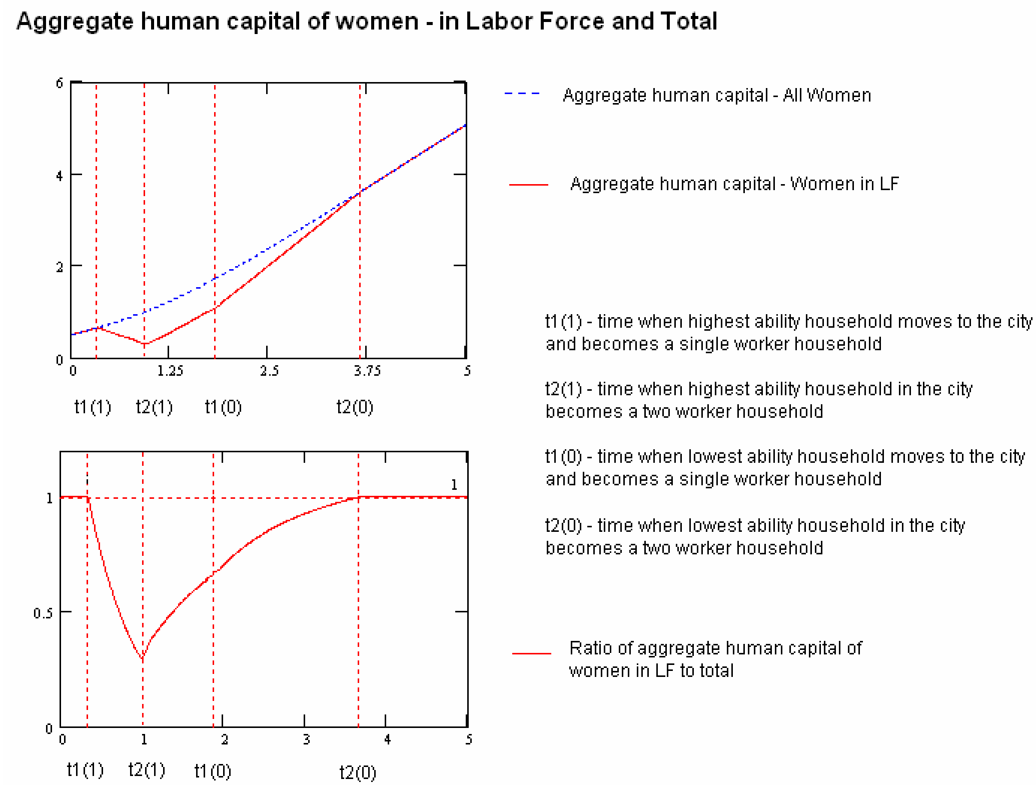
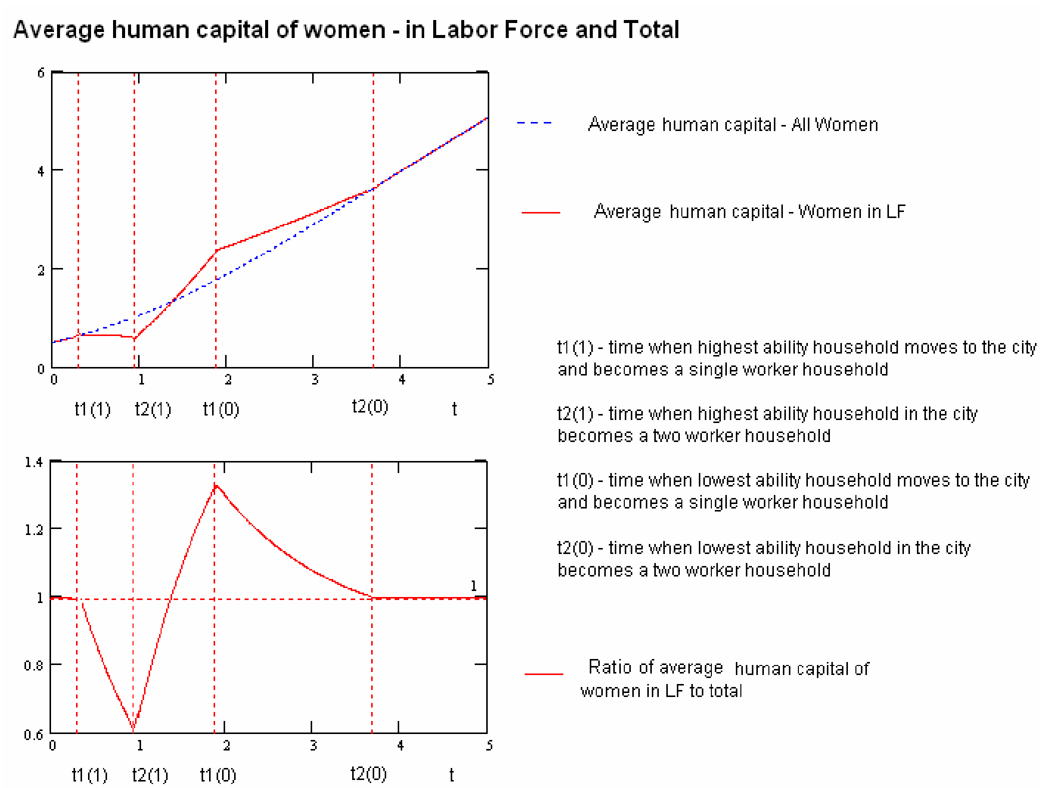


