Liquidity Biases in Asset Pricing Tests

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Abstract

This paper examines how microstructure biases arising from "bid-ask bounce" affect empirical asset pricing tests. The focus is mainly on tests of whether liquidity is priced, but the analysis also provides new insights regarding tests of whether systematic risk is priced. We present theory and simulation-based evidence indicating that bid-ask spreads and endogenous trade or no-trade decisions lead to biases in observable risk and return measures that affect the reliability of asset pricing tests. The most robust finding is that these frictions can lead to upward bias in estimates of the return premium for illiquidity. We exploit the fact that CRSP has reported closing quotes for Nasdaq National Market System stocks since 1983 to verify empirically that the estimated return premium related to the bid-ask spread is significantly larger when returns are computed from closing prices rather than quote midpoints. We also document that, depending on research design, microstructure considerations potentially obscure the relation between average returns and betas. We discuss possible methodological corrections for these microstructure biases, and conditions under which they may be effective.

I. Introduction

A substantial recent literature has addressed the question of whether liquidity affects asset returns. Amihud and Mendelson (1986) and Acharya and Pedersen (2005), among others, present theoretical models implying that illiquidity is priced as a security characteristic and/or a risk factor. However, models presented by Constantinides (1986), Heaton and Lucas (1996), and Vayanos (1998) imply that the potential effects of illiquidity on prices should not be substantial, because agents will adjust their portfolio trading frequencies to mitigate illiquidity costs.[1] Numerous papers have addressed the issue empirically, and the emerging consensus (Amihud, Pedersen, and Mendelson (2005)) appears to be that liquidity does affect asset returns.

This paper examines how microstructure biases arising from "bid-ask bounce" affects empirical asset pricing tests. The focus is mainly on tests of whether liquidity is priced, but the analysis also provides new insights regarding tests of whether systematic risk is priced. We present theory, simulation-based evidence and empirical evidence relying on CRSP data for Nasdaq National Market System (NMS hereafter) stocks indicating that bid-ask spread and endogenous trade or no-trade decisions lead to biases in observable risk and return measures that affect the reliability of asset pricing tests. The most robust finding is that these frictions can lead to upward bias in the estimated return premium for illiquidity, measured as a security characteristic by bid-ask spreads. We show that the bias in mean returns can be avoided under certain assumptions if researchers employ quote midpoint returns, make an explicit adjustment to transaction returns for bid-ask bounce, or employ log transaction returns. We also document that, depending on research design, microstructure considerations potentially obscure the relation between average returns and betas.

The microstructure-based biases we study arise because most empirical asset pricing studies rely on return series that are created from transaction prices. In the Center for Research in Security Prices (CRSP) daily and monthly databases, the closing price reflects the last transaction prior to the close if trading occurred that day, or (the opposite of)[2] the closing

[1] However, Hasbrouck (2004, p.154) notes that actual share turnover is an order of magnitude larger than that implied by Constantinides' analysis.

[2] For Nasdaq NMS securities CRSP always reported the negative of the bid-ask midpoint prior to November 1, 1982, and for Nasdaq Small Cap securities CRSP always reported the quote midpoint prior to June 15, 1992.

quote midpoint if no trading occurred.[3] Microstructure theory implies that market buy orders are typically completed at an effective ask price that exceeds the true value of the asset, while market sell orders are completed at a bid price that is less than the true asset value. As a consequence, when trade occurs, observed returns differ from true returns due to "bid-ask bounce".

In addition to allowing for bid-ask bounce, we endogenize trading decisions by assuming that investors compare the potential gain from trade to the cost of trading, and on some days choose to refrain from trading. In the presence of non-trading, the relation between observable bid-ask spreads and bid-ask bounce is complex. Wider spreads lead to greater bid-ask bounce when trades occur, but also discourage trading, *ceteris paribus.* On days without trade the reporting of the quote midpoint reduces the amount of bid-ask bounce in the observed time series of returns. We show that under some assumptions bid-ask bounce can be greater for securities with narrower bid-ask spreads. More generally, our analysis shows that the biases in asset pricing tests attributable to bid-ask bounce need not be monotone in observable spreads.

Several of the results obtained here build on the Blume and Stambaugh (1983) insight that bid-ask bounce imparts an upward bias to mean returns measured from transaction prices, due to Jensen's inequality. We extend their analysis to include the effect of endogenous non-trading. More importantly, we study the effects of bid-ask spread and non-trading on inferences drawn from tests of whether both beta risk and illiquidity are priced.[4] In addition to addressing effects on mean returns, we document that that bid-ask bounce leads to bias in beta estimates. Further, bid-ask bounce increases the noise in beta estimates. This is relevant because the widely-used Fama and MacBeth (1973) method and similar procedures involve regressions of returns on

[3] Compustat, Datastream, Worldscope, and Compustat Global and Emerging markets also report daily returns computed from closing prices. Unlike CRSP, on non-trading days Datastream reports again the prior closing price.

[4] Lesmond, Ogden, and Trzcinka (1999) also consider the effects of trading costs on the decision to refrain from trading. However, the focus of their study is on how the observed frequency of non-trading can be used to infer the magnitude of trading costs. They do not consider how bid-ask bounce or non-trading affects asset pricing tests. The biases in asset pricing tests documented here are also distinct from those noted by Keim (1989) or Ferson, Sarkissian, and Simin (1999). Keim (1989) documents that some calendar-based empirical patterns in stock returns are attributable to systematic clustering of transaction prices at either the bid or the ask. In contrast, our analysis assumes that transactions occur randomly at either the ask or bid. Ferson *et al.* document that biases can arise in asset pricing tests when researchers form portfolios on the basis of stock price attributes that are found in the data to be related to returns. We document that biases related to bid-ask bounce are actually most pronounced when the analysis is conducted at the individual security rather than the portfolio level.

estimated rather than true betas.[5] Measurement error in regressors will most typically bias coefficient estimates toward zero. Thus, the increased noise in beta estimates attributable to bid-ask bounce can cause a downward bias in Fama-MacBeth risk premium estimates that is distinct from the effect of microstructure-induced bias in the beta estimates.

Recent studies, e.g. Bessembinder (2003), have reported that quoted spreads on U.S. equity markets are quite narrow, particularly subsequent to the 2001 shift from fractional to decimal pricing. This evidence might be viewed as suggestive that measurement errors attributable to bid-ask bounce are a minor concern. However, bid-ask spreads reported for earlier years by Chalmers and Kadlec (1998) for NYSE/AMEX stocks and by Fortin, Grube, and Joy (1989) for Nasdaq stocks are markedly wider than spreads estimated from the recent data. Further, researchers have and will continue to study asset pricing and liquidity in non-U.S. markets, which generally have wider spreads.[6] Also, quoted spreads need only apply to orders up to the quote size, so larger orders may be completed at average prices outside the quotes. We are concerned with temporary changes in transaction prices resulting from actual orders, including large institutional orders.

To illustrate the potential magnitude of the microstructure biases, we conduct a series of simulations. The key advantage of a simulation approach is that we know the true relation between returns, betas, and liquidity, and we can assess whether estimates obtained when applying versions of the Fama-MacBeth method to the simulated data reveal the true pricing structure or not. The simulations rely on spreads calibrated to those reported for actual U.S. markets, and indicates that microstructure biases in asset pricing tests can be substantial. The most robust finding of the simulations is that bid-ask spread can appear to be positively related to average returns even when it is not. We also exploit the fact that CRSP has reported closing quotes for Nasdaq NMS stocks since 1983 to verify empirically that estimated premia for illiquidity are significantly larger when returns are computed from closing prices rather than quote midpoints.

Finally, we document that the increased noise in beta estimates attributable to microstruc-

[5]Shanken and Zhou (2006) report that the Fama-MacBeth methodology is applied in at least 735 papers.

[6]For example, Table 2 in Jain (2001) indicates bid-ask spreads that average 6.10% as recently as year 2000 for a sample of forty seven non-U.S. markets that includes both developed and developing economies.

ture effects is sufficient that the accompanying errors-in-variables problem can, depending on research design, lead to significant downward biases in the market price of beta risk. This effect is most pronounced when the cross-sectional Fama-MacBeth regressions are estimated by regressing individual security returns on estimated individual security betas or when portfolio returns are regressed on estimated portfolio betas, as in Eleswarapu (1997) and Fama and French (1996). However, the bias is minimal when individual security returns are regressed on betas estimated on a portfolio basis, as in Fama and French (1992). Since Fama and French (1992) do not detect a significant premium on beta after controlling for firm size, we conclude that microstructure biases alone do not explain the empirical failure of the CAPM.

II. The Related Literature

The relation between average returns and measures of liquidity has been the subject of considerable research interest. Stoll and Whalley (1983) first suggested that transaction costs are a "missing factor" in empirical tests of the Capital Asset Pricing Model (CAPM). Amihud and Mendelson (1986) develop a theory that implies that average returns should increase with spreads, and report evidence consistent with their implications for NYSE-listed stocks. However, Eleswarapu and Reinganum (1993) find a statistically significant relation between average return and bid-ask spread for NYSE stocks only in January. Chen and Kan (1996) report that the Amihud and Mendelson (1986) findings are specific to the multivariate methodology they employ, and that application of the Fama and MacBeth method in the same data does not result in a reliable return-spread relationship. Chalmers and Kadlec (1998) examine the amortized spread (which incorporates also investors' holding periods), for NYSE and AMEX stocks and find that the relation between average returns and illiquidity is stronger for amortized than for unamortized spreads. Barclay, Kandel and Marx (1998) find that transaction costs significantly reduce trading volume, but do not detect a significant effect on prices.

Eleswarapu (1997) tests the Amihud and Mendelson (1986) model using Nasdaq stocks. His results support the model and are much stronger than for the New York Stock Exchange (NYSE), as reported by Chen and Kan (1989) and Eleswarapu and Reinganum (1993). Amihud (2002) introduces a measure of illiquidity that relies only on return and trading volume measures, and can therefore be computed from most daily databases, in the absence of data on bid-ask spreads. He provides evidence of a significant positive relation between average returns and this illiquidity measure for NYSE common stocks over the interval 1964-1997.

In addition to the studies that focus on illiquidity as a potentially-priced stock characteristic, Pastor and Stambaugh (2003) and Acharya and Pedersen (2005) provide evidence that systematic liquidity *risk* affects average returns. Korajczyk and Sadka (2006) provide an integrated analysis documenting that both systematic liquidity risk and levels of idiosyncratic liquidity affect average returns, while Fujimoto and Watanabe (2006) use a regime-shifting model to document that liquidity risk varies over time and that the estimated liquidity risk premium is larger at times of high return sensitivities to an aggregate liquidity factor. Bekaert, Harvey and Lundblad (2005) find that in emerging markets unexpected liquidity shocks are positively correlated with contemporaneous returns and negatively correlated with the dividend yield, consistent with liquidity being a priced factor.

The current paper also relates to the literature on nonsynchronicity. Scholes and Williams (1977), Dimson (1979) and Cohen *et al.* (1983) show that nonsynchronous trading leads to bias in betas estimated by standard procedures, and introduce techniques that provide consistent beta estimates, under their assumptions. However, these authors do not consider the effect of bid-ask bounce. We document that implementing the method recommended by Scholes-Williams to correct beta estimates for the effects of nonsynchronous trading does not generally mitigate the biases that arise in asset pricing tests due to bid-ask bounce. This result is to be expected, as nonsynchronous trading differs from the nontrading effect studied here, which arises even if securities trade at the same time, as in batch trading. CRSP data on days with trading is affected by both non-synchronicity and bid-ask bounce, while CRSP data on non-trade days is affected by neither.

Finally, our analysis is related to that of Brennan and Wang (2006), who also consider how return measurment errors can affect asset pricing tests. Like Blume and Stambaugh (1983), they rely on a Jensen inequality argument to establish that mean observed returns are upward biased when observed prices differ from underlying value. However, Brennan and Wang focus on market pricing errors, due for example to investors' underreaction to new information, as the source of the measurement error, while we assume markets are efficient in the sense that quote midpoints are equal to true values, so that the return measurement error is solely attributable to bid-ask bounce. Both analyses lead to the implication that the estimated return premium associated with illiquidity is likely to be upward biased. In Brennan and Wang the conclusion follows from the observation that mispricing, and hence measured return biases, are likely to be greater for illiquid stocks due to impediments to arbitrage, while in our case the conclusion arises directly from bid-ask bounce, even without mispricing. The bias in the illiquidity premium estimate obtained in actual data potentially includes both effects.

III. A Model of Returns and Betas with Microstructure Frictions

The analysis of mean returns presented here follows Blume and Stambaugh (1983), except that we also allow for endogenous nontrading. We also assess how bid-ask bounce affects covariances and beta estimates, issues which Blume and Stambaugh did not address. The simulation results reported in Section V addresses estimation error as well.

Assume that the true return for security i in excess of the risk free interest rate in period t is generated by the following stochastic model:

$$r_{i,t} = \beta_i r_{M,t} + e_{i,t}, \tag{1}$$

where $r_{M,t}$ is a normal random variable with $E[r_{M,t}] \geq 0$ and variance $var[r_{M,t}]$, that is independently and identically distributed across t. The disturbance term $e_{i,t}$ is also independent and identically distributed $E[e_{i,t}] = 0$ with variance $var[e_{i,t}]$ for all i and t. Further, $r_{M,t}$ and $e_{i,\tau}$ are independent for all i, t and τ. Hence, $r_{M,t}$ is a common factor affecting all securities and $e_{i,t}$ reflects zero-mean security-specific information. Denote the true price at time t of stock i as $v_{i,t}$, the price at which, in the absence of transaction costs, a share of stock could be both bought and sold.

The true price evolves as:

$$v_{i,t} = v_{i,t-1}(1 + r_{i,t}). \tag{2}$$

The observed closing price, $v^o_{i,t}$, deviates from the true price $v_{i,t}$ due to bid-ask spread.

Define $\delta_{i,t}$ as the signed half spread relative to the true price. It is positive for market buy orders and negative for market sell orders. We assume that $\delta_{i,t}$ is symmetric for buy and sell orders, or equivalently that the midpoint of the effective bid and ask prices is the true price, $v_{i,t}$. Assuming that each security trades every period, the observed price, $v^o_{i,t}$ can be expressed as:

$$v^o_{i,t} = (1 + \delta_{i,t})\, v_{i,t}. \tag{3}$$

We assume that $E[\delta_{i,t}] = 0$, that $\delta_{i,t}$ is independently distributed across t, and that $\delta_{i,t}$ is independent of $v_{i,\tau}$ for all τ. The relative total bid-ask spread (not signed) at period t for security i is $2|\delta_{i,t}|$.

We assume that investors will compare the cost of trading to their perceived benefits of doing so. To allow for endogenous no-trade decisions, we introduce a variable, c, that quantifies the potential gain from trade. Investors will trade only if the percentage trading cost is less than c. We also introduce the variable $\bar{\delta}_{i,t}$:

$$\bar{\delta}_{i,t} = \delta_{i,t} I\{|\delta_{i,t}| \leq c\}, \tag{4}$$

where $I\{.\}$ is an indicator variable equal to unity when $-c \leq \delta_{i,t} \leq c$, i.e. when trade occurs, and zero otherwise. Consistent with the reporting conventions of the CRSP database, we assume that when $\bar{\delta}_{i,t} = 0$, i.e. no trade occurs, the researcher observes the midpoint.

Using Eq.(2), Eq.(3) and Eq.(4) the observed return, $r^o_{i,t,}$ in the case of endogenous nontrading is:

$$r^o_{i,t} = \frac{v^o_{i,t} - v^o_{i,t-1}}{v^o_{i,t-1}} = \frac{\left(1 + \bar{\delta}_{i,t}\right)}{\left(1 + \bar{\delta}_{i,t-1}\right)} (1 + r_{i,t}) - 1. \tag{5}$$

Taking expectations of Eq.(5) gives[7]:

[7] If however $\bar{\delta}_{i,t}$ is not independent of price then taking expectations yields an extra term:
$$E\left[r^o_{i,t}\right] = E\left[\frac{1 + \bar{\delta}_{i,t}}{1 + \bar{\delta}_{i,t-1}}\right] \{1 + E[r_{i,t}]\} + cov\left[\frac{1 + \bar{\delta}_{i,t}}{1 + \bar{\delta}_{i,t-1}}, r_{i,t}\right] - 1.$$

$$E\left[r^{o}_{i,t}\right] = E\left[\frac{1+\bar{\delta}_{i,t}}{1+\bar{\delta}_{i,t-1}}\right]\left\{1+E\left[r_{i,t}\right]\right\} - 1. \tag{6}$$

We show in the appendix that the first expectation on the right side of Eq.(6) exceeds one due to Jensen's inequality, i.e. that bid-ask bounce imparts an upward bias to observed mean returns. The upward bias in mean returns is attributable to the fact that $f(x) = 1/x$ is convex in x. Following Blume and Stambaugh (1983) the bias in mean returns can be approximated as:

$$E\left[r^{o}_{i,t}\right] \approx E\left[r_{i,t}\right] + var\left[\bar{\delta}_{i,t}\right]. \tag{7}$$

where $var\left[\bar{\delta}_{i,t}\right]$ denotes the variance of $\bar{\delta}_{i,t}$ and measures the amount of bid-ask bounce. If securities trade every period those with wider bid-ask spreads will have higher bid-ask bounce. In the case of endogenous nontrading, i.e. when $\bar{\delta}_{i,t}$ is zero at some periods, the relation between bid-ask spread and bid-ask bounce is more complex. We next provide some results regarding the amount of bid-ask bounce with endogenous nontrading.

The variance of $\bar{\delta}_{i,t}$ can be expressed as:

$$\begin{aligned} var\left[\bar{\delta}_{i,t}\right] &= E\left[\delta^{2}_{i,t} I\left\{\left|\delta_{i,t}\right| \leq c\right\}\right] - \left\{E\left[\delta_{i,t} I\left\{\left|\delta_{i,t}\right| \leq c\right\}\right]\right\}^{2} \\ &= \int_{-c}^{c} x_i^2 f\left(x_i\right) dx_i - \left(\int_{-c}^{c} x_i f\left(x_i\right) dx_i\right)^{2}, \end{aligned} \tag{8}$$

where $f\left(x_i\right)$ is the density function of $\delta_{i,t}$ for security i. From the fact that $\delta_{i,t}$ has a symmetric density, i.e. $f\left(x_i\right) = f\left(-x_i\right)$ it follows that the second term on the right hand side in equation (8) is:

$$\int_{-c}^{c} x_i f\left(x_i\right) dx_i = 0.$$

Hence, Eq.(8) transforms into:

$$var\left[\bar{\delta}_{i,t}\right] = \int_{-c}^{c} x_i^2 f\left(x_i\right) dx_i. \tag{9}$$

The variable $var\left[\bar{\delta}_{i,t}\right]$ quantifies the amount of bid-ask bounce in observed returns, given that securities sometimes do not trade. The amount of bid-ask bounce depends both on spread widths and traders' potential gains from trade, c. To obtain more detailed implications requires additional structure. We analyze some possible relations between spread widths and bid-ask bounce in the Appendix, showing that in some circumstances securities with narrower spreads can have more bid-ask bounce, and that in general the relation between the amount of bid-ask bounce and bid-ask spreads need not be monotone.

We next assess how bid-ask bounce affects measures of return covariances and systematic risk. The covariance between the observed returns for security i, $r^o_{i,t}$ and security j, $r^o_{j,t}$, derived in the appendix, is:

$$(10)\qquad cov\left[r^o_{i,t}, r^o_{j,t}\right] = \left\{1 + var\left[\bar{\delta}_{j,t-1}\right]\right\}\left\{1 + var\left[\bar{\delta}_{i,t-1}\right\}\right]\beta_i\beta_j var\left[r_{M,t}\right],$$

which is an increasing function of the bid-ask bounce of both security i and security j. Summing over i and j in Eq.(10) gives the variance of the observed equal-weighted market return:

$$(11)\qquad var\left[r^o_{M,t}\right] = \frac{1}{N^2} var\left[r_{M,t}\right]\sum_{i=1}^{N}\left\{1 + var\left[\bar{\delta}_{i,t-1}\right]\right\}\beta_i\sum_{j=1}^{N}\left\{1 + var\left[\bar{\delta}_{j,t-1}\right]\right\}\beta_j.$$

A standard market model regression using observed returns would not provide an estimate of true security i beta, but rather would provide an estimate of:

$$\beta^o_i = \frac{1}{N}\frac{\sum_{j=1}^{N}\left\{1 + var\left[\bar{\delta}_{i,t-1}\right]\right\}\left\{1 + var\left[\bar{\delta}_{j,t-1}\right]\right\}\beta_i\beta_j var\left[r_{M,t}\right]}{var\left[r^o_{M,t}\right]}.$$

Using Eq.(11) the above expression simplifies to:

$$(12)\qquad \beta^o_i = \frac{N\left\{1 + var\left[\bar{\delta}_{i,t}\right]\right\}\beta_i}{\sum_{j=1}^{N}\left\{1 + var\left[\bar{\delta}_{j,t-1}\right]\right\}\beta_j}.$$

Examining this expression, betas obtained from observed return will be increased relative to true betas for securities with higher-than-average bid-ask bounce, and vice versa. The bias documented in Eq.(12) arises from bid-ask bounce, which was not considered in the analyses of nonsynchronous trading of Scholes and Williams (1977) or Dimson (1979), and arises even when all securities trade (or not) at the same moment.

In summary, from Eq.(7) and Eq.(12), securities with high bid-ask bounce will tend to have both their estimated mean returns and their estimated betas biased upwards. Further, the bid-ask bounce is a function of the bid-ask spread. The potential for spurious results in asset pricing tests involving returns, beta estimates, and spreads is readily apparent.