Figure 9



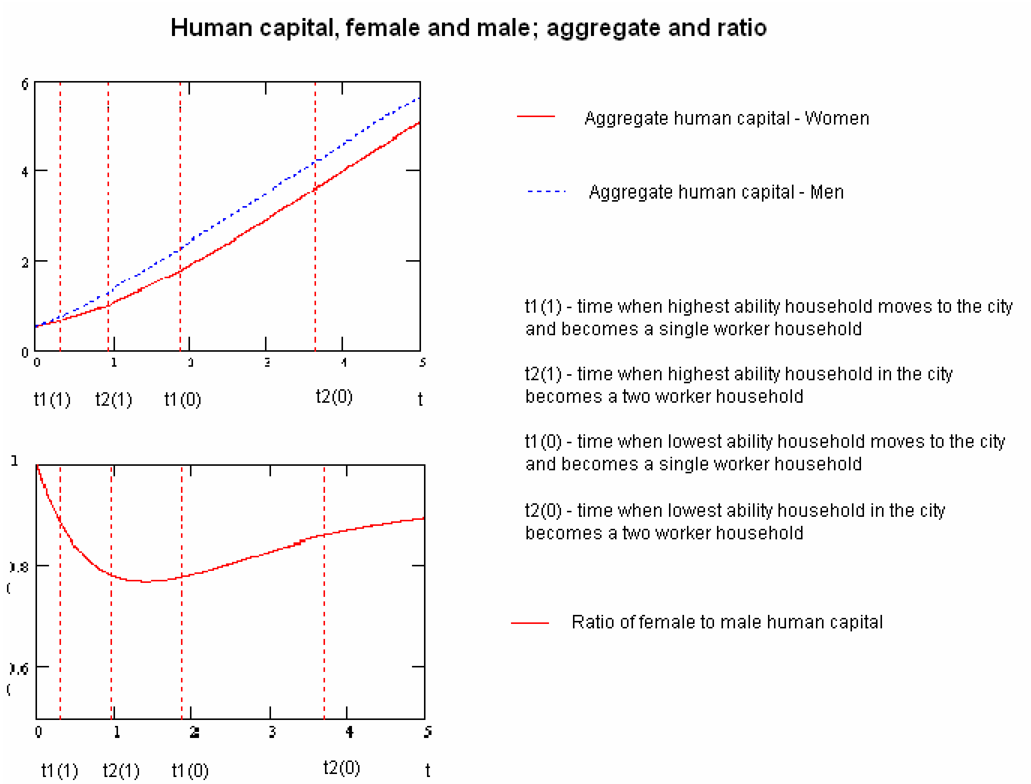
4.3 Stylized Fact 3

Outside of the developed countries the educational attainment of men exceeds that of women. Even in the upper income countries the essential equality in years of schooling of the two genders is a fairly recent phenomenon. In general as a country begins to industrialize male education increases first, and only later does female education begins to rise as well (Goldin paper pg 15.).