The magnitude of the relative bias in the observed beta estimate can be approximated as:

$$\frac{\beta_i^o}{\beta_i} \approx \frac{1}{N} \frac{var\left[r_{M,t}\right] \sum_{j=1}^{N} \left\{1 + var\left[\bar{\delta}_{j,t-1}\right]\right\} \beta_j}{var\left[r_{M,t}^o\right]} \left\{1 + var\left[\bar{\delta}_{i,t-1}\right]\right\}. \tag{13}$$

Substituting Eq.(11) into Eq.(13) gives:

$$\frac{\beta_i^o}{\beta_i} \approx N \frac{1 + var\left[\bar{\delta}_{i,t}\right]}{\sum_{j=1}^{N} \left\{1 + var\left[\bar{\delta}_{j,t-1}\right]\right\} \beta_j}. \tag{14}$$

Thus, the relative bias in beta is larger for securities with high bid-ask bounce. The covariance across stocks between the bias in observed beta and total spread is:

$$cov\left[\frac{\beta_i^o}{\beta_i}, 2|\bar{\delta}_{i,t}|\right] = 2N cov\left[\frac{1 + var\left[\bar{\delta}_{i,t}\right]}{\sum_{j=1}^{N} \left\{1 + var\left[\bar{\delta}_{j,t-1}\right]\right\} \beta_j}, |\bar{\delta}_{i,t}|\right]. \tag{15}$$

Further, the covariance across stocks between the bias in relative returns and the bias in relative betas can be expressed as,

(16)

$$cov\left[\frac{E\left[r_{i,t}^o\right]}{E\left[r_{i,t}\right]}, \frac{\beta_i^o}{\beta_i}\right] = cov\left[\frac{E\left[r_{i,t}\right] + var\left[\bar{\delta}_{i,t}\right]}{E\left[r_{i,t}\right]}, N \frac{1 + var\left[\bar{\delta}_{i,t}\right]}{\sum_{j=1}^{N} \left\{1 + var\left[\bar{\delta}_{j,t-1}\right]\right\} \beta_j}\right].$$

Since the security i bid-ask bounce appears in the numerator of both terms in the covariance on the right side of this expression, we can infer that bid-ask bounce will tend to induce a spurious positive relation between observed returns and observed beta. Further, the covariance between the bias in expected returns and the total spread can be expressed as:

$$cov\left[\frac{E\left[r^o_{i,t}\right]}{E\left[r_{i,t}\right]}, 2|\bar{\delta}_{i,t}|\right] = cov\left[\frac{E\left[r_{i,t}\right] + var\left[\bar{\delta}_{i,t}\right]}{E\left[r_{i,t}\right]}, 2|\bar{\delta}_{i,t}|\right]. \tag{17}$$

Since the variance and the absolute value of $\delta_{i,t}$ appear in the two terms within the covariance on the right side of this expression, we can also anticipate the possibility that spurious relations between observed returns and average spreads can arise.

In summary, securities with more bid-ask bounce will tend to have both their average returns and their beta estimates biased upward. This implies the potential for upward bias in estimated relations between mean returns, betas, and any measure correlated with bid-ask bounce. However, due to non-trading, the relation between observable bid-ask spreads and bid-ask bounce need not be monotone.

IV. Potential Solutions

We have shown that bid-ask bounce biases both mean returns and beta estimates. In this section we consider some solutions that researchers might adopt, assumptions under which the solutions may be appropriate, and discuss practical limitations on adopting these solutions.

A. Quotation Midpoint Returns

We assume in our theoretical analysis that the quote midpoint is equal to the true value of the security. If this assumption is accurate, then a simple empirical solution is to compute returns from quote midpoints instead of from reported closing prices. Unfortunately, this solution may be difficult to implement due to data availability. Quotation data for stocks listed on the NYSE and Nasdaq is available from the Trade and Quote (TAQ) database from 1993 onward. However, as noted in the prior section, a lack of statistical power is an important issue in asset pricing applications, and the available time series of TAQ-based quotation returns may not be sufficient for many applications. CRSP reports closing quote midpoints for Nasdaq NMS stocks from November 1983 onward, providing a somewhat longer time series for this subset of stocks. We report in Section VI below the results of some tests of whether there is a return premium associated with the bid-ask spread using this data source.

Further, while the assumption that the quote midpoint is equal to the true asset value is analytically convenient for presenting a model of biases arising from bid-ask bounce, it is not clear that the assumption is accurate for actual data. Many models (e.g. Ho and Stoll, (1980)) imply that liquidity providers will move quotations away from asset value in order to manage inventory. Further, as Brennan and Wang (2006) emphasize, return measurement errors arising from market misvaluations may also bias tests of the relation between average returns and illiquidity measures.

B. Adjust Closing Returns for Bid-Ask Bounce

Eq.(7) in the preceding section shows that the bias in the mean return for a given stock is approximately equal to $var\left[\bar{\delta}_{i,t}\right]$. Blume and Stambaugh (1983), assuming that securities trade each period and that the bid-ask spread is constant over time for a given stock, show (their expression (7)) that $var\left[\bar{\delta}_{i,t}\right]$ is equal to the square of half the proportional bid-ask spread. Under their assumptions the bias in mean returns (though not the bias in beta estimates) can be approximately eliminated by deducting this quantity from every return observation.[8]

However, securities do not trade every period, and the bid-ask spread is not constant over time, implying that a simple adjustment such deducting the square of half the average proportional bid-ask spread is unlikely to yield zero bias when implemented in actual data. On a more positive note, if time series of both trade and quotation data are available then, assuming that the quote midpoint equals the true security value at the time of the last trade, $\bar{\delta}_{i,t}$ can be measured for security i on day t as the difference between the last trade price and the quote midpoint. The variance of $\bar{\delta}_{i,t}$ can then be estimated from a time series of observations for a given security. We report in Section VI the results of implementing this correction in Nasdaq NMS data, for which both quotations and trade prices are available.

[8]Amihud and Mendelson (1980) reference expression (7) in Blume and Stambaugh, and report that they implement a correction for bid-ask bounce as a sensitivity test. However, they are not specific as the correction implemented. To our knowledge, Amihud and Mendelson are the only authors who report implementing any sort of correction for bid-ask bounce.

C. The Use of Continuously Compounded Returns

As noted in Section III, the bias in mean returns stemming from bid-ask bounce is attributable to Jensen's inequality. Notably, this bias does not exist under reasonable assumptions if the focus is on continuously compounded rather than holding period returns. Taking expectations of the logarithms of both sides of Eq.(6), we have:

$$E\left[ln\left[1+r_{i,t}^{o}\right]\right] = E\left[ln\left[(1+\delta_{i,t})\right]\right] - E\left[ln\left[(1+\delta_{i,t-1})\right]\right] + E\left[ln\left[1+r_{i,t}\right]\right]$$

Assuming further that $\delta_{i,t}$'s are identical it follows that $E\left[ln\left[(1+\delta_{i,t})\right]\right] = E\left[ln\left[(1+\delta_{i,t-1})\right]\right]$ then we have:

$$E\left[ln\left[1+r_{i,t}^{o}\right]\right] = E\left[ln\left[1+r_{i,t}\right]\right],$$

implying that the mean of the observed continuously compounded returns equals the mean of the true continuously compounded returns.

The preceding insight implies that the inference problems in asset pricing tests attributable to upward bias in mean returns (though not those attributable to bias in betas) stemming from bid-ask bounce can be avoided by using continuously compounded rather than simple returns. However, this solution is appropriate *only* if the asset pricing theory being tested makes predictions regarding mean continuously compounded returns, as in Merton (1971). It is not appropriate when testing theories, e.g. the discrete time Capital Asset Pricing Model or the Arbitrage Pricing Theory, whose implications regard mean holding period returns. Ferson and Korajczyk (1995) articulate several reasons that it is not appropriate to use continuously compounded returns when testing discrete-time asset pricing models, including: (1) that wealth depends on the simple return to the investors' portfolios, (2) that continuously compounded portfolio returns are not the portfolio-weighted averages of the securities' continuously compounded returns, and most importantly (3) that the mean continuously compounded return is less than the mean simple return, with the differential increasing in the return variance. We have verified using simulations that estimates of the price of beta risk are downward biased if returns are generated by a discrete time Capital Asset Pricing Model, but tests are conducted using continuously compounded returns. Nevertheless, we also report as a sensitivity test in Section VI the results obtained when Fama-MacBeth tests are conducted in the Nasdaq NMS data, using continuously compounded returns.

V. Simulation Evidence

To quantify the direction and the magnitude of the potential biases in asset pricing applications that arise from bid-ask bounce and non-trading, we create a series of simulated data sets in which underlying parameters are known, and then obtain empirical estimates of the parameters from the simulated data. Shanken and Zhou (2006) also report results of asset pricing simulations, but with a different focus. Their intent is to evaluate the relative merits of several alternative estimation techniques (including the Fama-MacBeth approach), but without consideration of return measurement errors. Our intent is to compare the outcomes of asset pricing tests conducted using the most common empirical methods, with and without measurement errors attributable to bid-ask bounce and endogenous nontrading.

Each simulated dataset contains return data covering 43 years (similar to the widely-studied CRSP daily dataset) for 1500 stocks. Some researchers, including Amihud (2002), Ang, Chen, and Xing (2006) and Lewellen and Nagel (2006), have studied asset pricing relations using daily return data. However, most asset pricing studies focus on monthly returns. We therefore focus primarily on the results of asset pricing tests conducted in simulated monthly returns.[9] The returns series span 516 months. The simulations are repeated 100 times, allowing us to consider both the mean and the volatility of the parameter estimates.

[9] Though we report results for simulated monthly returns, we caution that the biases and measurement errors attributable to bid-ask bounce are even more important in daily data. CRSP has recently made available daily returns for NYSE stocks dating to December 1925. We anticipate that the issues discussed here will be particularly important in potential asset pricing studies that rely on this data.

A. The simulated data, and beta estimates

Within each simulation, the return on the common factor, $r_{M,t}$ and the true return to stock i in period t, are created as:

$$
\begin{aligned}
r_{M,t} &= \gamma_M + \eta_{M,t} \\
r_{i,t} &= \beta_i r_{M,t} + \gamma_s (s_i - \bar{s}) + \eta_{i,t}
\end{aligned}
$$

where β_i is stock i's true beta coefficient, s_i is the spread parameter for stock i, $\bar{s}$ is the mean of the s_i distribution, γ_s and γ_M are premia for illiquidity and for market risk, respectively, $\eta_{i,t}$ is a firm-specific disturbance, and $\eta_{M,t}$ is the random component of the common factor return.[10] Each disturbance term is generated as a zero-mean random normal variable. When simulating monthly returns the firm-specific and market standard deviations are 4.5% and 5.5%, respectively. The β_i are generated as normal random variables[11] with mean 1.0 and standard deviation of 0.4, with the betas independent of other variables.

To obtain reasonable assessments of the potential biases attributable to bid-ask bounce it is particularly important to select bid-ask spread parameters that are representative of actual spreads in the data typically used to test asset pricing models. We rely on estimates of spreads for NYSE and AMEX stocks as reported by Chalmers and Kadlec (1998) and for Nasdaq stocks as reported by Fortin, Grube, and Joy (1989). In particular, Table 1, Panel B of Chalmers and Kadlec (1998) reports average effective spreads (absolute value of trade price less quote midpoint) in percent for ten deciles of NYSE/AMEX securities, estimated over the interval 1983 to 1992. Similarly, Table 1, Panel B of Fortin, Grube, and Joy reports average inside (lowest ask from any dealer minus highest bid from any dealer) spreads for five quintiles of Nasdaq stocks

[10] The market-wide premium γ_M is constant across all simulations. The factor $r_{M,t}$ is the same across securities for each period t within each simulation, but varies across t and across simulations. Unique spread, β_i and c_i parameters are assigned for each stock in each simulation, but within a simulation these parameters are constant across time periods, t.

[11] The cross-sectional standard deviation of the betas is in line with the estimates reported by Blume (1971) and Kolb and Rodriguez (1990).

over the period July 1980 to December 1985.[12]

We construct observed return series based on two research scenarios. The first is that asset pricing tests are conducted using NYSE and AMEX securities, in which case spreads are assigned to individual stocks based on parameters as identified in Table I, Panel A, such that simulated mean spreads match the means by stock deciles as reported by Chalmers and Kadlec. The second scenario focuses on tests conducted using fifty percent Exchange-listed (NYSE and AMEX) stocks and fifty percent Nasdaq stocks. In this case spreads are assigned to individual stocks based on parameters identified in Table I, Panel B, such that spreads for the first half of the simulated sample correspond to mean spreads reported by Chalmers and Kadlec, while spreads for the second half of the sample correspond to mean spreads by quintile as reported by Fortin, Grube, and Joy. The focus on simulated spreads that match empirical estimates from the 1980s and early 1990s is likely to be conservative, in the sense that spreads were likely wider during earlier decades.