In fact the present model generates a u-shaped path for the female to male education ratio. Initially male investment in education is greater, and increases faster, then investment in female education. This causes the ratio, which was initially one, to sharply fall. Eventually male investment levels off at $\frac{A_C}{\beta}$ while female investment continues to increase and the ratio begins to increase. The u-shape however is essentially an artifact of the assumption that males and females start out with the same initial level of human capital. If we assumed an initial inequality, the trough would occur very quickly and would result in a mostly monotonic path over time.

Proposition 3 – Starting from a position of equal human capital endowments, men's education rises faster than women's. Hence initially the ratio of male to female education falls. Eventually this reverses itself and the ratio approaches one as time goes to infinity.

Figure 10



For the parameters used the ratio reaches a minimum of around .75. The value of the minimum depends negatively on P and positively on both city and farm wages. The first and third effects are intuitive:

A higher productivity in the household means that a higher level of female education is needed to justify market work for women. Hence households will spend a longer time on the farm and in the city as single worker families, where female human capital does not matter for current consumption. This in turn will slow down investment in female education without affecting investment in male education, hence resulting in, at the extreme, a larger gap between the educations of the two groups.

A higher market wage in the city works similarly except that it speeds up the investment of both men and women. However, since the boundary $\frac{P}{A_C}$ shifts more than the boundary $\frac{2A_F}{A_C}$, less time will be spent by each household in Area 2 (in other words, (t2-t1) gets smaller for all j). This implies that the increase in female education investment due to a higher market wage will be greater than the increase in male education investment, resulting in a smaller gap.

An increase in the farm wage has the effect of delaying the shift from farm to the city. As long as the increase in A_F is not too big (so that $P > 2A_F$ continues to hold) this will lower investment in male education for farm households, while leaving female education investment unchanged (since this depends only on A_C and P). Therefore, the gap between the two members of the households will be less pronounced.

4.4 Stylized Facts 4 and 5: Income and wage effects

Historically, for the United States, the effect of changes in income (basically understood as a change in the husband's income or wages for married women) and wages on female labor force participation rate has not remained constant (pg.133). Specifically, at the turn of the century the income effect was negative and substantial (the increase in the average man's income tended to push women out of the labor force), while the wage effect was positive albeit small. As Goldin puts it "A married women was not easily enticed into the labor force by higher wages, but she was, at the same time, encouraged to leave by higher earnings of her husband and other family members." By the time of WW2 the income effect has declined in size while the wage effect increased. Finally, presently both wage and income effects are small in magnitude. Once again, we illustrate below that the model replicates these facts.

To analyze the effects of income and wage increases on female labor force participation rate in the model we will distinguish here between male and female wages in the city although so far we have assumed that these are equal (we retain the assumption of equal wages on the farm). Hence the relevant boundaries become $\frac{2A_F}{w_C^M}$ and $\frac{P}{w_C^F}$, where w_C^M is the male wage in the city and w_C^F is the analogous female wage. We then identify income effects with the change in female labor force participation due to a change in w_C^M, $\frac{\Delta FLFPR}{\Delta w_C^M}$, and wage effects with the change in female labor force participation due to a change in w_C^F, $\frac{\Delta FLFPR}{\Delta w_C^F}$. This is consistent with standard empirical and theoretical work in labor economics which identifies income effects on women's labor supply decision with the changes in the husband's salary and wage effects with the change in woman's own income. We consider discrete changes in wages because taking a partial derivative, while accurate locally, will run into problems near the "kinks" – households who are close to switching between the Areas.

When considering shocks to wages we, as always, need to distinguish between anticipated and unanticipated shocks. Unanticipated shocks will have the effect of only changing a given household's optimal location decision, whereas anticipated ones will also affect the entire path of investment and human capital. For simplicity, below we focus on unanticipated shocks.

4.4.1 Income effect

The magnitude of the income effect depends on the distribution of household at the time it occurs. An increase in the men's city wage has the effect of shifting the boundary, $\frac{2A_F}{w_C^M}$, leftward. This may cause some households on the farm to move to the city and become single worker families, or, if human capital is high enough, it may have no effect at all. Similarly when households have very little human capital and the change in city wages for men is small there will be no discernible effect on female labor force participation rate (the shock is not big enough to induce families to move). This means that in the early part of the development process, provided there are some households near the boundary $\frac{2A_F}{w_C^M}$, the income effect will be negative. However if we look at the same shock once some human capital has been accumulated the negative effect of the same shock will be smaller. Finally, once human capital is high enough, an increase in men's wages will have no effect on female labor force participation. A fall in men's wages will work in the opposite direction.

Let city wages for men increase by δ %. The change in the female labor force is then given by:

$$\frac{\Delta FLFPR}{\Delta w_C^M} = \left\{ \begin{array}{c} 0 \Leftrightarrow t \le t_1^1 - dt^M \\ j1(t)^* - 1 \Leftrightarrow t \in [t_1^1 - dt^M, t_1^1] \\ j1(t)^* - j1(t) \Leftrightarrow t \in [t_1^1, t_1^0 - dt^M] \\ -j1(t) \Leftrightarrow t \in [t_1^0 - dt^M, t_1^0] \\ 0 \Leftrightarrow t \ge t_1^0 \end{array} \right\}$$

where

$$dt^M = -\frac{1}{\beta}\ln\left(\frac{1+\delta}{1+\delta c^M}\right) \text{ and } c^M = 1 - \frac{\beta^2}{w_C^M}\frac{2A_F}{w_C^M}.$$

Note however that now t1(1) becomes the time that the highest ability household *expected* to enter Area 2, since we are considering an unanticipated shock. The same is true for t1(0). Hence $j1(t)^*$ is the household, if any, which finds itself on the boundary between farm and city-single-worker, after the shock has occurred. Calculating it in the manner analogous to above we have:

$$\frac{2A_F}{w_C(1+\delta)} = h^M(j) = j + \frac{w_C}{\beta^2}(e^{\beta t}-1)e^{-\beta t_1^j}, \text{ which implies}$$

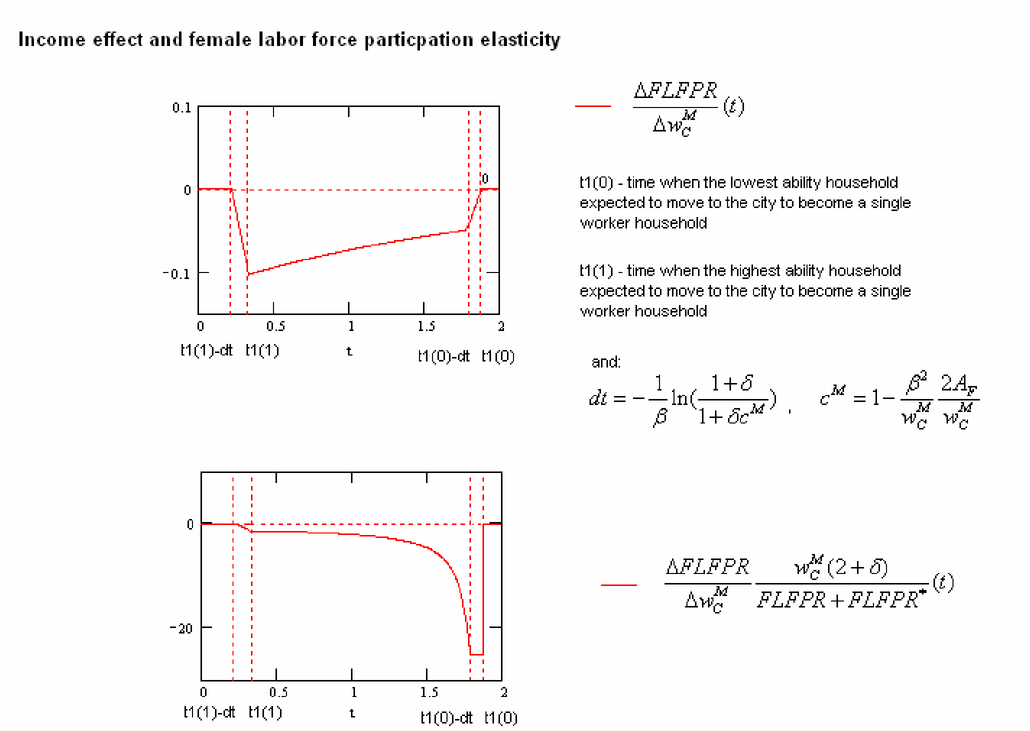
$$j1(t)^* = \frac{2A_F}{(1+\delta)w_C^M}e^{-\beta t} + \frac{w_C^M}{\beta^2}c^M(1-e^{-\beta t})$$