We consider return series where neither beta nor spread is priced ($\gamma_M = 0$ and $\gamma_s = 0$) as well as series where beta or both beta and spread are priced. When simulating monthly returns we allow for premia of $\gamma_M = 0.8\%$, and $\gamma_s = 6.0\%$. The beta premium of 0.8% per month equates to about 9.6% per year for a stock with a beta of one. The spread premium of 6.0% per month equates to an annual premium of about 6% for a stock in the widest Chalmers-Kadlec spread decile relative to a stock with an average bid-ask spread near zero (the annual premium is computed as 6% times the .083 average spread for this decile, times 12).

Observed return series are then constructed from the true return series as follows. For each stock i the actual spread on day t, $\delta_{i,t}$, is a random draw from the uniform distribution on the interval $[-s_i, s_i]$. We assume for the simulations reported that securities always trade. We have also conducted simulations that incorporate endogenous trade or no-trade decisions, with parameters selected to induce non-trading at frequencies consistent with those observed in the actual data, and verified that our conclusions are not alterred. The observed return $r^o_{i,t,}$ is

[12]The use of effective spread measures for NYSE and AMEX stocks is likely conservative, in that this measure is computed on a trade-by-trade basis. To the extent that large orders are split into smaller trades this measure likely understates total trading costs for the order. Using quoted spreads for Nasdaq stocks may overstate execution costs to the extent that institutions could negotiate trade prices inside the quotes. However, quotes were only binding for a fixed number of shares (typically 1000), and larger orders may also have paid larger execution costs.

computed from Eq.(5). The time series of observed and true market returns are computed as the simple cross-sectional averages of observed and true security returns.

We consider the results of asset pricing tests when using several distinct measures of beta. While research must necessarily be conducted using the observable data, the simulations allow us to gain additional insights into the sources of biases in asset pricing tests by also using beta measures that would not be observable in real data. The beta measures we examine include:

- The true beta. This can be used in conjunction with true returns to assess the statistical power of an estimation technique under ideal circumstances. Further, it can be used to assess whether a particular empirical methodology leads to biased results even in the absence of microstructure considerations.

- The estimated true beta. This is the estimate obtained from a market model regression of true stock returns on true market returns. By comparing to results obtained with the true beta, this can be used to assess the impact of estimation error in betas, in the absence of any other measurement problems.

- The computed beta. This is the beta obtained by computing Eq.(12). This can be used to assess the effect of biases in beta estimates that arise from bid-ask bounce and non-trading, in the absence of estimation error.

- The estimated observed beta. This is the beta estimate obtained by a market model regression of observed security returns on observed market returns. It corresponds to the estimate that would be obtained by a researcher using CRSP return data, and is subject to both microstructure biases and estimation error.

- The Scholes-Williams beta. This is the beta estimate obtained if one implements the Scholes and Williams (1977) method to estimate beta, using observed security returns and observed market returns. This can be used to assess whether the Scholes-Williams correction improves inference.

B. Simulated Asset Pricing Tests When Securities Always Trade

Tables II through V report results obtained when implementing versions of the Fama and MacBeth (1973) method under varying assumptions regarding spread widths, and assuming that securities always trade.[13] The research designs selected here are intended to be similar to those that are widely used in the literature. However, since all parameters are stable across time within any simulation we omit those aspects of research designs (e.g. the use of beta estimates obtained over rolling five year windows, illiquidity measures estimated over the most recent year or scaled by market liquidity) that are intended to accommodate time variation in parameters. The Tables report mean coefficient estimates and mean t-values across the one hundred simulations, as well as standard errors of the mean estimates. On each Table, Panel A reports results obtained when both beta and spread are priced, while Panel B reports results obtained when neither is priced. For results reported in Part I of panels A and B the cross-sectional regression includes only beta, while for results reported in Part II of each panel the cross-sectional regression includes both beta and spread.

B.1. Fama-MacBeth Estimation in Individual Securities

Tables II and III report results obtained when implementing the Fama-MacBeth approach in individual securities, with spreads calibrated to NYSE/AMEX levels and NYSE/AMEX/Nasdaq levels, respectively. The first 5 years of simulated monthly data are used to estimate beta coefficients for each of the 1500 stocks. We then run cross-sectional regressions of individual security returns on betas, with and without including the stock specific spread measure, s_i, for each of the remaining time periods. Final estimates from each simulation are obtained as the time series mean and t-statistic of the monthly cross-sectional parameter estimates.

Several key results can be observed on Table II. First, and most important, the estimated premium on the spread is upward biased in all specifications that rely on observed rather than true returns. Note that this result is obtained when spreads are assigned randomly to securities, so it cannot reflect correlations between spreads and any priced variable. Focusing on Panel

[13] We do not mimic the original Fama-MacBeth procedure precisely, but rather choose research designs to match recent implementations of their procedure, including the approaches of Fama and French (1992) and Eleswarapu (1997).

B.II, where returns do not contain a true spread premium, the estimated spread premium averages about 3.1% per month, with an average t-statistic exceeding 7.5. The mean spread premium exceeds the true parameter of zero by over one hundred times the standard error of the mean. Consistent with the theory developed in Section III, bid-ask bounce resulting from spreads calibrated at NYSE/AMEX levels lead to a strong but spurious observed cross-sectional relation between average returns and spreads.

Results reported in Column (1) of Table II verify that the Fama-MacBeth procedure can, in principle, reveal true asset pricing parameters on average. When true returns are regressed on true betas the estimated premium on beta risk is 0.78% (both with and without the spread included in the regression), which differs from the true parameter of 0.80% by less than one standard error. Similarly, the average estimated premium on the spread reported in column (1) of Panel A.II is 5.96%, which is close to the true parameter of 6.0%.

However, even in the idealized situation where returns and betas are perfectly measured, statistical power is an issue. Although the average t-statistic for the beta premium reported in column (1) of Panel A.1 exceeds 3.0, there is substantial variation across the one hundred simulations. Despite the inclusion of a relatively large (0.8% per month or 9.6% per year) beta premium in the simulated return data, in fifteen of the 100 simulations the estimated t-statistic on the beta premium is less than 1.96, so a researcher would have failed to detect the true positive relation between average returns and beta at the conventional 5% significance level. The finding that the power of the Fama-MacBeth technique to detect beta pricing relations is low is broadly consistent with results reported by Affleck-Graves and Bradfield (1993) and Shanken and Zhou (2006).

Comparing results across Columns (1) and (2) of Table II reveals that estimation error is also a potentially important issue. When using betas estimated by OLS regressions of true monthly returns on the true market return instead of true betas in the Fama-MacBeth regressions, the estimated risk premium reported in Panel A.I is reduced from 0.781% per month to 0.729% per month. This result can be understood in terms of a standard errors-in-variables (EIV) analysis, where the coefficient on a variable measured with error (beta) will be biased toward zero, *ceteris paribus*. Shanken and Zhou (2006) and Chen and Kan (2004) also document downward bias attributable to EIV in risk premium estimates obtained by use of the Fama-MacBeth method. They note that this bias persists even if the cross-sectional regressions are estimated by generalize least squares or maximum likelihood techniques.

Of course, in practice researchers do not have the option of using true returns to estimate betas, but must rely on observed returns. The Fama-MacBeth coefficient estimates obtained from regressing observed monthly returns on observed betas are biased further toward zero. In particular, the beta risk premium estimate obtained when regressing observed returns on observed betas as reported in column (5) of Panel A.II is 0.710% per month, compared to 0.729% in column (2) and 0.781% in column (1).

However, note also the results in columns (3) and (4) of Panel A.II. When observed monthly returns are regressed on either true betas or computed betas, the downward bias in the estimate of the market price of beta risk is no longer observed. We can therefore conclude that the downward bias in the estimated market price of beta risk observed in column (5) of Panel A.II is attributable to estimation error in betas that is made worse by bid-ask bounce, as opposed to bias in monthly betas that is attributable to bid-ask bounce.

Comparing results across Table III and Table II reveals that the inclusion in the simulated sample of securities with spreads representative of Nasdaq stocks worsens the biases. The upward bias in the estimated spread premium jumps to about 8.8% per month (Panel B.II. of Table III), as compared to 3.1% per month when the simulated sample included only NYSE and AMEX securities. Further, the downward bias in the estimated premium for beta risk is worsened; for example the estimated beta premium when regressing observed returns on betas estimated from observed returns drops from 0.71% in column (5), Panel A.II of Table II to 0.60% on Table III.

Implementing the Scholes-Williams correction when estimating betas in observed returns does not eliminate the microstructure-related biases in the asset pricing tests. In fact, the downward bias in the estimate of the beta premium is worsened, as the coefficient estimates reported in column (6), Panels A.I and A.II of Tables II and III are yet lower than the corresponding estimates reported in column (5), where betas were estimated without the Scholes-Williams correction. Further, the upward bias in spread premium estimates is not mitigated by use of Scholes-Williams betas. The ineffectiveness of the Scholes - Williams correction is to be expected, as the correction was not designed to correct the effects of bid-ask bounce, which are present in the simulated (as well as real, e.g. CRSP) data. Rather, the correction is intended to address *nonsychronicity*, which affect observed returns differently than either bid-ask bounce or nontrading.

We do not report the results of implementing the Scholes-Williams method in subsequent Tables, since the conclusion that the Scholes-Williams correction is not generally useful in correcting for the effects of bid-ask bounce remains the same. Also, since the results are most often repetitive, we do not report results obtained when using true returns and true betas, observed returns and computed betas, or observed returns and true betas on Tables that follow, but simply mention these results when noteworthy.

B.2. Fama-MacBeth Estimation in Portfolios

Researchers at least since Fama and MacBeth (1973) have been aware that measurement errors inherent in the use of estimated betas can cause downward bias in risk premium estimates obtained in cross-sectional regressions. This consideration among others motivated the use of portfolios rather than individual stocks to estimate betas, under the reasoning that diversification would reduce measurement errors for portfolio betas. Accordingly, we adopt portfolio approaches for the remainder of this study.

Some researchers, including Fama and MacBeth (1973) and Eleswarapu (1997), form portfolios of stocks, and estimate premia by cross-sectional regressions of portfolio returns on estimated portfolio betas. Other researchers, including Amihud (2002), Fama and French (1992), and Easley, O'Hara, and Hvidkjaer (2002) assign estimated portfolio betas to all stocks in a portfolio, and estimate premia by cross-sectional regressions of individual stock returns on portfolio betas. We assess the effects of microstructure biases using a version of each approach, referring to the former as the "Portfolio Return/Portfolio Beta" approach and the latter as the "Individual Return/Portfolio Beta" approach.

The details of our implementation of the "Portfolio Return/Portfolio Beta" approach are generally similar to the research design used by Eleswarapu (1997), whose paper is of particular interest due to the finding of a strong positive cross-sectional relation between mean returns and bid-ask spreads. In particular, we form forty nine portfolios by first assigning stocks to seven portfolios based on estimated (by OLS in the first five years of observed returns) betas, and then separating each beta portfolio into seven spread portfolios based on the s_i parameter. Mean portfolio returns are computed on a monthly basis as the equal-weighted average of the portfolio's component stock returns, and the portfolio spread is computed as the mean of the portfolio's component stock spreads. The estimated portfolio beta is obtained by an OLS regression of portfolio returns on market returns, using the first five years of monthly data. Premia for beta risk and spread are estimated by cross-sectional monthly regressions (excluding the first five years) of portfolio returns on estimated portfolio betas and spreads.

Table IV reports results obtained when we implement the "Portfolio Return/Portfolio Beta" approach in simulated return data where spreads are calibrated to NYSE/AMEX levels (columns 1 and 2) and to NYSE/AMEX/Nasdaq levels (columns 3 and 4). The broad conclusion that can be drawn is that the Portfolio Return/Portfolio Beta approach reduces, but only slightly, the magnitude of the various microstructure biases. Specifically, the premium on the spread estimated from observed returns for NYSE/AMEX stocks when the true premium is zero is reduced from about 3.11% on Panel B.II of Table II to 2.86% on Panel B.II of Table V. The corresponding figures for the simulated NYSE/AMEX/Nasdaq sample indicate a reduction in the bias on the spread premium estimate from 8.83% to 8.10%. However, the upward bias in the spread premium remains highly significant in a statistical sense, as the mean estimated spread premium continues to exceed the true parameter of zero by over one hundred times the standard error of the mean in each sample. The downward bias in the estimated beta premium attributable to errors-in-variables is essentially unchanged by the shift to the portfolio return/portfolio beta approach.

The results regarding the pricing of the spread parameter in the simulated NYSE / AMEX / Nasdaq sample are of particular interest, since the methodology is very similar to that employed by Eleswarapu (1997), who reported strong evidence of a positive relation between average observed returns and bid-ask spreads. The estimated bias on the spread premium of 8.10% in this sample is large in economic terms. Fortin, Grube, and Joy (1989) estimate the average spread for the largest-spread quintile of Nasdaq stocks to be 23.7%. The point estimates in column (4), Panel B.II of Table V therefore equate to an estimated monthly return premium for this portfolio of 0.237x8.1 = 1.92%, as compared to stocks with a spread near zero. The point estimates reported here actually exceed those reported by Eleswarapu (1997, Table III), which average about 3.5%. This reflects in part that we at this point continue to assume that securities always trade, while the actual data includes non-trading, which mitigates bid-ask bounce. Also, Eleswarapu uses Nasdaq stock data over the 1973-1990 period. CRSP return data for Nasdaq NMS stocks was based on quote midpoints rather than trade prices prior to November, 1982.

We next turn our attention to the level of microstructure biases observed when using the "Individual Return/Portfolio Beta" approach. Our implementation of this approach is similar to the research design employed by Fama and French (1992). More specifically, we form one hundred portfolios by first assigning stocks to ten portfolios based on estimated (by OLS in the first five years of observed returns) betas, and then separating each beta portfolio into ten spread portfolios based on the s_i parameter. Mean portfolio returns are computed on a monthly basis as the equal-weighted average of component stock returns. The estimated portfolio betas are obtained by OLS regressions of portfolio returns on market returns, using the full time series of monthly data. The estimated portfolio beta is assigned to each stock in the portfolio. Premia for beta risk and spread are estimated as the time series mean of coefficients obtained in cross-sectional monthly regressions (still excluding the first five years, to avoid increasing the effective sample size as compared to prior approaches) of portfolio returns on estimated portfolio betas and individual stock spreads. Table V reports results obtained when we implement this method in simulated return data where spreads are calibrated to NYSE/AMEX levels (columns 1 and 2) and NYSE/AMEX/Nasdaq levels (columns 3 and 4).

The theory developed in Section III indicated the possibility of an upward bias in the beta risk premium estimate, due to both returns and betas for stocks with more bid-ask bounce being upward upward biased. This upward bias is weakly apparent in the average beta premium estimate of 0.84%, as reported in Column (4) of Panel A.I. That the upward bias in the risk premium estimate reflects bias in the betas and not just the upward bias in observed returns can be verified by comparing to the corresponding estimate obtained when regressing observed returns on betas estimated from true returns, which is 0.77% (not reported in the Table).[14]

The key insight that arises from comparing results reported on Table V to corresponding results obtained when we used the "Portfolio Return/Portfolio Beta" approach as reported on Table IV is that the use of individual returns and portfolio betas essentially eliminates the downward bias in the beta risk premium estimate. Each mean beta risk premium reported in Panel A of Table V is within about 1.1 standard errors of the true parameter of 0.80%. Thus our simulation evidence supports the Fama and French (1992, p. 432) conjecture that "...the precision of the full-period post-ranking portfolios betas, relative to the imprecise beta estimates that would be obtained for individual stocks, more than makes up for the fact that true betas are not the same for all stocks in a portfolio."[15] However, in contrast to the improved results regarding estimation of the beta risk premium, the bias in the estimated spread premium is not lessened by the Individual Stock/Portfolio Beta approach.

C. How Large Must Spreads be to Introduce Bias?

The simulation results reported in Sections B and C above indicate that bid-ask bounce consistently leads to upward bias in estimated premia for illiquidity, when spreads are calibrated to those actually observed during the 1980's. A natural question is whether such bias could be avoided by focusing on samples with narrower spreads, either by restricting the analysis to more recent data or by omitting stocks with large spreads. To shed some light on this issue we report on Table VI a set of analyses where stocks are excluded from the simulated sample as a function of spread widths. These results are based on the Individual Return/Portfolio beta method, with all securities trading each day, observed returns regressed on betas estimated from observed returns, and an actual spread premium equal to 6.0%.

The first row of Table VI reproduces results reported in Table V, Panel A.II., Column 4.

[14]To gain additional insight as to the magnitude of the potential bias in betas attibutable to bid-ask bounce, we use the simulated monthly return data underlying Table V, sort stocks into 10 portfolios by spread, and then compute the beta bias within each portfolio as 100*(true beta-computed beta)/computed beta. We document an upward bias in the computed beta of 1.58% in the high-spread portfolio and a downward bias in beta for the low-spread portfolio of 0.34%. These biases would be larger in simulated daily data.

[15]We investigate whether the reduced EIV bias stems from the use of portfolio beta estimates in lieu of individual stock beta estimates, or from the use of the full sample rather than just five years to estimate the betas. We find that portfolio betas estimated using only the first five years perform almost as well as betas estimated from the full sample. We therefore conclude that the reduced bias in our simulations is largely attributable to the former explanation.

The mean estimated spread premium of 14.92% exceeds the true premium of 6.0% by 450 times the standard error of the mean. The second row of Table VI reports results when ten percent of stocks have been excluded from the sample, the third row reports results when twenty percent of stocks are excluded, etc. Stocks are excluded sequentially based on average spread widths as reported on Panel B of Table I. The first set excluded is comprised of the Nasdaq 5th quintile (average spread 23.7%), the second set excluded is comprised of the Nasdaq 4th quintile (average spread 12.1%), the third set excluded is the Nasdaq 3rd quintile (average spread 7.3%), the fourth set excluded is comprised of the NYSE/AMEX 5th quintile (average spread 6.4%), etc.

The results on Table VI indicate that the bias in the spread premium declines rapidly as the widest-spread stocks are excluded from the analysis. When only the widest-spread decile is excluded the estimated spread premium declines to 10.21%, as compared to 14.92% when all stocks were included. Excluding the second widest-spread decile further reduces the estimated spread premium to 8.88%. When the analysis includes only the narrowest-spread decile (the first NYSE/AMEX quintile), the estimated premium of 6.25% is only slightly greater than the true parameter of 6.0%.

However, statistical power to detect the true positive relation between returns and spreads also declines rapidly as securities are omitted from the analysis. This reduction in power reflects both the smaller sample size and the reduced cross-sectional variation in spreads. The tradeoff between reduced bias and reduced power is illustrated vividly when focusing on results obtained when eighty percent of sample stocks are excluded (i.e. only the narrowest 40% of NYSE/AMEX stocks are included). In this case the average estimated spread premium across the one hundred simulations is 6.97%. Though the magnitude is relatively small, the mean is still upward biased, exceeding the true parameter of 6.0% by 1.78 times the standard error of the mean. At the same time, the average t-statisic on the spread is only 1.20 across the 100 simulations. In 77 of the 100 individual simulations the t-statistic on the spread is less than 1.96, implying that a researcher would have failed to detect the actual positive spread premium at conventional significance levels. We therefore conclude that the upward bias in the estimated spread premium can be reduced, but not eliminated, by excluding wide-spread securities, but at a cost in statistical power such that the researcher may well not be able to detect a true positive illiquidity premium if it exits.

VI. Empirical Evidence from Nasdaq

To this point, we have relied on simulation evidence, the results of which indicate that biases in asset pricing tests arising from microstructure considerations can be substantial. To assess the magnitude of microstructure-based biases using actual data requires that we compare results obtained using observed returns to results obtained using true returns, which unfortunately cannot be observed. However, the CRSP database reports closing bid and ask quotations as well as closing trade prices for Nasdaq NMS (National Market System) stocks subsequent to November l, 1982. The magnitude of the biases discussed here can be assessed by comparing results of asset pricing tests conducted in closing trade returns to those obtained using closing midpoint returns, at least to the extent that quotation midpoints can be viewed as reasonable proxies for true asset values.

We construct a sample consisting of Nasdaq NMS stocks from November 1982 through December 31, 2005. We include only ordinary common shares (CRSP share code $shrcd = 10, 11, 12$). The sample includes daily returns computed by CRSP from closing prices (or midpoints on nontrading days), as well as daily returns that we compute from quotations.[16] Monthly returns are computed by compounding the daily returns within each month. The percentage spread for each stock is calculated by averaging the daily spread relative to the quote midpoint. Stocks are excluded from a given month if closing (trade or quote midpoint) data is available for less than fifteen days. We also exclude the one percent of stock/days with the largest absolute difference between the closing trade price and the closing quote midpoint, as these likely reflect errors in the closing quotation data.

We implement asset pricing tests using the methodology of Eleswarapu (1997), Table III. Specifically, we form forty nine equal weighted portfolios. Stocks are sorted into portfolios based on (1) the stock's average spread in the previous year and (2) the stock's beta estimated using the preceding 36 months. The portfolio spread in each month of the cross-sectional regression is the average of the firms' spreads the prior month. The portfolio beta is estimated using the

[16] We obtain quotations from the CRSP data as follows. If the CRSP variable *prc* is positive then the ask is set equal to the CRSP variable *nmsask* and bid is set equal to the CRSP variable *nmsbid*. If *prc* is negative the ask is set equal to the CRSP variable *askhi* and the bid is set equal to the CRSP variable *bidlo*. Midpoints are the simple average of the ask and bid quotes. Midpoint returns are computed from midpoints, after adjustments for cash dividends, distributions, and splits as reported by CRSP.

entire 240-month test period. A cross-sectional regression of portfolio return on portfolio beta and average portfolio bid-ask spread is estimated for each month of the test period, and final estimates are the time series means of the monthly estimates.