Proposition 4 – The elasticity of female labor force participation with respect to men's city wages is negative and initially small in magnitude. It then increases in magnitude, reaches a minimum and begins to decrease in magnitude until it reaches zero.

Hence the income effect is small at the beginning and end of the "development process" and large in the middle. Goldin finds that at the beginning and in the first two decades of the twentieth century, the income effect appeared to be large in magnitude but has since moved closer to zero, especially in the post WWII era. At the same time the bottom of the U-shape for the female labor force participation also very likely occurred during the same time period. Roughly speaking then, the turn of the century represents the middle of the "development process". Hence the model is consistent with the empirical finding of a large negative income effect at the turn of the century, which subsequently diminishes. Furthermore it provides an explanation for this phenomenon: a change in male wages in the industrial sector is likely to have large effects if there is a large number of households who are close to being indifferent between remaining in the traditional sector and moving to the city. The resulting rural-urban migration induces households to become single worker families in the city as married women specialize in household work and child rearing. On the other hand, if most households have already moved to the cities then an increase in wages will not have a large effect. Similarly, if the skills of the workers on the farm are incompatible with those demanded by firms in the city (i.e. the workers have low levels of human capital appropriate for work in the industrial sector) then the marginal benefit of higher city work will be outweighed by the marginal cost due to loss off farm wages and economies of scope in family production.

The figures below illustrate the level change and the elasticity of female labor force participation with respect to male city wages. The parameters are the same as used above and the shock is an unanticipated increase of 10% in city wages (note that because we are considering discrete changes we compute elasticities via the "midpoint method").

Figure 11



As can be seen from the second panel, for the parameters chosen, at its most extreme, a 10% change in men's city wages will decrease female labor force participation by roughly 25%. This is the case where the change in wages basically sweeps almost all households remaining on the farm into the cities, pushing female market attachment down to zero.

4.4.2 Wage Effect.

The wage effect works in a way opposite, but similar, to the income effect. When households have very low human capital a discrete change in female wages will have no effect on the location decision of households and hence no effect on female labor force participation rate. As households move northwest in the h^F / h^M space a female city wage shock however, which shifts the $\frac{P}{w_C^F}$ boundary downward, will cause some single worker households in the city to begin supplying female labor to the market. This effect initially increases steeply as more households approach the boundary, and then begins to decreases. In percentage terms, the own-wage elasticity of female labor force participation is initially small and increases over the course of development. At its peak, which roughly occurs sometime after the trough in the female labor market attachment is attained, a 10% increase in female city wages results in about a 25% increase in female labor force.

The wage effect is given by:

$$\frac{\Delta FLFPR}{\Delta w_C^F} = \left\{ \begin{array}{c} 0 \Leftrightarrow t \le t_2^1 - dt^F \\ 1 - j2t^* \Leftrightarrow t \in [t_2^1 - dt^F, t_2^1] \\ j2(t) - j2(t)^* \Leftrightarrow t \in [t_2^1, t_2^0 - dt^{\mathrm{F}}] \\ j2(t) \Leftrightarrow t \in [t_2^0 - dt^F, t_2^0] \\ 0 \Leftrightarrow t \ge t_2^0 \end{array} \right\}$$

where t2(1) and t2(0) are defined similar to above except with respect to the expectations regarding the $\frac{P}{w_C^F}$ boundary. Furthermore $j2(t)^*$ is given by:

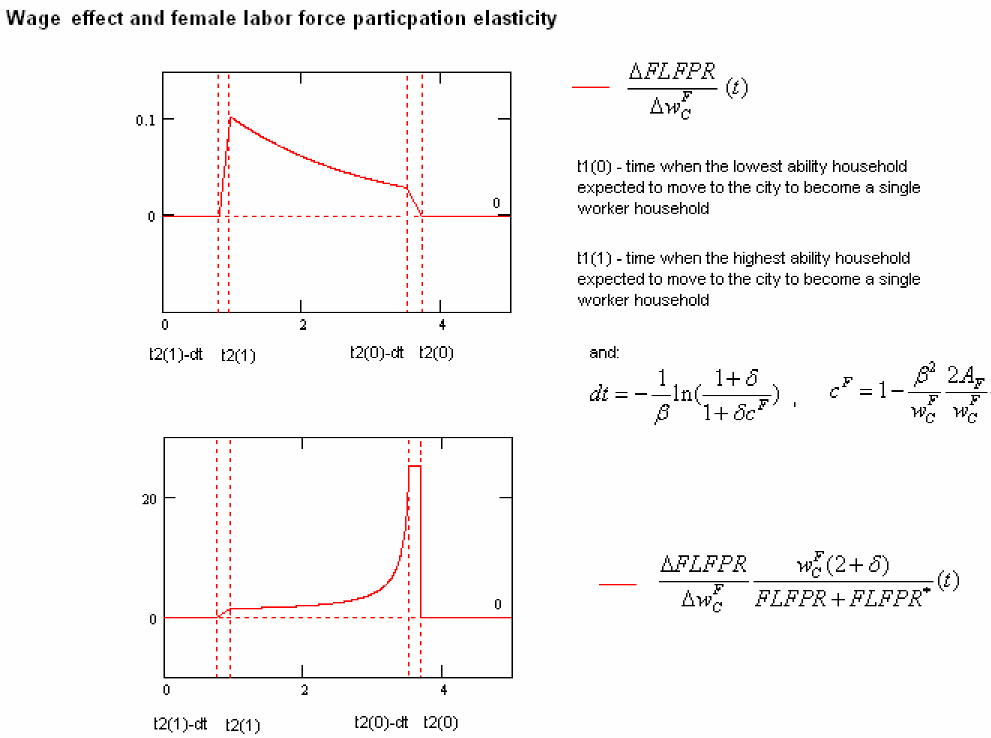
$$j2(t)^* = \frac{P}{(1+\delta)w_C^F} e^{-\beta t} + \frac{w_C^F}{\beta^2} c^F (1 - e^{-\beta t}) \quad \text{where now}$$

$$dt = -\frac{1}{\beta} \ln\left(\frac{1+\delta}{1+\delta c^F}\right) \quad \text{and} \quad c^F = 1 - \frac{\beta^2}{w_C^F} \frac{2A_F}{w_C^F}.$$

Proposition 5 - The elasticity of female labor force participation with respect to women's city wages is positive and initially small in magnitude. It then increases, levels off, and becomes zero once all households have moved to the city.

The figures below illustrate the effect of a shock to female wages on female force participation rate as a function of time (that is, as a function of the distribution of households at a particular moment in time).

Figure 12



6. Conclusion

In this paper we developed a dynamic model of female participation rate and education which appears to do a good job of broadly matching several stylized facts found in the data. For empirical inspiration and support we have mostly relied on Claudia Goldin's "Understanding the Gender Gap". Hopefully we have provided an intuitive justification for the processes and dynamic phenomenon which are summarized in the five stylized facts listed in the introduction; the existence of economies of scope in the traditional sector and the resulting tradeoff between locating one's productive activities "on the farm" or "in the city" and choosing between operating as a single earner or two earner household.

The paper provides falsifiable results which could in principle be tested econometrically. In a companion paper we hope to analyze changes in the female labor force participation, marriage patterns and education for British women in much the same way as "Understanding The Gender Gap" did for the history of American women. The major challenges that the present framework faces is that it is notoriously difficult to estimate the value of household production, to accurately measure productivity in the traditional sector or to successfully detect the presence of economies of scope in such a sector.

The model can be readily extended in several ways. For one, we can explicitly incorporate unmarried women in to the model and see if the resulting dynamics produce the married/unmarried substitution evident in history (intuition and some preliminary work seem to indicate that this is indeed the case). More substantially, the model could be expanded to include an explicit fertility choice and a production function for children's human capital. In this case an over lapping generations framework might be better suited rather than the continuous time model presented here.

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