Results are reported on Table VII. The first row reports results based on returns reported by CRSP, which rely on trade prices on those days with trading and quote midpoints on non-trading days. Consistent with the results reported by Eleswarapu (1997), we estimate a positive (0.422) and statistically significant (t-statistic = 2.01) cross-sectional relation between average returns and average bid-ask spreads for Nasdaq NMS stocks. However, when we repeat the Fama-MacBeth regressions using quote midpoint returns we no longer observe any significant relation between mean returns and mean spreads: the estimated coefficient is -0.22, with a t-statistic of -0.52. These results are consistent with the theory and simulation evidence presented here implying that bid-ask bounce biases upwards the estimated relation between average returns and bid-ask spreads. The results are also consistent with the reasoning that there is no relationship between mean returns and illiquidity. However, the sample employed contains a narrow set of stocks observed over a relatively short interval, implying that the absense of an observed relation after adjustments for microstructure biases could be sample-specific. Indeed, He and Kryzanowski (2006) report a positive and significant illiquidity premium for Canadian stocks, even when employing quote-midpoint returns.

We also assess the effect of implementing the other two potential corrections for bid-ask bounce discussed in Section IV. The third row of Table VII reports results obtained when the Fama-MacBeth regressions are implemented in continuously compounded returns, created for each stock day as the natural logarithm of one plus the return reported by CRSP. This correction also yields an estimate of the spread premium indistinguishable from zero (0.056, with a t-statistic of 0.16).

The fourth row of VII reports results obtained when each individual stock return is adjusted by deducting an estimate of $var\left[\bar{\delta}_{i,t}\right]$. $\bar{\delta}_{i,t}$ is measured for each stock-day as the percentage difference between the closing trade price and the closing quote midpoint, and the variance of the measured $\bar{\delta}_{i,t}$ is then computed for each stock-year. Adjusted returns are those reported by CRSP less the variance of $\bar{\delta}_{i,t}$ for that stock-year. Results reported on the fourth row of Table

VII also indicate the absence of a significant cross-sectional relation between returns adjusted for bid-ask bounce and spreads (coefficient = -0.099, t-statistic = -1.13).

In summary, we estimate relations between average stock returns and average bid-ask spreads in one of the few databases where both closing trade prices and closing quote midpoints are available. Consistent with the model and the simulation results developed here, we find evidence that bid-ask bounce in returns computed from trade prices biases upwards the estimated relation between average returns and bid-ask spreads.

VII. Conclusion and Further Research

Empirical studies that use the CRSP data to examine relations between average return, risk, and liquidity rely on observed return measures that are constructed from a combination of closing trade prices (which are affected by bid-ask bounce) and closing quote midpoints (which are not). Our analysis shows that bid-ask bounce biases both mean return and beta measures, and also increases noise in beta estimates, and that each of these effects potentially imparts biases in asset pricing tests. Further, the relation between the biases attributable to bid-ask bounce and observable bid-ask spreads need not be monotone, as it depends on those factors that determine non-trading.

To illustrate the potential magnitude of the microstructure biases, we conduct a series of simulations, where parameters are calibrated to the CRSP data. The key advantage of a simulation approach is that we know the true relation between returns, betas, and liquidity. The most robust finding of the simulations is that bid-ask spread can appear to be positively related to average returns even when they are not. We also assess via simulation whether a researcher can avoid the biases documented here by excluding securities with wide spreads from the analysis. Results indicate that the upward bias in the estimated spread premium can be reduced, but not eliminated, by excluding wide-spread securities, but at a cost in statistical power such that the researcher may well not be able to detect a true positive illiquidity premium if it exits.

We also examine data for Nasdaq NMS securities after 1983, for which CRSP reports both closing trade prices and closing quote midpoints. By comparing results obtained when using closing price returns to those obtained using closing quote midpoint returns, we verify empirically that bid-ask bounce biases upward the estimated relation between returns and bid-ask spreads.

We document that the increased noise in beta estimates attributable to microstructure effects is sufficient that the accompanying errors-in-variables problem can, depending on research design, lead to significant downward biases in the market price of beta risk. However, the bias is minimal when individual security returns are regressed on betas estimated on a portfolio basis, as in Fama and French (1992). Since Fama and French (1992) do not detect a significant premium on beta after controlling for firm size, we conclude that microstructure biases alone do not explain the empirical failure of the CAPM.

We consider a series of possible cures for the biases that arise due to bid-ask bounce. Two of the possible approaches, the use of quote midpoint returns or a direct adjustment of returns for the amount of bid-ask bounce, require data on both trade prices and quotations. Quotation databases that span enough years to be useful for asset pricing tests are not readily available. The third approach is to use continuously compounded rather than simple returns. While this can be implemented in any database, it is only appropriate when testing models with implications for continuously compounded returns, not when testing models like the discrete-time CAPM that have implications regarding holding period returns.

The results reported here need not imply that illiquidity has no effect on average asset returns. However, the results do imply that the estimated empirical relations between mean returns and illiquidity measures reported in the literature are likely to be larger than the true economic relations.

Our analysis of biases in asset pricing tests with regard to illiquidity is limited to only one asset characteristic, i.e. bid-ask spread. Since the results indicate the likelihood of obtaining an upward biased coefficient estimate on this characteristic, a natural question is whether biased results might also be obtained in tests of whether other measures of liquidity as firm characteristic or measures of liquidity risk are priced. Additionally, the question arises as to whether asset characteristics that might be correlated with bid-ask bounce such as firm size may also appear to be priced when they are not. Further, since bid-ask bounce also increases observed return volatility, our results may have relevance for the recent literature (see, for example Spiegel and

Wang (2005) and the papers referenced there) that assess whether idiosyncratic return volatility is priced.

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Appendix

Equation (7):

We have that:

$$E\left\{r_{i,t}^{o}\right\} = E\left\{\frac{1+\bar{\delta}_{i,t}}{1+\bar{\delta}_{i,t-1}}\right\}\left[1+E\left\{r_{i,t}\right\}\right]-1=$$
$$= E\left[1+\bar{\delta}_{i,t}\right]E\left[\frac{1}{1+\bar{\delta}_{i,t-1}}\right]\left[1+E\left\{r_{i,t}\right\}\right]-1.$$

because $\delta_{i,t}$'s and consequently $\bar{\delta}_{i,t}$'s are independent.

Given Jensen's inequality, i.e. if $f(x)$ is concave then $Ef(x) \leq f(Ex)$ and if $f(x)$ is convex then $Ef(x) \geq f(Ex)$. The function $f(x) = 1/x$ is convex, thus, $E(1/x) \geq 1/Ex$. Thus,

$$E\left[\frac{1}{1+\bar{\delta}_{i,t-1}}\right] \geq \frac{1}{E\left[1+\bar{\delta}_{i,t-1}\right]}.$$

It follows that

$$E\left\{\frac{1+\bar{\delta}_{i,t}}{1+\bar{\delta}_{i,t-1}}\right\} = E\left[1+\bar{\delta}_{i,t}\right]E\left[\frac{1}{1+\bar{\delta}_{i,t-1}}\right] \geq \frac{E\left[1+\bar{\delta}_{i,t}\right]}{E\left[1+\bar{\delta}_{i,t-1}\right]} = \frac{1+E\left[\bar{\delta}_{i,t}\right]}{1+E\left[\bar{\delta}_{i,t-1}\right]} = 1$$

Therefore given the last expression $E\left[r_{i,t}^{o}\right] > E\left[r_{i,t}\right]$. Further, using a Taylor series:

$$E\left[\frac{1}{1+\bar{\delta}_{i,t-1}}\right] = E\left[1-\bar{\delta}_{i,t-1}+\bar{\delta}_{i,t-1}^{2}-\ldots\right] \approx 1+var\left[\bar{\delta}_{i,t-1}\right], \tag{A.1}$$

where $var\left[\bar{\delta}_{i,t-1}\right]$ denotes the variance of $\bar{\delta}_{i,t}$. If the third- and higher-ordered odd moments of $\bar{\delta}_{i,t}$ are zero then the variance in Eq.(A.1) provides a lower bound for the bias induces by the bid-ask effect. Eq.(A.1) assumes that $\bar{\delta}_{i,t}$ is small. Combining the above equation with Eq.(6) and dropping the cross product term gives Eq.(7). This argument requires $-1 < \bar{\delta}_{i,t-1} < 1$.

Bid Ask Spreads vs. Bid Ask Bounce

Assume that actual spreads are uniformly distributed. In particular, $\delta_{i,t}$ is uniformly distributed on the interval $[-s_i, s_i]$. For the moment we assume that

$$c \leq s_i, \tag{A.2}$$

for all i, which implies that each security sometimes does not trade.

Given the uniform distribution assumption, the average unsigned spread for security i is equal to s_i. In particular, $E\left[spread_{i,t}\right] = 2E\left|\delta_{i,t}\right| = s_i$. If quotations are observable and those trades that do occur are priced at the quotes, then s_i can be estimated as the average (across both trading and non-trading intervals) quoted spread. If trades occur every period but at prices other than the quotes, then s_i corresponds to the average effective spread (twice the percentage absolute difference between trade price and value). If trade prices can differ from the quotes and securities endogenously do not trade on some dates then the average effective spread observed in the data will be less than s_i, due to the censoring of observations when spreads are widest.

The amount of bid-ask bounce given endogenous nontrading in Eq.(9) can be reexpressed as:

$$\text{(A.3)} \qquad var\left[\bar{\delta}_{i,t}\right] = \frac{1}{2s_i}\int_{-c}^{c} x_i^2 dx_i = \left.\frac{x_i^3}{6s_i}\right|_{-c}^{c} = \frac{c^3}{3s_i}.$$

By comparison, the bid-ask bounce in the case when security i always trades is:

$$\text{(A.4)} \qquad var\left[\delta_{i,t}\right] = \int_{-s_i}^{s_i} x_i^2 f\left(x_i\right) dx_i = \frac{1}{2s_i}\int_{-s_i}^{s_i} x_i^2 dx_i = \left.\frac{x_i^3}{6s_i}\right|_{-s_i}^{s_i} = \frac{s_i^2}{3}.$$

From Eq.(A.2), Eq.(A.3) and Eq.(A.4) it follows that:

$$\text{(A.5)} \qquad var\left[\bar{\delta}_{i,t}\right] \leq var\left[\delta_{i,t}\right],$$

formalizing that non-trading reduces bid-ask bounce. Now assume that for security j, $\delta_{j,t}$ has a uniform distribution on the interval $[-s_j, s_j]$. Analogous to Eq.(A.3),

$$var\left[\bar{\delta}_{j,t}\right] = \frac{c^3}{3s_j}.$$

Assuming further that $s_i \leq s_j$, i.e. that the average spread for security i is less than for security j, it follows that:

$$\text{(A.6)} \qquad var\left[\bar{\delta}_{j,t}\right] \leq var\left[\bar{\delta}_{i,t}\right].$$

Thus, we have what may appear to be a counterintuitive result. If the gains from trade are constant across securities, spreads are uniformly distributed, and spreads for all securities sometimes exceed the gains from trade, then the security with a wider average quoted spread will actually have less bid-ask bounce in observed returns. This occurs because securities with wider average spreads will trade less frequently, leading to more frequent observation of the quote midpoint.

We next consider the case where all securities are *potentially* subject to nontrading, but some securities always trade because their spread never gets wide enough to cause nontrading. Assume that for security k, $\delta_{k,t}$ has a uniform distribution on the interval $[-s_k, s_k]$, and that $s_k \leq c \leq s_i \leq s_j$. Thus, security k always trades while securities i and j are subject to endogenous nontrading. Analogous to Eq.(A.4):

$$\text{(A.7)} \qquad var\left[\bar{\delta}_{k,t}\right] = var\left[\delta_{k,t}\right] = \frac{s_k^2}{3}.$$

With endogenous nontrading the relationship between the bid-ask bounces for security i and j is given by Eq.(A.6). The inequality

$$var\left[\bar{\delta}_{k,t}\right] \geq var\left[\bar{\delta}_{i,t}\right],$$

can, using Eq.(A.3) and Eq.(A.7) be restated as:

$$c \leq \sqrt[3]{s_k^2 s_i}.$$

Thus, as long as the above inequality is true the security with a smaller average quoted bid-ask spread will have greater bid-ask bounce in observed returns, and vice versa.

The preceding analysis relied on the assumption that the gain from trade, c, is the same across all securities. This may be reasonable if c reflects liquidity needs; cash can be raised equally well by selling any security. If, however, c is motivated by information then c could well vary across assets. One simple but useful specification is to assume that for any security i the gain to trade is related to the average quoted spread:

$$c_i = g(s_i). \tag{A.8}$$

While it is simplistic to link c_i and s_i mechanically, it seems plausible that these variables would naturally be related. For example, spreads and gains from trade are likely to both be linked to the frequency with which investors have non-public information.

Assume further that $s_i \leq s_j$. Consider first the possibility that $g(.)$ is a decreasing function, so that gains from trade are greater for low-spread securities. Then, $g(s_i) \geq g(s_j)$, and using Eq.(A.3) and Eq.(A.8) it follows that inequality Eq.(A.6) is always true. In this case securities with high bid-ask spread will continue to have less bid-ask bounce.

Alternately, consider the possibility that $g(.)$ is an increasing function, so that gains from trade are greater for high-spread securities. Then, $g(s_i) \leq g(s_j)$. In this case, if

$$\frac{g(x)}{x}$$

is a nondecreasing (decreasing) function then securities with high bid-ask spread will have low (high) bid-ask bounce. Higher average spreads for securities with greater gains from trade might arise because information asymmetries increase both spreads and gains from trade. Since this scenario is plausible, this analysis implies that, while it is possible to specify the direction of the asset pricing biases caused by larger bid-ask bounce, it may not possible to make simple inferences about the magnitude of bid-ask bounce and the direction of microstructure biases in asset pricing tests on the basis of observable bid-ask spreads. The relations will depend on detailed parameters and observed non-trading frequencies.

Equation (10):

Let's derive an expression for cross-covariance. Given that

$$cov\left(r^o_{i,t}, r^o_{j,t}\right) = E\left[r^o_{i,t} r^o_{j,t}\right] - E\left[r^o_{i,t}\right] E\left[r^o_{j,t}\right]. \tag{A.9}$$

Define

$$x = \frac{1+\bar{\delta}_{i,t}}{1+\bar{\delta}_{i,t-1}}; y = \frac{1+\bar{\delta}_{j,t}}{1+\bar{\delta}_{j,t-1}}. \tag{A.10}$$

The cross-covariance could be obtained by first calculating the uncentered moment:

$$\begin{aligned} E\left[r^o_{i,t} r^o_{j,t}\right] &= E\left[\left\{x\left(1+r_{i,t}\right)-1\right\}\left\{y\left(1+r_{j,t}\right)-1\right\}\right] = \\ &= E[xy] + E[xyr_{j,t}] + E[xyr_{i,t}] + E[xyr_{i,t}r_{j,t}] - E[x(1+r_{i,t})] - E[y(1+r_{j,t})] + 1 \end{aligned}$$

The second term on the right hand side in Eq.(A.9) could be expressed as:

$$\begin{aligned} E\left[r^o_{i,t}\right] E\left[r^o_{j,t}\right] &= \left\{E[x]\left\{1+E[r_{i,t}]\right\}-1\right\}\left\{E[y]\left\{1+E[r_{j,t}]\right\}-1\right\} = \\ &= E[x]\left\{1+E[r_{i,t}]\right\}E[y]\left\{1+E[r_{j,t}]\right\} - E[x]\left\{1+E[r_{i,t}]\right\} - E[y]\left\{1+E[r_{j,t}]\right\} + 1. \end{aligned}$$

The common factor induces contemporaneous cross-sectional correlation between the virtual returns of securities i and j. Using the fact that

$$cov\left(r_{i,t}, r_{j,t}\right) = \beta_i \beta_j var\left[r_{M,t}\right]$$

and Eq.(A.10) and Eq.(A.1) then yields the following:

$$cov\left(r^o_{i,t}, r^o_{j,t}\right) = \left\{1 + var\left[\bar{\delta}_{i,t}\right]\right\}\left\{1 + var\left[\bar{\delta}_{j,t}\right]\right\}\beta_i \beta_j var\left[r_{M,t}\right] \tag{A.11}$$

Equation (12):

$$\begin{aligned}\beta^o_i = \frac{cov\left(r^o_{i,t}, r^o_{M,t}\right)}{var\left(r^o_{M,t}\right)} &= \frac{cov\left(r^o_{i,t}, \frac{1}{N}\sum_{j=1}^N r^o_{j,t}\right)}{var\left(r^o_{M,t}\right)} = \\ = \quad \frac{1}{N}\frac{cov\left(r^o_{i,t}, \sum_{j=1}^N r^o_{j,t}\right)}{var\left(r^o_{M,t}\right)} &= \frac{1}{N}\frac{\sum_{j=1}^N cov\left(r^o_{i,t}, r^o_{j,t}\right)}{var\left(r^o_{M,t}\right)}.\end{aligned}$$

Thus,

$$\begin{aligned}\beta^o_i = \frac{1}{N}\frac{\sum_{j=1}^N E\left[\frac{1+\delta_{i,t}}{1+\delta_{i,t-1}}\right] E\left[\frac{1+\delta_{j,t}}{1+\delta_{j,t-1}}\right]\beta_i \beta_j var\left[r_{M,t}\right]}{var\left(r^o_{M,t}\right)} = \\ = \quad \frac{1}{N}\frac{\sum_{j=1}^N \left\{1 + var\left[\delta_{i,t-1}\right]\right\}\left\{1 + var\left[\delta_{j,t-1}\right]\right\}\beta_i \beta_j var\left[r_{M,t}\right]}{var\left[r^o_{M,t}\right]}\end{aligned}$$

Table I. Bid-Ask Spread Parameters Used in the Simulations.

For each individual stock i the parameter s_i is assigned as a random draw from a uniform distribution on the interval from the indicated lower to upper bound. The (unsigned) half-spread on day t for stock i is a random draw from a uniform distribution on the interval 0 to s_i. The average half-spread for stock i is therefore $s_i/2$, while the average full spread is s_i. Parameters in Panel A are selected so that mean spreads for each set of stocks match means for deciles of NYSE/AMEX stocks as reported by Chalmers and Kadlec (1998). Parameters in Panel B are selected so that mean spreads for the first five sets of stocks match means for NYSE/AMEX securities, while mean spreads for the second five sets of stocks match means for quintiles of Nasdaq securities, as reported by Fortin, Grube and Joy (1989).

Panel A: When replicating NYSE-AMEX spreads

Stocks	Spread Lower Bound	Spread Upper Bound	Spread Mean	Portfolio Replicated
1-150	0.0040	0.0060	0.0050	NYSE/AMEX 1st Decile
151-300	0.0060	0.0080	0.0070	NYSE/AMEX 2nd Decile
301-450	0.0080	0.0092	0.0086	NYSE/AMEX 3rd Decile
451-600	0.0092	0.0120	0.0106	NYSE/AMEX 4th Decile
601-750	0.0120	0.0144	0.0132	NYSE/AMEX 5th Decile
751-900	0.0144	0.0176	0.0160	NYSE/AMEX 6th Decile
901-1050	0.0176	0.0216	0.0196	NYSE/AMEX 7th Decile
1051-1200	0.0216	0.0356	0.0286	NYSE/AMEX 8th Decile
1201-1350	0.0356	0.0548	0.0452	NYSE/AMEX 9th Decile
1351-1500	0.0548	0.1116	0.0832	NYSE/AMEX 10th Decile

Panel B: When replicating NYSE-AMEX-Nasdaq spreads

Stocks	Spread Lower Bound	Spread Upper Bound	Spread Mean	Portfolio Replicated
1-150	0.00400	0.00800	0.00600	NYSE/AMEX 1st Quintile
151-300	0.00800	0.01120	0.00960	NYSE/AMEX 2nd Quintile
301-450	0.01120	0.01800	0.01460	NYSE/AMEX 3rd Quintile
451-600	0.01800	0.03020	0.02410	NYSE/AMEX 4th Quintile
601-750	0.03032	0.09832	0.06432	NYSE/AMEX 5th Quintile
751-900	0.01200	0.03200	0.02200	Nasdaq 1st Quintile
901-1050	0.03200	0.05600	0.04400	Nasdaq 2nd Quintile
1051-1200	0.05600	0.09000	0.07300	Nasdaq 3rd Quintile
1201-1350	0.09000	0.15200	0.12100	Nasdaq 4th Quintile
1351-1500	0.15200	0.32200	0.23700	Nasdaq 5th Quintile

Table II. Fama-MacBeth analysis using 1500 individual securities, when securities always trade and spreads are calibrated to NYSE/AMEX levels.

Monthly returns are regressed on betas, with and without inclusion of spreads, and premia are estimated as the time series mean of the monthly estimates. The Table reports the average premium in percent, average t-statistic, and the standard error of the mean premium across 100 simulations. The columns report results based on: Col 1: True returns and true betas, Col 2: True returns and betas estimated by OLS from true returns, Col 3: Observed returns and betas computed from text expression 12, Col 4: Observed returns and true betas, Col 5: Observed returns and betas estimated by OLS from observed returns, Col 6: Observed returns and betas estimated by the Scholes-Williams method from observed returns.

	(1)	(2)	(3)	(4)	(5)	(6)
Return measure	True	True	Obs.	Obs.	Obs.	Obs.
Beta measure	True	Est.True	Computed	True	Est.Obs	Est.SW
Panel A: When beta (premium = 0.8%) and spread (premium = 6.0%) are priced						
A.I. Returns regressed on beta only						
Mean Beta Premium	0.780	0.729	0.782	0.781	0.711	0.605
Mean t-statistic	(3.03)	(3.04)	(3.03)	(3.03)	(3.04)	(3.03)
Standard Error of Mean	0.025	0.023	0.025	0.025	0.023	0.021
A.II. Returns regressed on beta and spread						
Mean Beta Premium	0.781	0.729	0.783	0.782	0.710	0.605
Mean t-statistic	(3.04)	(3.04)	(3.04)	(3.04)	(3.04)	(3.03)
Standard Error of Mean	0.025	0.023	0.025	0.025	0.023	0.020
Mean Spread Premium	5.962	5.956	9.093	9.117	9.066	9.089
Mean t-statistic	(25.99)	(25.76)	(22.52)	(22.57)	(22.24)	(22.23)
Standard Error of Mean	0.027	0.028	0.027	0.027	0.033	0.034
Panel B: When neither beta or spread is priced						
B.I. Returns regressed on beta only						
Mean Beta Premium	-0.018	-0.017	-0.018	-0.019	-0.015	-0.014
Mean t-statistic	(-0.07)	(-0.07)	(-0.07)	(-0.07)	(-0.07)	(-0.07)
Standard Error of Mean	0.025	0.023	0.025	0.025	0.023	0.019
B.II. Returns regressed on beta and spread						
Mean Beta Premium	-0.018	-0.017	-0.018	-0.018	-0.015	-0.014
Mean t-statistic	(-0.07)	(-0.07)	(-0.07)	(-0.07)	(-0.07)	(-0.07)
Standard Error of Mean	0.025	0.023	0.025	0.025	0.022	0.019
Mean Spread Premium	-0.037	-0.035	3.081	3.080	3.084	3.086
Mean t-statistic	(-0.17)	(-0.16)	(7.69)	(7.69)	(7.62)	(7.61)
Standard Error of Mean	0.027	0.027	0.027	0.027	0.028	0.028

Table III. Fama-MacBeth analysis using 1500 individual securities, when securities always trade and spreads are calibrated to NYSE/AMEX/Nasdaq levels.

Monthly returns are regressed on betas, with and without inclusion of spreads, and premia are estimated as the time series mean of the monthly estimates. The Table reports the average premium in percent, average t-statistic, and the standard error of the mean premium across 100 simulations. The columns report results based on: Col 1: True returns and true betas, Col 2: True returns and betas estimated by OLS from true returns, Col 3: Observed returns and betas computed from text expression 12, Col 4: Observed returns and true betas, Col 5: Observed returns and betas estimated by OLS from observed returns, Col 6: Observed returns and betas estimated by the Scholes-Williams method from observed returns.

	(1)	(2)	(3)	(4)	(5)	(6)
Return measure	True	True	Obs.	Obs.	Obs.	Obs.
Beta measure	True	Est.True	Computed	True	Est.Obs	Est.SW
Panel A: When beta (premium = 0.8%) and spread (premium = 6.0%) are priced						
A.I. Returns regressed on beta only						
Mean Beta Premium	0.779	0.727	0.818	0.776	0.652	0.528
Mean t-statistic	(3.03)	(3.03)	(3.15)	(2.99)	(3.26)	(3.14)
Standard Error of Mean	0.025	0.024	0.027	0.027	0.022	0.021
A.II. Returns regressed on beta and spread						
Mean Beta Premium	0.781	0.729	0.788	0.784	0.604	0.505
Mean t-statistic	(3.04)	(3.04)	(3.03)	(3.02)	(3.03)	(2.99)
Standard Error of Mean	0.025	0.023	0.025	0.025	0.020	0.018
Mean Spread Premium	5.986	5.982	14.952	15.018	14.924	14.967
Mean t-statistic	(77.66)	(77.00)	(43.39)	(43.47)	(43.07)	(43.16)
Standard Error of Mean	0.009	0.009	0.018	0.018	0.024	0.023
Panel B: When neither beta or spread is priced						
B.I. Returns regressed on beta only						
Mean Beta Premium	-0.018	-0.017	0.003	-0.024	0.020	0.002
Mean t-statistic	(-0.07)	(-0.07)	(0.01)	(-0.09)	(0.10)	(0.02)
Standard Error of Mean	0.025	0.023	0.026	0.026	0.020	0.017
B.II. Returns regressed on beta and spread						
Mean Beta Premium	-0.018	-0.016	-0.015	-0.019	-0.008	-0.012
Mean t-statistic	(-0.07)	(-0.07)	(-0.06)	(-0.07)	(-0.05)	(-0.07)
Mean of St. Errors of Coeff.	0.025	0.023	0.025	0.025	0.019	0.016
Mean Spread Premium	-0.014	-0.014	8.825	8.824	8.826	8.826
Mean t-statistic	(-0.19)	(-0.18)	(26.16)	(26.09)	(25.98)	(25.99)
Standard Error of Mean	0.009	0.009	0.018	0.018	0.019	0.019

Table IV. Fama-MacBeth analysis using Portfolio Return/Portfolio Beta approach, when securities always trade.

Monthly returns are regressed on betas, with and without inclusion of spreads, and premia are estimated as the time series mean of the monthly estimates. The Table reports the average premium in percent, average t-statistic, and the standard error of the mean premium across 100 simulations. Columns (1) and (2) report results for a simulated NYSE/AMEX sample, while columns (3) and (4) report results for a simulated NYSE/AMEX/Nasdaq sample. Results in columns (1) and (3) are obtained using true returns and betas estimated from true returns, while results in columns (2) and (4) are obtained using observed returns and betas estimated from observed returns.

	NYSE/AMEX		**NYSE/AMEX/Nasdaq**	
	(1)	(2)	(3)	(4)
Return measure	True	Obs.	True	Obs.
Beta measure	Est.True	Est.Obs	Est.True	Est.Obs
Panel A: When beta (premium = 0.8%) and spread (premium = 6.0%) are priced				
A.I. Returns regressed on beta only				
Mean Beta Premium	0.729	0.711	0.728	0.655
Mean t-statistic	(3.04)	(3.04)	(3.03)	(3.26)
Standard Error of Mean	0.023	0.023	0.024	0.023
A.II. Returns regressed on beta and spread				
Mean Beta Premium	0.730	0.711	0.730	0.611
Mean t-statistic	(3.04)	(3.04)	(3.04)	(3.04)
Standard Error of Mean	0.023	0.023	0.023	0.019
Mean Spread Premium	5.952	8.838	5.981	14.165
Mean t-statistic	(24.19)	(22.86)	(72.37)	(46.17)
Standard Error of Mean	0.028	0.033	0.009	0.023
Panel B: When neither beta or spread is priced				
B.I. Returns regressed on beta only				
Mean Beta Premium	-0.016	-0.015	-0.016	0.019
Mean t-statistic	(-0.07)	(-0.07)	(-0.07)	(0.09)
Standard Error of Mean	0.023	0.023	0.023	0.020
B.II. Returns regressed on beta and spread				
Mean Beta Premium	-0.016	-0.015	-0.016	-0.006
Mean t-statistic	(-0.07)	(-0.07)	(-0.07)	(-0.03)
Standard Error of Mean	0.023	0.023	0.023	0.019
Mean Spread Premium	-0.037	2.858	-0.014	8.096
Mean t-statistic	(-0.15)	(7.44)	(-0.17)	(26.89)
Standard Error of Mean	0.028	0.029	0.009	0.019

Table V. Fama-MacBeth analysis using Individual Return/Portfolio Beta approach, when securities always trade.

Monthly returns are regressed on betas estimated from the full sample, with and without inclusion of spreads, and premia are estimated as the time series mean of the monthly estimates. The Table reports the average premium in percent, average t-statistic, and the standard error of the mean premium across 100 simulations. Columns (1) and (2) report results for a simulated NYSE/AMEX sample, while columns (3) and (4) report results for a simulated NYSE/AMEX/Nasdaq sample. Results in columns (1) and (3) are obtained using true returns and betas estimated from true returns, while results in columns (2) and (4) are obtained using observed returns and betas estimated from observed returns.

	NYSE/AMEX		**NYSE/AMEX/Nasdaq**	
	(1)	(2)	(3)	(4)
Return measure	True	Obs.	True	Obs.
Beta measure	Est.True	Est.Obs	Est.True	Est.Obs
Panel A: When beta (premium = 0.8%) and spread (premium = 6.0%) are priced				
A.I. Returns regressed on beta only				
Mean Beta Premium	0.777	0.776	0.776	0.836
Mean t-statistic	(3.04)	(3.04)	(3.03)	(3.29)
Standard Error of Mean	0.025	0.025	0.026	0.027
A.II. Returns regressed on beta and spread				
Mean Beta Premium	0.777	0.776	0.778	0.771
Mean t-statistic	(3.04)	(3.04)	(3.04)	(3.03)
Standard Error of Mean	0.025	0.025	0.025	0.025
Mean Spread Premium	5.96	9.077	5.980	14.916
Mean t-statistic	(25.96)	(22.48)	(77.34)	(43.30)
Standard Error of Mean	0.027	0.027	0.009	0.019
Panel B: When neither beta or spread is priced				
B.I. Returns regressed on beta only				
Mean Beta Premium	-0.017	-0.017	-0.017	0.026
Mean t-statistic	(-0.07)	(-0.07)	(-0.07)	(0.10)
Standard Error of Mean	0.025	0.025	0.025	0.026
B.II. Returns regressed on beta and spread				
Mean Beta Premium	-0.017	-0.017	-0.017	-0.013
Mean t-statistic	(-0.07)	(-0.07)	(-0.07)	(-0.05)
Standard Error of Mean	0.025	0.025	0.025	0.024
Mean Spread Premium	-0.037	3.082	-0.013	8.288
Mean t-statistic	(-0.16)	(7.70)	(-0.18)	(26.18)
Standard Error of Mean	0.027	0.027	0.009	0.019

Table VI. The Threshold of Spread-Induced Bias: Fama-MacBeth analysis using the Individual Return/Portfolio Beta approach, when some wide-spread securities are excluded from the analysis.

The analysis replicates that of Table 6, column 4, where monthly observed returns are regressed on betas estimated from observed returns and spreads. The first row is based on the full sample of 1500 simulated securities, while each subsequent row reports results after excluding another 10 percent of the sample, beginning with the widest average spreads reported on Panel B of Table 2. Premia are estimated as the time series mean of the monthly estimates, and the Table reports the average premium in percent, average t-statistic, and the standard error of the mean premium across 100 simulations.

NYSE/AMEX/Nasdaq When beta (premium = 0.8%) and spread (premium = 6.0%) are priced						
	Mean Beta Premium	*Mean t-statistic*	*Standard Error of Mean*	*Mean Spread Premium*	*Mean t-statistic*	*Standard Error of Mean*
All	0.771	(3.03)	0.025	14.916	(43.30)	0.019
10% stocks excluded	0.771	(3.02)	0.024	10.214	(31.47)	0.020
20% stocks excluded	0.775	(3.03)	0.025	8.879	(23.84)	0.026
30% stocks excluded	0.775	(3.03)	0.025	8.746	(18.11)	0.030
40% stocks excluded	0.777	(3.03)	0.025	7.773	(11.88)	0.055
50% stocks excluded	0.776	(3.03)	0.025	7.200	(6.70)	0.099
60% stocks excluded	0.777	(3.03)	0.025	7.413	(5.43)	0.116
70% stocks excluded	0.776	(3.02)	0.025	7.298	(2.73)	0.257
80% stocks excluded	0.776	(3.00)	0.025	6.968	(1.20)	0.544
90% stocks excluded	0.773	(2.96)	0.025	6.252	(0.41)	1.561

Table VII. Empirical evidence from Nasdaq 1983-2005

Reported are results of implementing cross-sectional Fama-MacBeth regressions of monthly returns to forty nine portfolios on estimated portfolio betas and bid-ask spreads. Data is for Nasdaq NMS stocks over the 1983 to 2005 interval. The methodology closely follows Eleswarapu (1997), Table III, Specification (B). Portfolios are formed on the basis of average bid-ask spreads during the prior year, and estimated betas over the prior three years. Closing price returns are those reported by CRSP based on the last trade if trade occurred or the closing quote midpoint if not. Quote midpoint returns are constructed from closing quotations, adjusted for dividends and stock distributions. Log returns are the natural logarithm of one plus the reported CRSP returns. Delta is defined for each stock day as the percentage difference between the closing trade price and the closing quote midpoint, and the variance of delta is computed for each stock/year. Adjusted returns are CRSP-reported returns less the variance of delta for that stock/year. Coefficients reported are the time-series means of the monthly cross-sectional regression estimates, with corresponding t-statistic.

	Mean Beta Premium (Mean t-statistic)	*Mean Spread Premium (Mean t-statistic)*
Simple Returns, from Closing Prices	0.096 *(1.01)*	0.422 *(2.01)*
Simple Returns, from Closing Quote Midpoints	-0.204 *(-1.00)*	-0.223 *(-0.52)*
Log Returns, from Closing Prices	-0.063 *(-0.44)*	0.056 *(0.16)*
Simple Returns, from Closing Prices, Less Var(delta)	0.040 *(1.18)*	-0.099 *(-1.13)*